Problem 2.9

Adding $1^{\underline{st}}$, $2^{\underline{nd}}$, and $4^{\underline{th}}$ equations $\Rightarrow W = 1$

Adding $2^{\underline{nd}}$ and $3^{\underline{rd}}$ equations $\Rightarrow Y = 1$

1^{st} equation $\Rightarrow X = 1$

$2^{\underline{nd}}$ equation $\Rightarrow Z = 0$.

Summary: $X = 1$, $Y = 1$, $Z = 0$, $W = 1$.

Problem 3.1

$$G = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$H = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 \end{pmatrix}$$

There are no 3 or less columns in H that add up to zero.

There are 4 columns in H that add up to zero.

Hence, $d_{min} = 4$.

Problem 3.9

The code has 16 codewords obtained by considering all linear combinations of the rows of G. The codewords and their weights are given by

codeword	weight	codeword	weight
00000000	0	10110001	4
01111000	4	11001001	4
11100100	4	01010101	4
10011100	4	00101101	4
11010010	4	01100011	4
10101010	4	00011011	4
00110110	4	10000111	4
01001110	4	11111111	8

Hence, the weight distribution of the code is given by:

$$A_0 = 1, \quad A_4 = 14, \quad A_8 = 1,$$

$$A_1 = A_2 = A_3 = A_5 = A_6 = A_7 = 0$$

The probability of an undetected error is

$$\sum_{i=1}^{n} A_i\, p^i (1-p)^{n-i} = 14\,(0.01)^4 (0.99)^4 + (0.01)^8$$

$$= 1.34 \times 10^{-7}$$

Problem 3.12

Syndromes	Correctable Error Patterns
0 0 0 0	0 0 0 0 0 0 0 0
1 0 0 0	1 0 0 0 0 0 0 0
0 1 0 0	0 1 0 0 0 0 0 0
0 0 1 0	0 0 1 0 0 0 0 0
0 0 0 1	0 0 0 1 0 0 0 0
0 1 1 1	0 0 0 0 1 0 0 0
1 1 1 0	0 0 0 0 0 1 0 0
1 1 0 1	0 0 0 0 0 0 1 0
1 0 1 1	0 0 0 0 0 0 0 1
1 1 0 0	1 1 0 0 0 0 0 0
1 0 1 0	1 0 1 0 0 0 0 0
0 1 1 0	0 1 1 0 0 0 0 0
1 0 0 1	1 0 0 1 0 0 0 0
0 1 0 1	0 1 0 1 0 0 0 0
0 0 1 1	0 0 1 1 0 0 0 0
1 1 1 1	1 0 0 0 1 0 0 0