Lecture 13:
Trees

Modern Parallel Computing
John Owens
EEC 289Q, UC Davis, Winter 2018

Credits

- Based on a lecture by Kerry Seitz

- Tero Karras
  - "Thinking Parallel" blog posts (https://devblogs.nvidia.com/parallelforall/thinking-parallel-part-i-collision-detection-gpu/)
  - "Maximizing Parallelism in the Construction of BVHs, Octrees, and k-d Trees (https://research.nvidia.com/publication/maximizing-parallelism-construction-bvhs-octrees-and-k-d-trees)

Overview

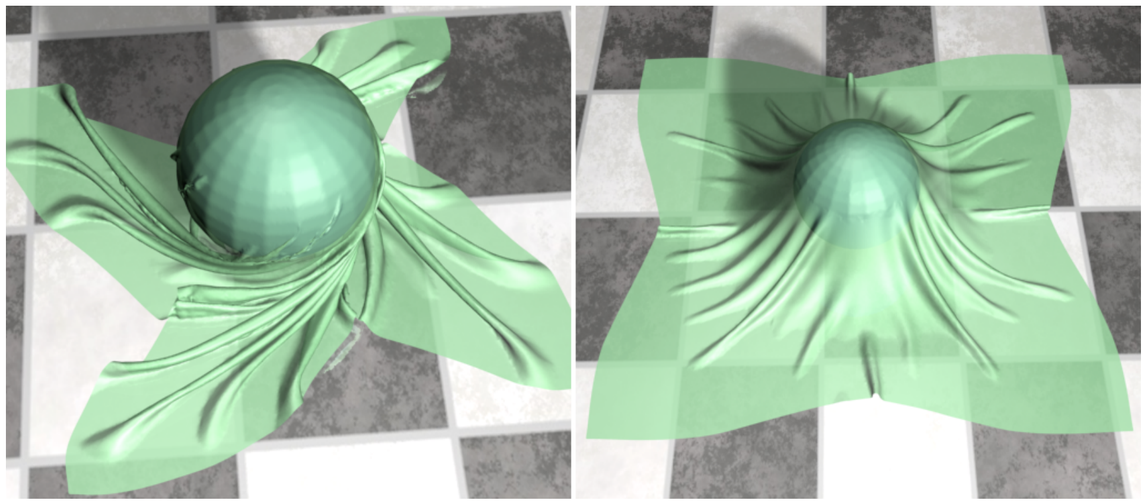
- Motivating Application: Collision Detection
- Bounding Volume Hierarchy (BVH)
- Tree Traversal
- Tree Construction

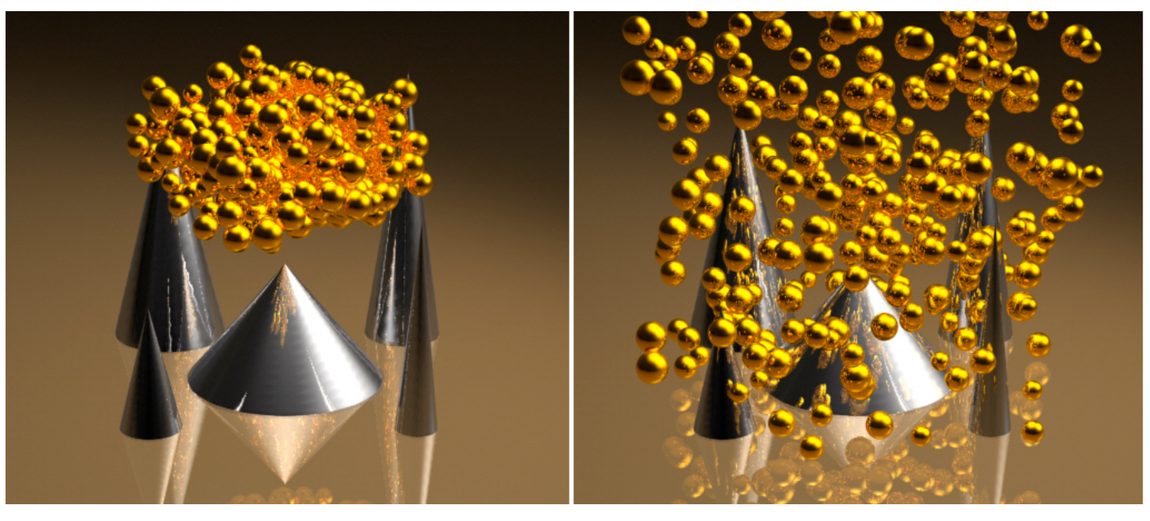
Collision Detection

- Query: Does object X intersect with anything in the scene?

- Difficulty: Both X and the scene are dynamic (constantly changing)

- Goal: Data structure that makes this query efficient (in parallel)





Images from HPCCD: Hybrid Parallel Continuous Collision Detection, Kim et al., Pacific Graphics 2009

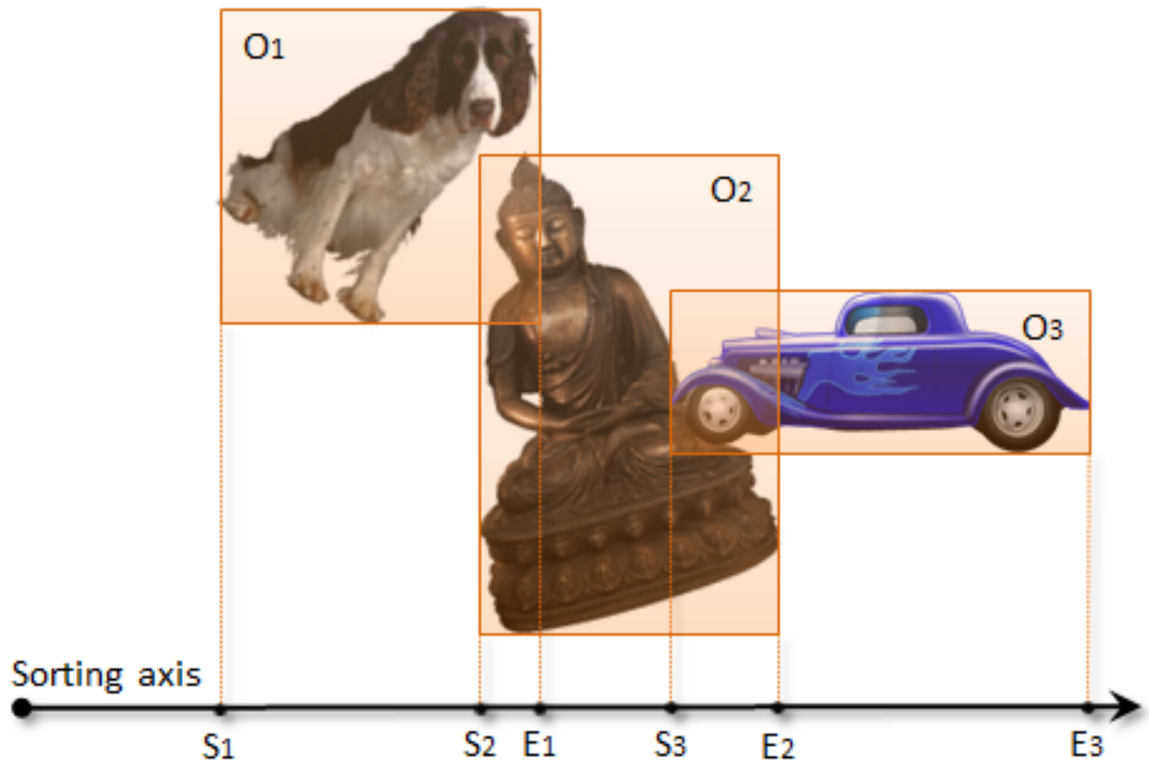
Collision Detection

- Naive approach: Full collision detection on all *n* items pairwise
  - $O(n^2)$ (this is a form of *n-body* computation)
  - Very inefficient

- Better approach: Use a "broad phase" and a "narrow phase"
  - Broad phase – identifies pairs of items that potentially collide
    - Often implemented on GPU, our focus today
  - Narrow phase – looks at the candidate pairs and determines actual collisions

Sort and Sweep

- Assign axis-aligned bounding boxes (AABB) for each object
- Project AABB to one axis
- Two items potentially collide if their projected boxes overlap

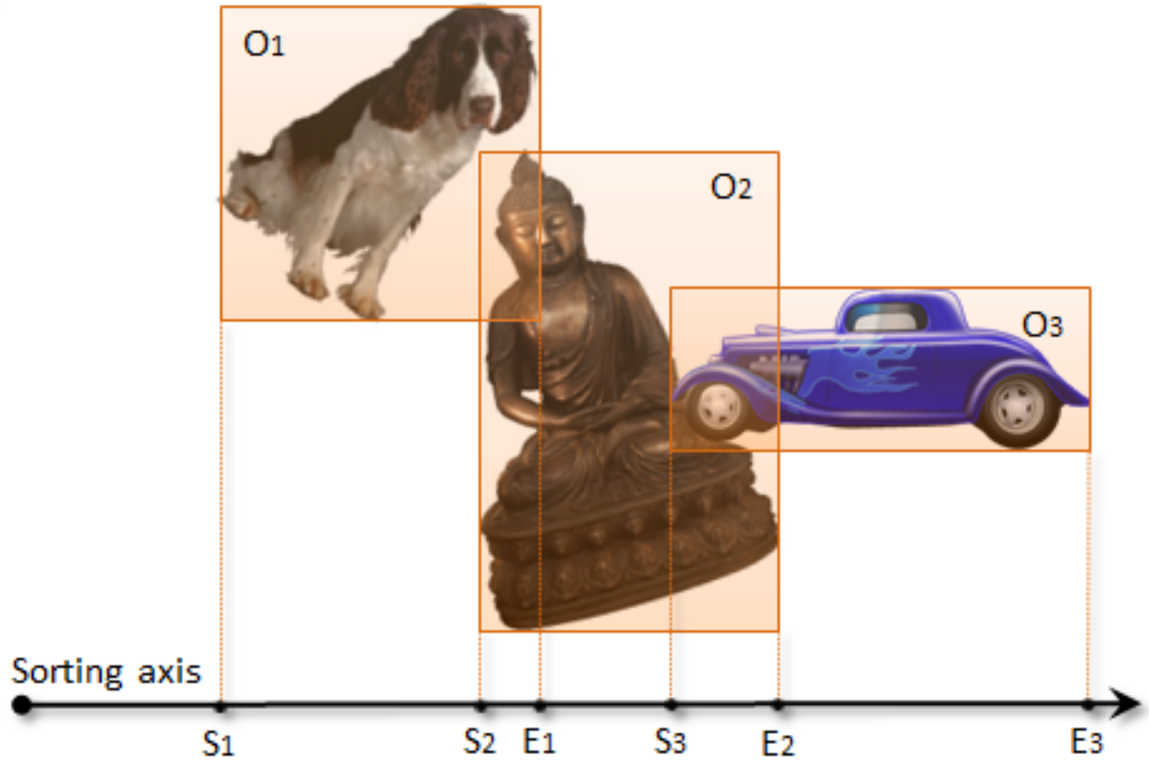


Sort and Sweep – Parallel

- Kernel 0: One thread per object
  - calculate AABB
  - project to axis
  - write start and end points into array

- Kernel 1: Sort the array

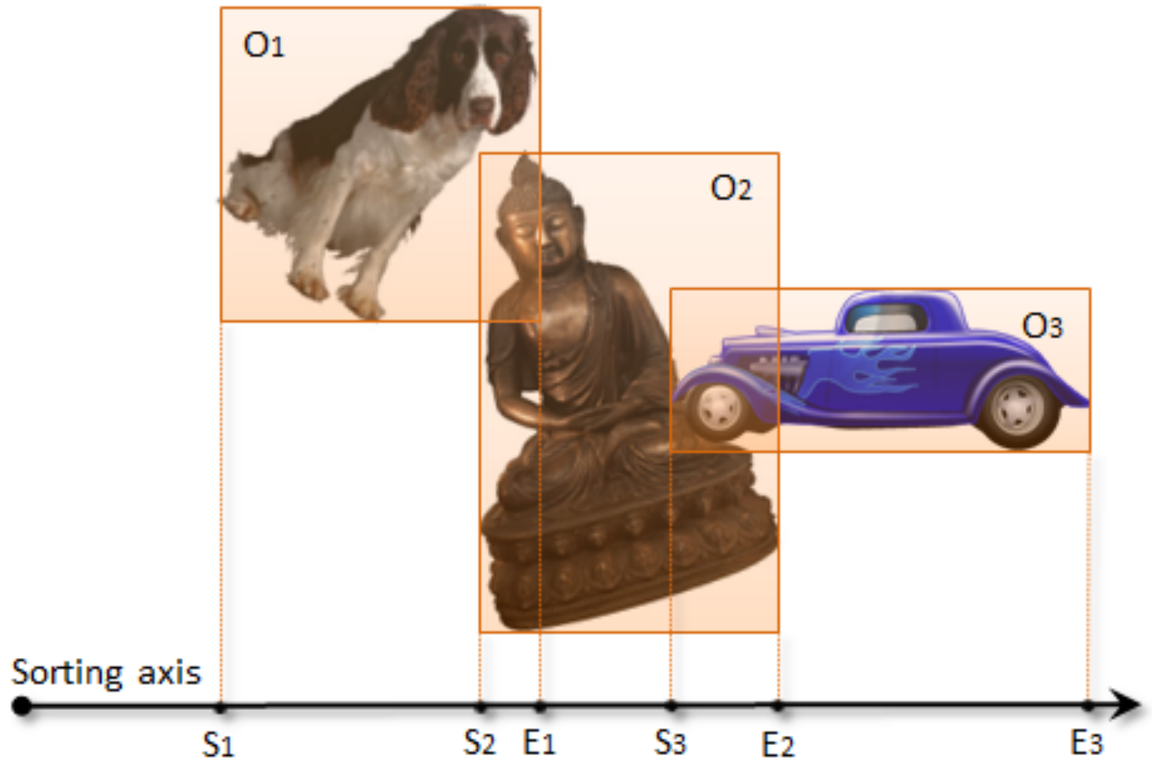
- Kernel 2: One thread per array element
  - If end point, exit
  - If start point, walk forward, outputting other start points until corresponding end point is reached



Sort and Sweep – Downsides

- Potentially output $O(n^2)$ pairs (if all object overlap on chosen axis)

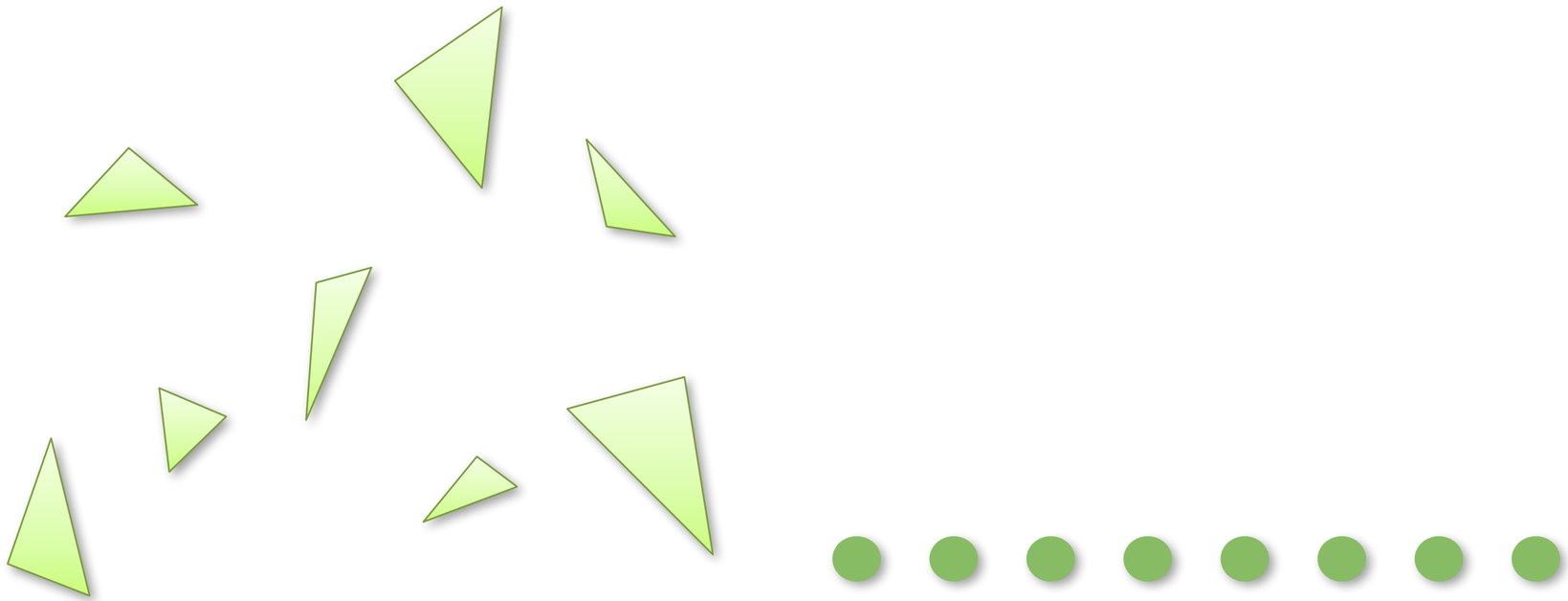
- Lots of divergence
  - Thread divergence
  - Data divergence (uncoalesced)



Bounding Volume Hierarchy

Bounding volume hierarchy

Bounding volume hierarchy



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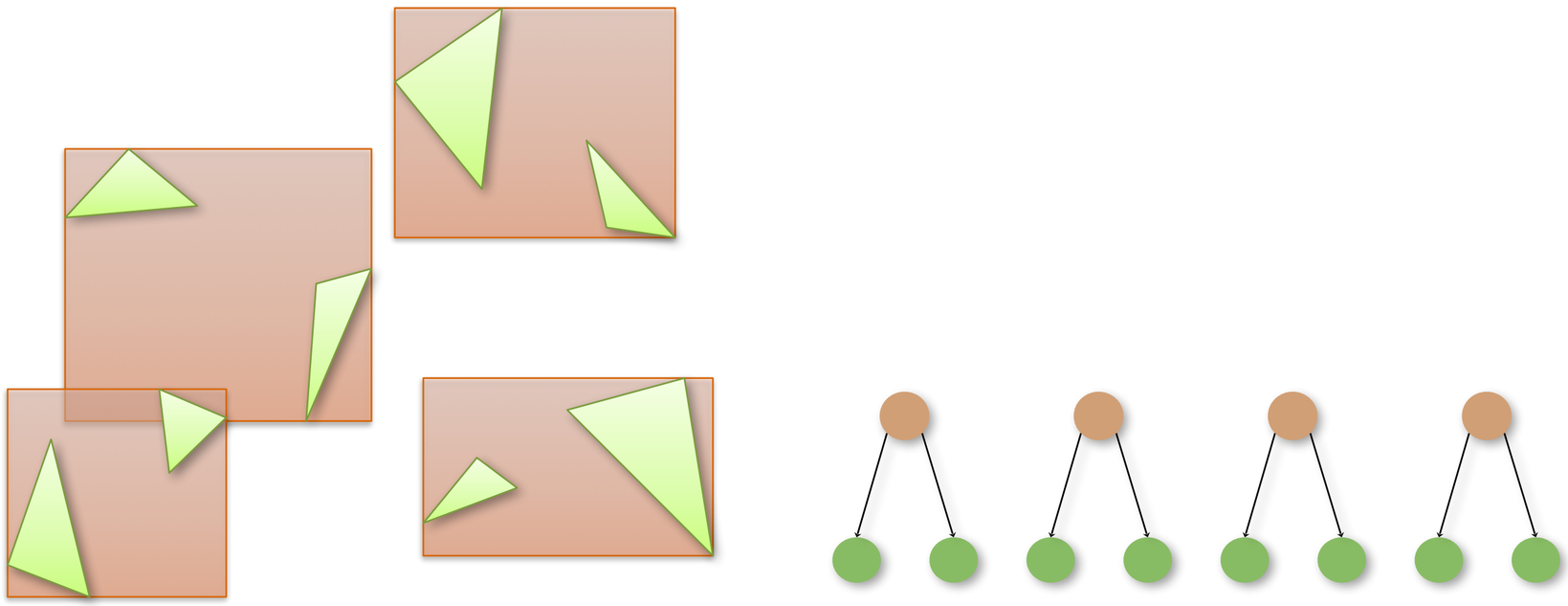

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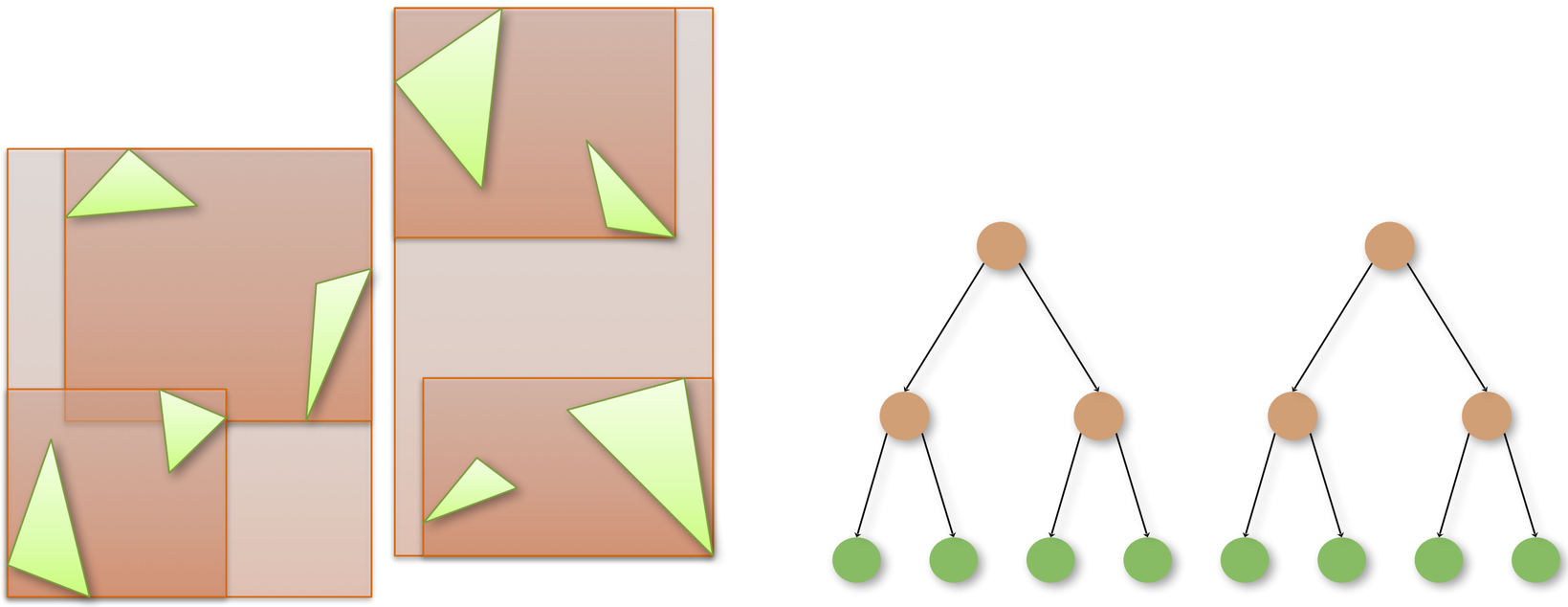

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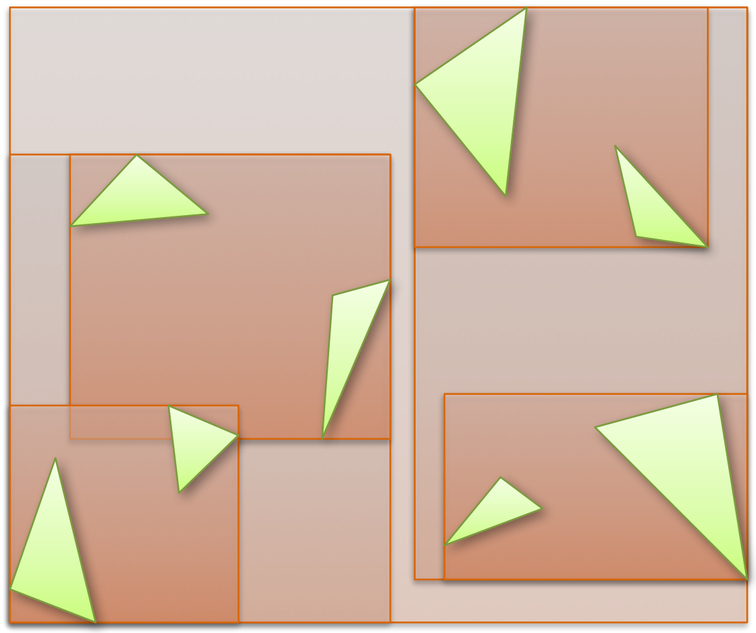

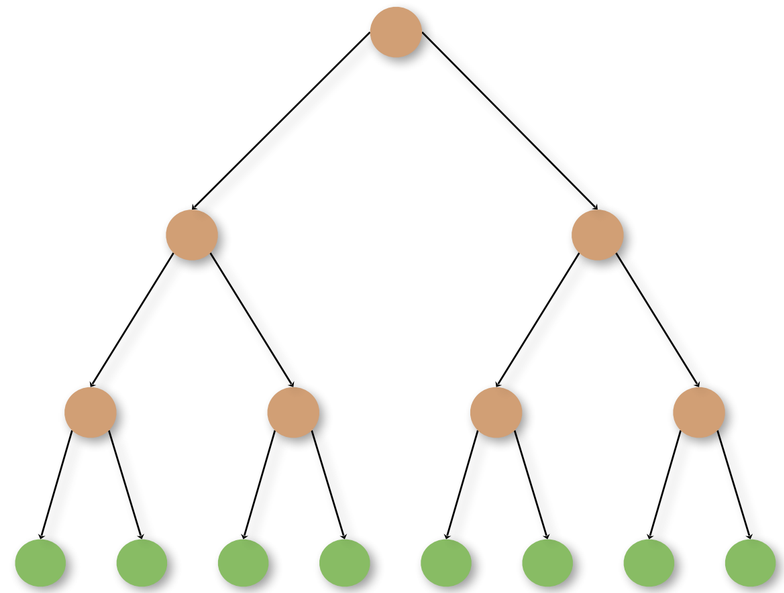

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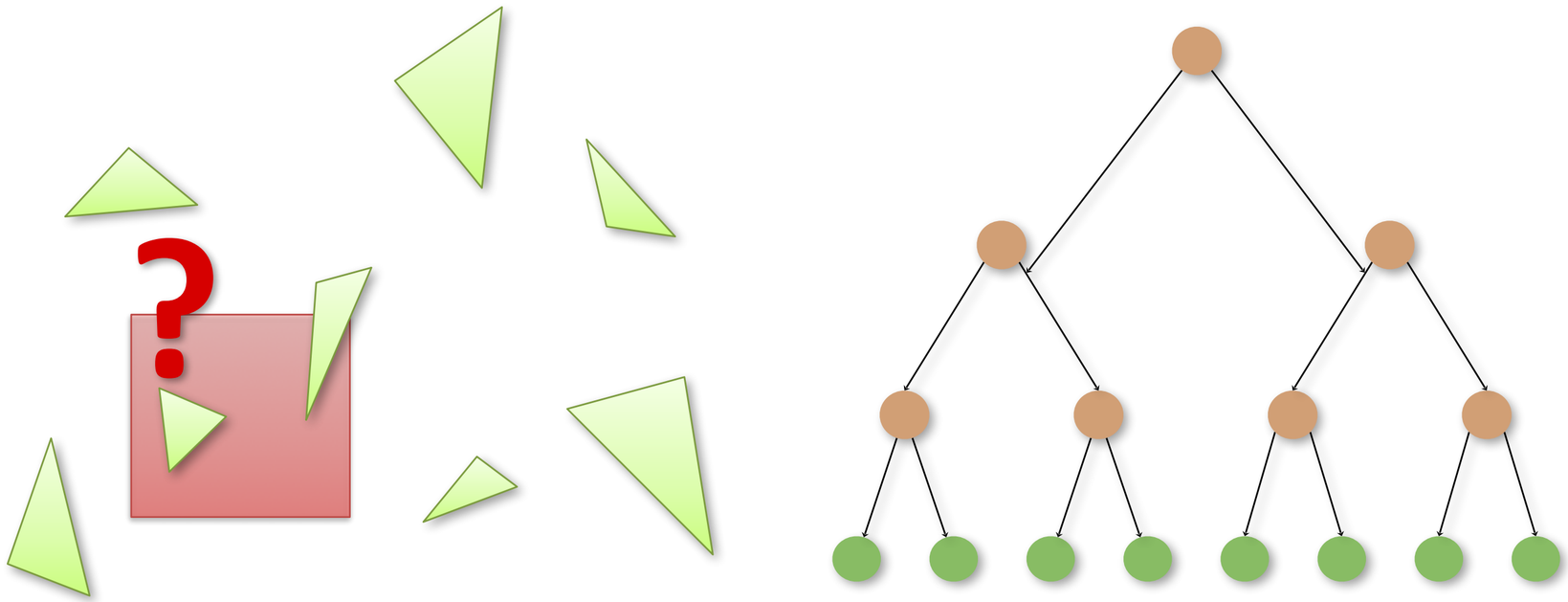

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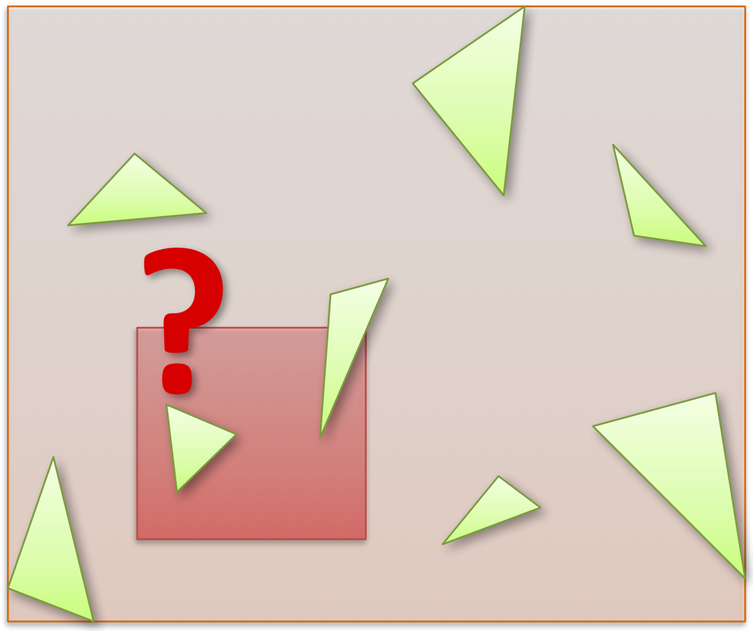

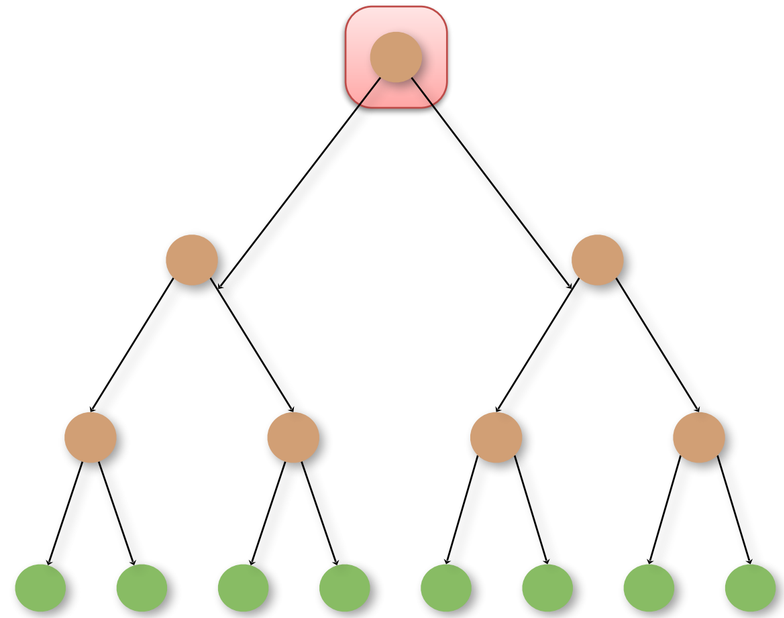

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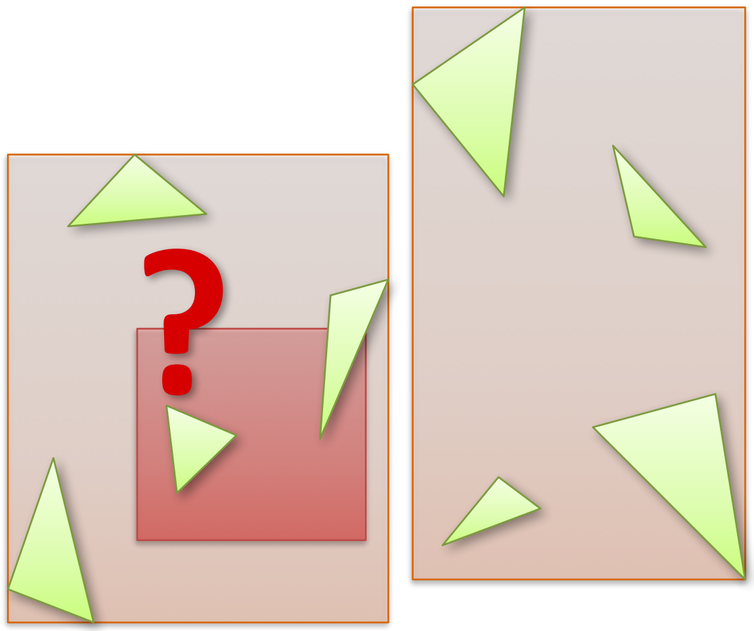

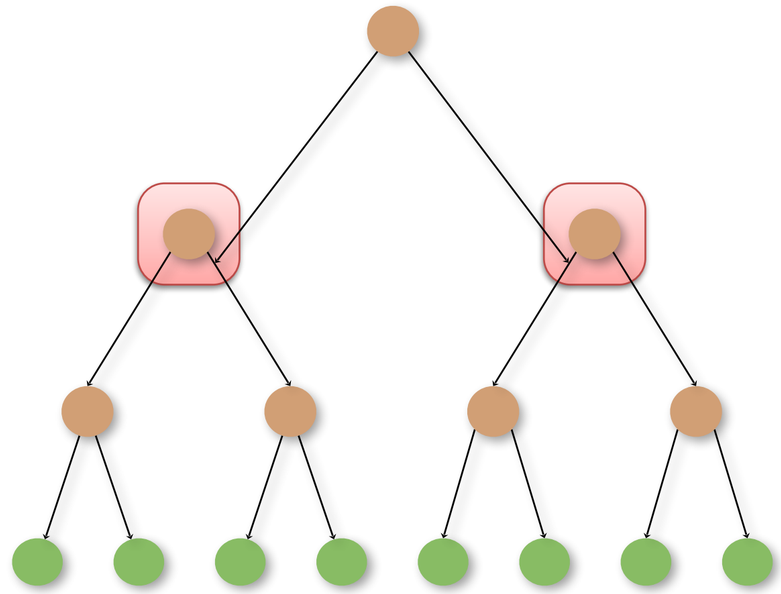

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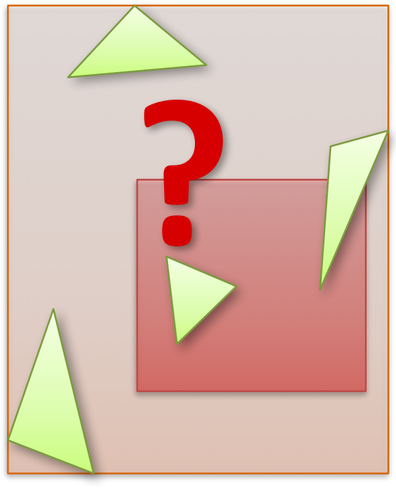

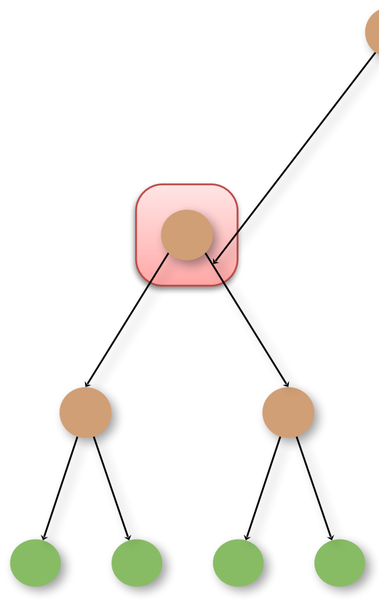

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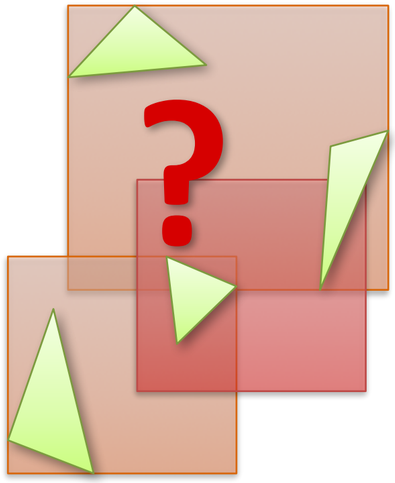

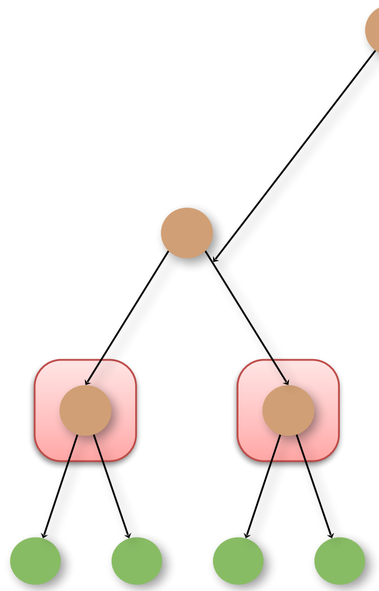

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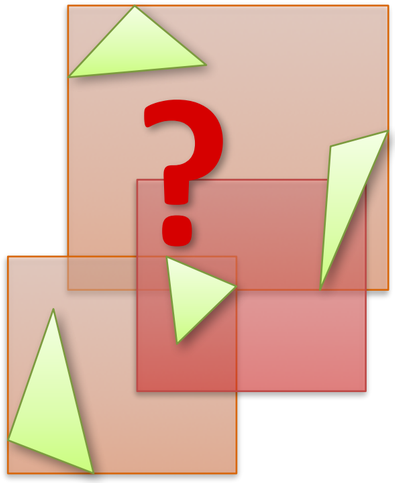

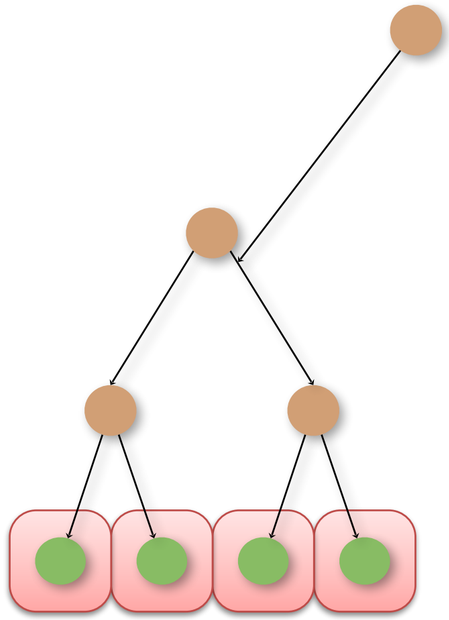

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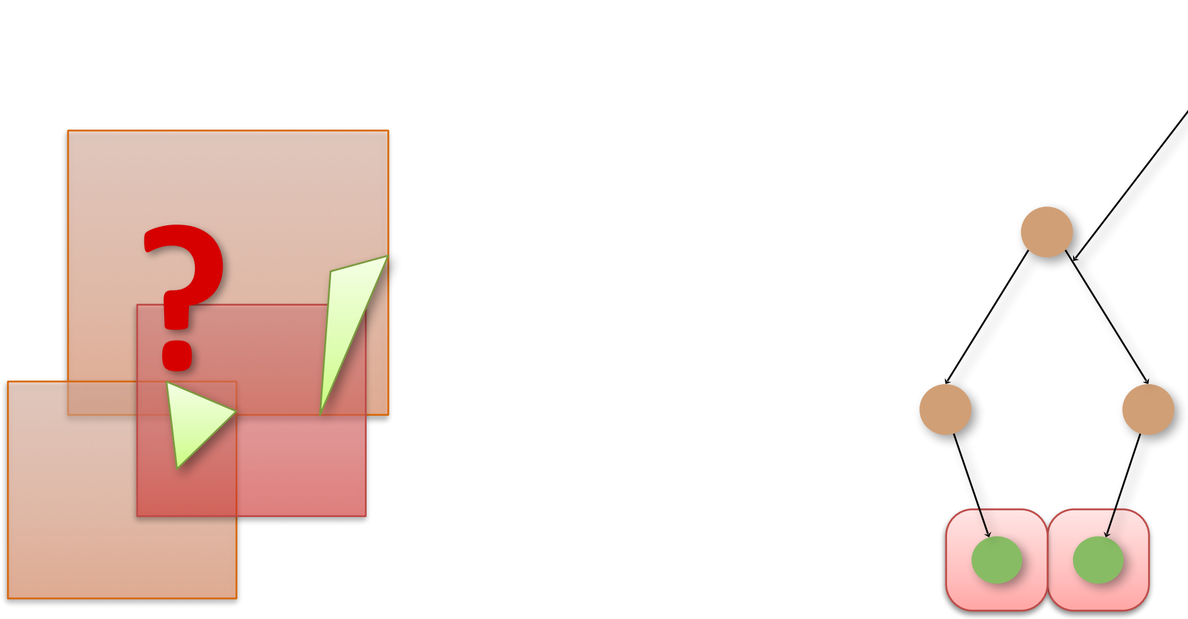
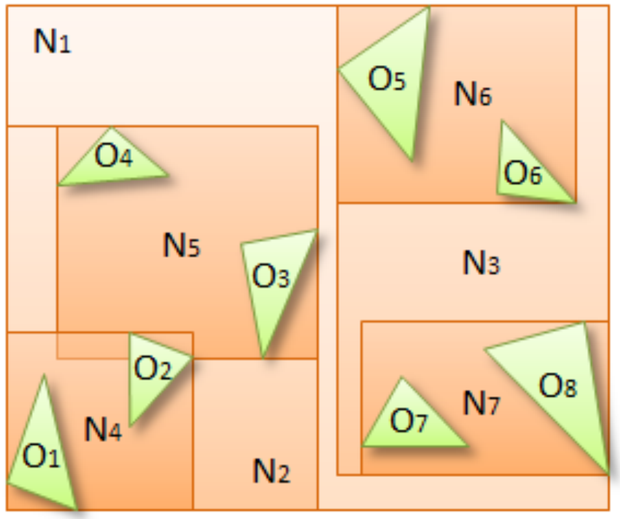
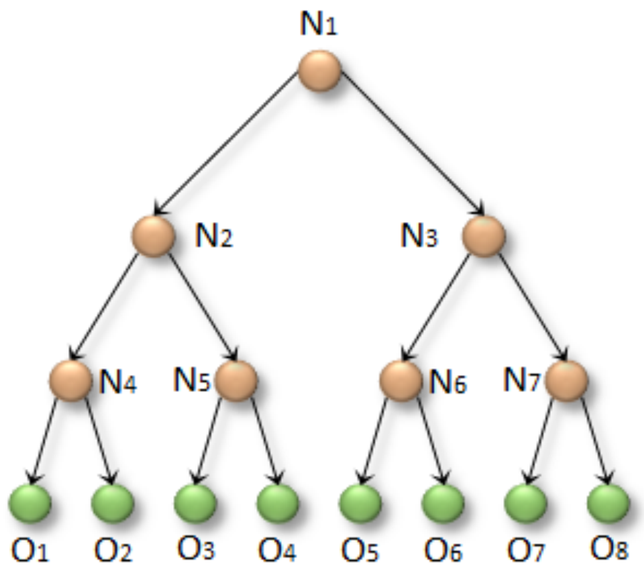


