Final Project - EEC254

Floor Planning via Convex Optimization

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In the project we would like to follow the lead of [3, 2, 4] where the floor planning problem was formulated as geometric programming problem. By doing that the global optimum can be found using standard convex optimization techniques. However, these previous work only considered the basic constraints i.e., the objects area constraints, width constraints, height constraints and aspect ration constraints. In this project we would like to also consider more constraints and try to inject these new constraints into the geometric programming formulation of the problem. Additional constraints we might consider includes the following

- Minimum spacing between the objects,
- Alignment constraints where we impose that two edges or objects centers to be aligned,
- Symmetry constraints about either axes of the bounding box,
- Similarity constraints where one object is a scaled version of another object, and
- Distance constraints where we impose constraints to limit the distance between pairs of objects.

Deliverable: Upon the complication of this project, we wish to deliver the following:

- Formulation of the basic floorplanning problem as a convex optimization problem
- Adding new constraints to the above formulation and re-formulating as convex optimization
- Providing a solution for the floorplanning with additional constraints based on the aforementioned formulation
- Providing few numerical experiments using MATLAB and CVX package

Our guarantee that the addition of new constraints will not disturb the convexity of the problems stems from Boyd's book [1] where it is mentioned that additional constraints can be introduced while maintaining the convexity of the problem. However, this was not explicitly detailed in the book.

1 Abstract

2 Introduction

Floor planning problem tries to find the optimal position and/or dimensions of geometric objects (commonly rectangles) within a space such that there is no overlap between them. We refer to the objects as *cells* which are axis-aligned. There could be some constraints on the cells e.g., area-constraints, aspect ratio constraints, and positions constraints. The objective is usually to minimize the size (e.g., area, volume, perimeter) of the bounding rectangle for fixed cells or maximize the size of the cells for a fixed bounding rectangle. Motivation.

3 System Model

We consider N axis-aligned cells $(c_1, \cdots, c_N)$ in 2D each specified by its lower-left corner (x_i, y_i), height h_i, and width w_i. The cells are configured inside a bounding rectangle (BR) of height H, width W, and the origin as its lower-left corner. The cells are always desired to not overlap. This constraint makes the general floor-planning a complicated combinatorial optimization problem. However, if the relative positioning of the cells is specified, the floor-planning problem can be cast as convex optimization.

Variables

The variables of the optimization problem are the position and size of the cells (x_i, y_i, h_i, w_i for $i = 1, \cdots, N$) and the size of the bounding rectangle (H, W).

Objectives

There are two different objectives that the problem of floor-planning may try to achieve either of them

1. Minimizing the are of the bounding rectangle of fixed-size cells. Here we consider minimizing the perimeter of the bounding rectangle since the area is a non-convex function and its log is concave function.

2. Maximizing the area of the cells for a fixed-size bounding rectangle. We take log are of the cell i.e., $max(log(w_i) + log(h_i))$ for $i = 1, \cdots, N$. This is a multi-criterion problem with N objective function which can be scalarized by take the sum of all objective function, thus maximizing $\sum_{i=1}^{N}(log(w_i) + log(h_i))$.

Constraints

Some or all of the following constraints maybe considered in the floor-planning problem

- non-overlapping constraints: or $\textbf{int}(c_i \cap c_j) = \phi$, $i \neq j$. We achieve this by imposing relative position instead using two binary relations on the indices $\{1, \cdots, N\}$: $\mathcal{L}$ and $\mathcal{U}$ means *left* *of* and *under* respectively. Two relations are sufficient because the left/right and under/above are pairs of anti-symmetric, transitive relations. For example, if $(i, j) \in \mathcal{L}$, then c_i is left of c_j and thus $x_i + w_i \leq x_j$. To ensure full description of all relative positions, we require for each (i, j) with $i \neq j$, one of the following holds

$$(i, j) \in \mathcal{L} \qquad (j, i) \in \mathcal{L} \qquad (i, j) \in \mathcal{U} \qquad (j, i) \in \mathcal{U}$$

This will give $N(N-1)/2$ inequalities. Using the transitivity of the relation, the number of inequalities can be greatly reduced.

- minimum spacing constraints: To avoid tight packing, imposing a minimum psotive spacing ρ is possible. This changes the interpretation of $(i, j) \in \mathcal{L}$ to mean $x_i + w_i + \rho \leq x_j$.
- bounding constraints: $x_i \geq 0, y_i \geq 0, x_i + w_i \leq W, y_i + h_i \leq H$ for $i = 1, \cdots, N$.
- minimum cell area constraints: formulated as $log(w_i) + log(h_i) \geq log(A_i^{min})$ or $sqrt(h_i w_i) \geq A_i^{min}$ where A_i^{min} is cell i minimum area constraint.
- aspect-ratio constraints: aspect-ratio is the ratio of height over width. The constraint is $\omega_i^{min} \cdot w_i \leq h_i \leq \omega_i^{max} \cdot w_i$ where ω_i^{min} and ω_i^{max} is the minimum and maximum aspect ratio constraints for the cell i.
- alignment constraints: used for aligning two edges of two cells together. For example, the horizontal center of cell i aligns with the top of the cell j when $y_i + \frac{w_i}{2} = y_j + w_j$.
- symmetry constraints: cells maybe required to be symmetric about a vertical or horizontal axis that can be fixed or floating. For example, to specify that cells i and j to be symmetric about vertical axis x_{axis}, the constraint is $x_{axis} - (x_i + \frac{w_i}{2}) = x_j + \frac{w_j}{2} - x_{axis}$.
- similarity constraints: cell i maybe required to be α-scaled version of cell j for fixed α i.e., $w_i = \alpha w_j$, $h_i = \alpha h_j$.
- containment constraints: A cell maybe required to contain a certain point (x_p, y_p) inside the bounding rectangle such that $0 \leq x_p - x_i \leq w_i$ and $0 \leq y_p - y_i \leq h_i$. It maybe also required that the block to be inside a given polyhedron.

4 Problem Formulation

Here we formulate the primal and dual optimization problems of the floor planning. We will examine the basic floor-planning in which we are trying to minimize the area of the bounding rectangle with no-overlapping and minimum area constraints only.

The relative positions relations $\mathcal{L}$ and $\mathcal{U}$ can be placed by matrices L and U such that

$$L_{ij} = \begin{cases} 1 & (i, j) \in \mathcal{L} \\ 0 & \text{otherwise} \end{cases} \qquad U_{ij} = \begin{cases} 1 & (i, j) \in \mathcal{U} \\ 0 & \text{otherwise} \end{cases}$$

5 Results

6 Conclusion

References

[1] Boyd, S. and Vandenberghe, L. (2004). *Convex optimization*. Cambridge university press.

[2] Chen, T. and Fan, M. K. (1998). On convex formulation of the floorplan area minimization problem. In *Proceedings of the 1998 international symposium on Physical design*, pages 124–128. ACM.

[3] Moh, T.-S., Chang, T.-S., and Hakimi, S. L. (1996). Globally optimal floorplanning for a layout problem. *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, 43(9):713–720.

[4] Young, F. Y., Chu, C. C., Luk, W., and Wong, Y. (2000). Floorplan area minimization using lagrangian relaxation. In *Proceedings of the 2000 international symposium on Physical design*, pages 174–179. ACM.