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UNIT - 2



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Fourier Series

Euler's formula.

$f(x) = f(x+2l) = \dots$ is a periodic function in $[c, c+2l]$

Fourier series expansion is given by

$$f(x) = \frac{a_0}{2} + \sum [a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l}]$$

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where $a_0 = \frac{1}{l} \int_c^{c+2l} f(x) dx$.

$$a_n = \frac{1}{l} \int_a^{c+2l} f(x) \cos \frac{n\pi x}{l} dx$$

$$b_n = \frac{1}{l} \int_c^{c+2l} f(x) \sin \frac{n\pi x}{l} dx$$

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1. Expand following functions into Fourier series in the given intervals.

(i) $f(x) = e^{-x}$ in $0 \leq x \leq 2\pi$.

$$f(x) = a_0 + \sum (a_n \cos nx + b_n \sin nx)$$

where $a_0 = \frac{1}{\pi} \int_0^{2\pi} f(x) dx$.

$$a_n = \frac{1}{\pi} \int_0^{2\pi} f(x) \cos nx dx.$$

Notes

$$b_n = \frac{1}{\pi} \int_0^{2\pi} f(x) \sin nx dx.$$

$$a_0 = \frac{1}{\pi} \int_0^{2\pi} e^{-x} dx = \frac{1}{\pi} \left[\frac{e^{-x}}{-1} \right]_0^{2\pi} = \frac{1 - e^{-2\pi}}{2\pi}.$$

$$a_n = \frac{1}{\pi} \int_0^{2\pi} e^{-x} \cos nx dx = \frac{1}{\pi} \left[\frac{e^{-x}}{1+n^2} (-\cos nx + n \sin nx) \right]_0^{2\pi}$$

$$= \frac{1}{\pi(1+n^2)} \left[e^{-2\pi} (-\cos 2n\pi + n \sin 2n\pi) - (-\cos 0 + 0) \right]$$

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$$a_n = \frac{1}{\pi} \left[e^{-2\pi} (-1) + 1 \right] \frac{1}{1+n^2}$$

$$= \frac{1 - e^{-2\pi}}{\pi(1+n^2)}$$

$$b_n = \frac{1}{\pi} \int_0^{2\pi} e^{-x} \sin nx \, dx :$$

$$= \frac{1}{\pi} \left[\frac{e^{-x}}{1+n^2} (-\sin nx - n \cos nx) \right]_0^{2\pi}$$

$$= \frac{1}{\pi} \left[\frac{e^{-2\pi} (-\sin 2n\pi - n \cos 2n\pi) - (0 - n)}{1+n^2} \right]$$

$$= \frac{1}{\pi} \left[\frac{e^{-2\pi}(-n) + n}{1+n^2} \right] = \frac{n(1-e^{-2\pi})}{\pi(1+n^2)}$$

$$= n(e^{-2\pi}) f(x) = \frac{1-e^{-2\pi}}{\pi} \left[\frac{1}{4} + \sum \frac{1}{1+n^2} (\cos nx + \frac{1}{n} \sin nx) \right]$$

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$$(ii) \quad f(x) = x - x^2 \quad \text{in } [-\pi, \pi]$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

$$\text{where } a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx.$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx.$$

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$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx. \quad 2$$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} (x - x^2) dx = \frac{1}{\pi} \left[\frac{x^2}{2} - \frac{x^3}{3} \right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi} \left[\frac{\pi^2}{2} - \frac{\pi^3}{3} - \frac{(-\pi)^2}{2} + \frac{(-\pi)^3}{3} \right] = \frac{-2\pi^3}{3}$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} (x - x^2) \cos nx dx$$