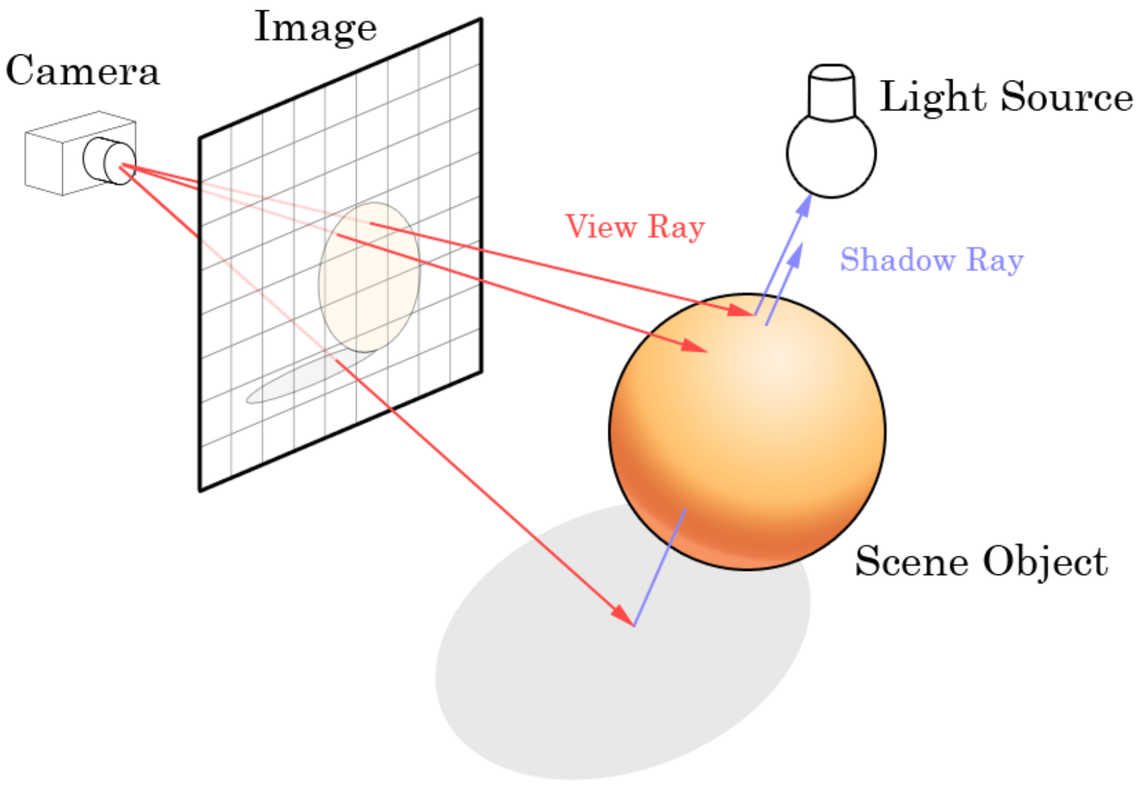
Bounding Volume Hierarchy (BVH)

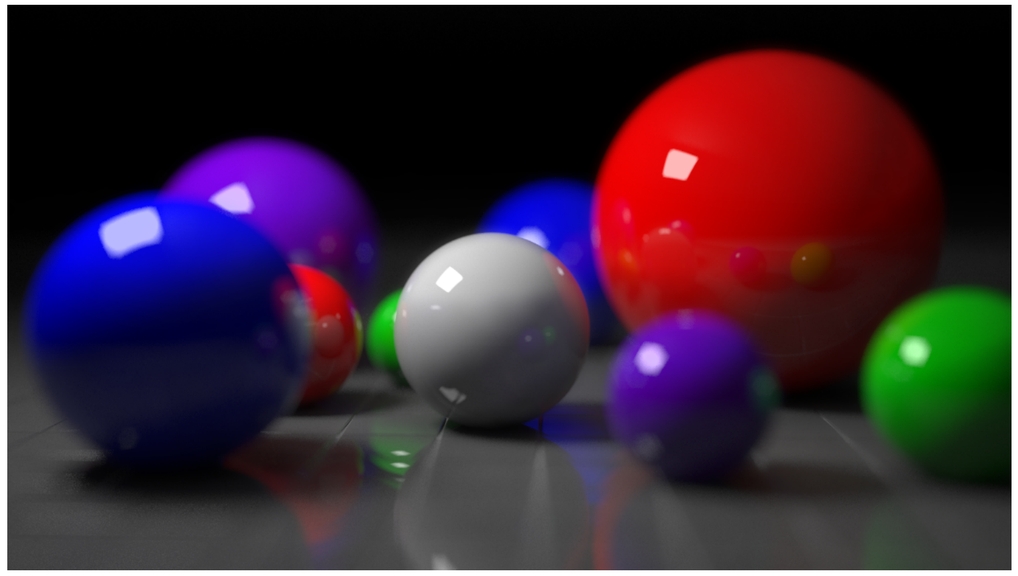
- Hierarchical grouping of 3D objects
- Nodes:
  - Leaf nodes – individual objects
  - Internal nodes – groupings of objects based on bounding volume
  - Root node – entire scene
- Bounding volumes can be any shape, but generally use axis-aligned bounding boxes (AABB) for simplicity
- Used in collision detection and ray tracing





BVH for Ray Tracing



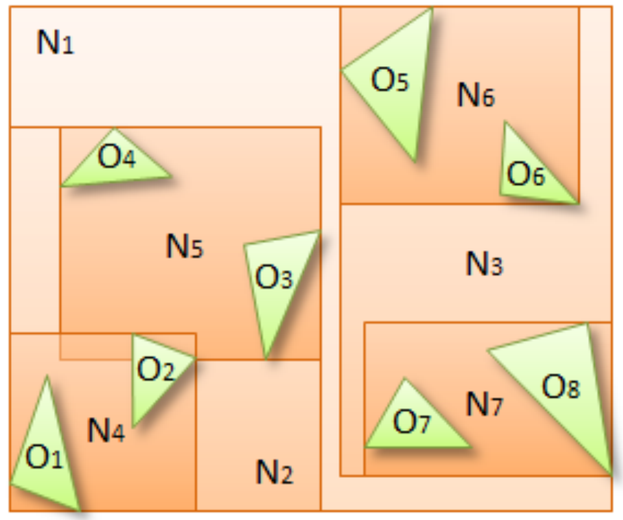


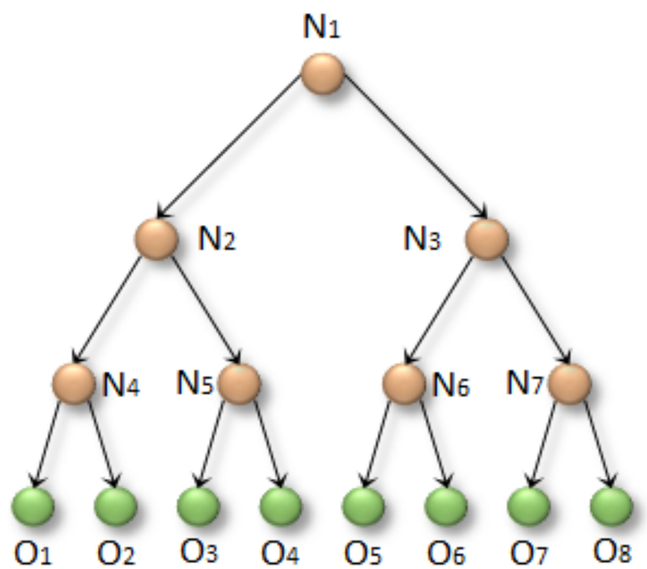
Tree Traversals

Scenario & things to consider

- Does AABB-specified object intersect any object in BVH?

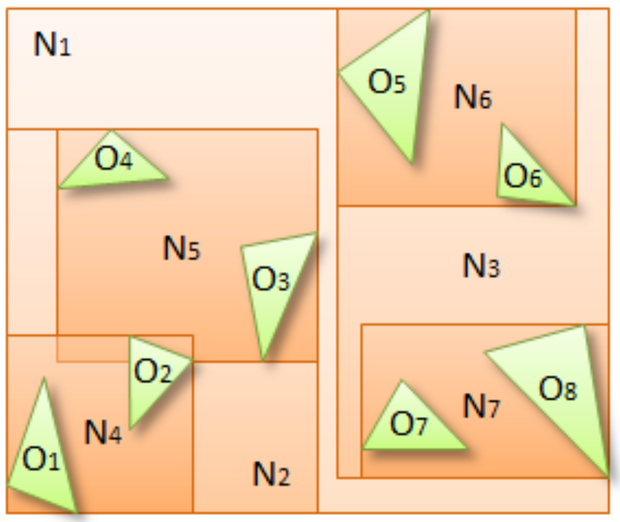
- Consider:
  - Amount of work
  - Occupancy
  - Thread divergence
  - Data access patterns

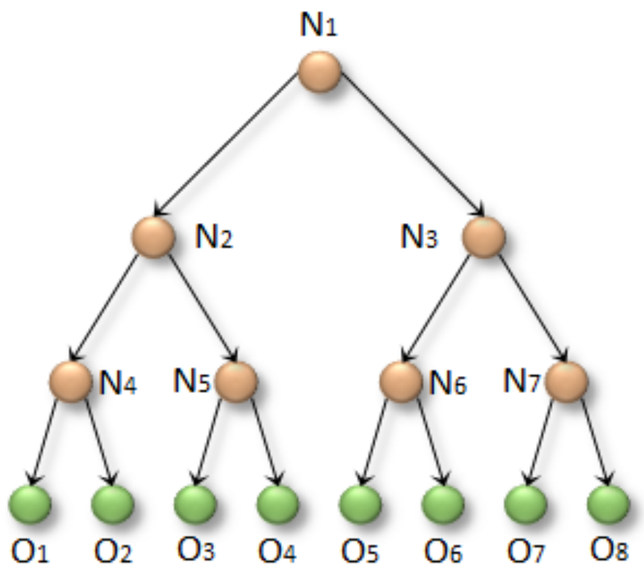




Options to Parallelize Traversal

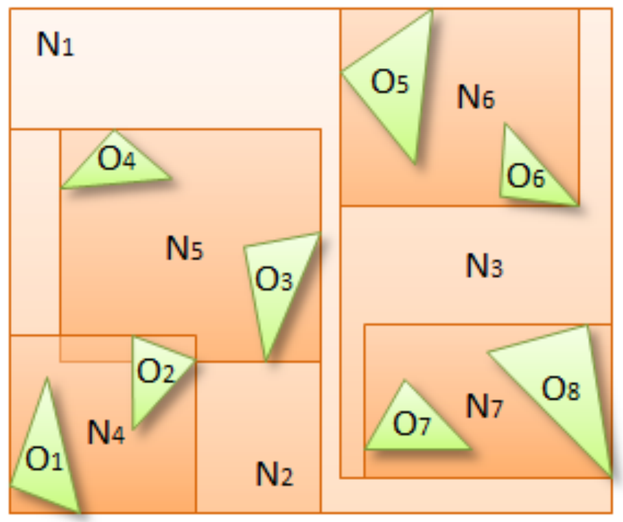
- One thread per object
  - E.g., [O_2, O_7, O_1, O_4, …]
  - Different objects take different paths down the tree

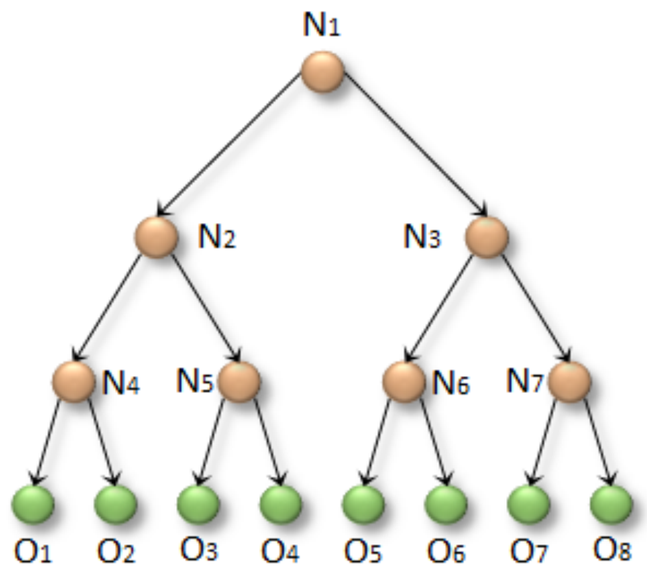




Options to Parallelize Traversal

- One thread per object
  - E.g., [O_2, O_7, O_1, O_4, …]
  - Different objects take different paths down the tree
  - Lots of divergence





BVH Traversal – First Attempt (C++)

query object

```
void traverseRecursive( CollisionList& list,
                        const BVH&     bvh,
                        const AABB&    queryAABB,
                        int            queryObjectIdx,
                        NodePtr        node) {
    // Bounding box overlaps the query => process node.
    if (checkOverlap(bvh.getAABB(node), queryAABB)) {
        // Leaf node => report collision.
        if (bvh.isLeaf(node))
            list.add(queryObjectIdx, bvh.getObjectIdx(node));
        // Internal node => recurse to children.
        else {
            NodePtr childL = bvh.getLeftChild(node);
            NodePtr childR = bvh.getRightChild(node);
            traverseRecursive(bvh, list, queryAABB,
                              queryObjectIdx, childL);
            traverseRecursive(bvh, list, queryAABB,
                              queryObjectIdx, childR);
        }
    }
}
```

BVH Traversal – First Attempt (CUDA)

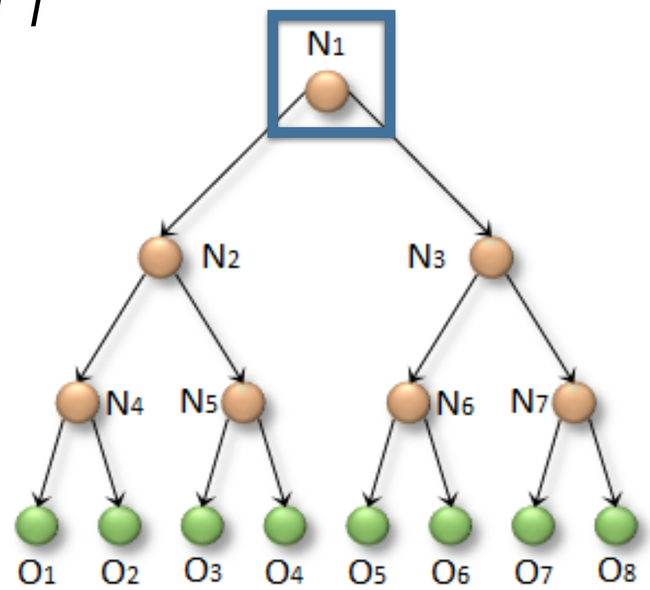
```
__device__ void traverseRecursive( CollisionList& list,
                                   const BVH&     bvh,
                                   const AABB&    queryAABB,
                                   int            queryObjectIdx,
                                   NodePtr        node)
{
    // same as before...
}
```

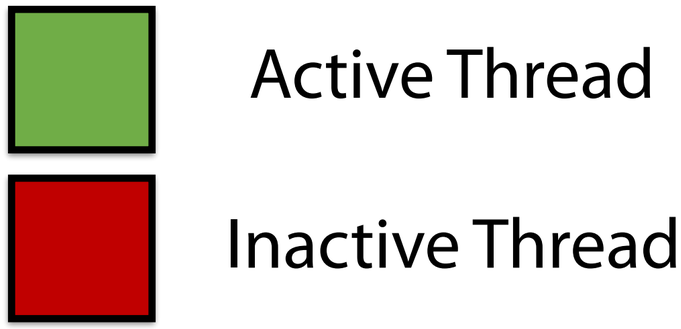
parallelize over query objects

```
__global__ void findPotentialCollisions( CollisionList list,
                                         BVH           bvh,
                                         AABB*         objectAABBs,
                                         int           numObjects)
{
    int idx = threadIdx.x + blockDim.x * blockIdx.x;
    if (idx < numObjects)
        traverseRecursive(bvh, list, objectAABBs[idx],
                          idx, bvh.getRoot());
}
```

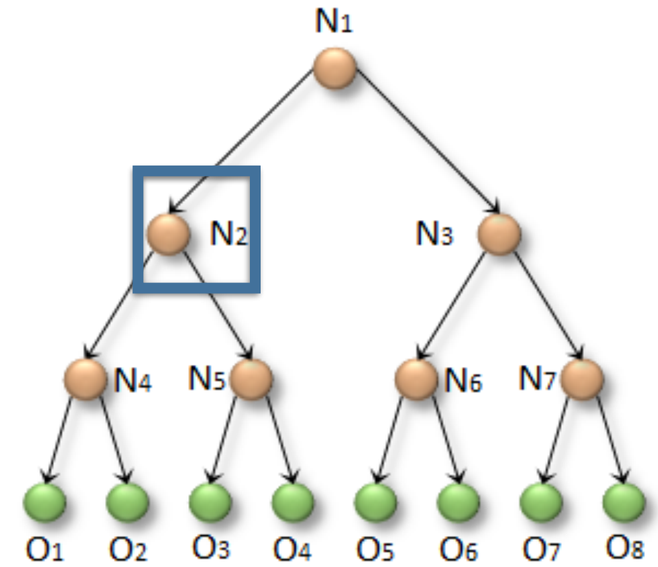
BVH Traversal – First Attempt

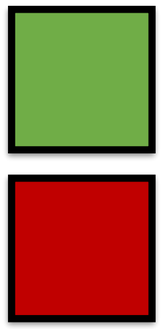
Note we're querying the same geometry that's in the BVH





BVH Traversal – First Attempt

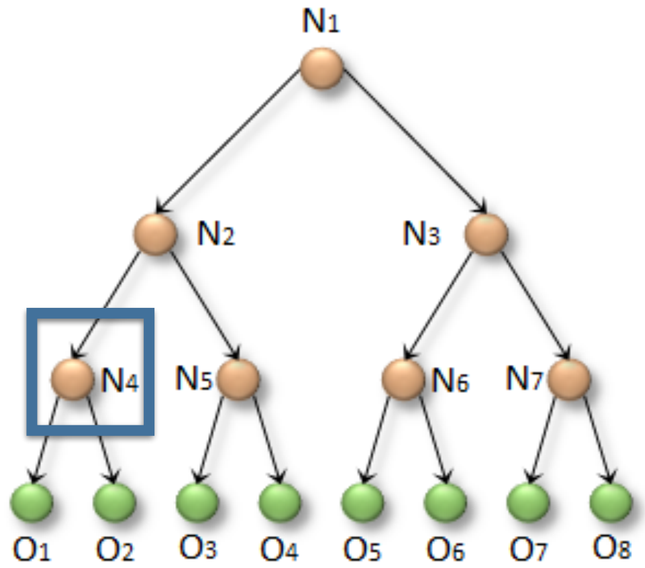


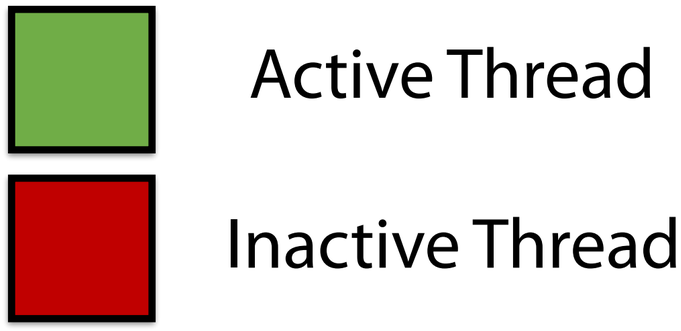


Active Thread

Inactive Thread

BVH Traversal – First Attempt





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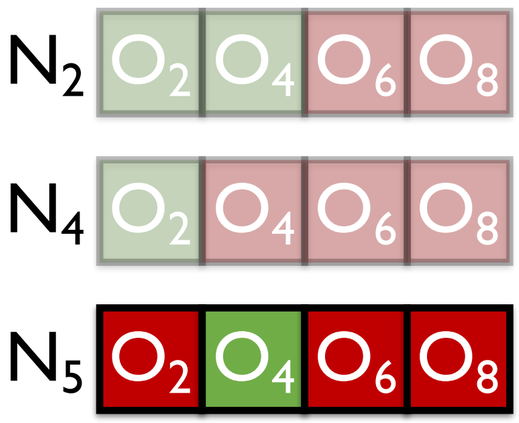

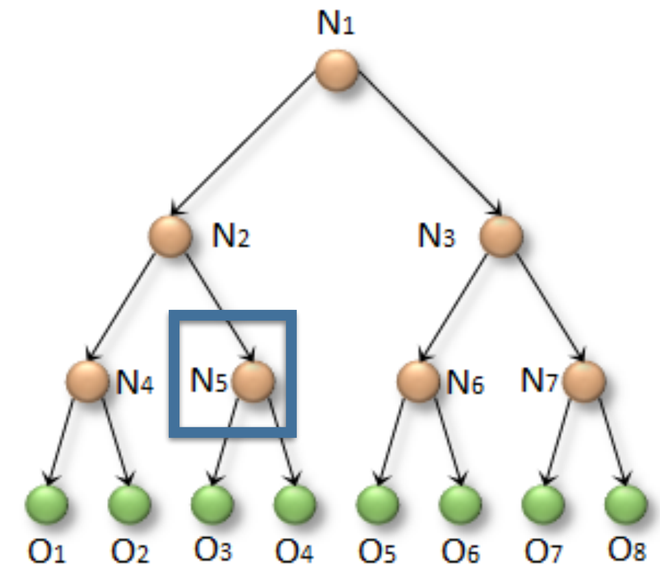

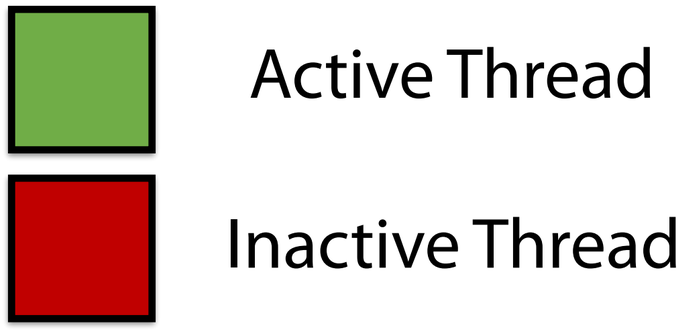

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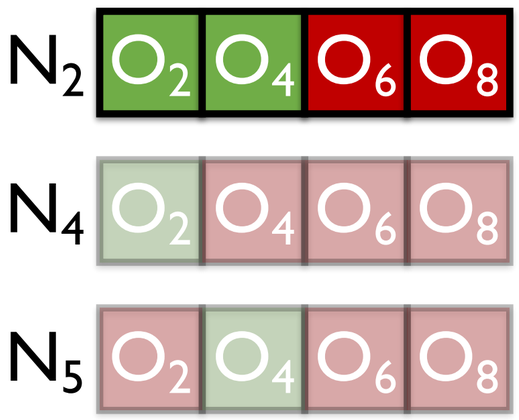
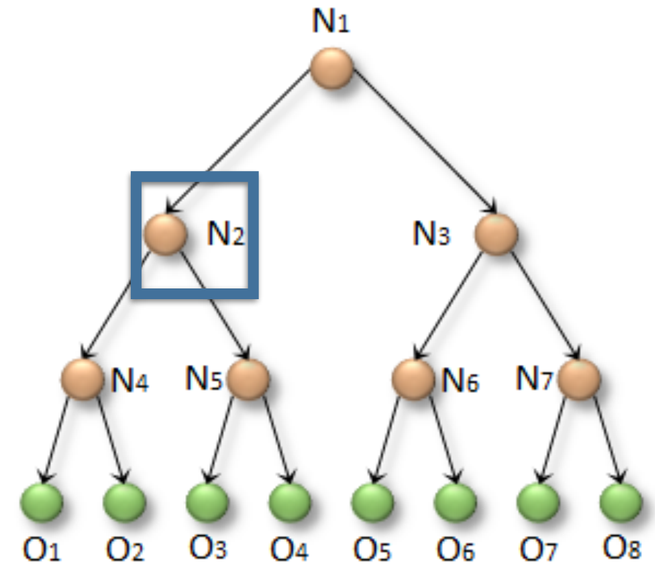
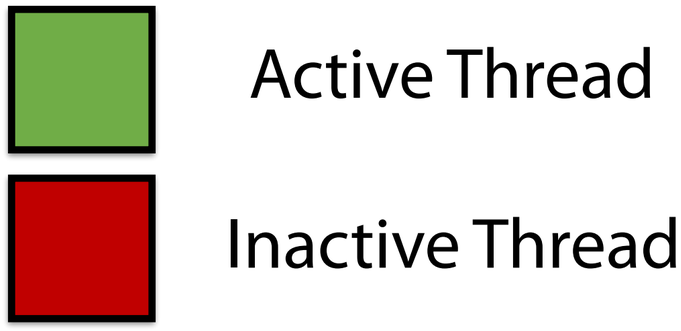
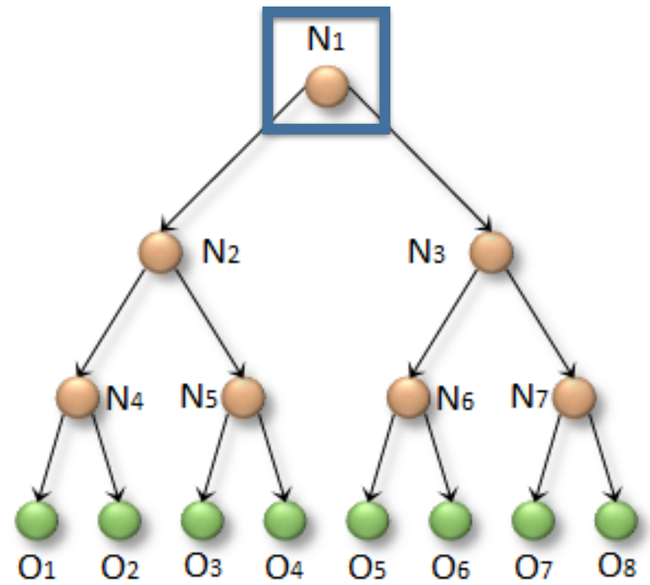
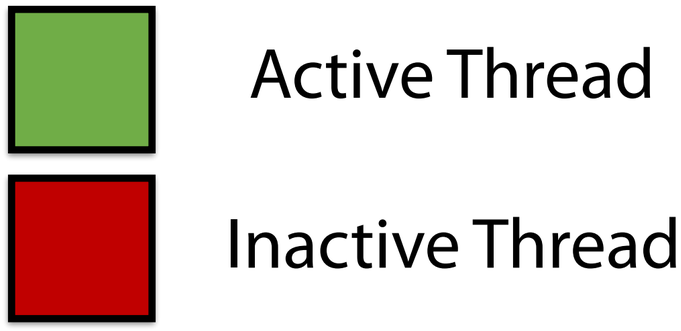




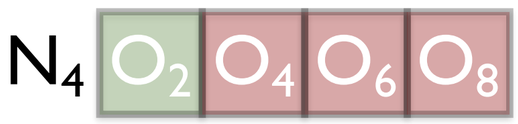


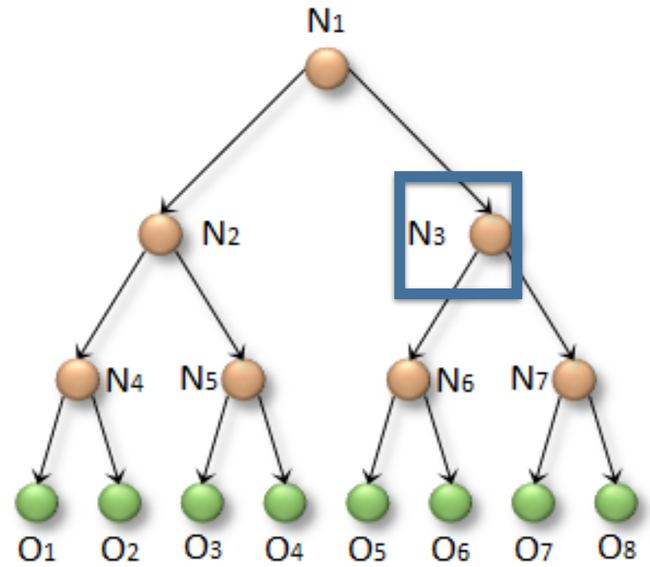
BVH Traversal – First Attempt

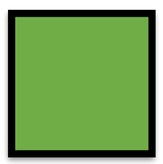




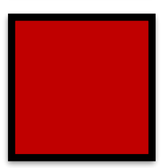
BVH Traversal – First Attempt





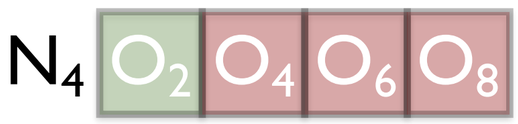


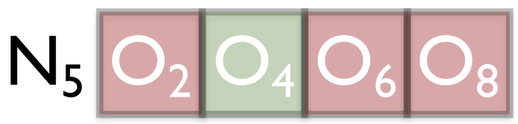
Active Thread

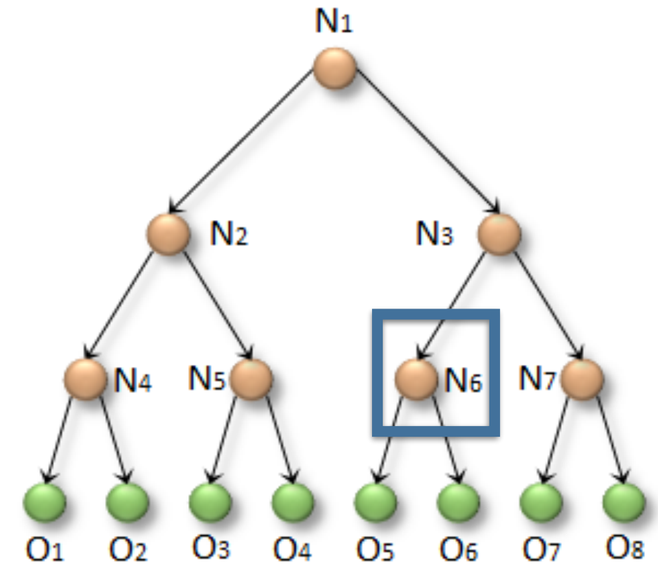


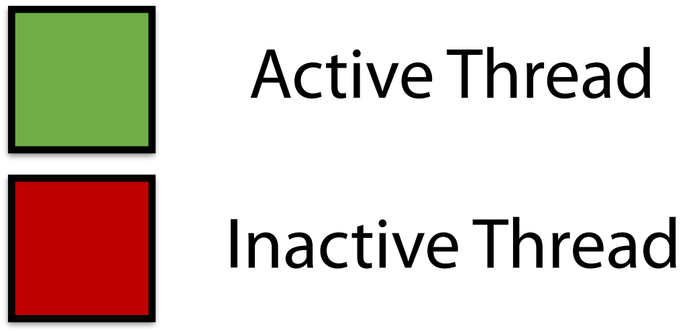
Inactive Thread

BVH Traversal – First Attempt

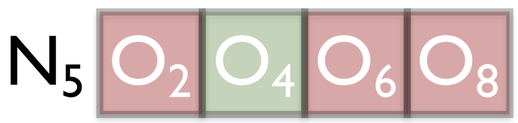


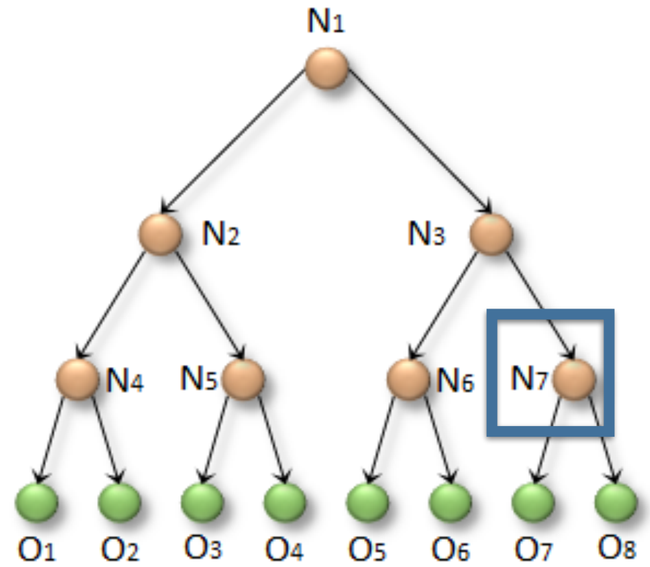


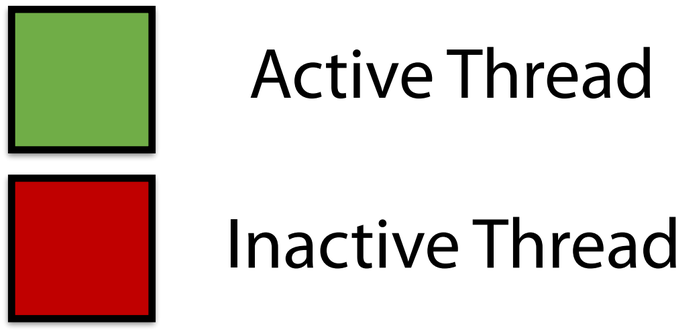




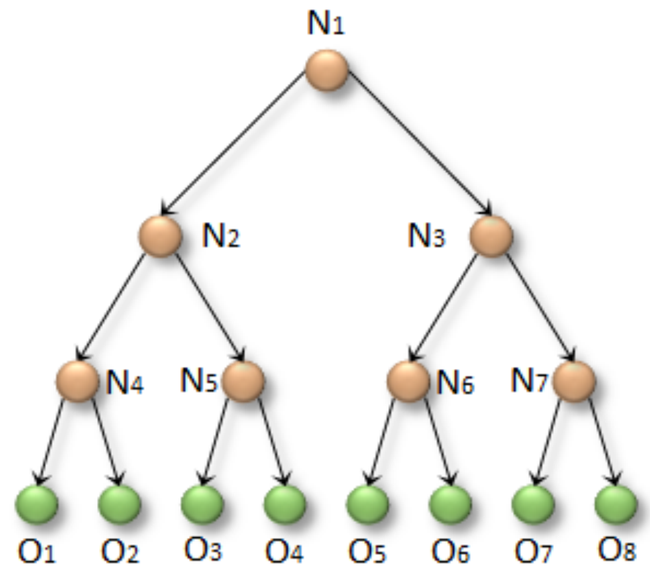
BVH Traversal – First Attempt

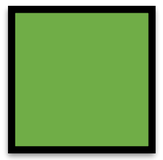




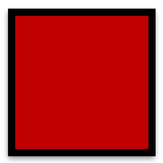


BVH Traversal – First Attempt





Active Thread



Inactive Thread

BVH Traversal – First Attempt Problems

- **Lots of thread divergence**

```
// Internal node => recurse to children.
else {
    NodePtr childL = bvh.getLeftChild(node);
    NodePtr childR = bvh.getRightChild(node);
    traverseRecursive(bvh, list, queryAABB, queryObjectIdx, childL);
    traverseRecursive(bvh, list, queryAABB, queryObjectIdx, childR);
}
```

If only 1 thread takes this path, the other 31 have to wait until that thread finishes its full depth traversal

BVH Traversal – First Attempt Performance

- Dataset from APEX Destruction
  - 12,486 objects
  - 73,704 pairs of potentially colliding objects
- Tested on NVIDIA GeForce GTX 690
- Execution time: 3.8 milliseconds
  - Very slow!
  - Only one part of collision detection
  - Only one part of a simulation ideally running at 60 FPS (16 ms/frame)
- Try 2: Convert to iterative; manage call stack explicitly

BVH Traversal – Second Attempt

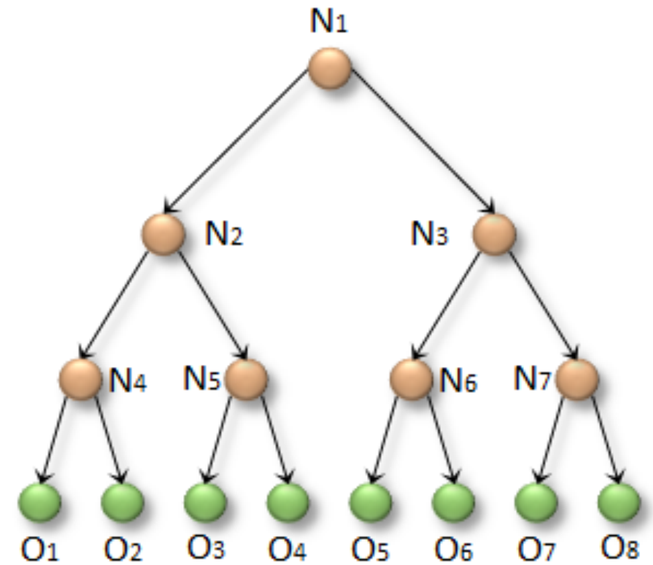
```
__device__ void traverseIterative( CollisionList& list,
                                   BVH& bvh,
                                   AABB& queryAABB,
                                   int queryObjectIdx)
{
    // Allocate traversal stack from thread-local memory,
    // and push NULL to indicate that there are no postponed nodes.
    NodePtr stack[64];
    NodePtr* stackPtr = stack;
    *stackPtr++ = NULL; // push

    // Traverse nodes starting from the root.
    NodePtr node = bvh.getRoot();
    do {
        // see next
    }
    while (node != NULL);
}
```

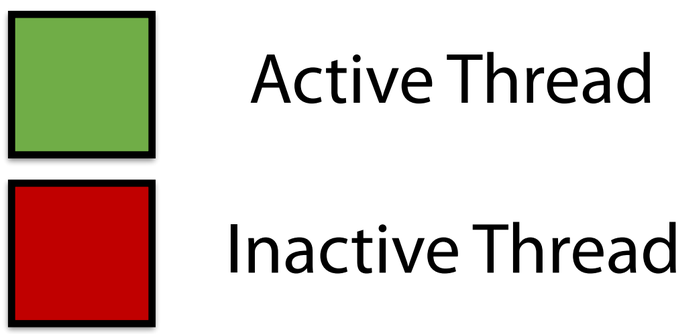
BVH Traversal – Second Attempt

```
__device__ void traverseIterative(

{
    // Allocate traversal stack fro
    // and push NULL to indicate th
    NodePtr stack[64];
    NodePtr* stackPtr = stack;
    *stackPtr++ = NULL; // push

    // Traverse nodes starting from
    NodePtr node = bvh.getRoot();
    do {
        // see next
    }
    while (node != NULL);
}
```

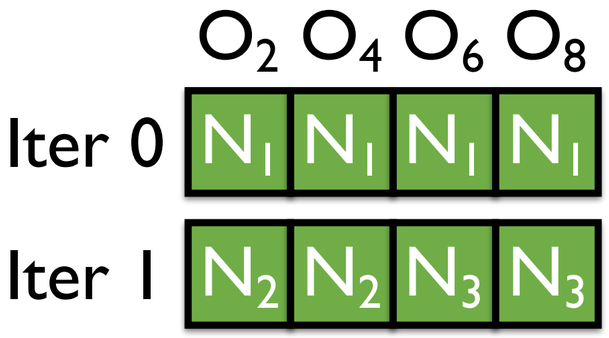
```
// Check each child node for overlap.
NodePtr childL = bvh.getLeftChild(node);
NodePtr childR = bvh.getRightChild(node);
bool overlapL = ( checkOverlap(queryAABB,
                               bvh.getAABB(childL)) );
bool overlapR = ( checkOverlap(queryAABB,
                               bvh.getAABB(childR)) );

// Query overlaps a leaf node => report collision.
if (overlapL && bvh.isLeaf(childL))
    list.add(queryObjectIdx, bvh.getObjectIdx(childL));

if (overlapR && bvh.isLeaf(childR))
    list.add(queryObjectIdx, bvh.getObjectIdx(childR));

// Query overlaps an internal node => traverse.
bool traverseL = (overlapL && !bvh.isLeaf(childL));
bool traverseR = (overlapR && !bvh.isLeaf(childR));

if (!traverseL && !traverseR)
    node = *--stackPtr; // pop
else {
    node = (traverseL) ? childL : childR;
    if (traverseL && traverseR)
        *stackPtr++ = childR; // push
}
```

