Example The $(15,7)$ code generated by $g(x) = 1 + X^4 + X^6 + X^7 + X^8$ is double error-correcting. Decode $r(X) = X^2 + X^6 + X^{11} + X^{12}$.

The following is a table of syndromes for the error patterns $0, 1, 1+X, 1+X^2, \ldots, 1+X^{14}$.

error pattern	syndrome
0	0
1	1
$1+X$	$1+X$
$1+X^2$	$1+X^2$
$1+X^3$	$1+X^3$
$1+X^4$	$1+X^4$
$1+X^5$	$1+X^5$
$1+X^6$	$1+X^6$
$1+X^7$	$1+X^7$
$1+X^8$	$X^4+X^6+X^7$
$1+X^9$	$X+X^4+X^5+X^6$
$1+X^{10}$	$1+X+X^2+X^5+X^6+X^7$
$1+X^{11}$	$X^2+X^3+X^4$
$1+X^{12}$	$1+X+X^3+X^4+X^5$
$1+X^{13}$	$1+X^2+X^4+X^5+X^6$
$1+X^{14}$	$1+X^3+X^5+X^6+X^7$

$r(x) = X^2 + X^6 + X^{11} + X^{12}$	$s(X) = 1 + X + X^5 + X^6$
$r^{(1)}(X) = Xr(X) \pmod{X^{15}+1}$	$s^{(1)}(X) = Xs(X) \pmod{g(X)} = X + X^2 + X^6 + X^7$
$r^{(2)}(X) = Xr^{(1)}(X) \pmod{X^{15}+1}$	$s^{(2)}(X) = Xs^{(1)}(X) \pmod{g(X)} = 1 + X^2 + X^3 + X^4 + X^6$
$r^{(3)}(X) = Xr^{(2)}(X) \pmod{X^{15}+1}$	$s^{(3)}(X) = Xs^{(2)}(X) \pmod{g(X)} = X + X^3 + X^4 + X^5 + X^7$
$r^{(4)}(X) = Xr^{(3)}(X) \pmod{X^{15}+1}$	$s^{(4)}(X) = Xs^{(3)}(X) \pmod{g(X)} = 1 + X^2 + X^5 + X^7$
$r^{(5)}(X) = Xr^{(4)}(X) \pmod{X^{15}+1}$	$s^{(5)}(X) = Xs^{(4)}(X) \pmod{g(X)} = 1 + X + X^3 + X^4 + X^7$
$r^{(6)}(X) = Xr^{(5)}(X) \pmod{X^{15}+1}$	$s^{(6)}(X) = Xs^{(5)}(X) \pmod{g(X)} = 1 + X + X^2 + X^5 + X^6 + X^7$

Hence, $e^{(6)}(X) = 1 + X^{10}$. Since $e^{(6)}(X) = X^6 e(X) \pmod{X^{15}+1}$,

$$e(X) = X^9 e^{(6)}(X) = X^9(1 + X^{10}) = X^4 + X^9 \pmod{X^{15}+1}.$$

The transmitted codeword is $v(X) = r(X) + e(X) = X^2 + X^4 + X^6 + X^9 + X^{11} + X^{12}$.