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$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} (x - x^2) \cos nx dx .$$

$$= \frac{1}{\pi} \left[\frac{(x - x^2) \sin nx}{n} \Big|_{-\pi}^{\pi} - \int_{-\pi}^{\pi} (-2x) \frac{\sin nx}{n} dx \right]$$

$$= \frac{1}{\pi} \left[\frac{(-2x) \cos nx}{n^2} \Big|_{-\pi}^{\pi} - \int_{-\pi}^{\pi} (-2) \frac{\sin nx}{n^2} dx \right]$$

$$4 = \frac{1}{\pi} \left[\frac{(1-2\pi) \cos n\pi}{n^2} - (1+2\pi) \frac{\cos n\pi}{n^2} \right] \text{ సోమవారము/MONDAY}$$

$$+ 2/n^2 \left[\frac{\sin nx}{n} \Big|_{-\pi}^{\pi} \right]$$

$$= \frac{1}{\pi n^2} [\cos n\pi - 2\pi \cos n\pi - \cos n\pi - 2\pi \cos n\pi]$$

$$= -\frac{4}{n^2} (-1)^n .$$

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$$\begin{aligned}
 b_n &= \frac{1}{\pi} \int_{-\pi}^{\pi} (x - x^2) \sin nx dx \\
 &= \frac{1}{\pi} \left[-(x - x^2) \frac{\cos nx}{n} \right]_{-\pi}^{\pi} - \int_{-\pi}^{\pi} (1-2x) \left(-\frac{\cos nx}{n} \right) dx \\
 &= \frac{1}{\pi} \left[-\frac{(\pi - \pi^2)}{n} \cos nx + \left(-\frac{\pi - \pi^2}{n} \right) \cos nx \right. \\
 &\quad \left. + \frac{1}{n} \int_{-\pi}^{\pi} (1-2x) \frac{\sin nx}{n^2} dx - \int_{-\pi}^{\pi} (-2) \frac{\sin nx}{n^2} dx \right]
 \end{aligned}$$

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$$\begin{aligned}
 &= \frac{1}{\pi} \left[-\frac{2\pi}{n} \cos nx + \frac{2}{n^2} (-\cos nx) \right]_{-\pi}^{\pi} \\
 &= \frac{1}{\pi} \left[-\frac{2\pi}{n} \cos nx - \frac{2}{n^2} (\cos n\pi - \cos(-n\pi)) \right] \\
 &= -\frac{2}{n} (-1)^n
 \end{aligned}$$

$$f(x) = -\frac{\pi^2}{3} + -\sum \frac{4}{n} (-1)^n (\cos nx + \frac{2}{n} (-1)^n \sin nx)$$

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$$\text{iii) } f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

$$\text{where } a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx.$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx.$$

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$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx.$$

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$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx = \frac{1}{\pi} \left[\int_{-\pi}^0 (-\pi) dx + \int_0^{\pi} x dx \right]$$

$$= -\pi x \Big|_{-\pi}^0 + \frac{x^2}{2} \Big|_0^{\pi} = -\pi^2 + \frac{\pi^2}{2} = \frac{-\pi^2}{2}$$

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$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx \cdot dx = \frac{1}{\pi} \left[\int_{-\pi}^0 (-\pi) \cos nx + \int_0^{\pi} x \cos nx \right]$$

$$a_n = \frac{1}{\pi} \left[-\pi \frac{\sin nx}{n} \Big|_{-\pi}^0 + \frac{x \sin nx}{n} \Big|_0^{\pi} - \int_0^{\pi} \frac{\sin nx}{n} dx \right]$$

$$= \frac{1}{\pi} \left[\frac{\pi \sin n\pi}{n} - 0 + \frac{\cos nx}{n^2} \Big|_0^{\pi} \right]$$

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$$= \frac{1}{\pi} \left[\frac{\cos n\pi - 1}{n^2} \right]$$

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$$b_n = \frac{1}{\pi} \left[\int_{-\pi}^{\pi} f(x) \sin nx dx \right]$$

$$= \frac{1}{\pi} \left[\int_{-\pi}^0 (-\pi) \sin nx dx + \int_0^{\pi} x \sin nx dx \right]$$

$$= \frac{1}{\pi} \left[\pi \frac{\cos nx}{n} \Big|_{-\pi}^0 + \frac{x \cos nx}{n} \Big|_0^{\pi} - \int_0^{\pi} \frac{\cos nx}{n^2} dx \right]$$

$$= \frac{1}{\pi} \left[\frac{\pi}{n} (1 - \cos n\pi) + \frac{\pi \cos n\pi}{n} \right] = \frac{1}{n} (1 - 2 \cos n\pi)$$

