CHAPTER 3

Diodes

INTRODUCTION

In the previous chapter we dealt almost entirely with linear circuits; any nonlinearity, such as that introduced by amplifier output saturation, was considered a problem to be solved by the circuit designer. However, there are many other signal-processing functions that can be implemented only by nonlinear circuits. Examples include the generation of dc voltages from the ac power supply and the generation of signals of various waveforms (e.g., sinusoids, square waves, pulses, etc.). Also, digital logic and memory circuits constitute a special class of nonlinear circuits.

The simplest and most fundamental nonlinear circuit element is the diode. Just like a resistor, the diode has two terminals; but unlike the resistor, which has a linear (straight-line) relationship between the current flowing through it and the voltage appearing across it, the diode has a nonlinear i–v characteristic.

This chapter is concerned with the study of diodes. In order to understand the essence of the diode function, we begin with a fictitious element, the ideal diode. We then introduce the silicon junction diode, explain its terminal characteristics, and provide techniques for the analysis of diode circuits. The latter task involves the important subject of device modeling.

Our study of modeling the diode characteristics will lay the foundation for our study of modeling transistor operation in the next two chapters.

Of the many applications of diodes, their use in the design of rectifiers (which convert ac to dc) is the most common. Therefore we shall study rectifier circuits in some detail and briefly look at a number of other diode applications. Further nonlinear circuits that utilize diodes and other devices will be found throughout the book, but particularly in Chapter 13.

To understand the origin of the diode terminal characteristics, we consider its physical operation. Our study of the physical operation of the *pn* junction and of the basic concepts of semiconductor physics is intended to provide a foundation for understanding not only the characteristics of junction diodes but also those of the field-effect transistor, studied in the next chapter, and the bipolar junction transistor, studied in Chapter 5.

Although most of this chapter is concerned with the study of silicon *pn*-junction diodes, we briefly consider some specialized diode types, including the photodiode and the light-emitting diode. The chapter concludes with a description of the diode model utilized in the SPICE circuit-simulation program. We also present a design example that illustrates the use of SPICE simulation.

3.1 THE IDEAL DIODE

3.1.1 Current–Voltage Characteristic

The ideal diode may be considered the most fundamental nonlinear circuit element. It is a two-terminal device having the circuit symbol of Fig. 3.1(a) and the i–v characteristic shown in Fig. 3.1(b). The terminal characteristic of the ideal diode can be interpreted as follows: If a

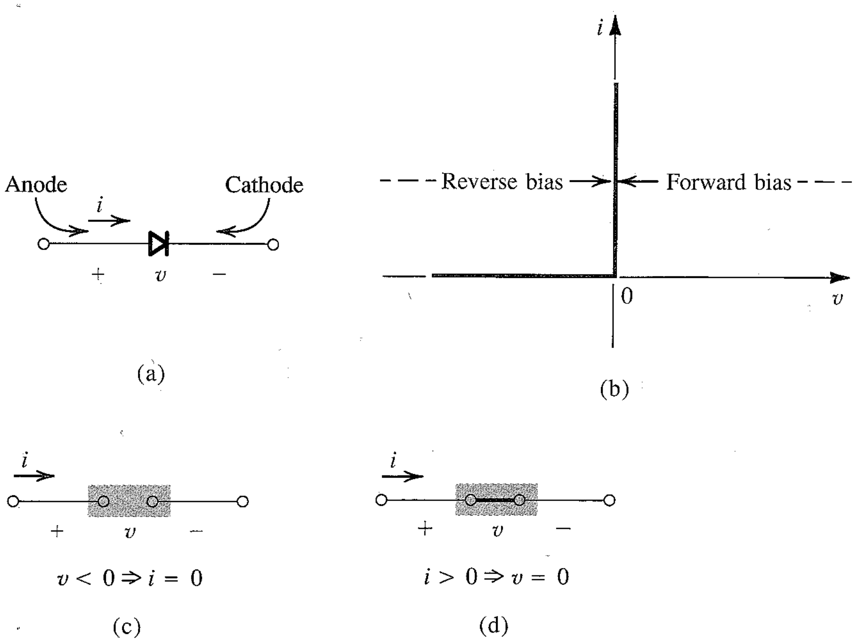

FIGURE 3.1 The ideal diode: **(a)** diode circuit symbol; **(b)** i–v characteristic; **(c)** equivalent circuit in the reverse direction; **(d)** equivalent circuit in the forward direction.

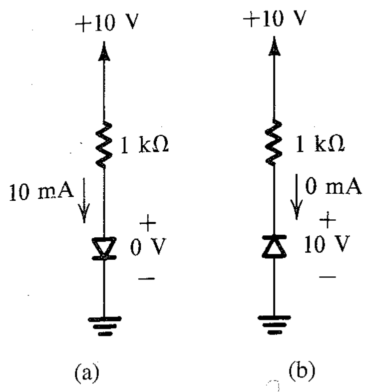

FIGURE 3.2 The two modes of operation of ideal diodes and the use of an external circuit to limit the forward current **(a)** and the reverse voltage **(b)**.

negative voltage (relative to the reference direction indicated in Fig. 3.1a) is applied to the diode, no current flows and the diode behaves as an open circuit (Fig. 3.1c). Diodes operated in this mode are said to be **reverse biased,** or operated in the reverse direction. An ideal diode has zero current when operated in the reverse direction and is said to be **cut off,** or simply **off.**

On the other hand, if a positive current (relative to the reference direction indicated in Fig. 3.1a) is applied to the ideal diode, zero voltage drop appears across the diode. In other words, the ideal diode behaves as a short circuit in the *forward* direction (Fig. 3.1d); it passes any current with zero voltage drop. A **forward-biased** diode is said to be **turned on,** or simply **on.**

From the above description it should be noted that the external circuit must be designed to limit the forward current through a conducting diode, and the reverse voltage across a cutoff diode, to predetermined values. Figure 3.2 shows two diode circuits that illustrate this point. In the circuit of Fig. 3.2(a) the diode is obviously conducting. Thus its voltage drop will be zero, and the current through it will be determined by the +10-V supply and the 1-kΩ resistor as 10 mA. The diode in the circuit of Fig. 3.2(b) is obviously cut off, and thus its current will be zero, which in turn means that the entire 10-V supply will appear as reverse bias across the diode.

The positive terminal of the diode is called the **anode** and the negative terminal the **cathode,** a carryover from the days of vacuum-tube diodes. The i–v characteristic of the ideal diode (conducting in one direction and not in the other) should explain the choice of its arrow-like circuit symbol.

As should be evident from the preceding description, the i–v characteristic of the ideal diode is highly nonlinear; although it consists of two straight-line segments, they are at 90° to one another. A nonlinear curve that consists of straight-line segments is said to be **piecewise linear.** If a device having a piecewise-linear characteristic is used in a particular application in such a way that the signal across its terminals swings along only one of the linear segments, then the device can be considered a linear circuit element as far as that particular circuit application is concerned. On the other hand, if signals swing past one or more of the break points in the characteristic, linear analysis is no longer possible.

3.1.2 A Simple Application: The Rectifier

A fundamental application of the diode, one that makes use of its severely nonlinear i–v curve, is the rectifier circuit shown in Fig. 3.3(a). The circuit consists of the series connection

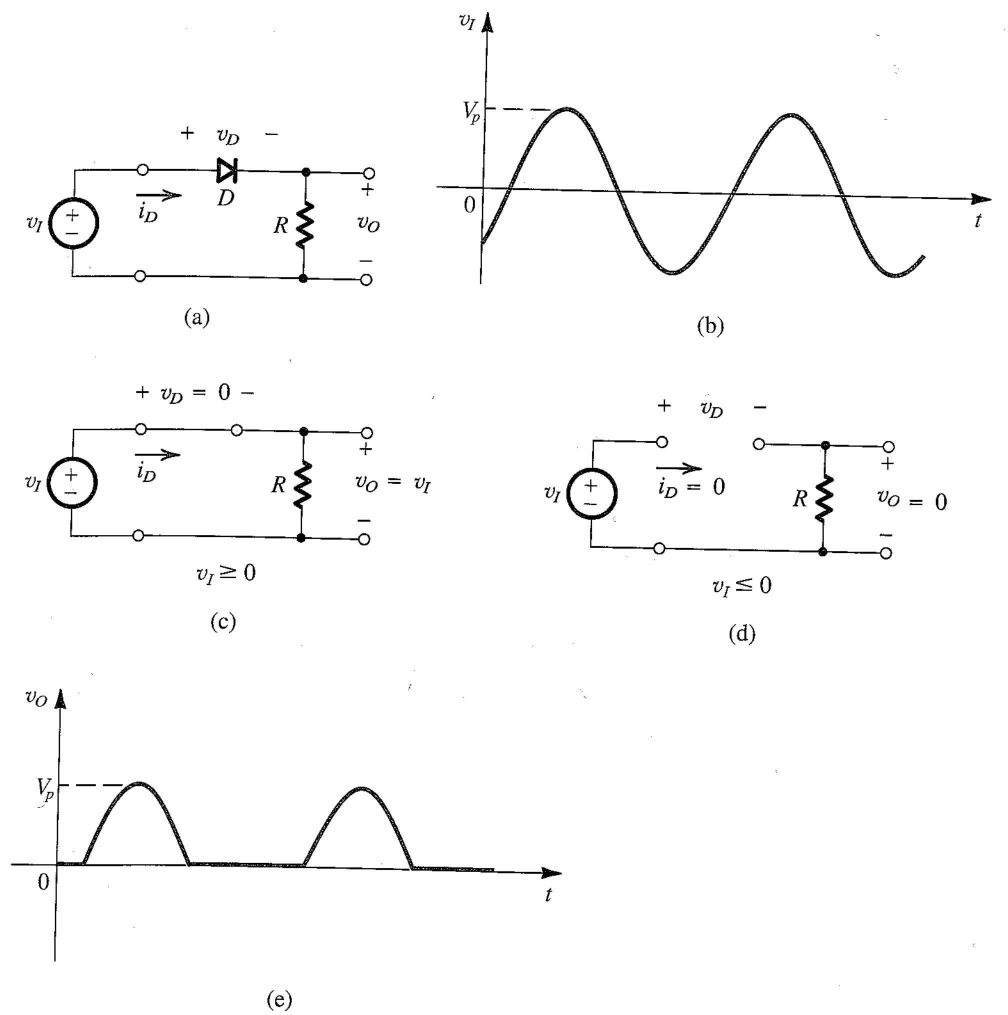

FIGURE 3.3 **(a)** Rectifier circuit. **(b)** Input waveform. **(c)** Equivalent circuit when $v_I \geq 0$. **(d)** Equivalent circuit when $v_I \leq 0$. **(e)** Output waveform.

of a diode D and a resistor R. Let the input voltage v_I be the sinusoid shown in Fig. 3.3(b), and assume the diode to be ideal. During the positive half-cycles of the input sinusoid, the positive v_I will cause current to flow through the diode in its forward direction. It follows that the diode voltage v_D will be very small—ideally zero. Thus the circuit will have the equivalent shown in Fig. 3.3(c), and the output voltage v_O will be equal to the input voltage v_I. On the other hand, during the negative half-cycles of v_I, the diode will not conduct. Thus the circuit will have the equivalent shown in Fig. 3.3(d), and v_O will be zero. Thus the output voltage will have the waveform shown in Fig. 3.3(e). Note that while v_I alternates in polarity and has a zero average value, v_O is unidirectional and has a finite average value or a *dc component*. Thus the circuit of Fig. 3.3(a) **rectifies** the signal and hence is called a **rectifier.** It can be used to generate dc from ac. We will study rectifier circuits in Section 3.5.

EXERCISES

3.1 For the circuit in Fig. 3.3(a), sketch the transfer characteristic v_O versus v_I.
Ans. See Fig. E3.1.

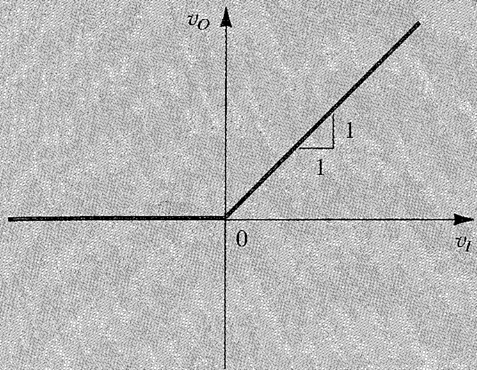

FIGURE E3.1

3.2 For the circuit in Fig. 3.3(a), sketch the waveform of v_D.
Ans. See Fig. E3.2.

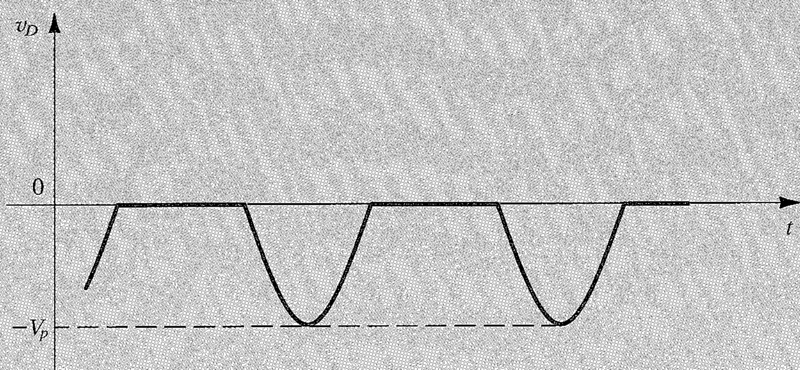

FIGURE E3.2

3.3 In the circuit of Fig. 3.3(a), let v_I have a peak value of 10 V and $R = 1\ \text{k}\Omega$. Find the peak value of i_D and the dc component of v_O.
Ans. 10 mA; 3.18 V

EXAMPLE 3.1

Figure 3.4(a) shows a circuit for charging a 12-V battery. If v_S is a sinusoid with 24-V peak amplitude, find the fraction of each cycle during which the diode conducts. Also, find the peak value of the diode current and the maximum reverse-bias voltage that appears across the diode.

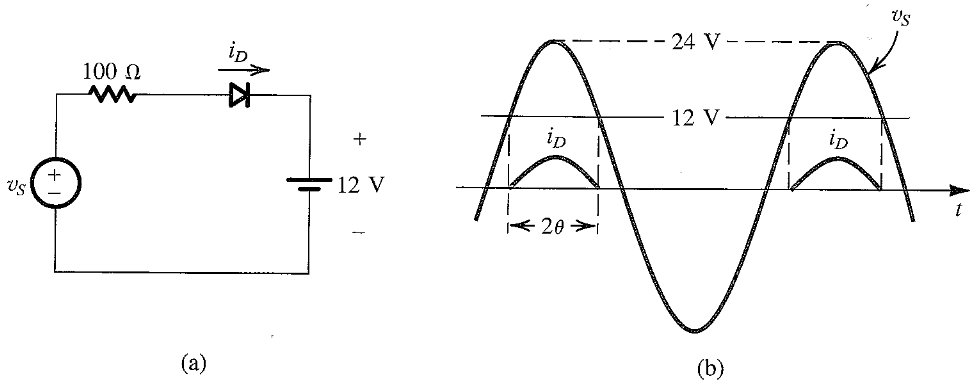

FIGURE 3.4 Circuit and waveforms for Example 3.1.

Solution

The diode conducts when v_S exceeds 12 V, as shown in Fig. 3.4(b). The conduction angle is 2θ, where θ is given by

$$24 \cos\theta = 12$$

Thus $\theta = 60°$ and the conduction angle is 120°, or one-third of a cycle.

The peak value of the diode current is given by

$$I_d = \frac{24 - 12}{100} = 0.12 \text{ A}$$

The maximum reverse voltage across the diode occurs when v_S is at its negative peak and is equal to $24 + 12 = 36$ V.

3.1.3 Another Application: Diode Logic Gates

Diodes together with resistors can be used to implement digital logic functions. Figure 3.5 shows two diode logic gates. To see how these circuits function, consider a positive-logic system in which voltage values close to 0 V correspond to logic 0 (or low) and voltage values

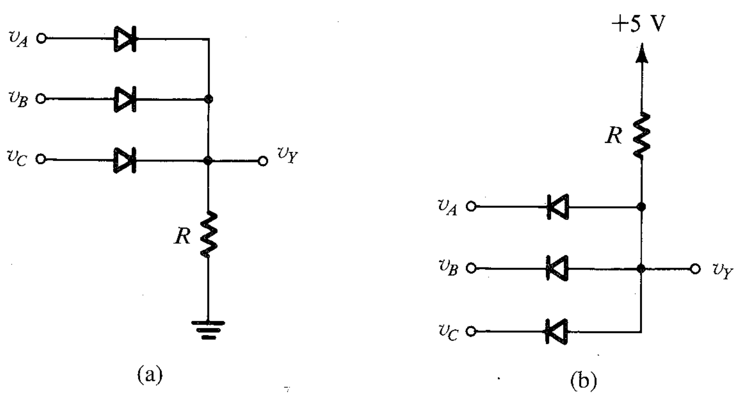

FIGURE 3.5 Diode logic gates: **(a)** OR gate; **(b)** AND gate (in a positive-logic system).

close to +5 V correspond to logic 1 (or high). The circuit in Fig. 3.5(a) has three inputs, v_A, v_B, and v_C. It is easy to see that diodes connected to +5-V inputs will conduct, thus clamping the output v_Y to a value equal to +5 V. This positive voltage at the output will keep the diodes whose inputs are low (around 0 V) cut off. Thus the output will be high if one or more of the inputs are high. The circuit therefore implements the **logic OR function,** which in Boolean notation is expressed as

$$Y = A + B + C$$

Similarly, the reader is encouraged to show that using the same logic system mentioned above, the circuit of Fig. 3.5(b) implements the **logic AND function,**

$$Y = A \cdot B \cdot C$$

EXAMPLE 3.2

Assuming the diodes to be ideal, find the values of I and V in the circuits of Fig. 3.6.

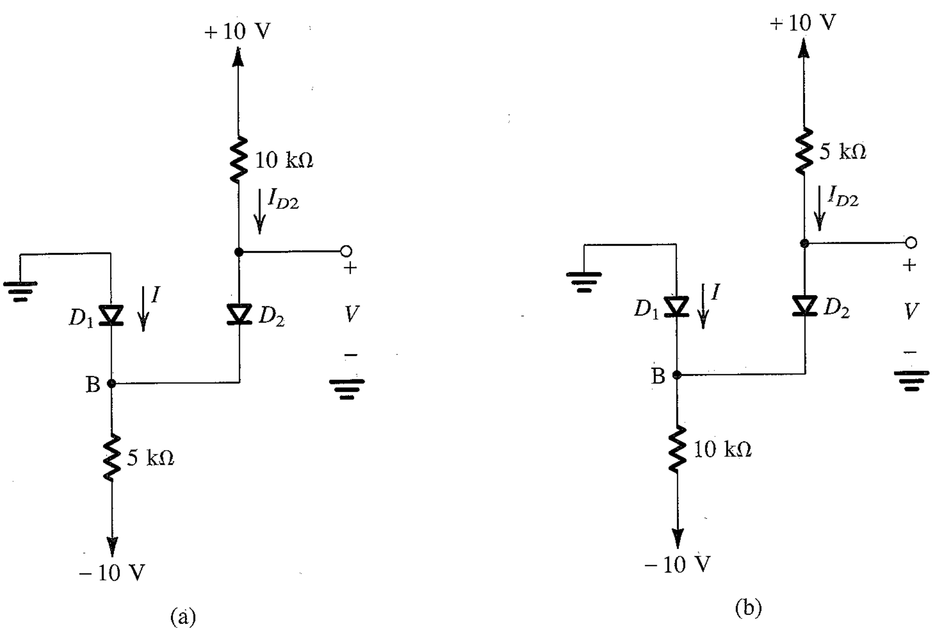

FIGURE 3.6 Circuits for Example 3.2.

