Lecture 12:

Structured Grids (Stencils)

Modern Parallel Computing

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EEC 289Q, UC Davis, Winter 2018

Credits

- Original lecture: Kerry Seitz, with material from:
  - Tobias Brandvik (Whittle Laboratory, University of Cambridge)
    - "Stencil operations in CUDA."
    - http://www.nvidia.com/content/PDF/isc-2011/Brandvik.pdf
  - Samuel Williams (Lawrence Berkeley National Laboratory)
    - "HPGMG-FV"
  - Jonathan Ragan-Kelley (MIT CSAIL; now at Berkeley)
    - "Decoupling Algorithms from the Organization of Computation for High-Performance Imaging"

What is a stencil?

- Class of iterative algorithms that update array elements according to some fixed pattern (stencil)

- Special case of a gather
  - Gather addresses are fixed offsets relative to the output location

- Used in simulations, solving PDEs, image processing, and cellular automata

- One of the "dwarfs" that we talked about several classes ago (structured grid)

What is a stencil?

- Code performs a sequence of sweeps (timesteps) through an array (usually 2D or 3D)

- In each timestep, all cells are updated using neighboring elements in a fixed pattern (stencil – shape is application-pecific)

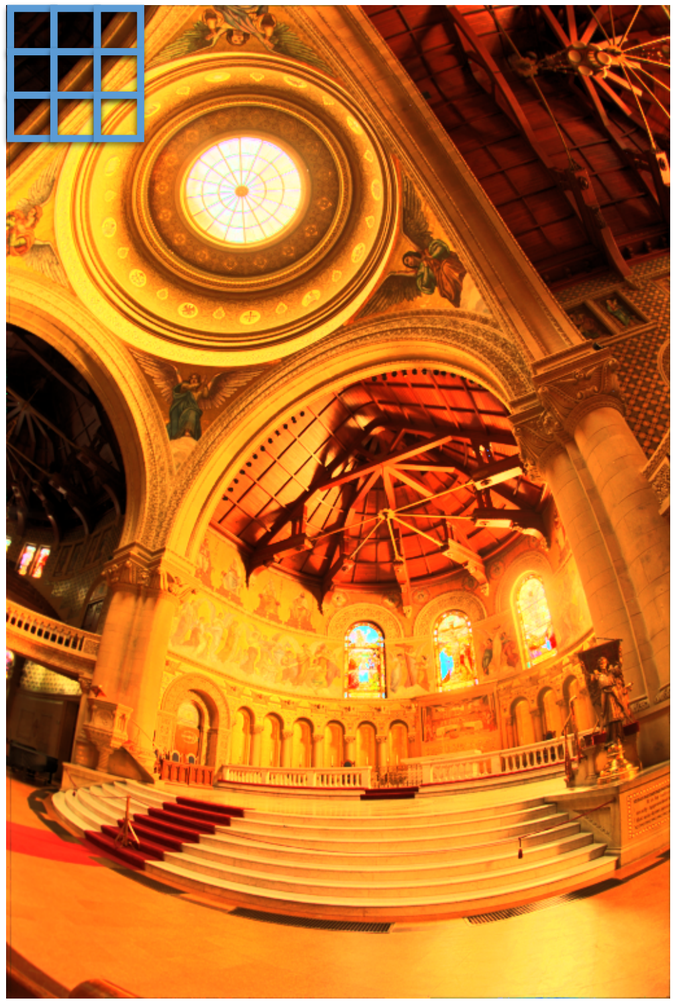
- Each timestep uses data from the previous timestep(s) only
  - Great for parallel processing

- Data access pattern is repeated

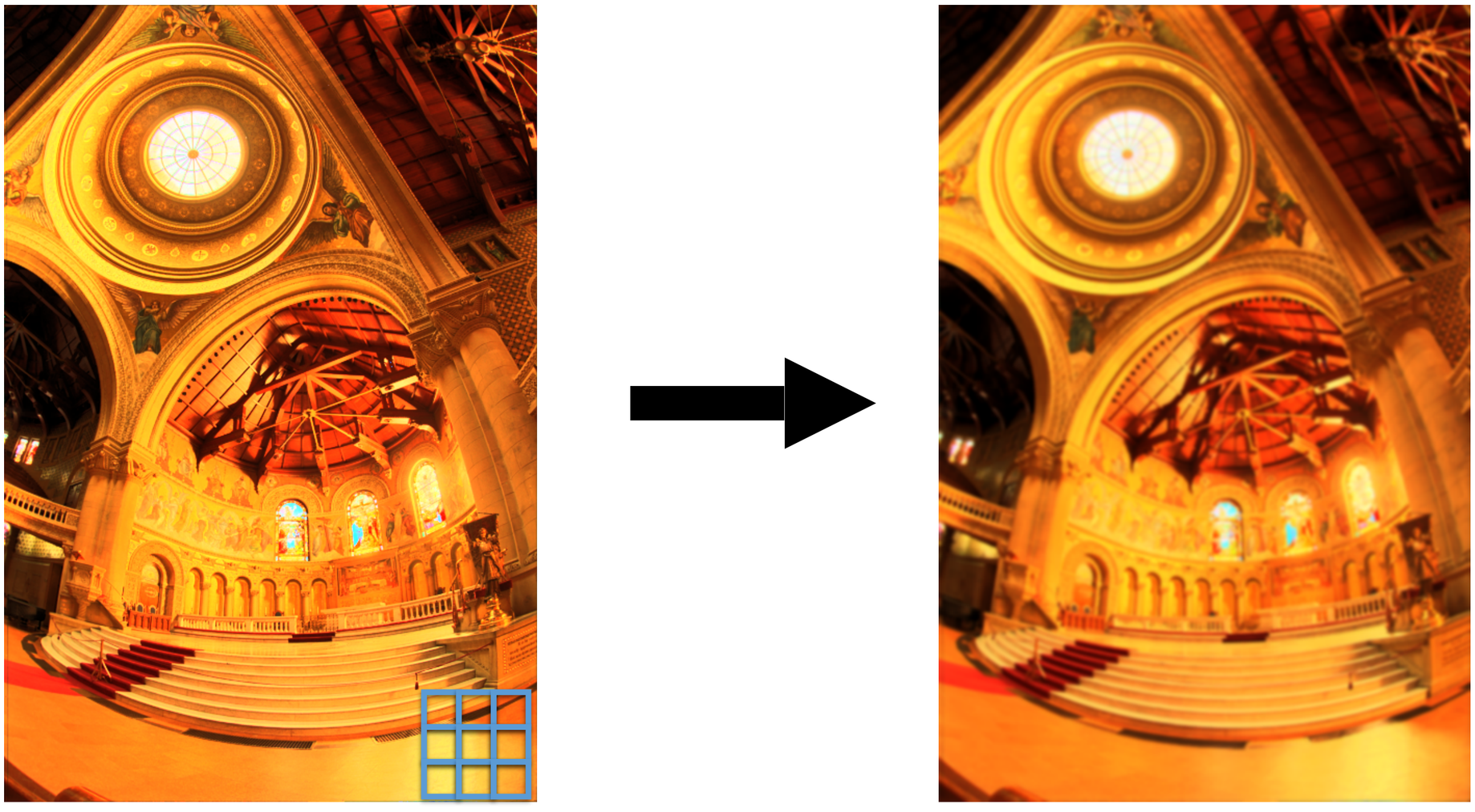
Stencil operations in one slide

- Parallelize over input or output?
  - Output
- What does each thread do?
  - Read all inputs in stencil
  - Do a math operation on those inputs
  - (Optionally) use some weights in that computation
    - Weights can be stored locally since they are reused, e.g., shared memory
- Likely limited by memory bandwidth
- Keys: efficient memory access; exploit locality

Box Blur (3x3)



Box Blur (3x3)



Box Blur (3x3)

```
void blur(Image &in, Image &blurred) {

  for(int y=0; y < in.height; ++y)
    for(int x=0; x < in.width; ++x)
      blurred(x,y) = (in(x-1, y-1)  + in(x, y-1)  + in(x+1, y-1)  +
                      in(x-1, y)    + in(x, y)    + in(x+1, y)    +
                      in(x-1, y+1) + in(x, y+1) + in(x+1, y+1)    ) / 9;
}
```

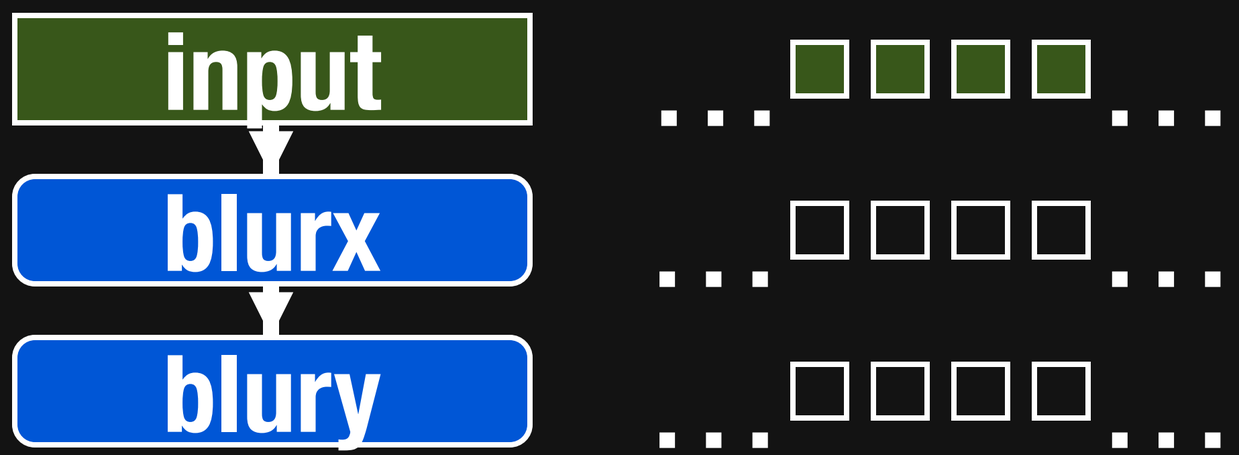
Separable Box Blur

```
void blur(Image &in, Image &blurred) {
  Image tmp(in.width(), in.height);

  for(int y=0; y < in.height; ++y)
    for(int x=0; x < in.width; ++x)
      tmp(x,y) = (in(x-1, y) + in(x, y) + in(x+1, y)) / 3;

  for(int y=0; y < in.height; ++y)
    for(int x=0; x < in.width; ++x)
      blurred(x,y) = (tmp(x, y-1) + tmp(x, y) + tmp(x, y+1)) / 3;
}
```

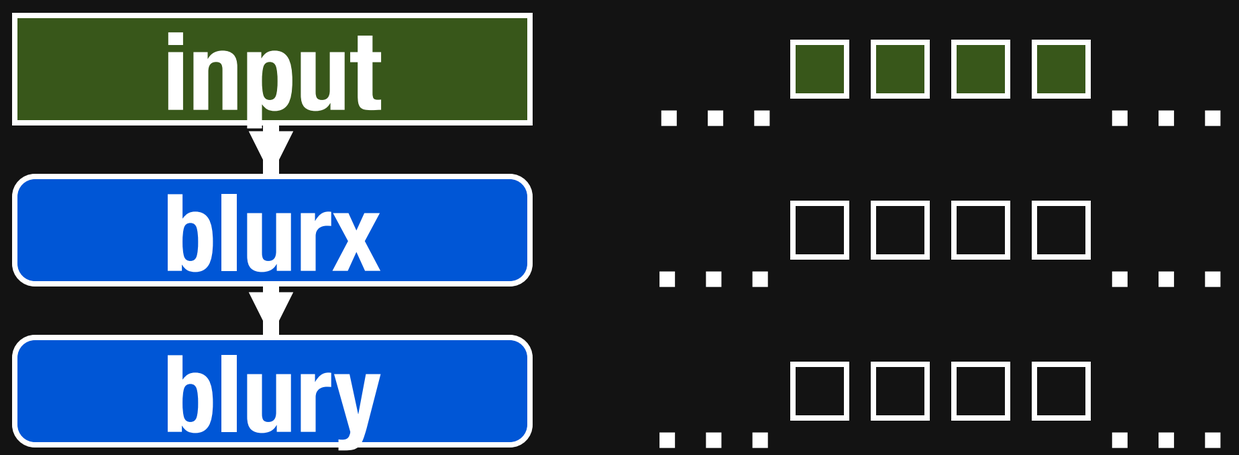
First option for GPU Implementation

- Do entire x-blur, then entire y-blur
- https://youtu.be/3uiEyEKji0M?t=6m32s

Organizing a data-parallel pipeline



Simple loops execute **breadth-first** across stages



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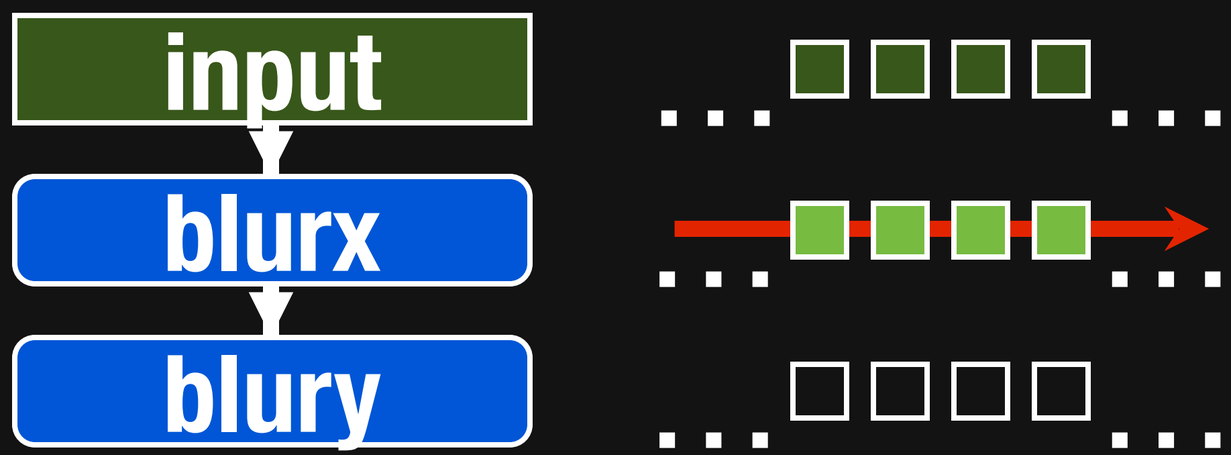

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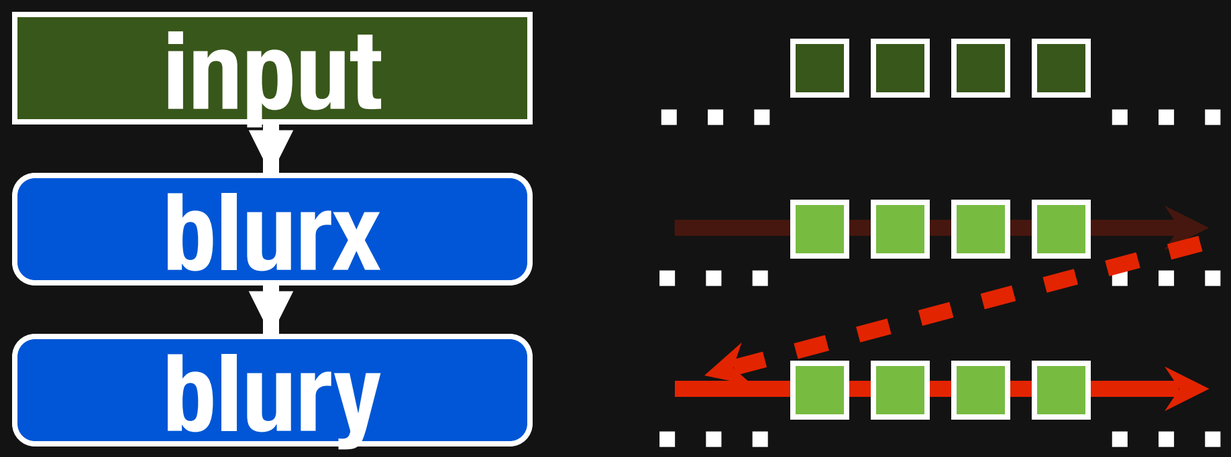

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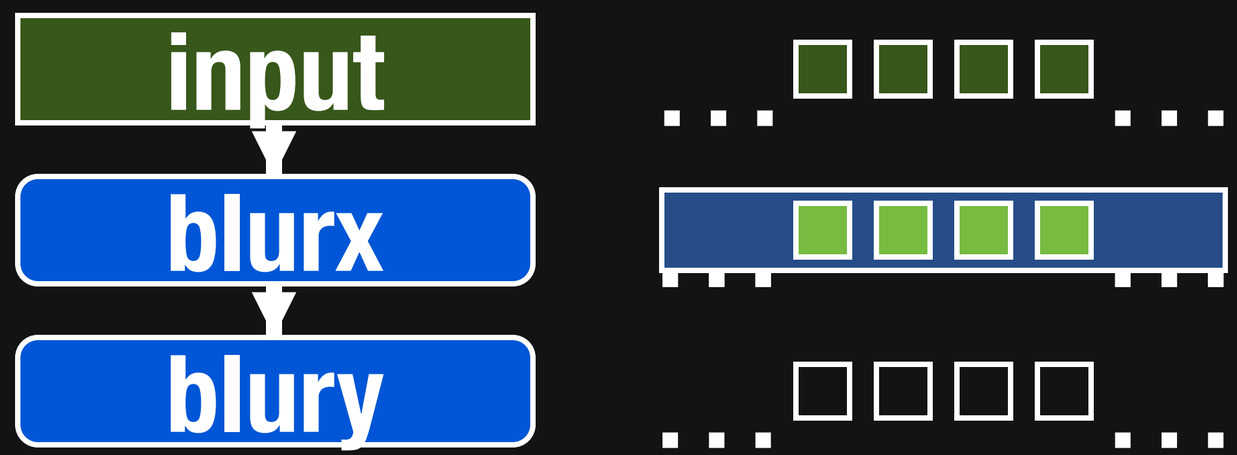

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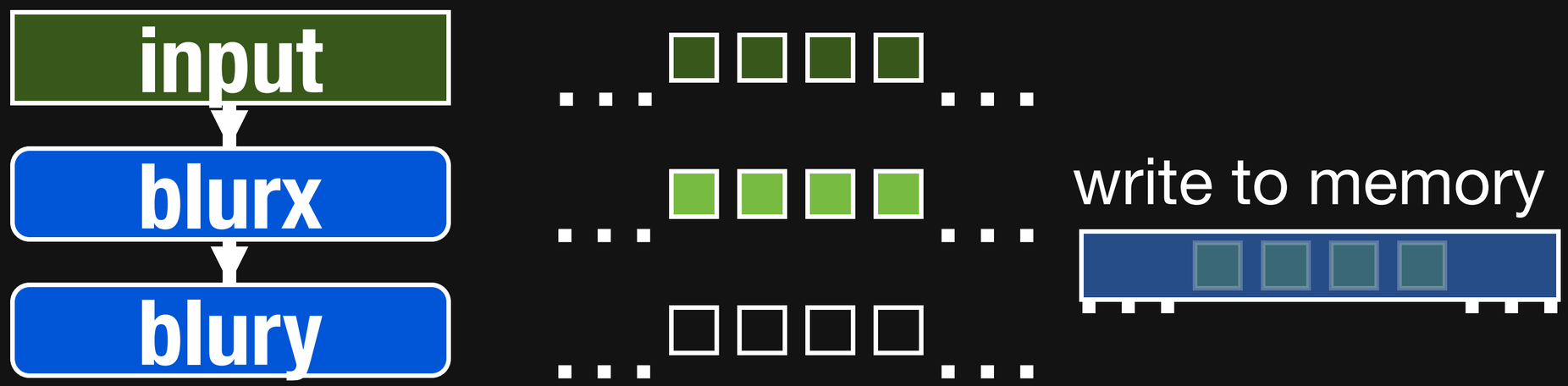

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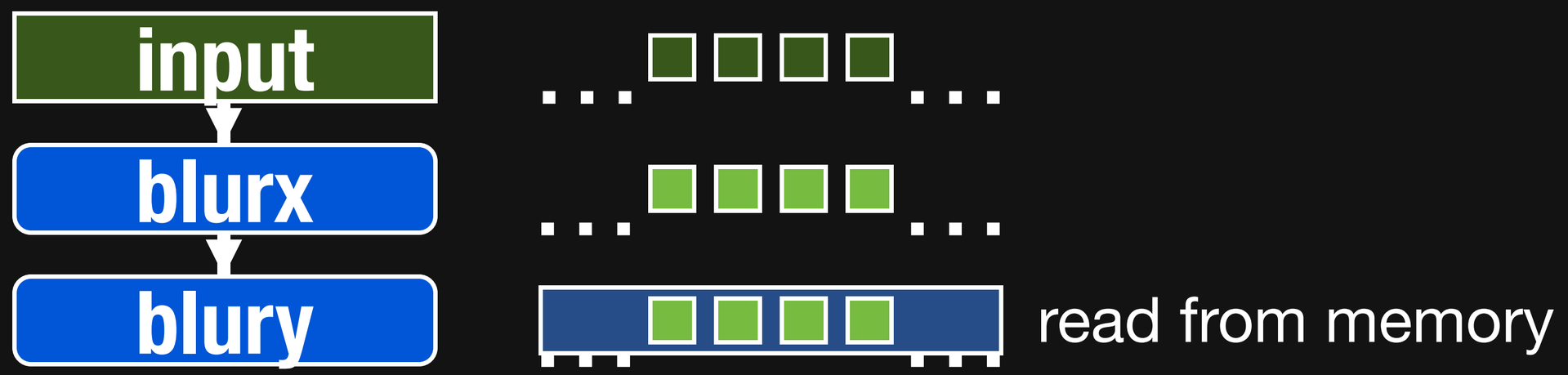
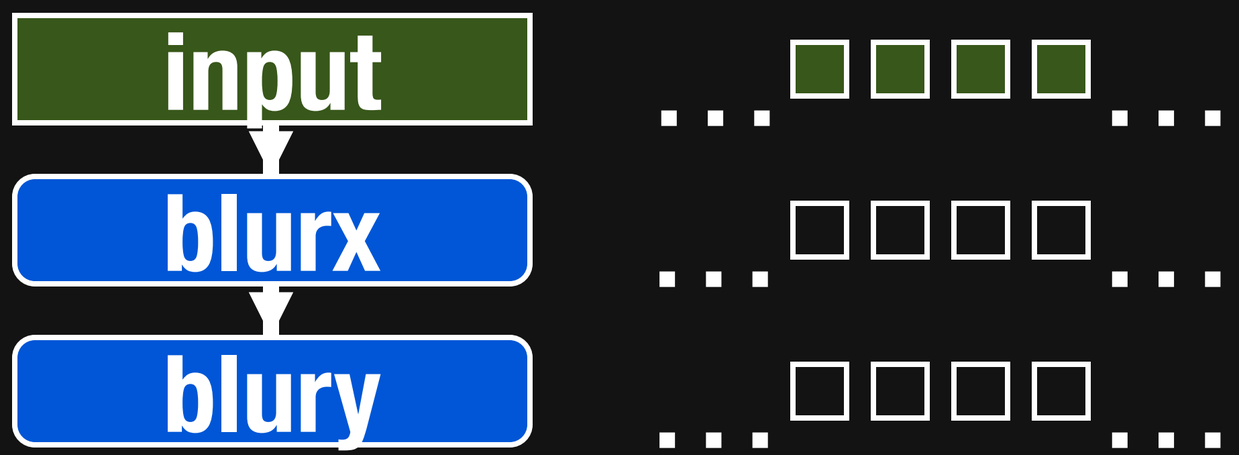


Breadth-first execution sacrifices locality



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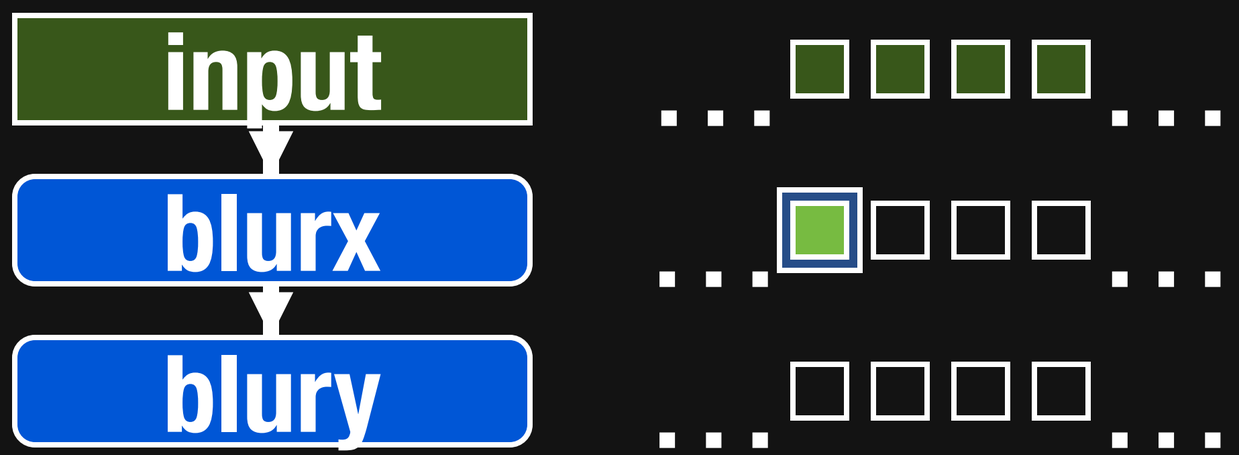

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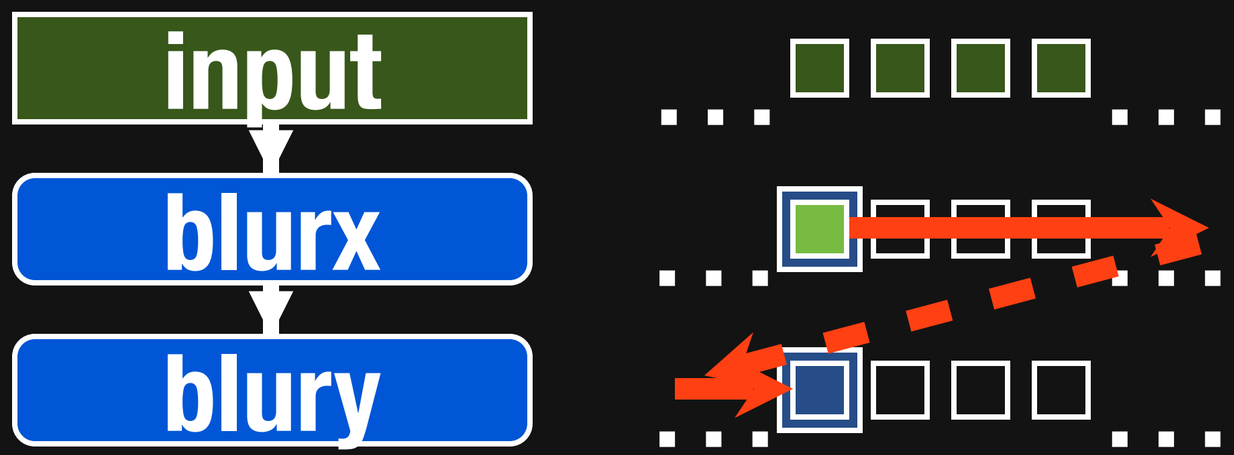

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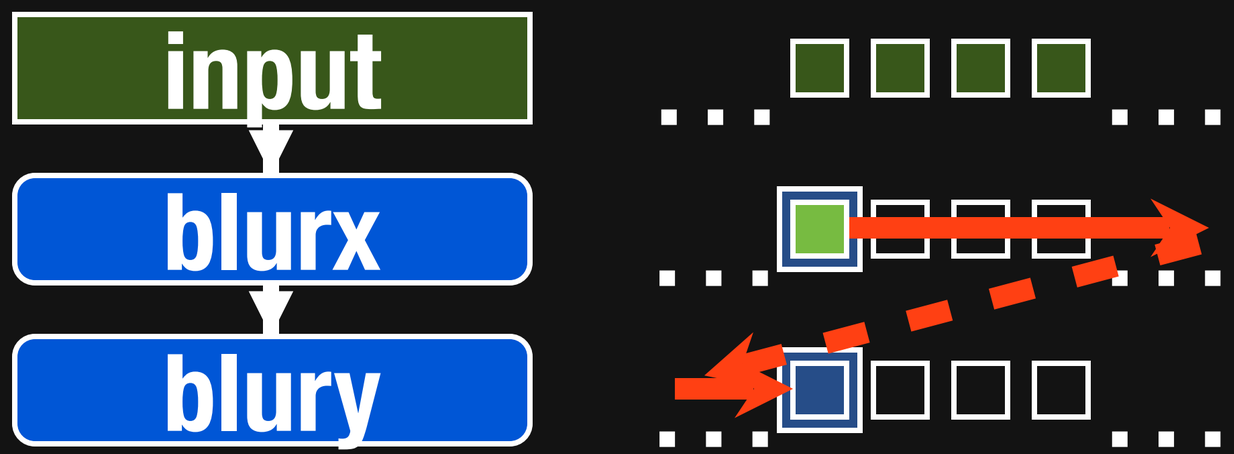


locality is a function of **reuse distance**

Two Options for GPU Implementation

- Do entire x-blur, then entire y-blur
  - https://youtu.be/3uiEyEKji0M?t=6m32s

- Do y-blur and compute x-blur as you go
  - https://youtu.be/3uiEyEKji0M?t=7m47s

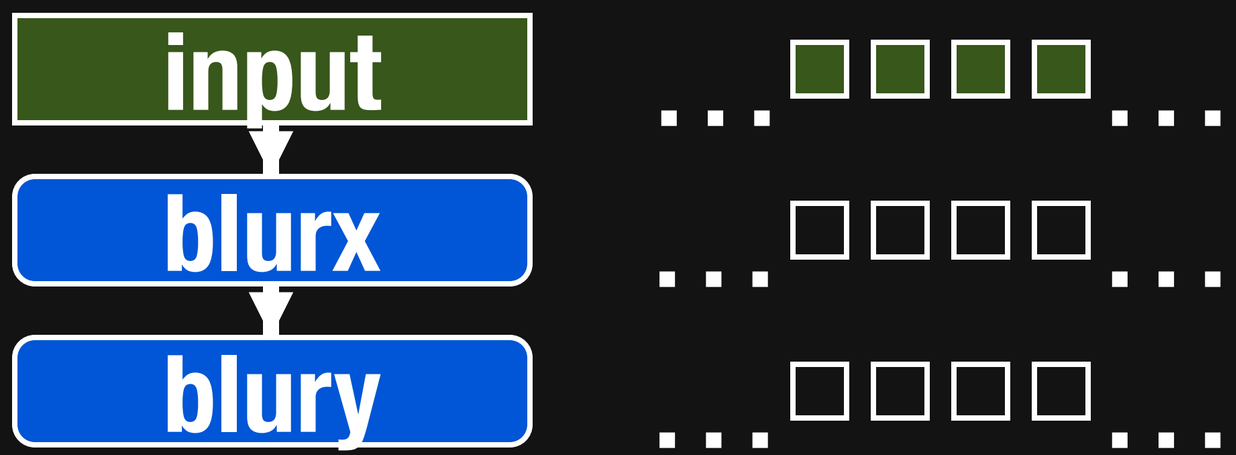
Compute X-Blur as needed

```
__global__ void blurX(Image &in, Image &out) {
  int x = threadIdx.x + blockIdx.x * blockDim.x
  int y = threadIdx.y + blockIdx.y * blockDim.y

  float tmp[3];
  for(int i = 0; i < 3; ++i) {
    int yi = y + i - 1;
    tmp[i] = (in(x-1, yi) + in(x, yi) + in(x+1, yi)) / 3;
  }

  out(x,y) = (tmp[0] + tmp[1] + tmp[2]) / 3;
}
```