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$$f(x) = -\frac{\pi^2}{4} + \frac{1}{\pi} \sum \frac{(-1)^n - 1}{n^2} \cos nx \\ + 2 \sum \frac{1 - 2(-1)^n}{n} \sin nx.$$

(iv) $f(x) = \begin{cases} -i & \text{for } -\pi < x < -\pi/2 \\ 0 & -\pi/2 < x < \pi/2 \\ i & \pi/2 < x < \pi \end{cases}$

12 $f(x) = \frac{a_0}{2} + \sum (a_n \cos nx + b_n \sin nx)$ మంగళవారము / TUESDAY

Where $a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx$, $a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx$$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx = \frac{1}{\pi} \left[\int_{-\pi}^{-\pi/2} (-1) dx + 0 \int_{-\pi/2}^{\pi/2} dx + \int_{\pi/2}^{\pi} 1 dx \right]$$

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$$a_0 = \frac{1}{\pi} \left[-x \Big|_{-\pi}^{-\pi/2} + x \Big|_{\pi/2}^{\pi} \right]$$

$$= \frac{1}{\pi} \left[\frac{\pi}{2} - \pi + \pi - \frac{\pi}{2} \right] = 0.$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx$$

$$= \frac{1}{\pi} \left[\int_{-\pi}^{-\pi/2} -\cos nx dx + 0 + \int_{\pi/2}^{\pi} \cancel{\sin nx dx} \right]$$

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$$= \frac{1}{\pi} \left[-\frac{\sin nx}{n} \Big|_{-\pi}^{-\pi/2} + \frac{\sin nx}{n} \Big|_{\pi/2}^{\pi} \right]$$

$$= \frac{1}{\pi n} \left[+\sin n\pi/2 - \sin n\pi + \sin n\pi - \sin n\pi/2 \right] = 0$$

$$a_1 = \frac{1}{\pi} \left[\int_{-\pi}^{-\pi/2} -\cos x dx + 0 + \int_{\pi/2}^{\pi} \cos x dx \right]$$

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$$a_1 = \frac{1}{\pi} \left[\sin x \Big|_{-\pi}^{-\pi/2} + \sin x \Big|_{\pi/2}^{\pi} \right] = \frac{1}{\pi} [-1 - 1] = -\frac{2}{\pi}$$

$$= \frac{1}{\pi} \left[-\cos x \Big|_{-\pi}^{-\pi/2} - \cos x \Big|_{\pi/2}^{\pi} \right]$$

$$= \frac{1}{\pi} \left[-\cos \pi/2 + \cos \pi - \cos \pi + \cos \pi/2 \right] = 0$$

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$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx$$

$$= \frac{1}{\pi} \left[\int_{-\pi}^{-\pi/2} -\sin nx dx + \int_{\pi/2}^{\pi} \sin nx dx \right]$$

$$= \frac{1}{\pi} \left[\frac{\cos nx}{n} \Big|_{-\pi}^{-\pi/2} - \frac{\cos nx}{n} \Big|_{\pi/2}^{\pi} \right]$$

$$= \frac{1}{\pi} \left[\frac{\cos n\pi/2}{n} - \frac{\cos n\pi}{n} - \frac{\cos n\pi}{n} + \frac{\cos n\pi/2}{n} \right]$$

$$= \frac{2}{n\pi} \left[\cos n\pi/2 - (-1)^n \right]$$

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$$f(x) = -\frac{2}{\pi} \cos x + \frac{2}{\pi} \sum \left\{ \cos n\pi/2 - (-1)^n \right\} \sin nx$$

$$(V) f(x) = \begin{cases} \pi x & 0 \leq x \leq 1 \\ \pi(2-x) & 1 \leq x \leq 2 \end{cases}$$

$$f(x) = \frac{a_0}{2} + \sum (a_n \cos nx + b_n \sin nx)$$

$$a_0 = \frac{1}{l} \int_{-l}^{+l} f(x) dx \quad l = 1$$

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$$a_n = \frac{1}{l} \int_{-l}^{+l} f(x) \cos \frac{n\pi}{l} x dx$$