Solution

In these circuits it might not be obvious at first sight whether none, one, or both diodes are conducting. In such a case, *we make a plausible assumption, proceed with the analysis, and then check whether we end up with a consistent solution.* For the circuit in Fig. 3.6(a), we shall assume that both diodes are conducting. It follows that $V_B = 0$ and $V = 0$. The current through D_2 can now be determined from

$$I_{D2} = \frac{10 - 0}{10} = 1 \text{ mA}$$

Writing a node equation at B,

$$I + 1 = \frac{0-(-10)}{5}$$

results in $I = 1$ mA. Thus D_1 is conducting as originally assumed, and the final result is $I = 1$ mA and $V = 0$ V.

For the circuit in Fig. 3.6(b), if we assume that both diodes are conducting, then $V_B = 0$ and $V = 0$. The current in D_2 is obtained from

$$I_{D2} = \frac{10-0}{5} = 2 \text{ mA}$$

The node equation at B is

$$I + 2 = \frac{0-(-10)}{10}$$

which yields $I = -1$ mA. Since this is not possible, our original assumption is not correct. We start again, assuming that D_1 is off and D_2 is on. The current I_{D2} is given by

$$I_{D2} = \frac{10-(-10)}{15} = 1.33 \text{ mA}$$

and the voltage at node B is

$$V_B = -10 + 10 \times 1.33 = +3.3 \text{ V}$$

Thus D_1 is reverse biased as assumed, and the final result is $I = 0$ and $V = 3.3$ V.

EXERCISES

3.4 Find the values of I and V in the circuits shown in Fig. E3.4.

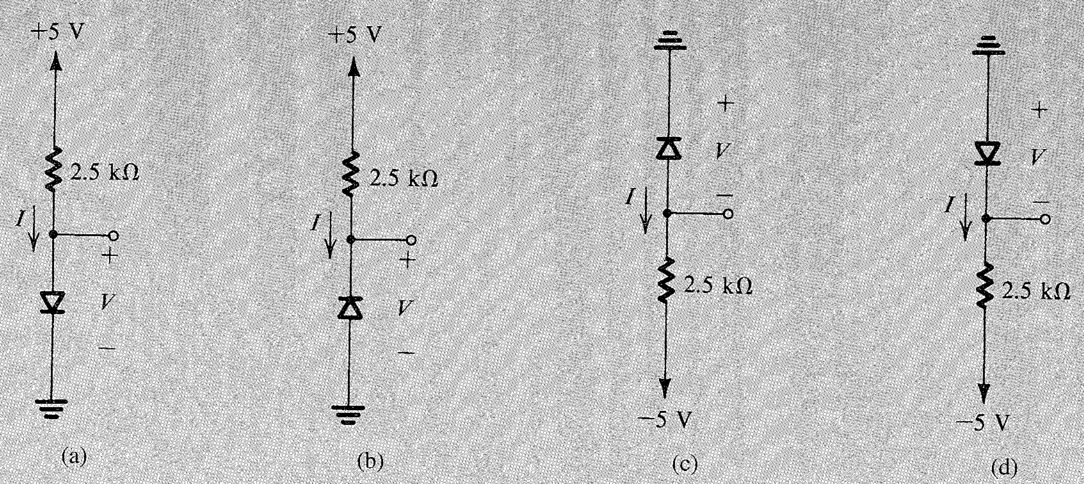

FIGURE E3.4

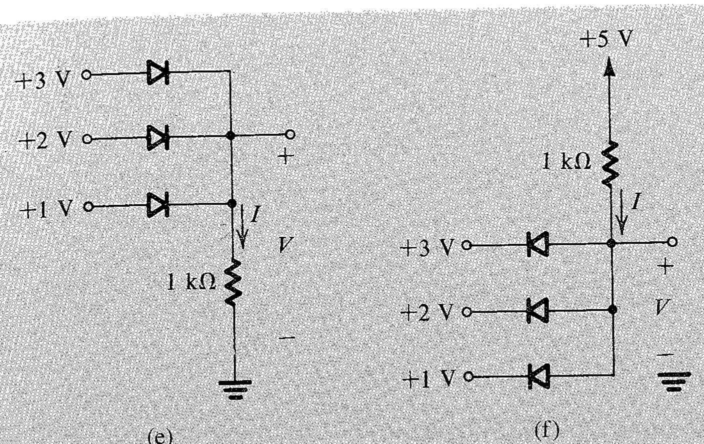

FIGURE E3.4 *(Continued)*

Ans. (a) 2 mA, 0 V; (b) 0 mA, 5 V; (c) 0 mA, 5 V; (d) 2 mA, 0 V; (e) 3 mA, +3 V; (f) 4 mA, +1 V.

3.5 Figure E3.5 shows a circuit for an ac voltmeter. It utilizes a moving-coil meter that gives a full-scale reading when the *average* current flowing through it is 1 mA. The moving-coil meter has a 50-Ω resistance.

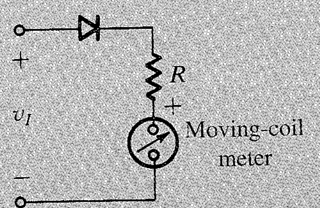

FIGURE E3.5

Find the value of R that results in the meter indicating a full-scale reading when the input sine-wave voltage v_I is 20 V peak-to-peak. (*Hint:* The average value of half-sine waves is V_p/π.)

Ans. 3.133 kΩ

3.2 TERMINAL CHARACTERISTICS OF JUNCTION DIODES

In this section we study the characteristics of real diodes—specifically, semiconductor junction diodes made of silicon. The physical processes that give rise to the diode terminal characteristics, and to the name "junction diode," will be studied in Section 3.7.

Figure 3.7 shows the i–v characteristic of a silicon junction diode. The same characteristic is shown in Fig. 3.8 with some scales expanded and others compressed to reveal details. Note that the scale changes have resulted in the apparent discontinuity at the origin.

As indicated, the characteristic curve consists of three distinct regions:

1. The forward-bias region, determined by $v > 0$
2. The reverse-bias region, determined by $v < 0$
3. The breakdown region, determined by $v < -V_{ZK}$

These three regions of operation are described in the following sections.

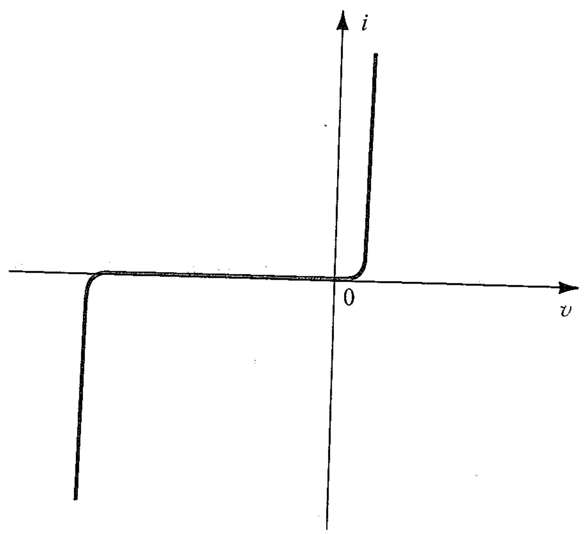

FIGURE 3.7 The i–v characteristic of a silicon junction diode.

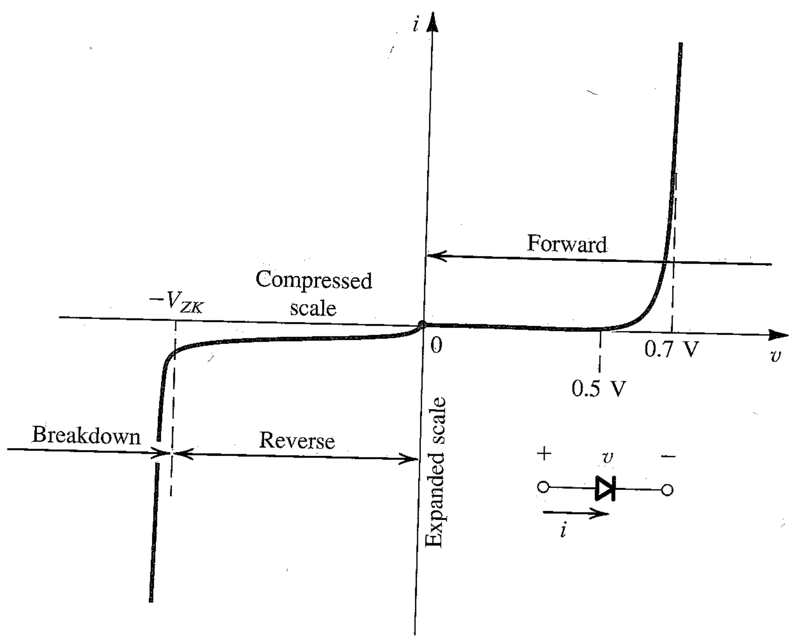

FIGURE 3.8 The diode i–v relationship with some scales expanded and others compressed in order to reveal details.

3.2.1 The Forward-Bias Region

The forward-bias—or simply forward—region of operation is entered when the terminal voltage v is positive. In the forward region the i–v relationship is closely approximated by

$$i = I_S(e^{v/nV_T} - 1) \tag{3.1}$$

In this equation I_S is a constant for a given diode at a given temperature. A formula for I_S in terms of the diode's physical parameters and temperature will be given in Section 3.7. The current I_S is usually called the **saturation current** (for reasons that will become apparent shortly). Another name for I_S, and one that we will occasionally use, is the **scale current.** This name arises from the fact that I_S is directly proportional to the cross-sectional area of the diode. Thus doubling of the junction area results in a diode with double the value of I_S and, as the diode equation indicates, double the value of current i for a given forward voltage v. For "small-signal" diodes, which are small-size diodes intended for low-power applications, I_S is on the order of 10^{-15} A. The value of I_S is, however, a very strong function of temperature. As a rule of thumb, I_S doubles in value for every 5°C rise in temperature.

The voltage V_T in Eq. (3.1) is a constant called the **thermal voltage** and is given by

$$V_T = \frac{kT}{q} \tag{3.2}$$

where

k = Boltzmann's constant = 1.38×10^{-23} joules/kelvin

T = the absolute temperature in kelvins = 273 + temperature in °C

q = the magnitude of electronic charge = 1.60×10^{-19} coulomb

At room temperature (20°C) the value of V_T is 25.2 mV. In rapid approximate circuit analysis we shall use $V_T \simeq 25$ mV at room temperature.[1]

In the diode equation the constant n has a value between 1 and 2, depending on the material and the physical structure of the diode. Diodes made using the standard integrated-circuit fabrication process exhibit $n = 1$ when operated under normal conditions.[2] Diodes available as discrete two-terminal components generally exhibit $n = 2$. In general, we shall assume $n = 1$ unless otherwise specified.

For appreciable current i in the forward direction, specifically for $i \gg I_S$, Eq. (3.1) can be approximated by the exponential relationship

$$i \simeq I_S e^{v/nV_T} \tag{3.3}$$

This relationship can be expressed alternatively in the logarithmic form

$$v = nV_T \ln \frac{i}{I_S} \tag{3.4}$$

where ln denotes the natural (base e) logarithm.

The exponential relationship of the current i to the voltage v holds over many decades of current (a span of as many as seven decades—i.e., a factor of 10^7—can be found). This is quite a remarkable property of junction diodes, one that is also found in bipolar junction transistors and that has been exploited in many interesting applications.

Let us consider the forward i–v relationship in Eq. (3.3) and evaluate the current I_1 corresponding to a diode voltage V_1:

$$I_1 = I_S e^{V_1/nV_T}$$

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[1] A slightly higher ambient temperature (25°C or so) is usually assumed for electronic equipment operating inside a cabinet. At this temperature, $V_T \simeq 25.8$ mV. Nevertheless, for the sake of simplicity and to promote rapid circuit analysis, we shall use the more arithmetically convenient value of $V_T \simeq 25$ mV throughout this book.

[2] In an integrated circuit, diodes are usually obtained by connecting a bipolar junction transistor (BJT) as a two-terminal device, as will be seen in Chapter 5.

Similarly, if the voltage is V_2, the diode current I_2 will be

$$I_2 = I_S e^{V_2/nV_T}$$

These two equations can be combined to produce

$$\frac{I_2}{I_1} = e^{(V_2 - V_1)/nV_T}$$

which can be rewritten as

$$V_2 - V_1 = nV_T \ln \frac{I_2}{I_1}$$

or, in terms of base-10 logarithms,

$$V_2 - V_1 = 2.3nV_T \log \frac{I_2}{I_1} \tag{3.5}$$

This equation simply states that for a decade (factor of 10) change in current, the diode voltage drop changes by $2.3nV_T$, which is approximately 60 mV for $n = 1$ and 120 mV for $n = 2$. This also suggests that the diode i–v relationship is most conveniently plotted on semilog paper. Using the vertical, linear axis for v and the horizontal, log axis for i, one obtains a straight line with a slope of $2.3nV_T$ per decade of current. Finally, it should be mentioned that not knowing the exact value of n (which can be obtained from a simple experiment), circuit designers use the convenient approximate number of 0.1 V/decade for the slope of the diode logarithmic characteristic.

A glance at the i–v characteristic in the forward region (Fig. 3.8) reveals that the current is negligibly small for v smaller than about 0.5 V. This value is usually referred to as the **cut-in voltage.** It should be emphasized, however, that this apparent threshold in the characteristic is simply a consequence of the exponential relationship. Another consequence of this relationship is the rapid increase of i. Thus, for a "fully conducting" diode, the voltage drop lies in a narrow range, approximately 0.6 V to 0.8 V. This gives rise to a simple "model" for the diode where it is assumed that a conducting diode has approximately a 0.7-V drop across it. Diodes with different current ratings (i.e., different areas and correspondingly different I_S) will exhibit the 0.7-V drop at different currents. For instance, a small-signal diode may be considered to have a 0.7-V drop at $i = 1$ mA, while a higher-power diode may have a 0.7-V drop at $i = 1$ A. We will study the topics of diode-circuit analysis and diode models in the next section.

EXAMPLE 3.3

A silicon diode said to be a 1-mA device displays a forward voltage of 0.7 V at a current of 1 mA. Evaluate the junction scaling constant I_S in the event that n is either 1 or 2. What scaling constants would apply for a 1-A diode of the same manufacture that conducts 1 A at 0.7 V?

Solution

Since

$$i = I_S e^{v/nV_T}$$

then

$$I_S = ie^{-v/nV_T}$$

For the 1-mA diode:

If $n = 1$: $I_S = 10^{-3}e^{-700/25} = 6.9 \times 10^{-16}$ A, or about 10^{-15} A

If $n = 2$: $I_S = 10^{-3}e^{-700/50} = 8.3 \times 10^{-10}$ A, or about 10^{-9} A

The diode conducting 1 A at 0.7 V corresponds to one-thousand 1-mA diodes in parallel with a total junction area 1000 times greater. Thus I_S is also 1000 times greater, being 1 pA and 1 μA, respectively for $n = 1$ and $n = 2$.

From this example it should be apparent that the value of n used can be quite important.

Since both I_S and V_T are functions of temperature, the forward i–v characteristic varies with temperature, as illustrated in Fig. 3.9. At a given constant diode current the voltage drop across the diode decreases by approximately 2 mV for every 1°C increase in temperature. The change in diode voltage with temperature has been exploited in the design of electronic thermometers.

EXERCISES

3.6 Consider a silicon diode with $n = 1.5$. Find the change in voltage if the current changes from 0.1 mA to 10 mA.

Ans. 172.5 mV

3.7 A silicon junction diode with $n = 1$ has $v = 0.7$ V at $i = 1$ mA. Find the voltage drop at $i = 0.1$ mA and $i = 10$ mA.

Ans. 0.64 V; 0.76 V

3.8 Using the fact that a silicon diode has $I_S = 10^{-14}$ A at 25°C and that I_S increases by 15% per °C rise in temperature, find the value of I_S at 125°C.

Ans. 1.17×10^{-8} A

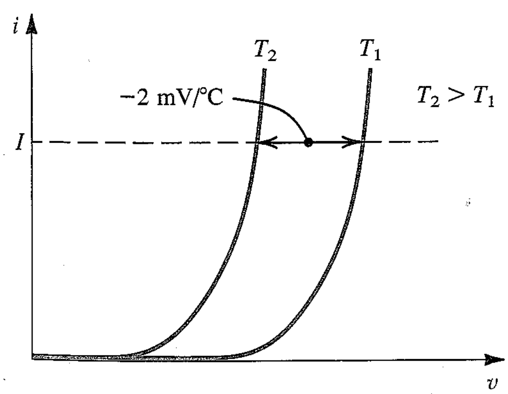


FIGURE 3.9 Illustrating the temperature dependence of the diode forward characteristic. At a constant current, the voltage drop decreases by approximately 2 mV for every 1°C increase in temperature.

3.2.2 The Reverse-Bias Region

The reverse-bias region of operation is entered when the diode voltage v is made negative. Equation (3.1) predicts that if v is negative and a few times larger than V_T (25 mV) in magnitude, the exponential term becomes negligibly small compared to unity, and the diode current becomes

$$i \simeq -I_S$$

That is, the current in the reverse direction is constant and equal to I_S. This constancy is the reason behind the term *saturation current.*

Real diodes exhibit reverse currents that, though quite small, are much larger than I_S. For instance, a small-signal diode whose I_S is on the order of 10^{-14} A to 10^{-15} A could show a reverse current on the order of 1 nA. The reverse current also increases somewhat with the increase in magnitude of the reverse voltage. Note that because of the very small magnitude of the current, these details are not clearly evident on the diode i–v characteristic of Fig. 3.8.

A large part of the reverse current is due to leakage effects. These leakage currents are proportional to the junction area, just as I_S is. Their dependence on temperature, however, is different from that of I_S. Thus, whereas I_S doubles for every 5°C rise in temperature, the corresponding rule of thumb for the temperature dependence of the reverse current is that it doubles for every 10°C rise in temperature.

EXERCISE

3.9 The diode in the circuit of Fig. E3.9 is a large high-current device whose reverse leakage is reasonably independent of voltage. If $V = 1$ V at 20°C, find the value of V at 40°C and at 0°C.

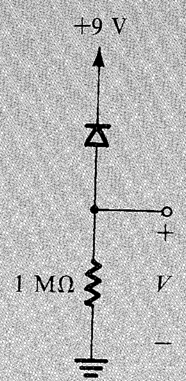


FIGURE E3.9

Ans. 4 V; 0.25 V

3.2.3 The Breakdown Region

The third distinct region of diode operation is the breakdown region, which can be easily identified on the diode i–v characteristic in Fig. 3.8. The breakdown region is entered when the magnitude of the reverse voltage exceeds a threshold value that is specific to the particular diode, called the **breakdown voltage.** This is the voltage at the "knee" of the i–v curve in

Fig. 3.8 and is denoted V_{ZK}, where the subscript Z stands for zener (to be explained shortly) and K denotes knee.