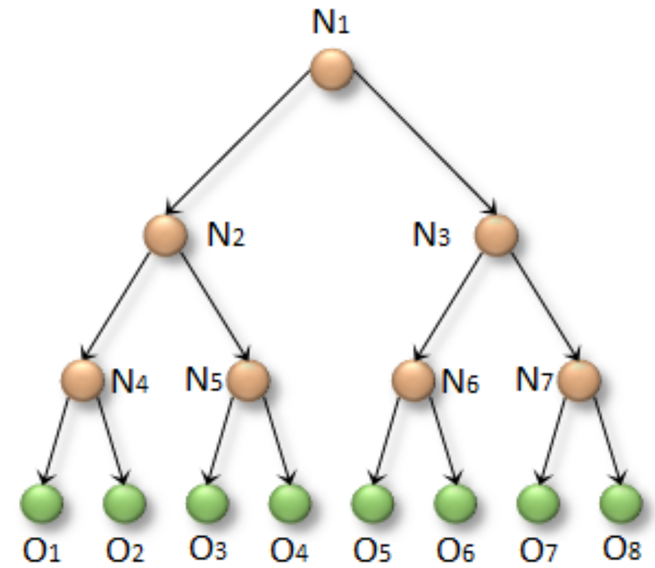
BVH Traversal – Second Attempt

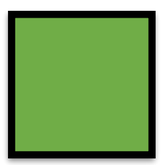




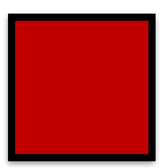
BVH Traversal – Second Attempt





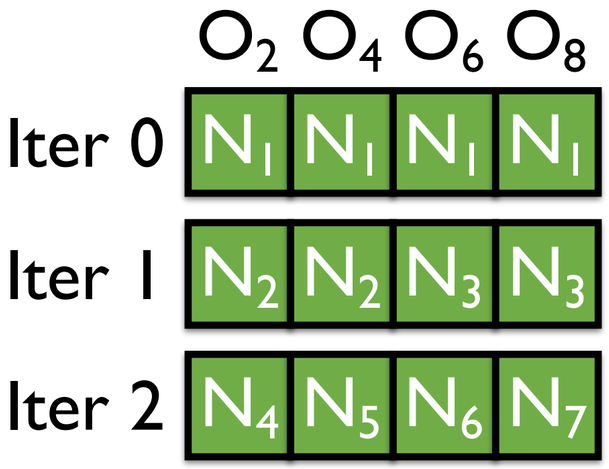


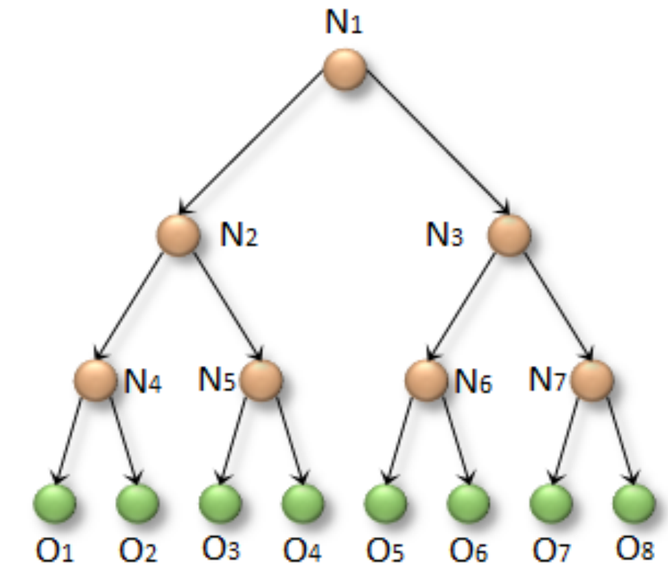
Active Thread

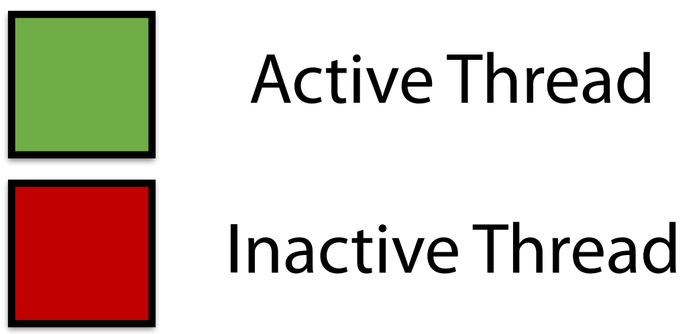


Inactive Thread

BVH Traversal – Second Attempt







BVH Traversal – Second Attempt Performance

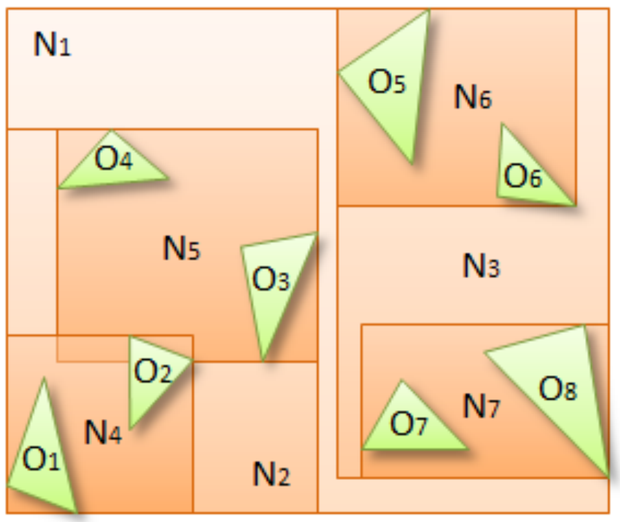
- Execution time: 0.91 milliseconds
  - vs. 3.8 ms for the recursive kernel

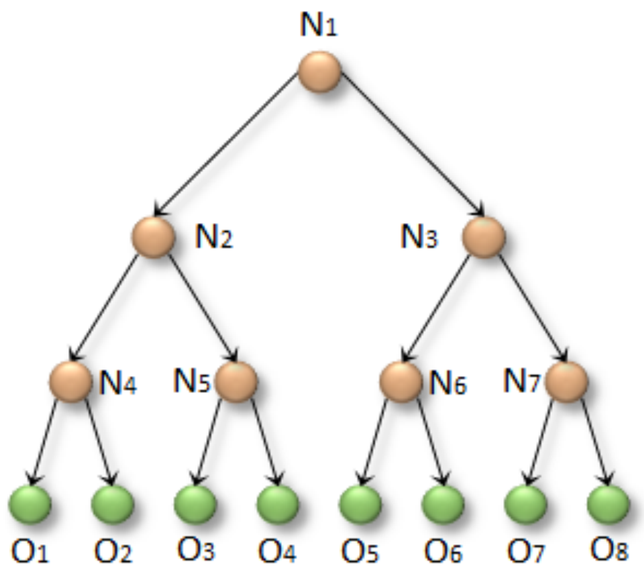
- Improvement over recursive: less thread divergence
  - Each thread executes the same loop repeatedly, regardless of traversal decisions

- Problem: data divergence
  - Threads might be accessing different parts of the tree
  - Threads may execute different number of iterations

Options to Parallelize Traversal

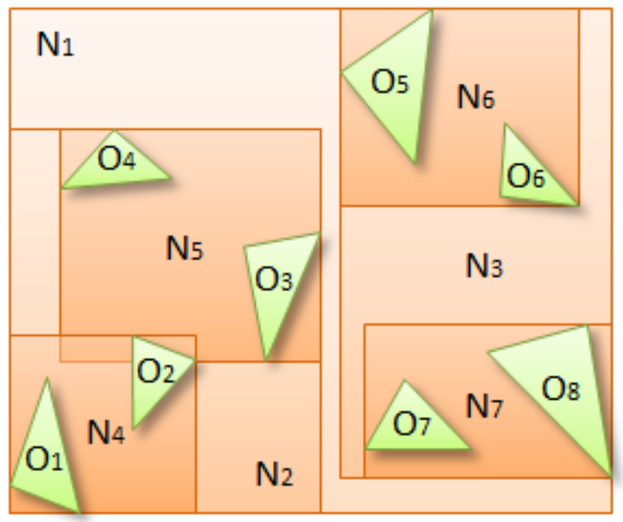
- One thread per object
  - E.g., [O_2, O_7, O_1, O_4, …]
  - Different objects take different paths down the tree

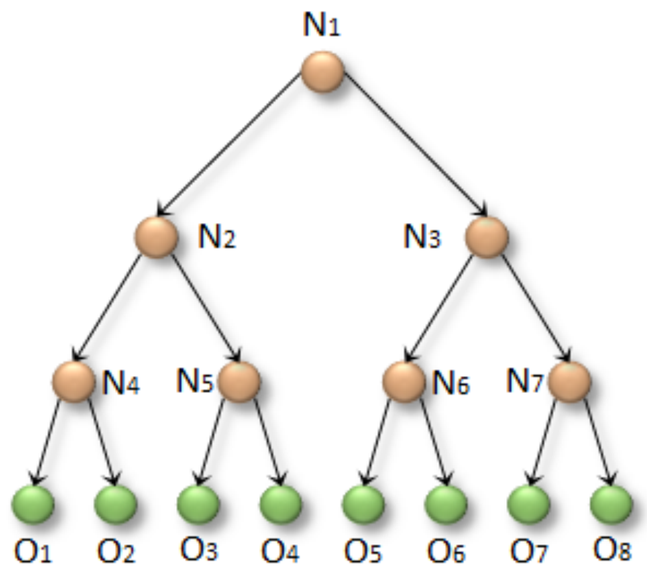




Options to Parallelize Traversal

- One thread per object
  - E.g., [O_2, O_7, O_1, O_4, …]
  - Different objects take different paths down the tree
  - Lots of divergence





Options to Parallelize Traversal

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  - E.g., [O_2, O_7, O_1, O_4, …]
  - Different objects take different paths down the tree
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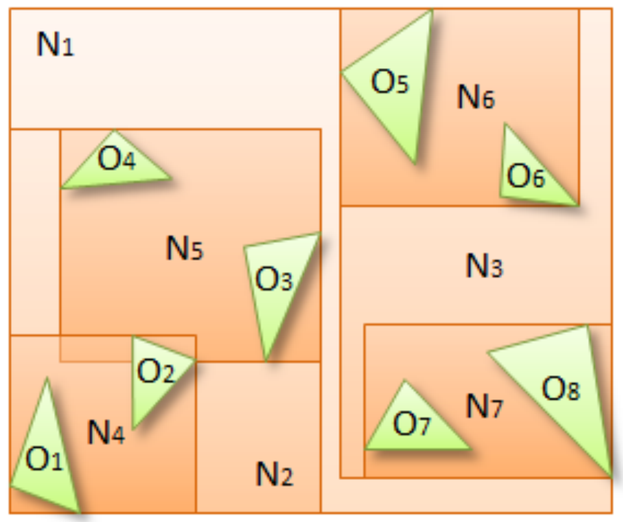
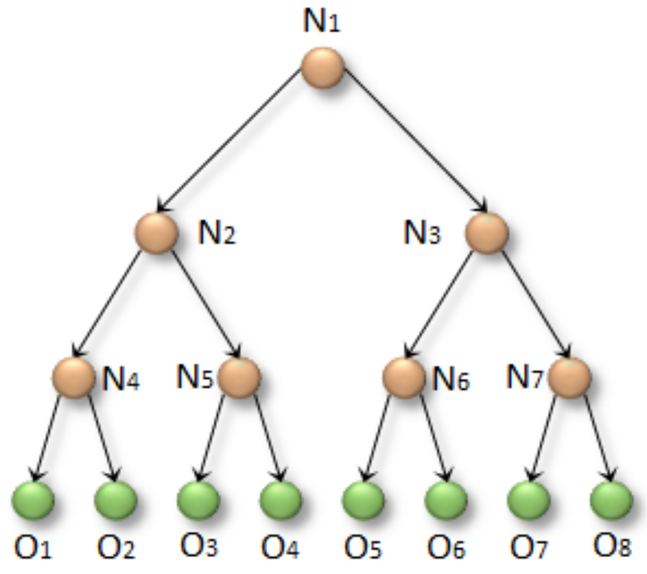
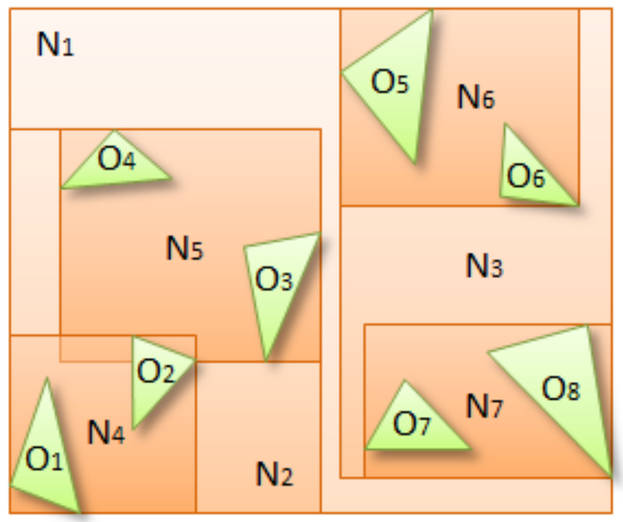


- One thread per leaf node
  - E.g., [O_1, O_2, O_3, O_4, …]
  - Still one thread per object

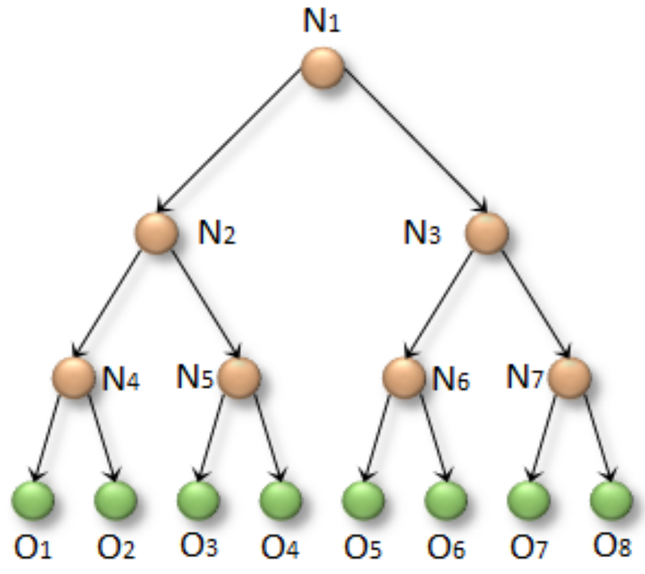


Options to Parallelize Traversal

- One thread per object
  - E.g., [O_2, O_7, O_1, O_4, …]
  - Different objects take different paths down the tree
  - Lots of divergence



- One thread per leaf node
  - E.g., [O_1, O_2, O_3, O_4, …]
  - Still one thread per object
  - But order of objects changed



BVH Traversal – Third Attempt

One thread per object (Second Attempt)

random order

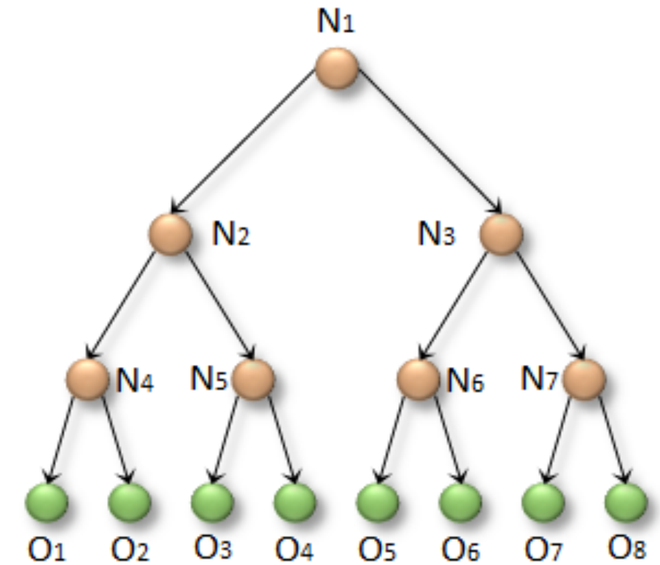
```
__global__ void findPotentialCollisions( CollisionList list, BVH bvh,
                                         AABB* objectAABBs, int numObjects) {
    int idx = threadIdx.x + blockDim.x * blockIdx.x;
    if (idx < numObjects)
        traverseIterative(list, bvh, objectAABBs[idx], idx);
}
```

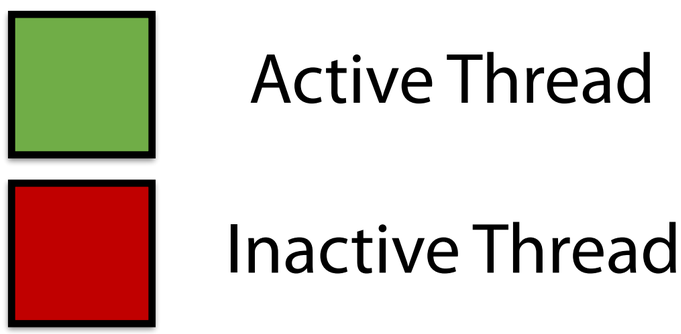
One thread per leaf node (Third Attempt)

```
__global__ void findPotentialCollisions( CollisionList list,
                                         BVH           bvh) {
    int idx = threadIdx.x + blockDim.x * blockIdx.x;
    if (idx < bvh.getNumLeaves())
    {
        NodePtr leaf = bvh.getLeaf(idx);
        traverseIterative(list, bvh,
                          bvh.getAABB(leaf),
                          bvh.getObjectIdx(leaf));
    }
}
```

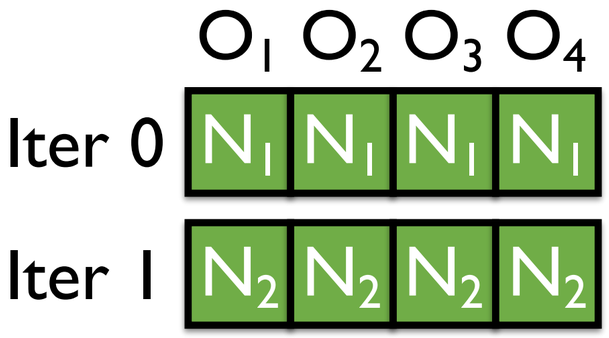
leaf order

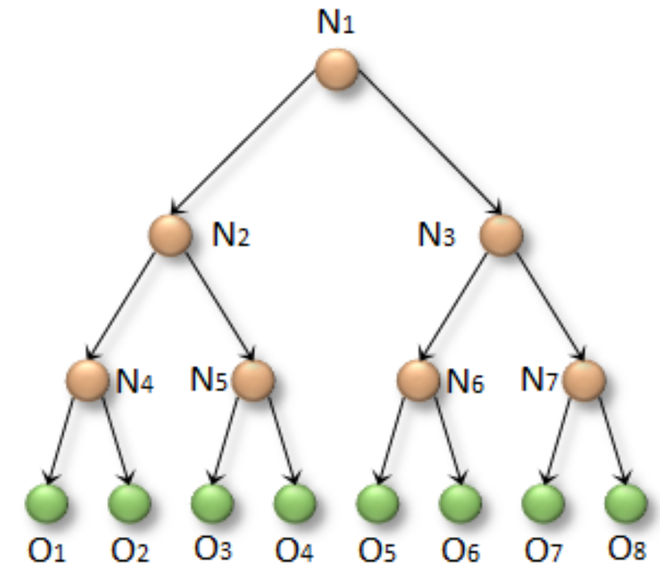
BVH Traversal – Third Attempt





BVH Traversal – Third Attempt

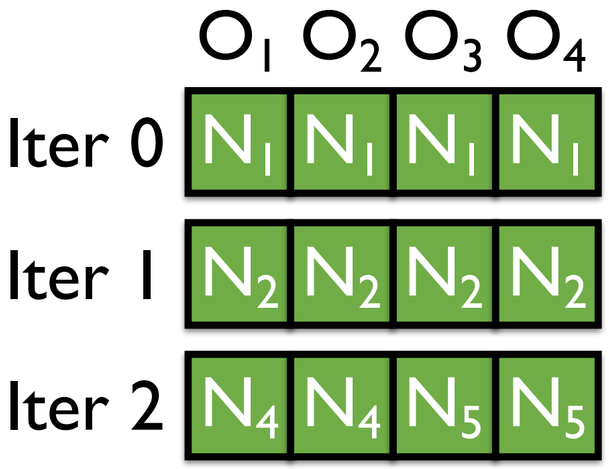


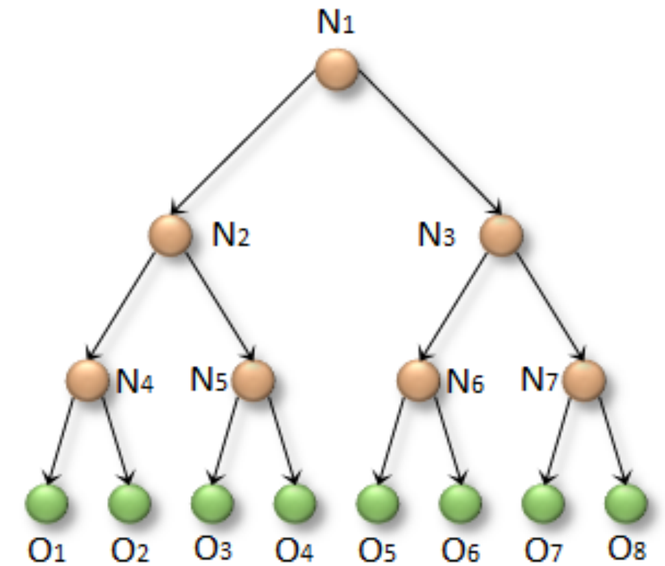


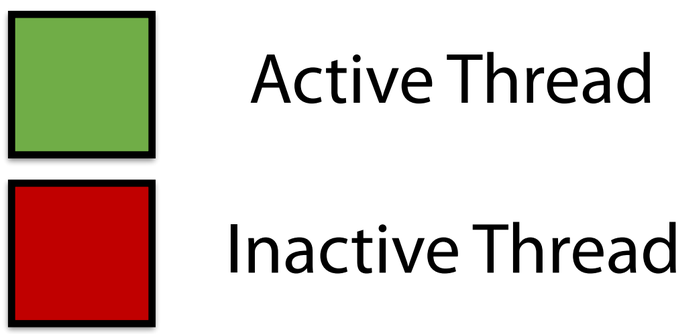
Active Thread

Inactive Thread

BVH Traversal – Third Attempt







BVH Traversal – Third Attempt Performance

- Execution time: 0.43 milliseconds
  - vs. 0.91 ms for second attempt
  - vs. 3.8 ms for the recursive kernel
- Improvement over second: better data access patterns
  - Objects close to each other (and, thus, likely to collide) are likely located in nearby leaf nodes
    - Improvement from coalescing, cache efficiency
- Problem: each collision reported twice
  - Twice as much work!
  - Solution: Require that A collides with B only if A appears before B in the tree (see blog post for detail)

Simultaneous Traversal

- Basic idea: group nearby objects together and traverse tree all at once for these groups
  - "Packet tracing"

- Works really well on CPU

- Worse performance on GPU
  - Individual traversal is faster
  - Divergence is the reason

Tree Construction

Motivation for GPU Tree Construction

- Constantly changing scene -> need to update BVH at every time step

- Could recycle the same BVH and make updates
  - But tree can deteriorate in unpredictable ways over time
  - Results in poor traversal performance
  - Also this is a challenging data structure problem

- Instead, we will recreate the BVH from scratch every time step

Linear BVH (LBVH)

- Basic algorithm:
  - Choose an order for leaf nodes (objects) to appear in the tree
  - Generate internal nodes that respect this order

For more details, see: C. Lauterbach, M. Garland, S. Sengupta, D. Luebke, and D. Manocha, "Fast BVH Construction on GPUs," Computer Graphics Forum, vol. 28, no. 2, 2009.

Linear BVH (LBVH)

- We want objects in close 3D proximity to be nearby in the BVH
- So, sort them along a space-filling curve
- Specifically, we will use a Z-order curve

Z-order Curve

- Maps multidimensional data to 1D
- Preserves locality of data points
- Defined using Morton codes
  - Interleave binary representation of its coordinate values

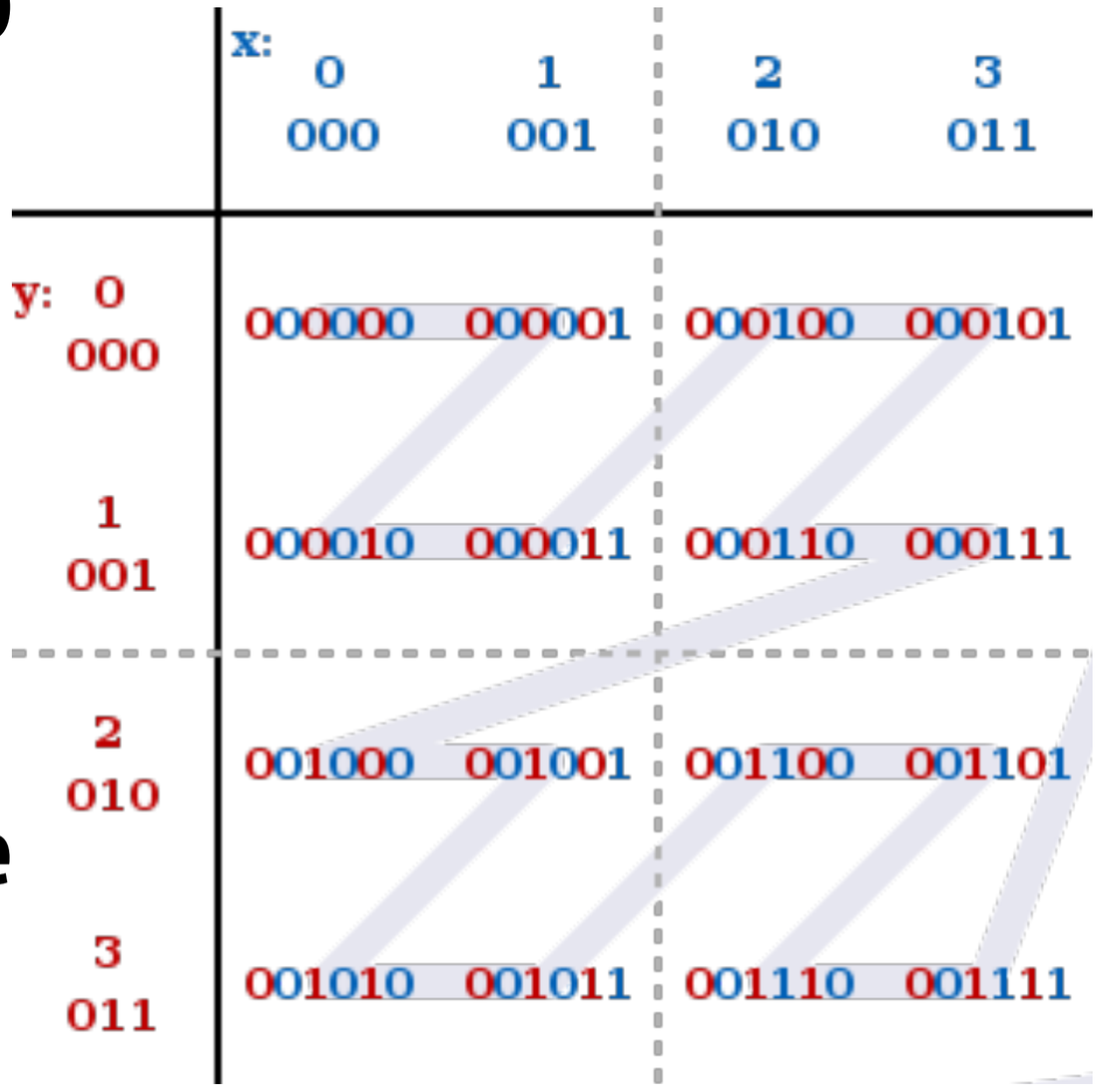


Image from https://en.wikipedia.org/wiki/Z-order_curve

Finding the leaf node order

- Compute Morton code for each object
  - Use the centroid of an object's AABB for its coordinates
  - Interleave the coordinates of the object as described previously
  - (Clever code to compute the codes can be found here: https://devblogs.nvidia.com/parallelforall/thinking-parallel-part-iii-tree-construction-gpu/)

- Sort the Morton codes
  - Radix sort will work well

LBVH - Lauterbach et al. [2009]

1. Assign Morton codes
2. Sort primitives
3. Generate hierarchy
4. Fit bounding boxes

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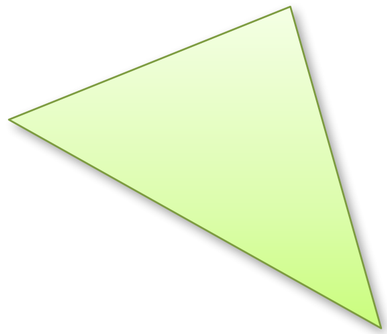

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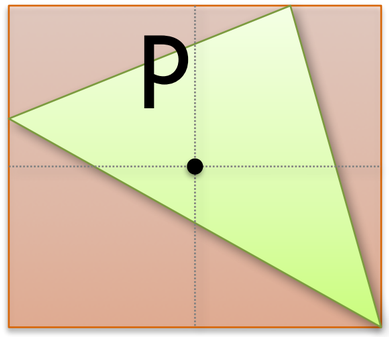

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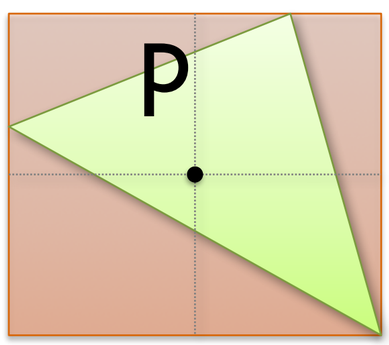

$p_x = 0.\ 1010$
$p_y = 0.\ 0111$
$p_z = 0.\ 1100$

LBVH - Lauterbach et al. [2009]

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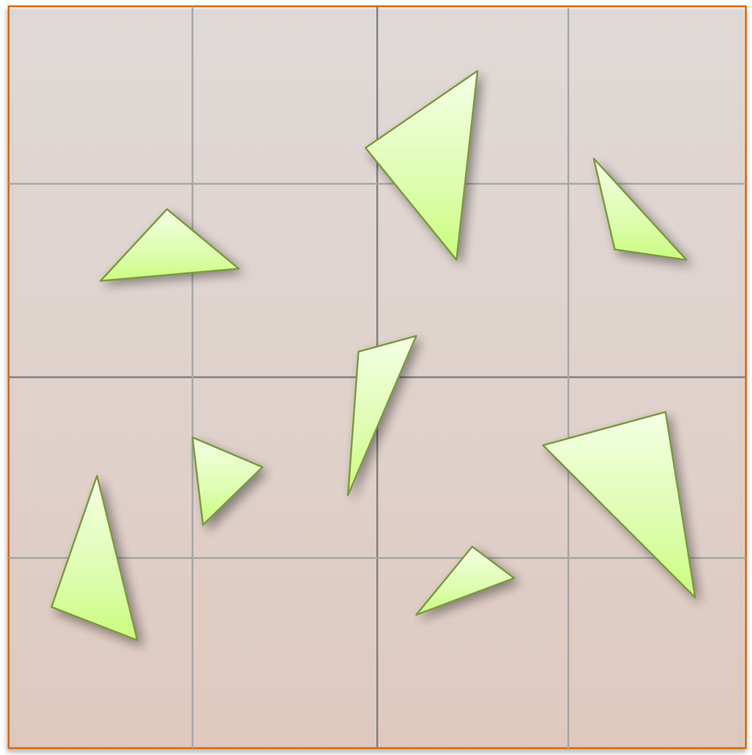

LBVH - Lauterbach et al. [2009]

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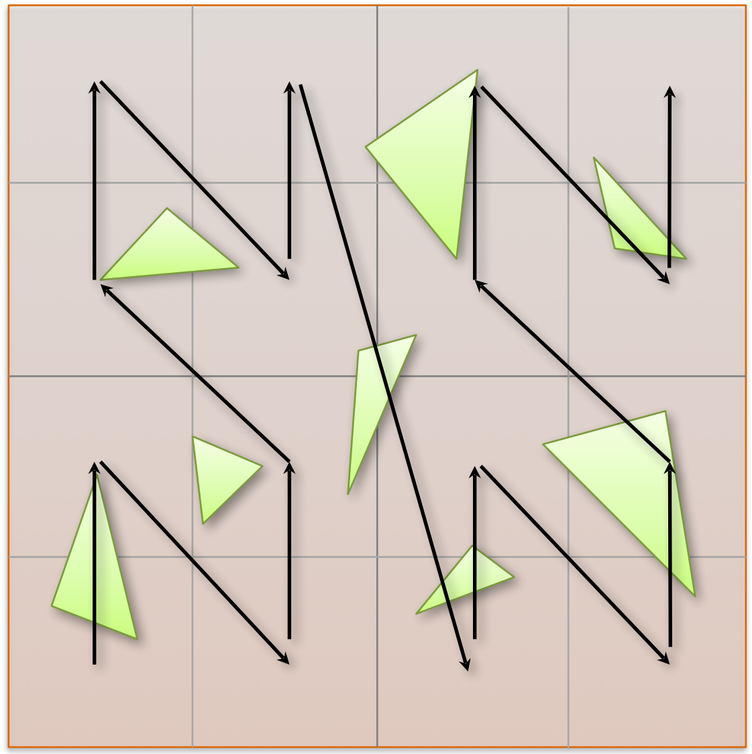


Finding the leaf node order: Performance

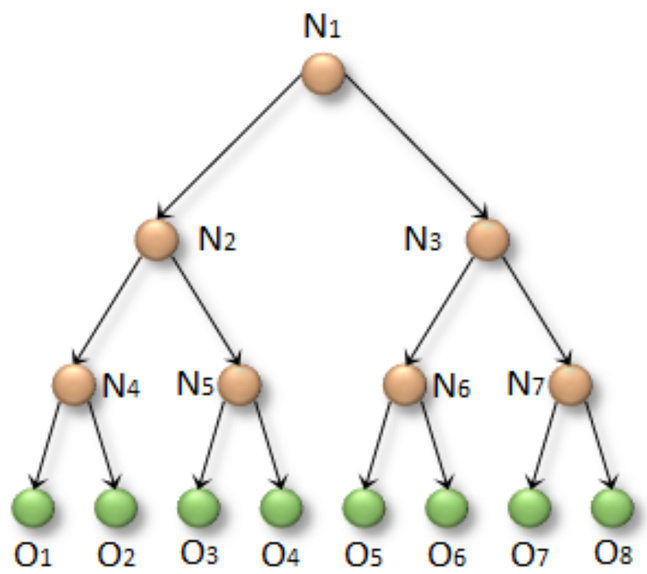
- Compute Morton code for each object
  - 0.02 milliseconds

- Sort the Morton codes
  - 0.18 milliseconds

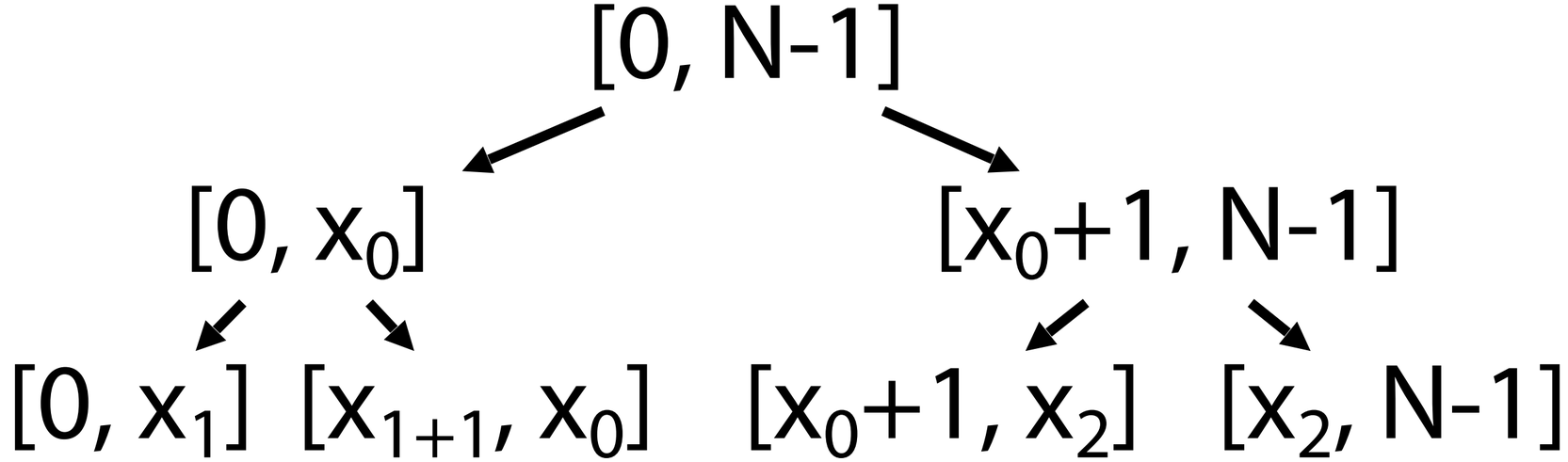
- Tested on GeForce GTX 690
  - Reminder: Traversal execution time is 0.25 milliseconds

Top-Down Hierarchy Generation

- Now that we have the nodes in order, we have to generate the hierarchy (the tree)
- With leaf nodes in order, internal nodes are just linear ranges over them
  - But not necessarily balanced linear ranges!
- N1: [01, 08]
- N2: [01, 04]
- N3: [05, 08]
- N4: [01, 02]
- etc.



Top-Down Hierarchy Generation

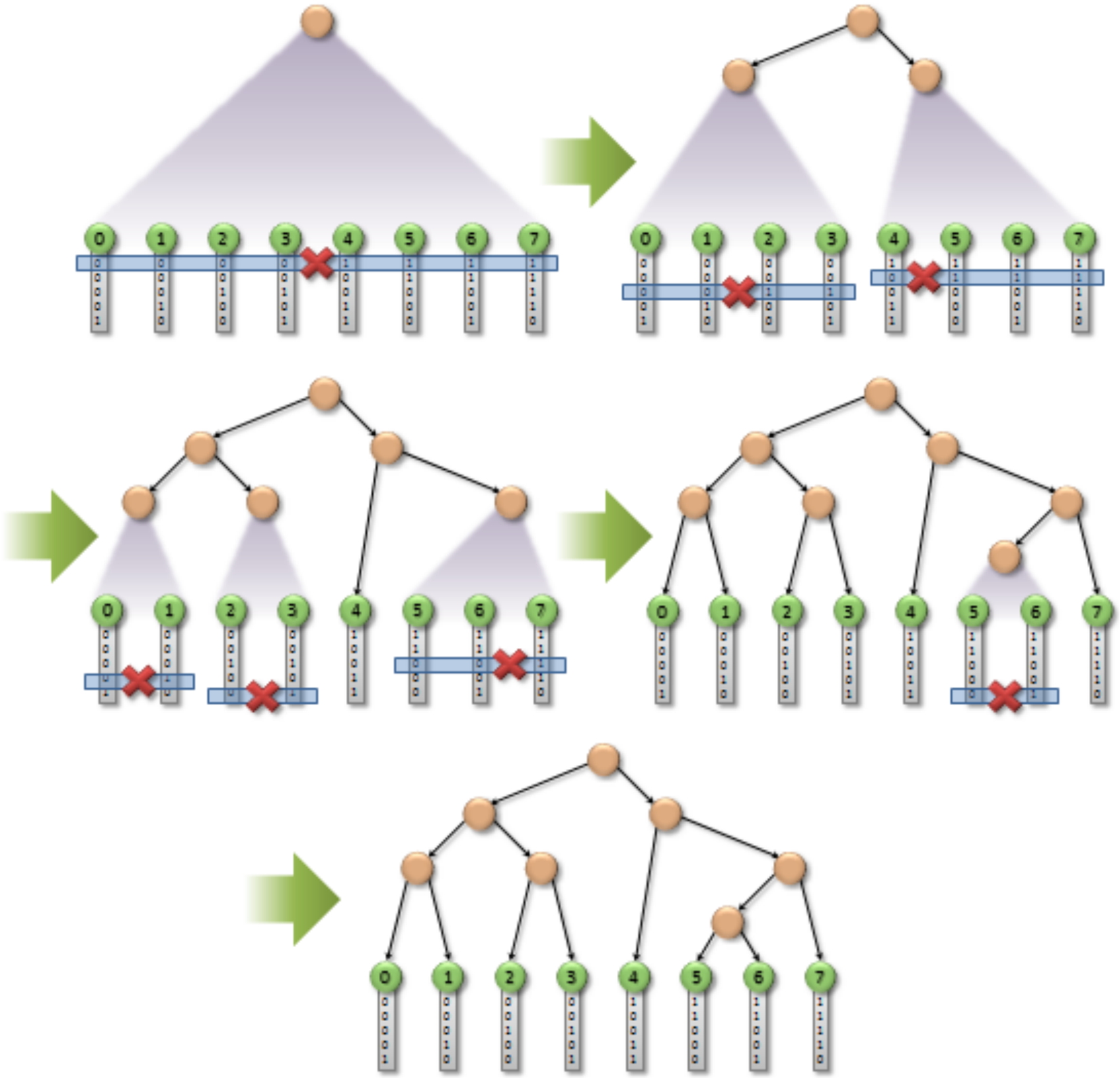


Top-Down Hierarchy Generation

```
Node* generateHierarchy( unsigned int* sortedMortonCodes,
                         int*          sortedObjectIDs,
                         int           first,
                         int           last)
{
    // Single object => create a leaf node.

    if (first == last)
        return new LeafNode(&sortedObjectIDs[first]);

    // Determine where to split the range.

    int split = findSplit(sortedMortonCodes, first, last);

    // Process the resulting sub-ranges recursively.

    Node* childA = generateHierarchy(sortedMortonCodes, sortedObjectIDs,
                                     first, split);
    Node* childB = generateHierarchy(sortedMortonCodes, sortedObjectIDs,
                                     split + 1, last);
    return new InternalNode(childA, childB);
}
```

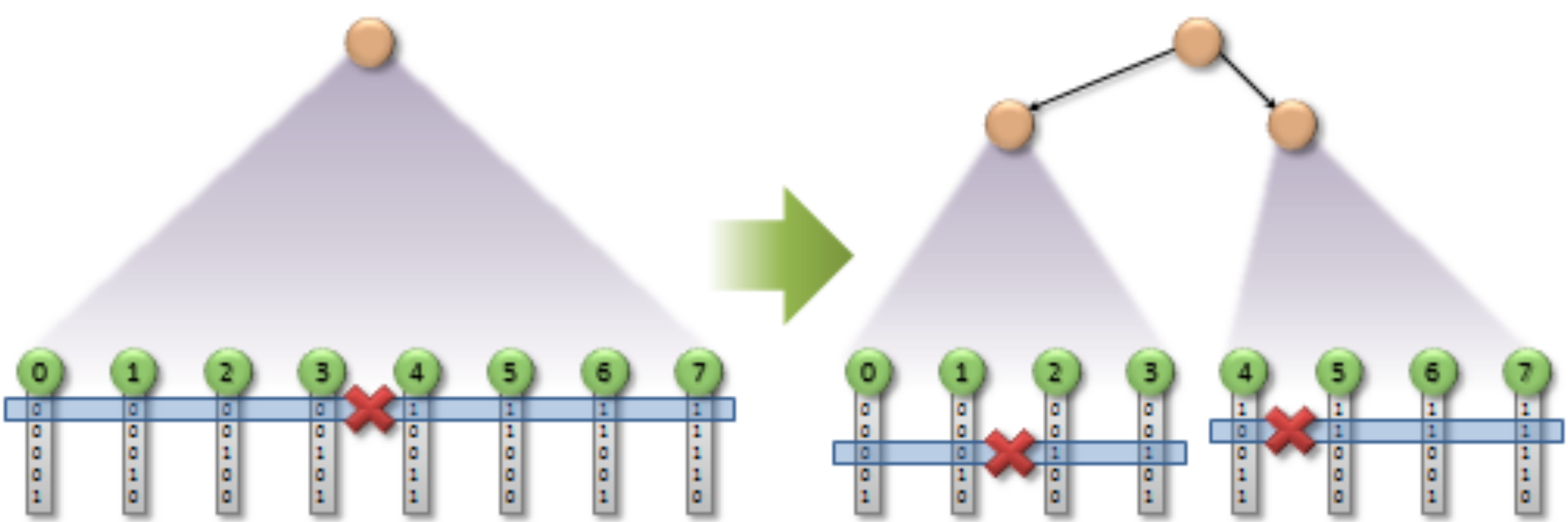
the interesting part!