$$b_n = \frac{1}{l} \int_{-l}^{+l} f(x) \sin \frac{n\pi}{l} x dx$$

$$a_0 = \int_0^2 f(x) dx = \int_0^1 \pi x dx + \int_1^2 \pi(2-x) dx$$

$$= \pi x^2/2 \Big|_0^1 + \pi (2x - x^2/2) \Big|_1^2$$

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$$a_0 = \frac{\pi}{2} + \pi [4 - 2 - 2 + \frac{1}{2}] = \pi'$$

$$a_n = \int_0^1 \pi x \cos n\pi x dx + \int_0^2 \pi(2-x) \cos n\pi x dx$$

$$= \pi x \left. \frac{\sin n\pi x}{n\pi} \right|_0^1 - \int_0^1 \pi \frac{\sin n\pi x}{n\pi} dx$$

$$+ \pi(2-x) \left. \frac{\sin n\pi x}{n\pi} \right|_1^2 - \int_0^2 \pi(-1) \frac{\sin n\pi x}{n\pi} dx.$$

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$$= \left. \frac{\cos n\pi x}{n^2\pi} \right|_0^1 + \left. \frac{\cos n\pi x}{n^2\pi} \right|_1^2$$

$$= \frac{\cos n\pi - 1}{n^2\pi} - \frac{\cos 2n\pi - \cos n\pi}{n^2\pi}$$

$$= \frac{2}{n^2\pi} \left\{ (-1)^n - 1 \right\},$$

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$$b_n = \int_0^1 \pi x \sin n\pi x dx + \int_1^2 \pi(2-x) \sin n\pi x dx.$$

$$= \left[\pi x \left(-\frac{\cos n\pi x}{n\pi} \right) \right]_0^1 + \int_0^1 \pi \cdot \left(-\frac{\cos n\pi x}{n\pi} \right) dx$$

$$+ \left[\pi(2-x) \left(-\frac{\cos n\pi x}{n\pi} \right) \right]_1^2 - \int_1^2 \pi(-1) \left(-\frac{\cos n\pi x}{n\pi} \right) dx$$

$$= -\frac{(-1)^n}{n} + \left. \frac{\sin n\pi x}{n^2\pi} \right|_0^1 + \left. \pi \frac{(-1)^x}{n\pi} - \frac{\sin^2 n\pi x}{n^2\pi} \right|_1^2$$

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$$= 0$$

$$= \frac{(-1)^n}{n}$$

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$$f(x) = \frac{\pi}{2} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2} \cos n\pi x.$$

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$$\text{iv) } f(t) = \begin{cases} 0 & -2 < t < -1 \\ 1+t & -1 < t < 0 \\ 1-t & 0 < t < 1 \\ 0 & 1 < t < 2 \end{cases}$$

$L=2$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi}{2} t + b_n \sin \frac{n\pi}{2} t \right)$$

$$a_0 = \frac{1}{2} \int_{-2}^{2} f(t) dt, \quad a_n = \frac{1}{2} \int_{-2}^{2} f(t) \cos \frac{n\pi}{2} t dt$$

$$24 \quad b_n = \frac{1}{2} \int_{-2}^{2} f(t) \sin \frac{n\pi}{2} t dt$$

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$$a_0 = \frac{1}{2} \left[\int_{-1}^0 (1+t) dt + \int_0^1 (1-t) dt \right]$$

$$= \frac{1}{2} \left[\left[t + \frac{t^2}{2} \right]_{-1}^0 + \left[t - \frac{t^2}{2} \right]_0^1 \right]$$

$$= \frac{1}{2} [0 - (-1 + \frac{1}{2}) + (1 - \frac{1}{2}) - 0]$$

$$= + \frac{1}{2}$$

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$$\begin{aligned}
 a_n &= \frac{1}{2} \left[\int_{-1}^0 (1+t) \cos \frac{n\pi}{2} t dt + \int_0^1 (1-t) \cos \frac{n\pi}{2} t dt \right] \\
 &= \frac{1}{2} \left[(1+t) \frac{\sin \frac{n\pi}{2} t}{n\pi/2} \Big|_{-1}^0 + \int_{-1}^0 1 \cdot \frac{\sin n\pi/2 t}{n\pi/2} dt \right. \\
 &\quad \left. + (1-t) \frac{\sin \frac{n\pi}{2} t}{n\pi/2} \Big|_0^1 - \int_0^1 (-1) \frac{\sin n\pi/2 t}{n\pi/2} dt \right] \\
 &= \frac{1}{2} \left[\frac{2}{n\pi} \frac{\cos n\pi/2 t}{n\pi/2} \Big|_{-1}^0 - \frac{2}{n\pi} \frac{\cos n\pi/2 t}{n\pi/2} \Big|_0^1 \right]
 \end{aligned}$$