Interleaved execution improves locality



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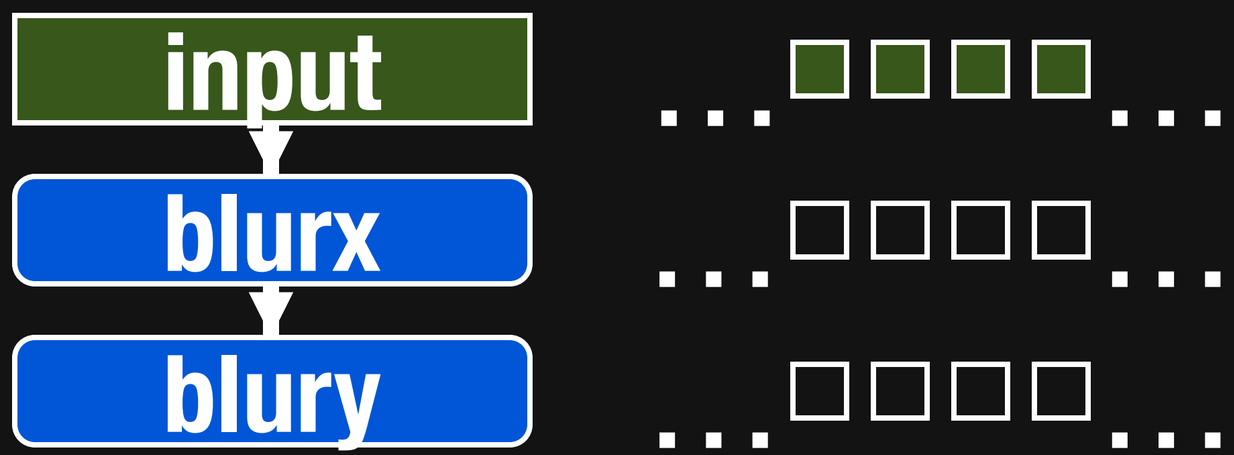

(simplified)

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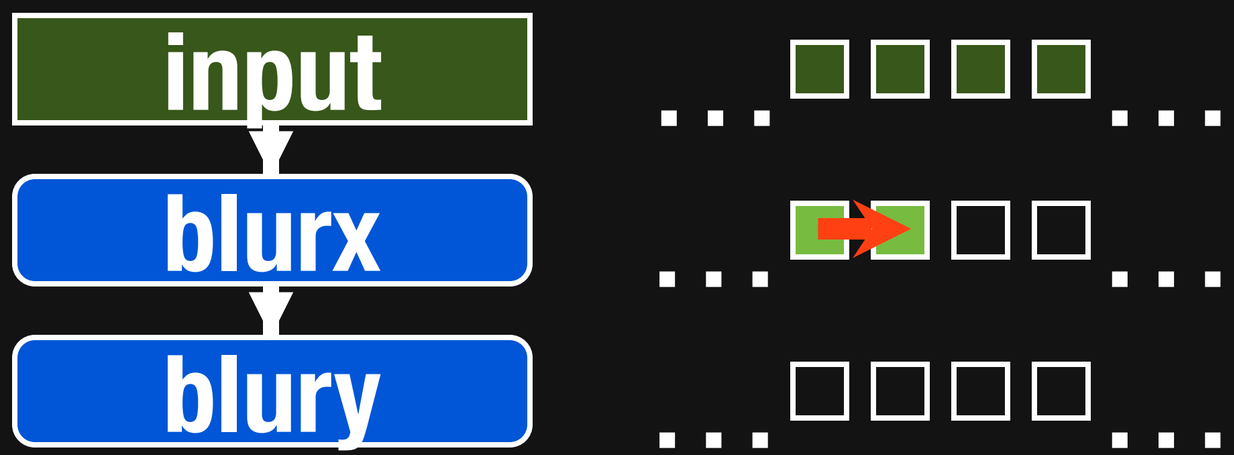

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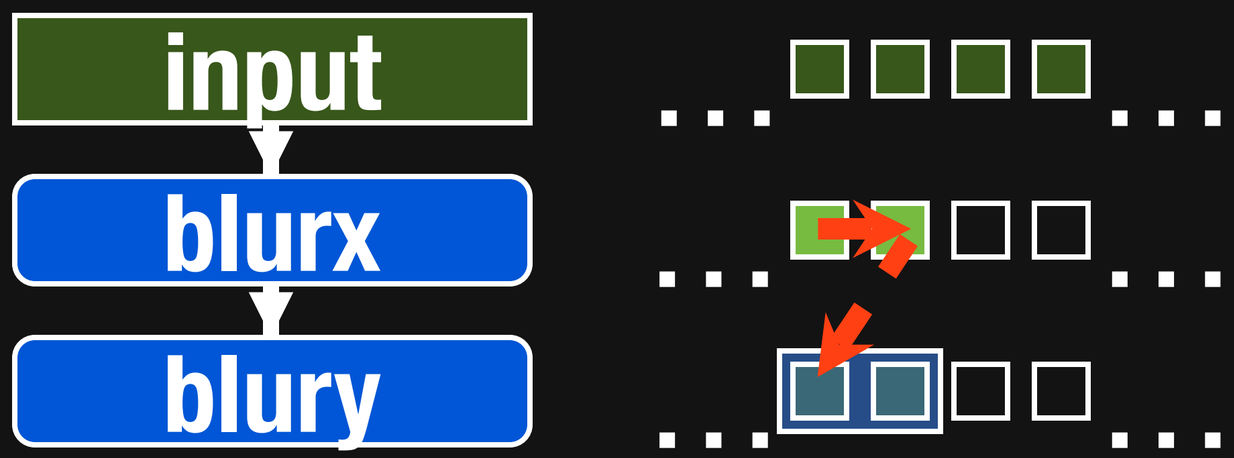

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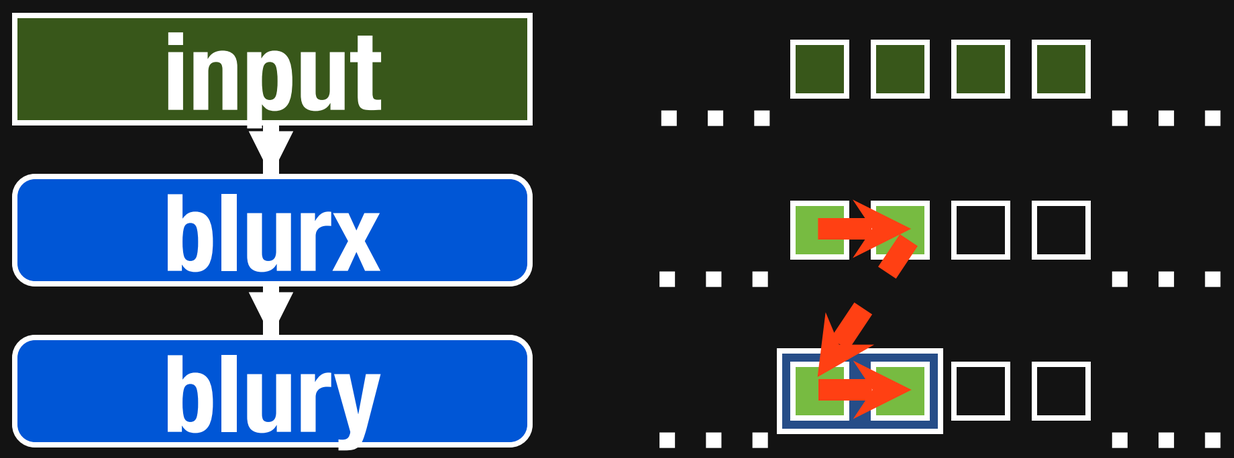

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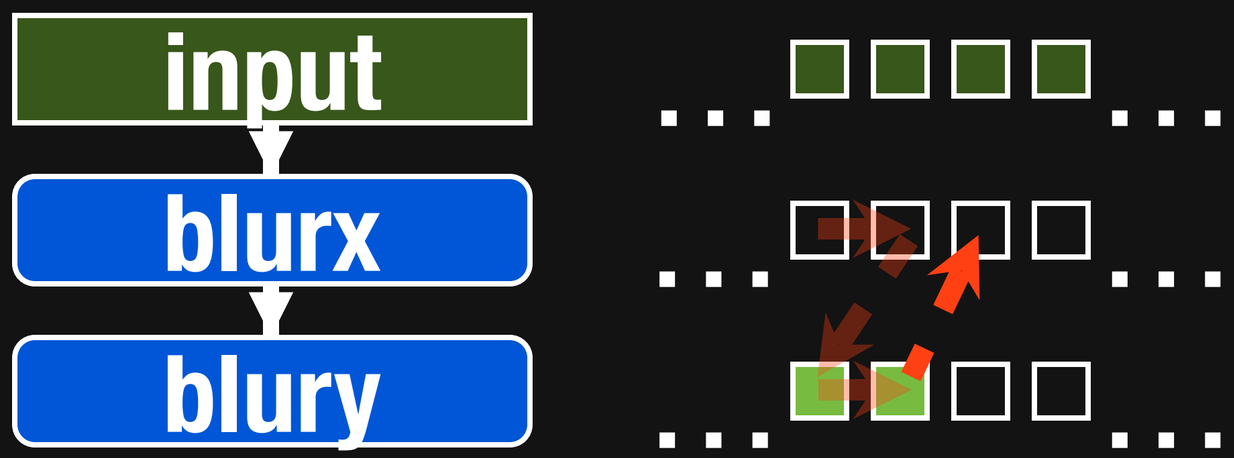

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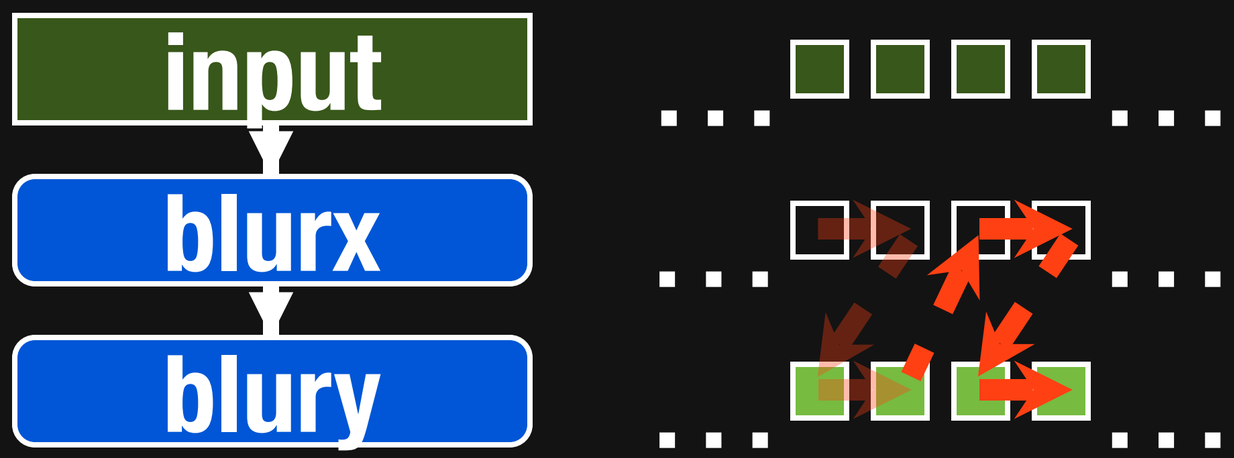

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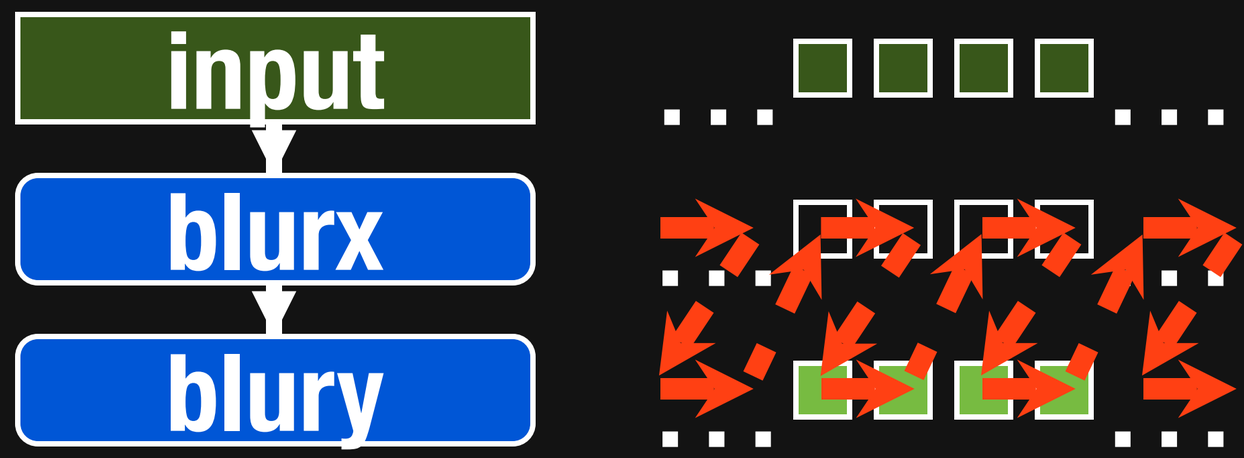

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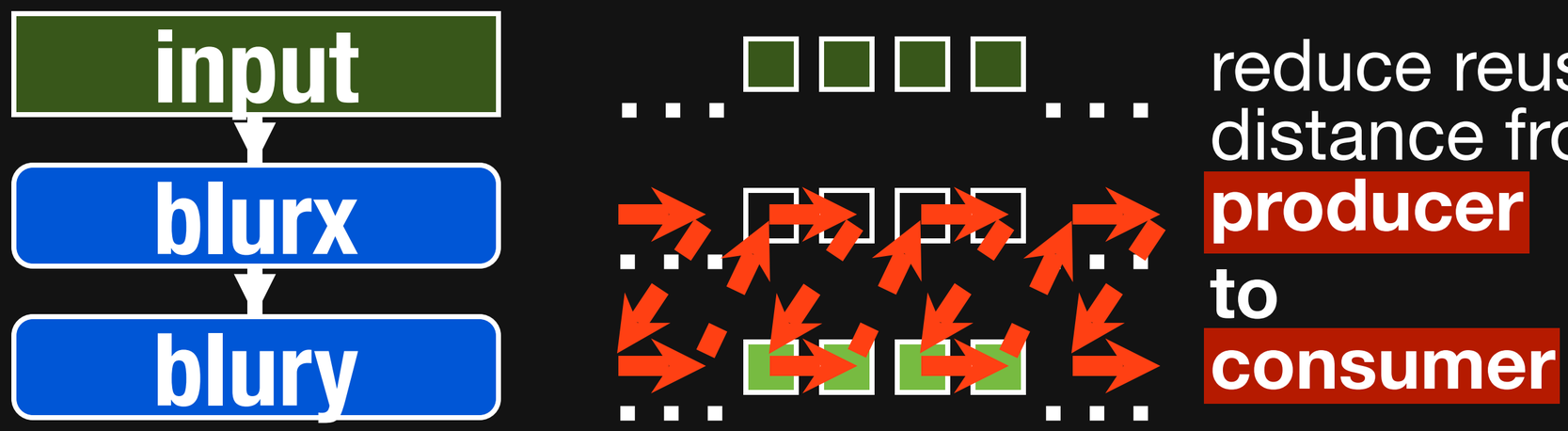
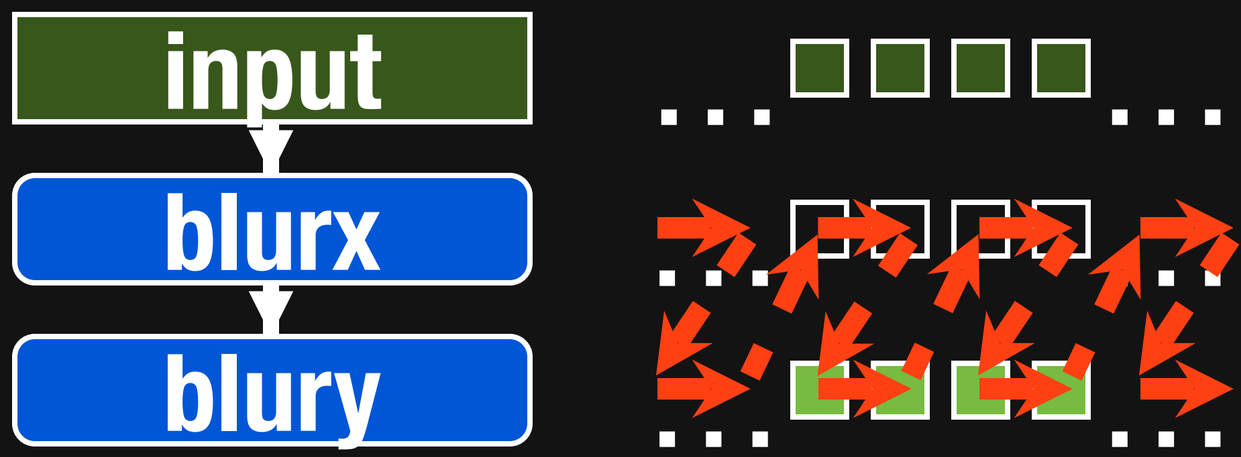


reduce reuse distance from **producer** to **consumer**

Fusion improves locality



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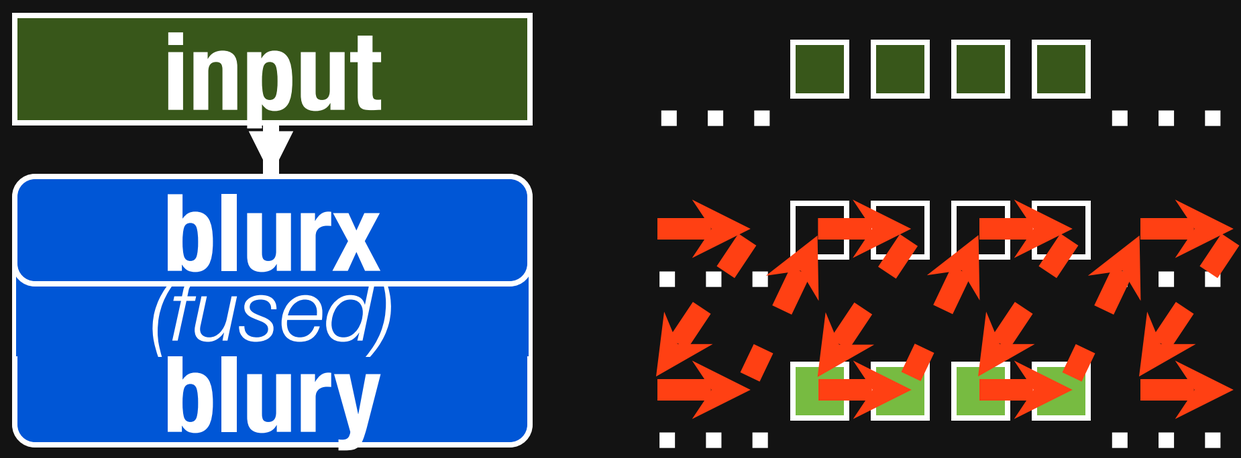
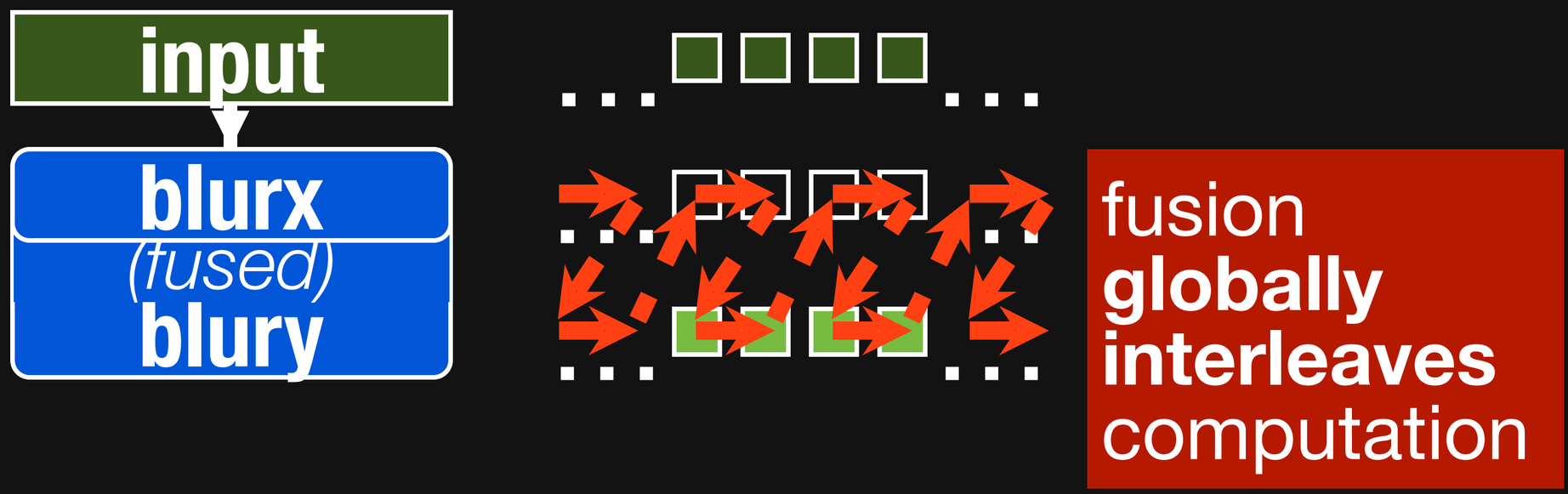


Fusion improves locality



Two Options for GPU Implementation

X-Blur then Y-Blur

- Requires communication between threads (kernel break)
- No redundant computation

X-Blur as needed

- No communication between threads
- Lots of redundant computation

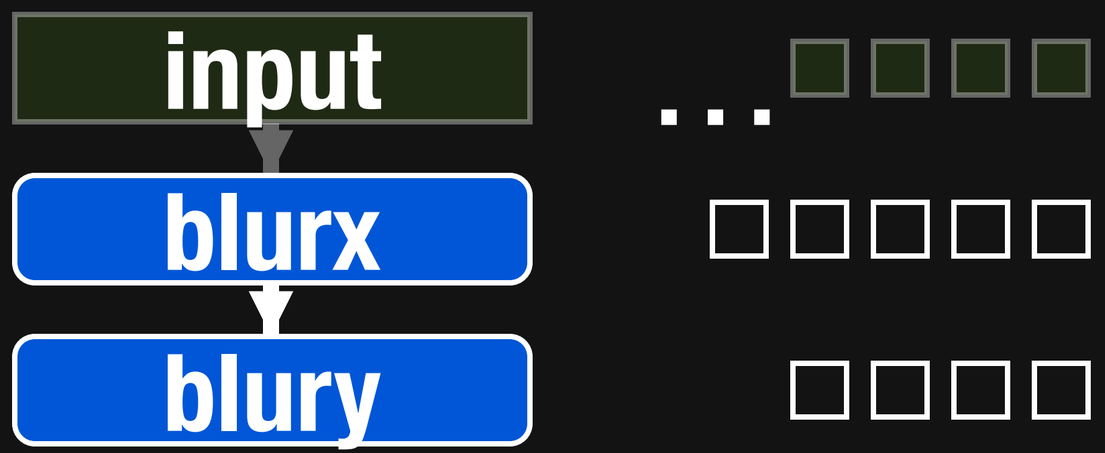
Understanding dependencies

Understanding dependencies

Understanding dependencies

Understanding dependencies

Stencils have overlapping dependencies



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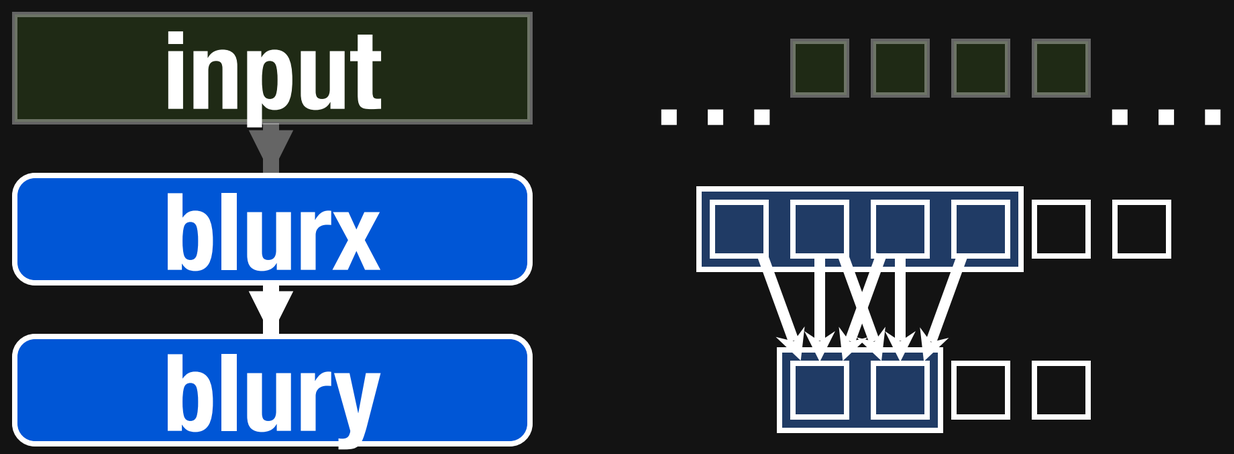

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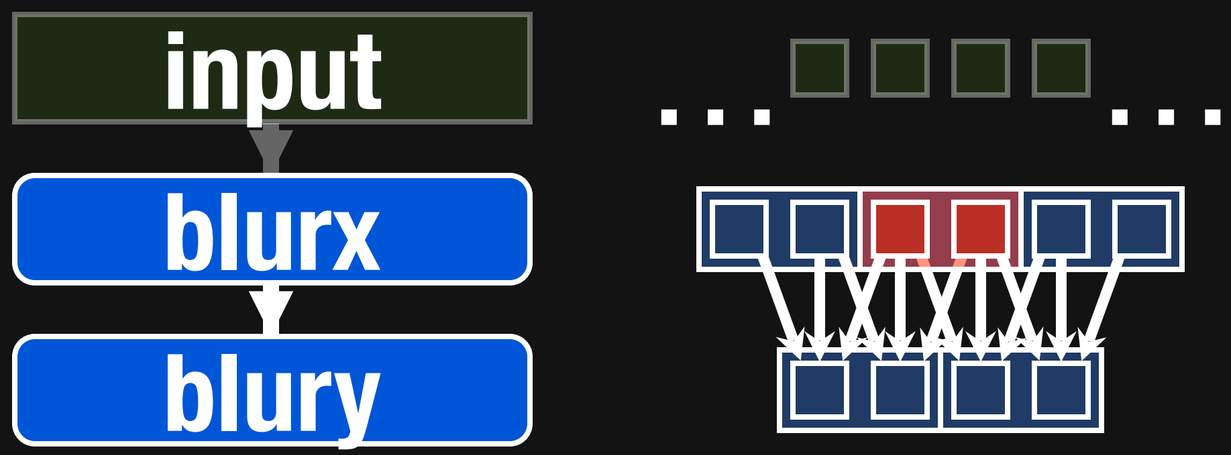

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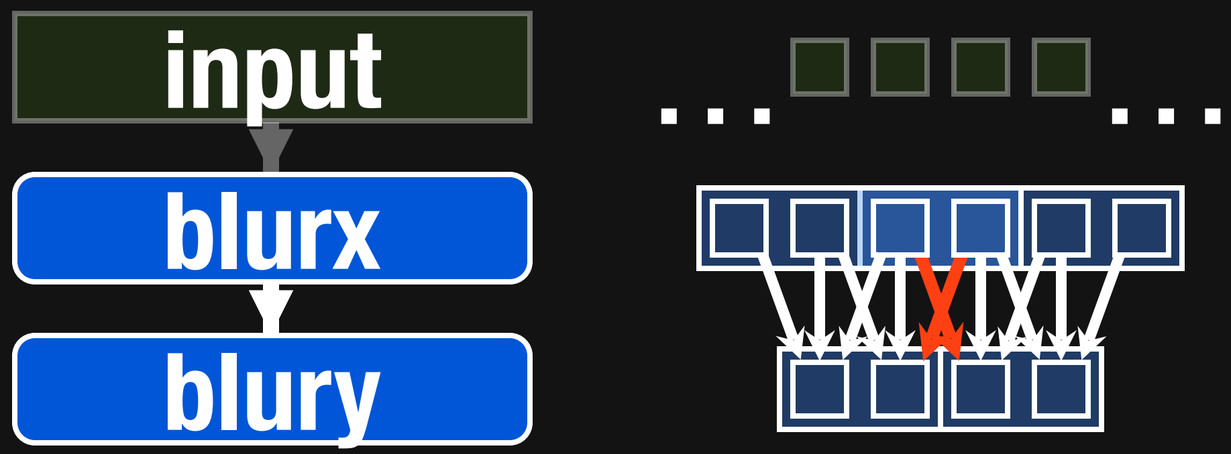
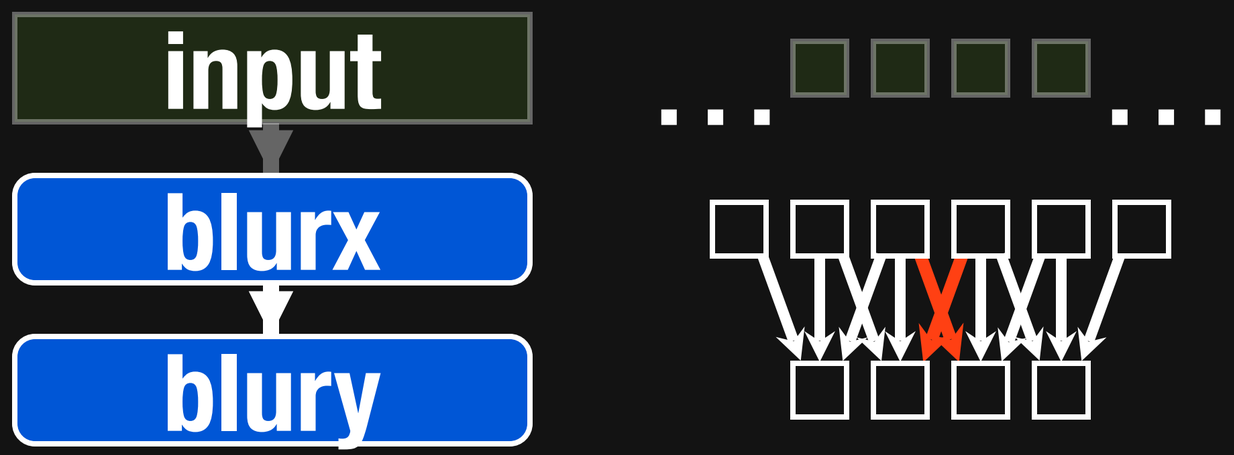