Top-Down Hierarchy Generation

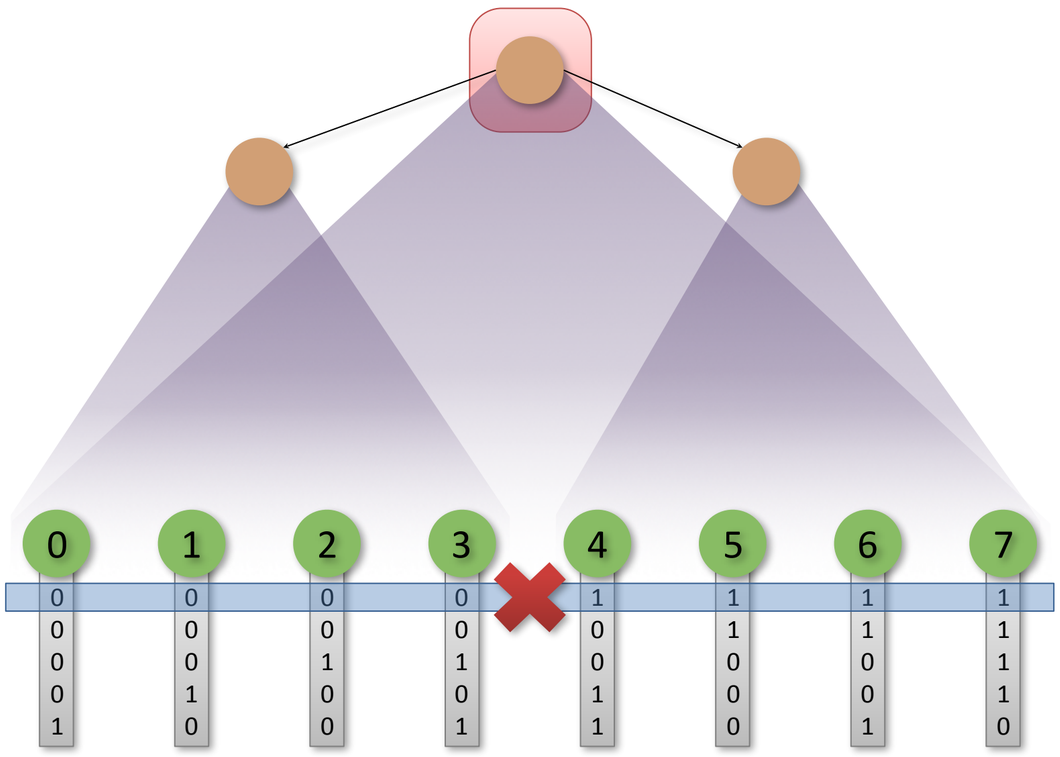


How should we choose the split points?

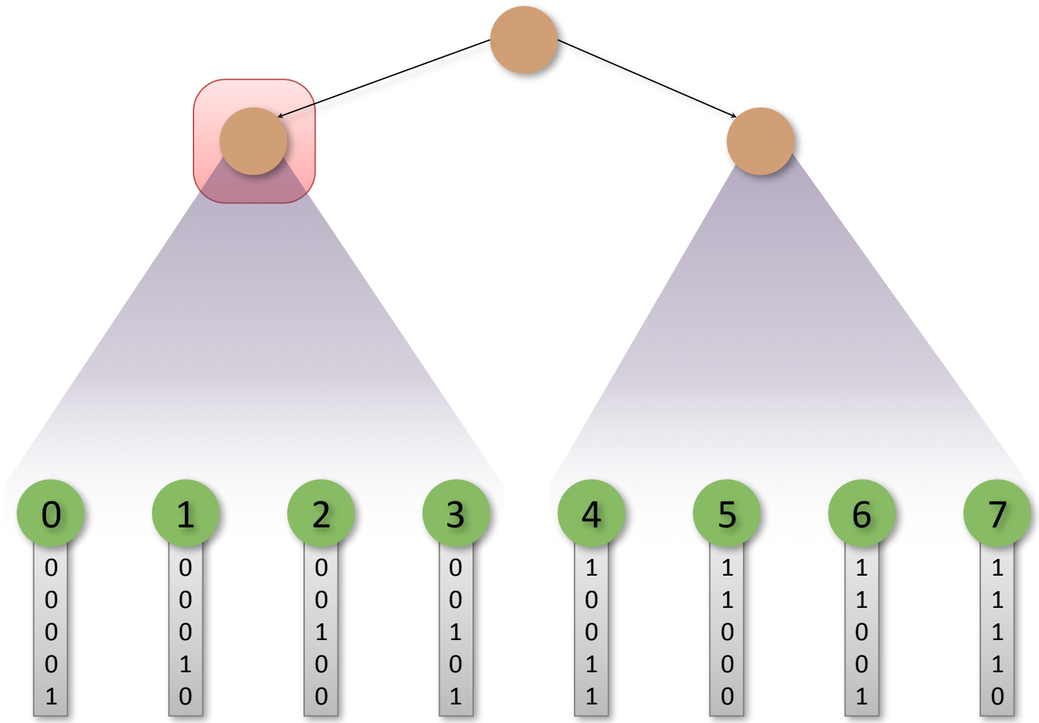
- LBVH chooses split point based on the highest differing bit



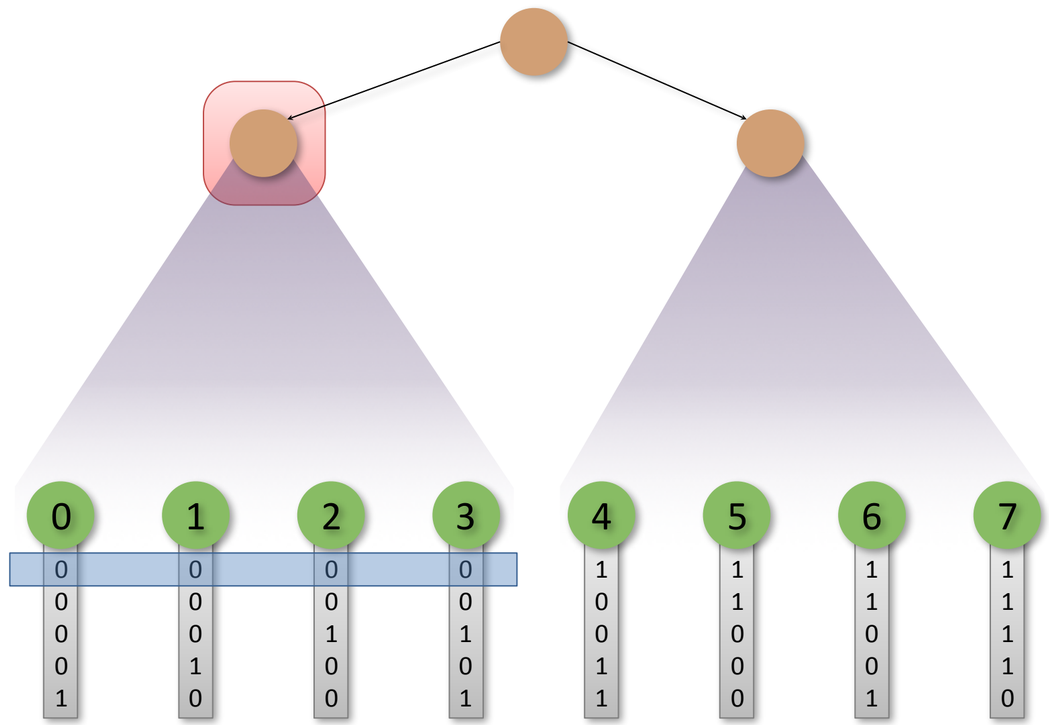
Binary radix tree



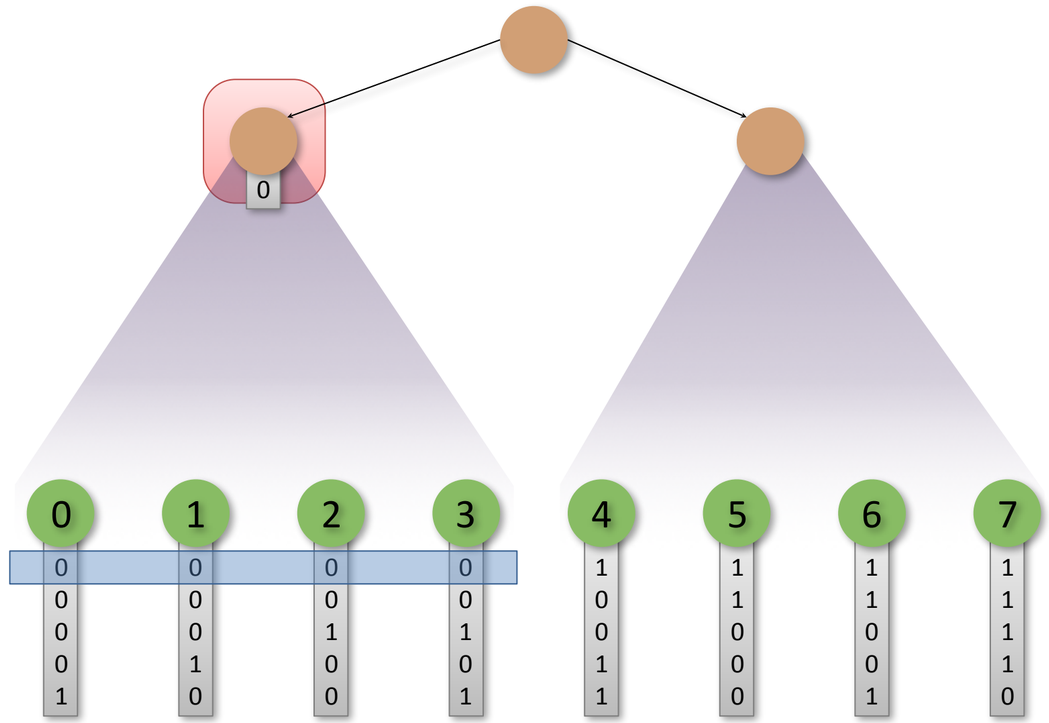
Binary radix tree



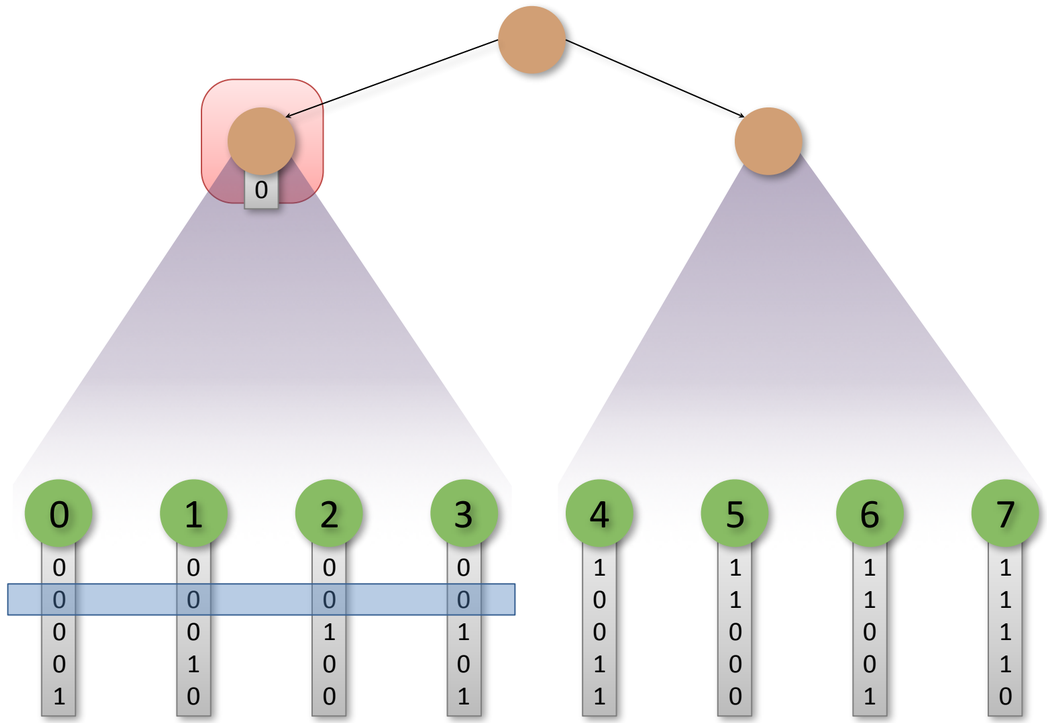
Binary radix tree



Binary radix tree



Binary radix tree



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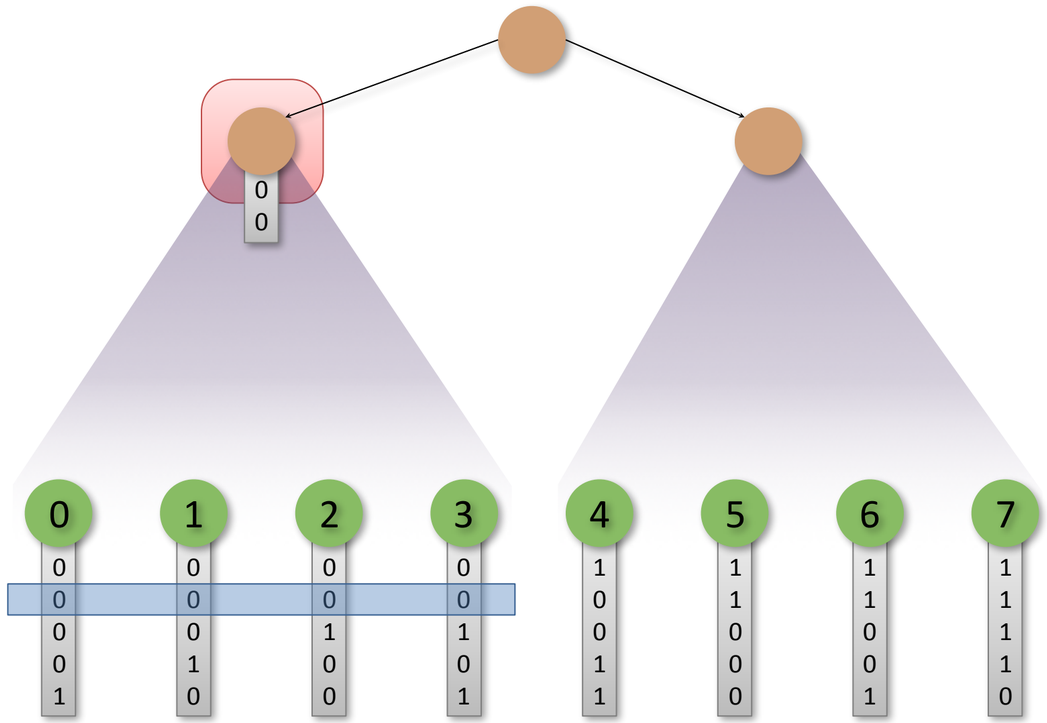

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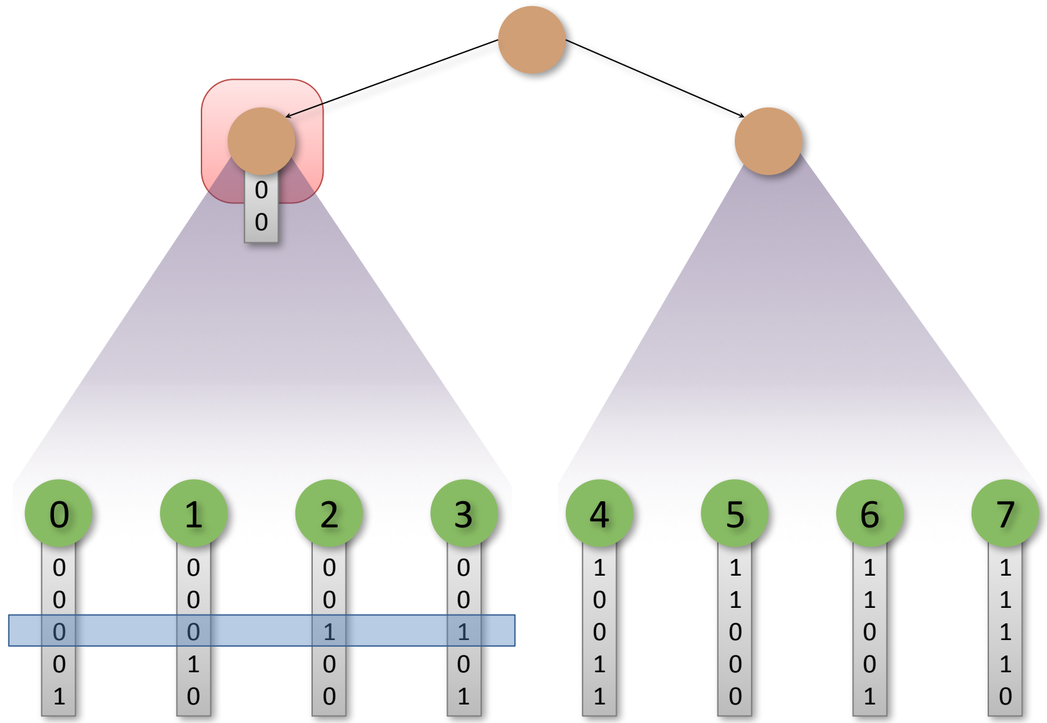
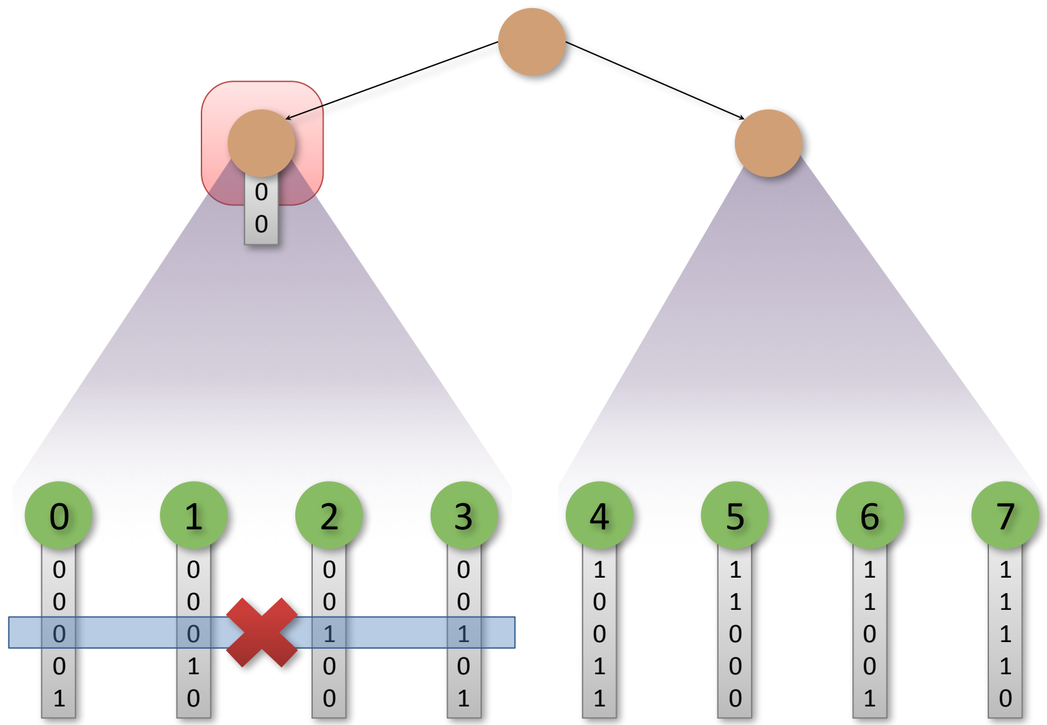


Binary radix tree



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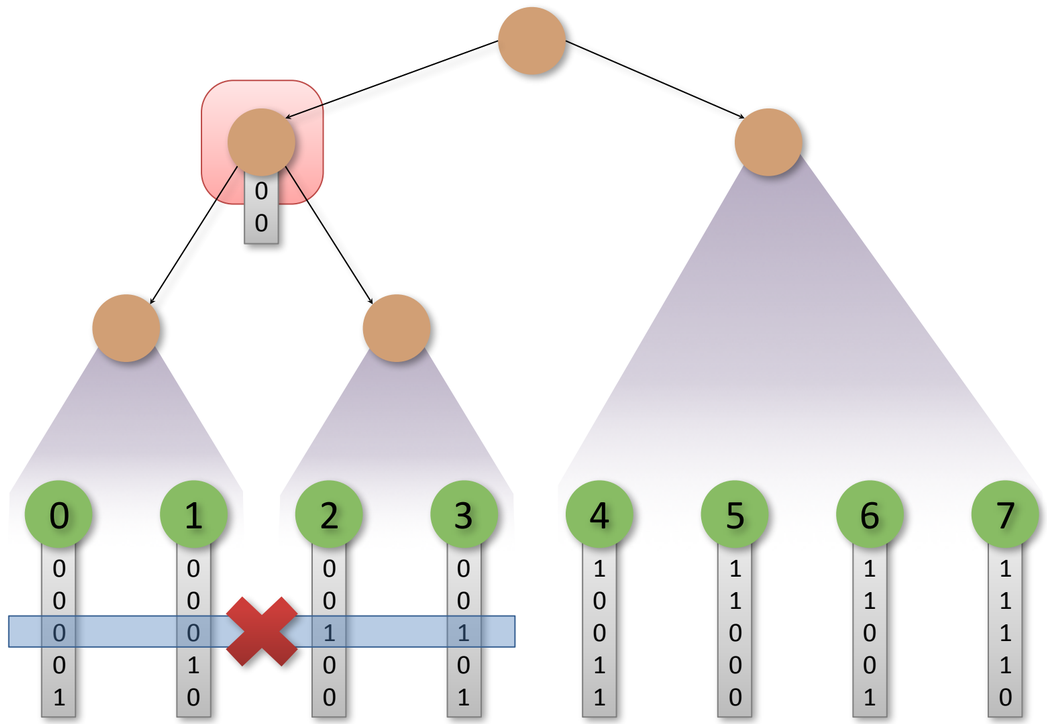

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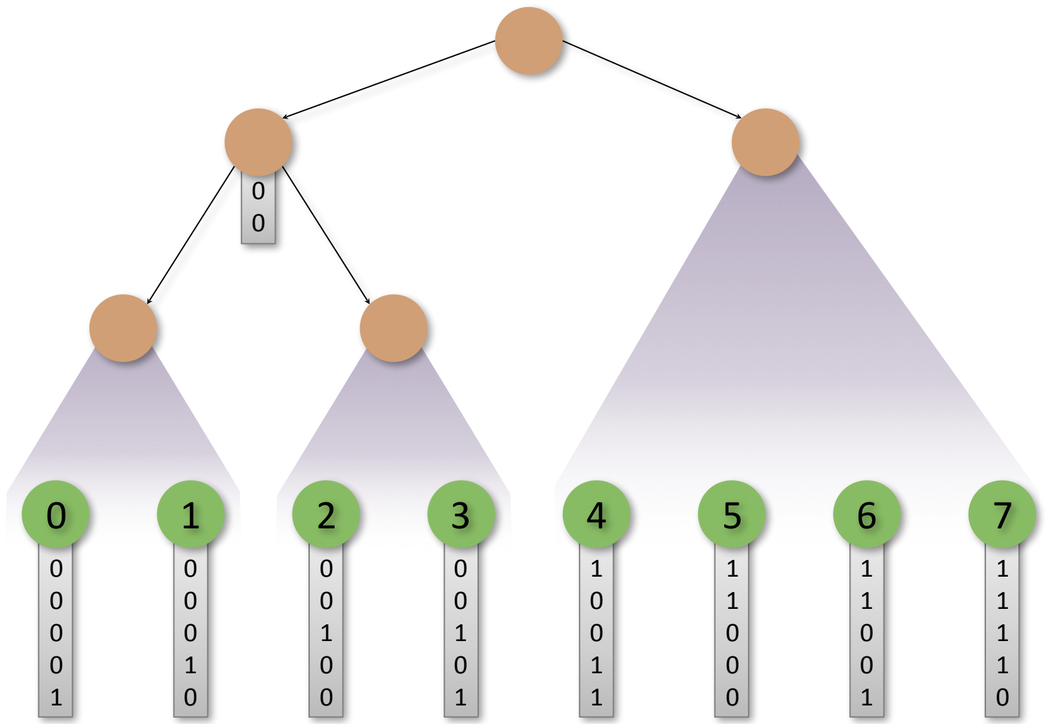

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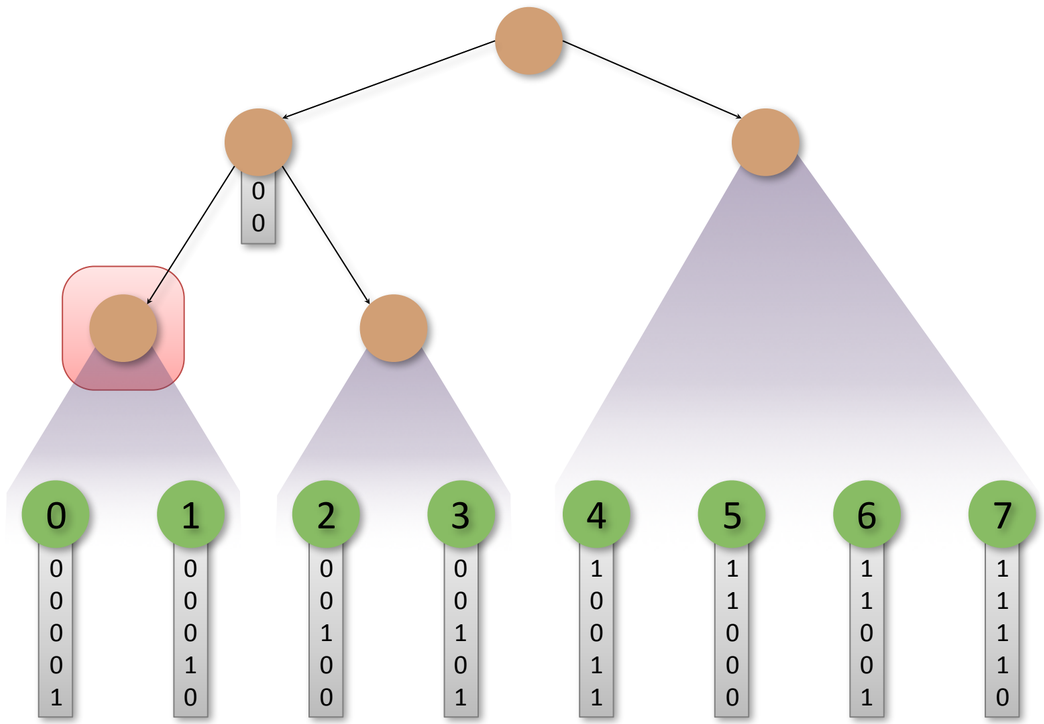

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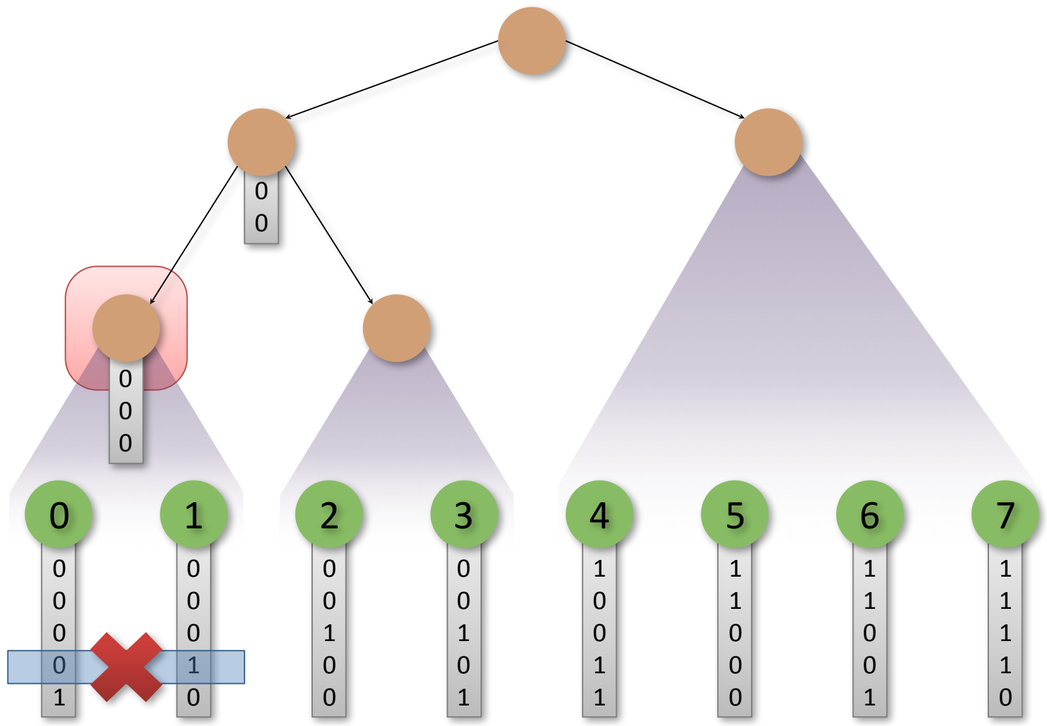

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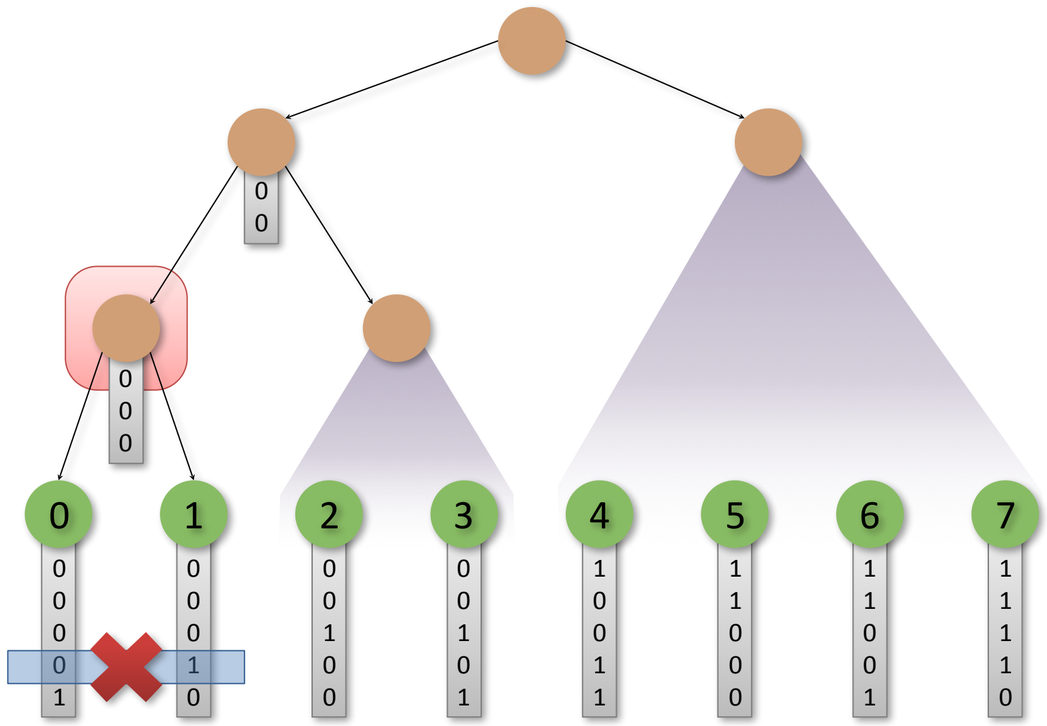

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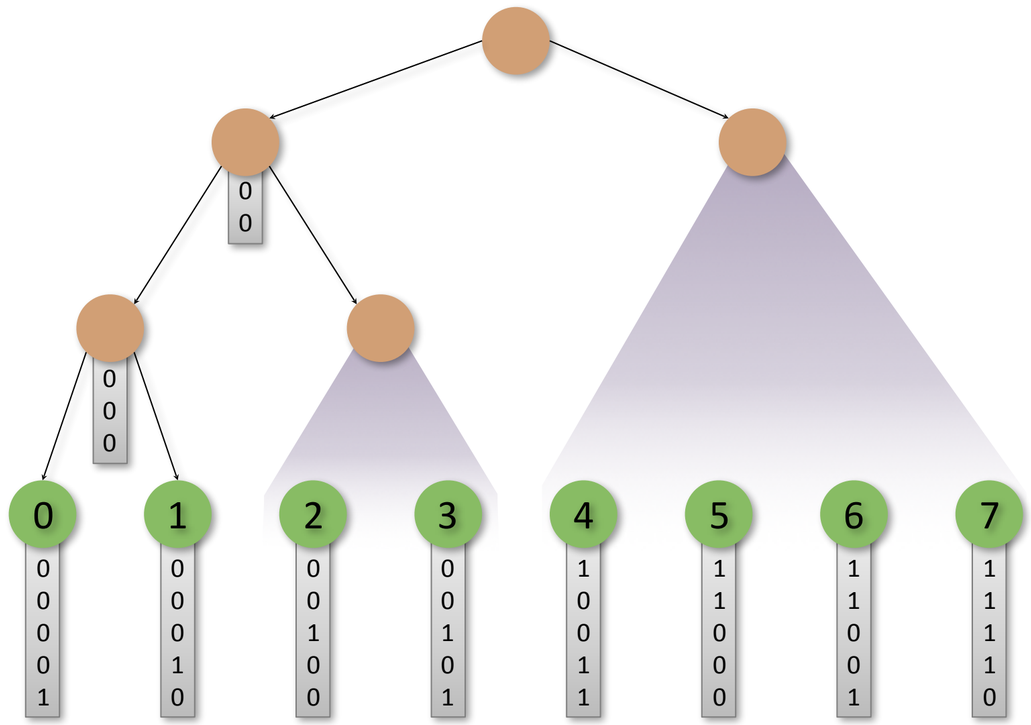

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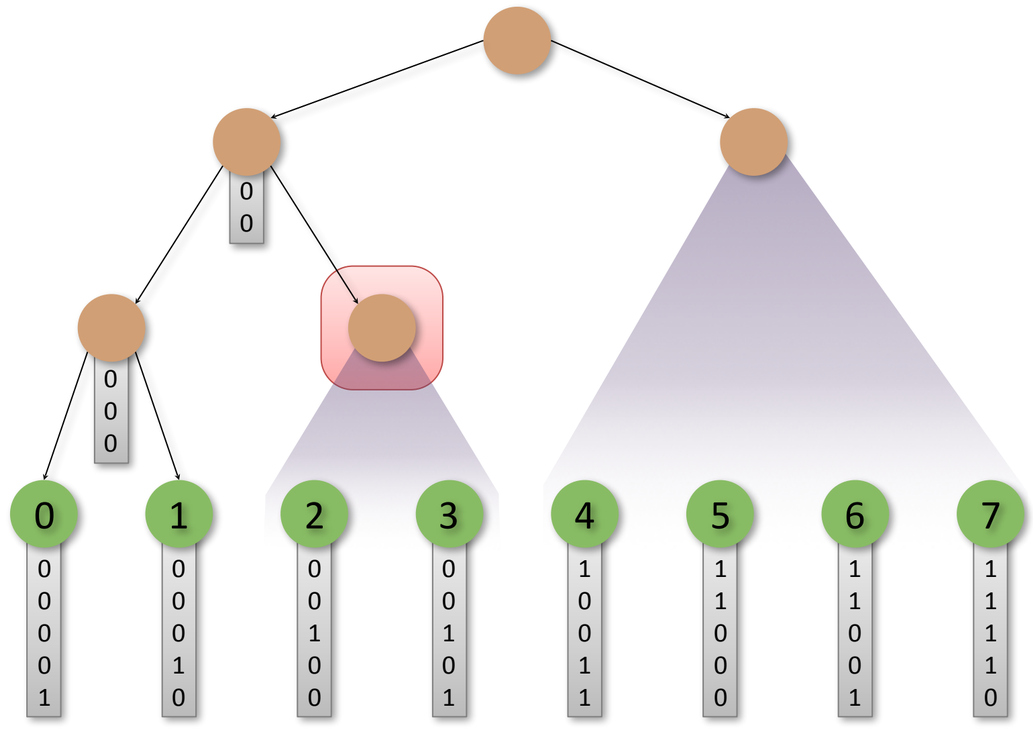

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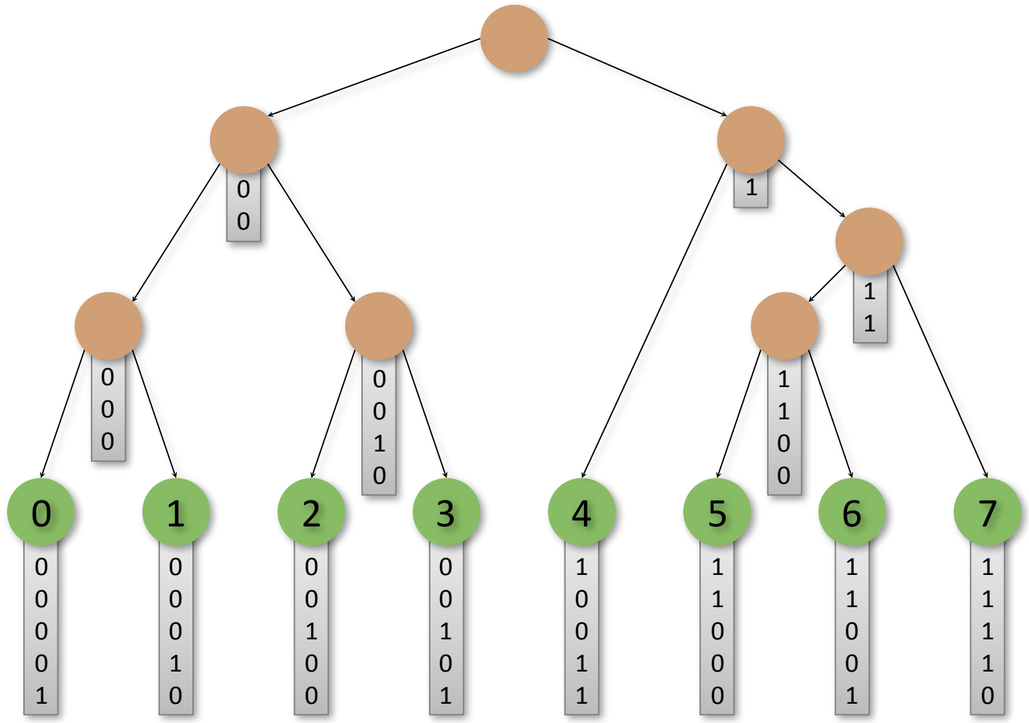

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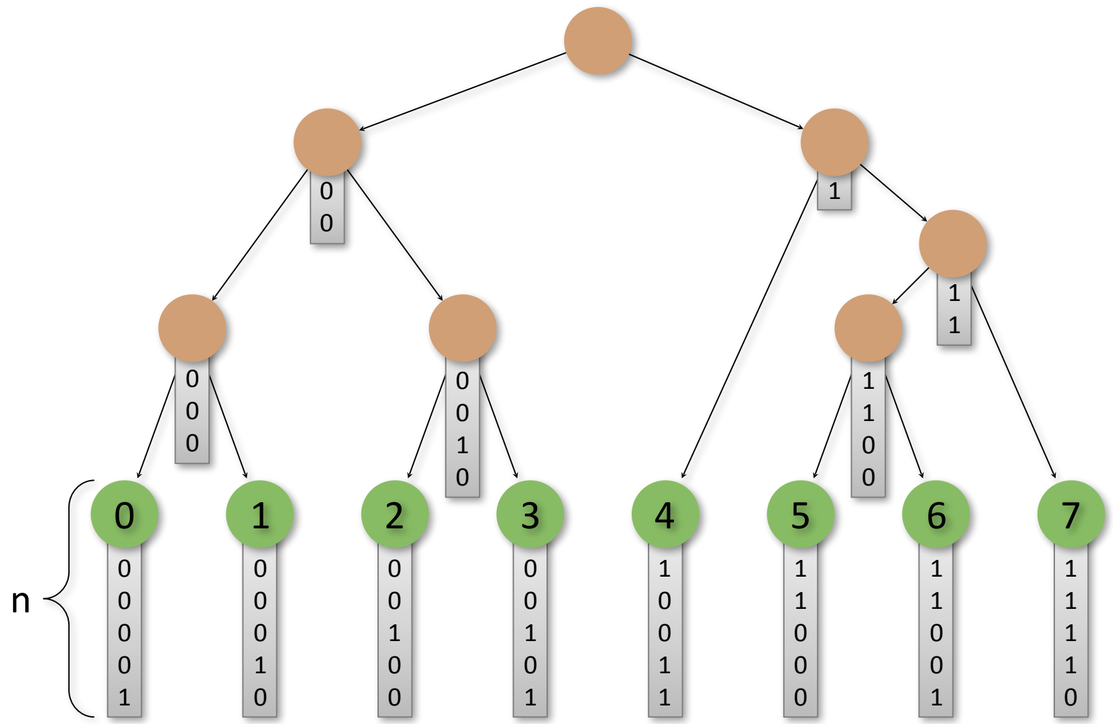

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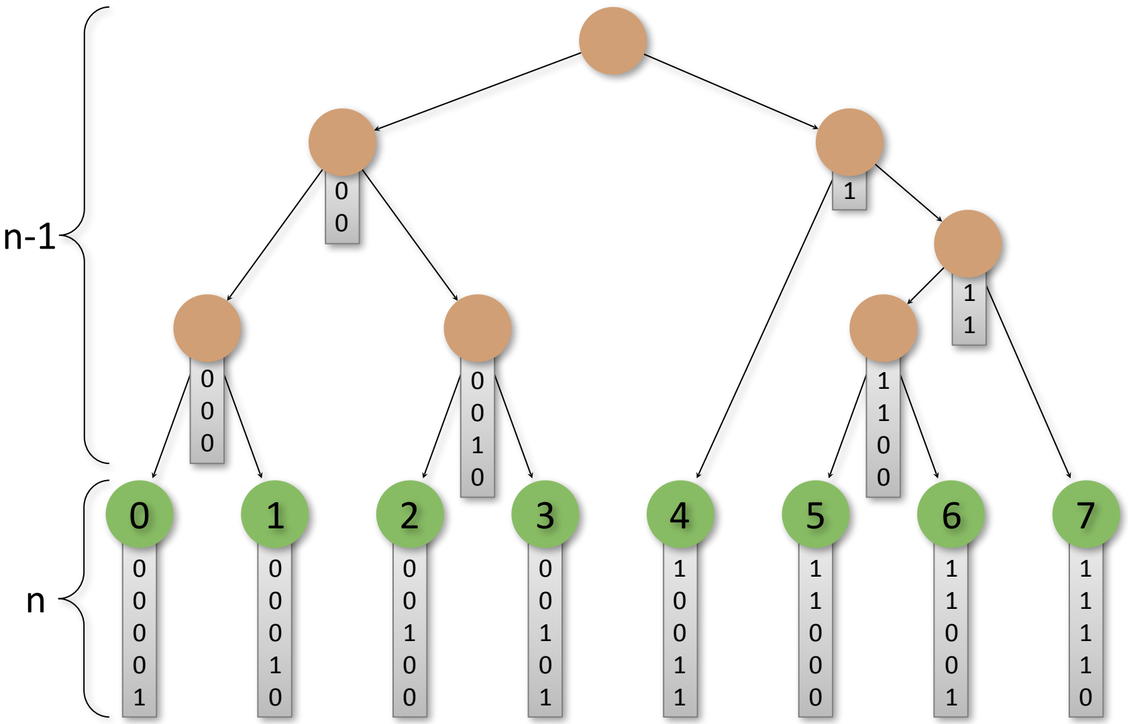


How should we choose the split points?