As can be seen from Fig. 3.8, in the breakdown region the reverse current increases rapidly, with the associated increase in voltage drop being very small. Diode breakdown is normally not destructive provided that the power dissipated in the diode is limited by external circuitry to a "safe" level. This safe value is normally specified on the device data sheets. It therefore is necessary to limit the reverse current in the breakdown region to a value consistent with the permissible power dissipation.

The fact that the diode i–v characteristic in breakdown is almost a vertical line enables it to be used in voltage regulation. This subject will be studied in Section 3.5.

3.3 MODELING THE DIODE FORWARD CHARACTERISTIC

Having studied the diode terminal characteristics we are now ready to consider the analysis of circuits employing forward-conducting diodes. Figure 3.10 shows such a circuit. It consists of a dc source V_{DD}, a resistor R, and a diode. We wish to analyze this circuit to determine the diode voltage V_D and current I_D. Toward that end we consider developing a variety of models for the operation of the diode. We already know of two such models: the ideal-diode model, and the exponential model. In the following discussion we shall assess the suitability of these two models in various analysis situations. Also, we shall develop and comment on a number of other models. This material, besides being useful in the analysis and design of diode circuits, establishes a foundation for the modeling of transistor operation that we will study in the next two chapters.

3.3.1 The Exponential Model

The most accurate description of the diode operation in the forward region is provided by the exponential model. Unfortunately, however, its severely nonlinear nature makes this model the most difficult to use. To illustrate, let's analyze the circuit in Fig. 3.10 using the exponential diode model.

Assuming that V_{DD} is greater than 0.5 V or so, the diode current will be much greater than I_S, and we can represent the diode i–v characteristic by the exponential relationship, resulting in

$$I_D = I_S e^{V_D/nV_T} \tag{3.6}$$

The other equation that governs circuit operation is obtained by writing a Kirchhoff loop equation, resulting in

$$I_D = \frac{V_{DD} - V_D}{R} \tag{3.7}$$

Assuming that the diode parameters I_S and n are known, Eqs. (3.6) and (3.7) are two equations in the two unknown quantities I_D and V_D. Two alternative ways for obtaining the solution are graphical analysis and iterative analysis.

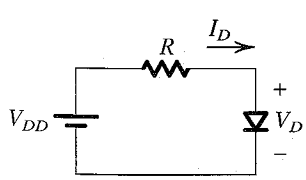


FIGURE 3.10 A simple circuit used to illustrate the analysis of circuits in which the diode is forward conducting.

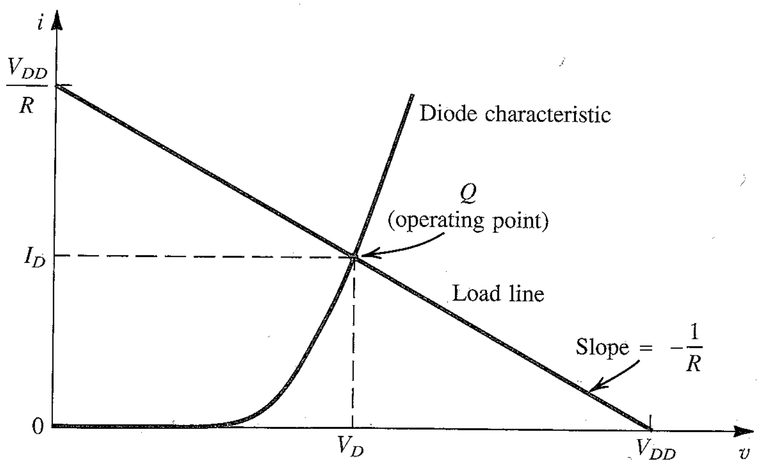


FIGURE 3.11 Graphical analysis of the circuit in Fig. 3.10 using the exponential diode model.

3.3.2 Graphical Analysis Using the Exponential Model

Graphical analysis is performed by plotting the relationships of Eqs. (3.6) and (3.7) on the i–v plane. The solution can then be obtained as the coordinates of the point of intersection of the two graphs. A sketch of the graphical construction is shown in Fig. 3.11. The curve represents the exponential diode equation (Eq. 3.6), and the straight line represents Eq. (3.7). Such a straight line is known as the **load line,** a name that will become more meaningful in later chapters. The load line intersects the diode curve at point Q, which represents the **operating point** of the circuit. Its coordinates give the values of I_D and V_D.

Graphical analysis aids in the visualization of circuit operation. However, the effort involved in performing such an analysis, particularly for complex circuits, is too great to be justified in practice.

3.3.3 Iterative Analysis Using the Exponential Model

Equations (3.6) and (3.7) can be solved using a simple iterative procedure, as illustrated in the following example.

EXAMPLE 3.4

Determine the current I_D and the diode voltage V_D for the circuit in Fig. 3.10 with $V_{DD} = 5$ V and $R = 1$ kΩ. Assume that the diode has a current of 1 mA at a voltage of 0.7 V and that its voltage drop changes by 0.1 V for every decade change in current.

Solution

To begin the iteration, we assume that $V_D = 0.7$ V and use Eq. (3.7) to determine the current,

$$I_D = \frac{V_{DD} - V_D}{R}$$

$$= \frac{5 - 0.7}{1} = 4.3 \text{ mA}$$

We then use the diode equation to obtain a better estimate for V_D. This can be done by employing Eq. (3.5), namely,

$$V_2 - V_1 = 2.3nV_T \log \frac{I_2}{I_1}$$

For our case, $2.3nV_T = 0.1$ V. Thus,

$$V_2 = V_1 + 0.1 \log \frac{I_2}{I_1}$$

Substituting $V_1 = 0.7$ V, $I_1 = 1$ mA, and $I_2 = 4.3$ mA results in $V_2 = 0.763$ V. Thus the results of the first iteration are $I_D = 4.3$ mA and $V_D = 0.763$ V. The second iteration proceeds in a similar manner:

$$I_D = \frac{5 - 0.763}{1} = 4.237 \text{ mA}$$

$$V_2 = 0.763 + 0.1 \log \left[\frac{4.237}{4.3}\right]$$

$$= 0.762 \text{ V}$$

Thus the second iteration yields $I_D = 4.237$ mA and $V_D = 0.762$ V. Since these values are not much different from the values obtained after the first iteration, no further iterations are necessary, and the solution is $I_D = 4.237$ mA and $V_D = 0.762$ V.

3.3.4 The Need for Rapid Analysis

The iterative analysis procedure utilized in the example above is simple and yields accurate results after two or three iterations. Nevertheless, there are situations in which the effort and time required are still greater than can be justified. Specifically, if one is doing a pencil-and-paper design of a relatively complex circuit, rapid circuit analysis is a necessity. Through quick analysis, the designer is able to evaluate various possibilities before deciding on a suitable circuit design. To speed up the analysis process one must be content with less precise results. This, however, is seldom a problem, because the more accurate analysis can be postponed until a final or almost-final design is obtained. Accurate analysis of the almost-final design can be performed with the aid of a computer circuit-analysis program such as SPICE (see Section 3.9). The results of such an analysis can then be used to further refine or "fine-tune" the design.

To speed up the analysis process, we must find simpler models for the diode forward characteristic.

3.3.5 The Piecewise-Linear Model

The analysis can be greatly simplified if we can find linear relationships to describe the diode terminal characteristics. An attempt in this direction is illustrated in Fig. 3.12, where the exponential curve is approximated by two straight lines, line A with zero slope and line B with a slope of $1/r_D$. It can be seen that for the particular case shown in Fig. 3.12, over the current range of 0.1 mA to 10 mA the voltages predicted by the straight-lines model shown differ from those predicted by the exponential model by less than 50 mV. Obviously the choice of these two straight lines is not unique; one can obtain a closer approximation by restricting the current range over which the approximation is required.

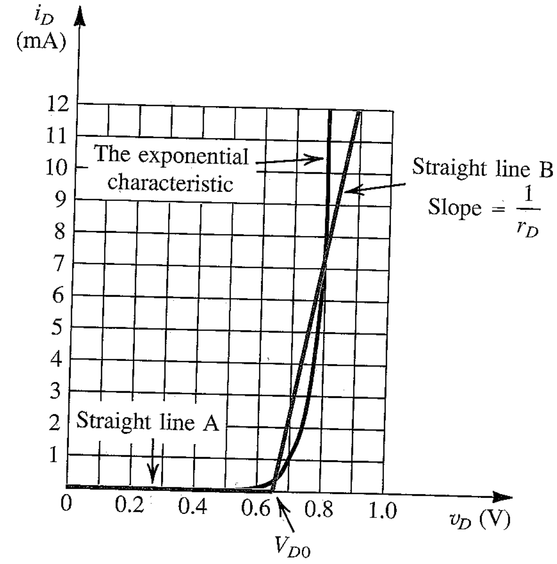


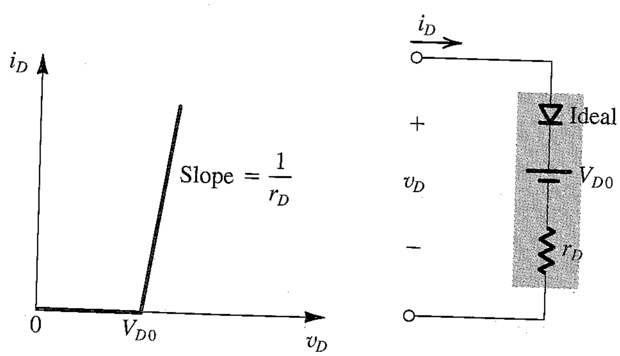
FIGURE 3.12 Approximating the diode forward characteristic with two straight lines: the piecewise-linear model.

The straight-lines (or piecewise-linear) model of Fig. 3.12 can be described by

$$
\begin{aligned}
i_D &= 0, \quad v_D \leq V_{D0} \\
i_D &= (v_D - V_{D0})/r_D, \quad v_D \geq V_{D0}
\end{aligned}
\tag{3.8}
$$

where V_{D0} is the intercept of line B on the voltage axis and r_D is the inverse of the slope of line B. For the particular example shown, $V_{D0} = 0.65$ V and $r_D = 20\ \Omega$.

The piecewise-linear model described by Eqs. (3.8) can be represented by the equivalent circuit shown in Fig. 3.13. Note that an ideal diode is included in this model to constrain i_D to flow in the forward direction only. This model is also known as the **battery-plus-resistance** model.



(a) (b)

FIGURE 3.13 Piecewise-linear model of the diode forward characteristic and its equivalent circuit representation.

EXAMPLE 3.5

Repeat the problem in Example 3.4 utilizing the piecewise-linear model whose parameters are given in Fig. 3.12 ($V_{D0} = 0.65$ V, $r_D = 20\ \Omega$). Note that the characteristics depicted in this figure are those of the diode described in Example 3.4 (1 mA at 0.7 V and 0.1 V/decade).

Solution

Replacing the diode in the circuit of Fig. 3.10 with the equivalent circuit model of Fig. 3.13 results in the circuit in Fig. 3.14, from which we can write for the current I_D,

$$I_D = \frac{V_{DD} - V_{D0}}{R + r_D}$$

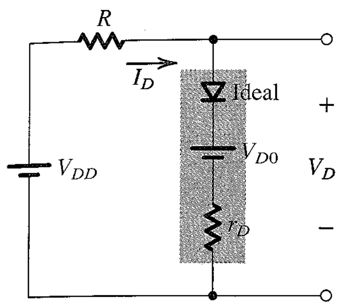


FIGURE 3.14 The circuit of Fig. 3.10 with the diode replaced with its piecewise-linear model of Fig. 3.13.

where the model parameters V_{D0} and r_D are seen from Fig. 3.12 to be $V_{D0} = 0.65$ V and $r_D = 20\ \Omega$. Thus,

$$I_D = \frac{5 - 0.65}{1 + 0.02} = 4.26 \text{ mA}$$

The diode voltage V_D can now be computed:

$$\begin{aligned} V_D &= V_{D0} + I_D r_D \\ &= 0.65 + 4.26 \times 0.02 = 0.735 \text{ V} \end{aligned}$$

3.3.6 The Constant-Voltage-Drop Model

An even simpler model of the diode forward characteristics can be obtained if we use a vertical straight line to approximate the fast-rising part of the exponential curve, as shown in Fig. 3.15. The resulting model simply says that *a forward-conducting diode exhibits a constant voltage drop V_D*. The value of V_D is usually taken to be 0.7 V. Note that for the particular diode whose characteristics are depicted in Fig. 3.15, this model predicts the diode voltage to within ±0.1 V over the current range of 0.1 mA to 10 mA. The constant-voltage-drop model can be represented by the equivalent circuit shown in Fig. 3.16.

The constant-voltage-drop model is the one most frequently employed in the initial phases of analysis and design. This is especially true if at these stages one does not have detailed information about the diode characteristics, which is often the case.

Finally, note that if we employ the constant-voltage-drop model to solve the problem in Examples 3.4 and 3.5, we obtain

$$V_D = 0.7 \text{ V}$$

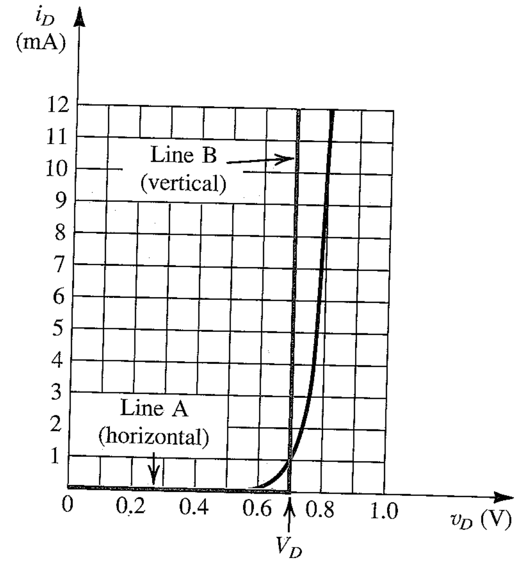


FIGURE 3.15 Development of the constant-voltage-drop model of the diode forward characteristics. A vertical straight line (B) is used to approximate the fast-rising exponential. Observe that this simple model predicts V_D to within ±0.1 V over the current range of 0.1 mA to 10 mA.

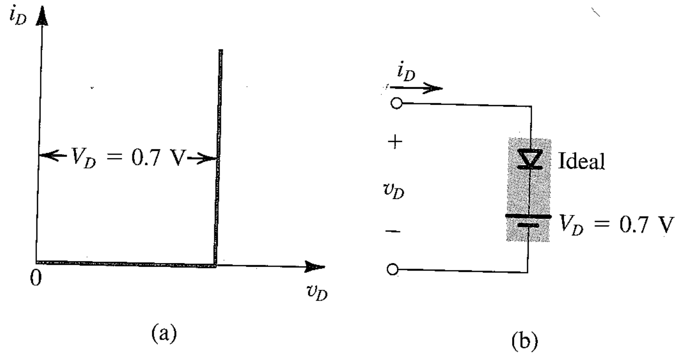


FIGURE 3.16 The constant-voltage-drop model of the diode forward characteristics and its equivalent-circuit representation.

and

$$I_D = \frac{V_{DD} - 0.7}{R}$$

$$= \frac{5 - 0.7}{1} = 4.3 \text{ mA}$$

which are not too different from the values obtained before with the more elaborate models.

3.3.7 The Ideal-Diode Model

In applications that involve voltages much greater than the diode voltage drop (0.6–0.8 V), we may neglect the diode voltage drop altogether while calculating the diode current. The result is the ideal-diode model, which we studied in Section 3.1. For the circuit in Examples 3.4 and 3.5 (i.e., Fig. 3.10 with $V_{DD} = 5$ V and $R = 1$ kΩ), utilization of the ideal-diode model leads to

$$V_D = 0 \text{ V}$$

$$I_D = \frac{5 - 0}{1} = 5 \text{ mA}$$

which for a very quick analysis would not be bad as a gross estimate. However, with almost no additional work, the 0.7-V-drop model yields much more realistic results. We note, however, that the greatest utility of the ideal-diode model is in determining which diodes are on and which are off in a multidiode circuit, such as those considered in Section 3.1.

EXERCISES

3.10 For the circuit in Fig. 3.10, find I_D and V_D for the case $V_{DD} = 5$ V and $R = 10$ kΩ. Assume that the diode has a voltage of 0.7 V at 1-mA current and that the voltage changes by 0.1 V/decade of current change. Use (a) iteration, (b) the piecewise-linear model with $V_{D0} = 0.65$ V and $r_D = 20\ \Omega$, (c) the constant-voltage-drop model with $V_D = 0.7$ V.

Ans. (a) 0.434 mA, 0.663 V; (b) 0.434 mA, 0.659 V; (c) 0.43 mA, 0.7 V

3.11 Consider a diode that is 100 times as large (in junction area) as that whose characteristics are displayed in Fig. 3.12. If we approximate the characteristics in a manner similar to that in Fig. 3.12 (but over a current range 100 times as large), how would the model parameters V_{D0} and r_D change?

Ans. V_{D0} does not change; r_D decreases by a factor of 100 to 0.2 Ω

D3.12 Design the circuit in Fig. E3.12 to provide an output voltage of 2.4 V. Assume that the diodes available have 0.7-V drop at 1 mA and that $\Delta V = 0.1$ V/decade change in current.

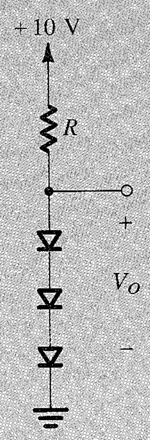


FIGURE E3.12

Ans. $R = 760\ \Omega$

3.13 Repeat Exercise 3.4 using the 0.7-V-drop model to obtain better estimates of I and V than those found in Exercise 3.4 (using the ideal-diode model).