Breaking dependencies introduces redundant work



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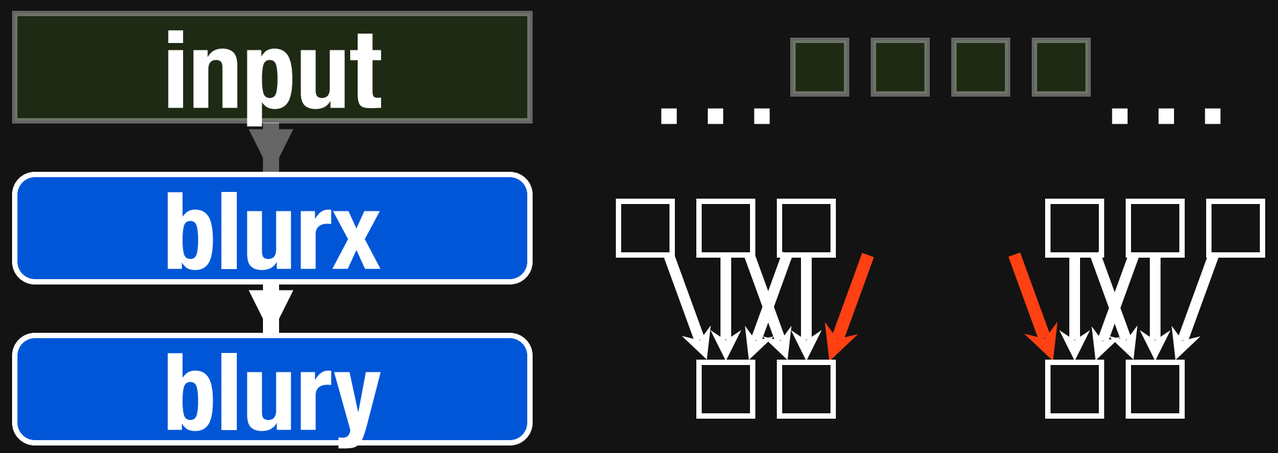

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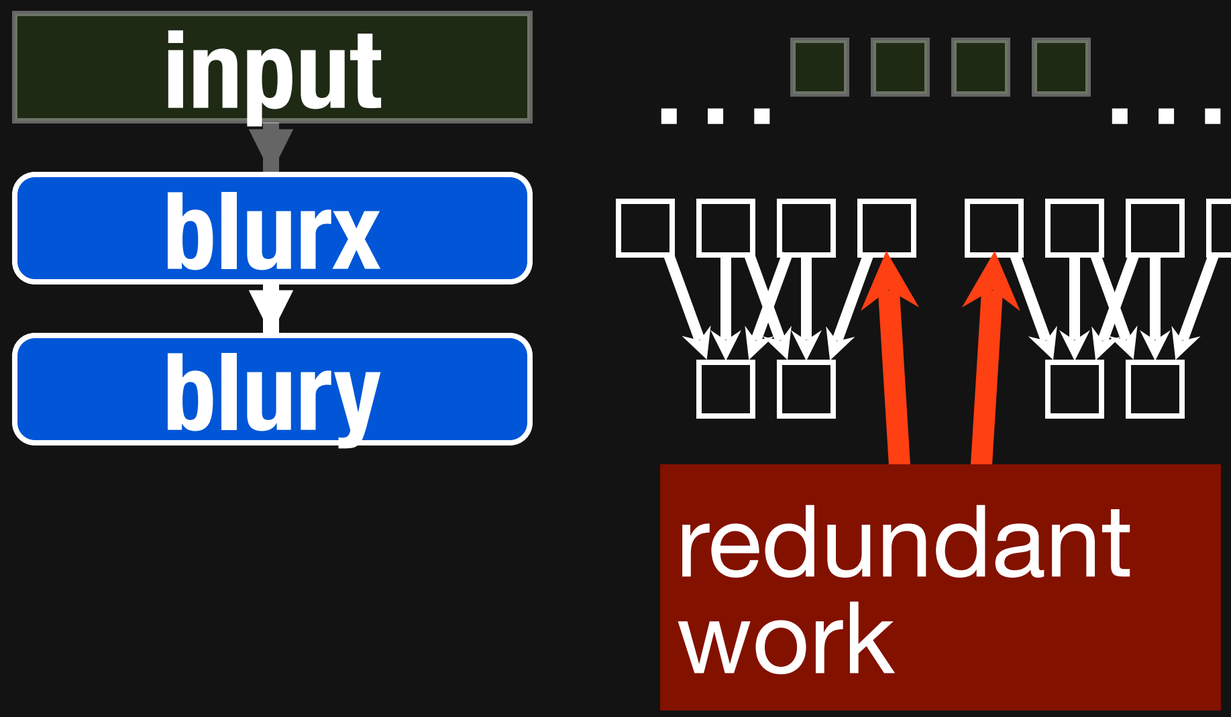
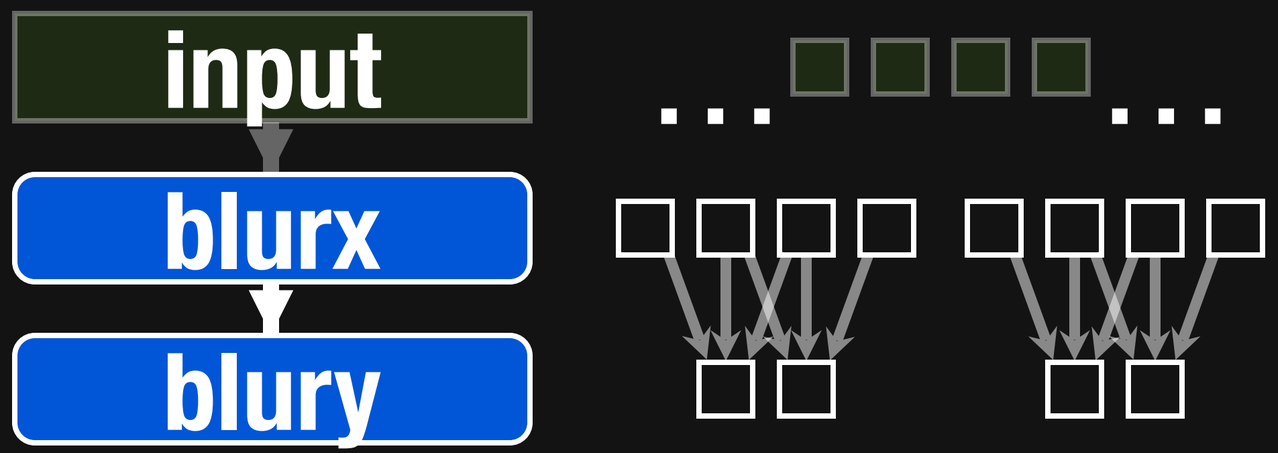


Decoupled tiles optimize parallelism & locality



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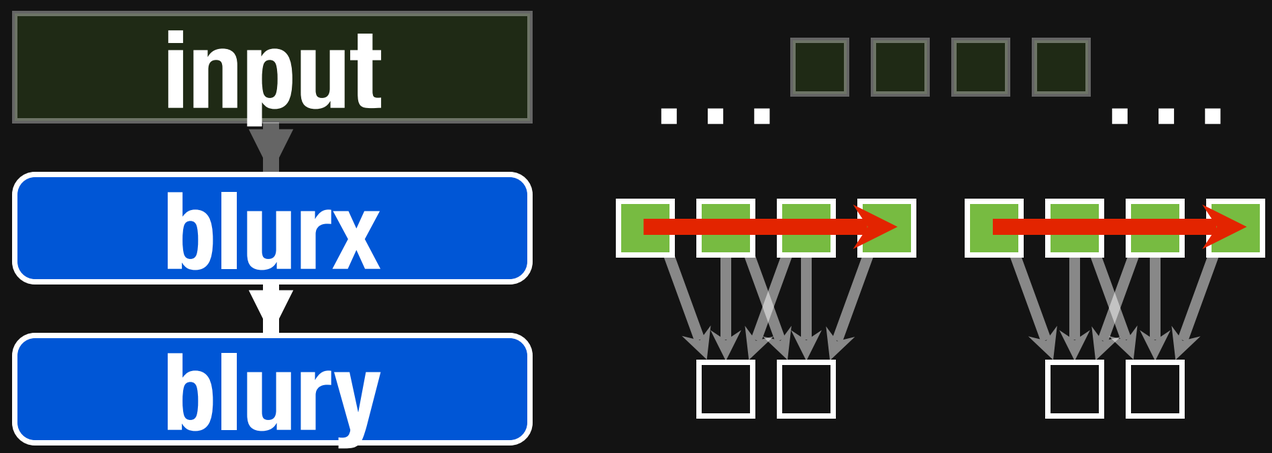

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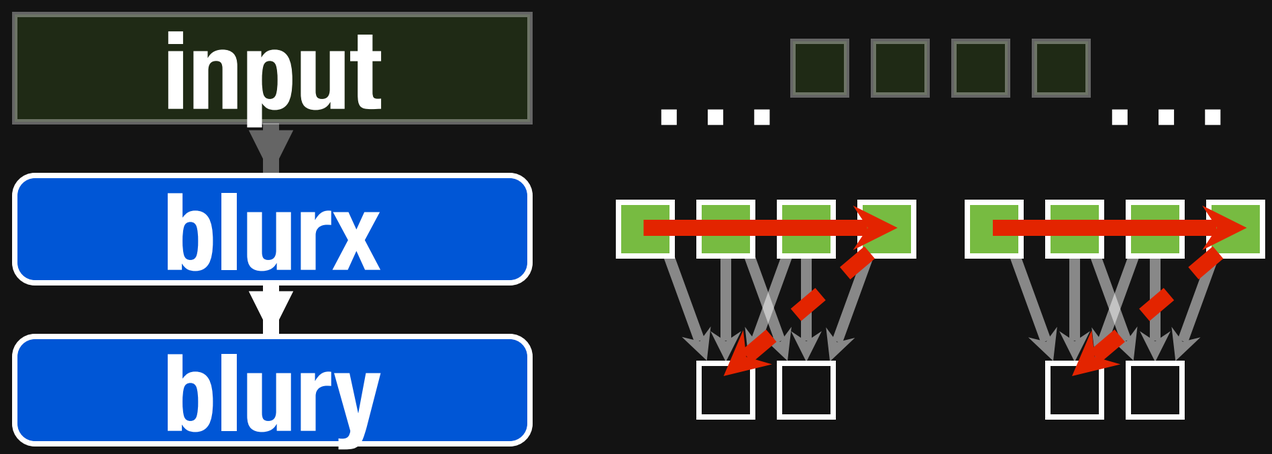

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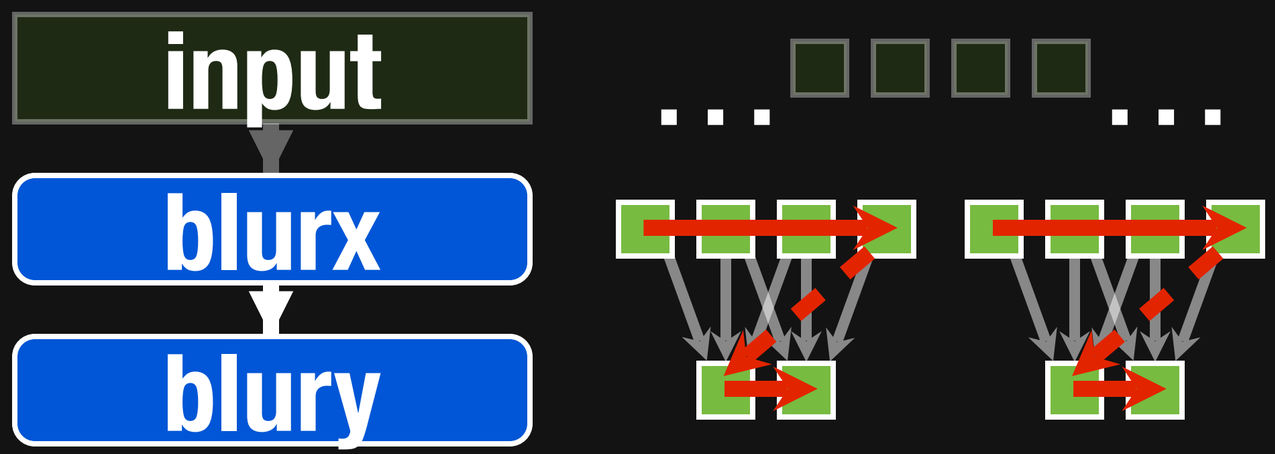

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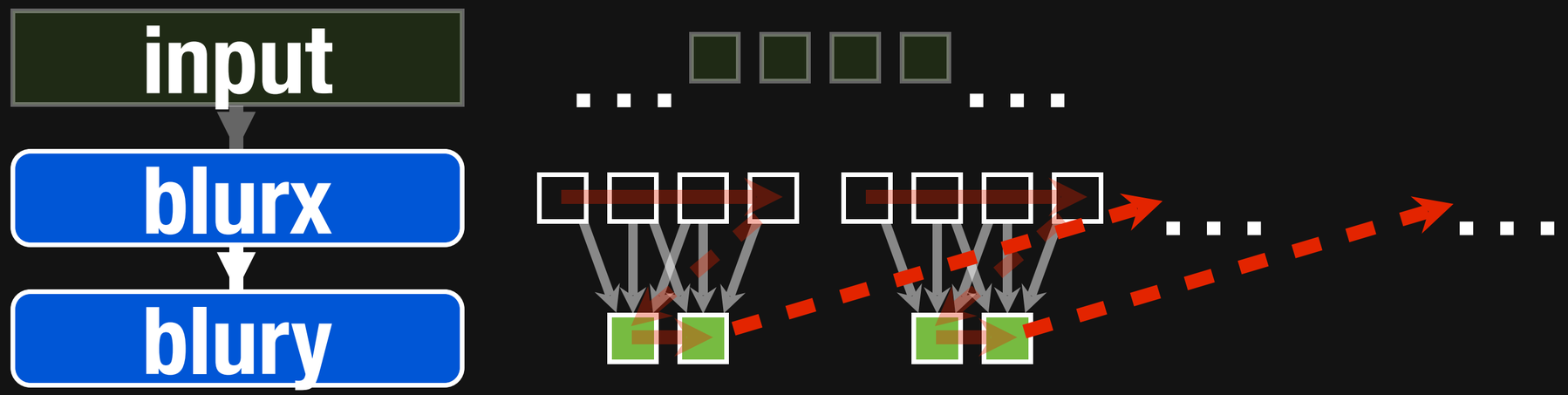

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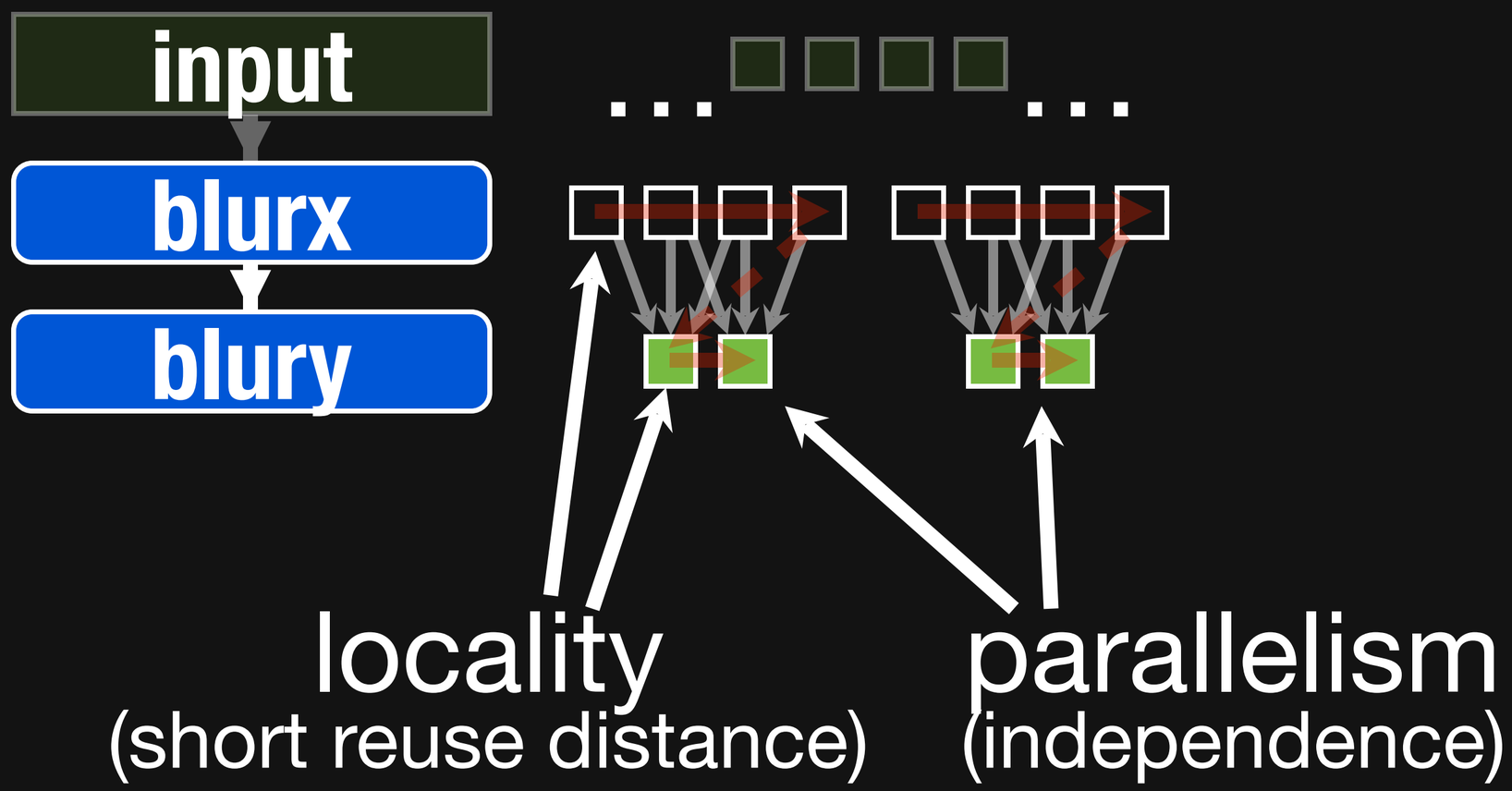
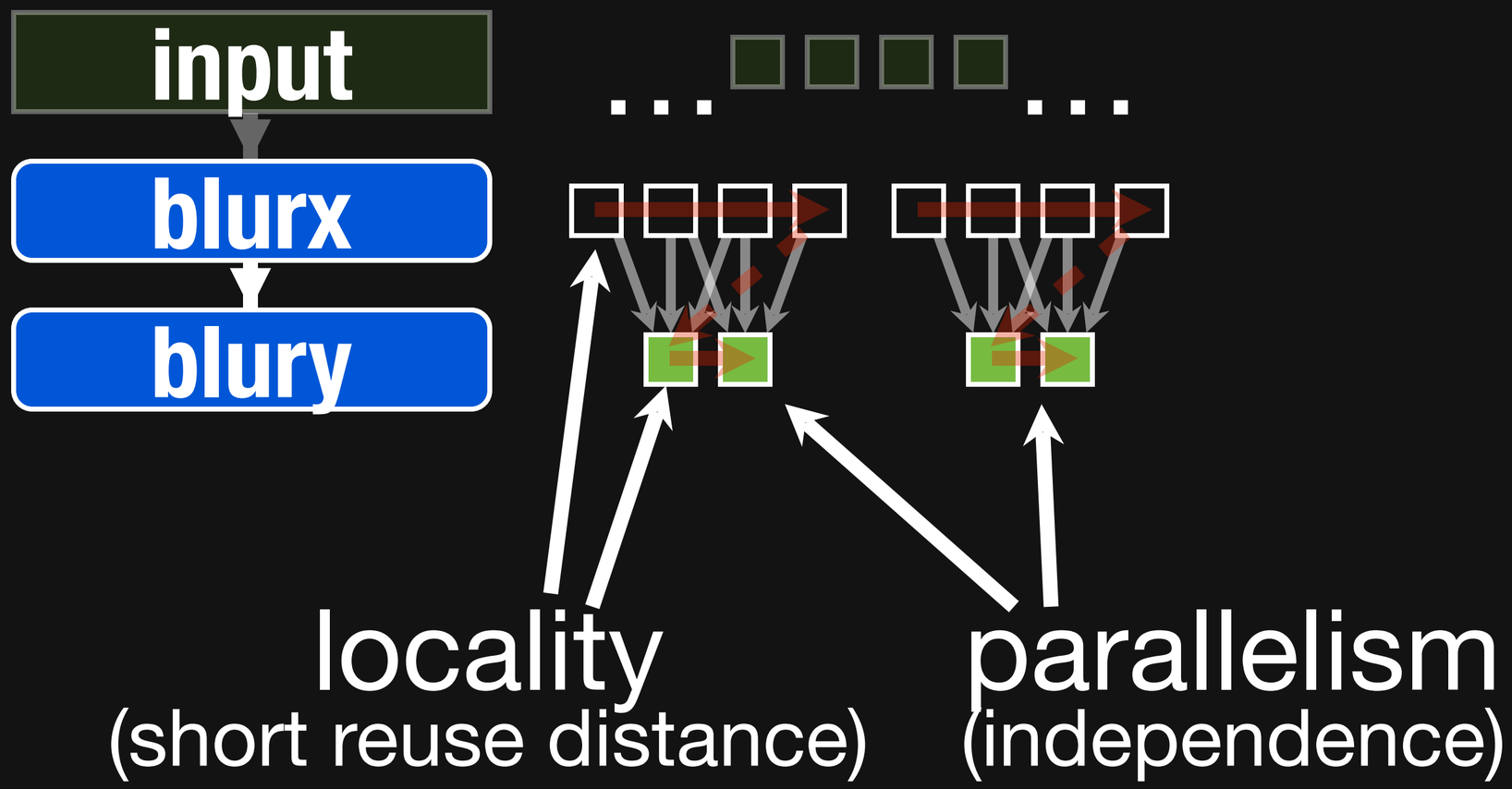


Decoupled tiles optimize parallelism & locality



... but at the cost of redundant work.

Halide

a new language & compiler for image processing

1. **Decouple *algorithm* from *schedule***

   Algorithm: *what* is computed

   Schedule: *where* and *when* it's computed

Halide

a new language & compiler for image processing

1. **Decouple *algorithm* from *schedule***

   Algorithm: *what* is computed

   Schedule: *where* and *when* it's computed

2. **Single, unified model for *all* schedules**

Halide

a new language & compiler for image processing

1. **Decouple *algorithm* from *schedule***

   Algorithm: *what* is computed
   Schedule: *where* and *when* it's computed

2. **Single, unified model for *all* schedules**

   Simple enough to search, expose to user
   Powerful enough to beat expert-tuned code

Aside: Halide

(a) Clean C++ : 9.94 ms per megapixel

```
void blur(const Image &in, Image &blurred) {
 Image tmp(in.width(), in.height());

 for (int y = 0; y < in.height(); y++)
  for (int x = 0; x < in.width(); x++)
   tmp(x, y) = (in(x-1, y) + in(x, y) + in(x+1, y))/3;
 for (int y = 0; y < in.height(); y++)
  for (int x = 0; x < in.width(); x++)
   blurred(x, y) = (tmp(x, y-1) + tmp(x, y) + tmp(x, y+1))/3;
}
```

(b) Fast C++ (for x86) : 0.90 ms per megapixel

```
void fast_blur(const Image &in, Image &blurred) {
 __m128i one_third = _mm_set1_epi16(21846);
 #pragma omp parallel for
 for (int yTile = 0; yTile < in.height(); yTile += 32) {
  __m128i a, b, c, sum, avg;
  __m128i tmp[(256/8)*(32+2)];
  for (int xTile = 0; xTile < in.width(); xTile += 256) {
   __m128i *tmpPtr = tmp;
   for (int y = -1; y < 32+1; y++) {
    const uint16_t *inPtr = &(in(xTile, yTile+y));
    for (int x = 0; x < 256; x += 8) {
     a = _mm_loadu_si128((__m128i*)(inPtr-1));
     b = _mm_loadu_si128((__m128i*)(inPtr+1));
     c = _mm_load_si128((__m128i*)(inPtr));
     sum = _mm_add_epi16(_mm_add_epi16(a, b), c);
     avg = _mm_mulhi_epi16(sum, one_third);
     _mm_store_si128(tmpPtr++, avg);
     inPtr += 8;
   }}
   tmpPtr = tmp;
   for (int y = 0; y < 32; y++) {
    __m128i *outPtr = (__m128i *)(&(blurred(xTile, yTile+y)));
    for (int x = 0; x < 256; x += 8) {
     a = _mm_load_si128(tmpPtr+(2*256)/8);
     b = _mm_load_si128(tmpPtr+256/8);
     c = _mm_load_si128(tmpPtr++);
     sum = _mm_add_epi16(_mm_add_epi16(a, b), c);
     avg = _mm_mulhi_epi16(sum, one_third);
     _mm_store_si128(outPtr++, avg);
}}}}}
```

(c) Halide : 0.90 ms per megapixel

```
Func halide_blur(Func in) {
 Func tmp, blurred;
 Var x, y, xi, yi;

 // The algorithm
 tmp(x, y) = (in(x-1, y) + in(x, y) + in(x+1, y))/3;
 blurred(x, y) = (tmp(x, y-1) + tmp(x, y) + tmp(x, y+1))/3;

 // The schedule
 blurred.tile(x, y, xi, yi, 256, 32)
        .vectorize(xi, 8).parallel(y);
 tmp.chunk(x).vectorize(x, 8);

 return blurred;
}
```

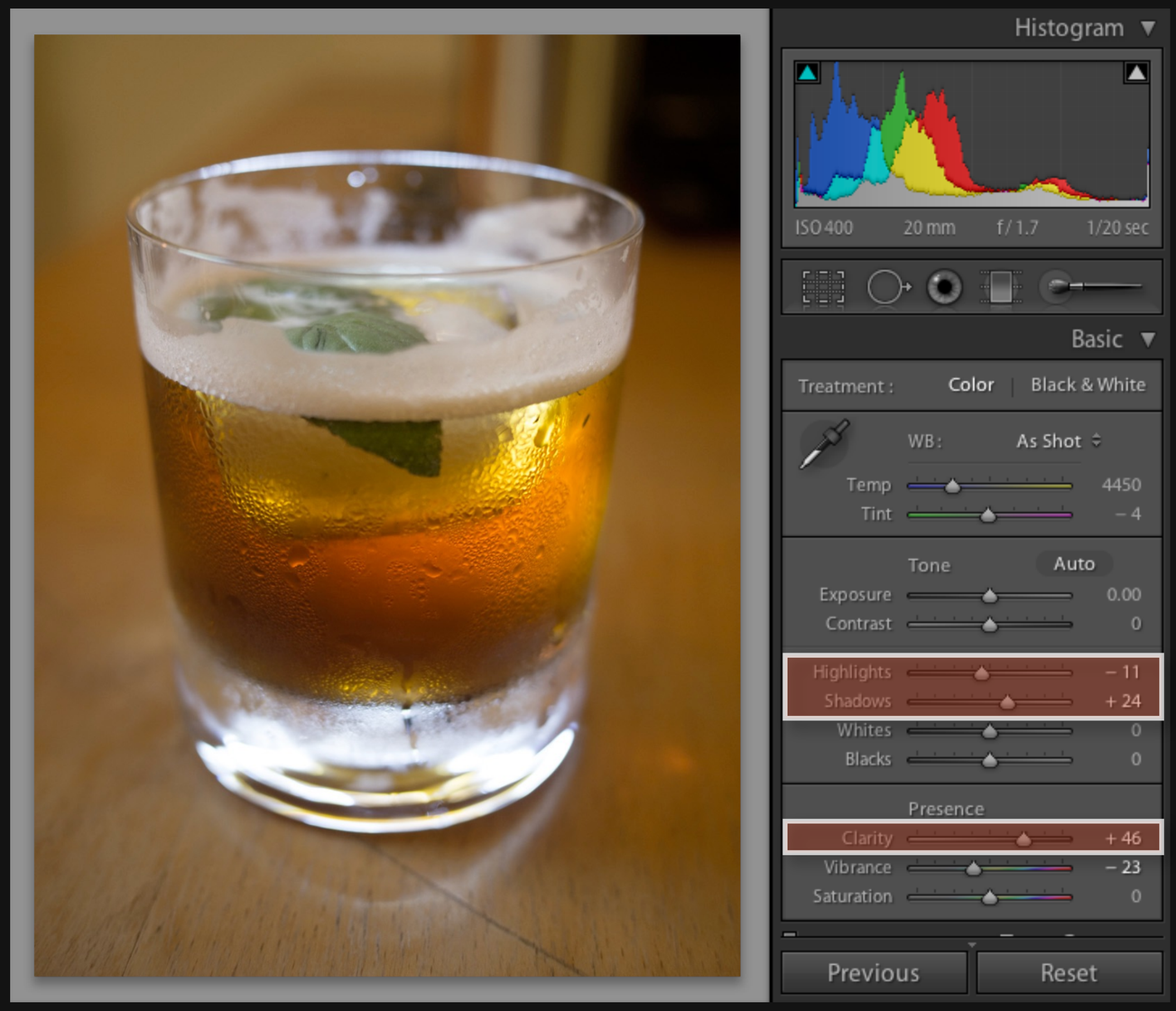
Local Laplacian Filters

Prototype for Adobe Photoshop Camera Raw / Lightroom

Adobe: 1500 lines of
expert-optimized C++
multithreaded, SSE
3 months of work
10x faster than original C++

Halide: 60 lines
1 intern-day

Halide vs. Adobe:
2x faster on same CPU,
9x faster on GPU



Aside: Halide

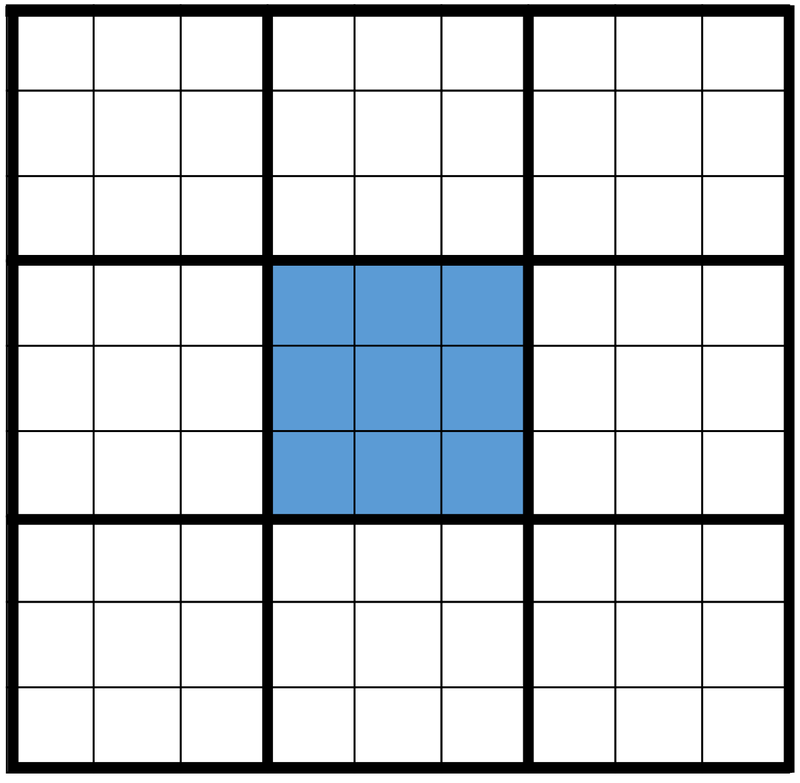
halide-lang.org

J. Ragan-Kelly, A. Adams, S. Paris, M. Levoy, S. Amarasinghe, and F. Durand. “Decoupling Algorithms from Schedules for Easy Optimization of Image Processing Pipelines.” ACM Transactions on Graphics 31(4) (In Proceedings of SIGGRAPH 2012).

