2.9 Solve the following simultaneous equations of X, Y, Z, and W with modulo-2 arithmetic:

$$\begin{aligned} X + Y \quad\quad + W &= 1, \\ X \quad\quad + Z + W &= 0, \\ X + Y + Z + W &= 1, \\ Y + Z + W &= 0. \end{aligned}$$

3.1 Consider a systematic (8, 4) code whose parity-check equations are

$$\begin{aligned} v_0 &= u_1 + u_2 + u_3, \\ v_1 &= u_0 + u_1 + u_2, \\ v_2 &= u_0 + u_1 + u_3, \\ v_3 &= u_0 + u_2 + u_3. \end{aligned}$$

where u_0, u_1, u_2, and u_3, are message digits, and v_0, v_1, v_2, and v_3 are parity-check digits. Find the generator and parity-check matrices for this code. Show analytically that the minimum distance of this code is 4.

3.9 Determine the weight distribution of the (8, 4) linear code given in Problem 3.1. Let the transition probability of a BSC be $p = 10^{-2}$. Compute the probability of an undetected error of this code.

3.12 The (8, 4) linear code given in Problem 3.1 is capable of correcting 16 error patterns (the coset leaders of a standard array). Suppose that this code is used for a BSC. Devise a decoder for this code based on the table-lookup decoding scheme. The decoder is designed to correct the 16 most probable error patterns.