- LBVH chooses split point based on the highest differing bit
  - Corresponds to classifying them on either size of an axis-aligned plane in 3D

- Find split points one level at a time

- Use binary search to find the highest differing bit

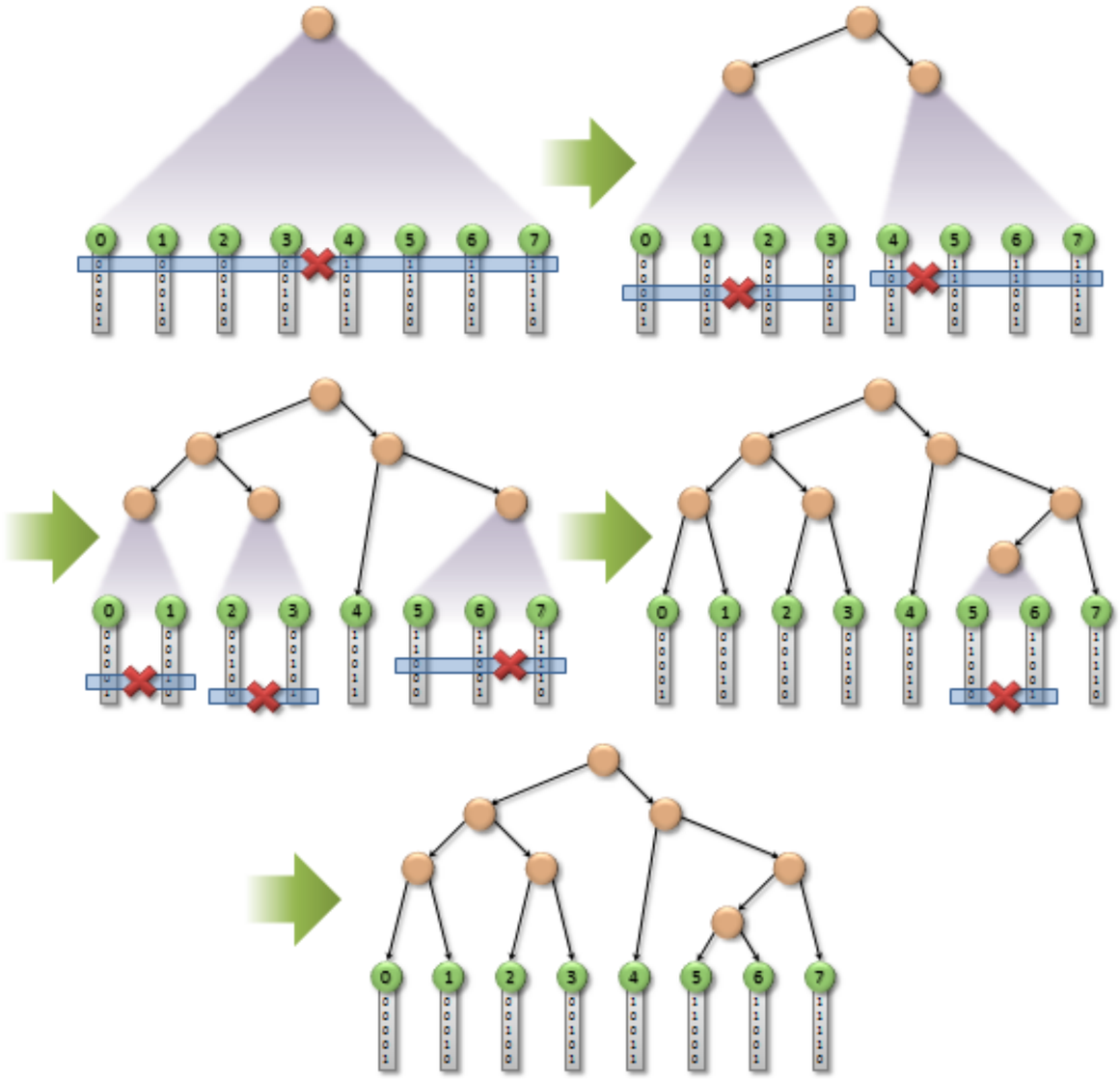
- Some clever code to find the split points can be found here: https://devblogs.nvidia.com/parallelforall/thinking-parallel-part-iii-tree-construction-gpu/

Parallel BVH Construction

- Process each level of nodes sequentially, starting from root

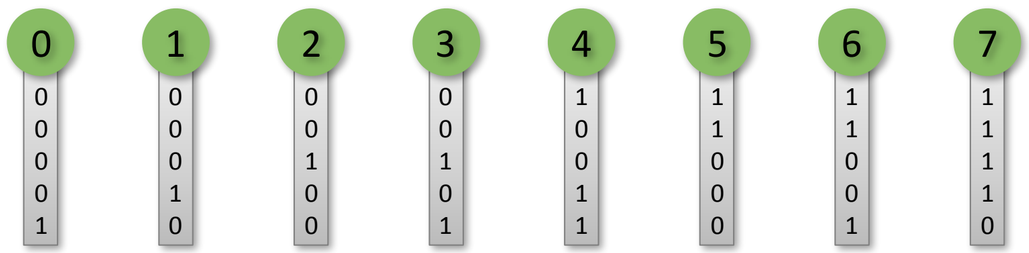
- Launch one thread per node in the level
  - Read first and last from input array
  - Call findSplit()
  - Append resulting child nodes to array using atomic counter
  - Write out corresponding sub-ranges

- Synchronize and repeat for subsequent levels

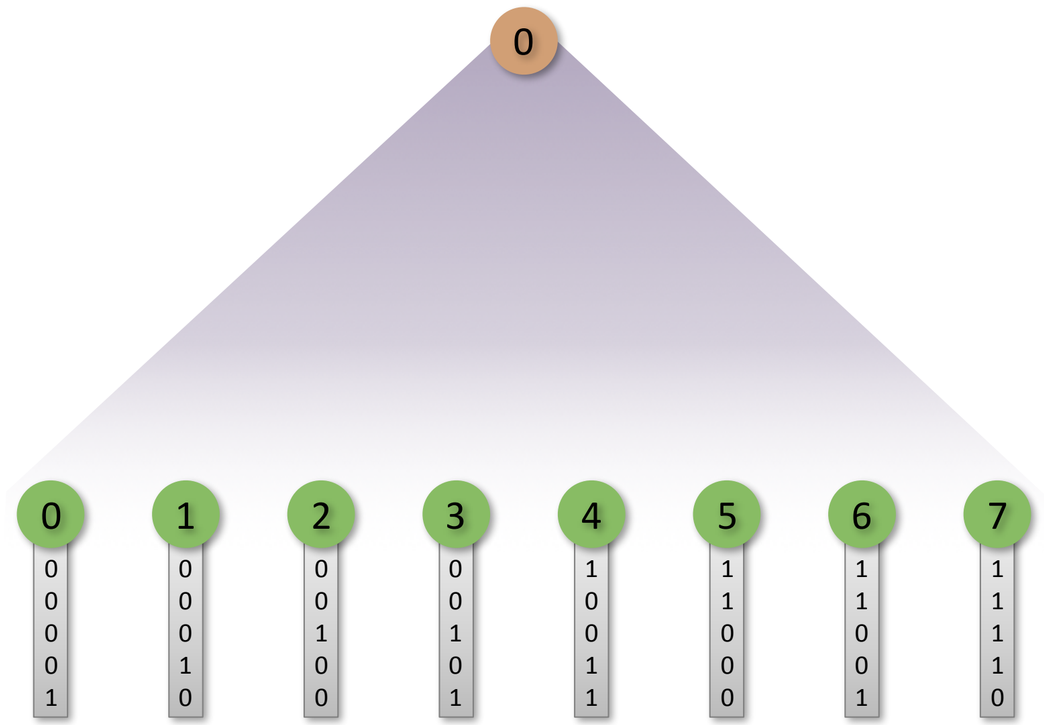


For more details, see: K. Garanzha, J. Pantaleoni, and D. McAllister, "Simpler and faster HLBVH with work queues," HPG'11: Proceedings of the ACM SIGGRAPH Symposium on High Performance Graphics

Garanzha et al. [2011]



Garanzha et al. [2011]



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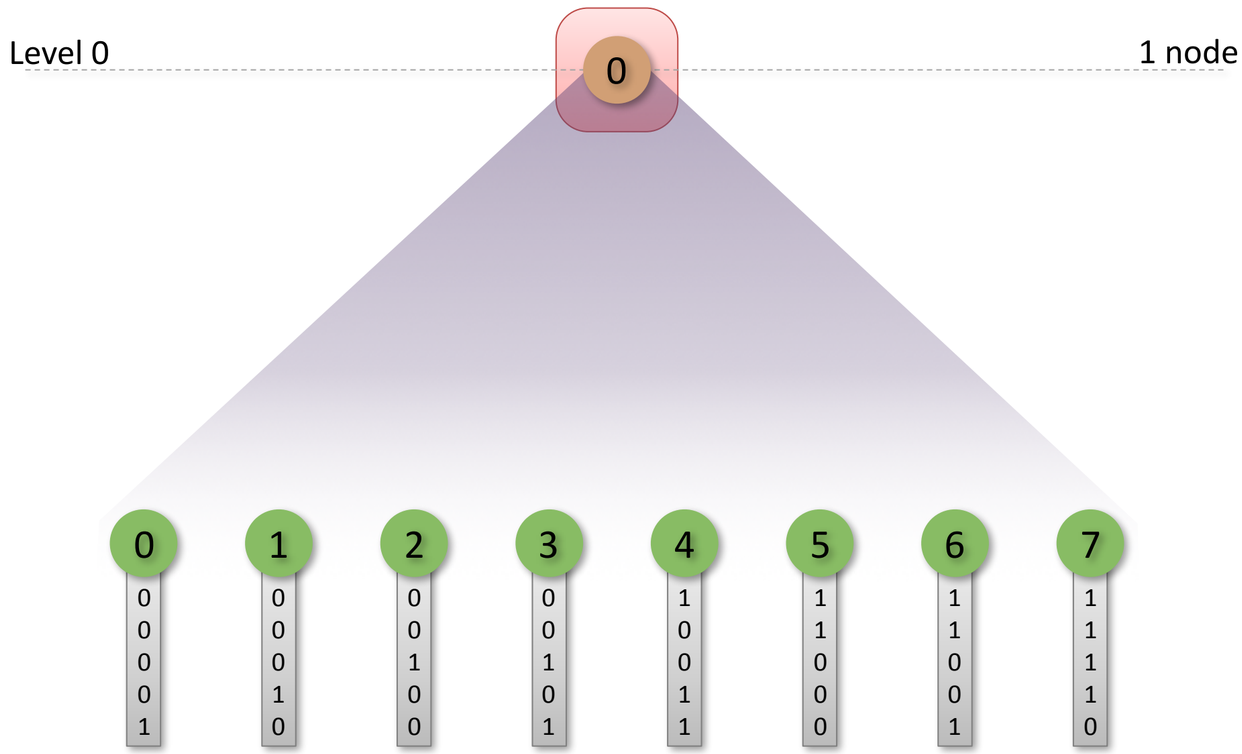

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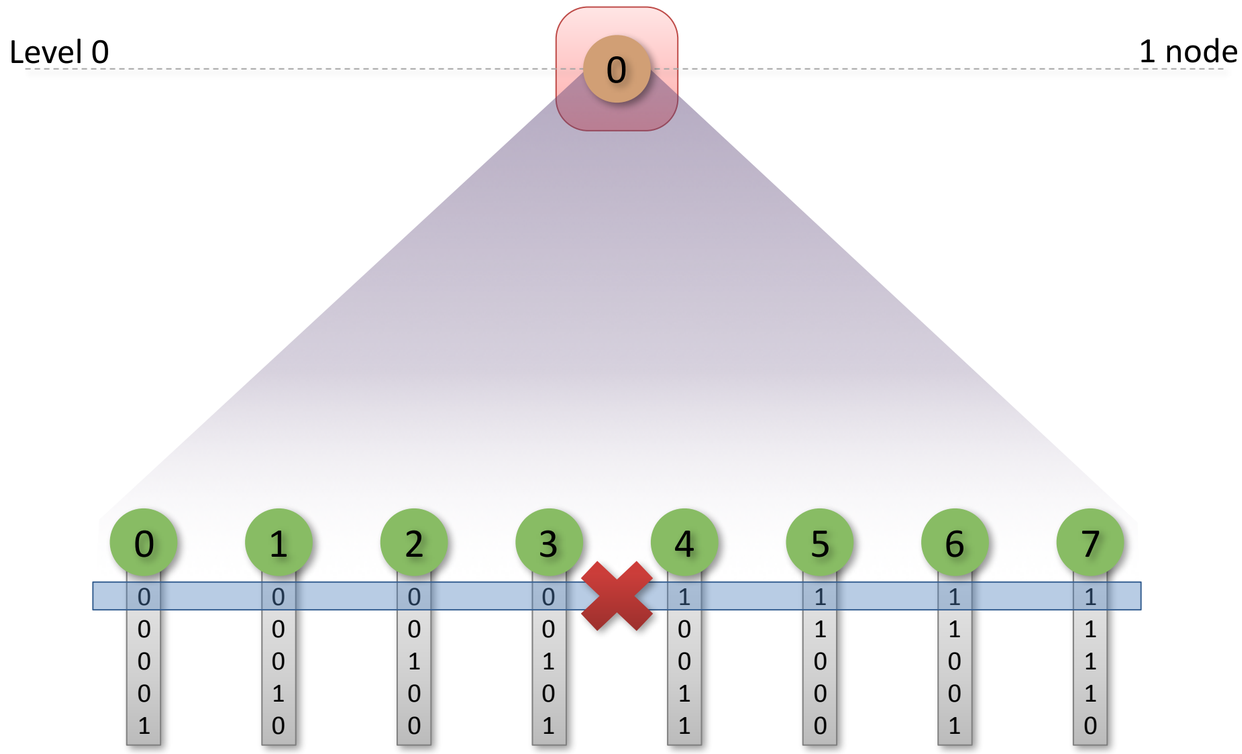
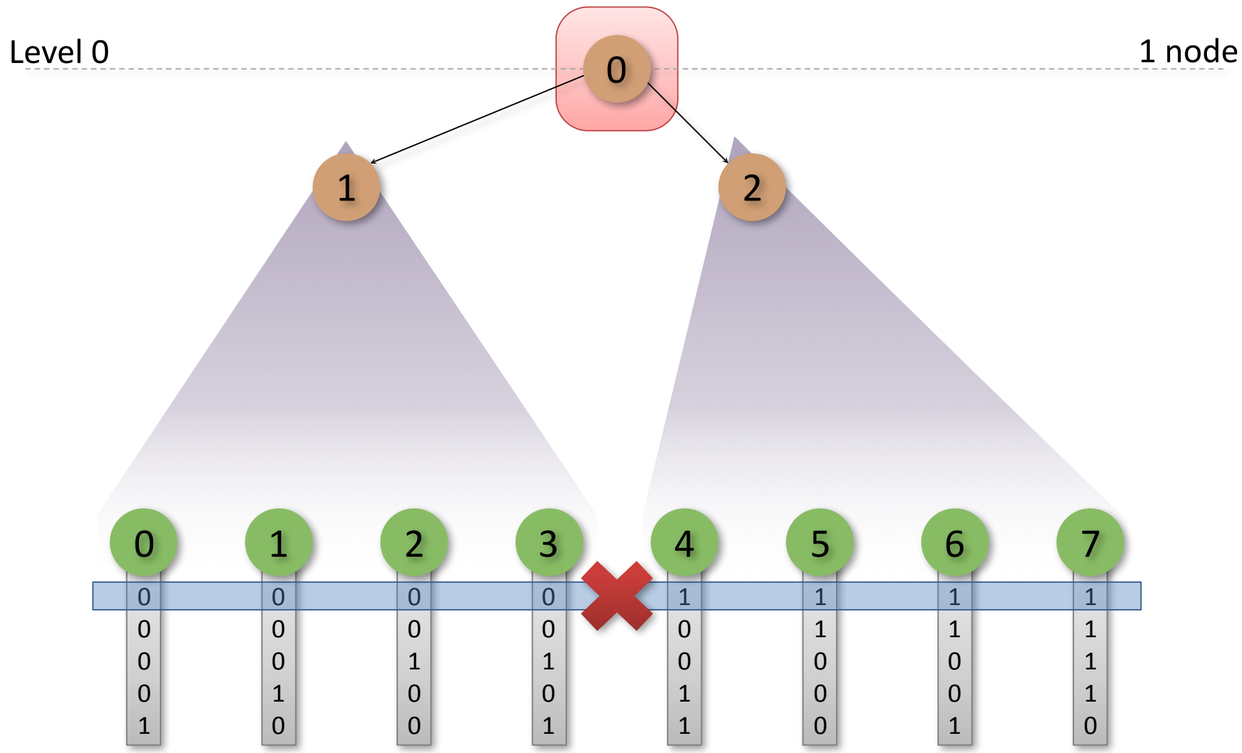


Garanzha et al. [2011]



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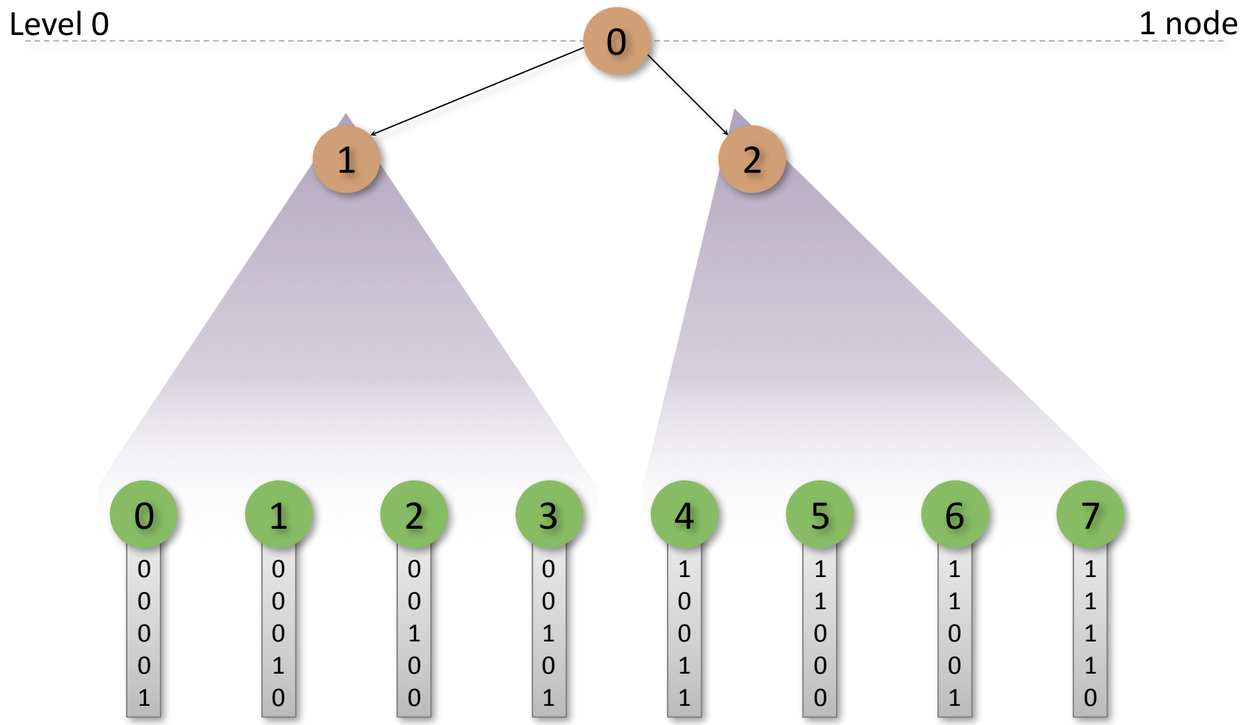

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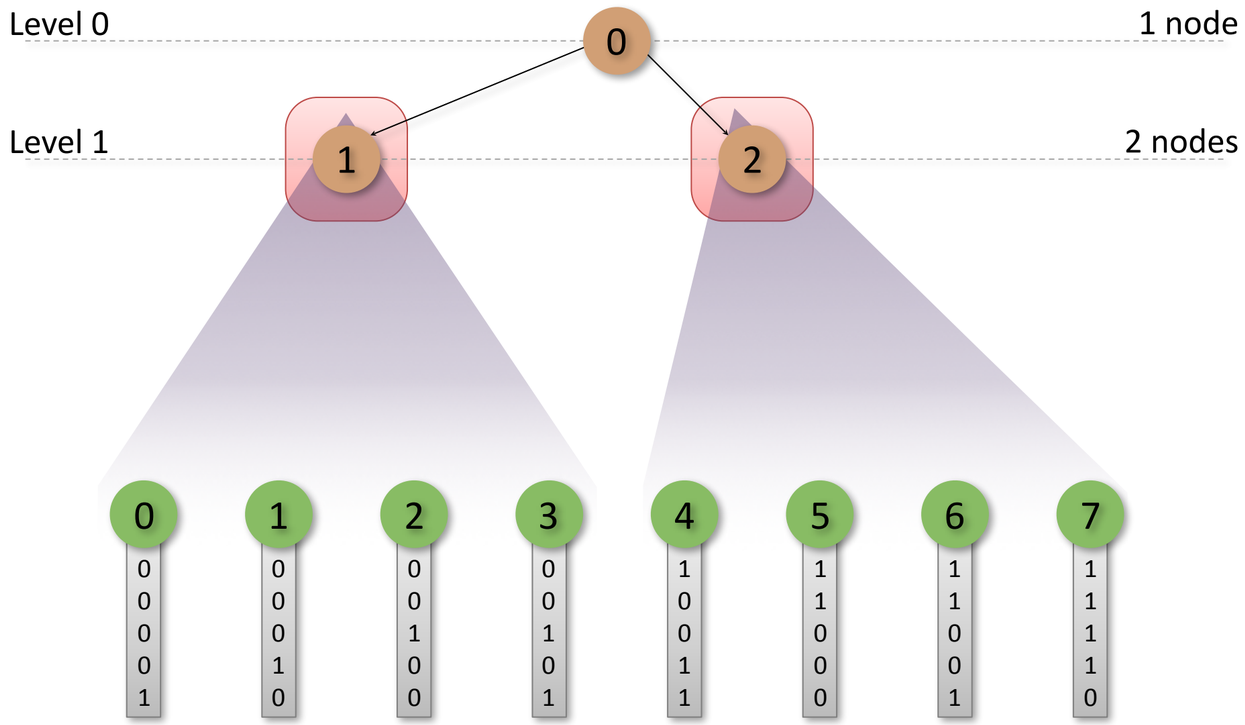

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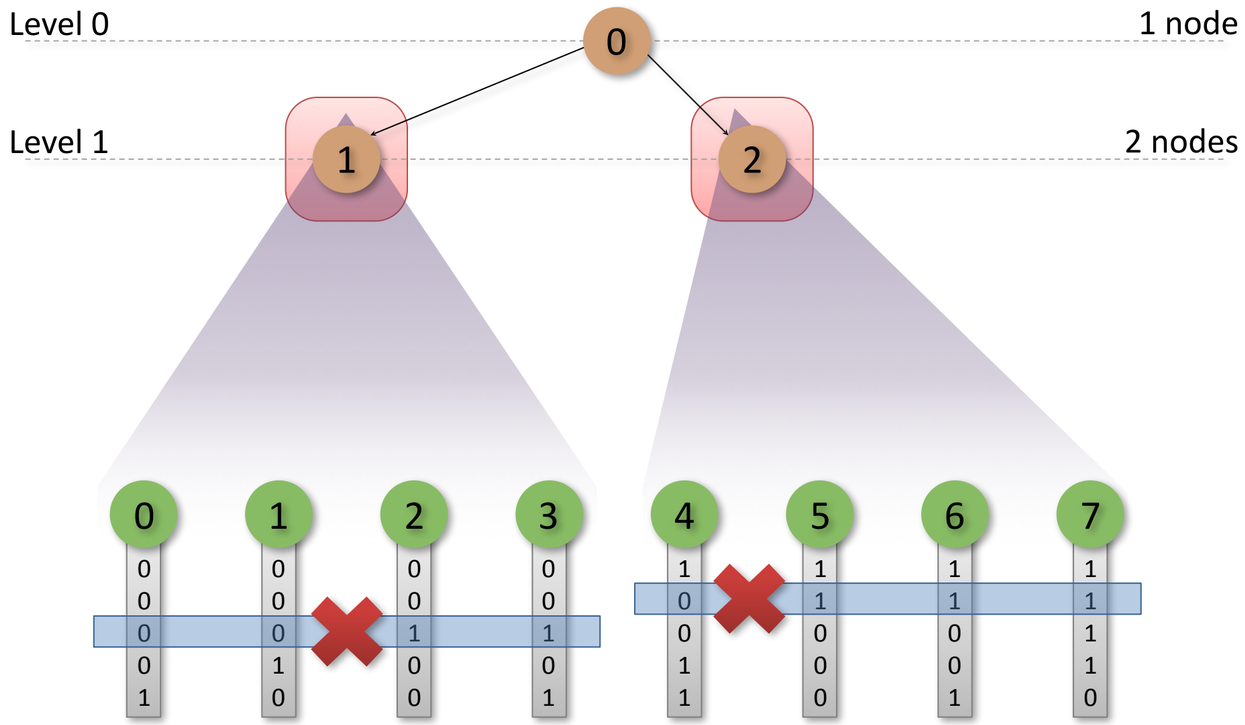

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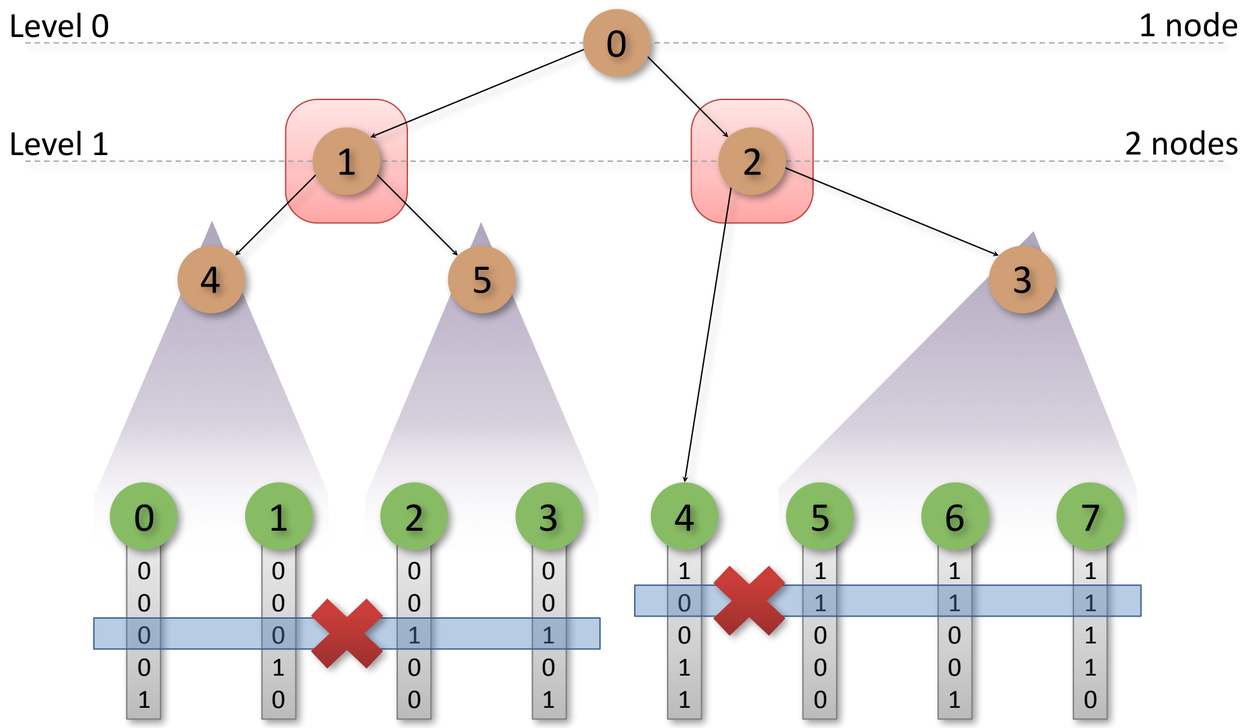

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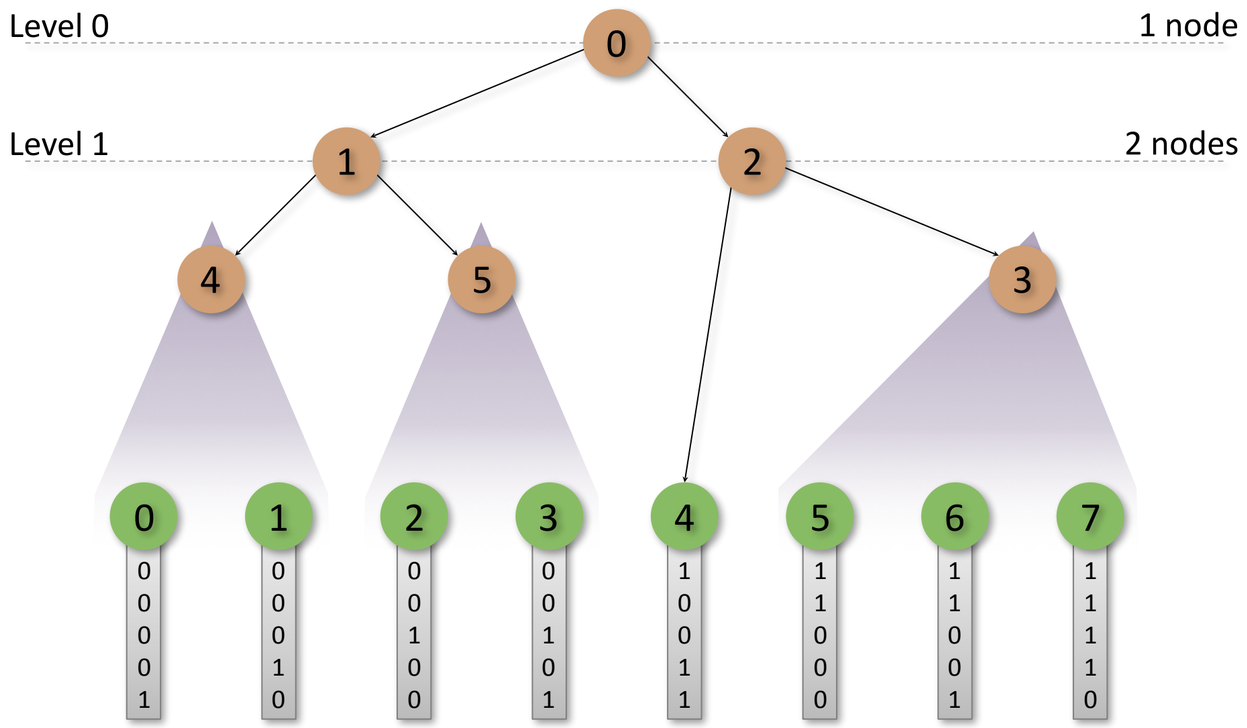

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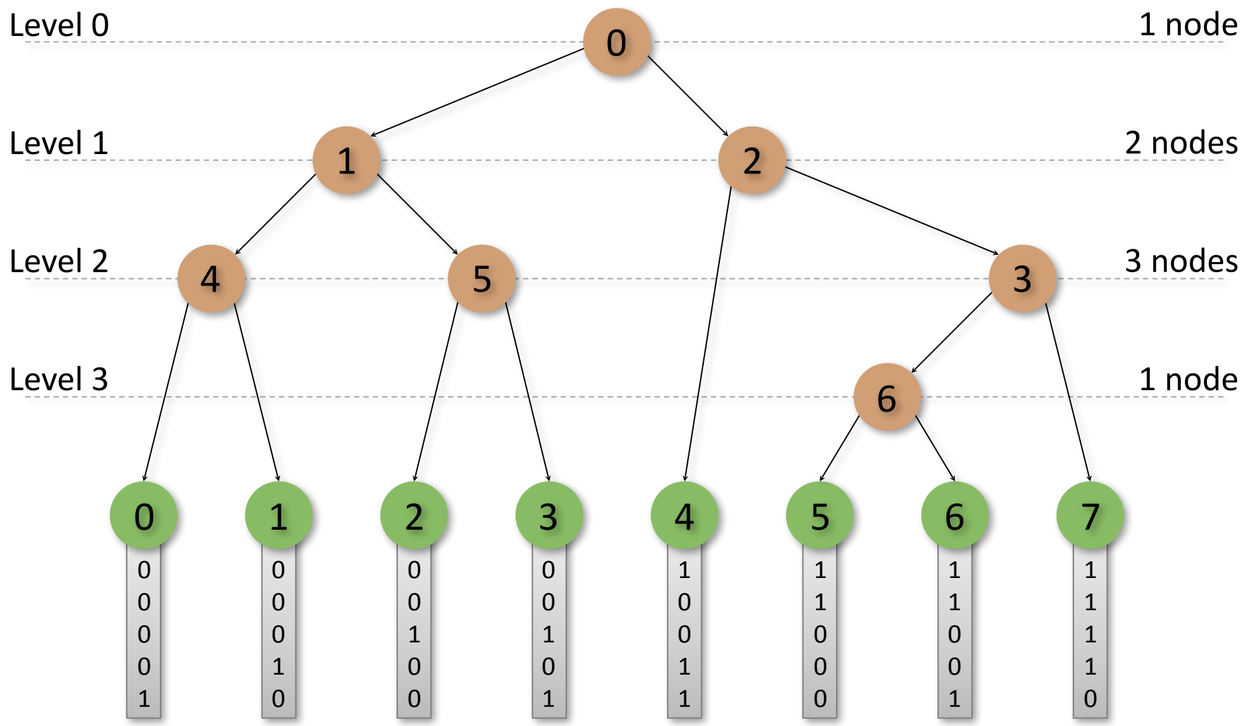

Parallel BVH Construction Performance

- When there are millions of objects, it is very fast
  - Algorithm spends majority of execution time the bottom levels of the tree, which contains plenty of work to fill the GPU
  - Some divergence at higher tree levels, but these levels take significantly less time considering overall execution time (Amdahl's Law)

- Less efficient for fewer objects

Parallel BVH Construction Performance

- When there are millions of objects, it is very fast
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  - Some divergence at higher tree levels, but these levels take significantly less time considering overall execution time (Amdahl's Law)

- Less efficient for fewer objects
  - E.g., 12K objects from the previous example
  - Execution time: 1.04 milliseconds

Occupancy

- # threads executing / maximum # threads the processor can support

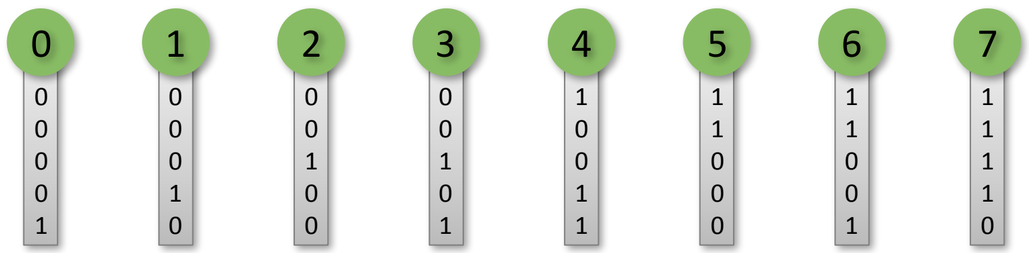
- Want to keep occupancy high
  - Then performance is limited by other factors
  - E.g., instruction throughput, memory bandwidth

Parallel BVH Construction Occupancy

- GeForce GTX 690 has maximum of 16k threads
- Application has 12k objects

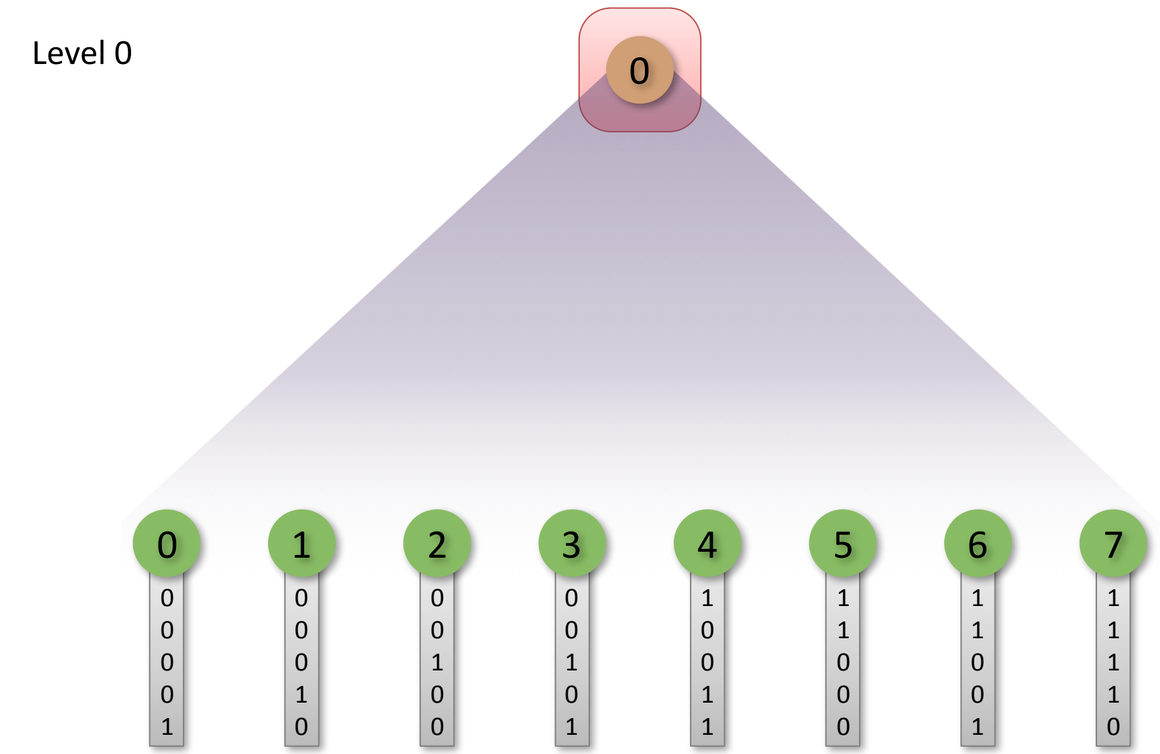
Tree Level	Number of active	Occupancy
0 (Root)	1	0.006%
1	2	0.013%
2	4	0.025%
3	8	0.050%
…	…	…
12	4,096	25.6%
13	8,192	52.2%
14	12,000	75%

Garanzha et al.: Challenge



- Building nodes seems serial by level
- Could we generate *all* internal nodes at once?