Ans. (a) 1.72 mA, 0.7 V; (b) 0 mA, 5 V; (c) 0 mA, 5 V; (d) 1.72 mA, 0.7 V; (e) 2.3 mA, +2.3 V; (f) 3.3 mA, +1.7 V

3.3.8 The Small-Signal Model

There are applications in which a diode is biased to operate at a point on the forward i–v characteristic and a small ac signal is superimposed on the dc quantities. For this situation, we first have to determine the dc operating point (V_D and I_D) of the diode using one of the models discussed above. Most frequently, the 0.7-V-drop model is utilized. Then, for

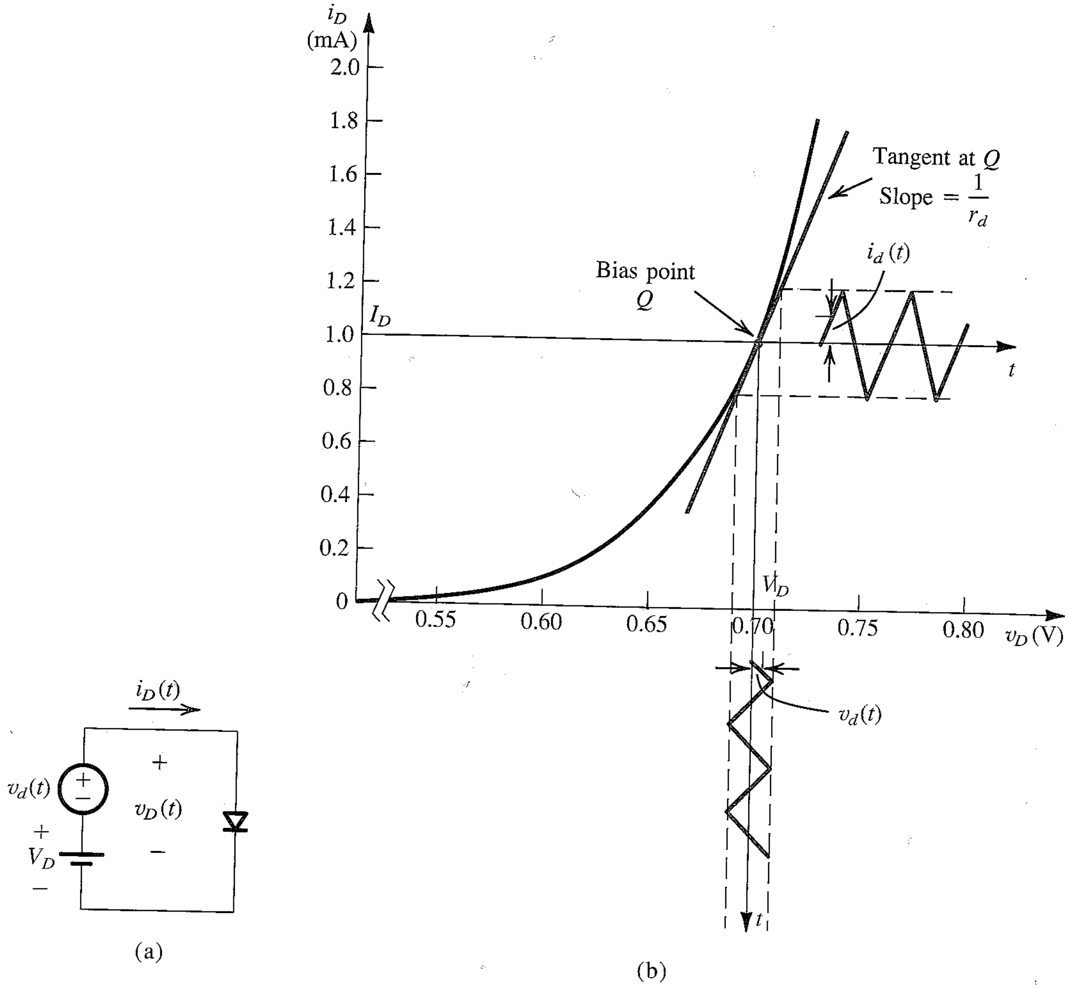


FIGURE 3.17 Development of the diode small-signal model. Note that the numerical values shown are for a diode with $n = 2$.

small-signal operation around the dc bias point, the diode is best modeled by a resistance equal to the inverse of the slope of the tangent to the exponential i–v characteristic at the bias point. The concept of biasing a nonlinear device and restricting signal excursion to a short, almost-linear segment of its characteristic around the bias point was introduced in Section 1.4 for two-port networks. In the following, we develop such a small-signal model for the junction diode and illustrate its application.

Consider the conceptual circuit in Fig. 3.17(a) and the corresponding graphical representation in Fig. 3.17(b). A dc voltage V_D, represented by a battery, is applied to the diode, and a time-varying signal $v_d(t)$, assumed (arbitrarily) to have a triangular waveform, is superimposed on the dc voltage V_D. In the absence of the signal $v_d(t)$ the diode voltage is equal to V_D, and correspondingly, the diode will conduct a dc current I_D given by

$$I_D = I_S e^{V_D/nV_T} \tag{3.9}$$

When the signal $v_d(t)$ is applied, the total instantaneous diode voltage $v_D(t)$ will be given by

$$v_D(t) = V_D + v_d(t) \tag{3.10}$$

Correspondingly, the total instantaneous diode current $i_D(t)$ will be

$$i_D(t) = I_S e^{v_D/nV_T} \tag{3.11}$$

Substituting for v_D from Eq. (3.10) gives

$$i_D(t) = I_S e^{(V_D+v_d)/nV_T}$$

which can be rewritten

$$i_D(t) = I_S e^{V_D/nV_T} e^{v_d/nV_T}$$

Using Eq. (3.9) we obtain

$$i_D(t) = I_D e^{v_d/nV_T} \tag{3.12}$$

Now if the amplitude of the signal $v_d(t)$ is kept sufficiently small such that

$$\frac{v_d}{nV_T} \ll 1 \tag{3.13}$$

then we may expand the exponential of Eq. (3.12) in a series and truncate the series after the first two terms to obtain the approximate expression

$$i_D(t) \simeq I_D\left(1 + \frac{v_d}{nV_T}\right) \tag{3.14}$$

This is the **small-signal approximation.** It is valid for signals whose amplitudes are smaller than about 10 mV for the case $n = 2$ and 5 mV for $n = 1$ (see Eq. 3.13 and recall that $V_T =$ 25 mV).[3]

From Eq. (3.14) we have

$$i_D(t) = I_D + \frac{I_D}{nV_T} v_d \tag{3.15}$$

Thus, superimposed on the dc current I_D, we have a signal current component directly proportional to the signal voltage v_d. That is,

$$i_D = I_D + i_d \tag{3.16}$$

where

$$i_d = \frac{I_D}{nV_T} v_d \tag{3.17}$$

The quantity relating the signal current i_d to the signal voltage v_d has the dimensions of conductance, mhos (℧), and is called the **diode small-signal conductance.** The inverse of this parameter is the **diode small-signal resistance,** or **incremental resistance,** r_d,

$$r_d = \frac{nV_T}{I_D} \tag{3.18}$$

Note that the value of r_d is inversely proportional to the bias current I_D.

[3] For $n = 2$, $v_d/nV_T = 0.2$ with $v_d = 10$ mV. Thus the next term in the series expansion of the exponential will be $\frac{1}{2} \times 0.2^2 = 0.02$, a factor of 10 lower than the linear term we kept. A better approximation can be achieved by keeping v_d smaller. Also, note that for $n = 1$, v_d should be limited to, say, 5 mV.

Let us return to the graphical representation in Fig. 3.17(b). It is easy to see that using the small-signal approximation is equivalent to assuming that *the signal amplitude is sufficiently small such that the excursion along the i–v curve is limited to a short almost-linear segment.* The slope of this segment, which is equal to the slope of the tangent to the i–v curve at the operating point Q, is equal to the small-signal conductance. The reader is encouraged to prove that the slope of the i–v curve at $i = I_D$ is equal to I_D/nV_T, which is $1/r_d$; that is,

$$r_d = 1 \Big/ \left[\frac{\partial i_D}{\partial v_D}\right]_{i_D=I_D} \tag{3.19}$$

From the preceding we conclude that superimposed on the quantities V_D and I_D that define the dc bias point, or **quiescent point,** of the diode will be the small-signal quantities $v_d(t)$ and $i_d(t)$, which are related by the diode small-signal resistance r_d evaluated at the bias point (Eq. 3.18). Thus the small-signal analysis can be performed separately from the dc bias analysis, a great convenience that results from the linearization of the diode characteristics inherent in the small-signal approximation. Specifically, after the dc analysis is performed, the small-signal equivalent circuit is obtained by eliminating all dc sources (i.e., short-circuiting dc voltage sources and open-circuiting dc current sources) and replacing the diode by its small-signal resistance. The following example should illustrate the application of the small-signal model.

EXAMPLE 3.6

Consider the circuit shown in Fig. 3.18(a) for the case in which $R = 10\ \text{k}\Omega$. The power supply V^+ has a dc value of 10 V on which is superimposed a 60-Hz sinusoid of 1-V peak amplitude. (This "signal" component of the power-supply voltage is an imperfection in the power-supply design. It is known as the **power-supply ripple.** More on this later.) Calculate both the dc voltage of the diode and the amplitude of the sine-wave signal appearing across it. Assume the diode to have a 0.7-V drop at 1-mA current and $n = 2$.

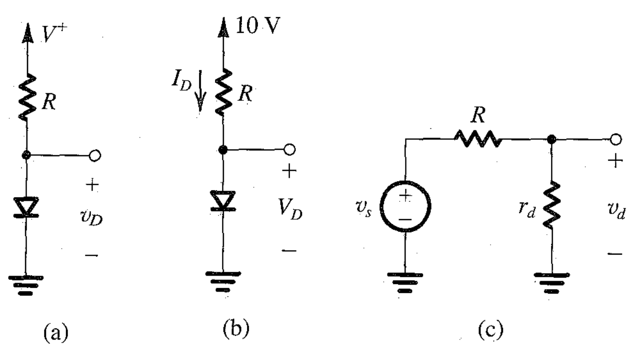


FIGURE 3.18 **(a)** Circuit for Example 3.6. **(b)** Circuit for calculating the dc operating point. **(c)** Small-signal equivalent circuit.

Solution

Considering dc quantities only, we assume $V_D \simeq 0.7$ V and calculate the diode dc current

$$I_D = \frac{10 - 0.7}{10} = 0.93\ \text{mA}$$

Since this value is very close to 1 mA, the diode voltage will be very close to the assumed value of 0.7 V. At this operating point, the diode incremental resistance r_d is

$$r_d = \frac{nV_T}{I_D} = \frac{2 \times 25}{0.93} = 53.8\ \Omega$$

The signal voltage across the diode can be found from the small-signal equivalent circuit in Fig. 3.18(c). Here v_s denotes the 60-Hz 1-V peak sinusoidal component of V^+, and v_d is the corresponding signal across the diode. Using the voltage-divider rule provides the peak amplitude of v_d as follows:

$$v_d\ (\text{peak}) = \hat{V}_s \frac{r_d}{R + r_d}$$

$$= 1 \frac{0.0538}{10 + 0.0538} = 5.35\ \text{mV}$$

Finally we note that since this value is quite small, our use of the small-signal model of the diode is justified.

3.3.9 Use of the Diode Forward Drop in Voltage Regulation

A further application of the diode small-signal model is found in a popular diode application, namely the use of diodes to create a regulated voltage. A voltage regulator is a circuit whose purpose is to provide a constant dc voltage between its output terminals. The output voltage is required to remain as constant as possible in spite of (a) changes in the load current drawn from the regulator output terminal and (b) changes in the dc power-supply voltage that feeds the regulator circuit. Since the forward voltage drop of the diode remains almost constant at approximately 0.7 V while the current through it varies by relatively large amounts, a forward-biased diode can make a simple voltage regulator. For instance, we have seen in Example 3.6 that while the 10-V dc supply voltage had a ripple of 2 V peak-to-peak (a ±10% variation), the corresponding ripple in the diode voltage was only about ±5.4 mV (a ±0.8% variation). Regulated voltages greater than 0.7 V can be obtained by connecting a number of diodes in series. For example, the use of three forward-biased diodes in series provides a voltage of about 2 V. One such circuit is investigated in the following example, which utilizes the diode small-signal model to quantify the efficacy of the voltage regulator that is realized.

EXAMPLE 3.7

Consider the circuit shown in Fig. 3.19. A string of three diodes is used to provide a constant voltage of about 2.1 V. We want to calculate the percentage change in this regulated voltage caused by (a) a ±10% change in the power-supply voltage and (b) connection of a 1-kΩ load resistance. Assume $n = 2$.

Solution

With no load, the nominal value of the current in the diode string is given by

$$I = \frac{10 - 2.1}{1} = 7.9\ \text{mA}$$

Thus each diode will have an incremental resistance of

$$r_d = \frac{nV_T}{I}$$

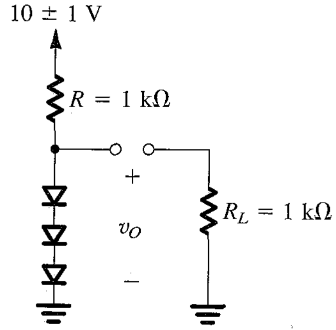


FIGURE 3.19 Circuit for Example 3.7.

Using $n = 2$ gives

$$r_d = \frac{2 \times 25}{7.9} = 6.3\ \Omega$$

The three diodes in series will have a total incremental resistance of

$$r = 3r_d = 18.9\ \Omega$$

This resistance, along with the resistance R, forms a voltage divider whose ratio can be used to calculate the change in output voltage due to a ±10% (i.e., ±1-V) change in supply voltage. Thus the peak-to-peak change in output voltage will be

$$\Delta v_O = 2\frac{r}{r+R} = 2\frac{0.0189}{0.0189+1} = 37.1\ \text{mV}$$

That is, corresponding to the ±1-V (±10%) change in supply voltage, the output voltage will change by ±18.5 mV or ±0.9%. Since this implies a change of about ±6.2 mV per diode, our use of the small-signal model is justified.

When a load resistance of 1 kΩ is connected across the diode string, it draws a current of approximately 2.1 mA. Thus the current in the diodes decreases by 2.1 mA, resulting in a decrease in voltage across the diode string given by

$$\Delta v_O = -2.1 \times r = -2.1 \times 18.9 = -39.7\ \text{mV}$$

Since this implies that the voltage across each diode decreases by about 13.2 mV, our use of the small-signal model is not entirely justified. Nevertheless, a detailed calculation of the voltage change using the exponential model results in $\Delta v_O = -35.5$ mV, which is not too different from the approximate value obtained using the incremental model.

EXERCISES

3.14 Find the value of the diode small-signal resistance r_d at bias currents of 0.1 mA, 1 mA, and 10 mA. Assume $n = 1$.

Ans. 250 Ω; 25 Ω; 2.5 Ω

3.15 Consider a diode with $n = 2$ biased at 1 mA. Find the change in current as a result of changing the voltage by (a) −20 mV, (b) −10 mV, (c) −5 mV, (d) +5 mV, (e) +10 mV, and (f) +20 mV. In each case, do the calculations (i) using the small-signal model and (ii) using the exponential model.

Ans. (a) −0.40, −0.33 mA; (b) −0.20, −0.18 mA; (c) −0.10, −0.10 mA; (d) +0.10, +0.11 mA; (e) +0.20, +0.22 mA; (f) +0.40, +0.49 mA

D3.16 Design the circuit of Fig. E3.16 so that $V_O = 3$ V when $I_L = 0$ and V_O changes by 40 mV per 1 mA of load current. Find the value of R and the junction area of each diode (assume all four diodes are identical) relative to a diode with 0.7-V drop at 1-mA current. Assume $n = 1$.

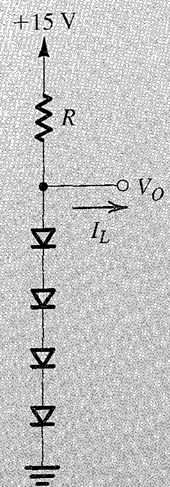


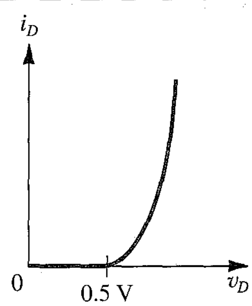
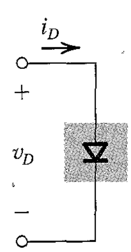
FIGURE E3.16

Ans. $R = 4.8$ kΩ; 0.34

3.3.10 Summary

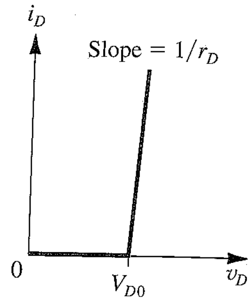
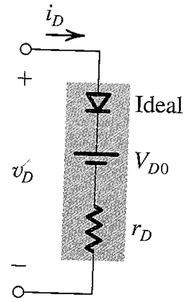
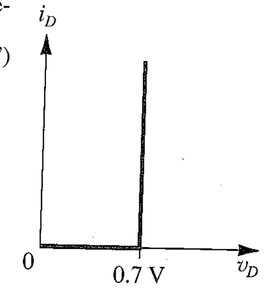
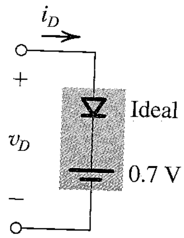
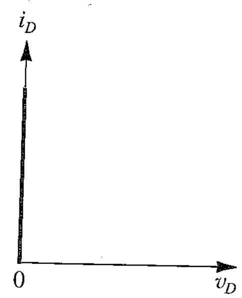
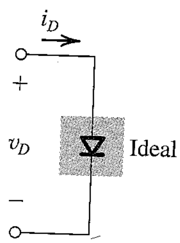
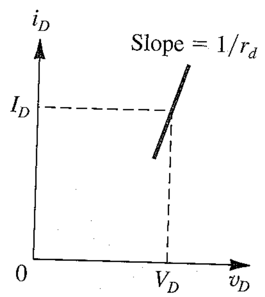
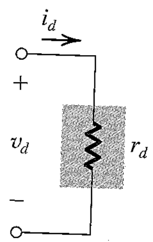
As a summary of this important section on diode modeling, Table 3.1 lists the five diode models studied and provides pertinent comments regarding each. These comments are intended to aid in the selection of an appropriate model for a particular application. The question "which model?" is one that circuit designers face repeatedly, not just with diodes but with every circuit element. The problem is finding an appropriate compromise between accuracy and speed of analysis. One's ability to select appropriate device models improves with practice and experience.

TABLE 3.1 Modeling the Diode Forward Characteristic

Model	Graph	Equations	Circuit	Comments
Exponential		$i_D = I_S e^{v_D/nV_T}$ $v_D = 2.3nV_T \log\left(\frac{i_D}{I_S}\right)$ $V_{D2} - V_{D1} = 2.3nV_T \log\left(\frac{I_{D2}}{I_{D1}}\right)$ $2.3nV_T = 60$ mV for $n = 1$ $2.3nV_T = 120$ mV for $n = 2$		$I_S = 10^{-12}$ A to 10^{-15} A, depending on junction area $V_T \cong 25$ mV $n = 1$ to 2 Physically based and remarkably accurate model Useful when accurate analysis is needed

(Continued)

TABLE 3.1 *(Continued)*

Model	Graph	Equations	Circuit	Comments
Piecewise-linear (battery-plus-resistance)		For $v_D \le V_{D0}$: $i_D = 0$ For $v_D \ge V_{D0}$: $i_D = \frac{1}{r_D}(v_D - V_{D0})$		Choice of V_{D0} and r_D is determined by the current range over which the model is required. For the amount of work involved, not as useful as the constant-voltage-drop model. Used only infrequently.
Constant-voltage-drop (or the "0.7-V model")		For $i_D > 0$: $v_D = 0.7$ V		Easy to use and very popular for the quick, hand analysis that is essential in circuit design.
Ideal-diode		For $i_D > 0$: $v_D = 0$		Good for determining which diodes are conducting and which are cutoff in a multiple-diode circuit. Good for obtaining very approximate values for diode currents, especially when the circuit voltages are much greater than V_D.
Small-signal		For small signals superimposed on V_D and I_D: $i_d = v_d/r_d$ $r_d = nV_T/I_D$ (For $n = 1$, v_d is limited to 5 mV; for $n = 2$, 10 mV)		Useful for finding the signal component of the diode voltage (e.g., in the voltage-regulator application). Serves as the basis for small-signal modeling of transistors (Chapters 4 and 5).