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$$\begin{aligned}
 &= \frac{1}{2} \left[\frac{4}{n^2\pi^2} \left(1 - \cos \frac{n\pi}{2} \right) - \frac{4}{n^2\pi^2} \left(\cos \frac{n\pi}{2} - 1 \right) \right] \\
 &= \frac{2}{n^2\pi^2} \left[1 - \cos \frac{n\pi}{2} - \cos \frac{n\pi}{2} + 1 \right] \\
 &= \frac{4}{n^2\pi^2} \left(1 - \cos \frac{n\pi}{2} \right)
 \end{aligned}$$

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$$b_n = \frac{1}{2} \left[\int_{-1}^0 (1+t) \sin \frac{n\pi}{2} t dt + \int_0^1 (1-t) \sin \frac{n\pi}{2} t dt \right]$$

$$= \frac{1}{2} \left[(1+t) \left(-\frac{\cos n\pi/2 t}{n\pi/2} \right) \Big|_{-1}^0 - \int_{-1}^0 -\frac{\cos n\pi/2 t}{n\pi/2} dt \right. \\ \left. + (1-t) \left(-\frac{\cos n\pi/2 t}{n\pi/2} \right) \Big|_0^1 - \int_0^1 (-1) \left(-\frac{\cos n\pi/2 t}{n\pi/2} \right) dt \right]$$

$$= \frac{1}{2} \left[\frac{2}{n\pi} (-1+0) + \frac{2}{n\pi} \frac{\sin n\pi/2 t}{n\pi/2} \Big|_{-1}^0 \right] + \text{గురువారమ్ / THURSDAY}$$

$$= \frac{2}{n\pi} (0+1) - \frac{2}{n\pi} \frac{\sin n\pi/2 t}{n\pi/2} \Big|_0^1]$$

$$= \frac{1}{2} \left[-\frac{2}{n\pi} + \frac{4}{n^2\pi^2} (0 + \cancel{\sin n\pi/2}) + \cancel{\frac{2}{n\pi}} - \frac{4}{n^2\pi^2} (\cancel{\sin n\pi/2} - 0) \right] \\ = \frac{4}{n^2\pi^2} \sin = 0.$$

$$f(x) = \frac{1}{4} + \frac{4}{\pi^2} \sum \frac{1 - \cos n\pi/2}{n^2} \cos \frac{n\pi}{2} t$$

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Even and odd function

If $f(x)$ is even in $[-l, l]$ then.

$$f(x) = \frac{a_0}{2} + \sum a_n \cos \frac{n\pi}{l} x, \quad b_n = 0.$$

$$\text{where } a_0 = \frac{2}{l} \int_0^l f(x) dx, \quad a_n = \frac{2}{l} \int_0^l f(x) \cos \frac{n\pi}{l} x dx.$$

If $f(x)$ is odd in $[-l, l]$

$$a_0 = a_n = 0$$

$$f(x) = \sum b_n \sin \frac{n\pi}{l} x.$$

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$$b_n = \frac{2}{l} \int_0^l f(x) \sin \frac{n\pi}{l} x dx.$$

1. Expand $f(x) = x^2$ into Fourier series in $(-l, l)$

b) $f(x) = x^2$ is even; $b_n = 0$.

$$f(x) = \frac{a_0}{2} + \sum a_n \cos \frac{n\pi}{l} x$$

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$$a_0 = \frac{2}{l} \int_0^l f(x) dx = \frac{2}{l} \int_0^l x^2 dx = \frac{2}{l} \cdot \frac{x^3}{3} \Big|_0^l \\ = \frac{2l^2}{3}$$

$$a_n = \frac{2}{l} \int_0^l x^2 \cos \frac{n\pi}{l} x dx$$

$$= \frac{2}{l} \left[\frac{x^2 \sin \frac{n\pi}{l} x}{n\pi/l} \Big|_0^l - \int_0^l 2x \cdot \frac{\sin \frac{n\pi}{l} x}{n\pi/l} dx \right]$$