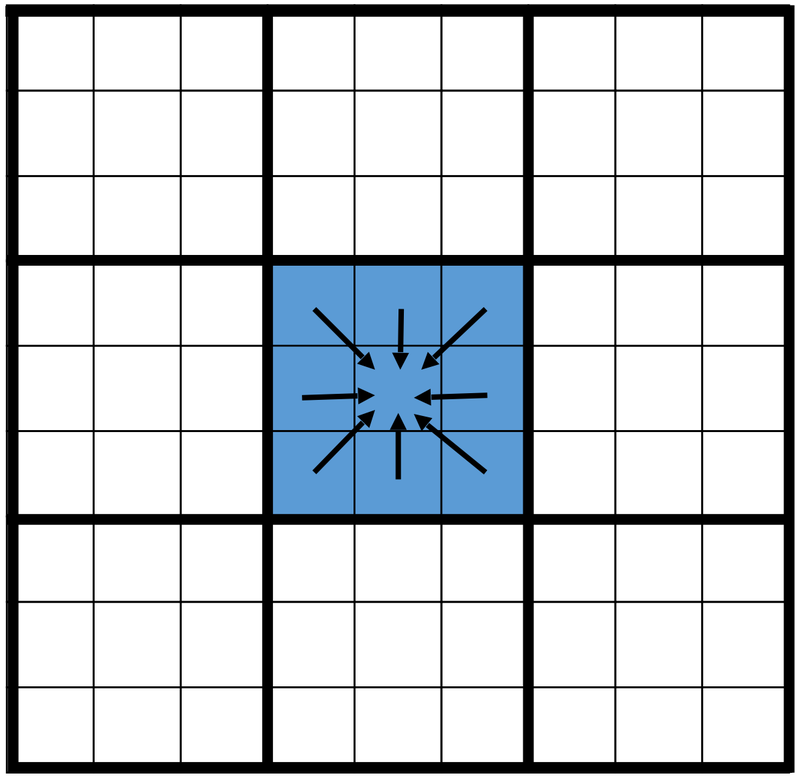
Third Option: Use tiling (with shared memory) to reduce redundant computation

- Read values into shared memory
- Local sync
- Compute x-blur on the shared memory values
- Local sync
- Use x-blur to compute y-blur

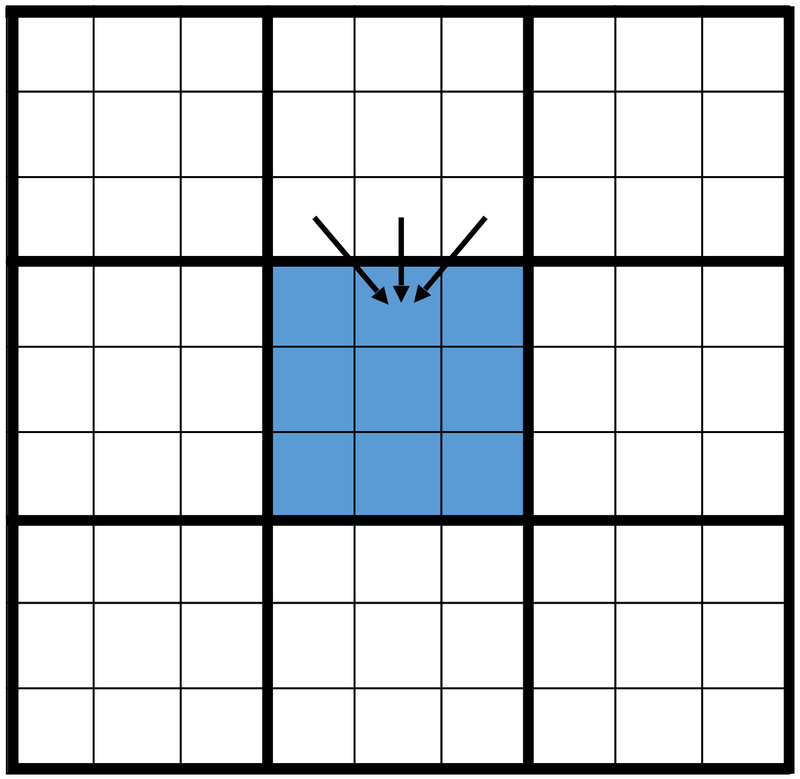
Blur with Tiling



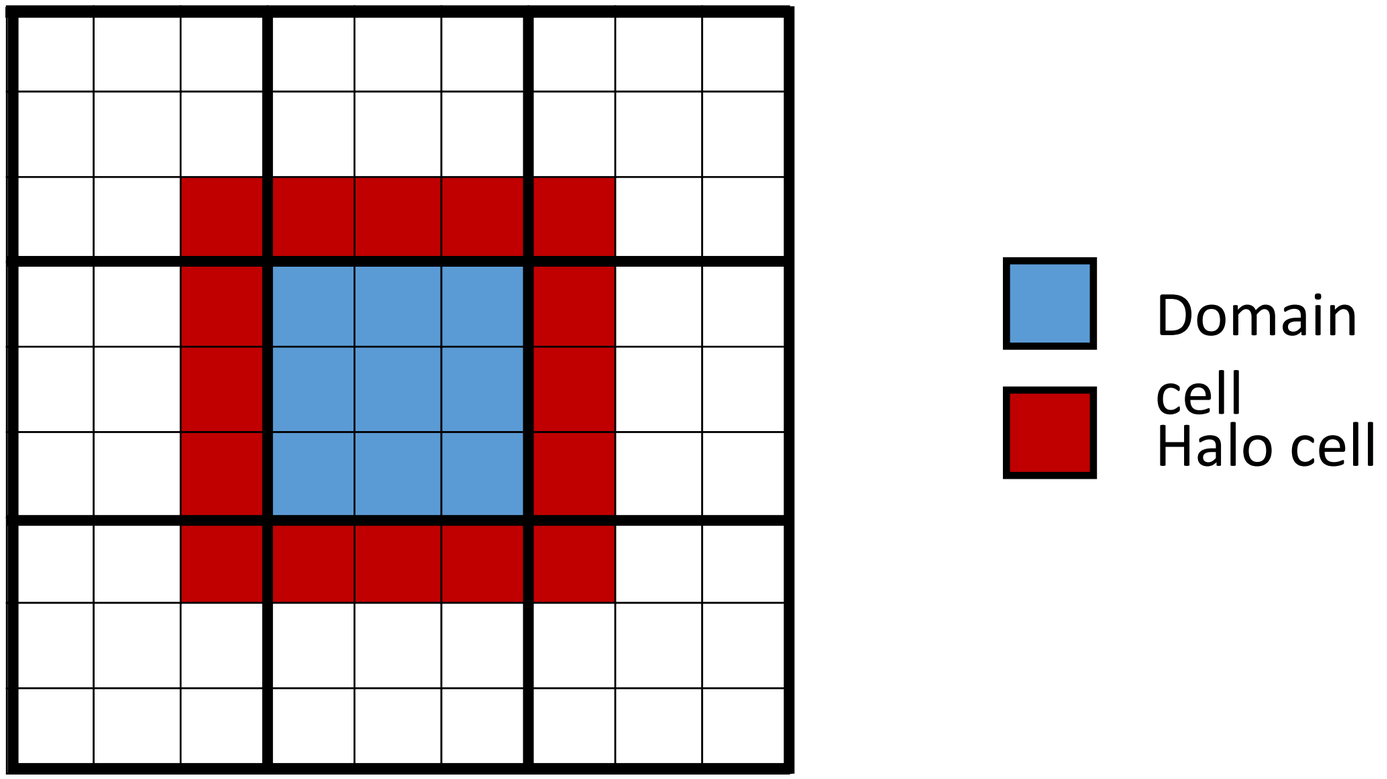
Blur with Tiling



Blur with Tiling



Halo/Ghost Region



Halo/Ghost Region

- A thread block needs data that “belongs” to a different block
- Read that extra data – known as “halo” or “ghost” region
  - Known as “overfetch”
- Some algorithms also require redundant computation
  - E.g., separable blur
  - Avoids communication

Tuning is required for best performance

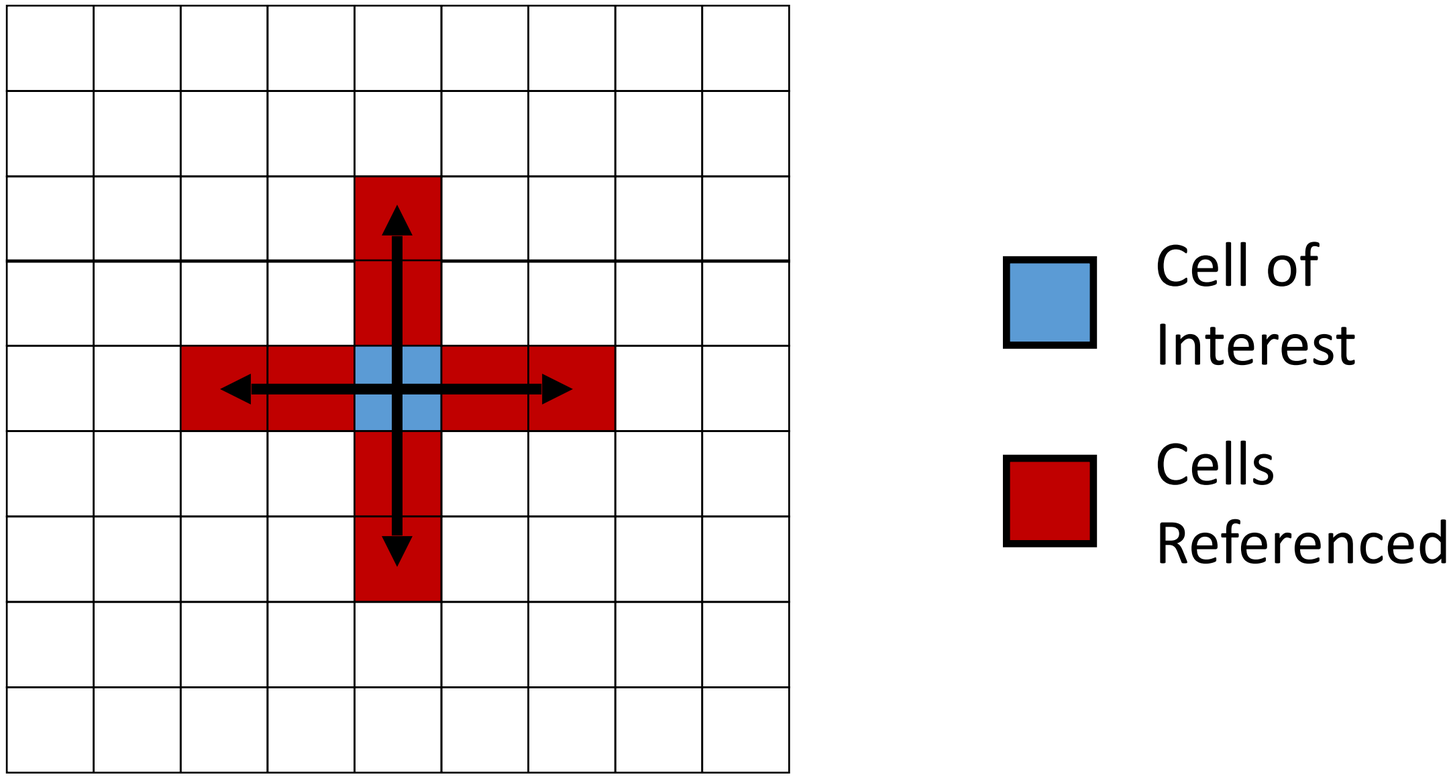
- Larger blocks = less overfetch (and fewer redundant computations)
- Smaller blocks = more parallelism across blocks (more latency hiding)
  less parallelism within a block
- Trade-off between data reuse and parallelism
  - Depends on the algorithm, data layout, and hardware specifics

Two types of locality

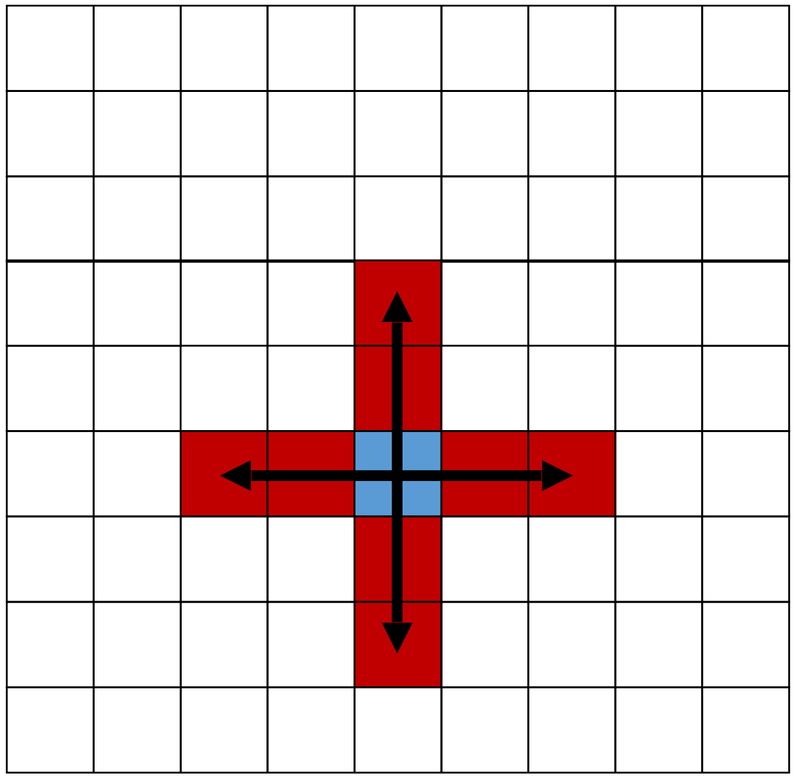
- Spatial locality
  - Elements stored near each other should be used at the same time
  - (coalescing)

- Temporal locality
  - Elements calculated recently should be reused by subsequent calculations

Spatial / temporal Locality



Spatial / temporal Locality

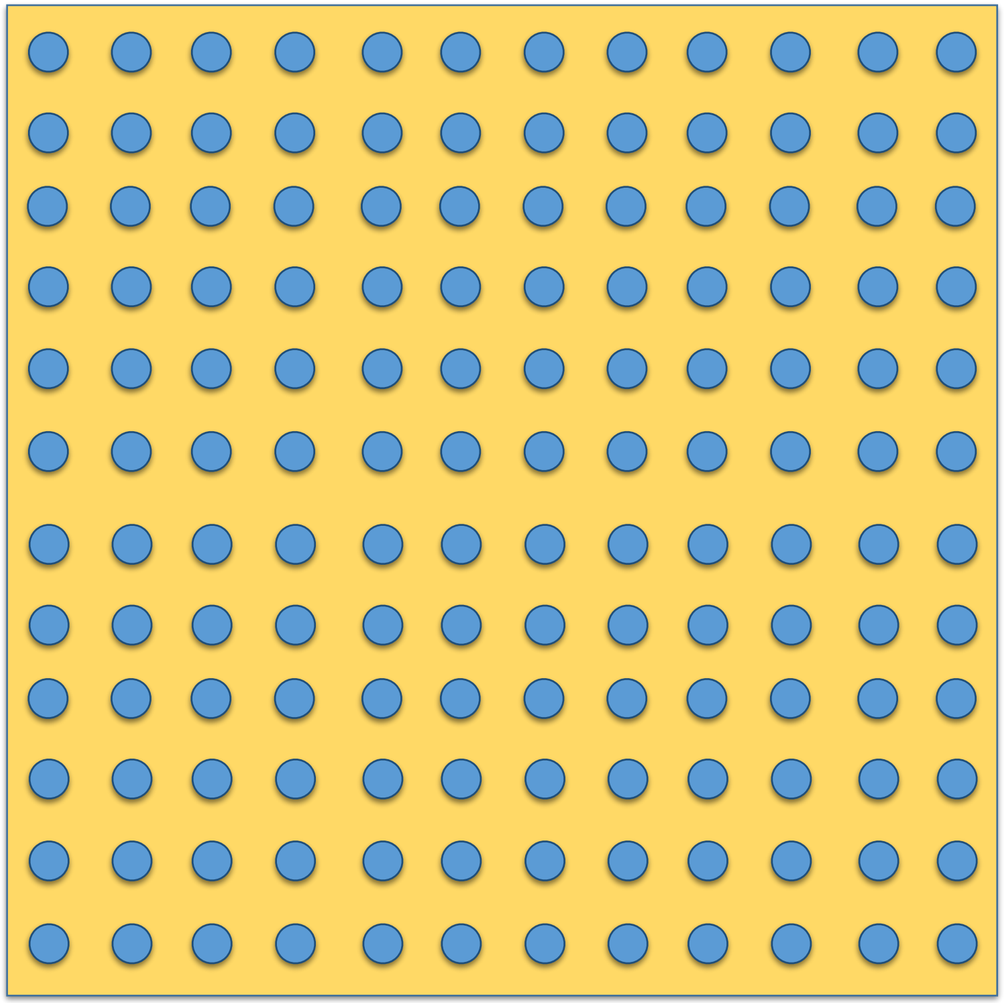


Cell of Interest

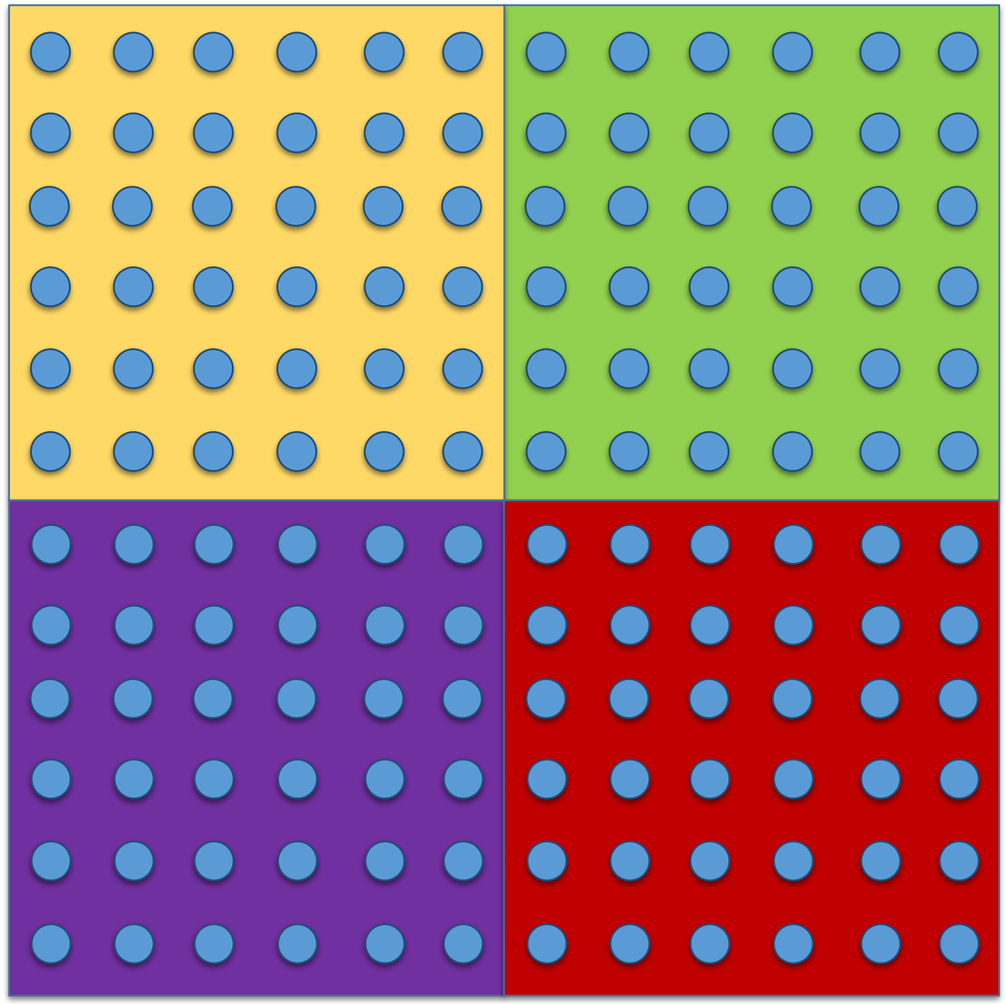
Cells Referenced

General Approach for Stencils

Divide grid into sub-blocks



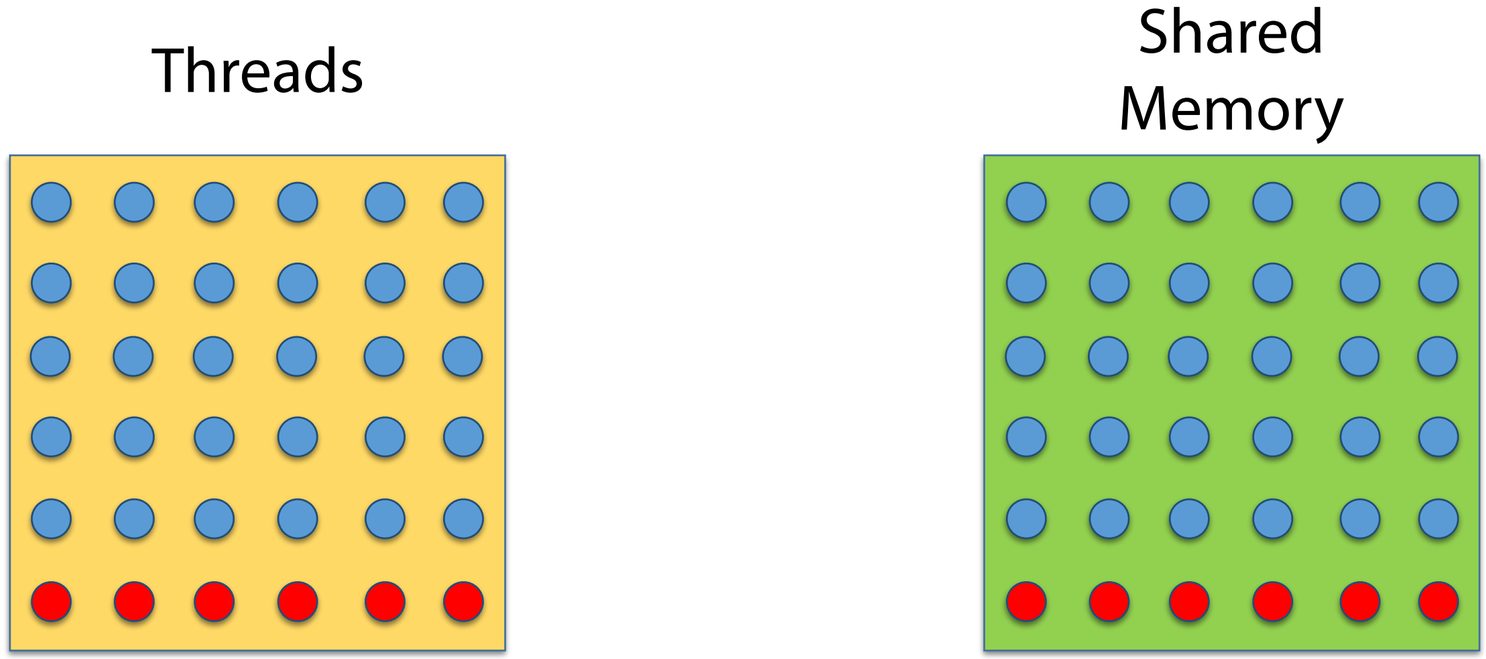
Divide grid into sub-blocks



Sub-block implementation

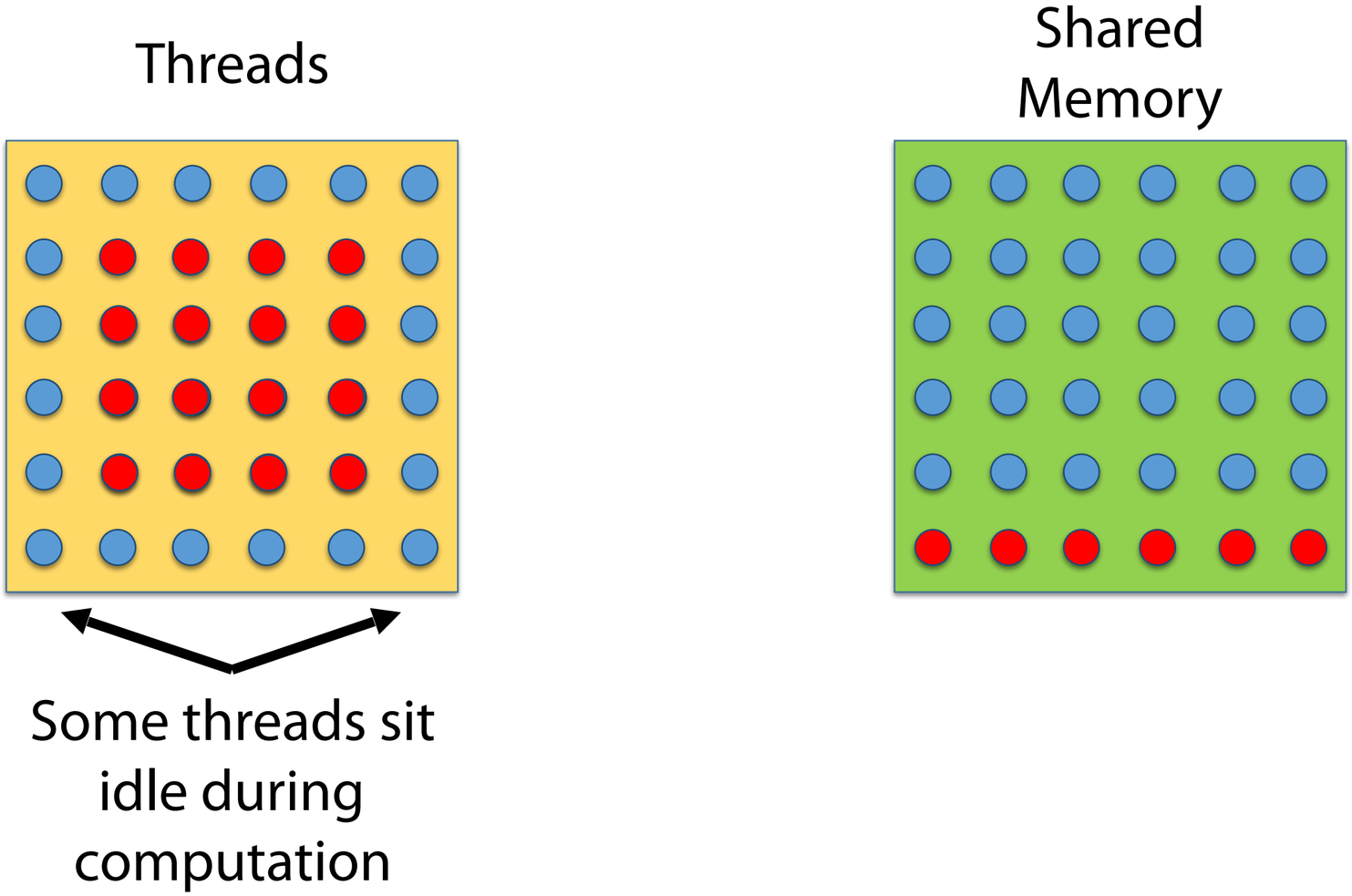
- Assign each thread block a different sub-block
- Each thread in a block reads sub-block data into shared memory
  - Use extra threads for halo/ghost region
- Update nodes in sub-block and output to global memory

Problem: Idle threads during computation

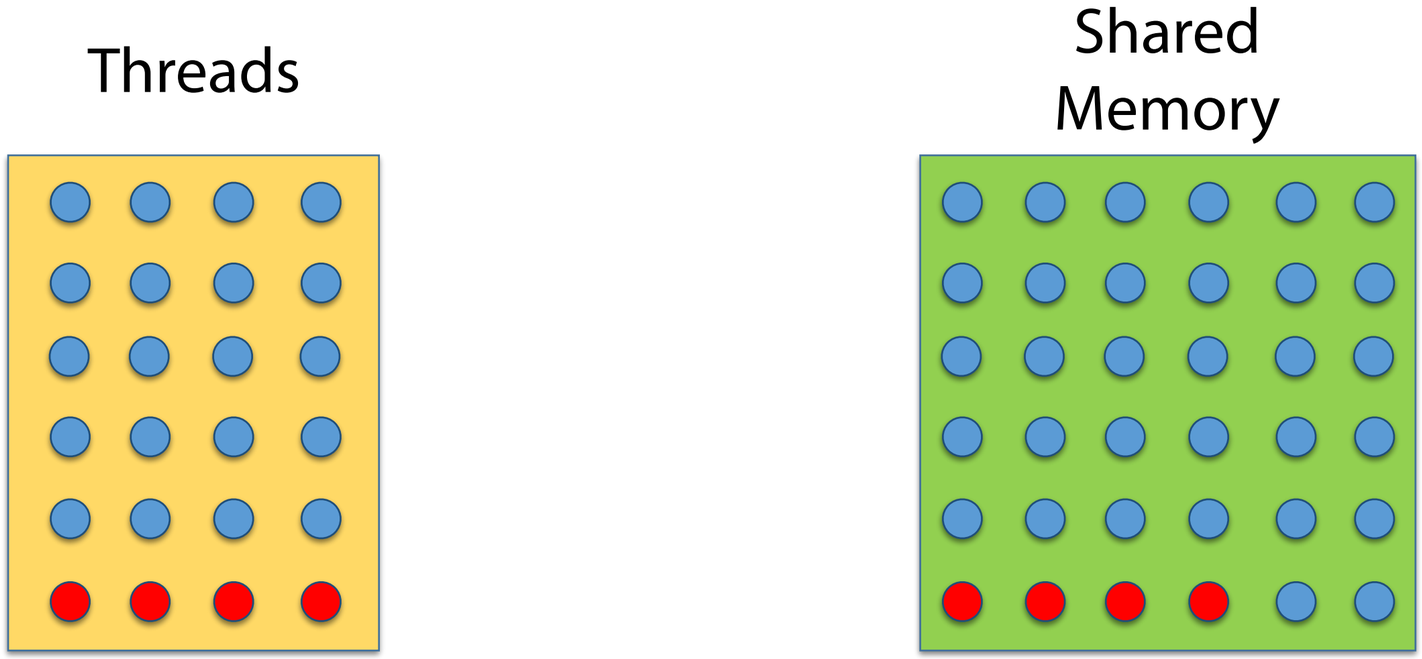


All threads active to read into shared memory

Problem: Idle threads during computation

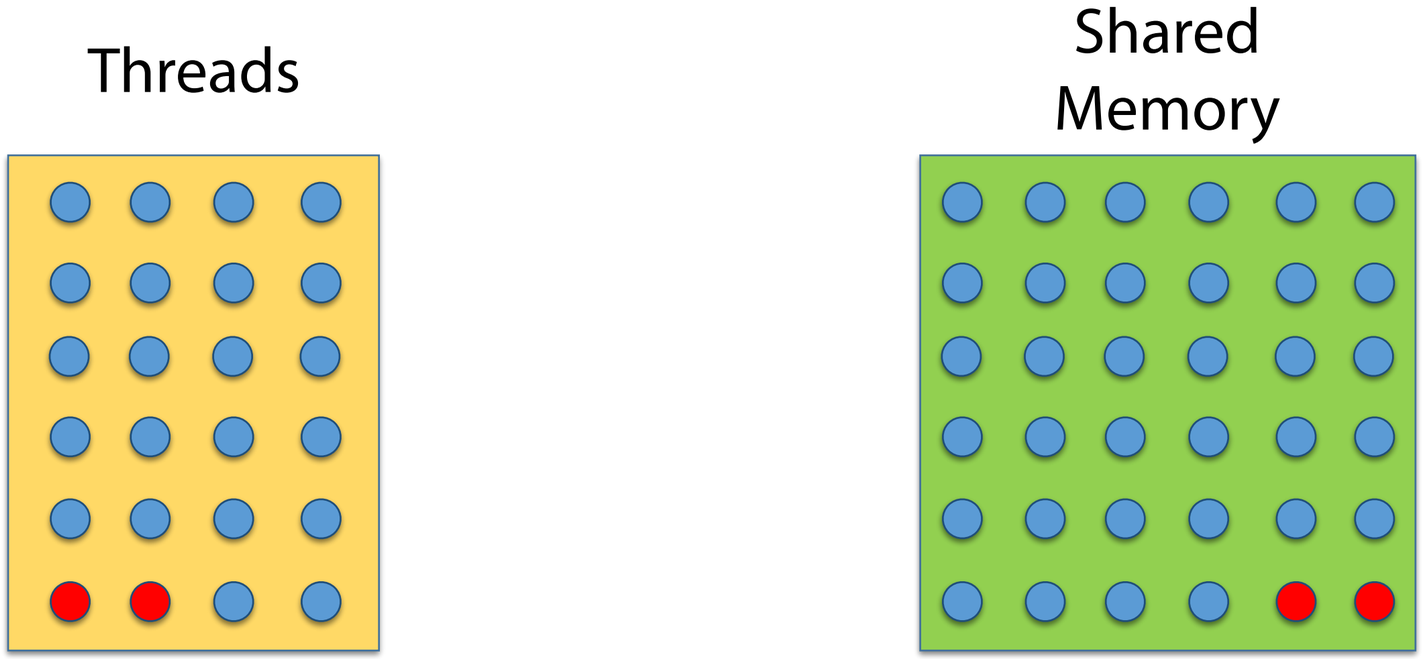


Optimization #1: No load-only threads



Launch fewer threads:
1st load – all threads read into Smem

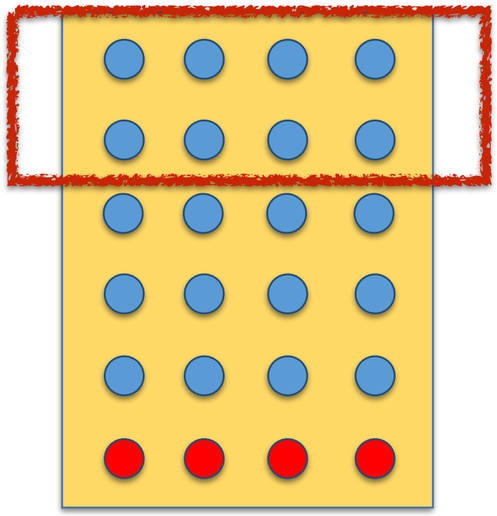
Optimization #1: No load-only threads

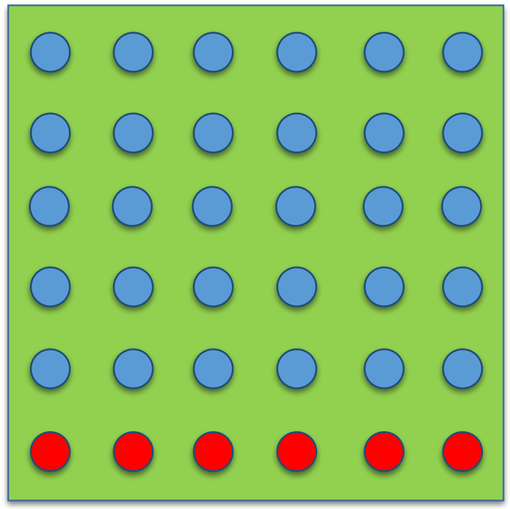


Launch fewer threads:
1st load – all threads read into Smem
2nd load – two threads read into Smem

Optimization #1: No load-only threads

Do you need these?





