- Suppose we have filled out all the rows up to and including the $\mu$-th row, we fill out the $(\mu + 1)$-th row as follows:

(1) If $d_\mu = 0$, then we set

$$\sigma^{(\mu+1)}(X) = \sigma^{(\mu)}(X).$$

(2) If $d_\mu \neq 0$, we find a row prior to the $\mu$-th row, say the $\rho$-th row, with partial solution $\sigma^{(\rho)}(X)$ such that $l_{\rho+1} > l_\rho$ and $\rho$ is maximal. ~~$d_\rho \neq 0$ and $2\rho - l_\rho$ is the largest~~. Then

$$\sigma^{(\mu+1)}(X) = \sigma^{(\mu)}(X) + d_\mu d_\rho^{-1} X^{2(\mu-\rho)} \sigma^{(\rho)}(X). \tag{5.37}$$

(3) Compute the discrepancy

$$d_{\mu+1} = S_{2\mu+3} + \sigma_1^{(\mu+1)} S_{2\mu+2} + \sigma_2^{(\mu+1)} S_{2\mu+1} + \cdots + \sigma_{l_{\mu+1}}^{(\mu+1)} S_{2\mu+3-l_{\mu+1}}. \tag{5.38}$$

where $l_{\mu+1}$ is degree of $\sigma^{(\mu+1)}(X)$.

- If $d_\mu \neq 0$, then $\sigma^{(\mu)}(X)$ needs to be adjusted to obtain a new minimum-degree polynomial $\sigma^{(\mu+1)}(X)$ whose coefficients satisfy the first $\mu + 1$ Newton's identities.

- **Correction**: Go back to the steps prior the $\mu$-th step and determine a step $\rho$ at which the partial solution is $\sigma^{(\rho)}(X)$ such that $l_{\rho+1} > l_\rho$ and $\rho$ is maximal, ~~$d_\rho \neq 0$ and $\rho - l_\rho$ has the largest value,~~ where $l_\rho$ is the degree of $\sigma^{(\rho)}(X)$. Then the solution at the $(\mu+1)$-th step of the iteration process for finding the error-location polynomial $\sigma(X)$ is

$$\sigma^{(\mu+1)}(X) = \sigma^{(\mu)}(X) + d_\mu d_\rho^{-1} X^{(\mu-\rho)} \sigma^{(\rho)}(X), \tag{5.32}$$

where

$$d_\mu d_\rho^{-1} X^{(\mu-\rho)} \sigma^{(\rho)}(X) \tag{5.33}$$

is the correction term with degree $\mu - (\rho - l_{\rho})$.

- ~~Since $\rho$ is chosen to maximize $\rho - l_\mu$, this choice of $\rho$ is equivalent to minimize the degree $\mu - (\rho - l_\rho)$ of the correction term.~~

- **Correction**: Go back to the steps prior to the $\mu$-th step and determine a polynomial $\sigma^{(\rho)}(X)$ such that $l_{\rho+1} > l_\rho$ and $\rho$ is maximum ~~$d_\rho \neq 0$ and $\rho - l_\rho$ has the largest value~~, where $l_\rho$ is the degree of $\sigma^{(\rho)}(X)$. Then

$$\sigma^{(\mu+1)}(X) = \sigma^{(\mu)}(X) - d_\mu d_\rho^{-1} X^{(\mu-\rho)} \sigma^{(\rho)}(X) \tag{6.19}$$

is the solution at the $(\mu + 1)$-th step of the iteration process.

- Continue the above iterative process until $2t$ steps have been completed. At the $2t$-th step, we have

$$\sigma(X) = \sigma^{(2t)}(X), \tag{6.20}$$

which is the minimum-degree polynomial whose coefficients satisfy the $2t - \nu$ generalized Newton's identities given by (6.13).

- If $\nu \leq t$ (the designed error-correcting capability), $\sigma^{(2t)}$ is unique and the true error-location polynomial with all its roots in GF($q^m$).

ρ is the maximum integer less than μ such that $l_{\rho+1} > l_\rho$.



TABLE 7.2: Steps for finding the error-location polynomial of the (15,9) RS code over $GF(2^4)$.

| $\mu$ | $\sigma^{(\mu)}(X)$ | $d_\mu$ | $l_\mu$ | $\mu - l_\mu$ |
|---|---|---|---|---|
| $-1$ | $1$ | $1$ | $0$ | $-1$ |
| $0$ | $1$ | $\alpha^{12}$ | $0$ | $0$ |
| $1$ | $1+\alpha^{12}X$ | $\alpha^7$ | $1$ | $0$(take $\rho = -1$) |
| $2$ | $1+\alpha^3X$ | $1$ | $1$ | $1$(take $\rho = 0$) |
| $3$ | $1+\alpha^3X+\alpha^3X^2$ | $\alpha^7$ | $2$ | $1$(take $\rho = 0$) |
| $4$ | $1+\alpha^4X+\alpha^{12}X^2$ | $\alpha^{10}$ | $2$ | $2$(take $\rho = 2$) |
| $1+\alpha^4X+\alpha^3X^2+\alpha^{13}X^3$ $5$ | $1+\alpha^7X+\alpha^4X^2+\alpha^6X^3$ | $\cancel{0}\,\alpha^{13}$ | $3$ | $2$(take $\rho = 3$) |
| $6$ | $1+\alpha^7X+\alpha^4X^2+\alpha^6X^3$ | — | — | — |

-1
0
6
2
2
4

**Step 2.** To find the error-location polynomial $\sigma(X)$, we fill out Table 7.1 and obtain Table 7.2. Thus, $\sigma(X) = 1+\alpha^7X+\alpha^4X^2+\alpha^6X^3$.

**Step 3.** By substituting $1, \alpha, \alpha^2, \cdots, \alpha^{14}$ into $\sigma(X)$, we find that $\alpha^3, \alpha^9$, and $\alpha^{12}$ are roots of $\sigma(X)$. The reciprocals of these roots are $\alpha^{12}, \alpha^6$, and $\alpha^3$, which are the error-location numbers of the error pattern $\mathbf{e}(X)$. Thus, errors occur at positions $X^3$, $X^6$, and $X^{12}$.

---

Next, we need to determine the error values, by finding the error-value evaluator. We define the syndrome polynomial $\mathbf{S}(X)$ as follows:

$$\begin{aligned}\mathbf{S}(X) &\triangleq S_1 + S_2X + \cdots + S_{2t}X^{2t-1} + S_{2t+1}X^{2t} + \cdots \\ &= \sum_{j=1}^{\infty} S_jX^{j-1}.\end{aligned} \tag{7.21}$$

Note that only the coefficients of the first $2t$ terms are known. For $1 \le j < \infty$, we also define

$$S_j = \sum_{l=1}^{\nu} \delta_l \beta_l^j. \tag{7.22}$$

The first $2t$ such $S_j$'s are simply the $2t$ equalities of (7.13). Combining (7.21) and (7.22), we can put $S(X)$ in the following form:

$$\begin{aligned}\mathbf{S}(X) &= \sum_{j=1}^{\infty} X^{j-1} \sum_{l=1}^{\nu} \delta_l \beta_l^j \\ &= \sum_{l=1}^{\nu} \delta_l \beta_l \sum_{j=1}^{\infty} (\beta_l X)^{j-1}.\end{aligned} \tag{7.23}$$

Note that

$$\frac{1}{(1-\beta_lX)} = \sum_{j=1}^{\infty} (\beta_lX)^{j-1}. \tag{7.24}$$

$l_\rho$ is the

(7.20)

ction of

lements

ch.

nerator

$\alpha^3 000$

e 2.8):