Notes

$$= \frac{2}{l} \left[+ \frac{2l}{n\pi} \int_0^l x \frac{\cos \frac{n\pi}{l} x}{n\pi/l} dx - \int_0^l l \cdot \frac{\cos \frac{n\pi}{l} x}{n\pi/l} dx \right]$$

$$= \frac{2}{l} \left[\frac{2l}{n\pi} \left\{ \frac{l \cos n\pi}{n\pi/l} \right\} - \frac{\sin \frac{n\pi}{l} x}{(n\pi/l)^2} \Big|_0^l \right]$$

$$= \frac{4l^2}{n^2\pi^2} (-1)^n$$

$$f(x) = \frac{l^2}{3} + \frac{4l^2}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos \frac{n\pi}{l} x$$

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$$2) f(x) = |x| \text{ in } (-\pi, \pi).$$

$|x|$ is even. $b_n = 0$, $f(x) = \frac{a_0}{2} + \sum a_n \cos nx$

$$a_0 = \frac{2}{\pi} \int_0^\pi f(x) dx, \quad a_n = \frac{2}{\pi} \int_0^\pi f(x) \cos nx dx.$$

$$a_0 = \frac{2}{\pi} \int_0^\pi x dx = \frac{2}{\pi} \left. \frac{x^2}{2} \right|_0^\pi = \pi$$

$$a_n = \frac{2}{\pi} \int_0^\pi x \cos nx dx = \frac{2}{\pi} \left[x \frac{\sin nx}{n} \Big|_0^\pi - \int_0^\pi \frac{\sin nx}{n} dx \right]$$

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$$= \frac{2}{\pi} \left[\frac{\cos nx}{n^2} \Big|_0^\pi \right] = \frac{2}{n^2 \pi} \{ (-1)^n - 1 \}.$$

$$f(x) = \frac{\pi}{2} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2} \cos nx$$

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$$3) f(x) = x \cos x \text{ in } (-\pi, \pi).$$

$f(x)$ is odd $\therefore a_0 = a_n = 0$.

$$f(x) = \sum b_n \sin nx \quad b_n = \frac{2}{\pi} \int_0^{\pi} f(x) \sin nx dx.$$

$$b_1 = \frac{2}{\pi} \int_0^{\pi} x \cos x \sin x dx.$$

$$= \frac{2}{\pi} \int_0^{\pi} x \sin 2x dx = \frac{1}{\pi} \left[-x \frac{\cos 2x}{2} \right]_0^{\pi} - \left[\frac{\cos 2x}{2} \right]_0^{\pi}$$

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$$= \frac{1}{\pi} \left[-\pi \cdot \frac{1}{2} - \frac{\sin 2x}{2} \Big|_0^{\pi} \right] = -\frac{1}{2}.$$

$$b_n = \frac{2}{\pi} \int_0^{\pi} x \cos x \sin nx dx.$$

$$= \frac{1}{\pi} \int_0^{\pi} x \{ \sin(n+1)x + \sin(n-1)x \} dx.$$

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$$\begin{aligned}
 &= \frac{1}{\pi} \left[-x \left\{ \frac{\cos(n+1)x}{n+1} + \frac{\cos(n-1)x}{n-1} \right\} \Big|_0^\pi \right. \\
 &\quad \left. - \int_0^\pi \left(\frac{\cos(n+1)x}{n+1} + \frac{\cos(n-1)x}{n-1} \right) dx \right], \\
 &= \frac{1}{\pi} \left[-\pi \left\{ \frac{\cos(n+1)\pi}{n+1} + \frac{\cos(n-1)\pi}{n-1} \right\} - 0 \right]
 \end{aligned}$$

$$= -\frac{2n(-1)^n}{n^2-1}$$

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$$f(x) = \frac{1}{2} \sin x + \sum_{n=2}^{\infty} \frac{(-1)^n n}{n^2-1} \sin nx.$$