3.4 OPERATION IN THE REVERSE BREAKDOWN REGION—ZENER DIODES

The very steep i–v curve that the diode exhibits in the breakdown region (Fig. 3.8) and the almost-constant voltage drop that this indicates suggest that diodes operating in the breakdown region can be used in the design of voltage regulators. From the previous section, the reader will recall that voltage regulators are circuits that provide constant dc output voltages in the face of changes in their load current and in the system power-supply voltage. This in fact turns out to be an important application of diodes operating in the reverse breakdown region, and special diodes are manufactured to operate specifically in the breakdown region. Such diodes are called **breakdown diodes** or, more commonly, **zener diodes,** after an early worker in the area.

Figure 3.20 shows the circuit symbol of the zener diode. In normal applications of zener diodes, current flows into the cathode, and the cathode is positive with respect to the anode. Thus I_Z and V_Z in Fig. 3.20 have positive values.

3.4.1 Specifying and Modeling the Zener Diode

Figure 3.21 shows details of the diode i–v characteristics in the breakdown region. We observe that for currents greater than the **knee current** I_{ZK} (specified on the data sheet of

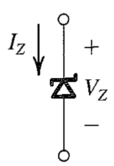


FIGURE 3.20 Circuit symbol for a zener diode.

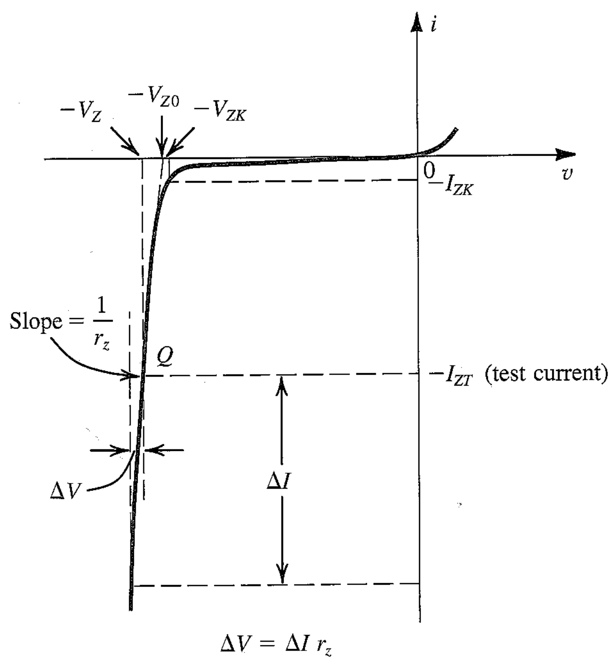


FIGURE 3.21 The diode i–v characteristic with the breakdown region shown in some detail.

the zener diode), the i–v characteristic is almost a straight line. The manufacturer usually specifies the voltage across the zener diode V_Z at a specified test current, I_{ZT}. We have indicated these parameters in Fig. 3.21 as the coordinates of the point labeled Q. Thus a 6.8-V zener diode will exhibit at 6.8-V drop at a specified test current of, say, 10 mA. As the current through the zener deviates from I_{ZT}, the voltage across it will change, though only slightly. Figure 3.21 shows that corresponding to current change ΔI the zener voltage changes by ΔV, which is related to ΔI by

$$\Delta V = r_z \Delta I$$

where r_z is the inverse of the slope of the almost-linear i–v curve at point Q. Resistance r_z is the **incremental resistance** of the zener diode at operating point Q. It is also known as the **dynamic resistance** of the zener, and its value is specified on the device data sheet. Typically, r_z is in the range of a few ohms to a few tens of ohms. Obviously, the lower the value of r_z is, the more constant the zener voltage remains as its current varies and thus the more ideal its performance becomes in the design of voltage regulators. In this regard, we observe from Fig. 3.21 that while r_z remains low and almost constant over a wide range of current, its value increases considerably in the vicinity of the knee. Therefore, as a general design guideline, one should avoid operating the zener in this low-current region.

Zener diodes are fabricated with voltages V_Z in the range of a few volts to a few hundred volts. In addition to specifying V_Z (at a particular current I_{ZT}), r_z, and I_{ZK}, the manufacturer also specifies the maximum power that the device can safely dissipate. Thus a 0.5-W, 6.8-V zener diode can operate safely at currents up to a maximum of about 70 mA.

The almost-linear i–v characteristic of the zener diode suggests that the device can be modeled as indicated in Fig. 3.22. Here V_{Z0} denotes the point at which the straight line of slope $1/r_z$ intersects the voltage axis (refer to Fig. 3.21). Although V_{Z0} is shown to be slightly different from the knee voltage V_{ZK}, in practice their values are almost equal. The equivalent circuit model of Fig. 3.22 can be analytically described by

$$V_Z = V_{Z0} + r_z I_Z \tag{3.20}$$

and it applies for $I_Z > I_{ZK}$ and, obviously, $V_Z > V_{Z0}$.

3.4.2 Use of the Zener as a Shunt Regulator

We now illustrate, by way of an example, the use of zener diodes in the design of shunt regulators, so named because the regulator circuit appears in parallel (shunt) with the load.

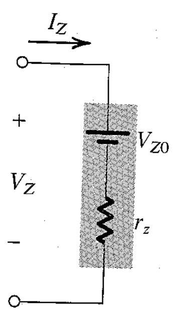


FIGURE 3.22 Model for the zener diode.

EXAMPLE 3.8

The 6.8-V zener diode in the circuit of Fig. 3.23(a) is specified to have $V_Z = 6.8$ V at $I_Z = 5$ mA, $r_z = 20\ \Omega$, and $I_{ZK} = 0.2$ mA. The supply voltage V^+ is nominally 10 V but can vary by ±1 V.

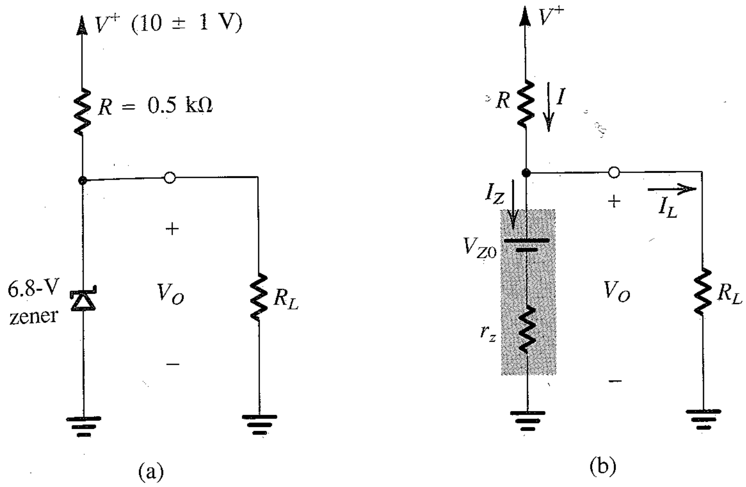


FIGURE 3.23 **(a)** Circuit for Example 3.8. **(b)** The circuit with the zener diode replaced with its equivalent circuit model.

(a) Find V_O with no load and with V^+ at its nominal value.

(b) Find the change in V_O resulting from the ±1-V change in V^+. Note that $(\Delta V_O/\Delta V^+)$, usually expressed in mV/V, is known as **line regulation.**

(c) Find the change in V_O resulting from connecting a load resistance R_L that draws a current $I_L =$ 1 mA, and hence find the **load regulation** $(\Delta V_O/\Delta I_L)$ in mV/mA.

(d) Find the change in V_O when $R_L = 2$ kΩ.

(e) Find the value of V_O when $R_L = 0.5$ kΩ.

(f) What is the minimum value of R_L for which the diode still operates in the breakdown region?

Solution

First we must determine the value of the parameter V_{Z0} of the zener diode model. Substituting $V_Z = 6.8$ V, $I_Z = 5$ mA, and $r_z = 20\ \Omega$ in Eq. (3.20) yields $V_{Z0} = 6.7$ V. Figure 3.23(b) shows the circuit with the zener diode replaced with its model.

(a) With no load connected, the current through the zener is given by

$$I_Z = I = \frac{V^+ - V_{Z0}}{R + r_z}$$
$$= \frac{10 - 6.7}{0.5 + 0.02} = 6.35 \text{ mA}$$

Thus,

$$V_O = V_{Z0} + I_Z r_z$$
$$= 6.7 + 6.35 \times 0.02 = 6.83 \text{ V}$$

(b) For a ±1-V change in V^+, the change in output voltage can be found from

$$\begin{aligned}\Delta V_O &= \Delta V^+ \frac{r_z}{R + r_z} \\ &= \pm 1 \times \frac{20}{500 + 20} = \pm 38.5 \text{ mV}\end{aligned}$$

Thus,

$$\text{Line regulation} = 38.5 \text{ mV/V}$$

(c) When a load resistance R_L that draws a load current $I_L = 1$ mA is connected, the zener current will decrease by 1 mA. The corresponding change in zener voltage can be found from

$$\begin{aligned}\Delta V_O &= r_z \Delta I_Z \\ &= 20 \times -1 = -20 \text{ mV}\end{aligned}$$

Thus the load regulation is

$$\text{Load regulation} \equiv \frac{\Delta V_O}{\Delta I_L} = -20 \text{ mV/mA}$$

(d) When a load resistance of 2 kΩ is connected, the load current will be approximately 6.8 V/2 kΩ = 3.4 mA. Thus the change in zener current will be $\Delta I_Z = -3.4$ mA, and the corresponding change in zener voltage (output voltage) will thus be

$$\begin{aligned}\Delta V_O &= r_z \Delta I_Z \\ &= 20 \times -3.4 = -68 \text{ mV}\end{aligned}$$

This calculation, however, is approximate, because it neglects the change in the current I. A more accurate estimate of ΔV_O can be obtained by analyzing the circuit in Fig. 3.23(b). The result of such an analysis is $\Delta V_O = -70$ mV.

(e) An R_L of 0.5 kΩ would draw a load current of $6.8/0.5 = 13.6$ mA. This is not possible, because the current I supplied through R is only 6.4 mA (for $V^+ = 10$ V). Therefore, the zener must be cut off. If this is indeed the case, then V_O is determined by the voltage divider formed by R_L and R (Fig. 3.23a),

$$\begin{aligned}V_O &= V^+ \frac{R_L}{R + R_L} \\ &= 10 \frac{0.5}{0.5 + 0.5} = 5 \text{ V}\end{aligned}$$

Since this voltage is lower than the breakdown voltage of the zener, the diode is indeed no longer operating in the breakdown region.

(f) For the zener to be at the edge of the breakdown region, $I_Z = I_{ZK} = 0.2$ mA and $V_Z \simeq V_{ZK} \simeq$ 6.7 V. At this point the lowest (worst-case) current supplied through R is $(9 - 6.7)/0.5 = 4.6$ mA, and thus the load current is $4.6 - 0.2 = 4.4$ mA. The corresponding value of R_L is

$$R_L = \frac{6.7}{4.4} \simeq 1.5 \text{ k}\Omega$$

3.4.3 Temperature Effects

The dependence of the zener voltage V_Z on temperature is specified in terms of its temperature coefficient TC, or **temco** as it is commonly known, which is usually expressed in mV/°C. The value of TC depends on the zener voltage, and for a given diode the TC varies with the operating current. Zener diodes whose V_Z are lower than about 5 V exhibit a negative TC. On the other hand, zeners with higher voltages exhibit a positive TC. The TC of a zener diode with a V_Z of about 5 V can be made zero by operating the diode at a specified current. Another commonly used technique for obtaining a reference voltage with low temperature coefficient is to connect a zener diode with a positive temperature coefficient of about 2 mV/°C in series with a forward-conducting diode. Since the forward-conducting diode has a voltage drop of $\simeq 0.7$ V and a TC of about –2 mV/°C, the series combination will provide a voltage of $(V_Z + 0.7)$ with a TC of about zero.

EXERCISES

3.17 A zener diode whose nominal voltage is 10 V at 10 mA has an incremental resistance of 50 Ω. What voltage do you expect if the diode current is halved? Doubled? What is the value of V_{Z0} in the zener model?

Ans. 9.75 V; 10.5 V; 9.5 V

D3.18 A zener diode exhibits a constant voltage of 5.6 V for currents greater than five times the knee current. I_{ZK} is specified to be 1 mA. The zener is to be used in the design of a shunt regulator fed from a 15-V supply. The load current varies over the range of 0 mA to 15 mA. Find a suitable value for the resistor R. What is the maximum power dissipation of the zener diode?

Ans. 470 Ω; 112 mW

3.19 A shunt regulator utilizes a zener diode whose voltage is 5.1 V at a current of 50 mA and whose incremental resistance is 7 Ω. The diode is fed from a supply of 15-V nominal voltage through a 200-Ω resistor. What is the output voltage at no load? Find the line regulation and the load regulation.

Ans. 5.1 V; 33.8 mV/V; –7 mV/mA

3.4.4 A Final Remark

Though simple and useful, zener diodes have lost a great deal of their popularity in recent years. They have been virtually replaced in voltage-regulator design by specially designed integrated circuits (ICs) that perform the voltage regulation function much more effectively and with greater flexibility than zener diodes.

3.5 RECTIFIER CIRCUITS

One of the most important applications of diodes is in the design of rectifier circuits. A diode rectifier forms an essential building block of the dc power supplies required to power electronic equipment. A block diagram of such a power supply is shown in Fig. 3.24. As indicated, the power supply is fed from the 120-V (rms) 60-Hz ac line, and it delivers a dc voltage V_O (usually in the range of 5–20 V) to an electronic circuit represented by the *load* block. The dc voltage V_O is required to be as constant as possible in spite of variations in the ac line voltage and in the current drawn by the load.

The first block in a dc power supply is the **power transformer.** It consists of two separate coils wound around an iron core that magnetically couples the two windings. The **primary winding,** having N_1 turns, is connected to the 120-V ac supply, and the **secondary winding,** having N_2 turns, is connected to the circuit of the dc power supply. Thus an ac voltage v_S

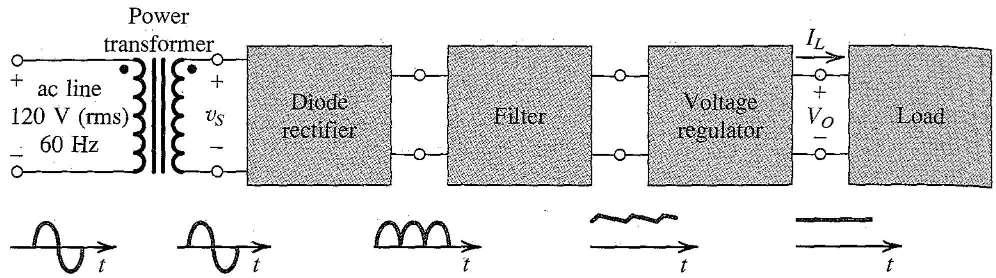


FIGURE 3.24 Block diagram of a dc power supply.

of $120(N_2/N_1)$ V (rms) develops between the two terminals of the secondary winding. By selecting an appropriate turns ratio (N_1/N_2) for the transformer, the designer can step the line voltage down to the value required to yield the particular dc voltage output of the supply. For instance, a secondary voltage of 8-V rms may be appropriate for a dc output of 5 V. This can be achieved with a 15:1 turns ratio.

In addition to providing the appropriate sinusoidal amplitude for the dc power supply, the power transformer provides electrical isolation between the electronic equipment and the power-line circuit. This isolation minimizes the risk of electric shock to the equipment user.

The diode rectifier converts the input sinusoid v_S to a unipolar output, which can have the pulsating waveform indicated in Fig. 3.24. Although this waveform has a nonzero average or a dc component, its pulsating nature makes it unsuitable as a dc source for electronic circuits, hence the need for a filter. The variations in the magnitude of the rectifier output are considerably reduced by the filter block in Fig. 3.24. In the following sections we shall study a number of rectifier circuits and a simple implementation of the output filter.

The output of the rectifier filter, though much more constant than without the filter, still contains a time-dependent component, known as **ripple.** To reduce the ripple and to stabilize the magnitude of the dc output voltage of the supply against variations caused by changes in load current, a voltage regulator is employed. Such a regulator can be implemented using the zener shunt regulator configuration studied in Section 3.4. Alternatively, and much more commonly at present, an integrated-circuit regulator can be used.

3.5.1 The Half-Wave Rectifier

The half-wave rectifier utilizes alternate half-cycles of the input sinusoid. Figure 3.25(a) shows the circuit of a half-wave rectifier. This circuit was analyzed in Section 3.1 (see Fig. 3.3) assuming an ideal diode. Using the more realistic battery-plus-resistance diode model, we obtain the equivalent circuit shown in Fig. 3.25(b), from which we can write

$$v_O = 0, \qquad v_S < V_{D0} \tag{3.21a}$$

$$v_O = \frac{R}{R+r_D}v_S - V_{D0}\frac{R}{R+r_D}, \qquad v_S \geq V_{D0} \tag{3.21b}$$

The transfer characteristic represented by these equations is sketched in Fig. 3.25(c). In many applications, $r_D \ll R$ and the second equation can be simplified to

$$v_O \simeq v_S - V_{D0} \tag{3.22}$$

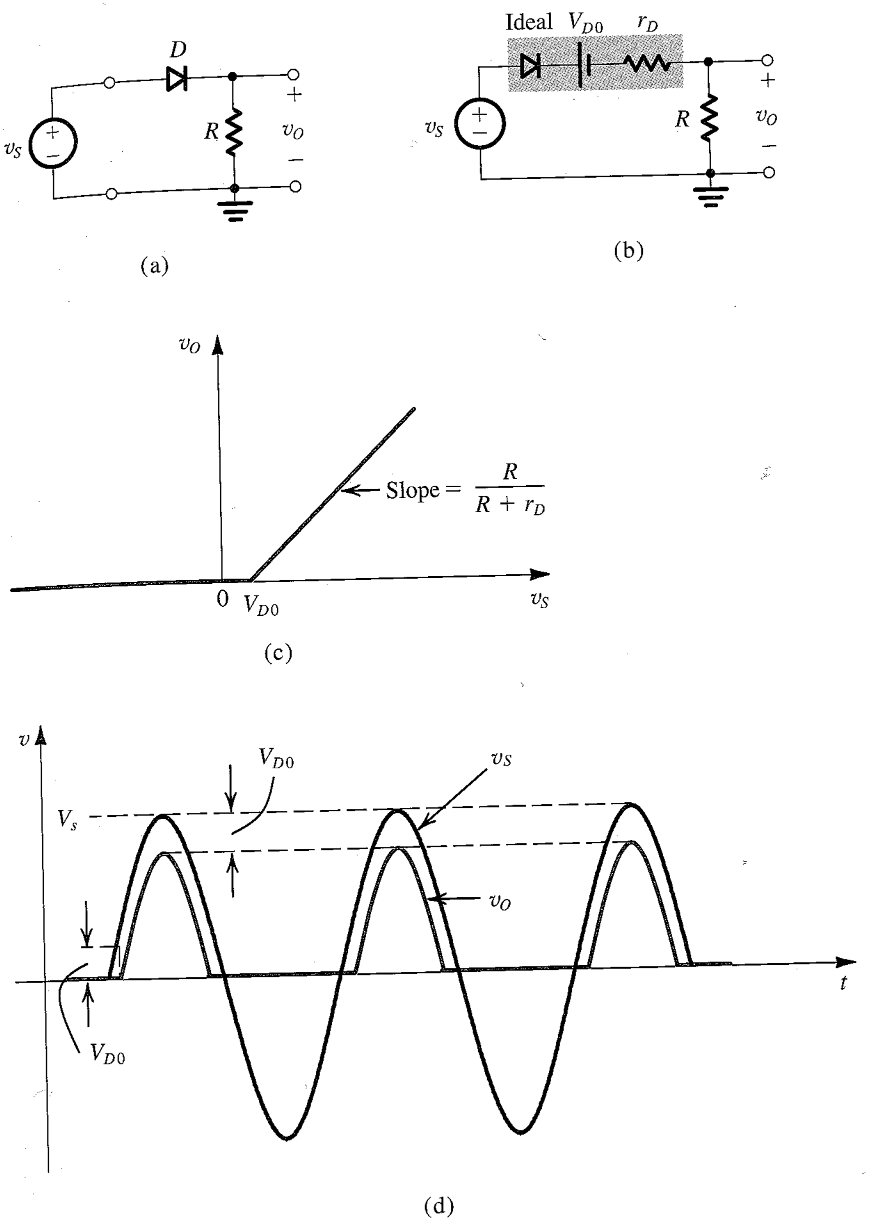


FIGURE 3.25 **(a)** Half-wave rectifier. **(b)** Equivalent circuit of the half-wave rectifier with the diode replaced with its battery-plus-resistance model. **(c)** Transfer characteristic of the rectifier circuit. **(d)** Input and output waveforms, assuming that $r_D \ll R$.

where $V_{D0} = 0.7$ V or 0.8 V. Figure 3.25(d) shows the output voltage obtained when the input v_S is a sinusoid.