Launch fewer threads:
1st load – all threads read into Smem
2nd load – two threads read into Smem
All threads active during computation

Problem: Limited memory size

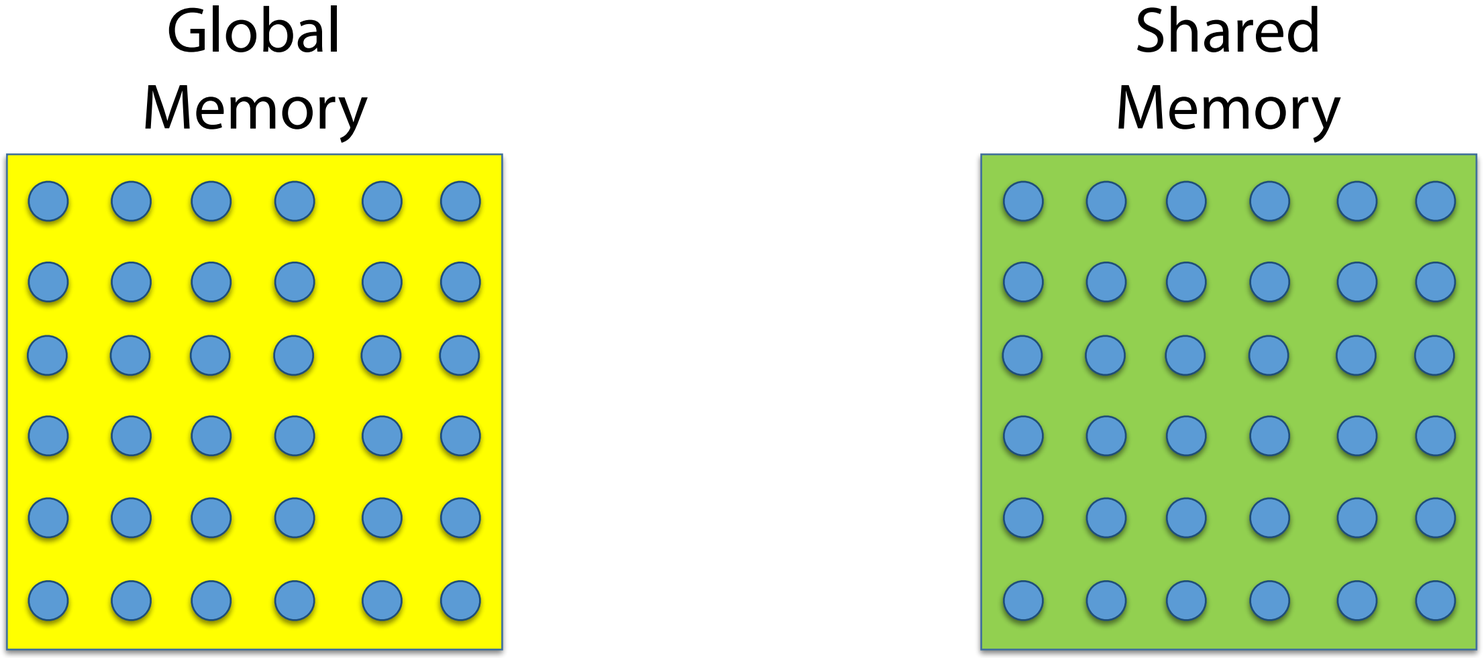
- Real applications often have many stencils, inputs, and outputs
- More data -> more shared memory per block -> fewer blocks per SM -> less latency hiding potential
- E.g.,
  - Inputs: A, B, C, D, E
  - Outputs: Y1, Y2, Y3
  - Stencils: f1, f2, f3, f4
  - $Y1 = f1(A, B, C) + f2(D, E)$
  - $Y2 = Y1 + f3(B, C, D)$
  - $Y3 = Y1 + Y2 + f4(A,E)$

Optimization #2: Use other memory resources

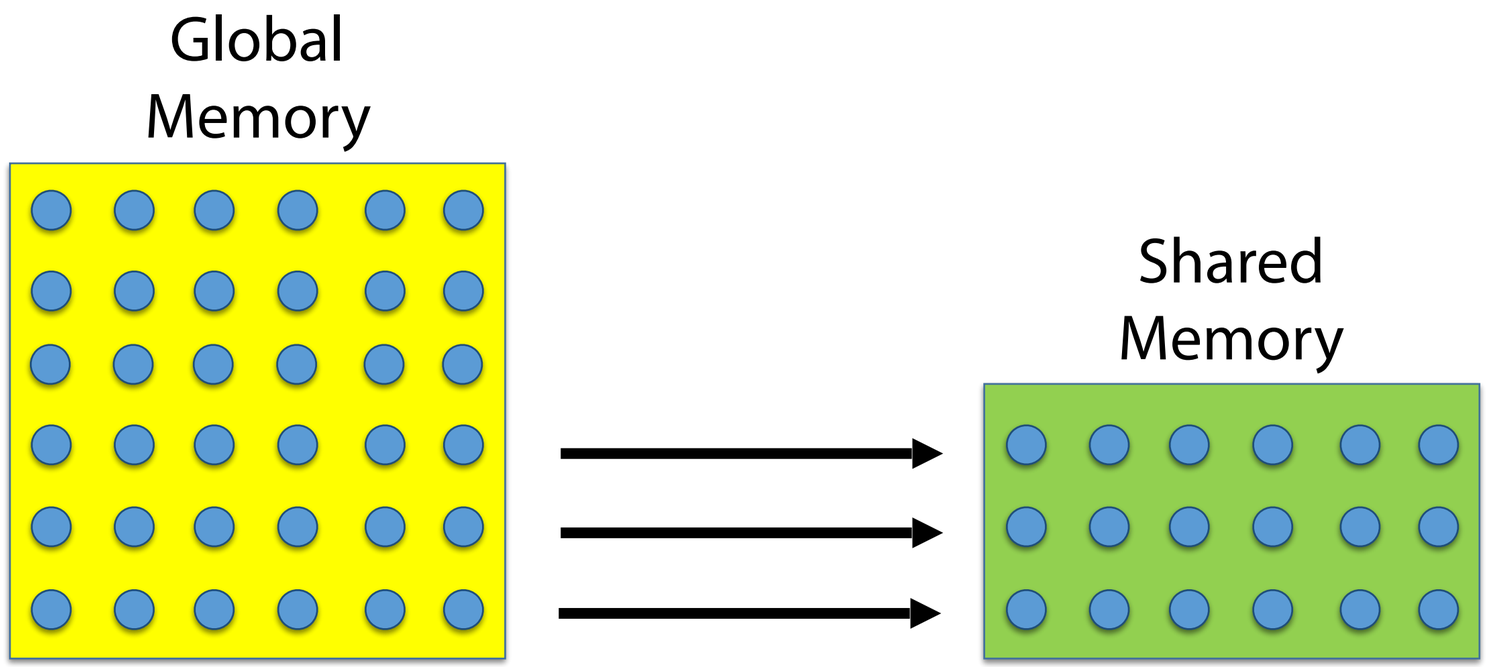
- Put some data into registers (more on this later)

- Store some inputs in texture memory
  - Use data directly from texture memory (don't load it into shared memory)
  - Optimized for 2D spatial locality
  - Faster than global memory for access patterns that exploit 2D spatial locality

Problem: Shared memory is limited, but we want to maximize reuse

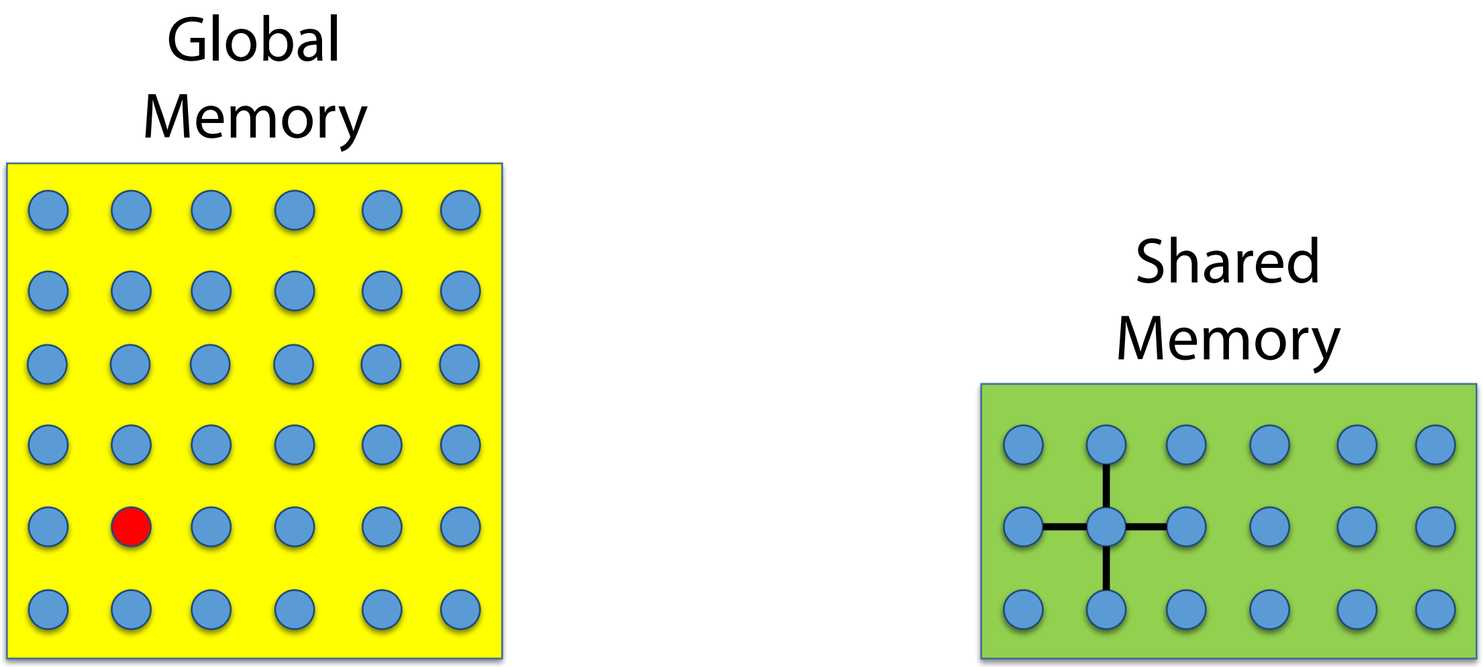


Optimization #3: March through domain plane-by-plane



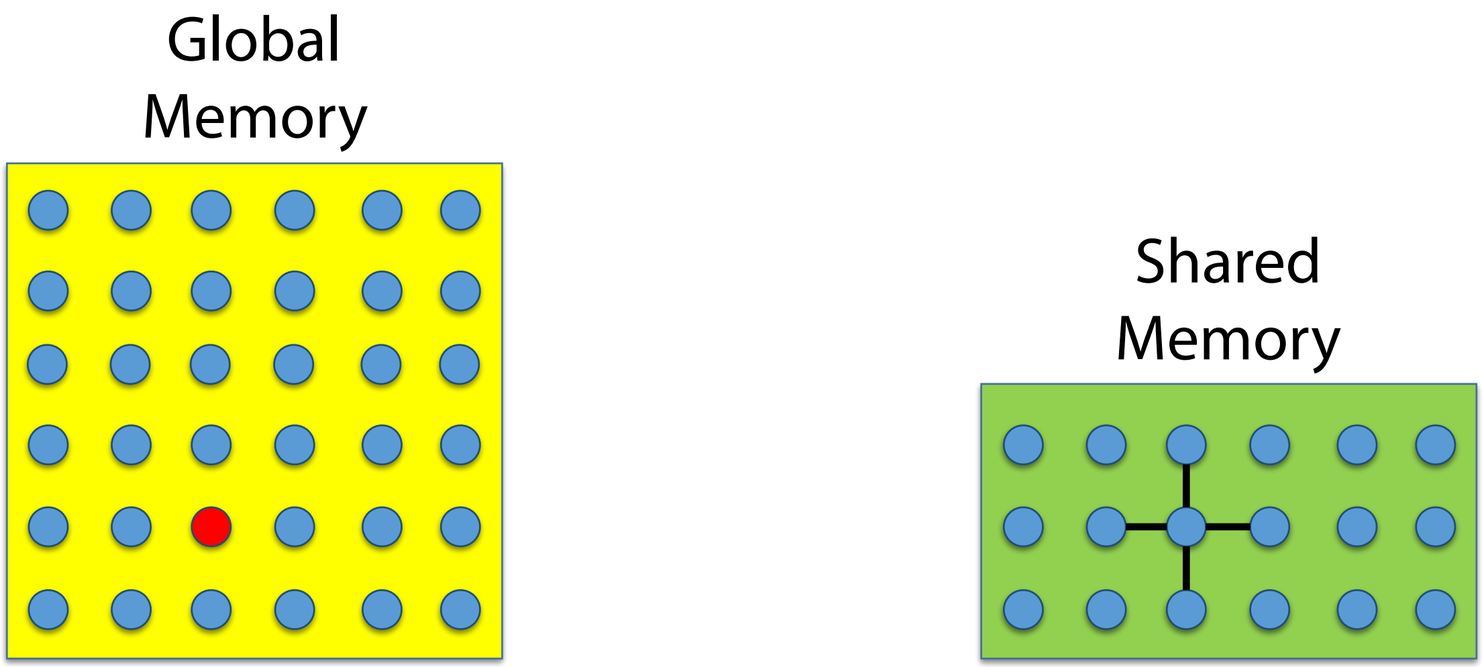
Fill shared memory array

Optimization #3: March through domain plane-by-plane



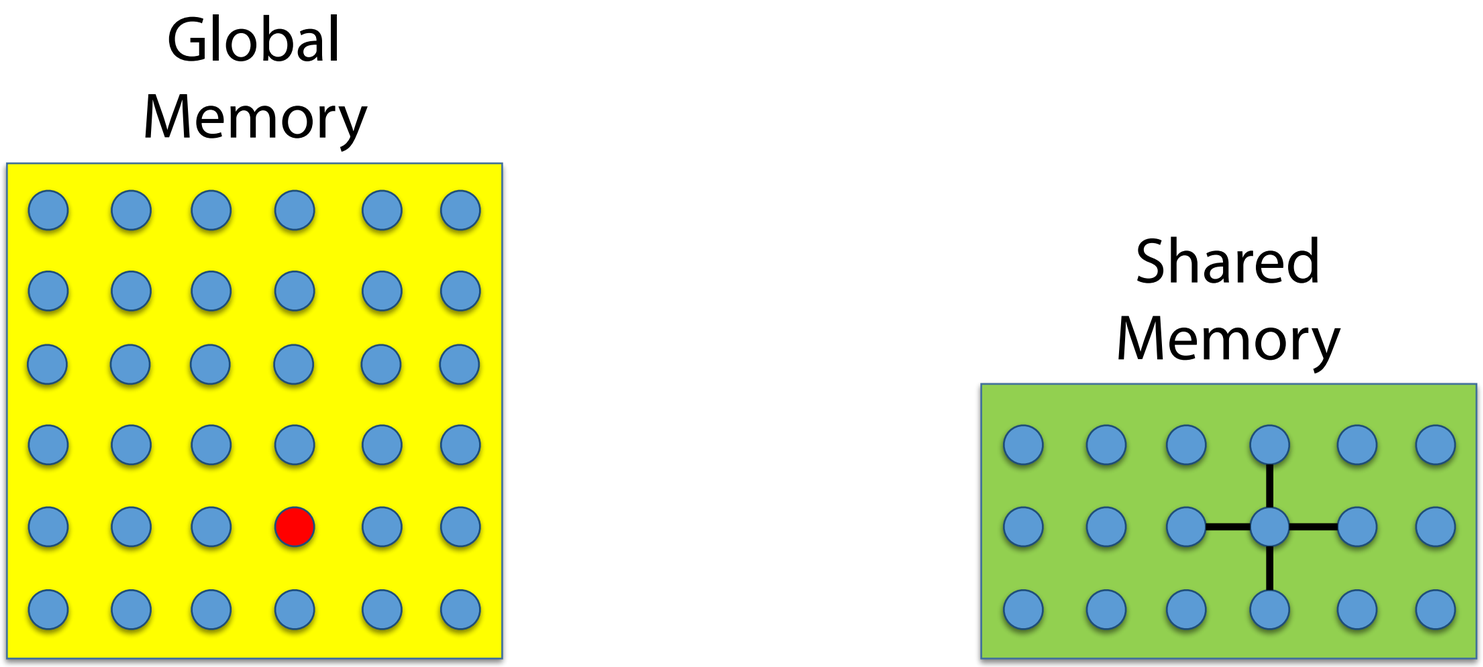
Evaluate stencil and store result in global memory

Optimization #3: March through domain plane-by-plane



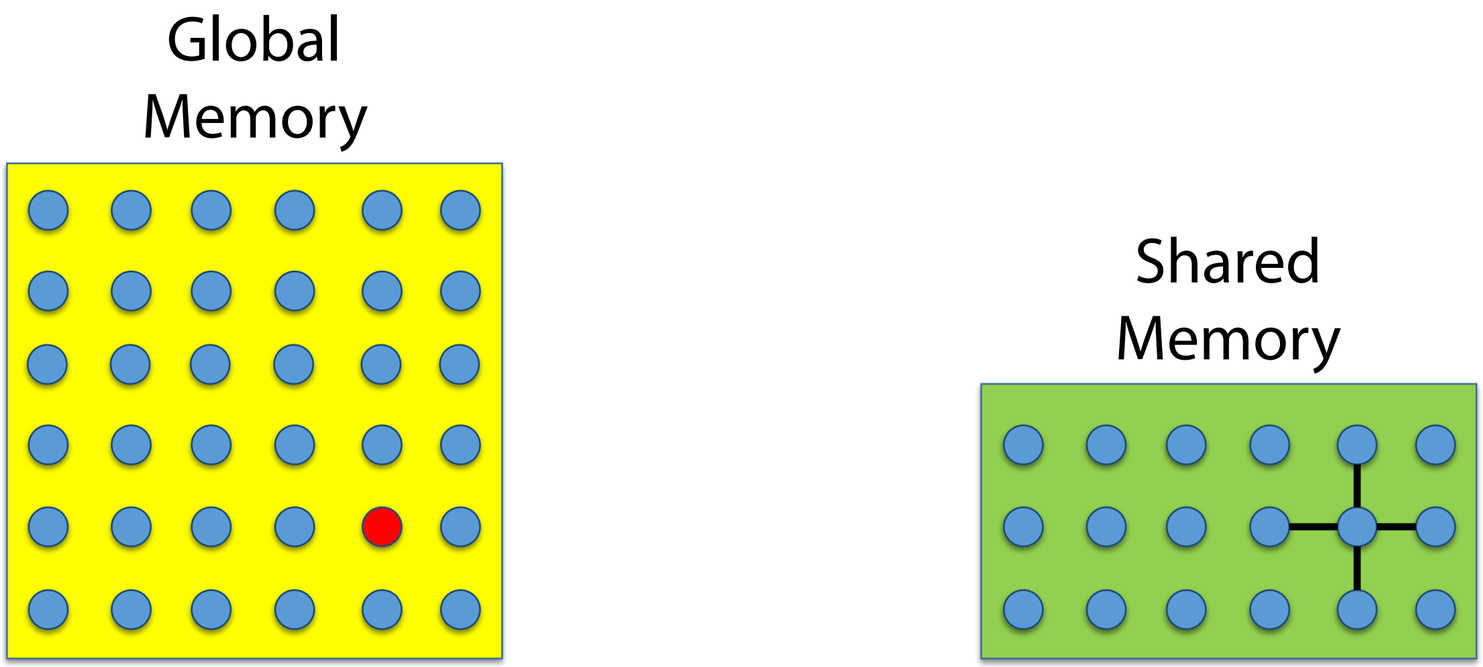
Evaluate stencil and store result in global memory

Optimization #3: March through domain plane-by-plane



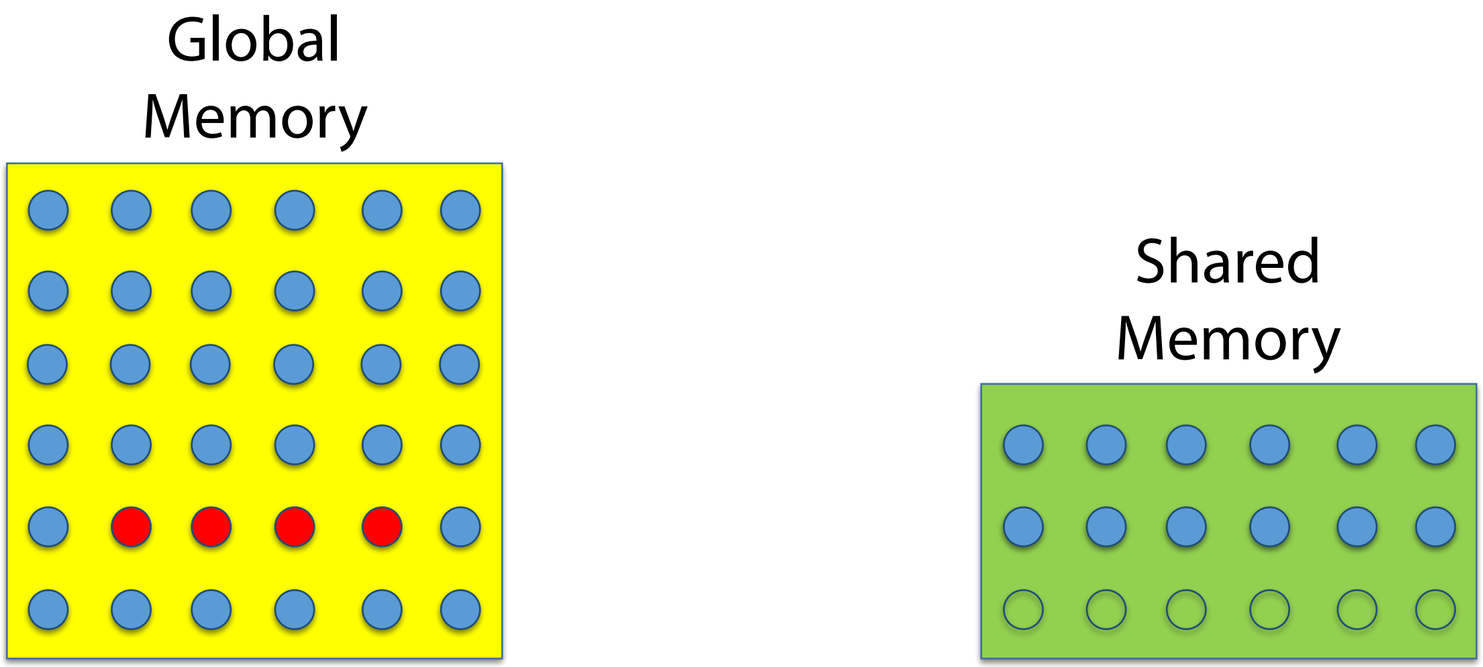
Evaluate stencil and store result in global memory

Optimization #3: March through domain plane-by-plane



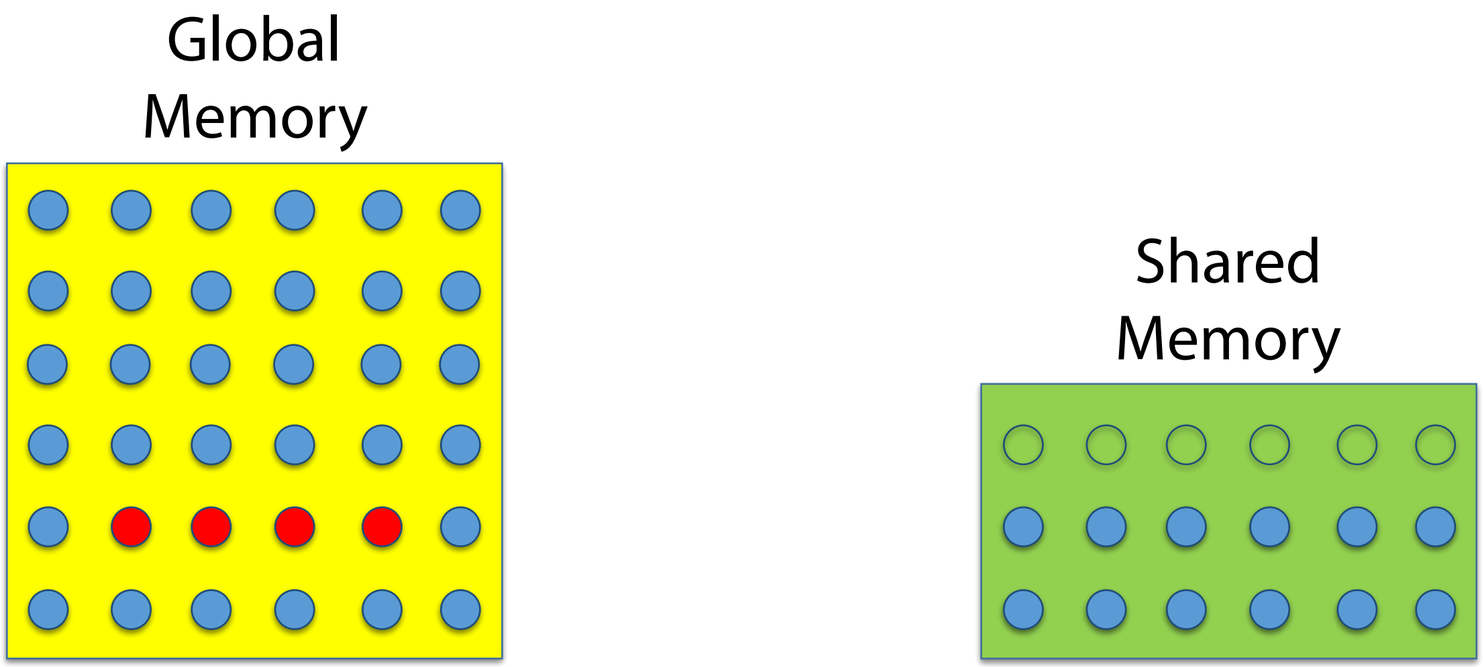
Evaluate stencil and store result in global memory