Garanzha et al.: Challenge



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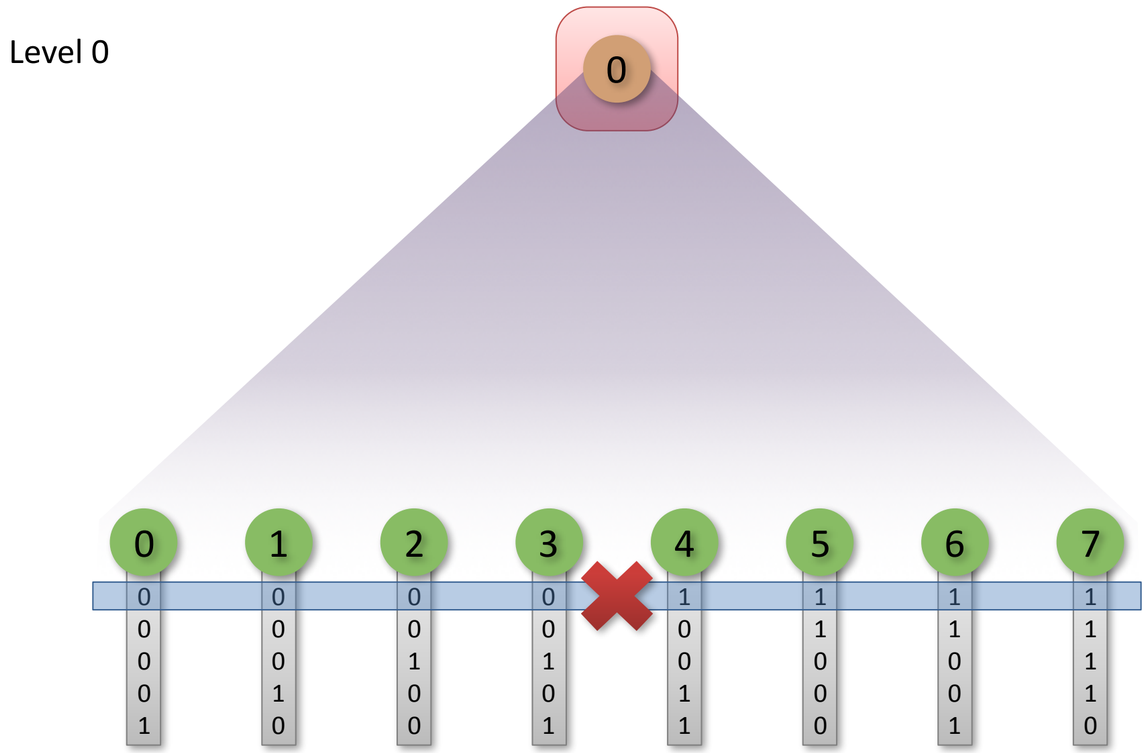
Garanzha et al.: Challenge



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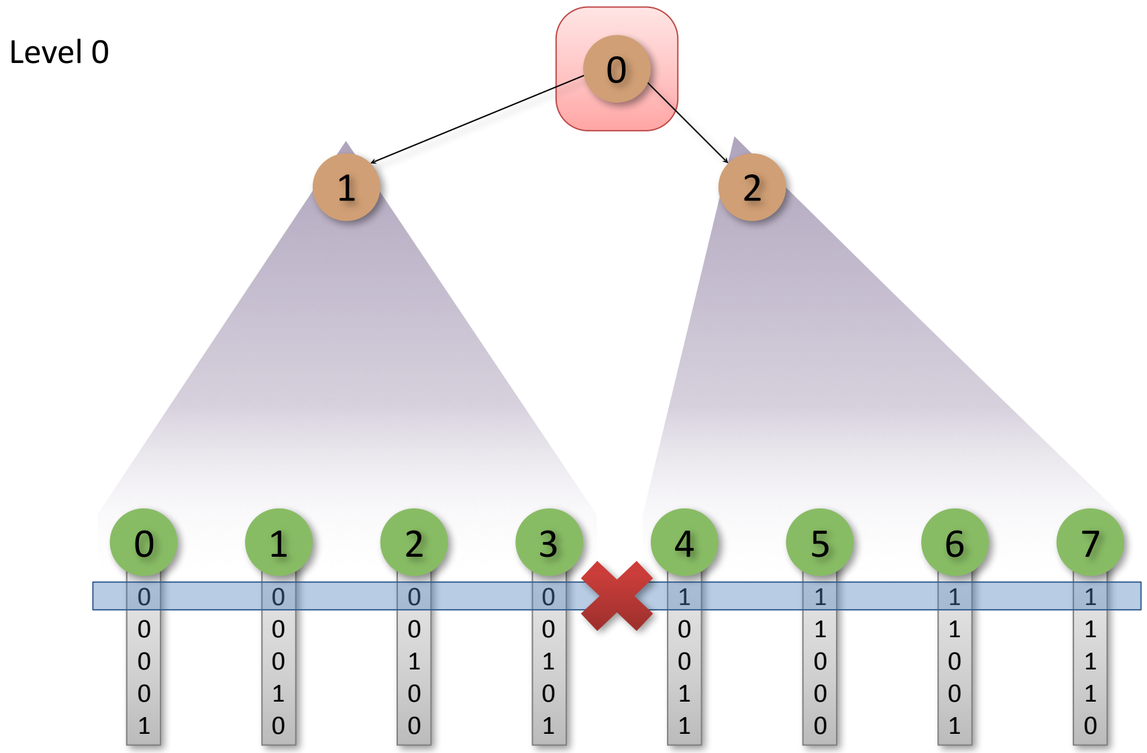
Garanzha et al.: Challenge



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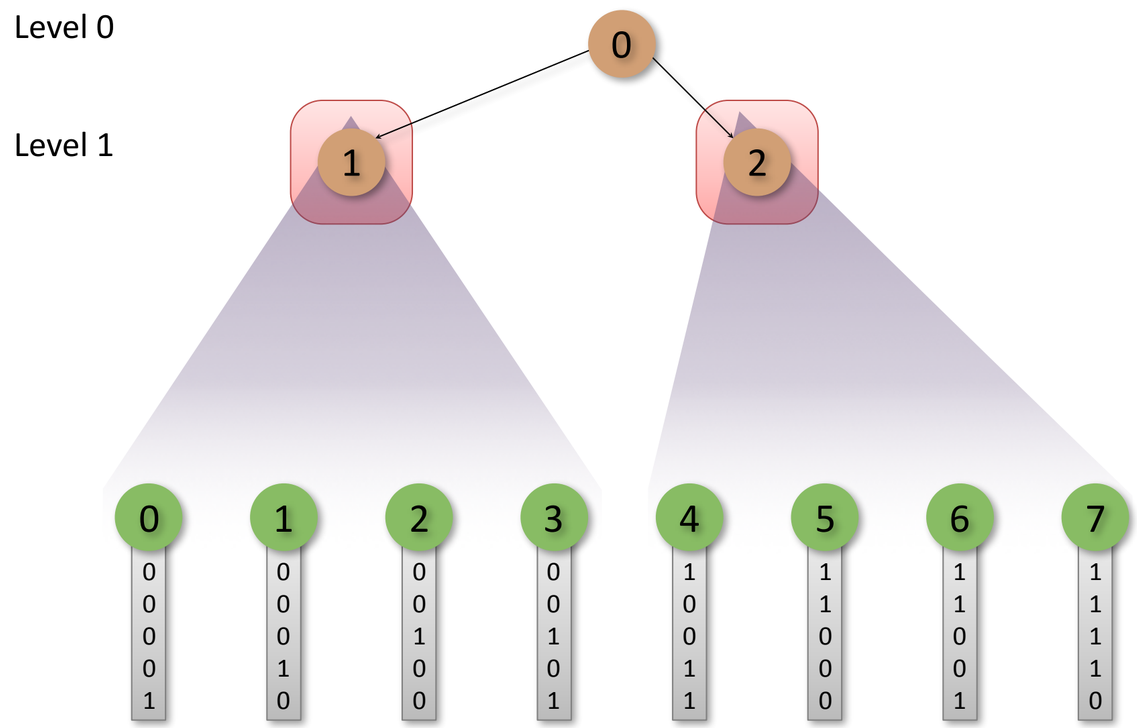
Garanzha et al.: Challenge



- Building nodes seems serial by level
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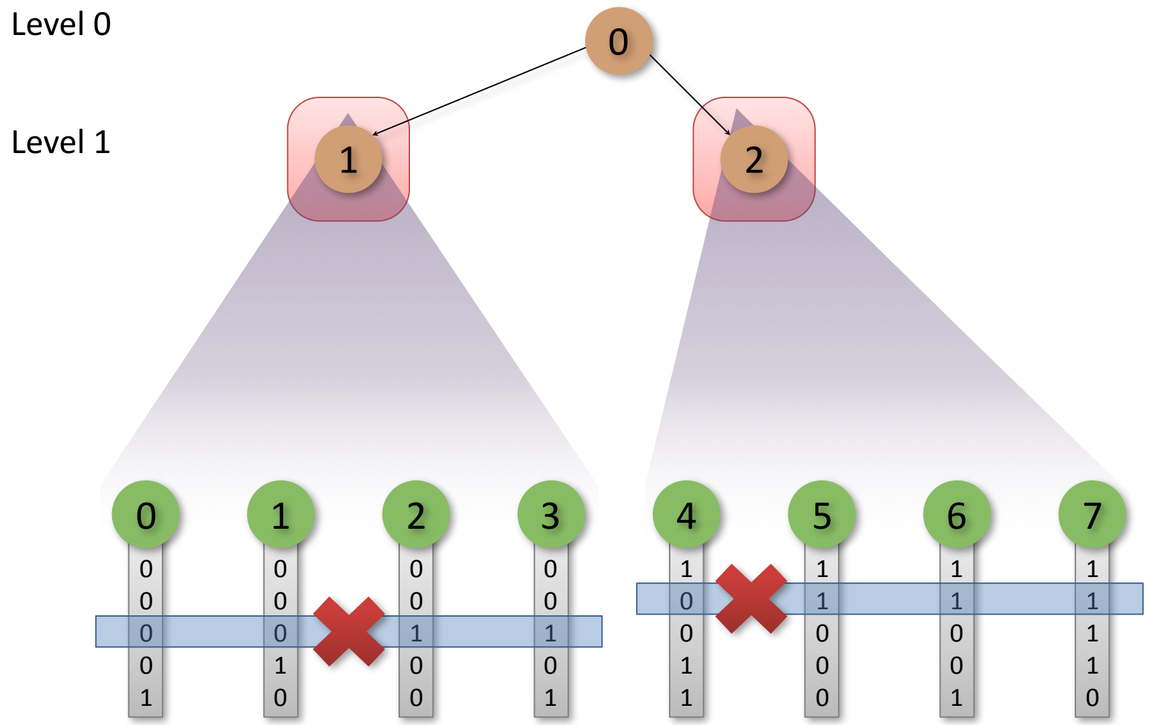
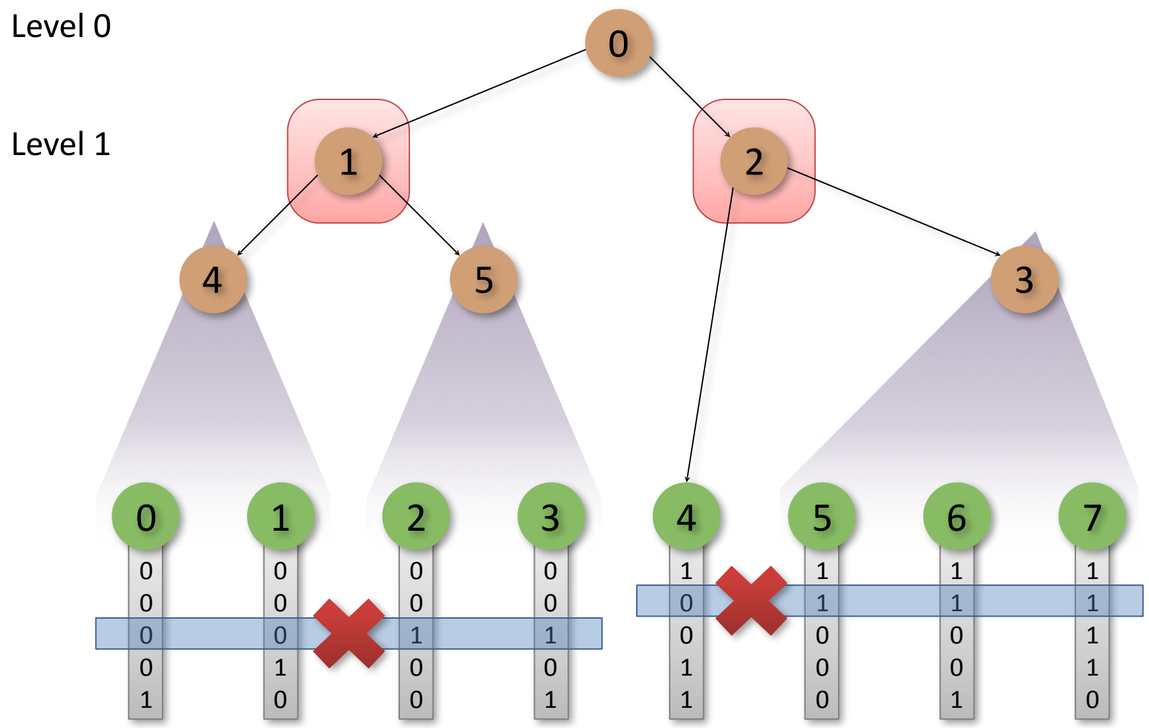
Garanzha et al.: Challenge



- Building nodes seems serial by level
- Could we generate *all* internal nodes at once?

Garanzha et al.: Challenge



- Building nodes seems serial by level
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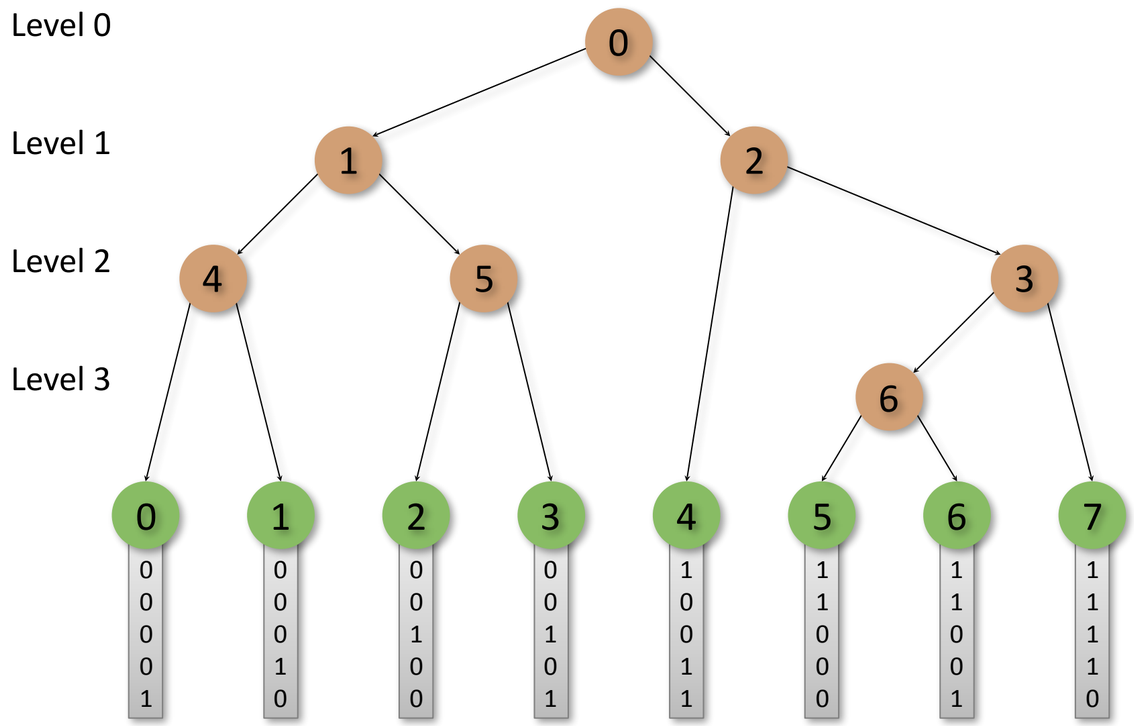
Garanzha et al.: Challenge



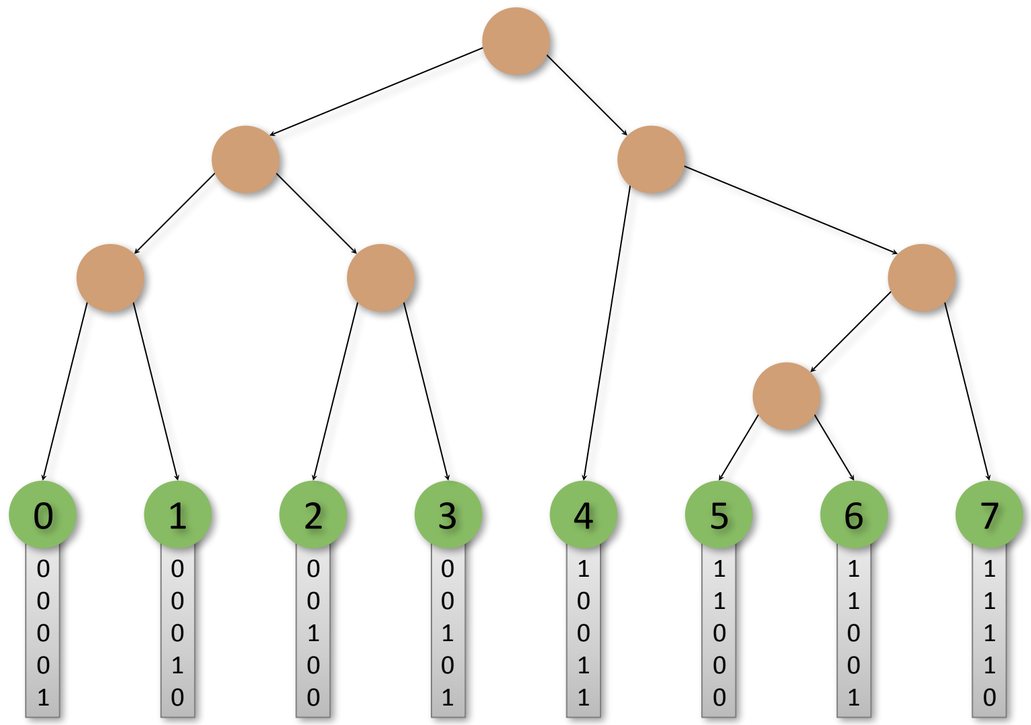
- Building nodes seems serial by level
- Could we generate *all* internal nodes at once?

Karras's method

- Define a numbering scheme for the internal nodes
  - Gain some knowledge of their identity
    - What keys do they cover?
  - Connect internal-node indices and leaf (key) indices
- Find the children of a given node
  - Only look at node index and nearby keys
- Do this for all nodes in parallel
  - If we do it right: No dependencies!

T. Karras, "Maximizing parallelism in the construction of BVHs, octrees, and k-d trees," HPG'12: Proceedings of the Fourth ACM SIGGRAPH / Eurographics Conference on High-Performance Graphics, 2012.

Numbering scheme



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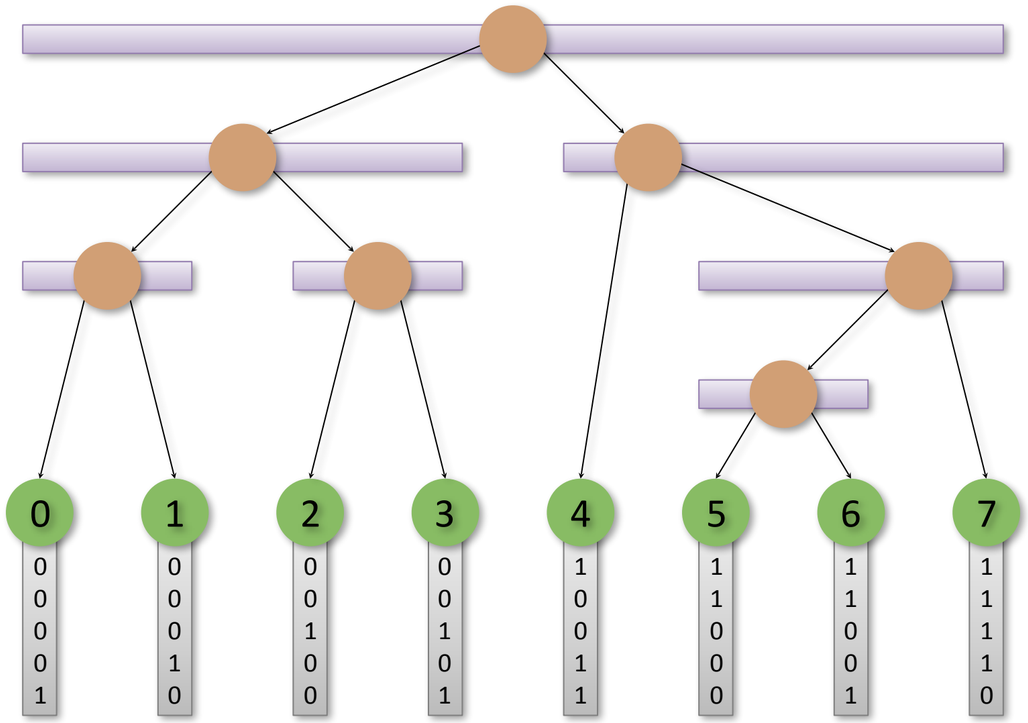

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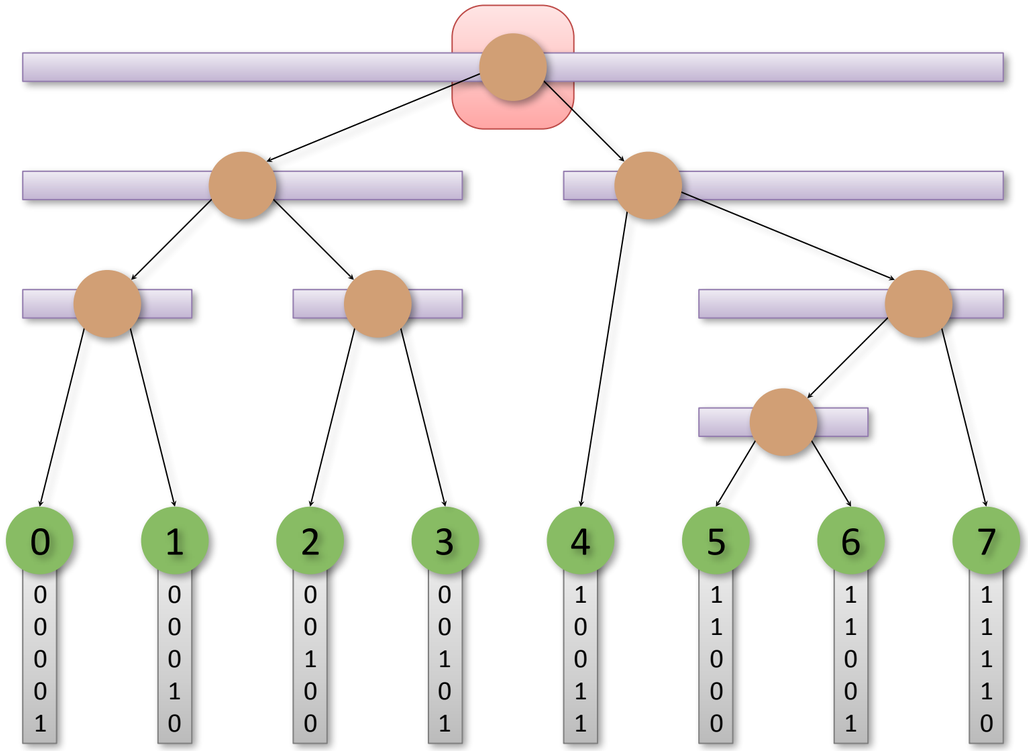

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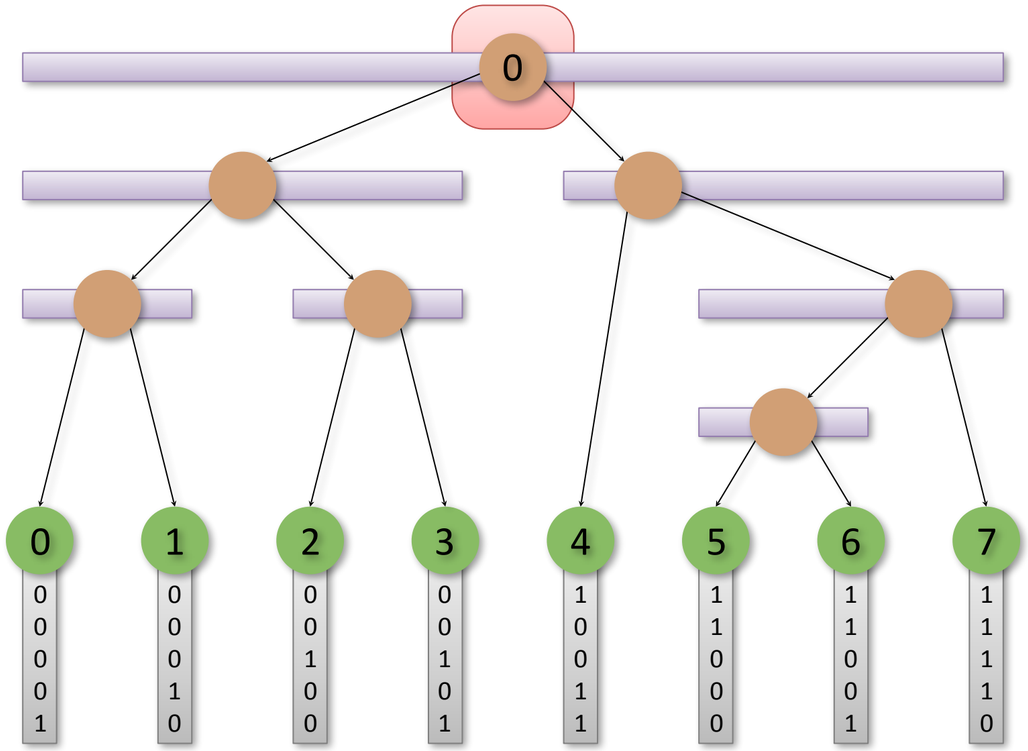

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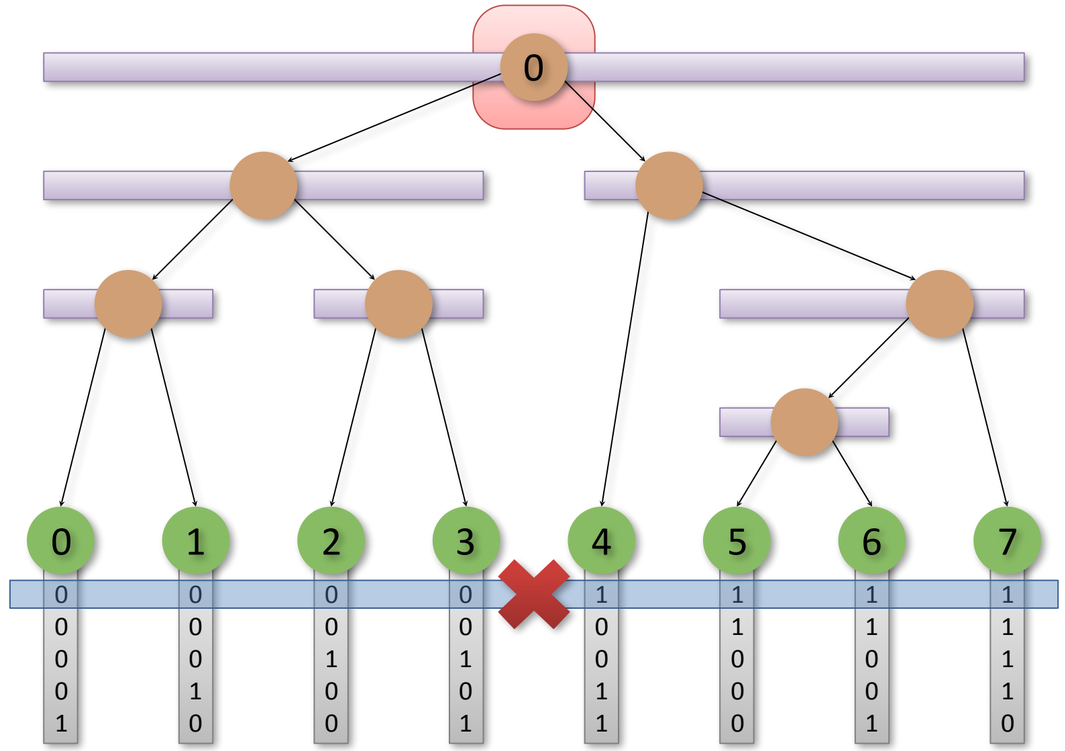
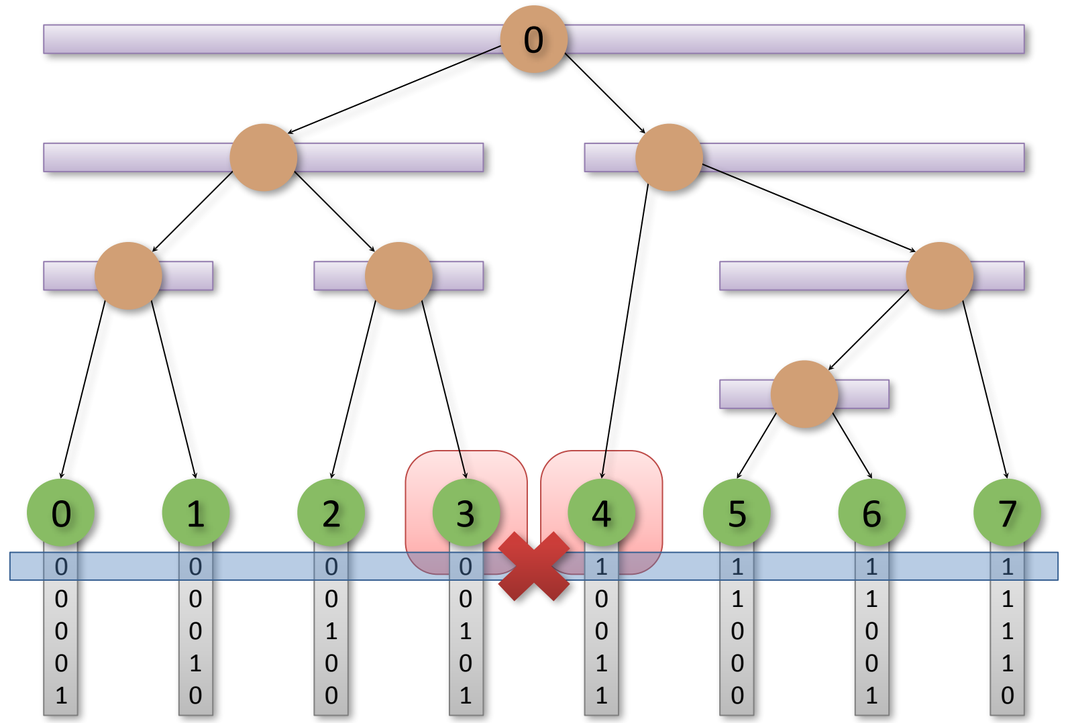
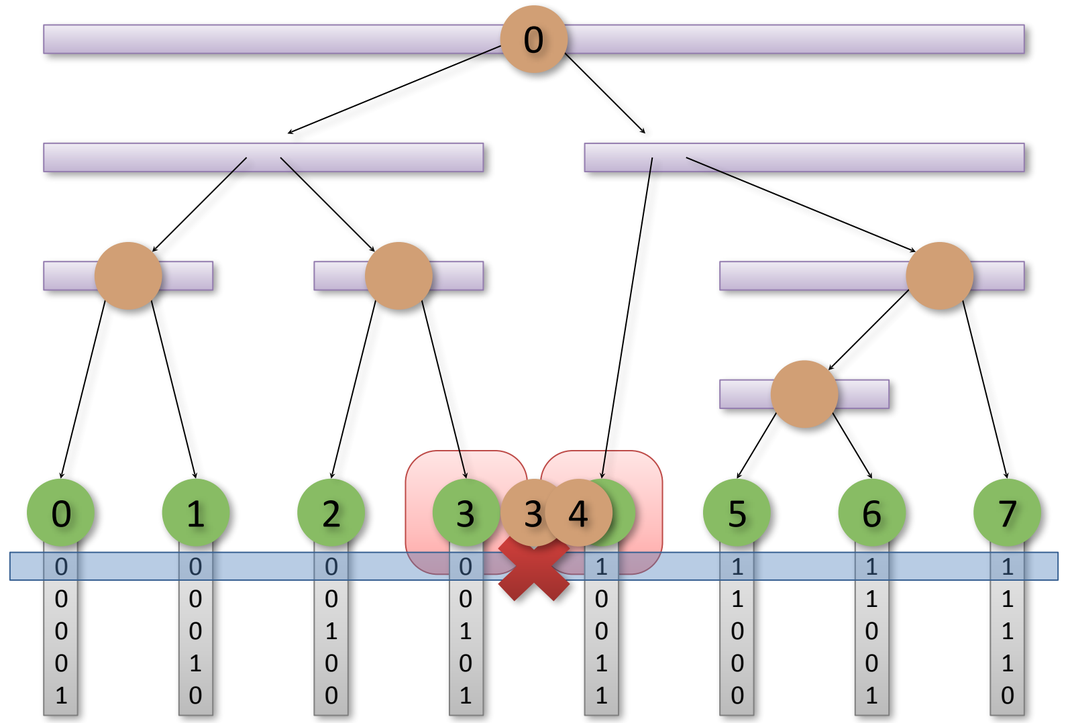


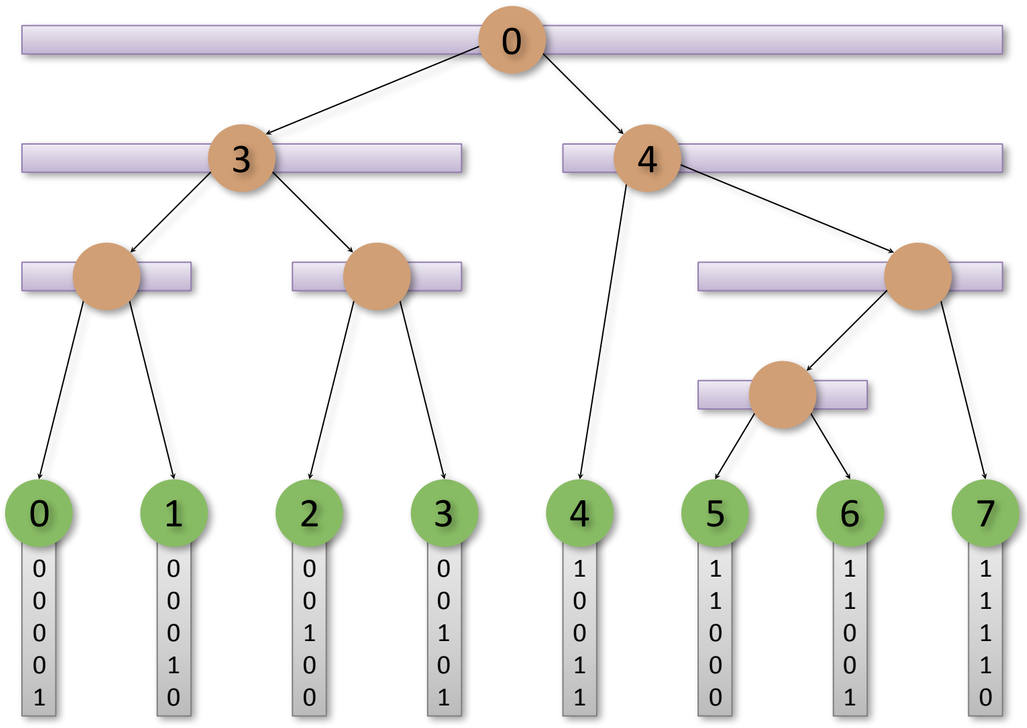
Numbering scheme



Numbering scheme



Numbering scheme



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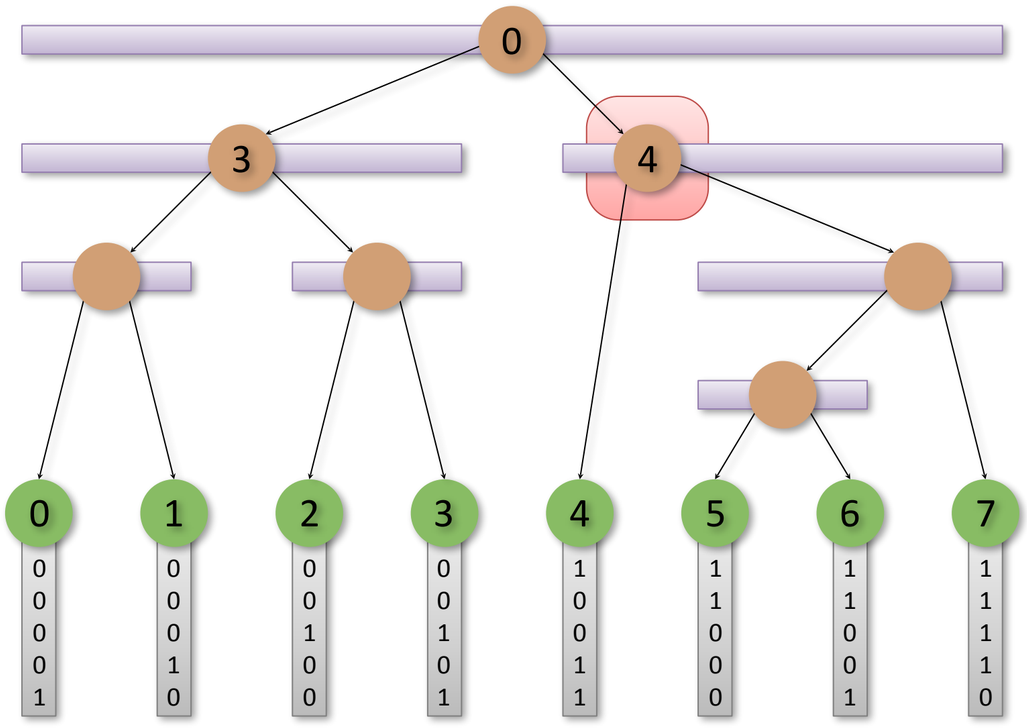

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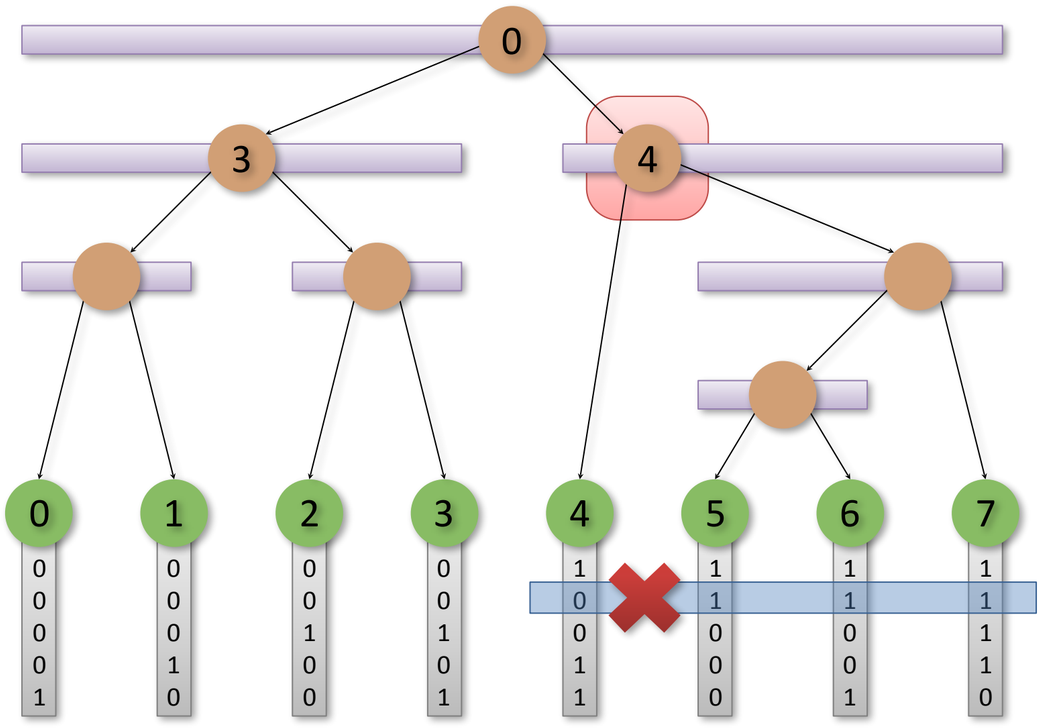
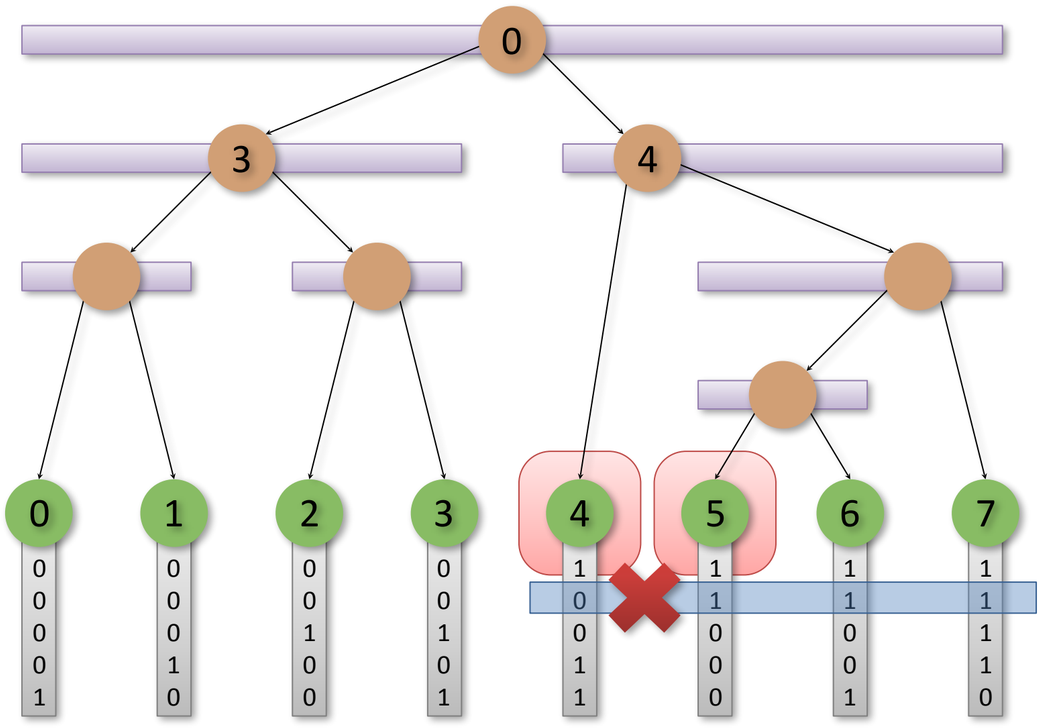