In selecting diodes for rectifier design, two important parameters must be specified: the current-handling capability required of the diode, determined by the largest current the diode is expected to conduct, and the **peak inverse voltage** (PIV) that the diode must be able to withstand without breakdown, determined by the largest reverse voltage that is expected to appear across the diode. In the rectifier circuit of Fig. 3.25(a), we observe that when v_S is negative the diode will be cut off and v_O will be zero. It follows that the PIV is equal to the peak of v_S,

$$\text{PIV} = V_s$$

It is usually prudent, however, to select a diode that has a reverse breakdown voltage at least 50% greater than the expected PIV.

Before leaving the half-wave rectifier, the reader should note two points. First, it is possible to use the diode exponential characteristic to determine the exact transfer characteristic of the rectifier (see Problem 3.73). However, the amount of work involved is usually too great to be justified in practice. Of course, such an analysis can be easily done using a computer circuit-analysis program such as SPICE (see Section 3.9).

Second, whether we analyze the circuit accurately or not, it should be obvious that this circuit does not function properly when the input signal is small. For instance, this circuit cannot be used to rectify an input sinusoid of 100-mV amplitude. For such an application one resorts to a so-called precision rectifier, a circuit utilizing diodes in conjunction with op amps. One such circuit is presented in Section 3.5.5.

EXERCISE

3.20 For the half-wave rectifier circuit in Fig. 3.25(a), neglecting the effect of r_D, show the following: (a) For the half-cycles during which the diode conducts, conduction begins at an angle $\theta = \sin^{-1}(V_{D0}/V_s)$ and terminates at $(\pi - \theta)$, for a total conduction angle of $(\pi - 2\theta)$. (b) The average value (dc component) of v_O is $V_O \simeq (1/\pi)V_s - V_{D0}/2$. (c) The peak diode current is $(V_s - V_{D0})/R$.

Find numerical values for these quantities for the case of 12-V (rms) sinusoidal input, $V_{D0} \simeq 0.7$ V, and $R = 100\ \Omega$. Also, give the value for PIV.

Ans. (a) $\theta = 2.4°$, conduction angle = 175°; (b) 5.05 V; (c) 163 mA; 17 V

3.5.2 The Full-Wave Rectifier

The full-wave rectifier utilizes both halves of the input sinusoid. To provide a unipolar output, it inverts the negative halves of the sine wave. One possible implementation is shown in Fig. 3.26(a). Here the transformer secondary winding is **center-tapped** to provide two equal voltages v_S across the two halves of the secondary winding with the polarities indicated. Note that when the input line voltage (feeding the primary) is positive, both of the signals labeled v_S will be positive. In this case D_1 will conduct and D_2 will be reverse biased. The current through D_1 will flow through R and back to the center tap of the secondary. The circuit then behaves like a half-wave rectifier, and the output during the positive half cycles when D_1 conducts will be identical to that produced by the half-wave rectifier.

Now, during the negative half cycle of the ac line voltage, both of the voltages labeled v_S will be negative. Thus D_1 will be cut off while D_2 will conduct. The current conducted by D_2 will flow through R and back to the center tap. It follows that during the negative half-cycles while D_2 conducts, the circuit behaves again as a half-wave rectifier. The important point, however, is that the current through R always flows in the same direction, and thus v_O will be unipolar, as indicated in Fig. 3.26(c). The output waveform shown is obtained

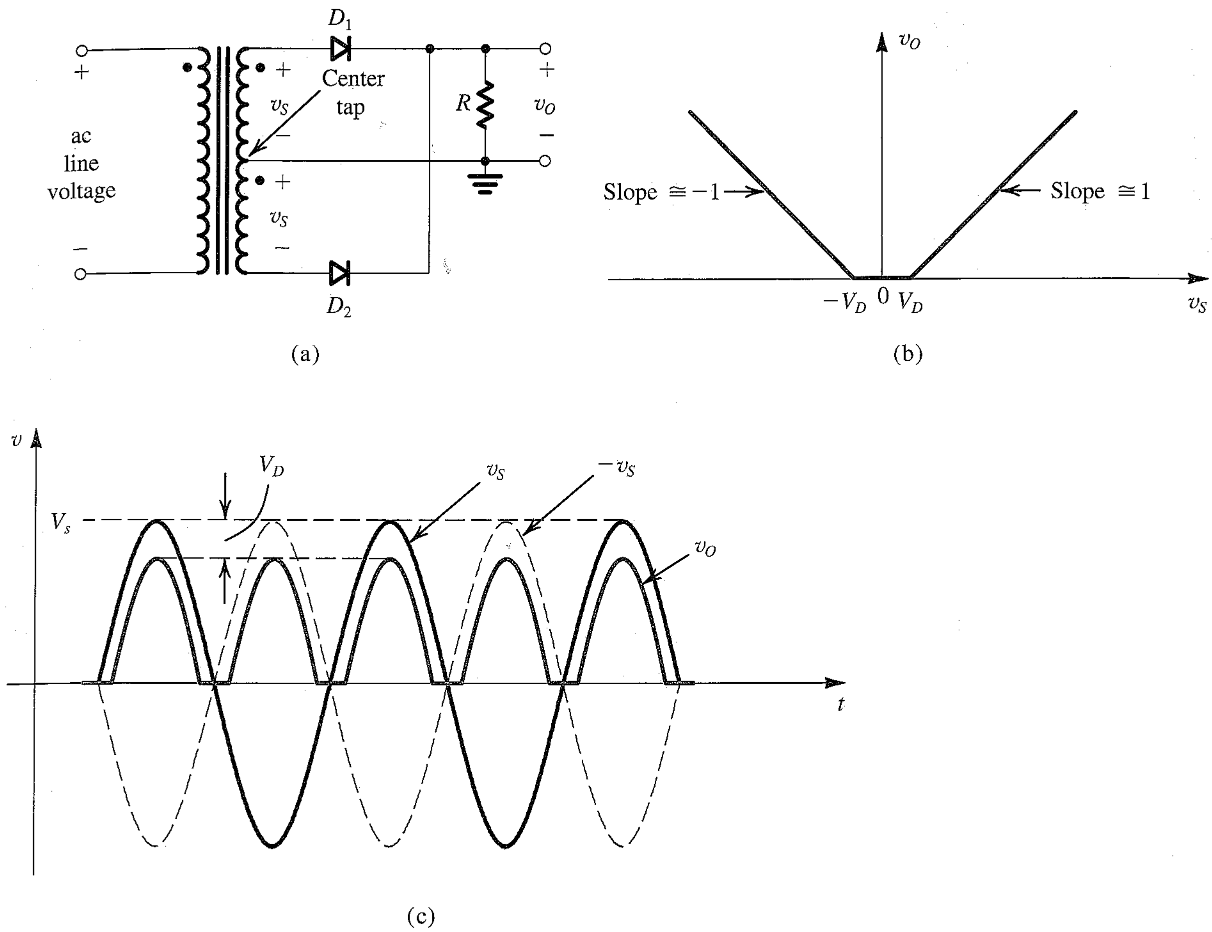


FIGURE 3.26 Full-wave rectifier utilizing a transformer with a center-tapped secondary winding: **(a)** circuit; **(b)** transfer characteristic assuming a constant-voltage-drop model for the diodes; **(c)** input and output waveforms.

by assuming that a conducting diode has a constant voltage drop V_D. Thus the transfer characteristic of the full-wave rectifier takes the shape shown in Fig. 3.26(b).

The full-wave rectifier obviously produces a more "energetic" waveform than that provided by the half-wave rectifier. In almost all rectifier applications, one opts for a full-wave type of some kind.

To find the PIV of the diodes in the full-wave rectifier circuit, consider the situation during the positive half-cycles. Diode D_1 is conducting, and D_2 is cut off. The voltage at the cathode of D_2 is v_O, and that at its anode is $-v_S$. Thus the reverse voltage across D_2 will be $(v_O + v_S)$, which will reach its maximum when v_O is at its peak value of $(V_s - V_D)$, and v_S is at its peak value of V_s; thus,

$$\text{PIV} = 2V_s - V_D$$

which is approximately twice that for the case of the half-wave rectifier.

EXERCISE

3.21 For the full-wave rectifier circuit in Fig. 3.26(a), neglecting the effect of r_D, show the following: (a) The output is zero for an angle of $2\sin^{-1}(V_D/V_s)$ centered around the zero-crossing points of the sine-wave input. (b) The average value (dc component) of v_O is $V_O \simeq (2/\pi)V_s - V_D$. (c) The peak current through each diode is $(V_s - V_D)/R$. Find the fraction (percentage) of each cycle during which $v_O > 0$, the value of V_O, the peak diode current, and the value of PIV, all for the case in which v_S is a 12-V (rms) sinusoid, $V_D \simeq 0.7$ V, and $R = 100\ \Omega$.

Ans. 97.4%; 10.1 V; 163 mA; 33.2 V

3.5.3 The Bridge Rectifier

An alternative implementation of the full-wave rectifier is shown in Fig. 3.27(a). The circuit, known as the bridge rectifier because of the similarity of its configuration to that of the Wheatstone bridge, does not require a center-tapped transformer, a distinct advantage over the full-wave rectifier circuit of Fig. 3.26. The bridge rectifier, however, requires four diodes as compared to two in the previous circuit. This is not much of a disadvantage, because diodes are inexpensive and one can buy a diode bridge in one package.

The bridge rectifier circuit operates as follows: During the positive half-cycles of the input voltage, v_S is positive, and thus current is conducted through diode D_1, resistor R, and

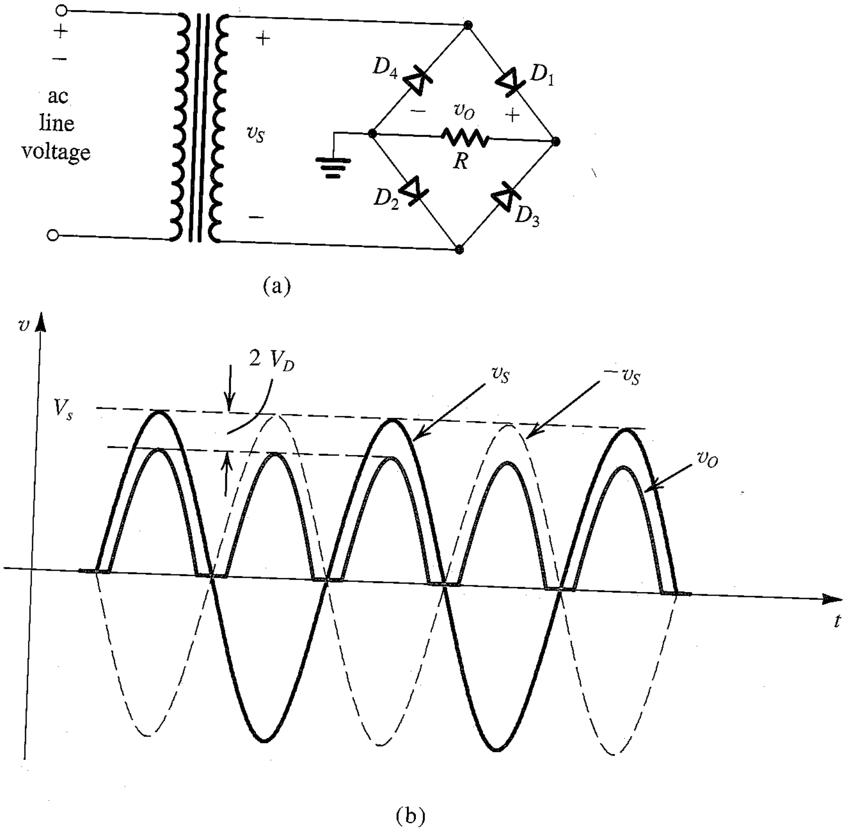


FIGURE 3.27 The bridge rectifier: **(a)** circuit; **(b)** input and output waveforms.

diode D_2. Meanwhile, diodes D_3 and D_4 will be reverse biased. Observe that there are two diodes in series in the conduction path, and thus v_O will be lower than v_S by two diode drops (compared to one drop in the circuit previously discussed). This is somewhat of a disadvantage of the bridge rectifier.

Next, consider the situation during the negative half-cycles of the input voltage. The secondary voltage v_S will be negative, and thus $-v_S$ will be positive, forcing current through D_3, R, and D_4. Meanwhile, diodes D_1 and D_2 will be reverse biased. The important point to note, though, is that during both half-cycles, current flows through R in the same direction (from right to left), and thus v_O will always be positive, as indicated in Fig. 3.27(b).

To determine the peak inverse voltage (PIV) of each diode, consider the circuit during the positive half-cycles. The reverse voltage across D_3 can be determined from the loop formed by D_3, R, and D_2 as

$$v_{D3}\text{ (reverse)} = v_O + v_{D2}\text{ (forward)}$$

Thus the maximum value of v_{D3} occurs at the peak of v_O and is given by

$$\text{PIV} = V_s - 2V_D + V_D = V_s - V_D$$

Observe that here the PIV is about half the value for the full-wave rectifier with a center-tapped transformer. This is another advantage of the bridge rectifier.

Yet one more advantage of the bridge rectifier circuit over that utilizing a center-tapped transformer is that only about half as many turns are required for the secondary winding of the transformer. Another way of looking at this point can be obtained by observing that each half of the secondary winding of the center-tapped transformer is utilized for only half the time. These advantages have made the bridge rectifier the most popular rectifier circuit configuration.

EXERCISE

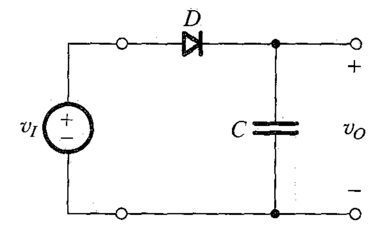
3.22 For the bridge rectifier circuit of Fig. 3.27(a), use the constant-voltage-drop diode model to show that (a) the average (or dc component) of the output voltage is $V_O \simeq (2/\pi)V_s - 2V_D$ and (b) the peak diode current is $(V_s - 2V_D)/R$. Find numerical values for the quantities in (a) and (b) and the PIV for the case in which v_S is a 12-V (rms) sinusoid, $V_D \simeq 0.7$ V, and $R = 100\ \Omega$.

Ans. 9.4 V; 156 mA; 16.3 V

3.5.4 The Rectifier with a Filter Capacitor—The Peak Rectifier

The pulsating nature of the output voltage produced by the rectifier circuits discussed above makes it unsuitable as a dc supply for electronic circuits. A simple way to reduce the variation of the output voltage is to place a capacitor across the load resistor. It will be shown that this **filter capacitor** serves to reduce substantially the variations in the rectifier output voltage.

To see how the rectifier circuit with a filter capacitor works, consider first the simple circuit shown in Fig. 3.28. Let the input v_I be a sinusoid with a peak value V_p, and assume the diode to be ideal. As v_I goes positive, the diode conducts and the capacitor is charged so that $v_O = v_I$. This situation continues until v_I reaches its peak value V_p. Beyond the peak, as v_I decreases the diode becomes reverse biased and the output voltage remains constant at the value V_p. In fact, theoretically speaking, the capacitor will retain its charge and hence its voltage indefinitely, because there is no way for the capacitor to discharge. Thus the circuit provides a dc voltage output equal to the peak of the input sine wave. This is a very encouraging result in view of our desire to produce a dc output.



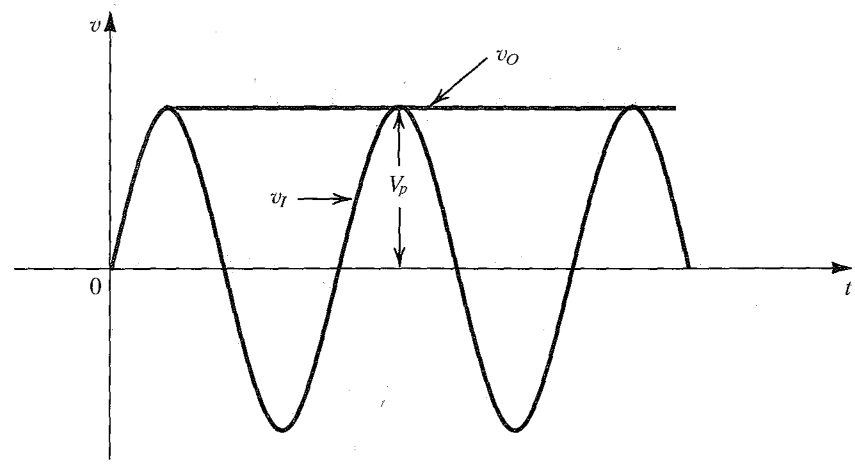


FIGURE 3.28 (a) A simple circuit used to illustrate the effect of a filter capacitor. (b) Input and output waveforms assuming an ideal diode. Note that the circuit provides a dc voltage equal to the peak of the input sine wave. The circuit is therefore known as a peak rectifier or a peak detector.

Next, we consider the more practical situation where a load resistance R is connected across the capacitor C, as depicted in Fig. 3.29(a). However, we will continue to assume the diode to be ideal. As before, for a sinusoidal input, the capacitor charges to the peak of the input V_p. Then the diode cuts off, and the capacitor discharges through the load resistance R. The capacitor discharge will continue for almost the entire cycle, until the time at which v_I exceeds the capacitor voltage. Then the diode turns on again and charges the capacitor up to the peak of v_I, and the process repeats itself. Observe that to keep the output voltage from decreasing too much during capacitor discharge, one selects a value for C so that the time constant CR is much greater than the discharge interval.