Optimization #3: March through domain plane-by-plane



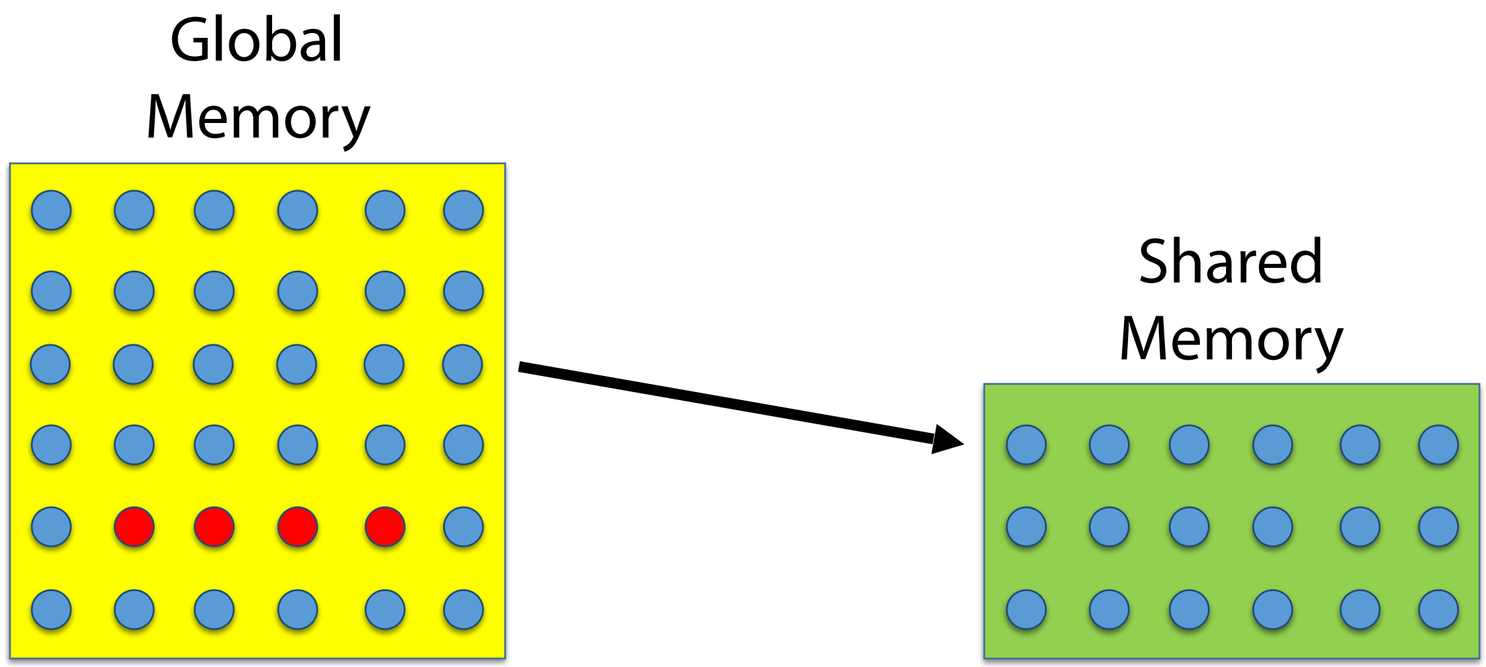
Load next row into shared memory

Optimization #3: March through domain plane-by-plane



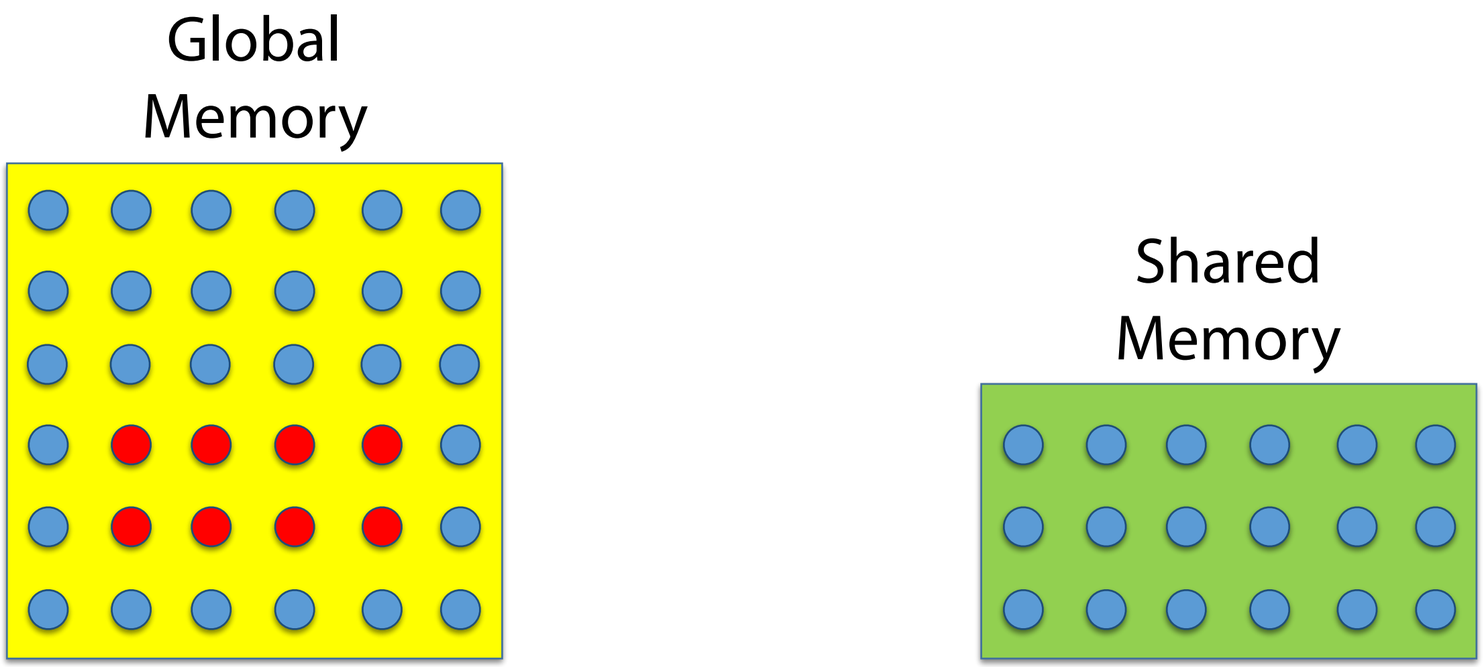
Load next row into shared memory

Optimization #3: March through domain plane-by-plane



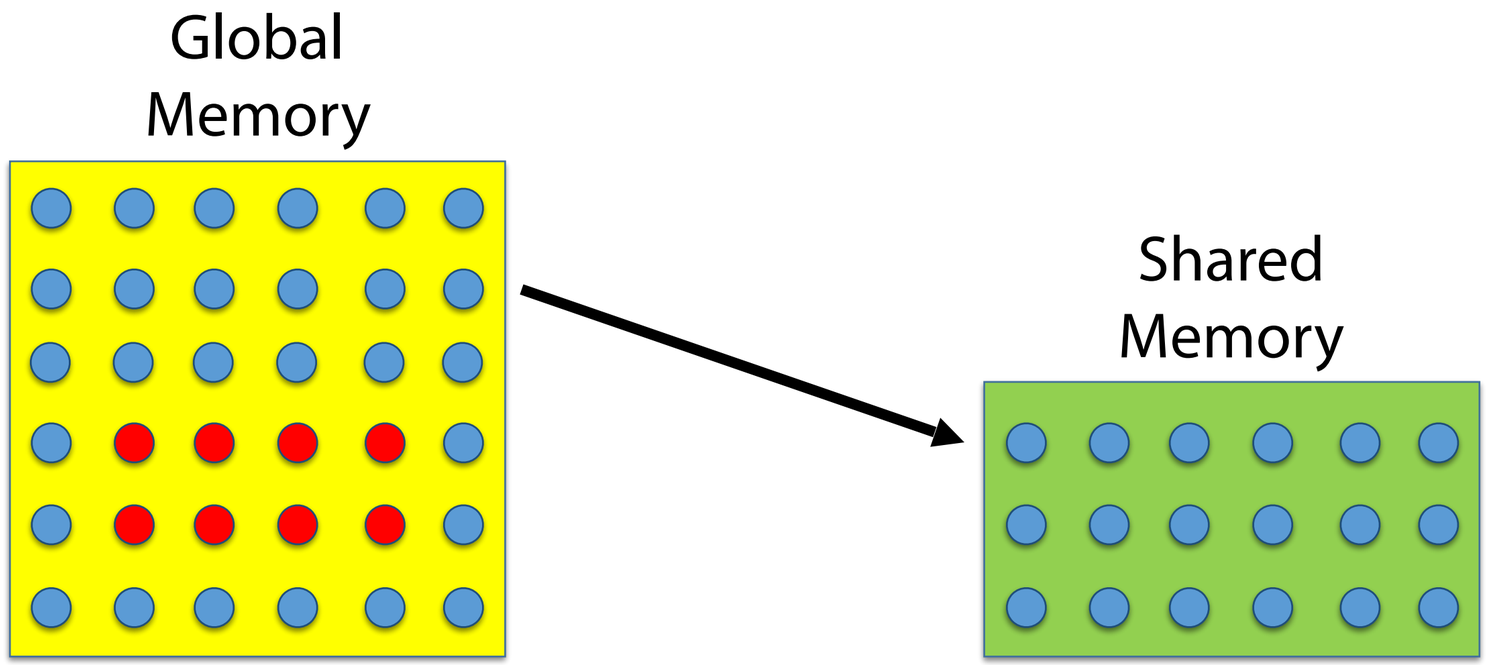
Load next row into shared memory

Optimization #3: March through domain plane-by-plane



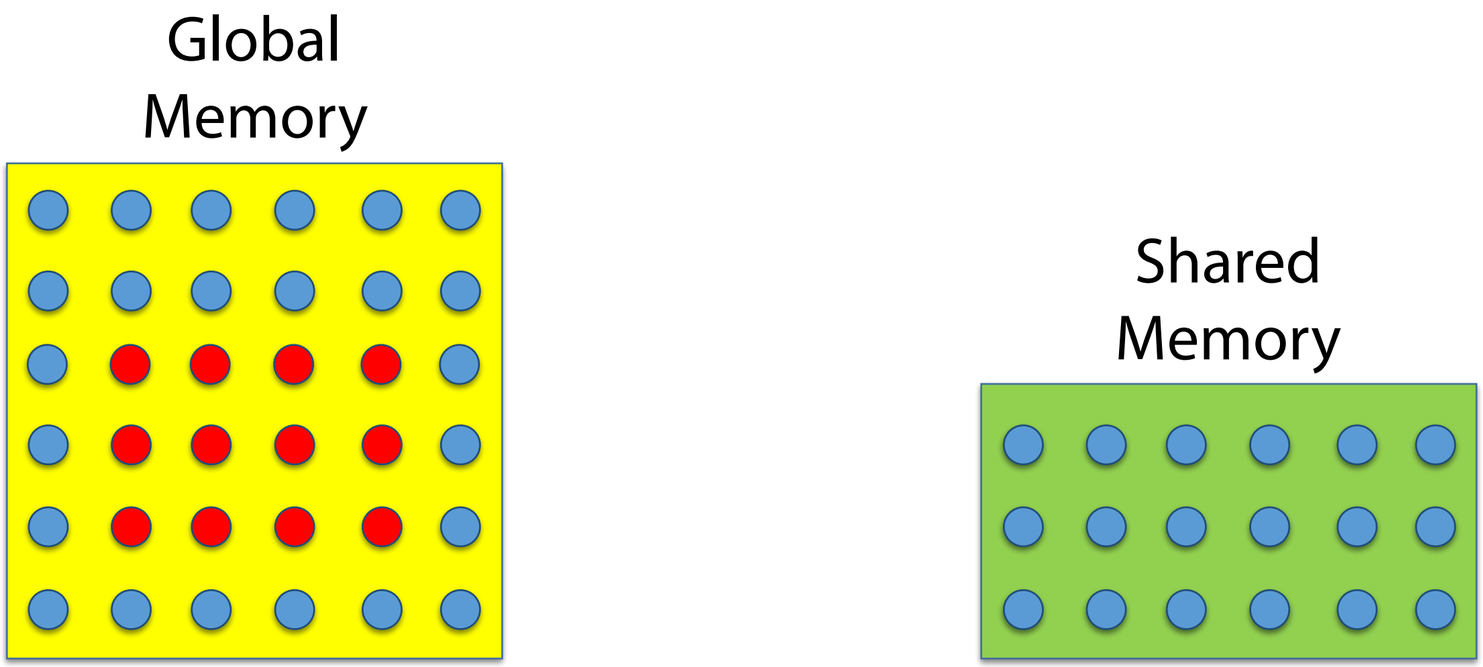
Load next row into shared memory

Optimization #3: March through domain plane-by-plane



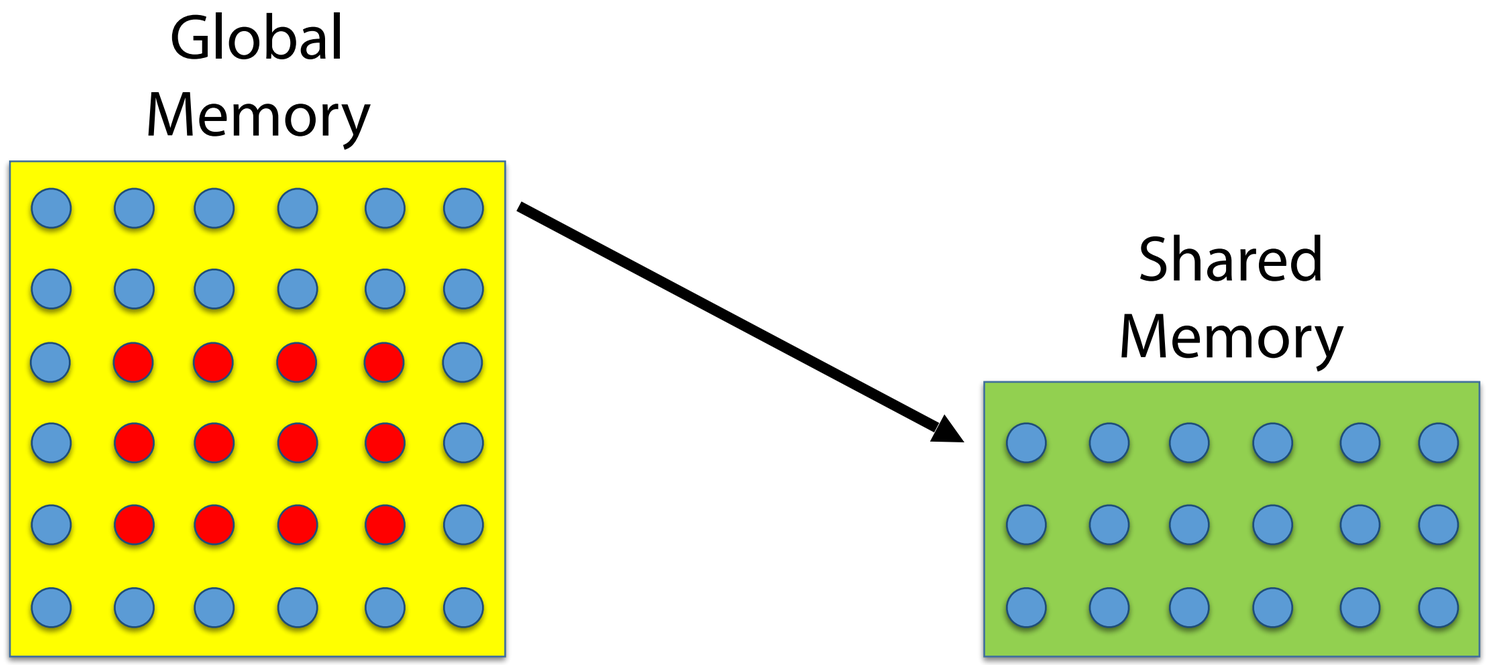
Load next row into shared memory

Optimization #3: March through domain plane-by-plane



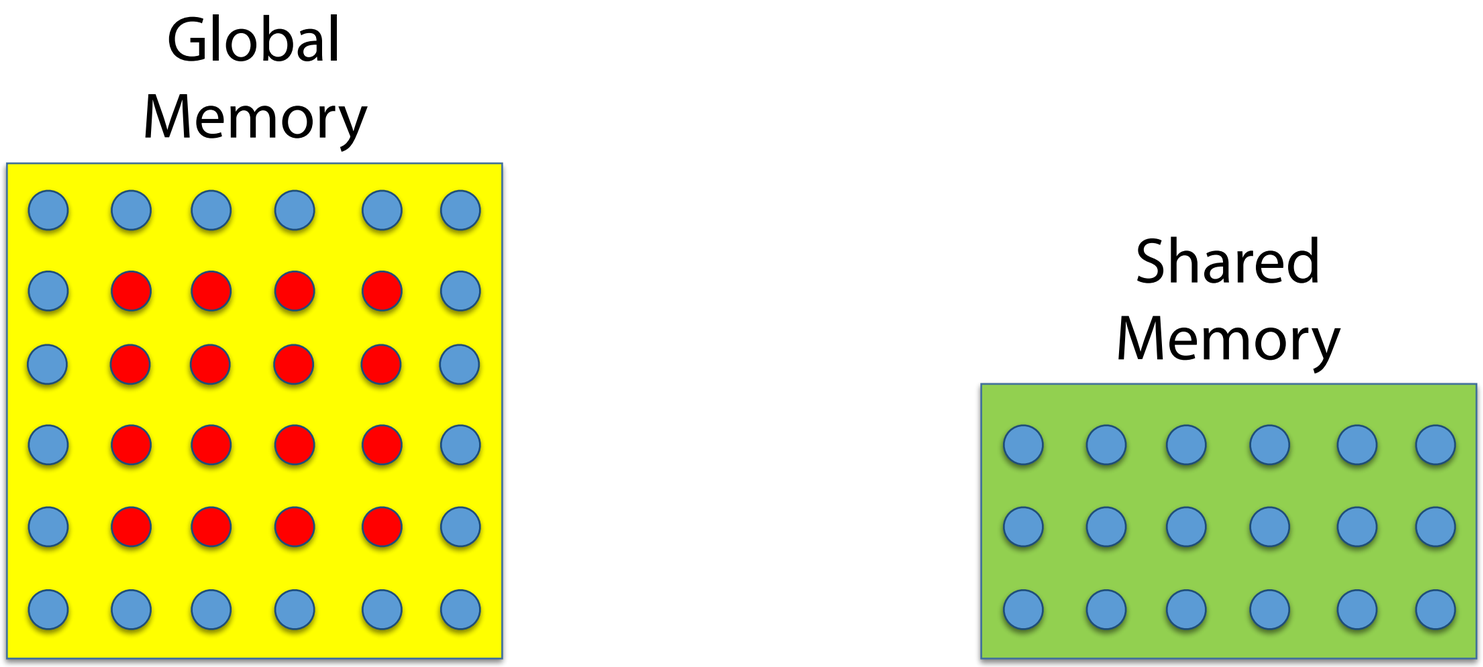
Load next row into shared memory

Optimization #3: March through domain plane-by-plane



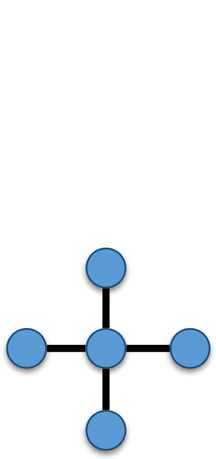
Load next row into shared memory

Optimization #3: March through domain plane-by-plane

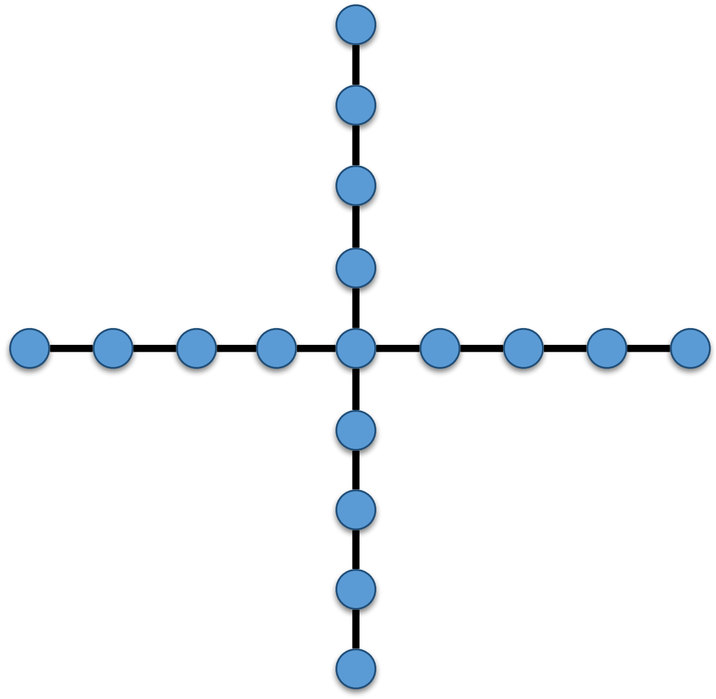


Load next row into shared memory

Optimization #4: High-order stencils

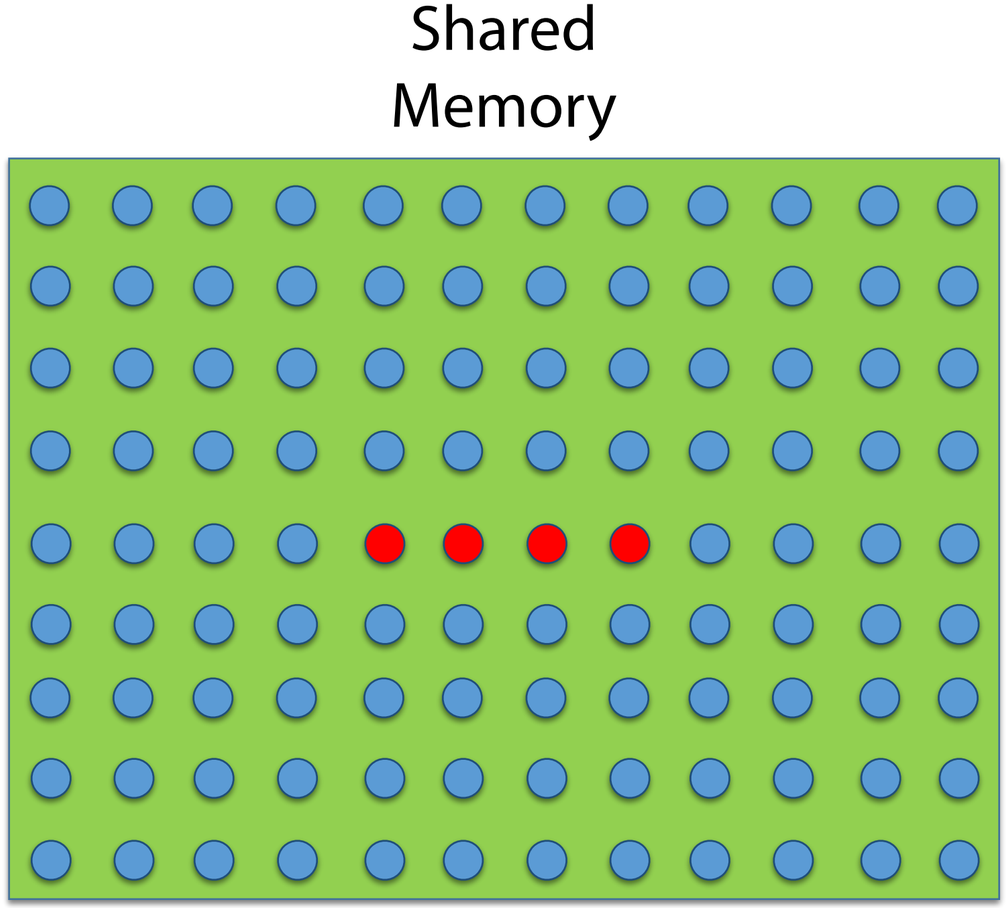


Second Order



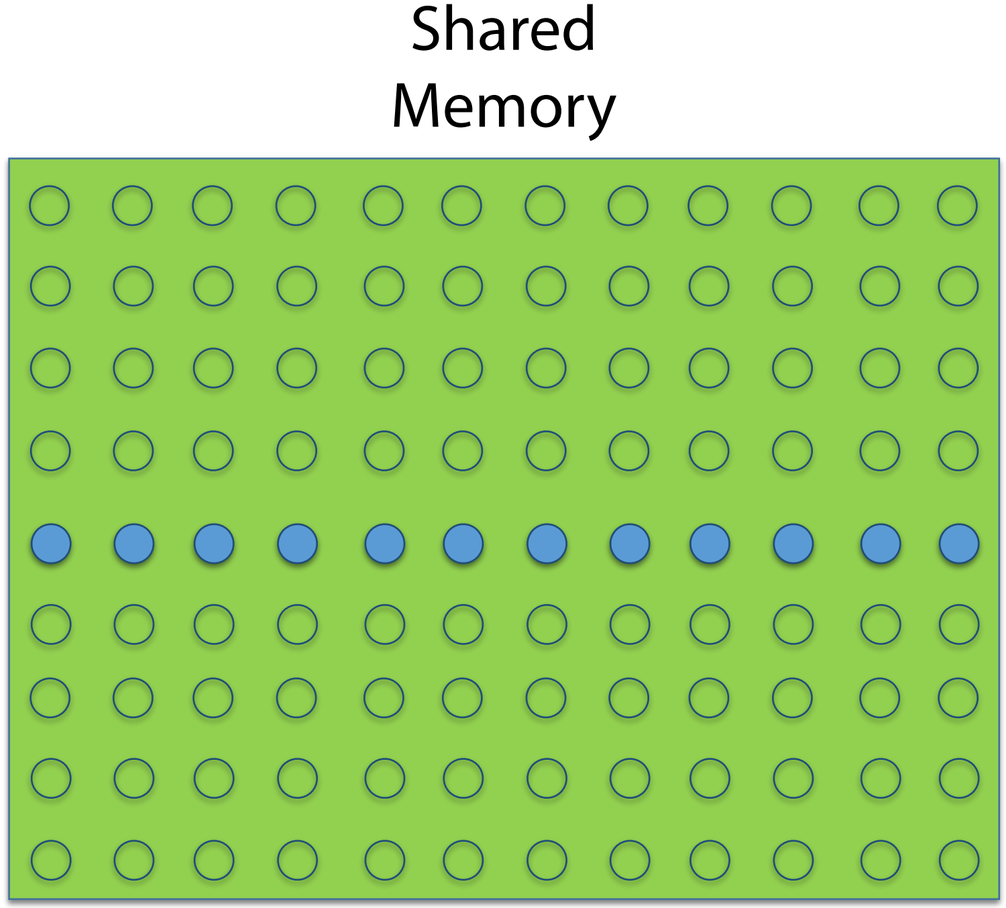
Eighth Order

Optimization #4: High-order stencils



Need 9 planes to compute 1

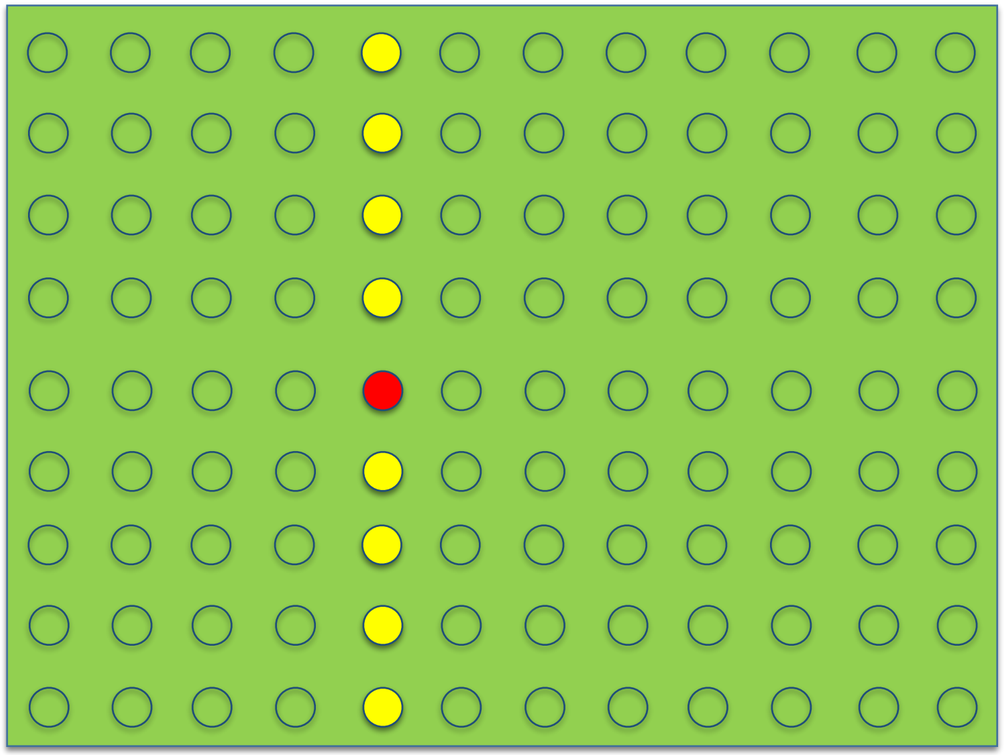
Optimization #4: High-order stencils



Only the grid nodes in one plane need to be shared to evaluate the stencil – store these in shared memory