Numbering scheme



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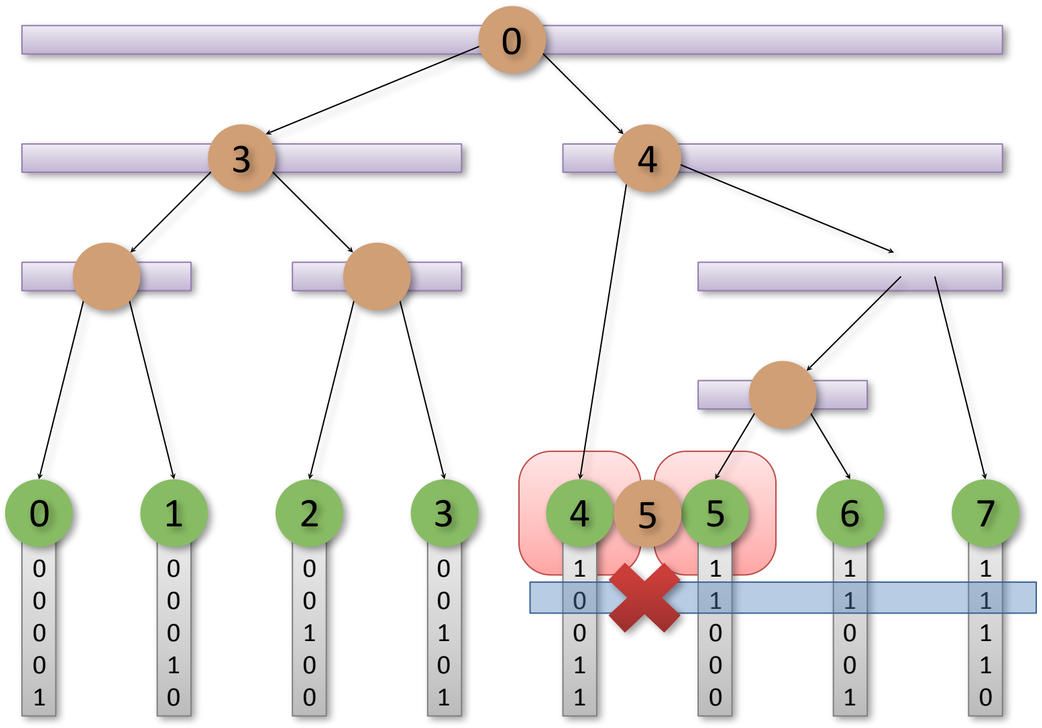
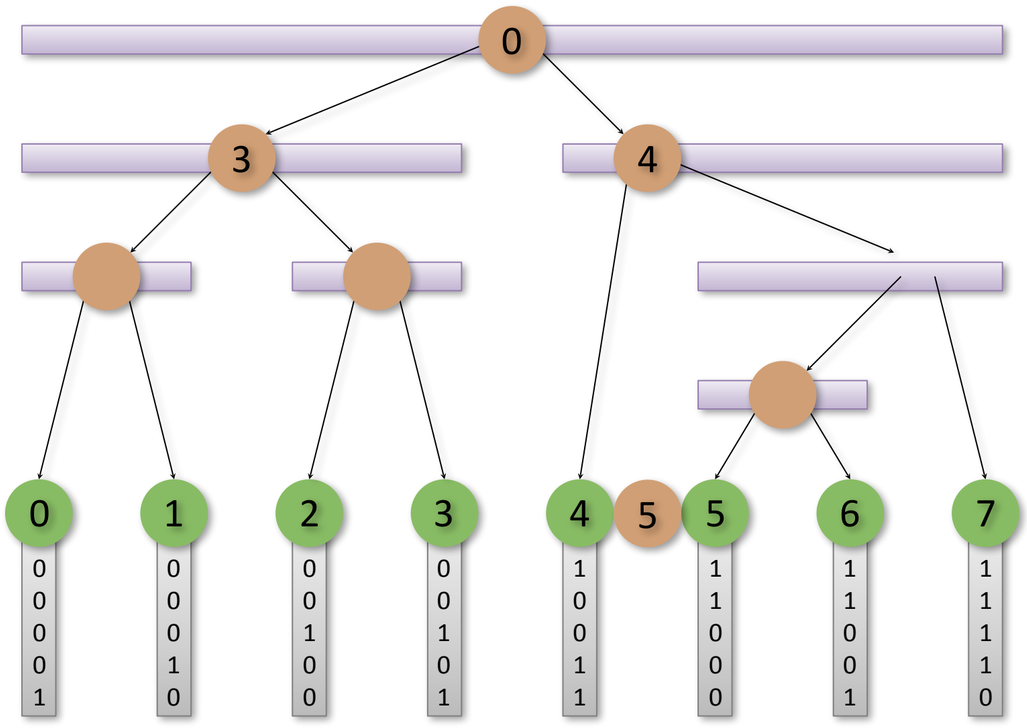
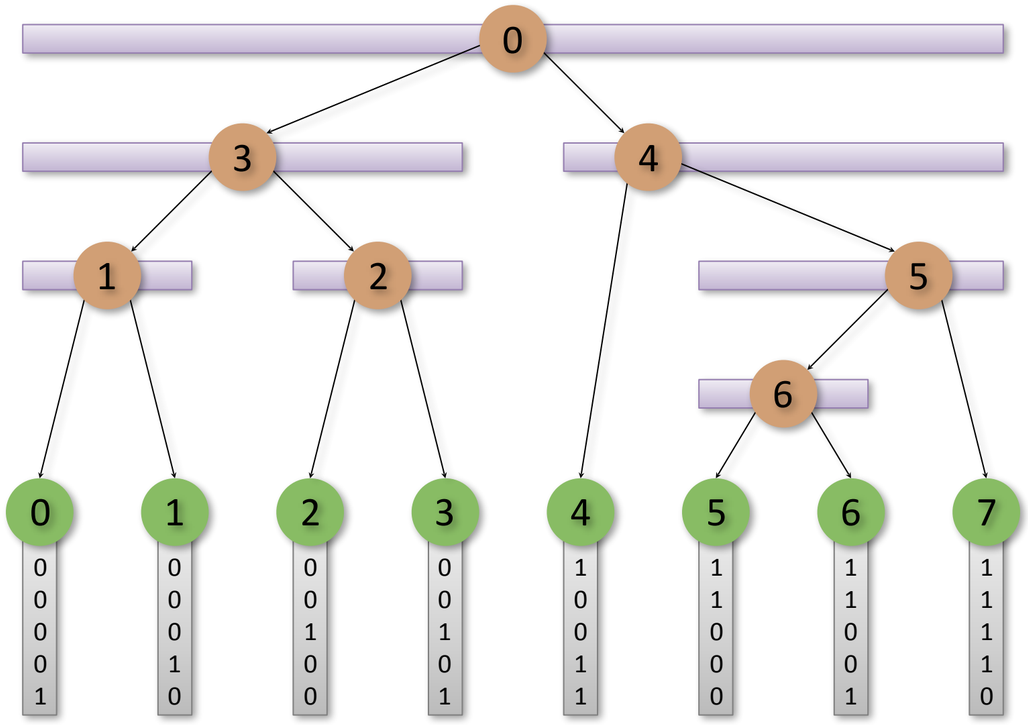


Numbering scheme



Numbering scheme



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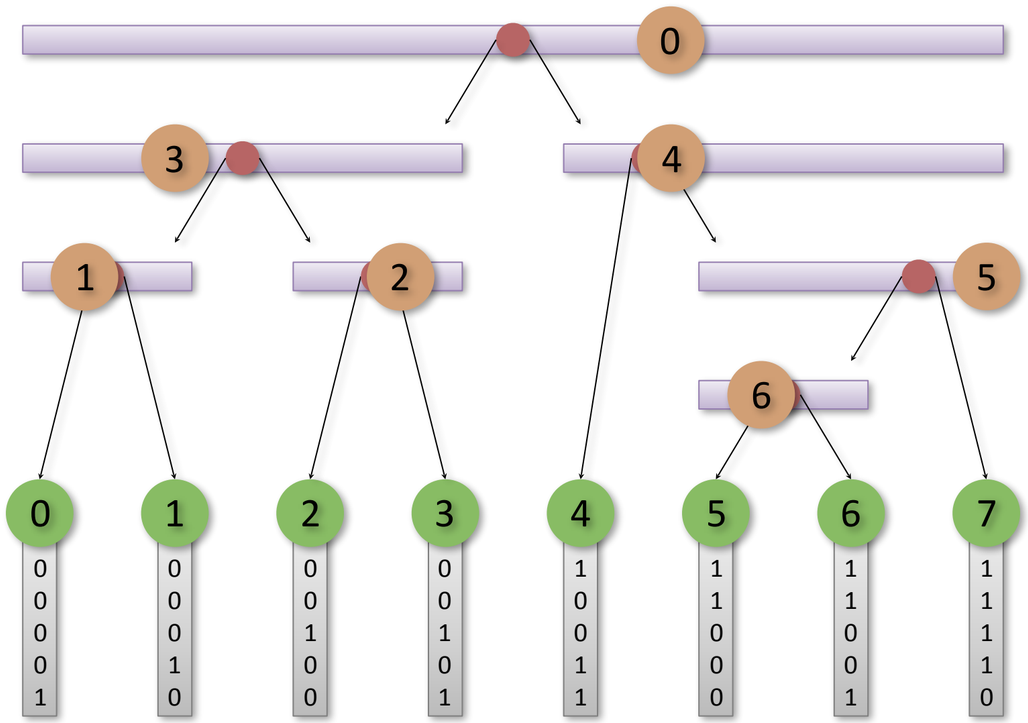

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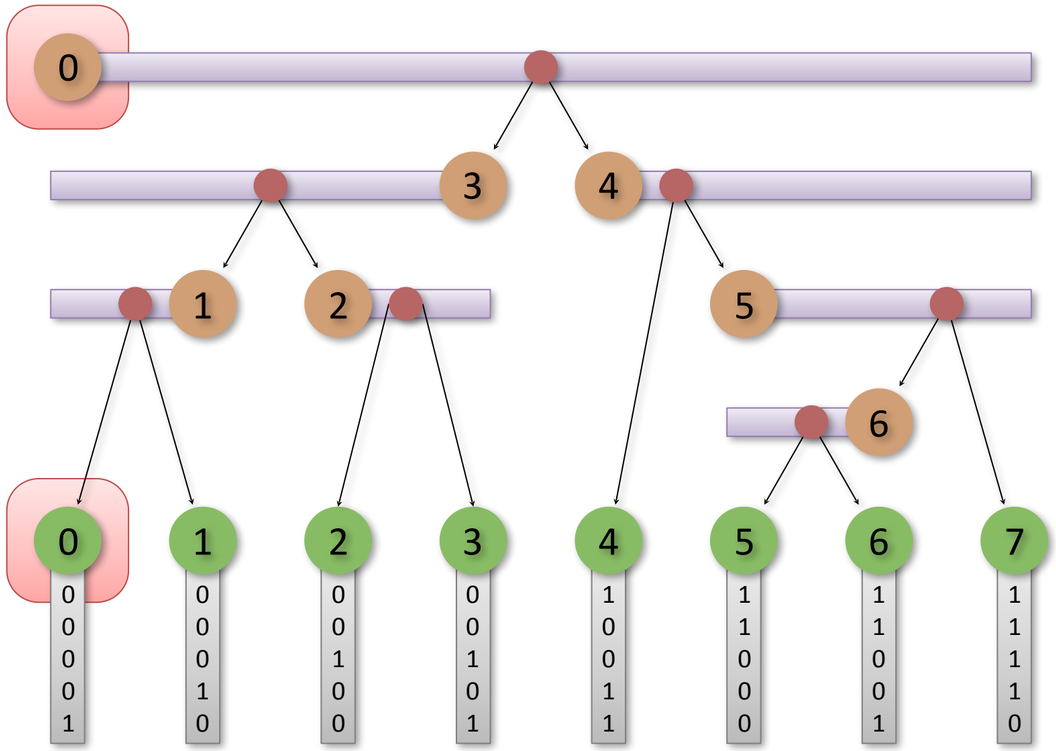

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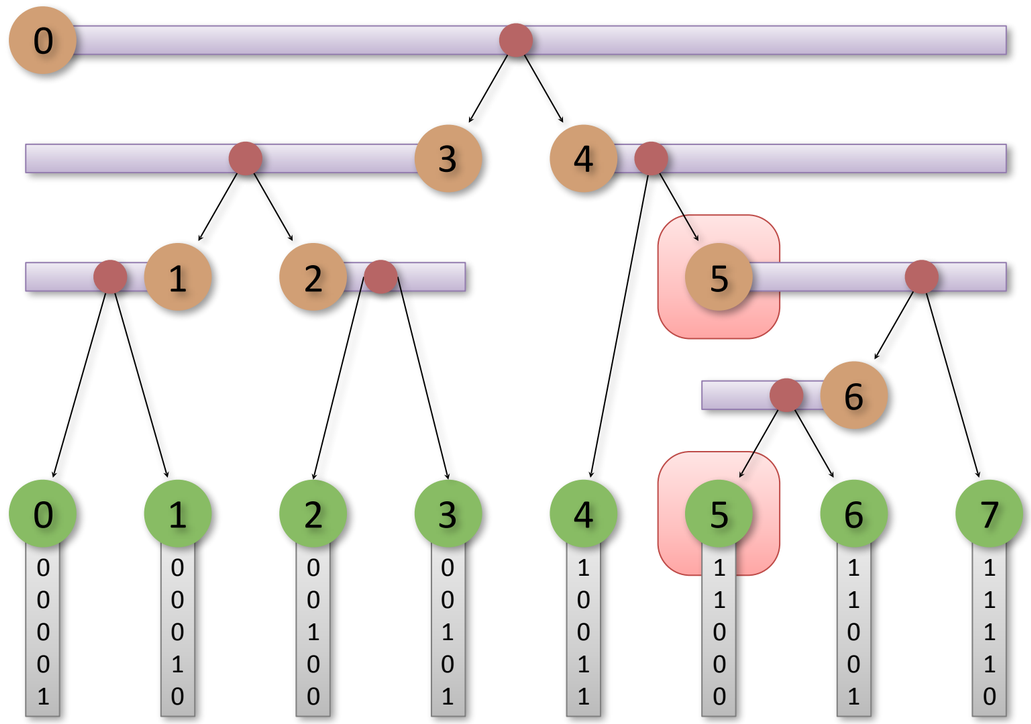

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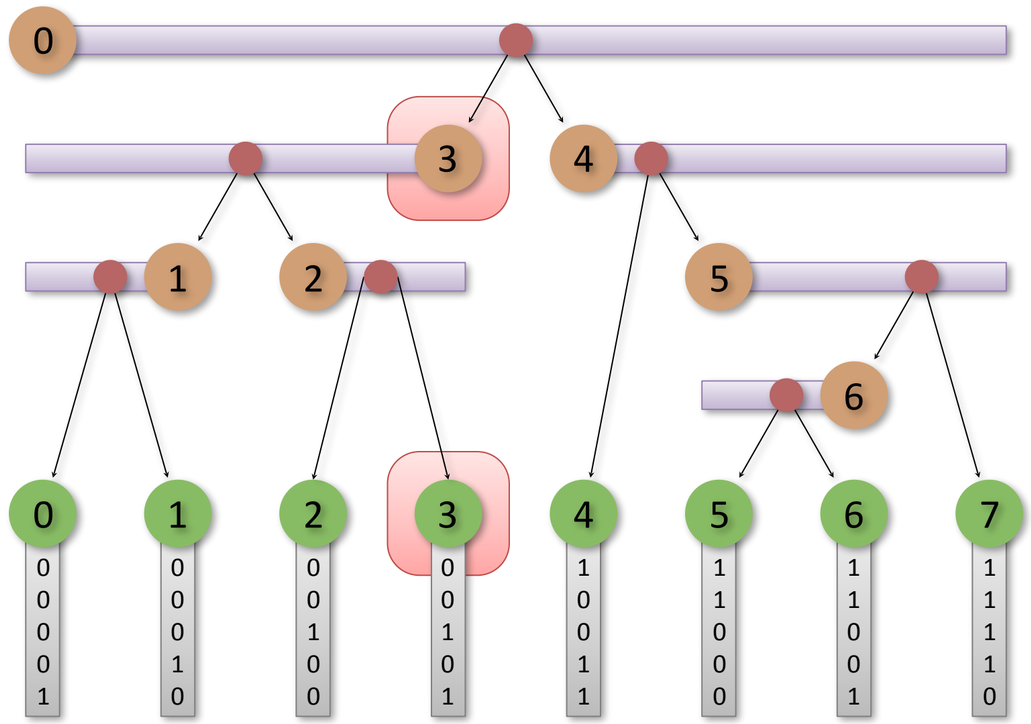

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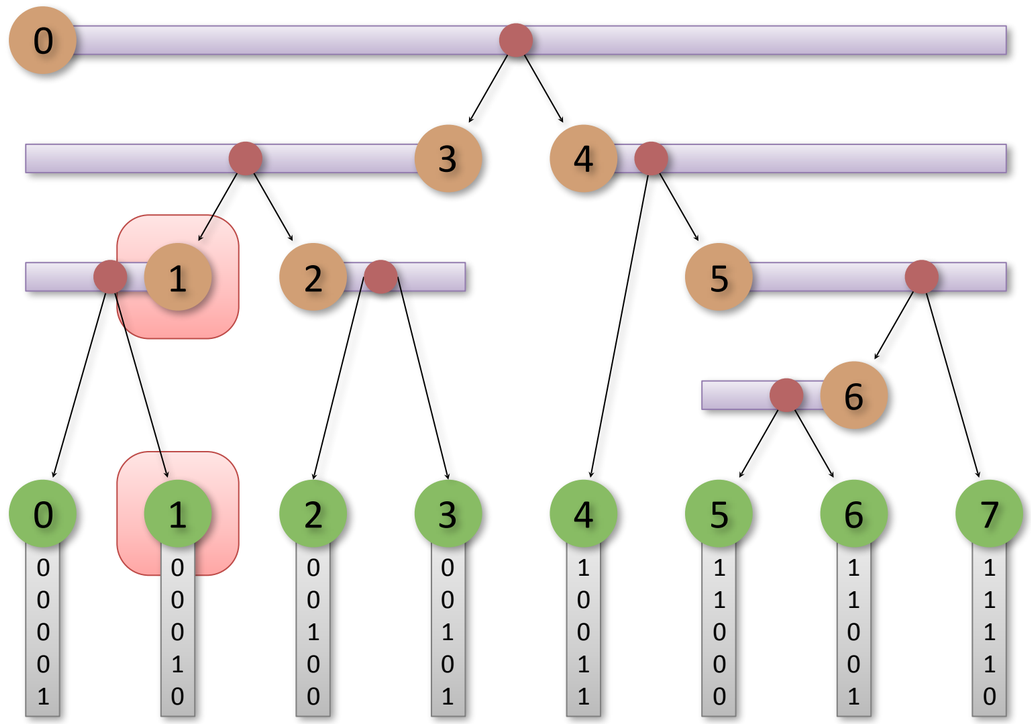

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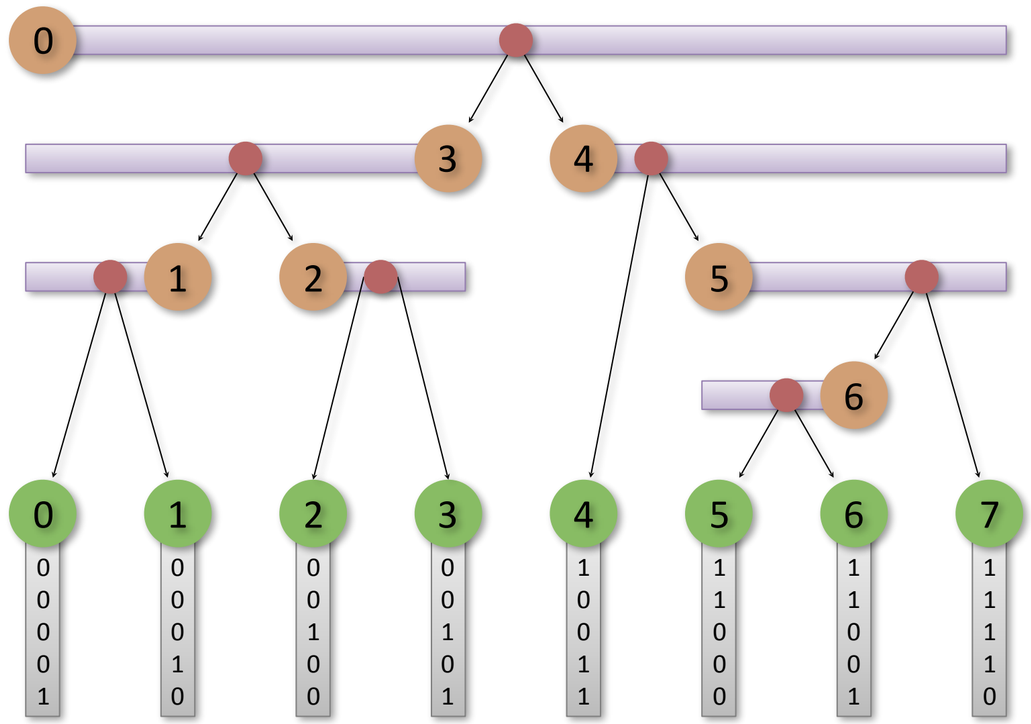

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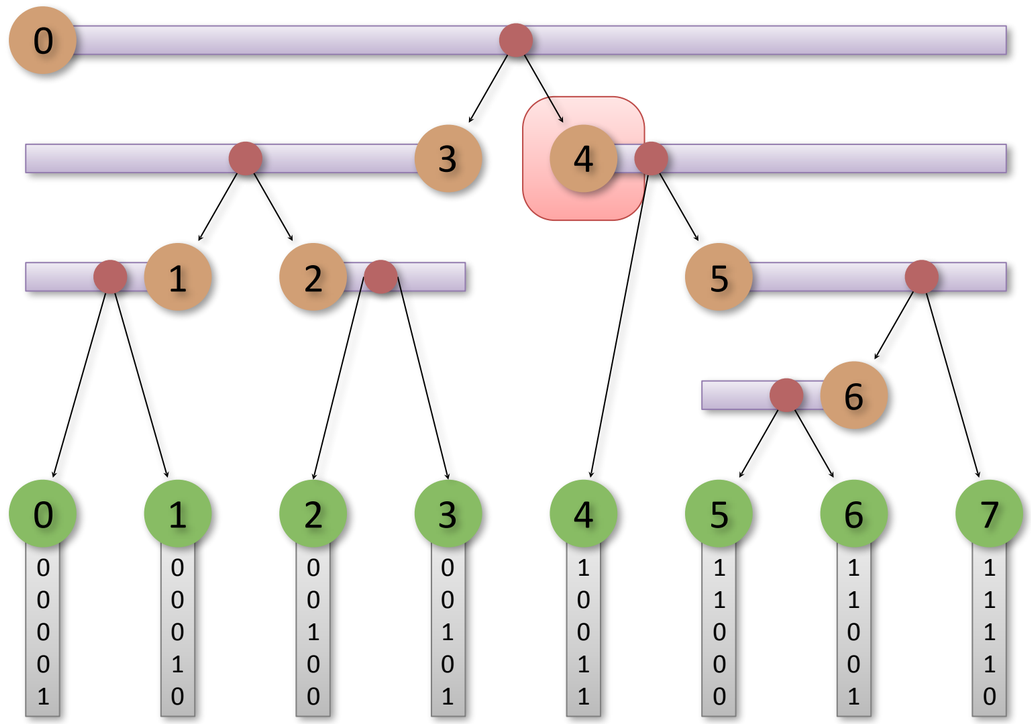

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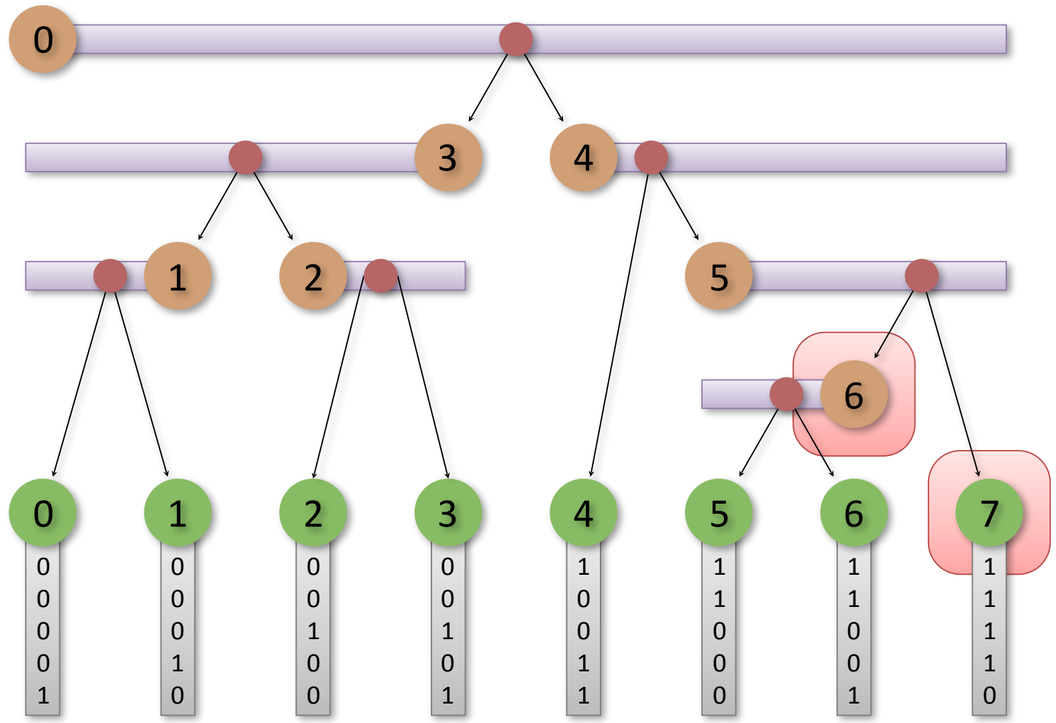

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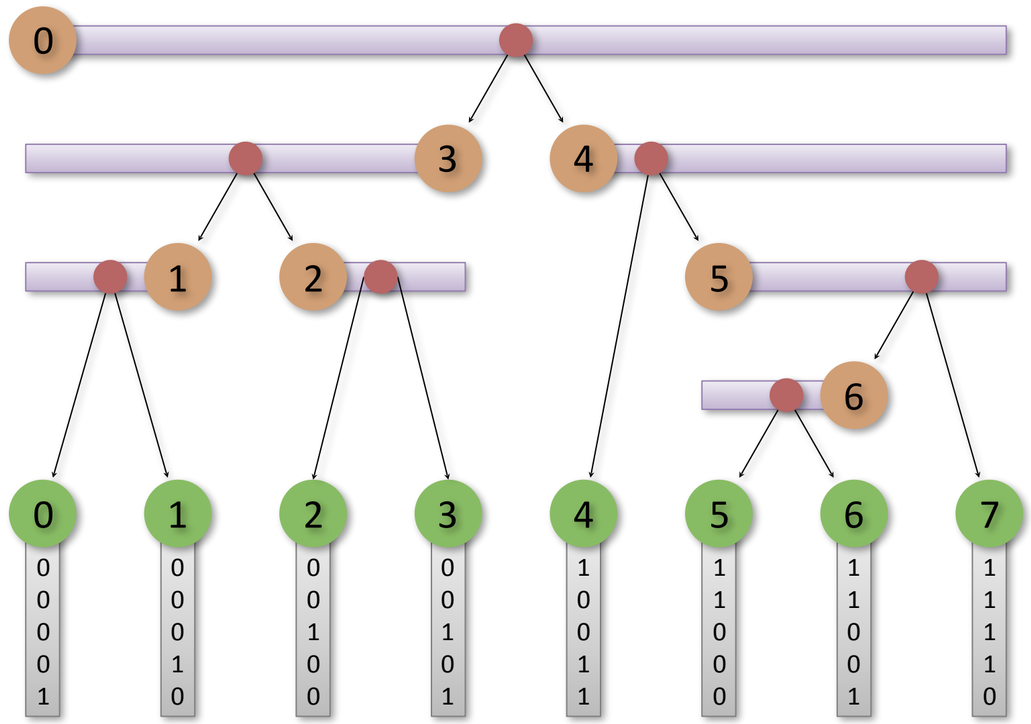
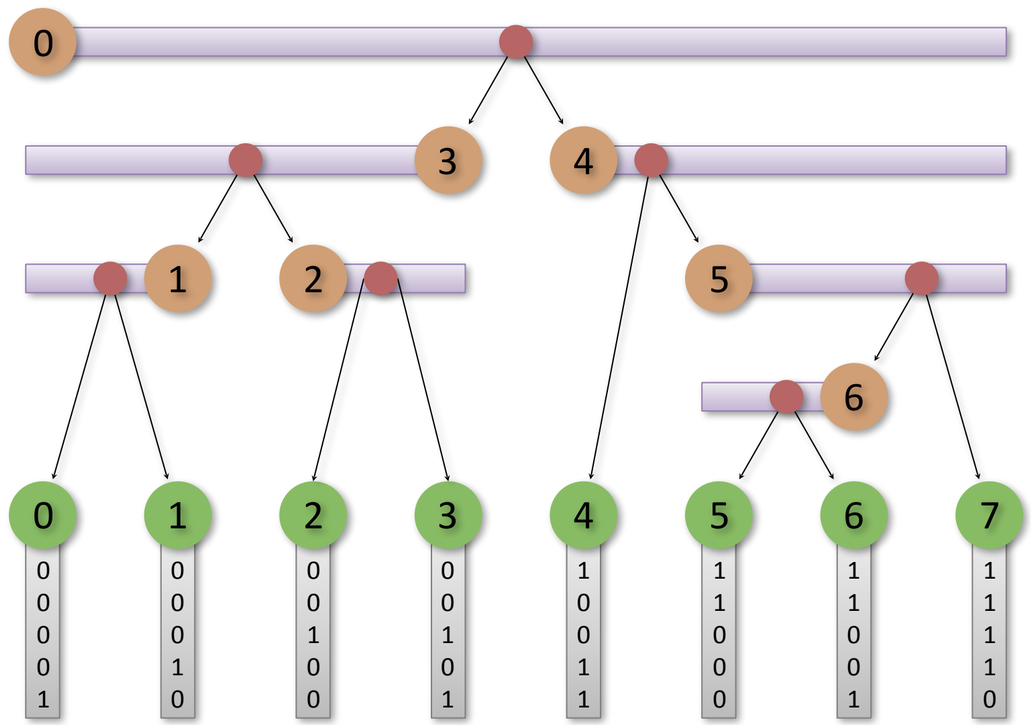


Algorithm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6
7
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0 0 1 0 0 0
0 0 1 0 1 1
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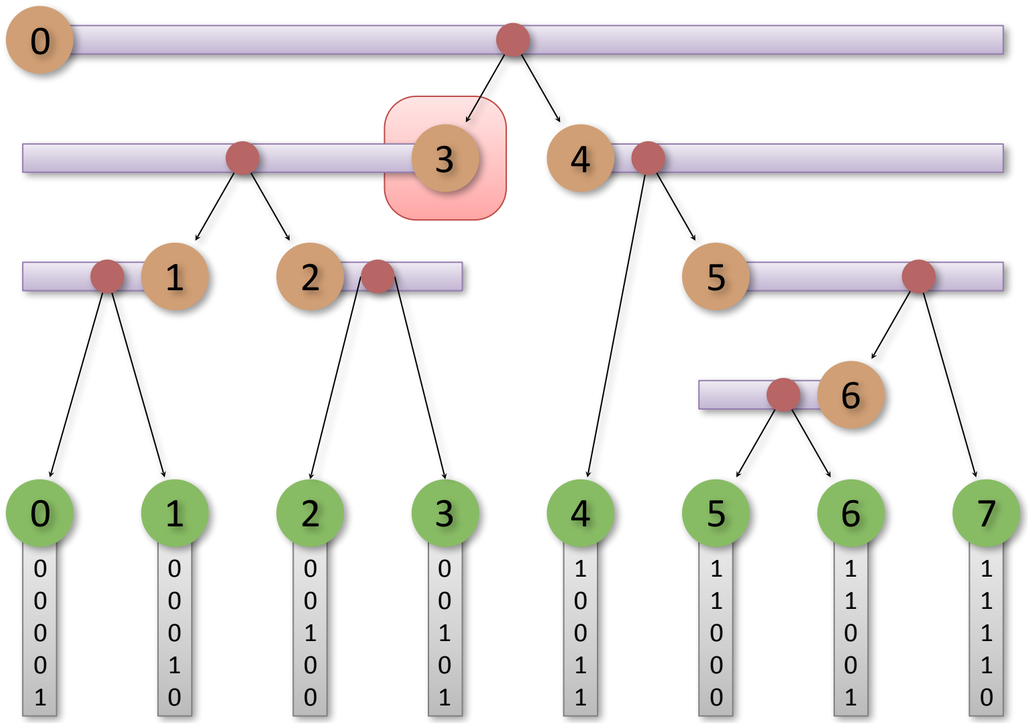
1 1 1 1 1 0

Algorithm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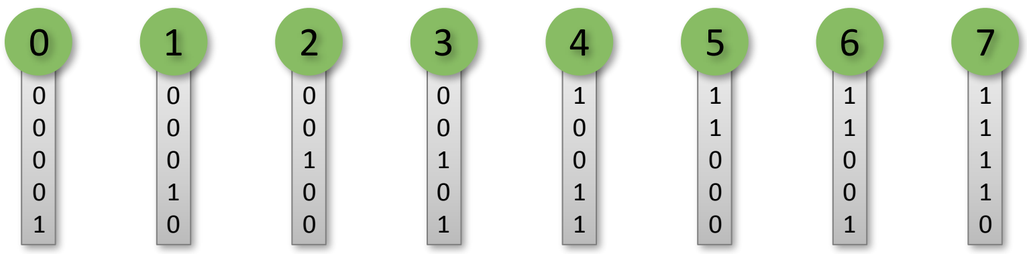
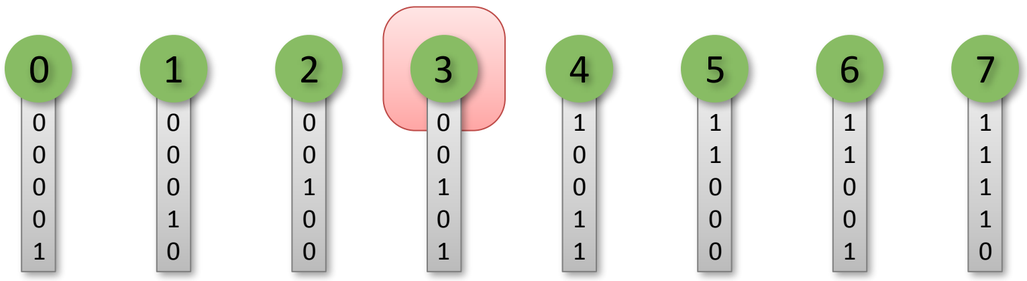
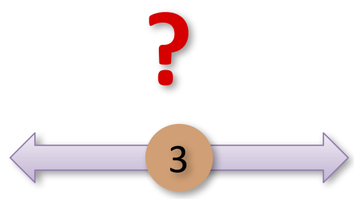
Algorithm

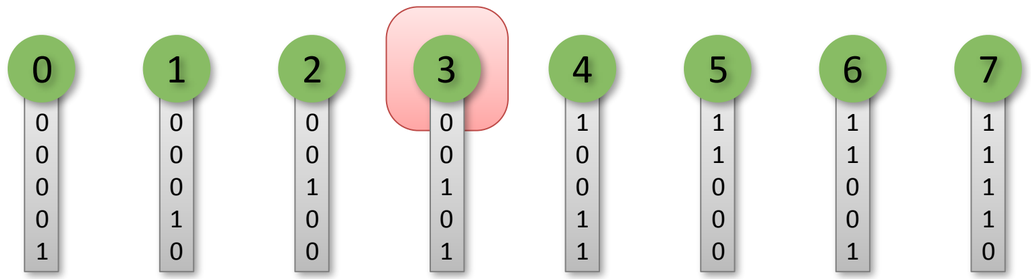


Algorithm



Algorithm





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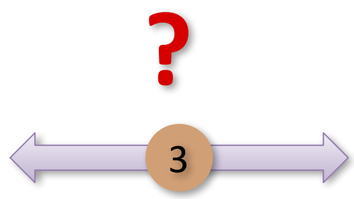
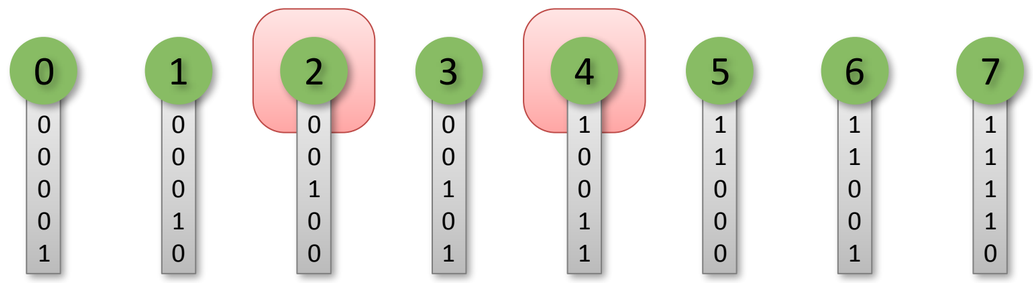
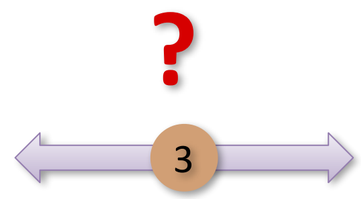
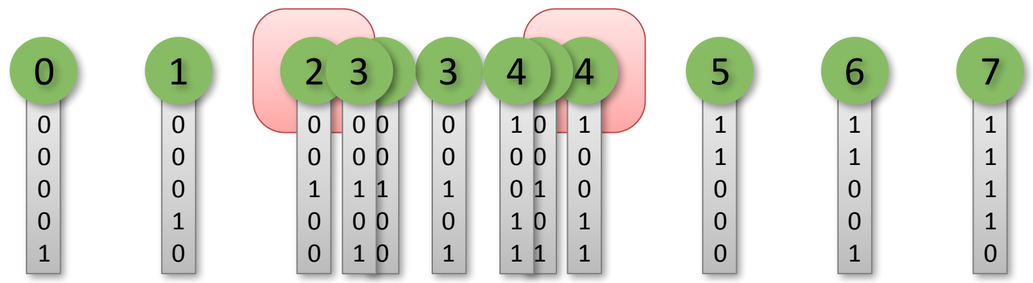




Algorithm


?
3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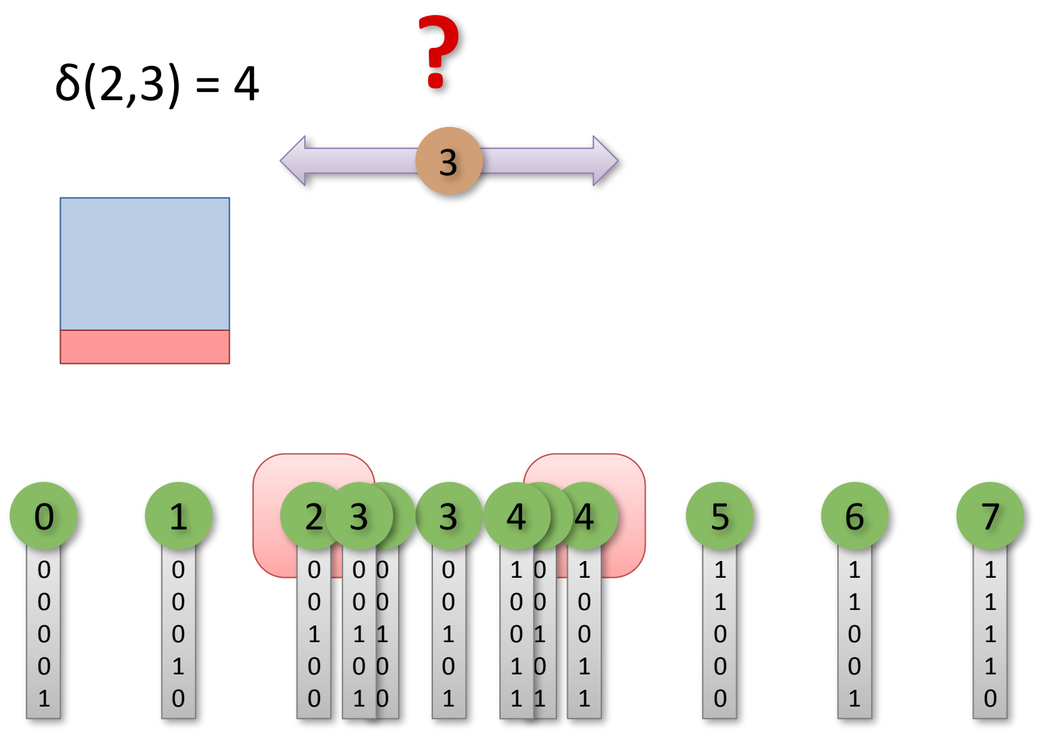
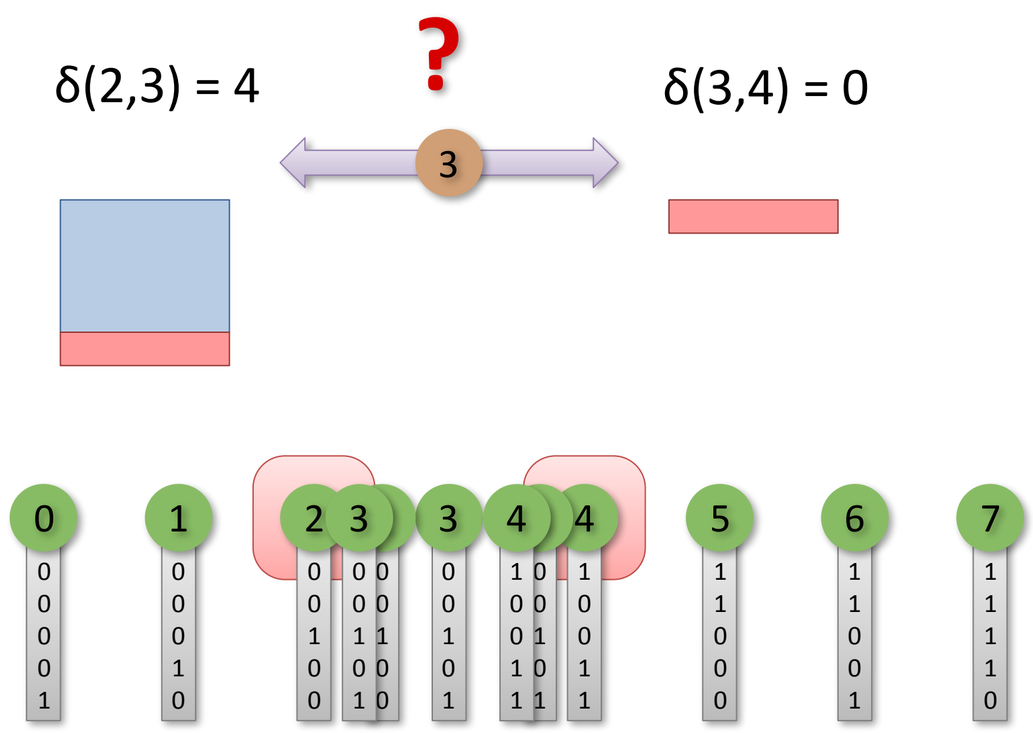
1 1 1 1 0

