UNIVERSITY OF CALIFORNIA, DAVIS
Error Correcting Codes I

PROBLEM SET 3

Reading Assignment: Lecture Notes: 4. Textbook: Chapter 5.

Solve problems by hand, i.e., do not use symbolic and/or numerical mathematics software package to solve the problems. However, you can use them, if you want, to check your answers.

Problem 5.1. Consider the $(15, 11)$ cyclic Hamming code generated by $\mathbf{g}(X) = 1 + X + X^4$.

a. Determine the parity-polynomial $\mathbf{h}(X)$ of this code. N.B. The parity polynomial is given by (4.5) in Lecture Notes 4.

b. Determine the generator polynomial of its dual code.

c. Find the generator and parity matrices in systematic forms for this code.

Problem 5.16 Make a table that gives the number of cyclic codes of length 15 and dimension k for each value of $k = 0, 1, \ldots, 15$. You do not need to construct the codes. (Hint: Using the fact that $X^{15} + 1$ has all the nonzero elements of $\mathrm{GF}(2^4)$ as roots and using Table 2.9 in textbook, factor $X^{15} + 1$ as a product of irreducible polynomials.)

Problem 5.x The $(15, 7)$ code generated by $g(X) = 1 + X + X^2 + X^4 + X^8$ is a double error correcting code.

1. Construct a table showing the syndromes of the error patterns $0, 1, 1 + X, 1 + X^2, \ldots, 1 + X^{14}$.
2. Decode the received word $1 + X + X^3 + X^6 + X^{10} + X^{11}$.
3. Decode the received word $X + X^2 + X^4 + X^7 + X^8 + X^9 + X^{11}$.

Problem 5.y

1. Show that if $g(X)$ generates a cyclic code of odd minimum distance d, then $(X + 1)g(X)$ generates a cyclic code of minimum distance at least $d + 1$.
2. The polynomial $g(X) = 1 + X + X^3$ generates a cyclic Hamming code of length 7. Find its dimension and minimum distance.
3. Find the dimension and minimum distance of the code of length 7 generated by the polynomial $(X + 1)g(X)$ where $g(X) = 1 + X + X^3$.