Optimization #4: High-order stencils

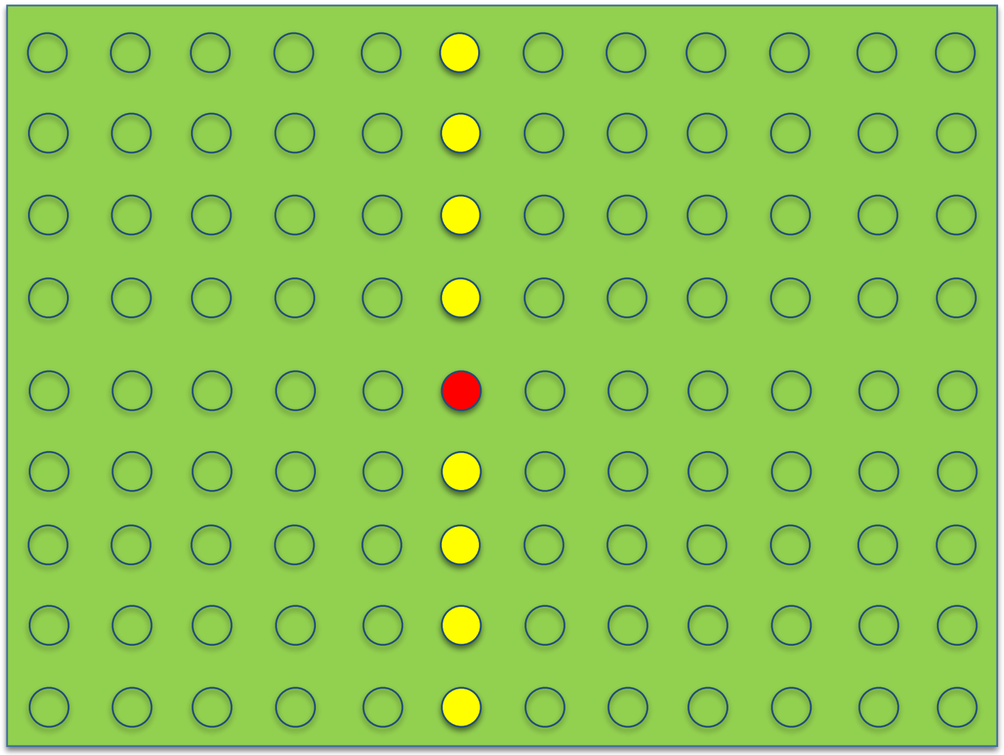
Registers



The other grid elements can be stored in registers for each thread

Optimization #4: High-order stencils

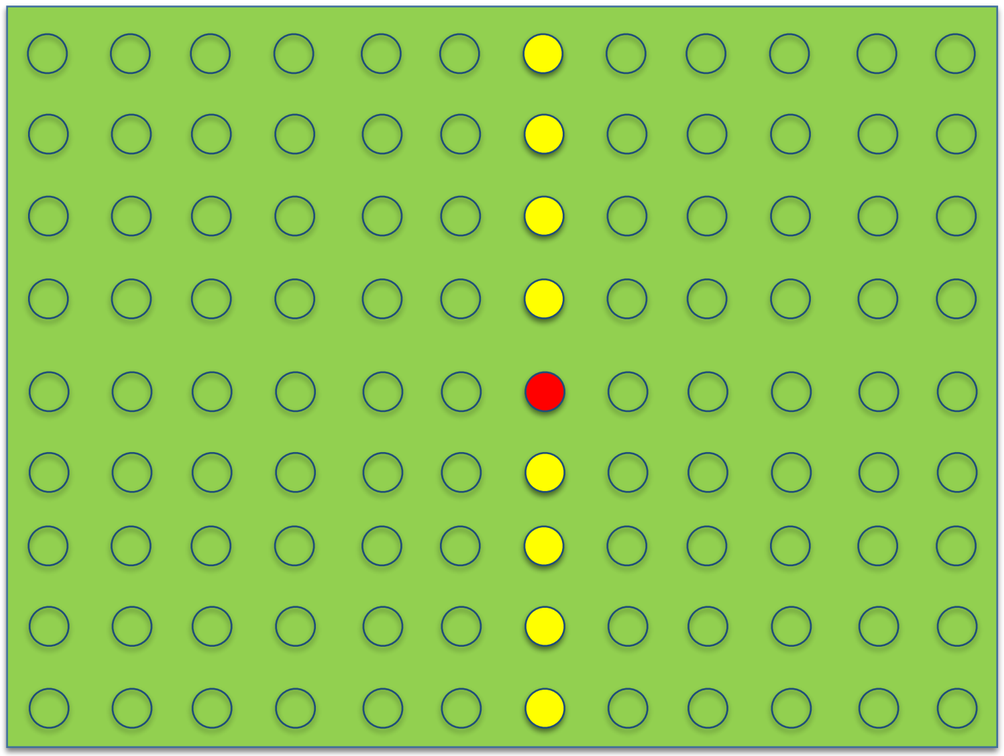
Registers



The other grid elements can be stored in registers for each thread

Optimization #4: High-order stencils

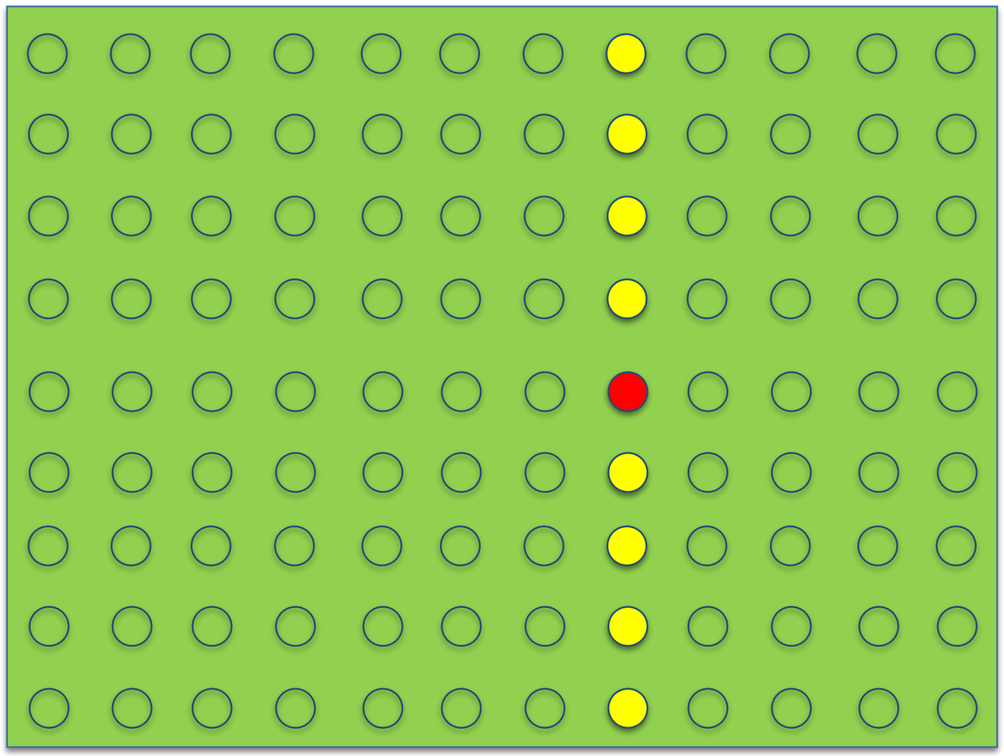
Registers



The other grid elements can be stored in registers for each thread

Optimization #4: High-order stencils

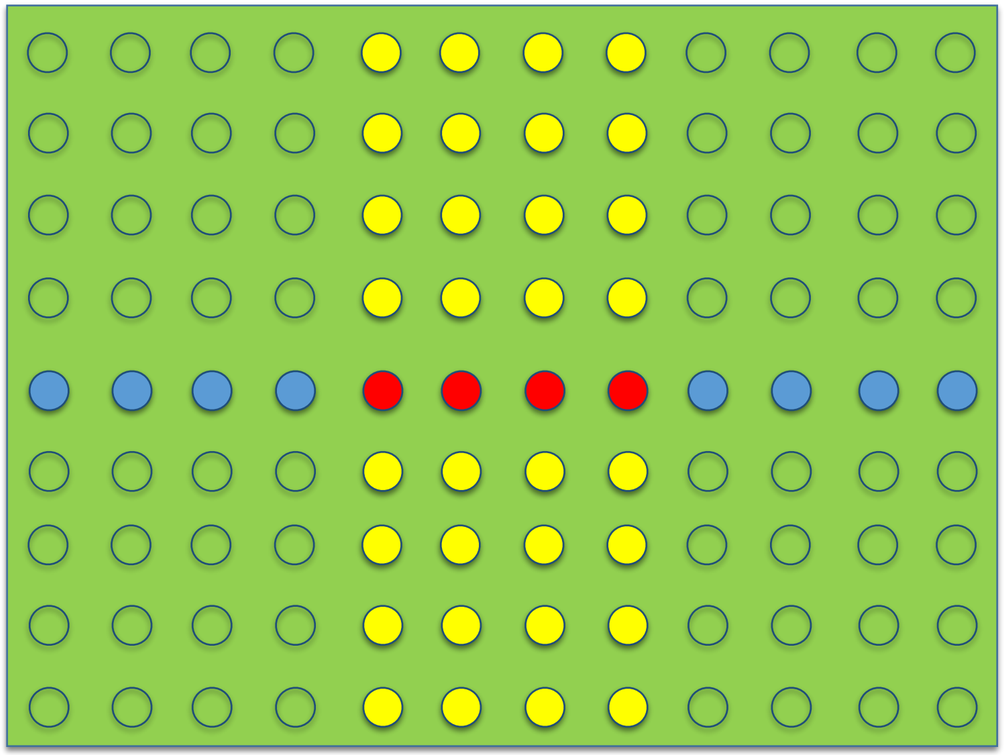
Registers



The other grid elements can be stored in registers for each thread

Optimization #4: High-order stencils

Shared Memory and
Registers



Final mix of shared memory and register storage

Application: Computational Fluid Dynamics (CFD)

- Uses numerical analysis and algorithms to solve and analyze fluid flow problems

- Solve system of partial differential equations (PDE) to determine flow

- Often use iterative solvers like Gauss-Seidel or Jacobi

https://en.wikipedia.org/wiki/Computational_fluid_dynamics

Application: Computational Fluid Dynamics (CFD)

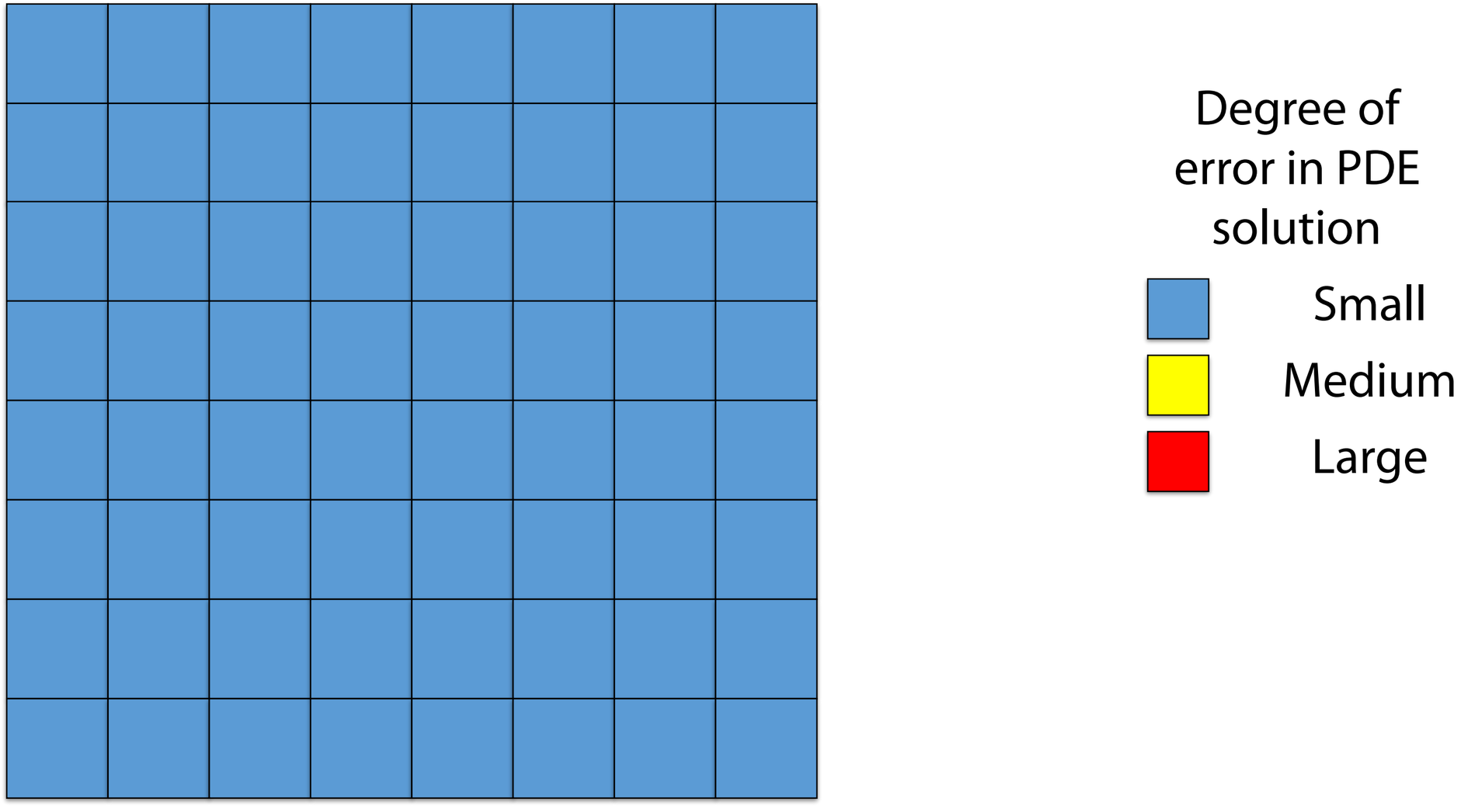
- Start with initial fluid state in each cell

- At each timestep, solve the PDEs
  - These are (basically) stencil operations

- Once PDEs converge, begin next timestep

https://en.wikipedia.org/wiki/Computational_fluid_dynamics

Application: Computational Fluid Dynamics (CFD)



Degree of error in PDE solution

Small

Medium

Large

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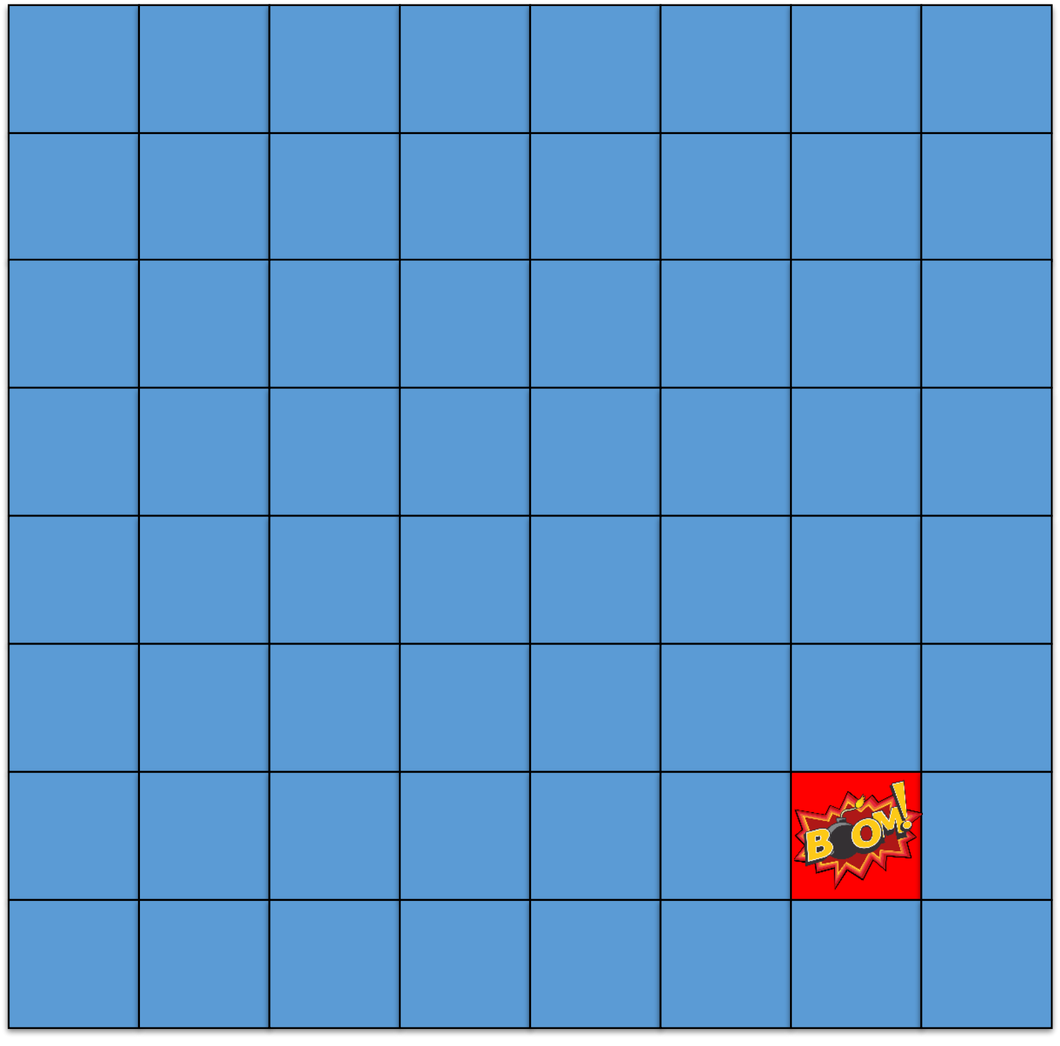

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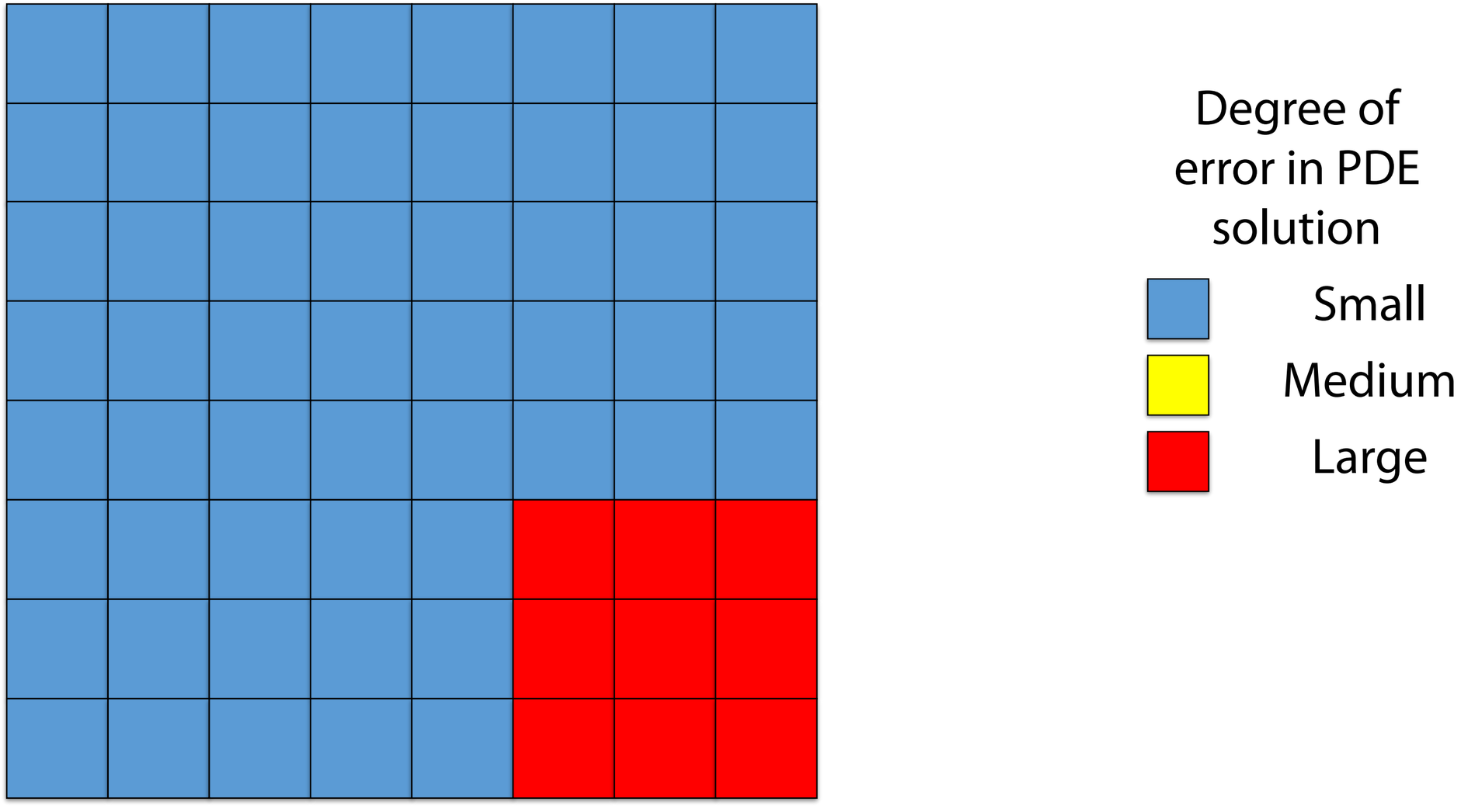

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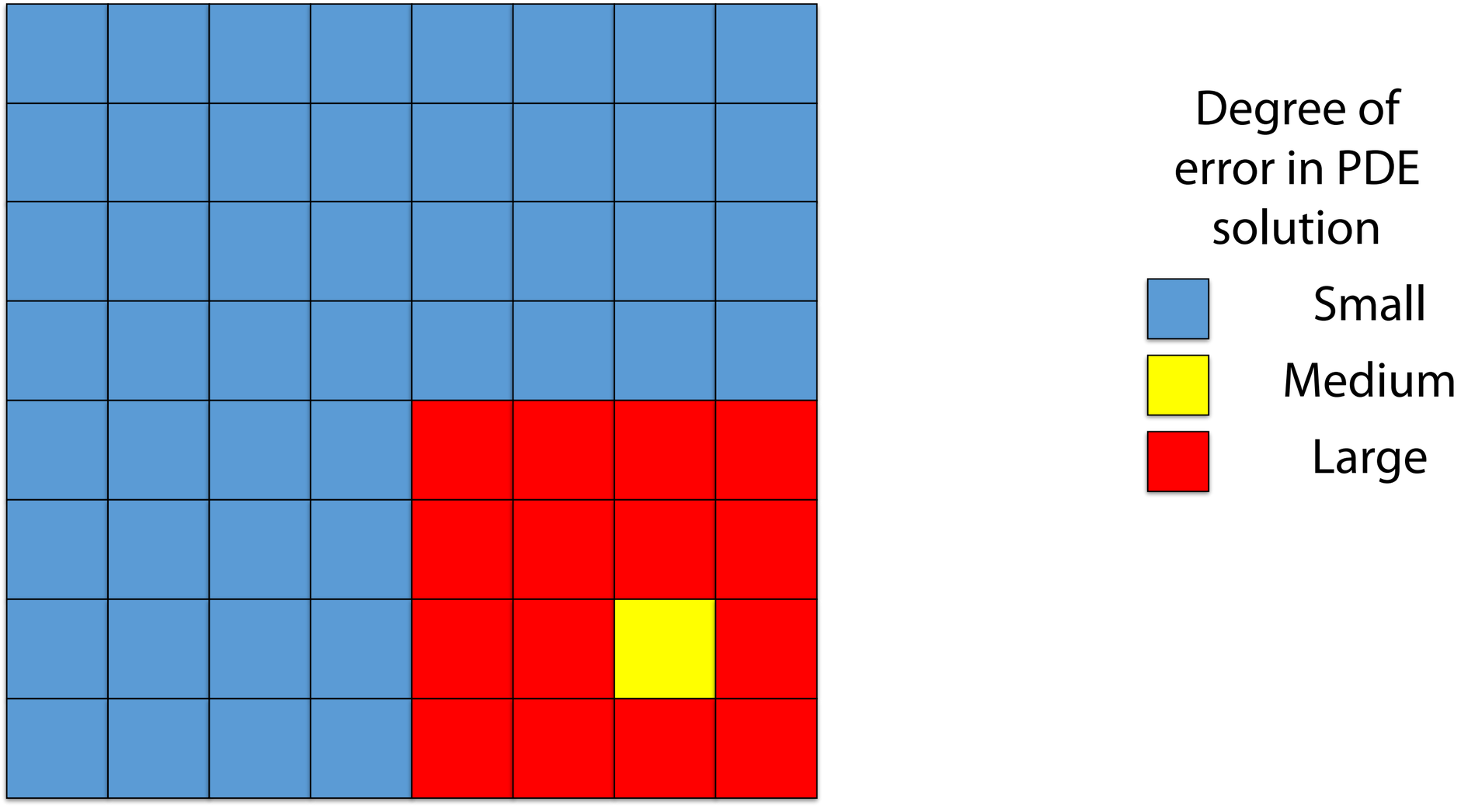

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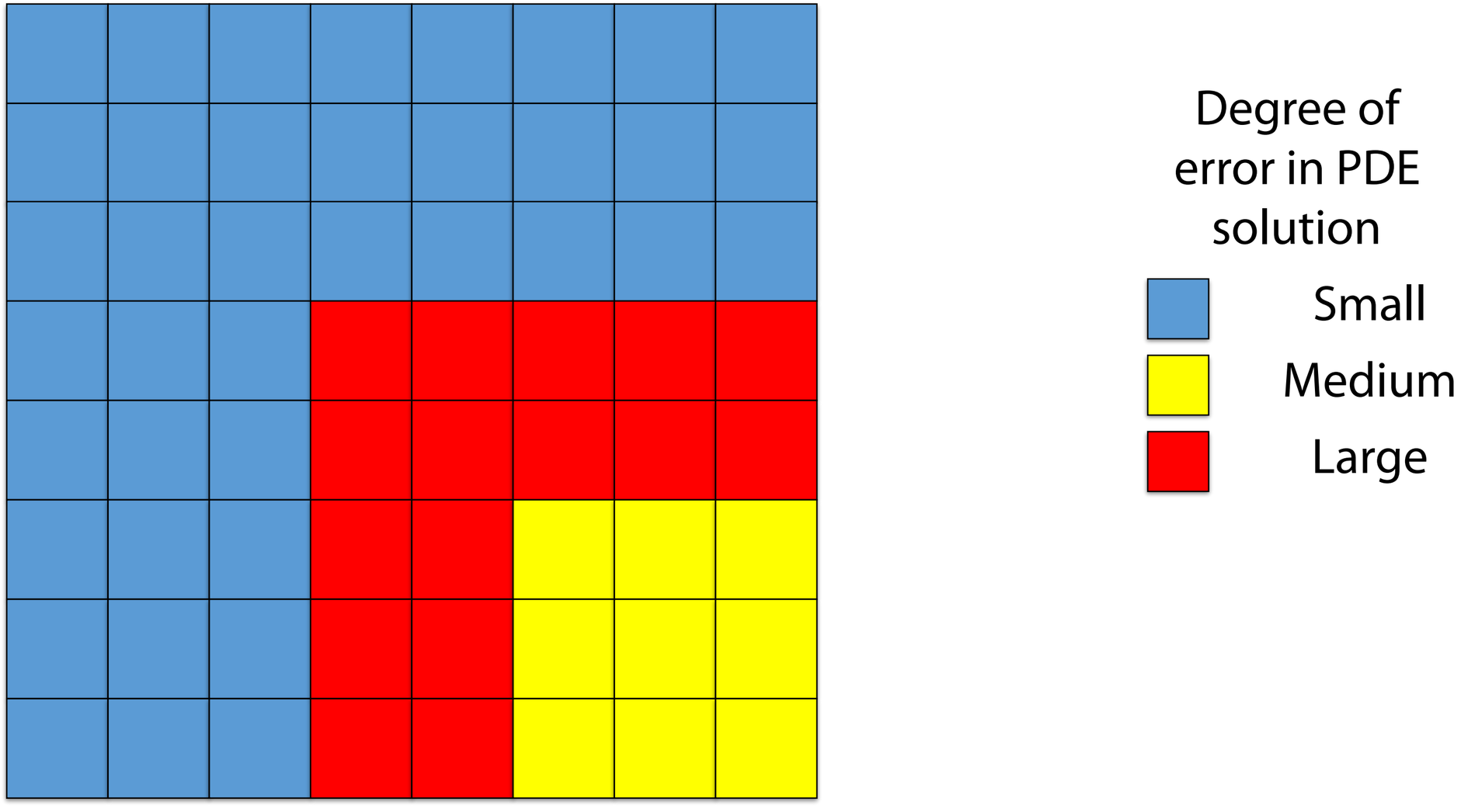

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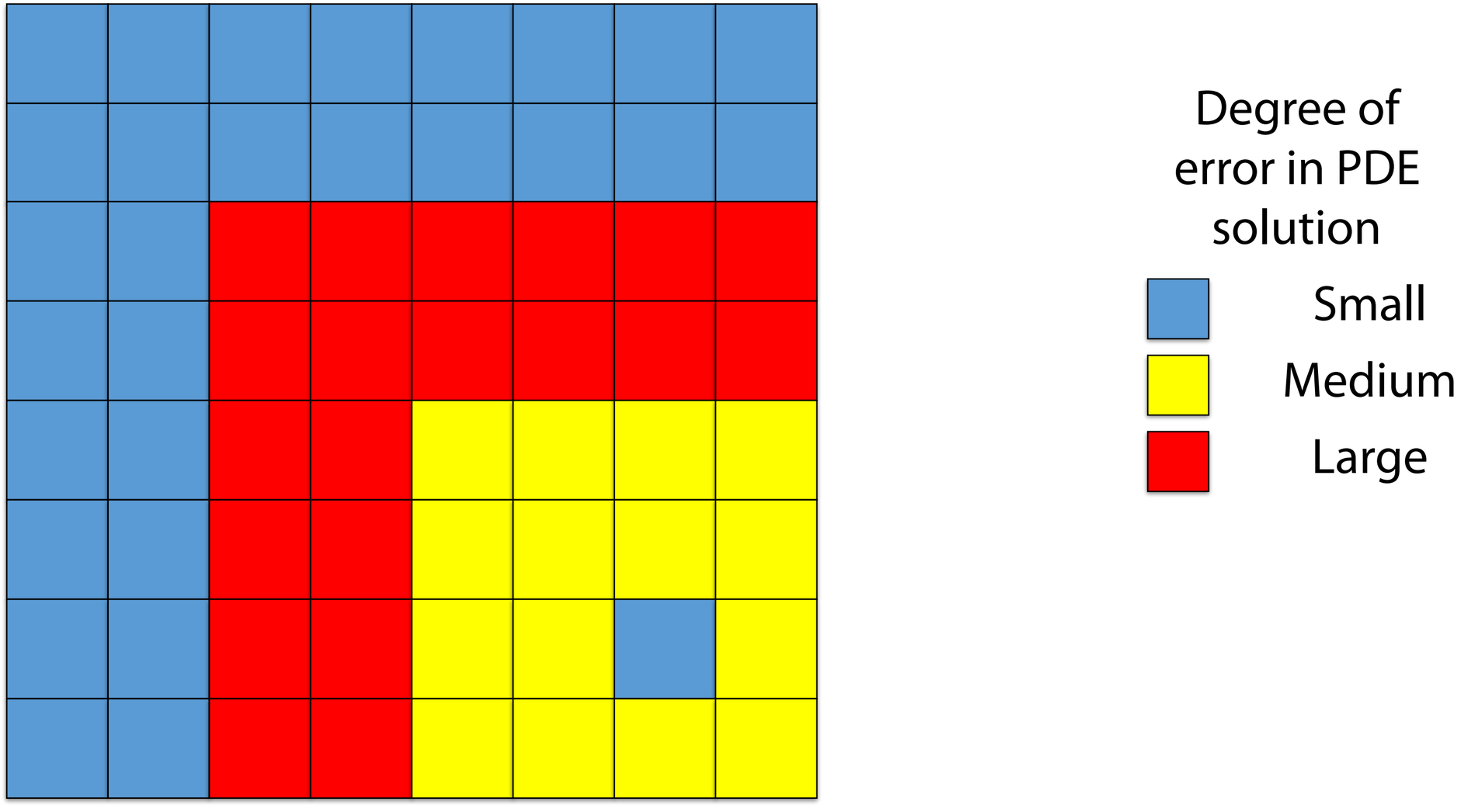