Algorithm



Algorithm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?
δ(3,4) = 0
3
0
1
2
3
3
4
4
5
6
7
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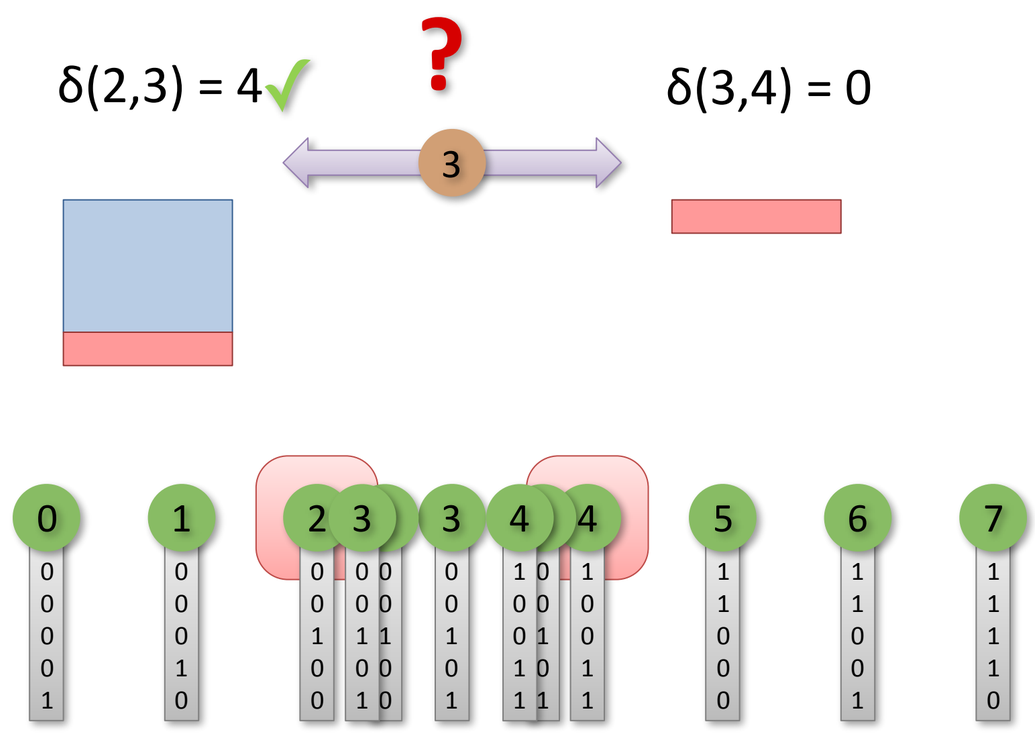
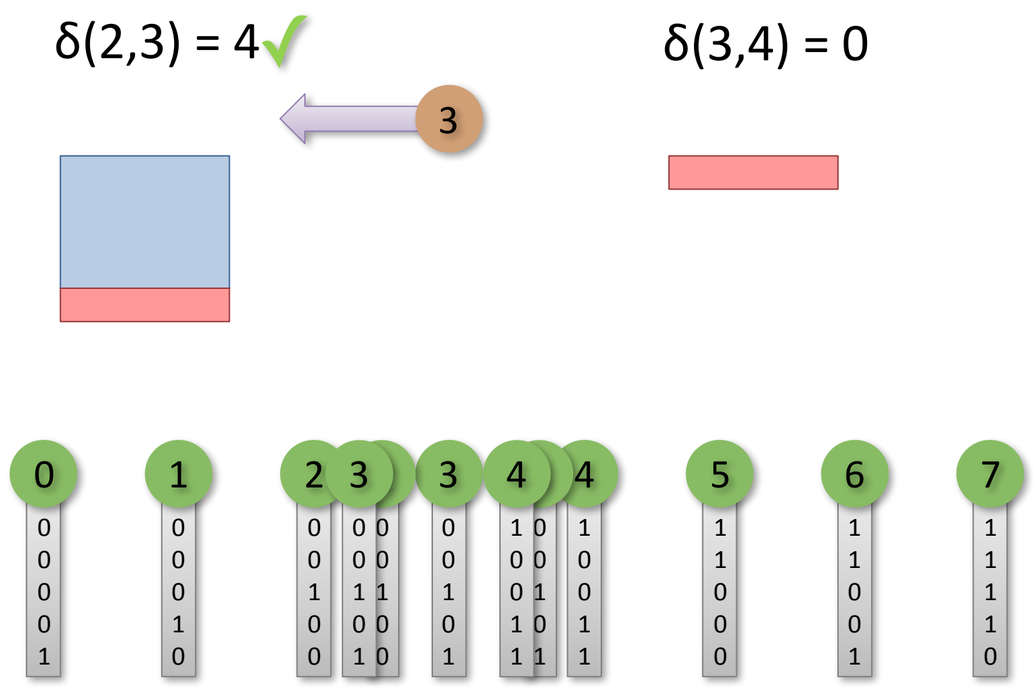
0 0 0 1 0
0 0 1 0 0
0 0 1 0 1
0 0 1 0 0
0 1 0 1
1 0 0 1 1
0 0 1 0 1
1 0 0 1 1
1 1 0 0 0
1 1 0 0 1
1 1 1 1 0

Algorithm



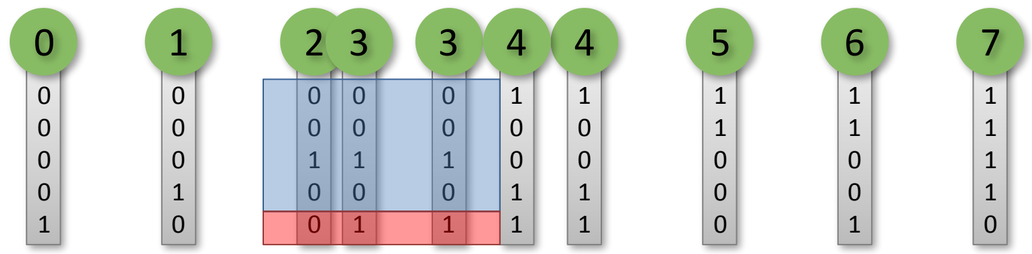
Algorithm


δ(2,3) = 4
δ(3,4) = 0
3
0
0 0 0 0 1
1
0 0 0 1 0
2
0 0 1 0 0
3
0 0 1 0 0
3
0 0 1 0 1
4
1 0 0 1 1
4
0 0 1 0 1
4
1 0 0 1 1
5
1 1 0 0 0
6
1 1 0 0 1
7
1 1 1 1 0

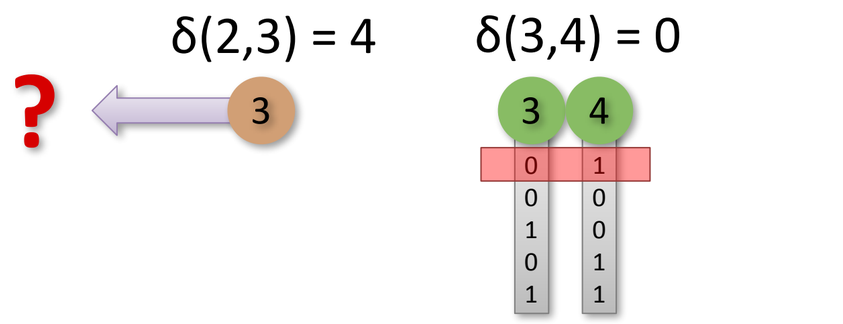
Algorithm

δ(2,3) = 4

δ(3,4) = 0



Algorithm


δ(2,3) = 4
δ(3,4) = 0
?
3
3
4
0
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1
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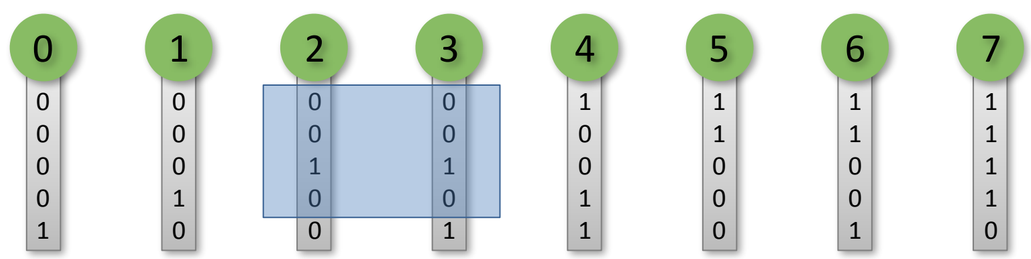
0
1
1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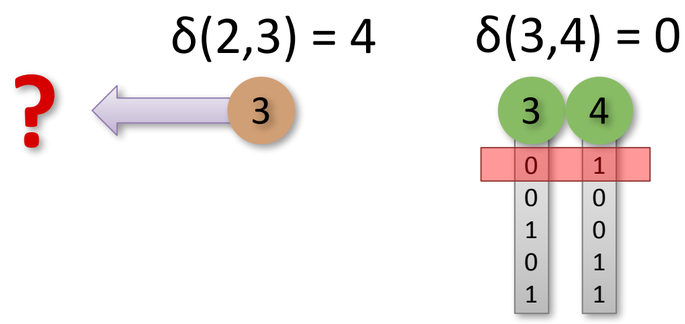
1 1 0 0 0
1 1 0 0 1
1 1 1 1 0

Algorithm


δ(2,3) = 4
δ(3,4) = 0
?
3
3
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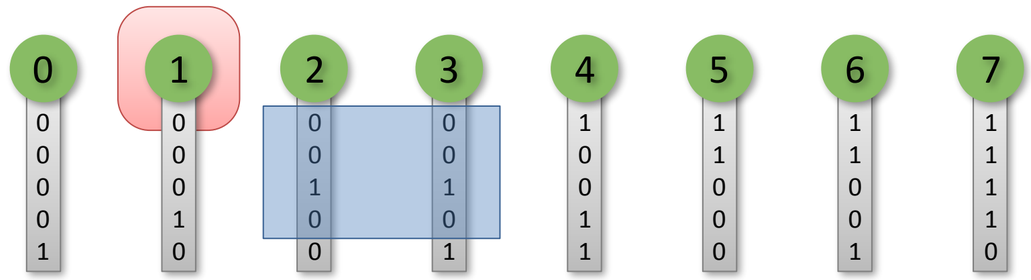
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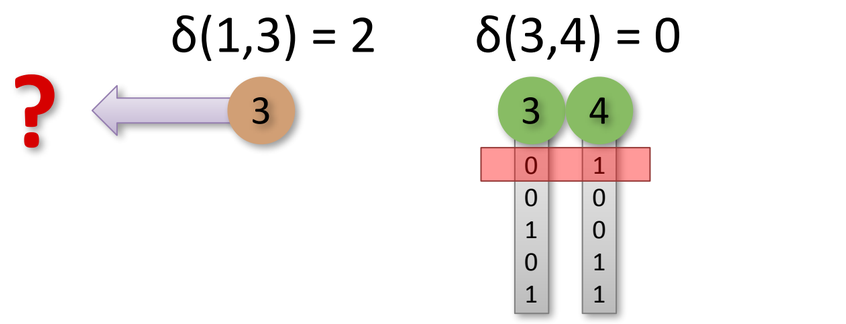
1 1 0 0 1
1 1 1 1 0

Algorithm


δ(1,3) = 2
δ(3,4) = 0
?
3
3
4
0
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1
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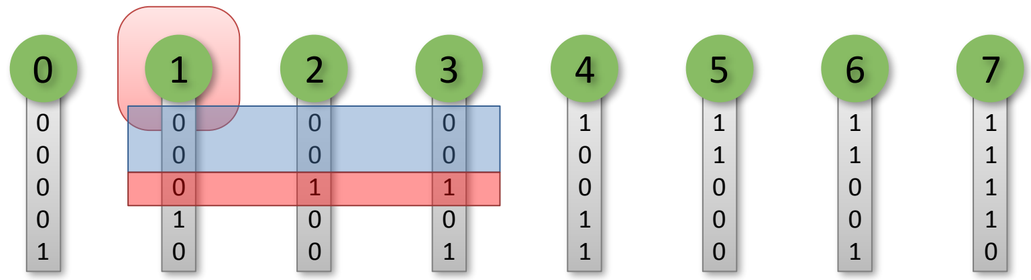
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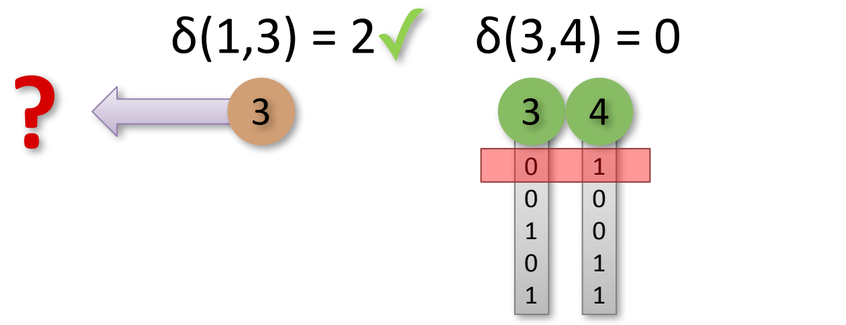
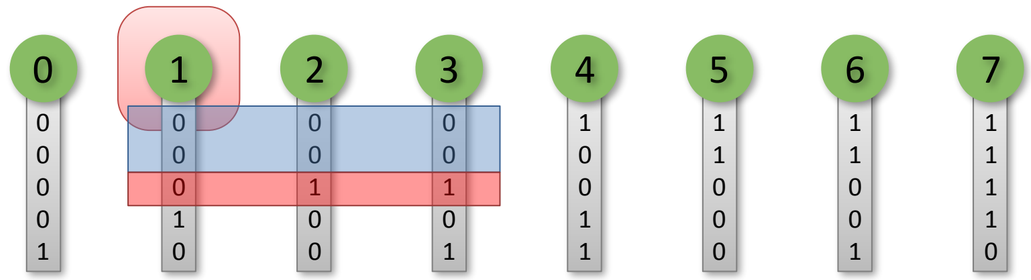
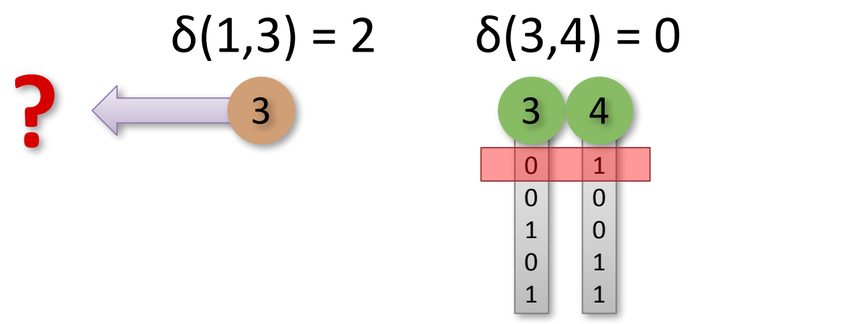
Algorithm





Algorithm


δ(1,3) = 2
δ(3,4) = 0
?
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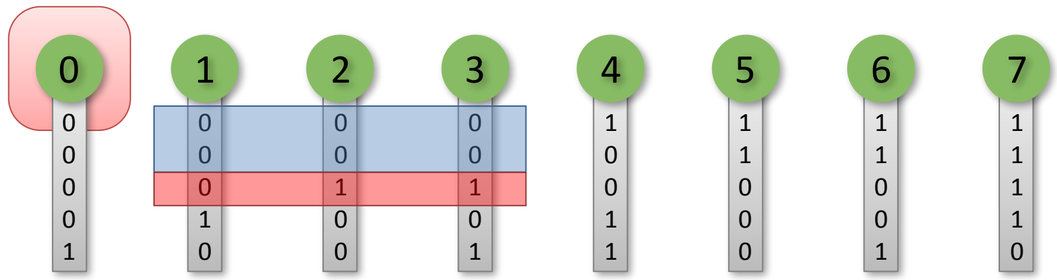
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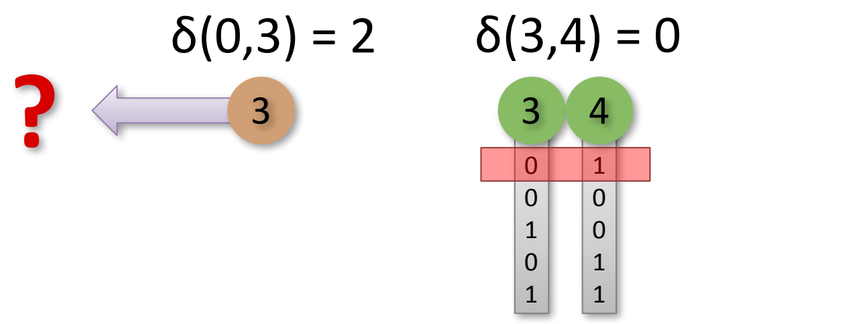
Algorithm


δ(0,3) = 2
δ(3,4) = 0
?
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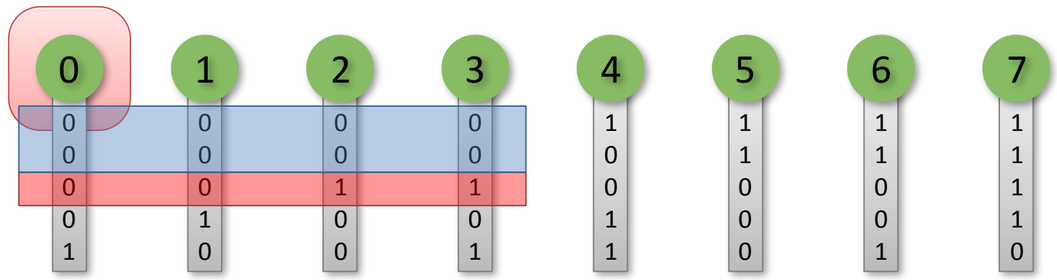
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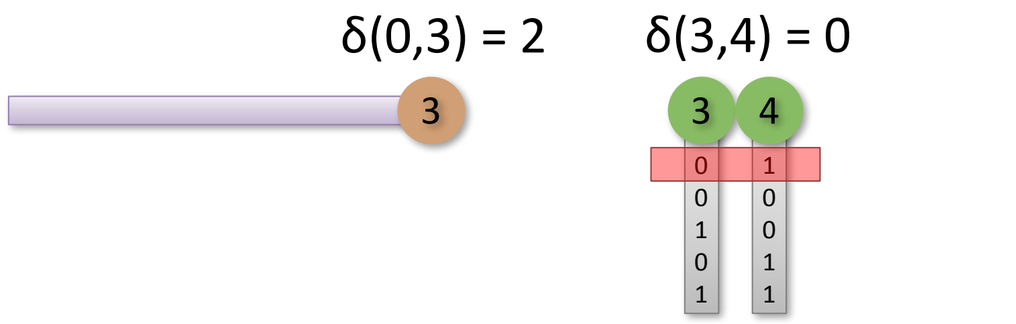
Algorithm


δ(0,3) = 2
δ(3,4) = 0
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3
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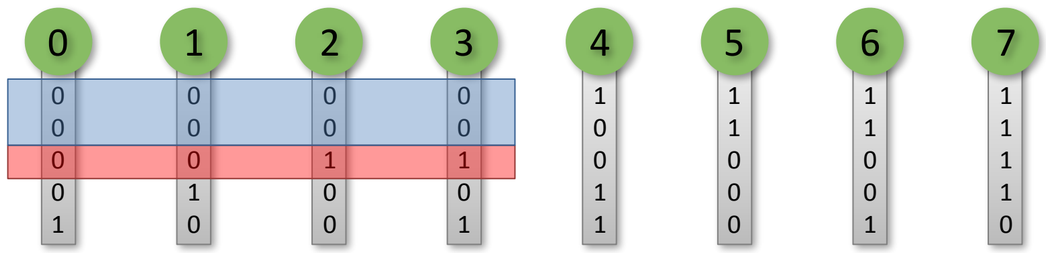
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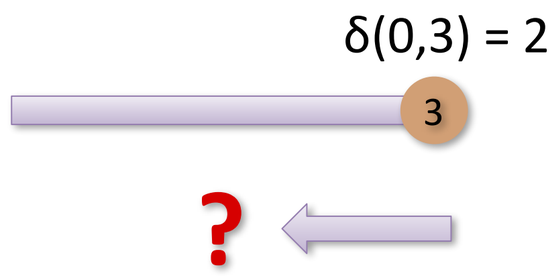
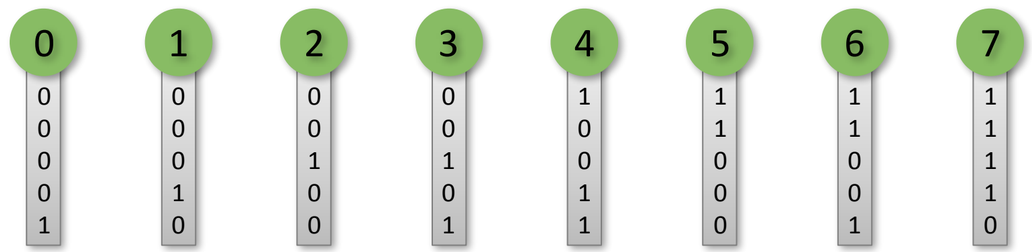
0

Algorithm


δ(0,3) = 2
3
?


0
0 0 0 0 1
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0 0 0 1 0
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0 0 1 0 0
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0 0 1 0 1
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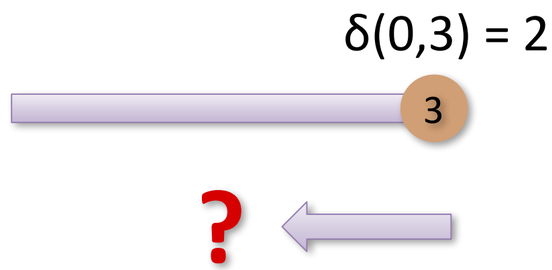
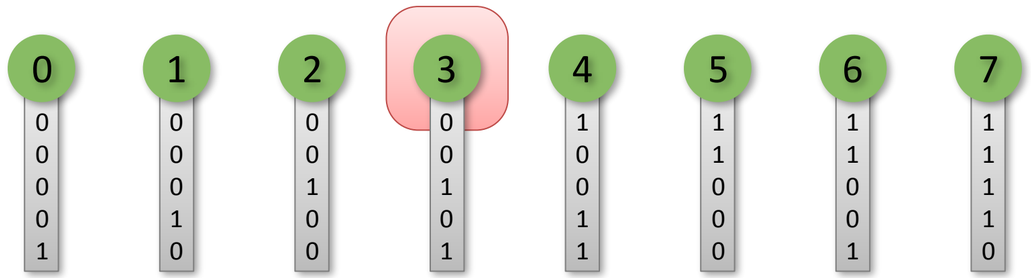
1 0 0 1 1
5
1 1 0 0 0
6
1 1 0 0 1
7
1 1 1 1 0

Algorithm


δ(0,3) = 2
3
?


0
0 0 0 0 1
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0 0 0 1 0
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1 0 0 1 1
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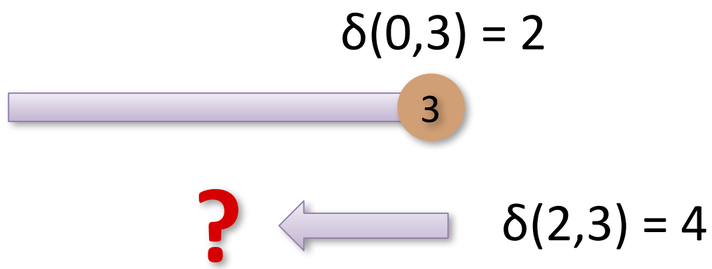
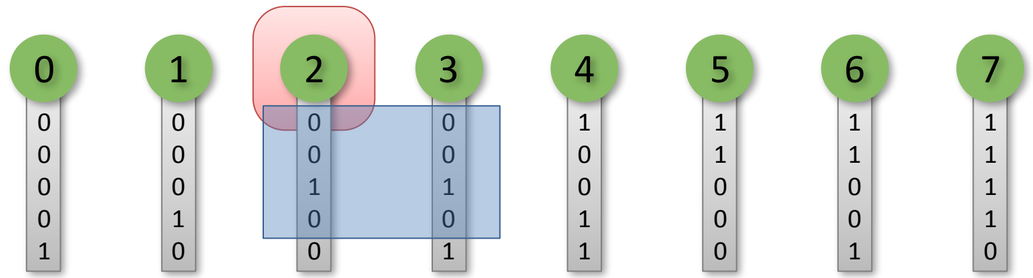
7
1 1 1 1 0

Algorithm


δ(0,3) = 2
3
?
δ(2,3) = 4


0
0 0 0 0 1
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0 0 0 1 0
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1 0 0 1 1
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1 1 0 0 0
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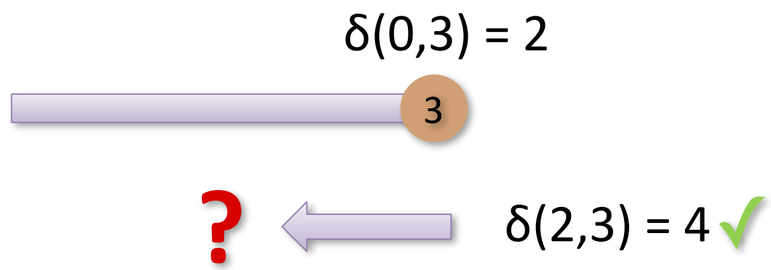
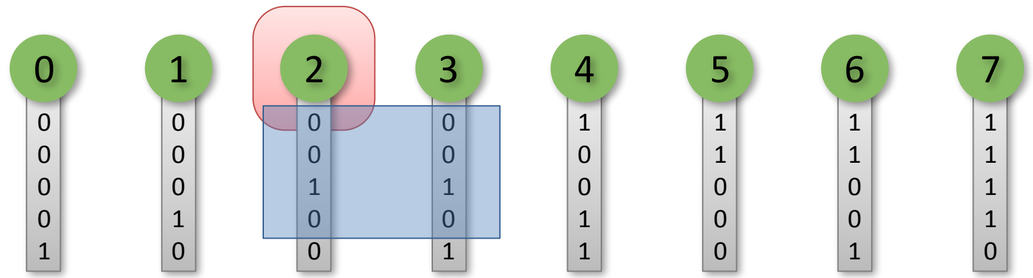
1 1 0 0 1
7
1 1 1 1 0

Algorithm


δ(0,3) = 2
3
?
δ(2,3) = 4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0 0 1 0 1
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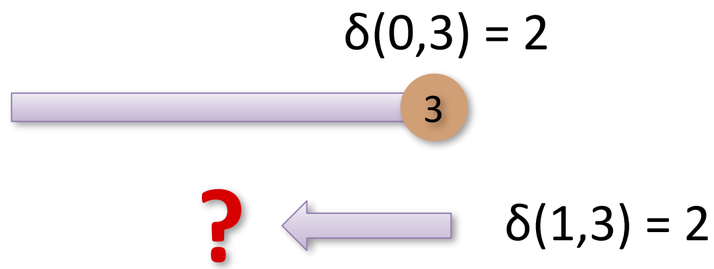
1 1 0 0 0
1 1 0 0 1
1 1 1 1 0

Algorithm


δ(0,3) = 2
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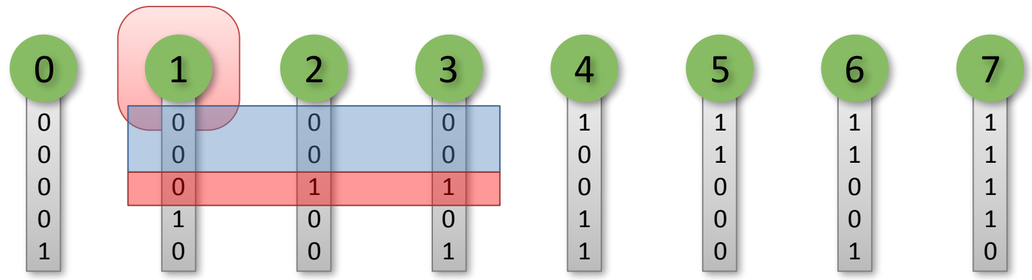
?
δ(1,3) = 2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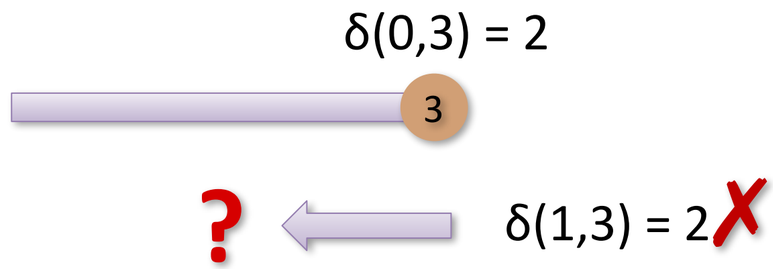
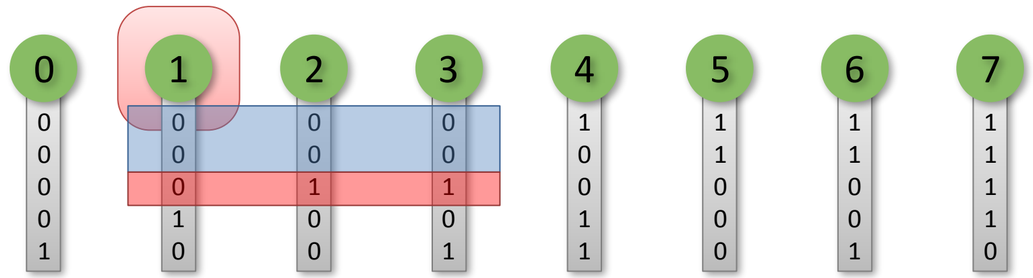
1 1 0 0 1
1 1 1 1 0

Algorithm


δ(0,3) = 2
3
?
δ(1,3) = 2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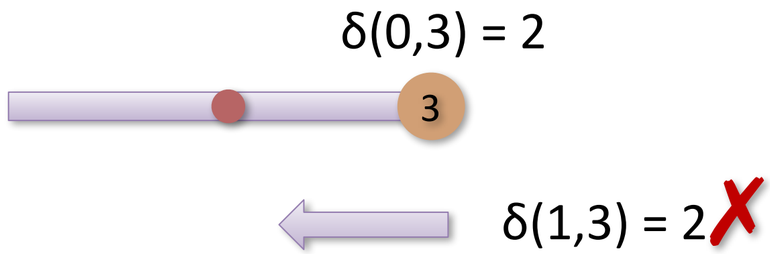
Algorithm


δ(0,3) = 2
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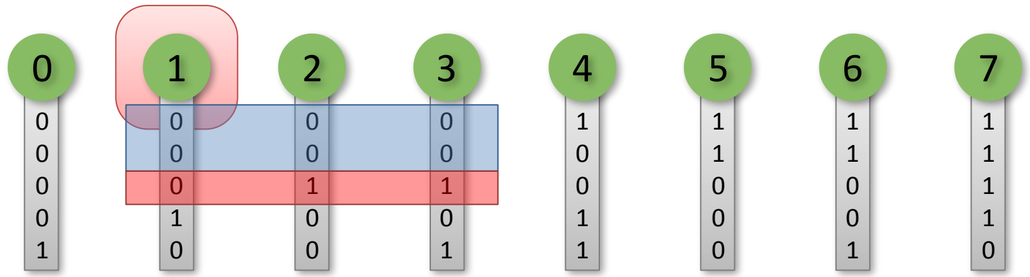
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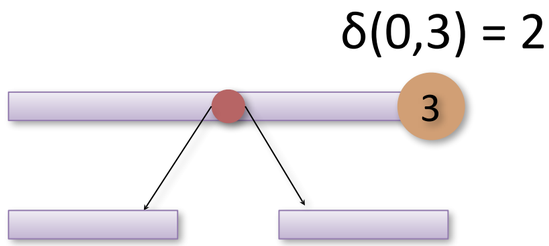
Algorithm


δ(0,3) = 2
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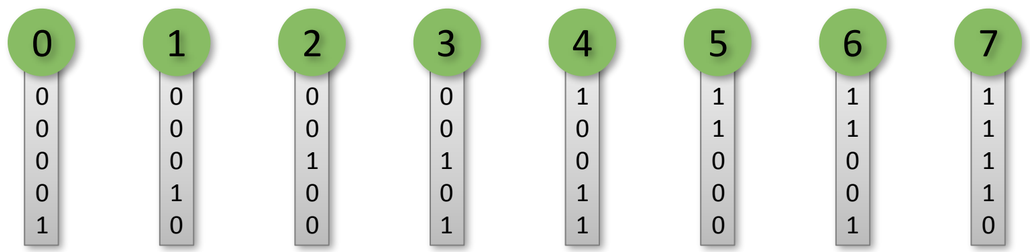
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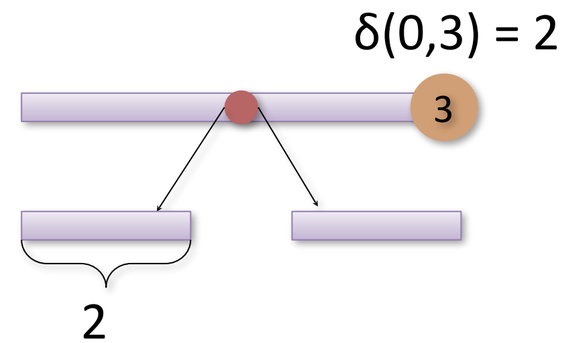
Algorithm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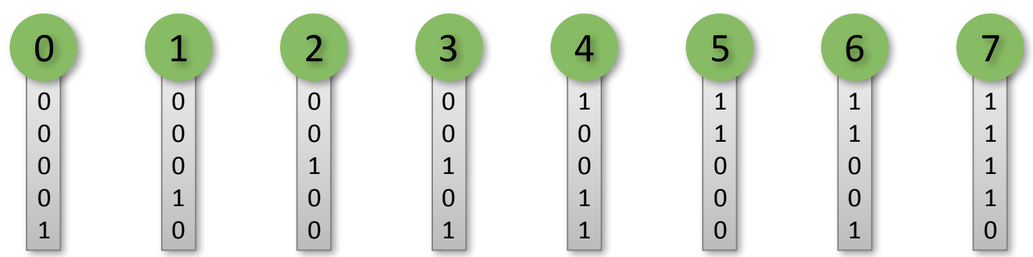
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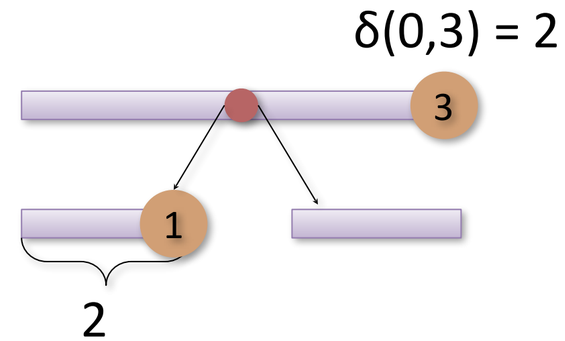
Algorithm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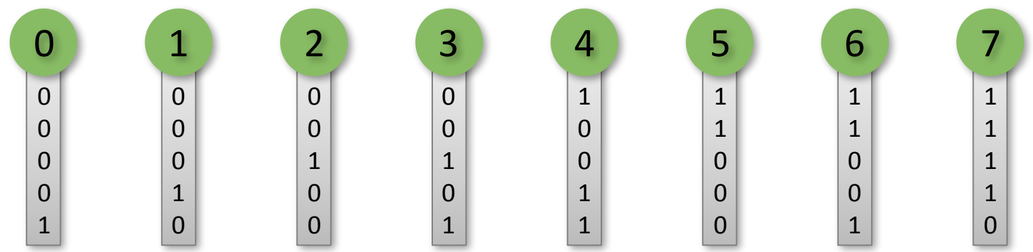
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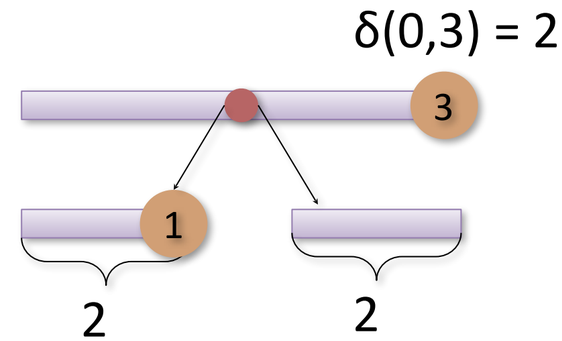
Algorithm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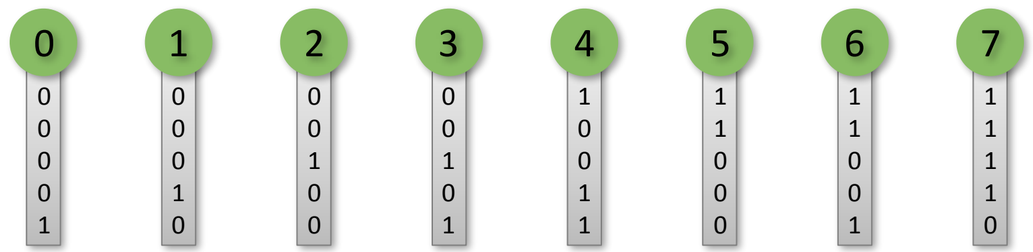
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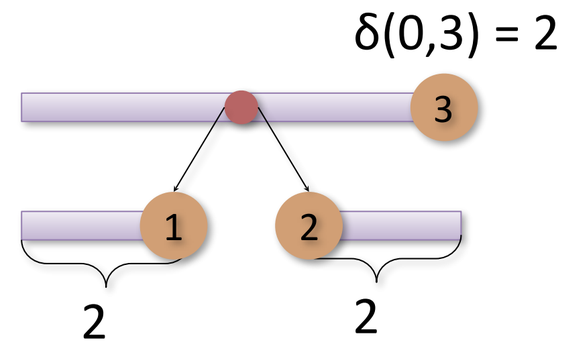
Algorithm


δ(0,3) = 2
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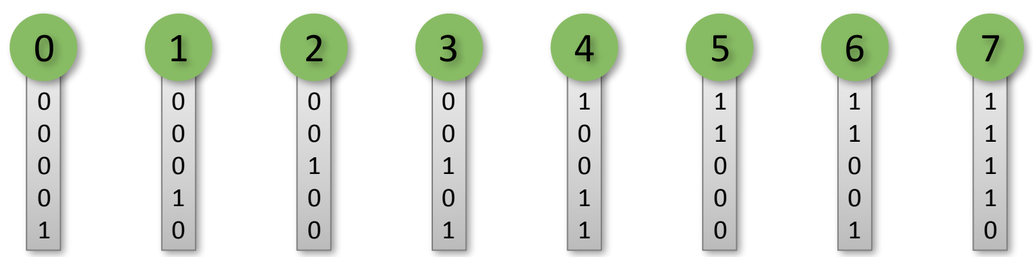
1
2
2
2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0
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7
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Algorithm

For each node i=0..n-2 in parallel:

1. Determine direction of the range
2. Expand the range as far as possible
3. Find where to split the range
4. Identify children

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Binary search

$$\mathcal{O}(n \log h)$$

Fully Parallel BVH Construction Performance

- Performs more work than previous approach
- But, keeps full occupancy because of increased parallelism
- Execution time: 0.02 milliseconds
  - 50x improvement over top-down algorithm
  - Still performs better than top-down as # objects increases
    - Due to less divergence in this algorithm

Calculating Bounding Boxes

- Traverse tree in parallel (bottom-up)
  - Start from leaves, advance towards root
  - Each node reads bb from its two children, computes bb, outputs bb
- Terminate threads using per-node atomic flags
  - Kill first thread that enters, allow second
  - Each node is processed once after both its children are done

Calculating Bounding Boxes Performance

- Execution time: 0.06 milliseconds

- Lots of divergence, but not really important because:
  - The execution time is still relatively low compared to other parts (e.g., sort)
  - The processing mainly consists of memory reads and writes (thus, memory bandwidth is the limiting factor)

Summary of BVH Traversal and Construction

- Traversal
  - One thread per leaf node: use iterative algorithm to find potential collisions (0.25 ms)
- Construction
  1. One thread per object: calculate bounding box and Morton code (0.02 ms)
  2. Parallel radix sort: sort the objects according to Morton code (0.18 ms)
  3. One thread per internal node: generate BVH node hierarchy (0.02 ms)
  4. One thread per object: calculate node bounding boxes by walking tree towards root (0.06 ms)
- Total time: 0.28 ms

Trees, More Broadly

- Lots of things are trees!
- *Dynamic* (runtime) decision-making
- Different tree structures:
  - Fixed vs. arbitrary depth
  - Static (known) structure vs. dynamic structure
  - Fixed vs. maximum vs. arbitrary branching factor
    - e.g., 2-deep + dynamic structure + arbitrary branching factor is nested data parallelism == sparse matrix
  - Recursive or not