**2.9** Solve the following simultaneous equations of $X, Y, Z$, and $W$ with modulo-2 arithmetic:

$$\begin{aligned} X + Y \quad\quad + W &= 1, \\ X \quad\quad + Z + W &= 0, \\ X + Y + Z + W &= 1, \\ Y + Z + W &= 0. \end{aligned}$$

**3.1** Consider a systematic (8, 4) code whose parity-check equations are

$$\begin{aligned} v_0 &= u_1 + u_2 + u_3, \\ v_1 &= u_0 + u_1 + u_2, \\ v_2 &= u_0 + u_1 + u_3, \\ v_3 &= u_0 + u_2 + u_3. \end{aligned}$$

where $u_0, u_1, u_2$, and $u_3$, are message digits, and $v_0, v_1, v_2$, and $v_3$ are parity-check digits. Find the generator and parity-check matrices for this code. Show analytically that the minimum distance of this code is 4.

**3.9** Determine the weight distribution of the (8, 4) linear code given in Problem 3.1. Let the transition probability of a BSC be $p = 10^{-2}$. Compute the probability of an undetected error of this code.

**3.12** The (8, 4) linear code given in Problem 3.1 is capable of correcting 16 error patterns (the coset leaders of a standard array). Suppose that this code is used for a BSC. Devise a decoder for this code based on the table-lookup decoding scheme. The decoder is designed to correct the 16 most probable error patterns.