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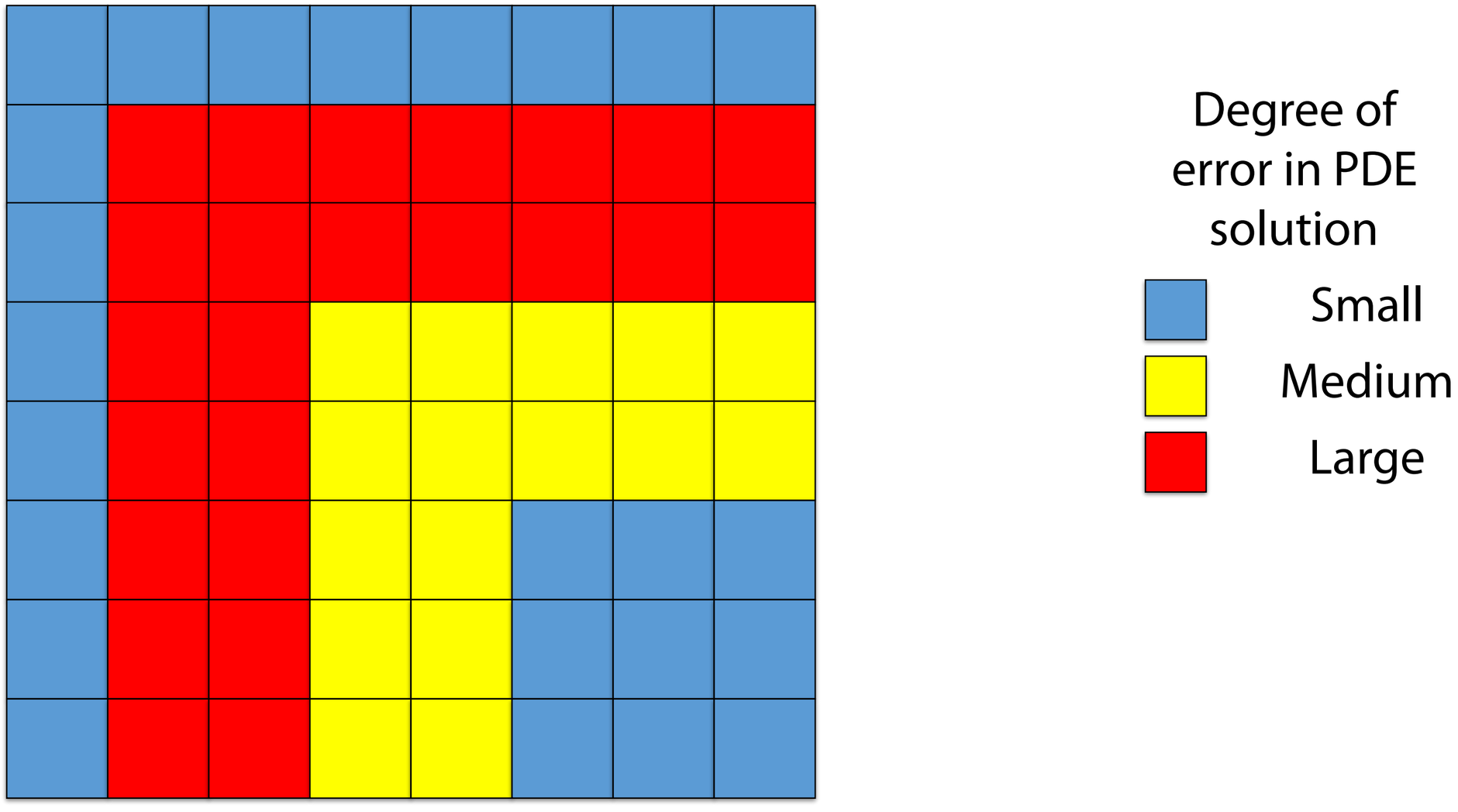

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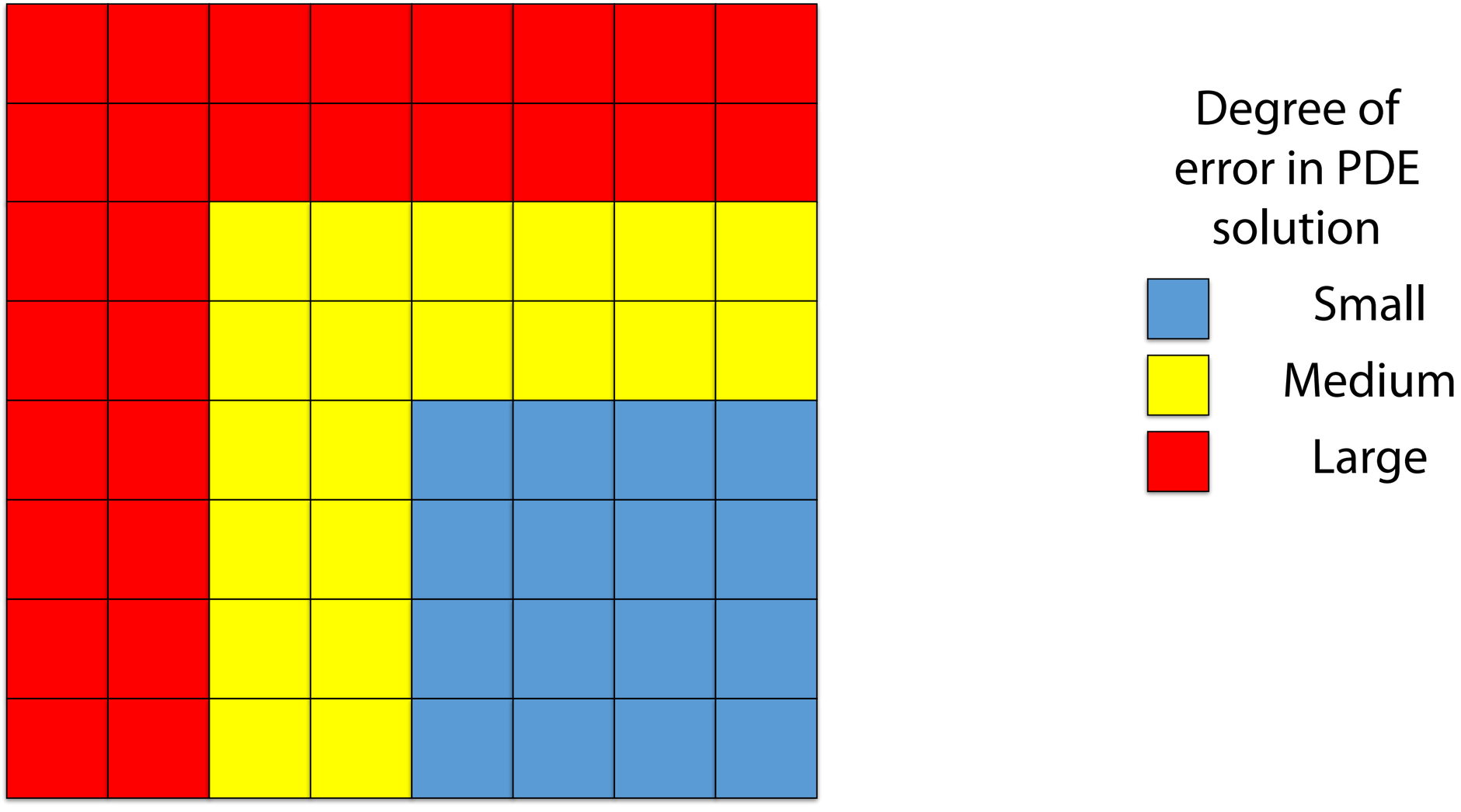

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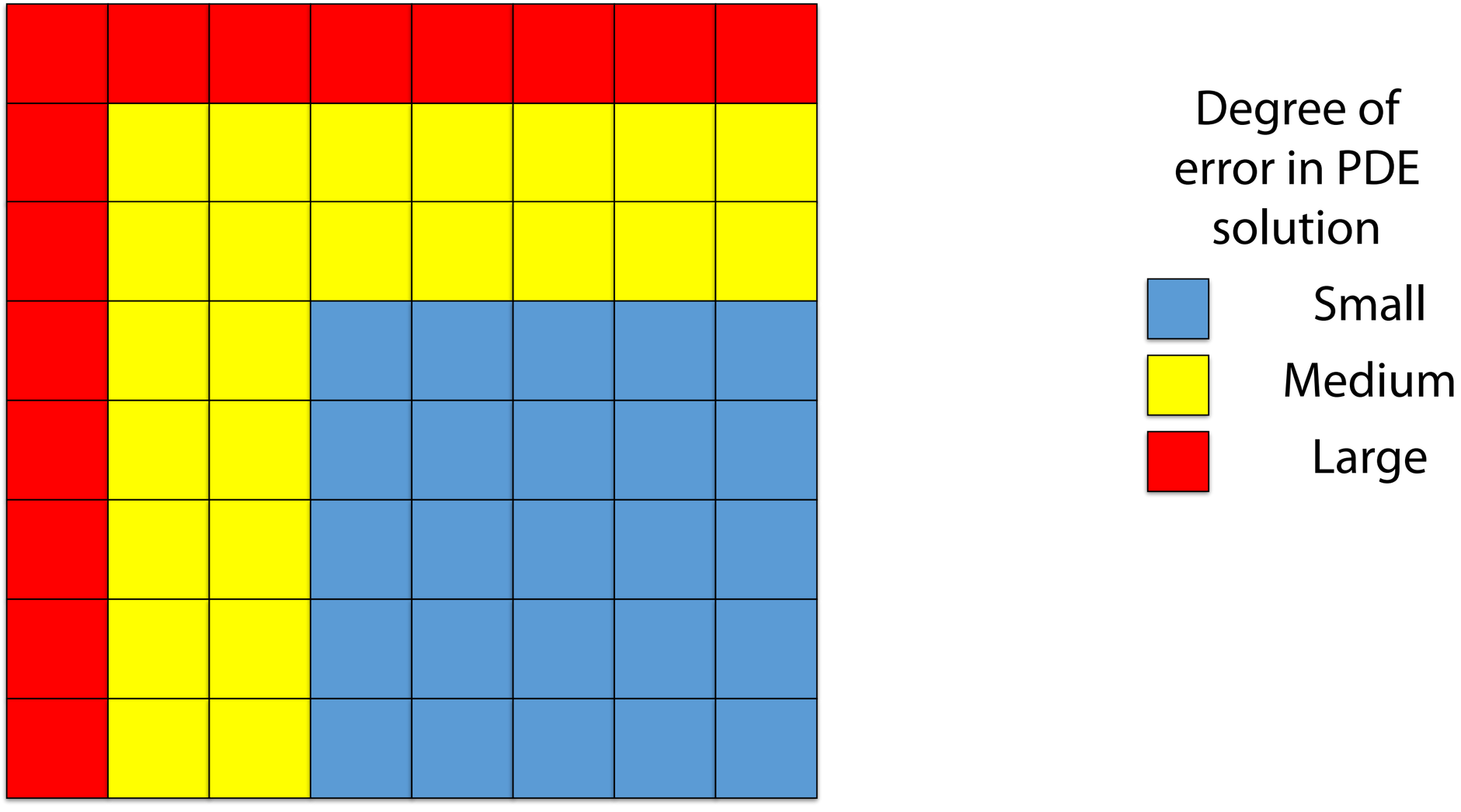

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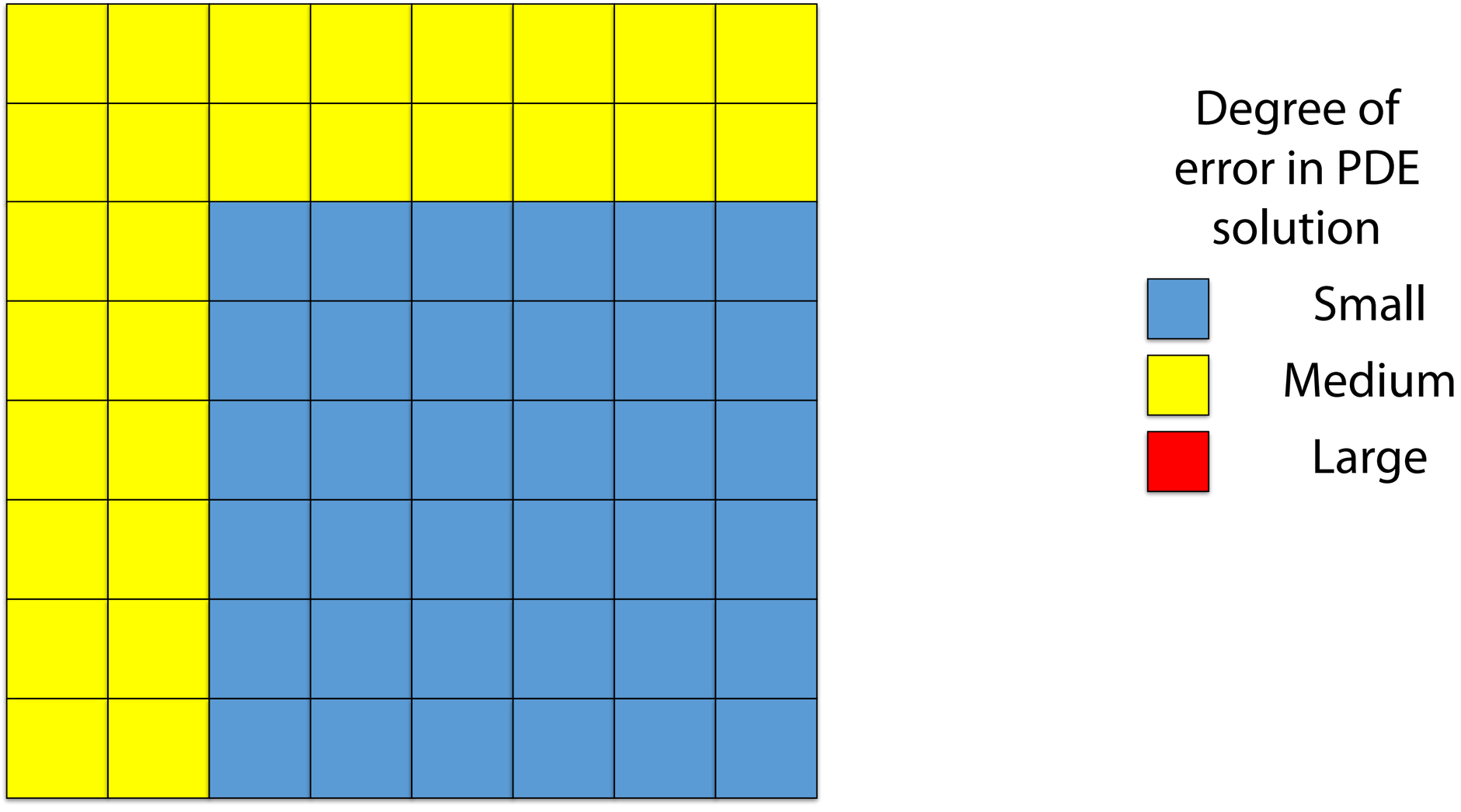

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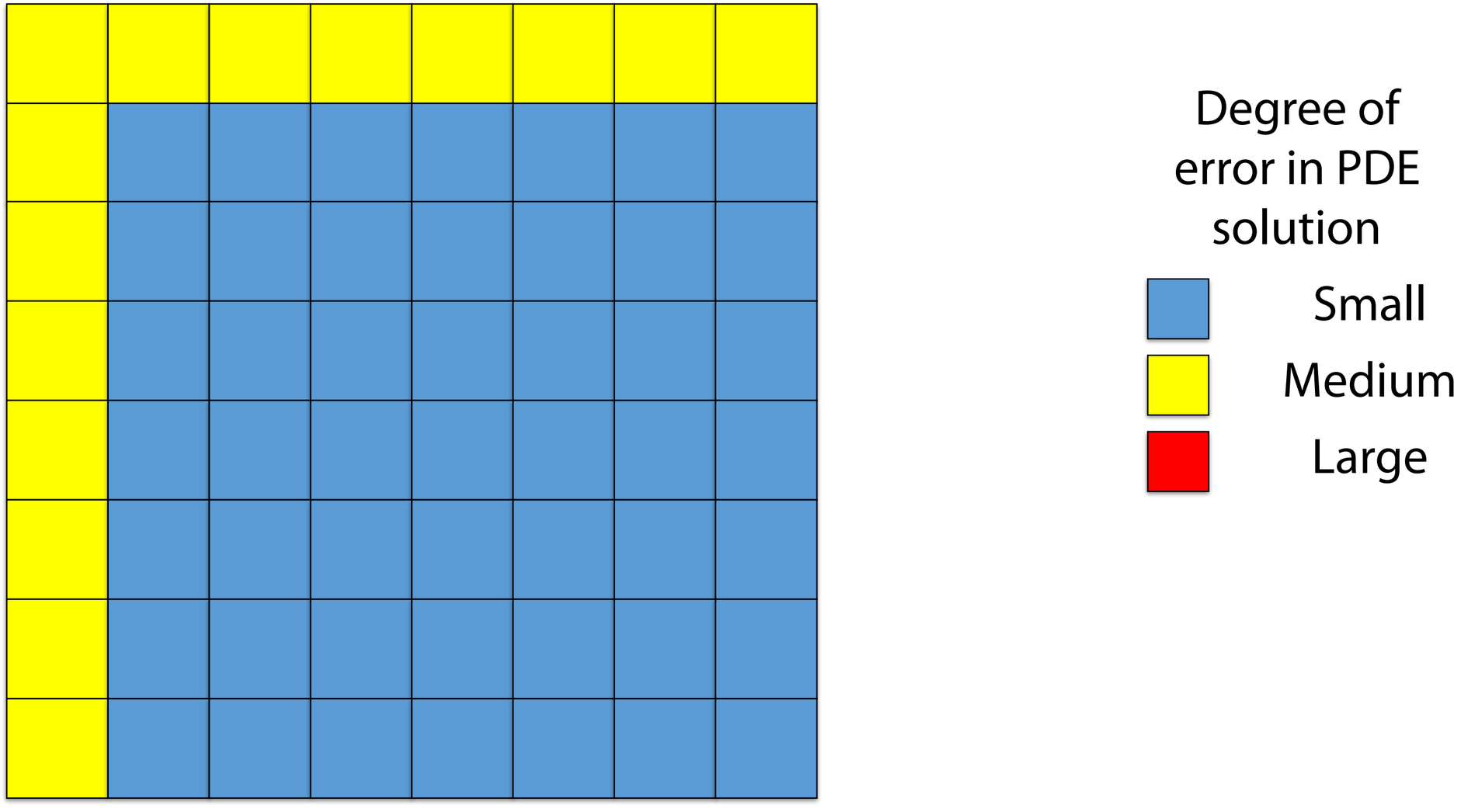

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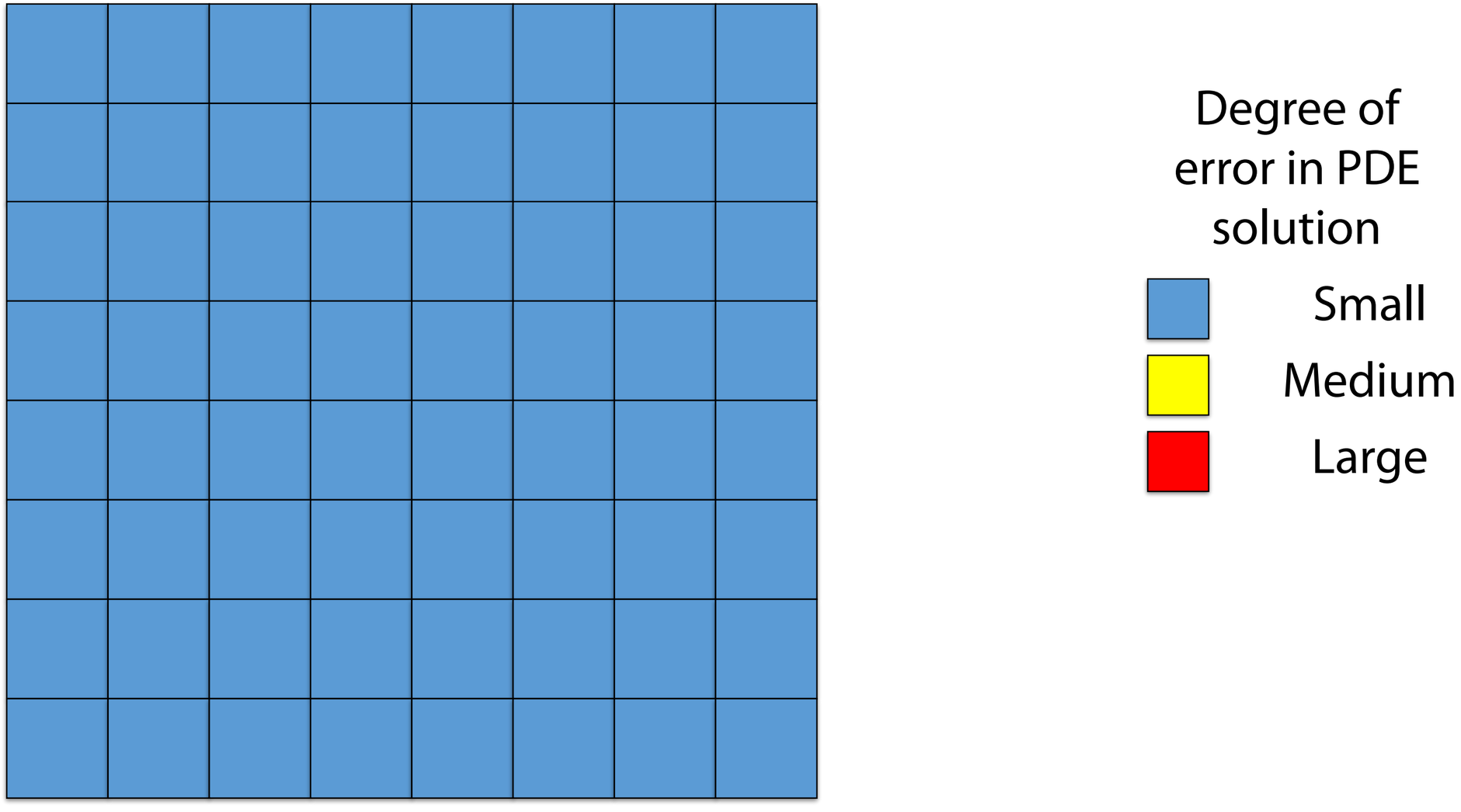

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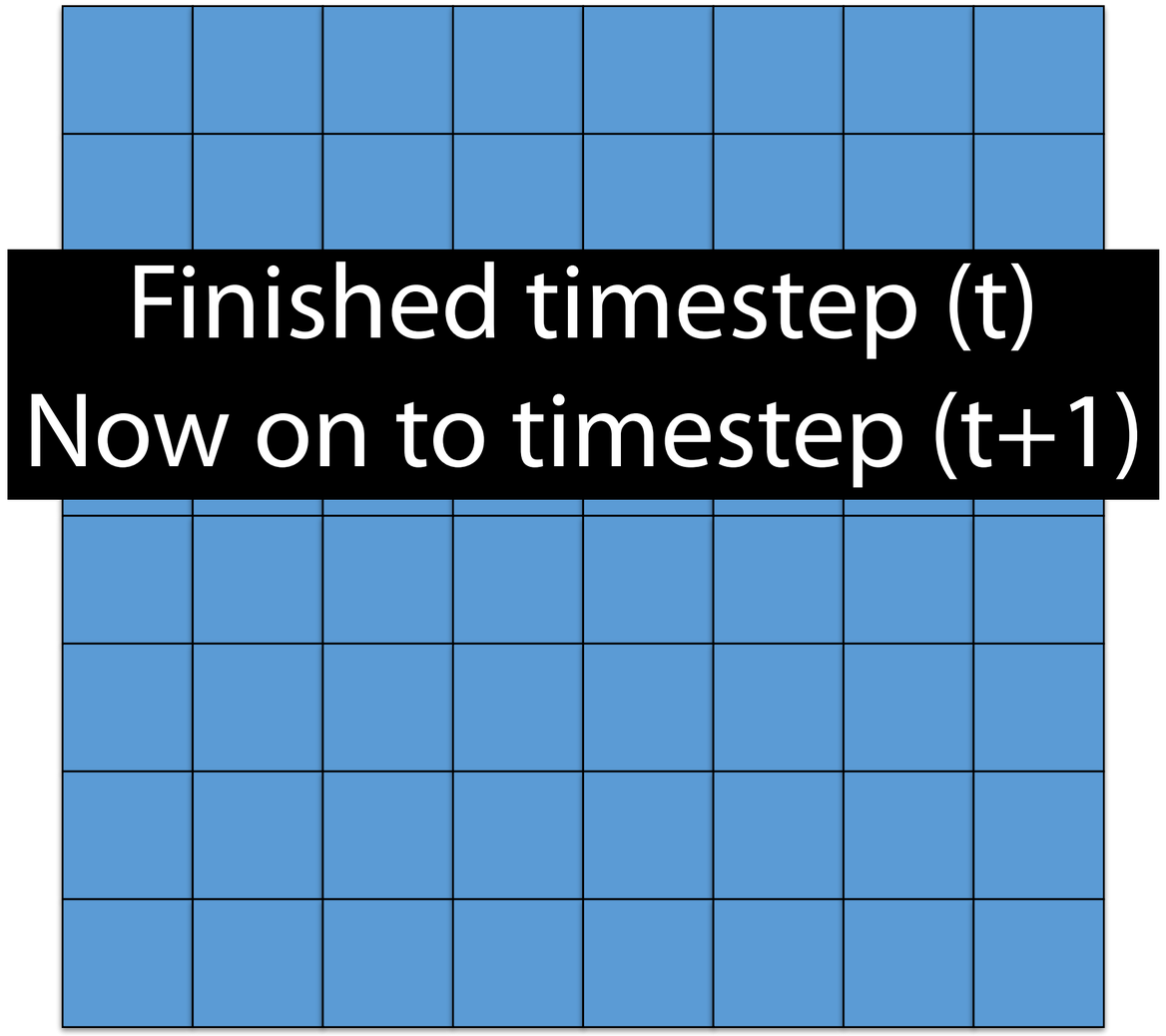

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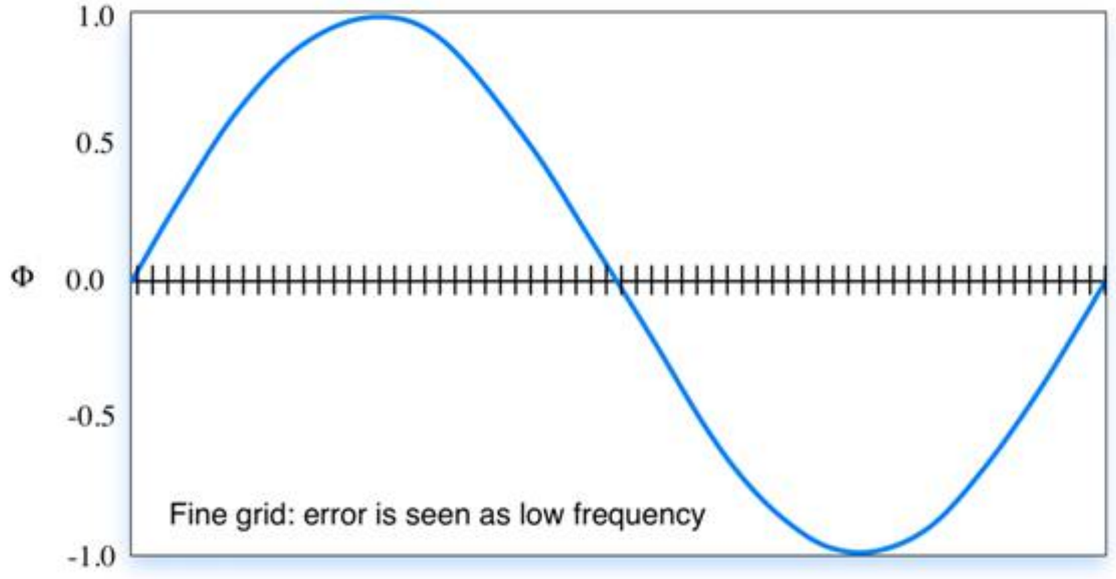
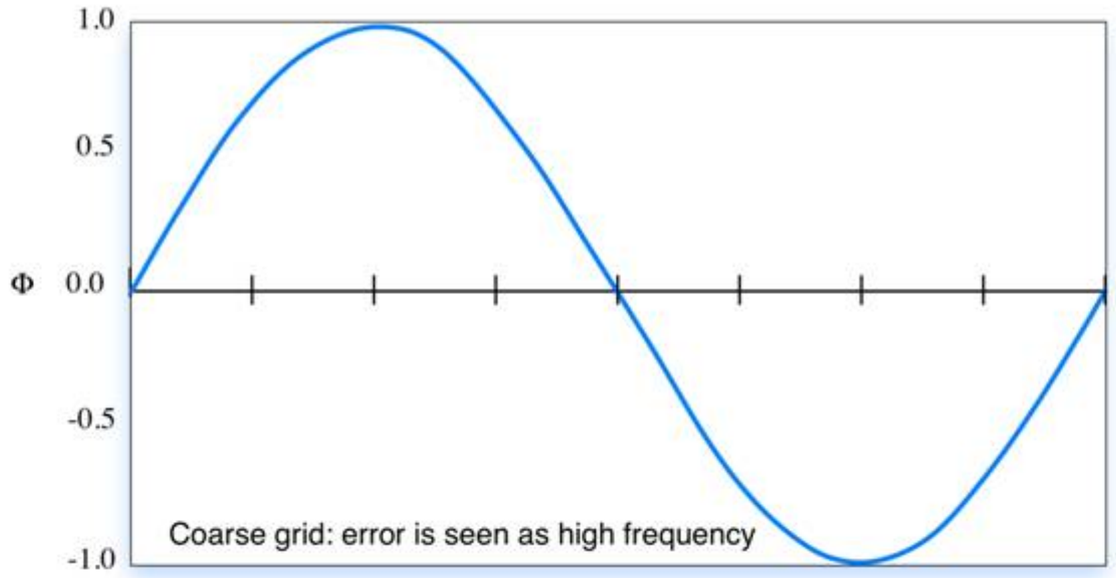
Multigrid

- Iterative solvers can take a long time to converge
  - Information must propagate, one cell at a time, to the entire grid

- Coarser grids converge more quickly
  - Information travels to entire grid in fewer iterations





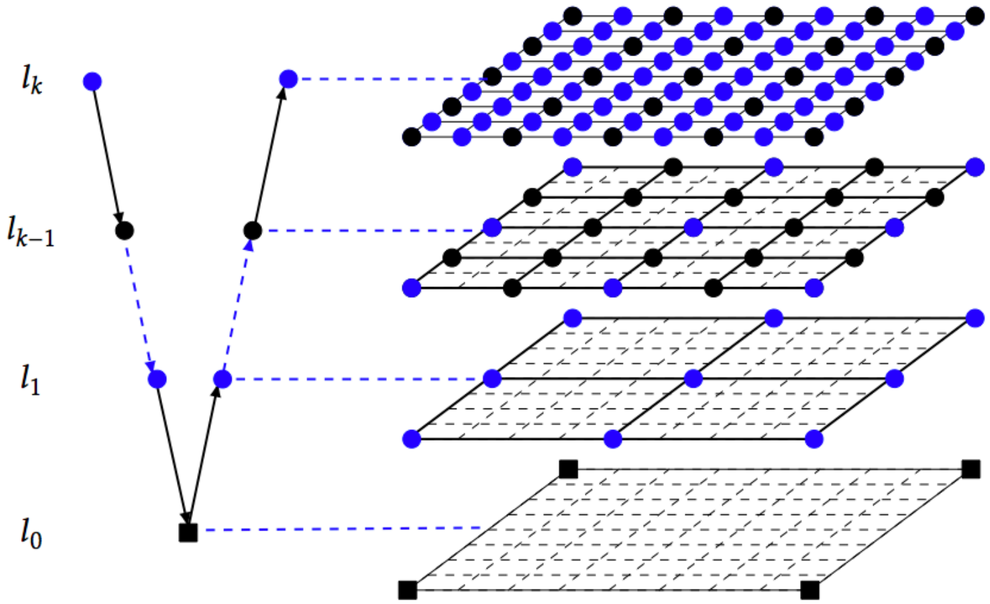
http://www.cfd-online.com/Wiki/Multigrid_methods

Three Main Steps of Multigrid

- **Smoothing – reduce high frequency errors using a few iterations of a solver (e.g., Gauss-Seidel)**

- **Restriction – downsample error to a coarser grid**

- **Interpolation (prolongation) – interpolate coarse grid error to finer grid**



http://web.utk.edu/~wfeng1/style/mggrid.png