

Fourier transform

- Mathematical technique that transform a time domain signal to frequency.
- Fourier series only work for periodic signals but Fourier transform breaks a signal into sum of sine and cosine waves by converting it to frequency domain.
- Formula: $F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$, $t \rightarrow$ time, $\omega \rightarrow$ angular frequency, $F(\omega) \rightarrow$ frequency domain representation

Example:

Fourier transform of $\cos(t)$

$\cos(t) \rightarrow$ frequency of 1 Hz

$$F(\omega) = \int_{-\infty}^{\infty} \cos t e^{-j\omega t} dt$$

$$F(\omega) = \int_{-\infty}^{\infty} (e^{jt} + e^{-jt}) e^{-j\omega t} dt$$

$$= \frac{1}{2} \int_{-\infty}^{\infty} e^{jt-j\omega t} + e^{-jt-j\omega t} dt$$

$$= \frac{1}{2} \int_{-\infty}^{\infty} e^{-j t(\omega-1)} + e^{-j t(\omega+1)} dt$$

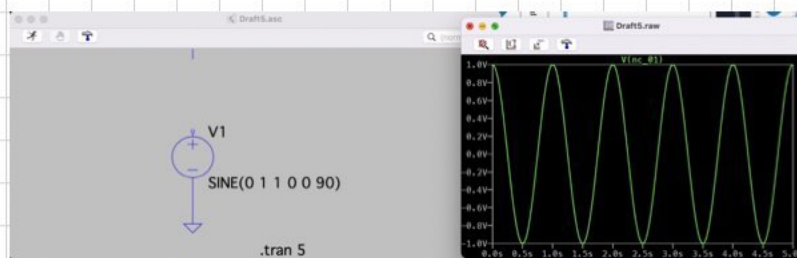
Property of Dirac delta function

$$\int_{-\infty}^{\infty} e^{jat} dt = 2\pi \delta(a)$$

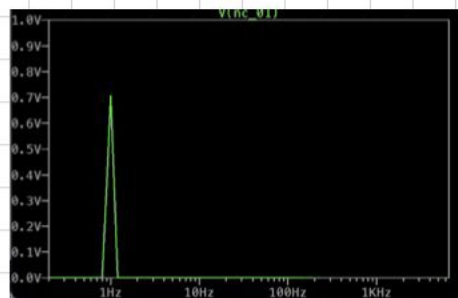
$$= \frac{1}{2} [2\pi \delta(\omega-1) + 2\pi \delta(\omega+1)]$$

$$= \pi \delta(\omega-1) + \pi \delta(\omega+1)$$

Simulating a cosin wave on LTSPICE Using a voltage source



FastFourier transform of the signal in linear co-ordinates



Since $\cos(t)$ is a function with its frequency being 1. A peak at 1 Hz can be found on the displayed graph of Fourier transform. Moreover $\delta(\omega-1)$ corresponds to a peak at 1 Hz.

If noise was now added to the cosine signal multiple small peaks would be observed.

Steps to remove noise

Fourier transform



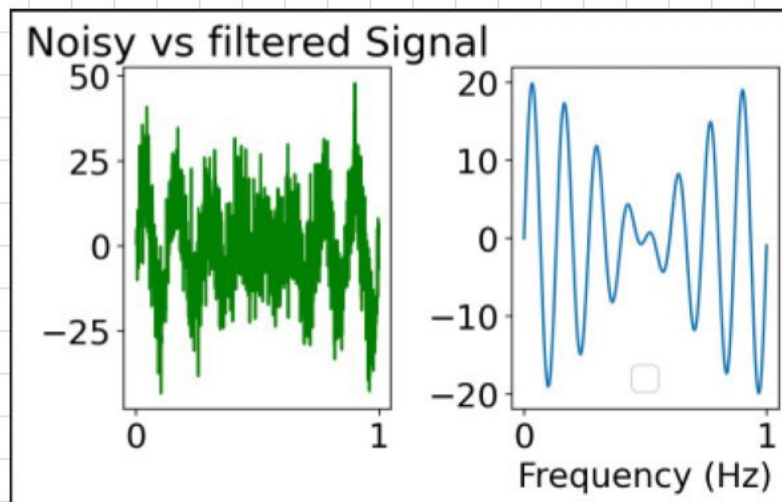
If magnitude of peak $<$ threshold * max-peak magnitude then the frequencies get attenuated



Inverse Fourier transform and get the filtered signal

Removing noise from a cosine signal in python

- Creating a time array.
- Create a cosine signal $x(t)$.
- Add noise to the signal by adding each element of the array to a scaled random number.
- Fourier transform the array with additional noise.
- Attenuate all frequencies that are below the threshold.
- Inverse Fourier transform to return back to Fourier transform.



Noisy vs filtered signal from python code.

Removing noise from an audio signal Using python

Convert the audio signal
into an array of
values.



Fourier transform the
signal array



Remove unwanted frequencies



Inverse Fourier transform to
convert back to time domain



Plot the array vs time and
convert the array into audio
and download.