We are now ready to analyze the circuit in detail. Figure 3.29(b) shows the steady-state input and output voltage waveforms under the assumption that $CR \gg T$, where T is the period of the input sinusoid. The waveforms of the load current

$$i_L = v_O/R \tag{3.23}$$

and of the diode current (when it is conducting)

$$i_D = i_C + i_L \tag{3.24}$$

$$= C\frac{dv_I}{dt} + i_L \tag{3.25}$$

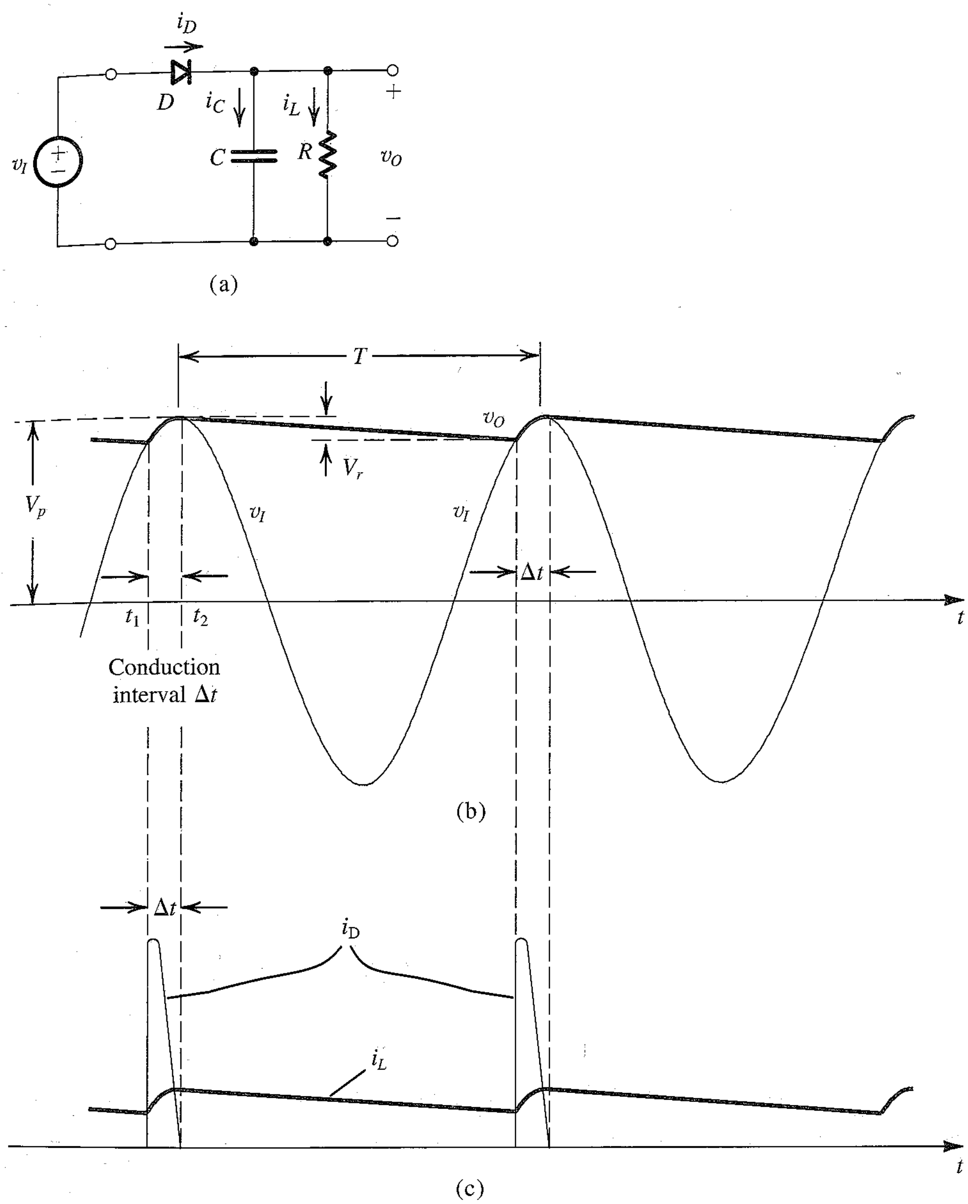


FIGURE 3.29 Voltage and current waveforms in the peak rectifier circuit with $CR \gg T$. The diode is assumed ideal.

are shown in Fig. 3.29(c). The following observations are in order:

1. The diode conducts for a brief interval, Δt, near the peak of the input sinusoid and supplies the capacitor with charge equal to that lost during the much longer discharge interval. The latter is approximately equal to the period T.
2. Assuming an ideal diode, the diode conduction begins at time t_1, at which the input v_I equals the exponentially decaying output v_O. Conduction stops at t_2 shortly after the peak of v_I; the exact value of t_2 can be determined by setting $i_D = 0$ in Eq. (3.25).

3. During the diode-off interval, the capacitor C discharges through R, and thus v_O decays exponentially with a time constant CR. The discharge interval begins just past the peak of v_I. At the end of the discharge interval, which lasts for almost the entire period T, $v_O = V_p - V_r$, where V_r is the peak-to-peak ripple voltage. When $CR \gg T$, the value of V_r is small.
4. When V_r is small, v_O is almost constant and equal to the peak value of v_I. Thus the dc output voltage is approximately equal to V_p. Similarly, the current i_L is almost constant, and its dc component I_L is given by

$$I_L = \frac{V_p}{R} \tag{3.26}$$

If desired, a more accurate expression for the output dc voltage can be obtained by taking the average of the extreme values of v_O,

$$V_O = V_p - \tfrac{1}{2}V_r \tag{3.27}$$

With these observations in hand, we now derive expressions for V_r and for the average and peak values of the diode current. During the diode-off interval, v_O can be expressed as

$$v_O = V_p e^{-t/CR}$$

At the end of the discharge interval we have

$$V_p - V_r \simeq V_p e^{-T/CR}$$

Now, since $CR \gg T$, we can use the approximation $e^{-T/CR} \simeq 1 - T/CR$ to obtain

$$V_r \simeq V_p \frac{T}{CR} \tag{3.28}$$

We observe that to keep V_r small we must select a capacitance C so that $CR \gg T$. The **ripple voltage** V_r in Eq. (3.28) can be expressed in terms of the frequency $f = 1/T$ as

$$V_r = \frac{V_p}{fCR} \tag{3.29a}$$

Using Eq. (3.26) we can express V_r by the alternate expression

$$V_r = \frac{I_L}{fC} \tag{3.29b}$$

Note that an alternative interpretation of the approximation made above is that the capacitor discharges by means of a constant current $I_L = V_p/R$. This approximation is valid as long as $V_r \ll V_p$.

Using Fig. 3.29(b) and assuming that diode conduction ceases almost at the peak of v_I, we can determine the **conduction interval** Δt from

$$V_p \cos(\omega \Delta t) = V_p - V_r$$

where $\omega = 2\pi f = 2\pi/T$ is the angular frequency of v_I. Since $(\omega \Delta t)$ is a small angle, we can employ the approximation $\cos(\omega \Delta t) \simeq 1 - \frac{1}{2}(\omega \Delta t)^2$ to obtain

$$\omega \Delta t \simeq \sqrt{2V_r/V_p} \tag{3.30}$$

We note that when $V_r \ll V_p$, the conduction angle $\omega \Delta t$ will be small, as assumed.

To determine the average diode current during conduction, i_{Dav}, we equate the charge that the diode supplies to the capacitor,

$$Q_{\text{supplied}} = i_{Cav}\,\Delta t$$

where from Eq. (3.24),

$$i_{Cav} = i_{Dav} - I_L$$

to the charge that the capacitor loses during the discharge interval,

$$Q_{\text{lost}} = CV_r$$

to obtain, using Eqs. (3.30) and (3.29a),

$$i_{Dav} = I_L(1 + \pi\sqrt{2V_p/V_r}) \tag{3.31}$$

Observe that when $V_r \ll V_p$, the average diode current during conduction is much greater than the dc load current. This is not surprising, since the diode conducts for a very short interval and must replenish the charge lost by the capacitor during the much longer interval in which it is discharged by I_L.

The peak value of the diode current, i_{Dmax}, can be determined by evaluating the expression in Eq. (3.25) at the onset of diode conduction—that is, at $t = t_1 = -\Delta t$ (where $t = 0$ is at the peak). Assuming that i_L is almost constant at the value given by Eq. (3.26), we obtain

$$i_{Dmax} = I_L(1 + 2\pi\sqrt{2V_p/V_r}) \tag{3.32}$$

From Eqs. (3.31) and (3.32), we see that for $V_r \ll V_p$, $i_{Dmax} \simeq 2i_{Dav}$, which correlates with the fact that the waveform of i_D is almost a right-angle triangle (see Fig. 3.29c).

EXAMPLE 3.9

Consider a peak rectifier fed by a 60-Hz sinusoid having a peak value $V_p = 100$ V. Let the load resistance $R = 10$ kΩ. Find the value of the capacitance C that will result in a peak-to-peak ripple of 2 V. Also, calculate the fraction of the cycle during which the diode is conducting and the average and peak values of the diode current.

Solution

From Eq. (3.29a) we obtain the value of C as

$$C = \frac{V_p}{V_r f R} = \frac{100}{2 \times 60 \times 10 \times 10^3} = 83.3\ \mu\text{F}$$

The conduction angle $\omega\,\Delta t$ is found from Eq. (3.30) as

$$\omega\Delta t = \sqrt{2 \times 2/100} = 0.2 \text{ rad}$$

Thus the diode conducts for $(0.2/2\pi) \times 100 = 3.18\%$ of the cycle. The average diode current is obtained from Eq. (3.31), where $I_L = 100/10 = 10$ mA, as

$$i_{Dav} = 10(1 + \pi\sqrt{2 \times 100/2}) = 324 \text{ mA}$$

The peak diode current is found using Eq. (3.32),

$$i_{Dmax} = 10(1 + 2\pi\sqrt{2 \times 100/2}) = 638 \text{ mA}$$

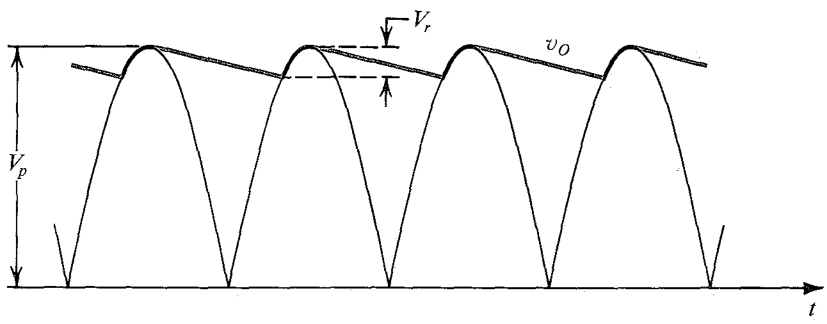


FIGURE 3.30 Waveforms in the full-wave peak rectifier.

The circuit of Fig. 3.29(a) is known as a half-wave **peak rectifier.** The full-wave rectifier circuits of Figs. 3.26(a) and 3.27(a) can be converted to peak rectifiers by including a capacitor across the load resistor. As in the half-wave case, the output dc voltage will be almost equal to the peak value of the input sine wave (Fig. 3.30). The ripple frequency, however, will be twice that of the input. The peak-to-peak ripple voltage, for this case, can be derived using a procedure identical to that above but with the discharge period T replaced by $T/2$, resulting in

$$V_r = \frac{V_p}{2fCR} \tag{3.33}$$

While the diode conduction interval, Δt, will still be given by Eq. (3.30), the average and peak currents in each of the diodes will be given by

$$i_{D\text{av}} = I_L(1 + \pi\sqrt{V_p/2V_r}) \tag{3.34}$$

$$i_{D\max} = I_L(1 + 2\pi\sqrt{V_p/2V_r}) \tag{3.35}$$

Comparing these expressions with the corresponding ones for the half-wave case, we note that for the same values of V_p, f, R, and V_r (and thus the same I_L), we need a capacitor half the size of that required in the half-wave rectifier. Also, the current in each diode in the full-wave rectifier is approximately half that which flows in the diode of the half-wave circuit.

The analysis above assumed ideal diodes. The accuracy of the results can be improved by taking the diode voltage drop into account. This can be easily done by replacing the peak voltage V_p to which the capacitor charges with $(V_p - V_D)$ for the half-wave circuit and the full-wave circuit using a center-tapped transformer and with $(V_p - 2V_D)$ for the bridge-rectifier case.

We conclude this section by noting that peak-rectifier circuits find application in signal-processing systems where it is required to detect the peak of an input signal. In such a case, the circuit is referred to as a **peak detector.** A particularly popular application of the peak detector is in the design of a demodulator for amplitude-modulated (AM) signals. We shall not discuss this application further here.

EXERCISES

3.23 Derive the expressions in Eqs. (3.33), (3.34), and (3.35).

D3.24 Consider a bridge-rectifier circuit with a filter capacitor C placed across the load resistor R for the case in which the transformer secondary delivers a sinusoid of 12 V (rms) having a 60-Hz frequency and assuming $V_D = 0.8$ V and a load resistance $R = 100\ \Omega$. Find the value of C that results in a ripple voltage no larger than 1 V peak-to-peak. What is the dc voltage at the output? Find the load current. Find the diodes' conduction angle. What is the average diode current? What is the peak reverse voltage across each diode? Specify the diode in terms of its peak current and its PIV.

Ans. 1281 μF; 15.4 V or (a better estimate) 14.9 V; 0.15 A; 0.36 rad (20.7°); 1.45 A; 2.74 A; 16.2 V. Thus select a diode with 3.5 A to 4 A peak current and a 20-V PIV rating.

3.5.5 Precision Half-Wave Rectifier—The Super Diode[4]

The rectifier circuits studied thus far suffer from having one or two diode drops in the signal paths. Thus these circuits work well only when the signal to be rectified is much larger than the voltage drop of a conducting diode (0.7 V or so). In such a case the details of the diode forward characteristics or the exact value of the diode voltage do not play a prominent role in determining circuit performance. This is indeed the case in the application of rectifier circuits in power-supply design. There are other applications, however, where the signal to be rectified is small (e.g., on the order of 100 mV or so) and thus clearly insufficient to turn on a diode. Also, in instrumentation applications, the need arises for rectifier circuits with very precise and predictable transfer characteristics. For these applications, a class of circuits has been developed utilizing op amps (Chapter 2) together with diodes to provide precision rectification. In the following discussion, we study one such circuit, leaving a more comprehensive study of op amp–diode circuits to Chapter 13.

Figure 3.31(a) shows a precision half-wave rectifier circuit consisting of a diode placed in the negative-feedback path of an op amp, with R being the rectifier load resistance. The op amp, of course, needs power supplies for its operation. For simplicity, these are not shown in the circuit diagram. The circuit works as follows: If v_I goes positive, the output voltage v_A of the op amp will go positive and the diode will conduct, thus establishing a closed feedback path between the op amp's output terminal and the negative input terminal.

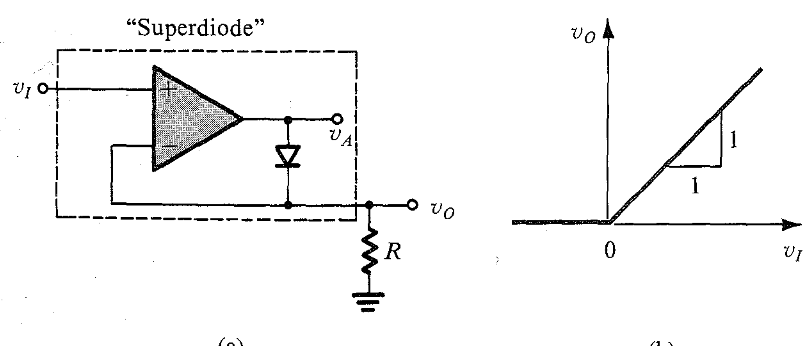


FIGURE 3.31 The "superdiode" precision half-wave rectifier and its almost-ideal transfer characteristic. Note that when $v_I > 0$ and the diode conducts, the op amp supplies the load current, and the source is conveniently buffered, an added advantage. Not shown are the op-amp power supplies.

[4] This section requires knowledge of operational amplifiers.

This negative-feedback path will cause a virtual short circuit to appear between the two input terminals. Thus the voltage at the negative input terminal, which is also the output voltage v_O, will equal (to within a few millivolts) that at the positive input terminal, which is the input voltage v_I,

$$v_O = v_I \qquad v_I \geq 0$$

Note that the offset voltage ($\simeq$0.6 V) exhibited in the simple half-wave rectifier circuit of Fig. 3.25 is no longer present. For the op-amp circuit to start operation, v_I has to exceed only a negligibly small voltage equal to the diode drop divided by the op amp's open-loop gain. In other words, the straight-line transfer characteristic v_O–v_I almost passes through the origin. This makes this circuit suitable for applications involving very small signals.

Consider now the case when v_I goes negative. The op amp's output voltage v_A will tend to follow and go negative. This will reverse-bias the diode, and no current will flow through resistance R, causing v_O to remain equal to 0 V. Thus, for $v_I < 0$, $v_O = 0$. Since in this case the diode is off, the op amp will be operating in an open-loop fashion, and its output will be at the negative saturation level.

The transfer characteristic of this circuit will be that shown in Fig. 3.31(b), which is almost identical to the ideal characteristic of a half-wave rectifier. The nonideal diode characteristics have been almost completely masked by placing the diode in the negative-feedback path of an op amp. This is another dramatic application of negative feedback, a subject we will study formally in Chapter 8. The combination of diode and op amp, shown in the dotted box in Fig. 3.31(a), is appropriately referred to as a "superdiode."

EXERCISES

3.25 Consider the operational rectifier or superdiode circuit of Fig. 3.31(a), with $R = 1\ \text{k}\Omega$. For $v_I = 10$ mV, 1 V, and −1 V, what are the voltages that result at the rectifier output and at the output of the op amp? Assume that the op amp is ideal and that its output saturates at ±12 V. The diode has a 0.7-V drop at 1-mA current, and the voltage drop changes by 0.1 V per decade of current change.

Ans. 10 mV, 0.51 V; 1 V, 1.7 V; 0 V, −12 V

3.26 If the diode in the circuit of Fig. 3.31(a) is reversed, find the transfer characteristic v_O as a function of v_I.

Ans. $v_O = 0$ for $v_I \geq 0$; $v_O = v_I$ for $v_I \leq 0$

3.6 LIMITING AND CLAMPING CIRCUITS

In this section, we shall present additional nonlinear circuit applications of diodes.

3.6.1 Limiter Circuits

Figure 3.32 shows the general transfer characteristic of a limiter circuit. As indicated, for inputs in a certain range, $L_-/K \leq v_I \leq L_+/K$, the limiter acts as a linear circuit, providing an output proportional to the input, $v_O = Kv_I$. Although in general K can be greater than 1, the circuits discussed in this section have $K \leq 1$ and are known as passive limiters. (Examples of active limiters will be presented in Chapter 13.) If v_I exceeds the upper *threshold* (L_+/K), the output voltage is *limited* or clamped to the upper limiting level L_+. On the other hand, if v_I is reduced below the lower limiting threshold (L_-/K), the output voltage v_O is limited to the lower limiting level L_-.

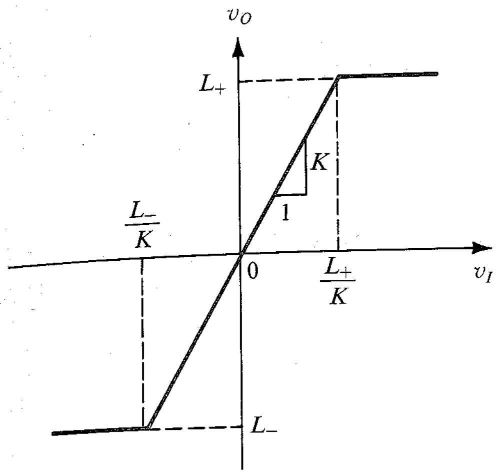


FIGURE 3.32 General transfer characteristic for a limiter circuit.

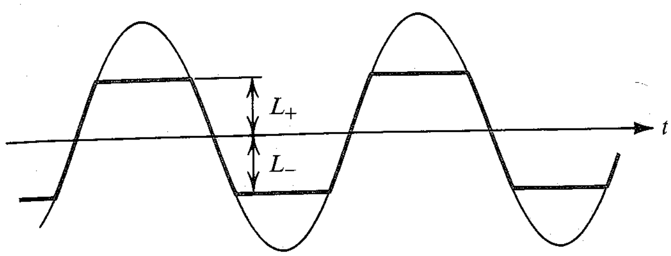


FIGURE 3.33 Applying a sine wave to a limiter can result in clipping off its two peaks.

The general transfer characteristic of Fig. 3.32 describes a **double limiter**—that is, a limiter that works on both the positive and negative peaks of an input waveform. **Single limiters,** of course, exist. Finally, note that if an input waveform such as that shown in Fig. 3.33 is fed to a double limiter, its two peaks will be *clipped off.* Limiters therefore are sometimes referred to as **clippers.**

The limiter whose characteristics are depicted in Fig. 3.32 is described as a **hard limiter. Soft limiting** is characterized by smoother transitions between the linear region and the saturation regions and a slope greater than zero in the saturation regions, as illustrated in Fig. 3.34. Depending on the application, either hard or soft limiting may be preferred.

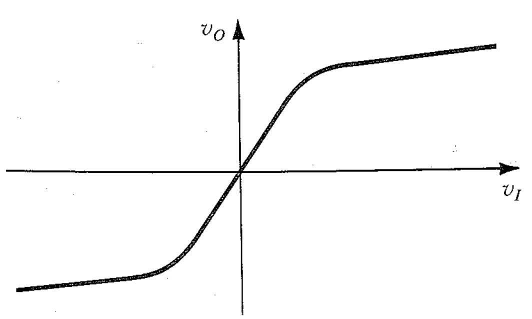


FIGURE 3.34 Soft limiting.

Limiters find application in a variety of signal-processing systems. One of their simplest applications is in limiting the voltage between the two input terminals of an op amp to a value lower than the breakdown voltage of the transistors that make up the input stage of the op-amp circuit. We will have more to say on this and other limiter applications at later points in this book.

Diodes can be combined with resistors to provide simple realizations of the limiter function. A number of examples are depicted in Fig. 3.35. In each part of the figure both the circuit and its transfer characteristic are given. The transfer characteristics are obtained using the constant-voltage-drop ($V_D = 0.7$ V) diode model but assuming a smooth transition between the linear and saturation regions of the transfer characteristic. Better approximations for the transfer characteristics can be obtained using the piecewise-linear diode model. If this is done, the saturation region of the characteristic acquires a slight slope (due to the effect of r_D).

The circuit in Fig. 3.35(a) is that of the half-wave rectifier except that here the output is taken across the diode. For $v_I < 0.5$ V, the diode is cut off, no current flows, and the voltage

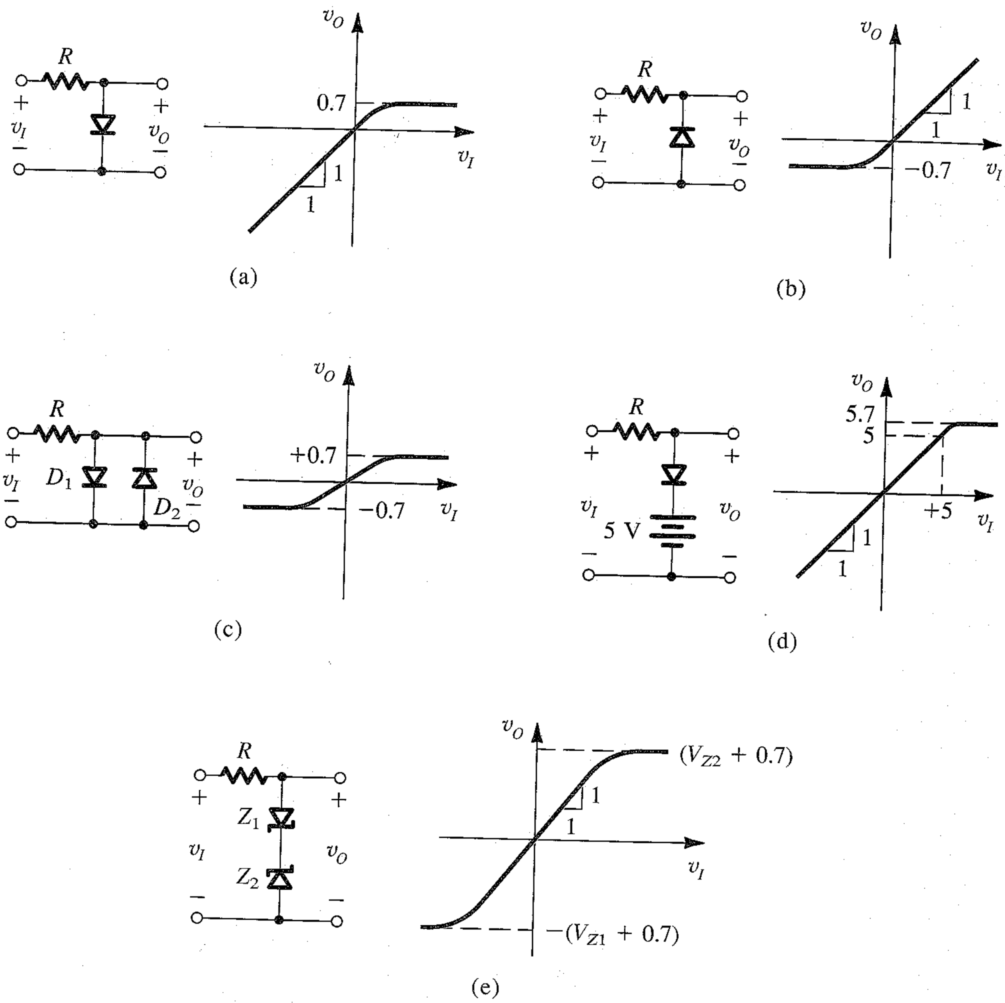


FIGURE 3.35 A variety of basic limiting circuits.

drop across R is zero; thus $v_O = v_I$. As v_I exceeds 0.5 V, the diode turns on, eventually limiting v_O to one diode drop (0.7 V). The circuit of Fig. 3.35(b) is similar to that in Fig. 3.35(a) except that the diode is reversed.

Double limiting can be implemented by placing two diodes of opposite polarity in parallel, as shown in Fig. 3.35(c). Here the linear region of the characteristic is obtained for $-0.5\text{ V} \le v_I \le 0.5\text{ V}$. For this range of v_I, both diodes are off and $v_O = v_I$. As v_I exceeds 0.5 V, D_1 turns on and eventually limits v_O to +0.7 V. Similarly, as v_I goes more negative than −0.5 V, D_2 turns on and eventually limits v_O to −0.7 V.

The thresholds and saturation levels of diode limiters can be controlled by using strings of diodes and/or by connecting a dc voltage in series with the diode(s). The latter idea is illustrated in Fig. 3.35(d). Finally, rather than strings of diodes, we may use two zener diodes in series, as shown in Fig. 3.35(e). In this circuit, limiting occurs in the positive direction at a voltage of $V_{Z2} + 0.7$, where 0.7 V represents the voltage drop across zener diode Z_1 when conducting in the *forward* direction. For negative inputs, Z_1 acts as a zener, while Z_2 conducts in the forward direction. It should be mentioned that pairs of zener diodes connected in series are available commercially for applications of this type under the name **double-anode zener.**

More flexible limiter circuits are possible if op amps are combined with diodes and resistors. Examples of such circuits are discussed in Chapter 13.

EXERCISE

3.27 Assuming the diodes to be ideal, describe the transfer characteristic of the circuit shown in Fig. E3.27.

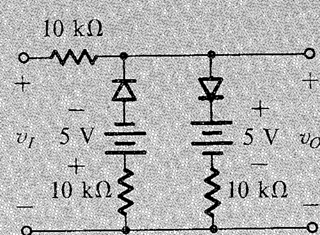


FIGURE E3.27

Ans. $v_O = v_I$ for $-5 \le v_I \le +5$

$v_O = \frac{1}{2}v_I - 2.5$ for $v_I \le -5$

$v_O = \frac{1}{2}v_I + 2.5$ for $v_I \ge +5$

3.6.2 The Clamped Capacitor or DC Restorer

If in the basic peak-rectifier circuit the output is taken across the diode rather than across the capacitor, an interesting circuit with important applications results. The circuit, called a dc restorer, is shown in Fig. 3.36 fed with a square wave. Because of the polarity in which the diode is connected, the capacitor will charge to a voltage v_C with the polarity indicated in Fig. 3.36 and equal to the magnitude of the most negative peak of the input signal. Subsequently, the diode turns off and the capacitor retains its voltage indefinitely. If, for instance, the input square wave has the arbitrary levels −6 V and +4 V, then v_C will be equal to 6 V.

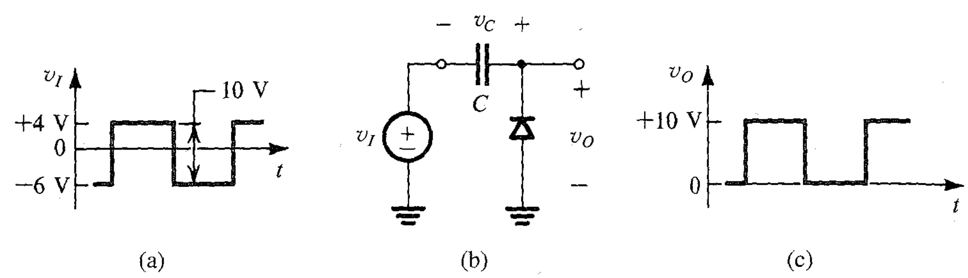


FIGURE 3.36 The clamped capacitor or dc restorer with a square-wave input and no load.

Now, since the output voltage v_O is given by

$$v_O = v_I + v_C$$

it follows that the output waveform will be identical to that of the input, except that it is shifted upward by v_C volts. In our example the output will thus be a square wave with levels of 0 V and +10 V.

Another way of visualizing the operation of the circuit in Fig. 3.36 is to note that because the diode is connected across the output with the polarity shown, it prevents the output voltage from going below 0 V (by conducting and charging up the capacitor, thus causing the output to rise to 0 V), but this connection will not constrain the positive excursion of v_O. The output waveform will therefore have its lowest peak *clamped* to 0 V, which is why the circuit is called a **clamped capacitor.** It should be obvious that reversing the diode polarity will provide an output waveform whose highest peak is clamped to 0 V. In either case, the output waveform will have a finite average value or dc component. This dc component is entirely unrelated to the average value of the input waveform. As an application, consider a pulse signal being transmitted through a capacitively coupled or ac-coupled system. The capacitive coupling will cause the pulse train to lose whatever dc component it originally had. Feeding the resulting pulse waveform to a clamping circuit provides it with a well-determined dc component, a process known as **dc restoration.** This is why the circuit is also called a **dc restorer.**

Restoring dc is useful because the dc component or average value of a pulse waveform is an effective measure of its duty cycle.[5] The duty cycle of a pulse waveform can be modulated (in a process called pulsewidth modulation) and made to carry information. In such a system, detection or demodulation could be achieved simply by feeding the received pulse waveform to a dc restorer and then using a simple *RC* low-pass filter to separate the average of the output waveform from the superimposed pulses.

When a load resistance R is connected across the diode in a clamping circuit, as shown in Fig. 3.37, the situation changes significantly. While the output is above ground, a net dc current must flow in R. Since at this time the diode is off, this current obviously comes from the capacitor, thus causing the capacitor to discharge and the output voltage to fall. This is shown in Fig. 3.37 for a square-wave input. During the interval t_0 to t_1, the output voltage falls exponentially with time constant CR. At t_1 the input decreases by V_a volts, and the output attempts to follow. This causes the diode to conduct heavily and to quickly charge the capacitor. At the end of the interval t_1 to t_2, the output voltage would normally be a few tenths of a volt negative (e.g., −0.5 V). Then, as the input rises by V_a volts (at t_2), the output follows, and the cycle repeats itself. In the steady state the charge lost by the capacitor during

[5] The duty cycle of a pulse waveform is the proportion of each cycle occupied by the pulse. In other words, it is the pulse width expressed as a fraction of the pulse period.

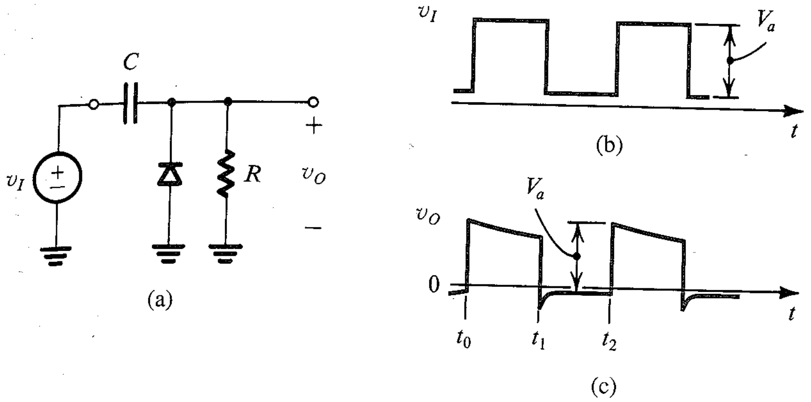


FIGURE 3.37 The clamped capacitor with a load resistance R.

the interval t_0 to t_1 is recovered during the interval t_1 to t_2. This charge equilibrium enables us to calculate the average diode current as well as the details of the output waveform.

3.6.3 The Voltage Doubler

Figure 3.38(a) shows a circuit composed of two sections in cascade: a clamp formed by C_1 and D_1, and a peak rectifier formed by D_2 and C_2. When excited by a sinusoid of amplitude V_p the clamping section provides the voltage waveform shown, assuming ideal diodes, in Fig. 3.38(b). Note that while the positive peaks are clamped to 0 V, the negative peak

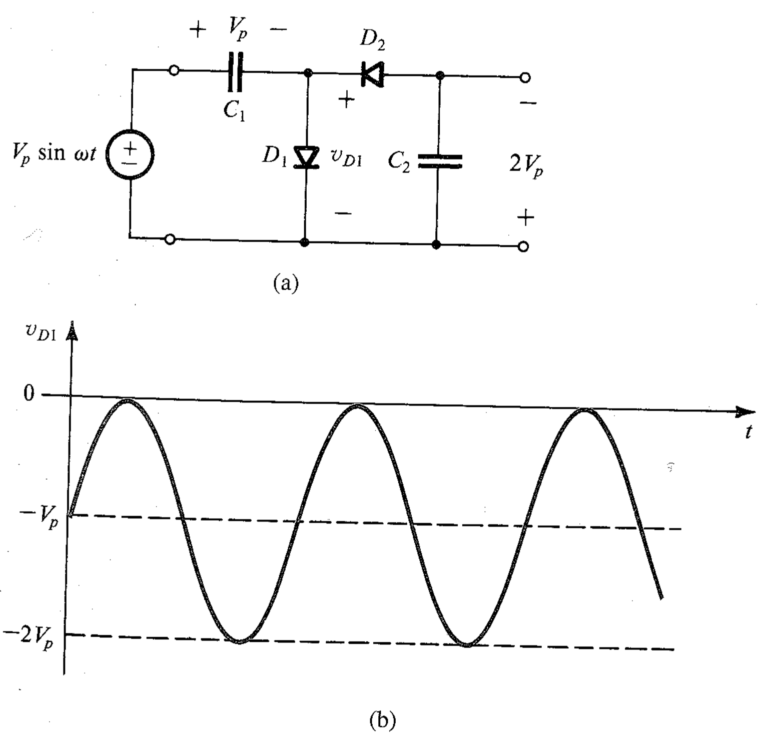


FIGURE 3.38 Voltage doubler: **(a)** circuit; **(b)** waveform of the voltage across D_1.

reaches $-2V_p$. In response to this waveform, the peak-detector section provides across capacitor C_2 a negative dc voltage of magnitude $2V_p$. Because the output voltage is double the input peak, the circuit is known as a voltage doubler. The technique can be extended to provide output dc voltages that are higher multiples of V_p.

EXERCISE

3.28 If the diode in the circuit of Fig. 3.36 is reversed, what will the dc component of v_O become?
Ans. −5 V

3.7 PHYSICAL OPERATION OF DIODES

Having studied the terminal characteristics and circuit applications of junction diodes, we will now briefly consider the physical processes that give rise to the observed terminal characteristics. The following treatment of device physics is somewhat simplified; nevertheless, it should provide sufficient background for a fuller understanding of diodes and for understanding the operation of transistors in the following two chapters.

3.7.1 Basic Semiconductor Concepts

The *pn* Junction The semiconductor diode is basically a *pn* junction, as shown schematically in Fig. 3.39. As indicated, the *pn* junction consists of *p*-type semiconductor material (e.g., silicon) brought into close contact with *n*-type semiconductor material (also silicon). In actual practice, both the *p* and *n* regions are part of the same silicon crystal; that is, the *pn* junction is formed within a single silicon crystal by creating regions of different "dopings" (*p* and *n* regions). Appendix A provides a brief description of the process employed in the fabrication of *pn* junctions. As indicated in Fig. 3.39, external wire connections to the *p* and *n* regions (i.e., diode terminals) are made through metal (aluminum) contacts.

In addition to being essentially a diode, the *pn* junction is the basic element of bipolar junction transistors (BJTs) and plays an important role in the operation of field-effect transistors (FETs). Thus an understanding of the physical operation of *pn* junctions is important to the understanding of the operation and terminal characteristics both of diodes and transistors.

Intrinsic Silicon Although either silicon or germanium can be used to manufacture semiconductor devices—indeed, earlier diodes and transistors were made of germanium—today's

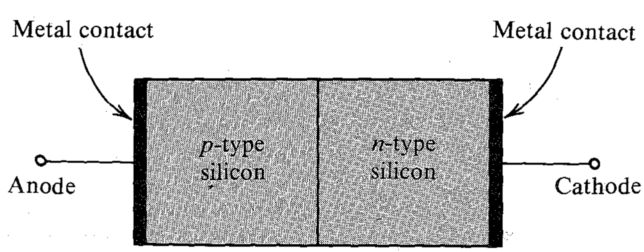


FIGURE 3.39 Simplified physical structure of the junction diode. (Actual geometries are given in Appendix A.)