MAT 226B Large Scale Matrix Computation
Homework 4

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Problem 1:

(a)

Problem 2:

The function `arnoldi_process` in `problem_2.m` implements Arnoldi process. It takes matrix A, initial r vector, and `KMAX` value and outputs matrices H_k and V_K. We test the function with $k = 5, 10,$ and 20. Table 1 shows the eignevalues of H_k for different values of k. Table 2 shows the eignevalues of A. To compare between the eigenvalue of H_{20} and A's eigenvalues, we computed $\| A\_eig - H_{20}\_eig \| = 2.022960706546113\text{e}{-14}$

K	Eigenvalues
5	$-1.667643118967301\text{e}{+00} + 3.322698092379202\text{e}{+00}i$
	$1.122189729586331\text{e}{+00} + -2.935868201795706\text{e}{+00}i$
	$-2.454291098365838\text{e}{+00} + -1.034091783975195\text{e}{-01}i$
	$1.901983203411739\text{e}{+00} + 2.938354695246812\text{e}{-02}i$
	$-1.488418331702275\text{e}{-01} + 4.856460403443859\text{e}{-03}i$
10	$-1.284055777449226\text{e}{+00} + +3.812474971159054\text{e}{+00}i$
	$2.481430445455568\text{e}{-01} + -3.792088063628674\text{e}{+00}i$
	$2.484678533435320\text{e}{+00} + -2.158804713951516\text{e}{+00}i$
	$-9.594670448581000\text{e}{-01} + -2.797385858005139\text{e}{+00}i$
	$-1.865849533089461\text{e}{+00} + +2.203044871147163\text{e}{+00}i$
	$-2.769085996727914\text{e}{+00} + -4.783604953147819\text{e}{-01}i$
20	$-1.246365732148964\text{e}{+00} + +3.879484437687161\text{e}{+00}i$
	$1.251730623491198\text{e}{-01} + -3.705917327586333\text{e}{+00}i$
	$2.594180629560022\text{e}{+00} + -2.284902242884260\text{e}{+00}i$
	$-1.983514535906369\text{e}{+00} + +2.443522984313272\text{e}{+00}i$
	$3.000394334727364\text{e}{+00} + -9.028020730501802\text{e}{-01}i$
	$-8.152087361523789\text{e}{-01} + -2.850024263184166\text{e}{+00}i$

Figure 1: H_k Eigenvalues

Eigenvalues
$-1.246365732148965\text{e}{+00} + +3.879484437687162\text{e}{+00}i$
$1.251730623491239\text{e}{-01} + -3.705917327586323\text{e}{+00}i$
$2.594180629560029\text{e}{+00} + -2.284902242884256\text{e}{+00}i$
$-1.983514535906377\text{e}{+00} + +2.443522984313265\text{e}{+00}i$
$3.000394334727376\text{e}{+00} + -9.028020730501798\text{e}{-01}i$
$-8.152087361523785\text{e}{-01} + -2.850024263184167\text{e}{+00}i$

Figure 2: A Eigenvalues

Problem 3:

(a) Function `hermitian_lanczos` implements the Hermitian Lanczos process where it takes the matrix A, vector r, and `KMAX` parameter and output T_k tridiagonal matrix in sparse format. We used this function to approximate eigenvalues of A from `make_3d_laplacian` function using $k = 7$. Table 3 shows the results where the first two columns show the exact eigenvalues of A (computed from the provided function) along its multiplicity and last column shows the approximate eigenvalues. We can see that the approximate solution is able to capture all the eigenvalues with very high accuracy.

Exact Eigenvalues	Multiplicity	Approximate Eigenvalues
1.757359312880715e+00	1	1.757359312880715e+00
3.171572875253810e+00	3	3.171572875253810e+00
4.585786437626905e+00	6	4.585786437626905e+00
6.000000000000000e+00	7	6.000000000000002e+00
7.414213562373095e+00	6	7.414213562373095e+00
8.828427124746190e+00	3	8.828427124746192e+00
1.024264068711928e+01	1	1.024264068711929e+01

Figure 3: Exact and approximate eigenvalues using Hermitian Lanczos process

Smallest Eigenvalues	Largest Eigenvalues
1.422787575075167e−02	1.170222368514153e+01
2.522292134967839e−02	1.175506886438500e+01
3.720946521956244e−02	1.180683457939747e+01
6.015341020592301e−02	1.184896392339356e+01
8.407846117839080e−02	1.188540931980204e+01
1.168498457907536e−01	1.191749739736500e+01
1.581734700307590e−01	1.194638621492721e+01
1.995732825474702e−01	1.196512279467138e+01
2.466691832816189e−01	1.198055273500129e+01
3.011244810109077e−01	1.199293004521624e+01

Figure 4: Approximate Eigenvalues using $K = 100$

(b)

Smallest Eigenvalues	Largest Eigenvalues
7.006926067695284e−03	1.191194541966088e+01
1.400816296330363e−02	1.192367820060319e+01
2.104749654401042e−02	1.193815033138952e+01
2.644416419643677e−02	1.194943725586547e+01
3.252921177956383e−02	1.195823115974695e+01
4.248144987518579e−02	1.196722905407029e+01
5.158180541911261e−02	1.197279443138257e+01
6.175958581860198e−02	1.197897280487774e+01
7.373377938324976e−02	1.198599186623671e+01
8.527634111033579e−02	1.199299336087879e+01

Figure 5: Approximate Eigenvalues using $K = 200$

Smallest Eigenvalues	Largest Eigenvalues
7.006639006043782e−03	1.195105825040107e+01
1.400782323540659e−02	1.195568924396476e+01
2.100900746475762e−02	1.195806895929420e+01
2.565829376859278e−02	1.196033932970904e+01
2.801019169415529e−02	1.196734052200119e+01
3.265947799840193e−02	1.197198980830585e+01
3.966069944862968e−02	1.197434170623140e+01
4.193108831261764e−02	1.197899099253526e+01
4.431079048315387e−02	1.198599217676459e+01
4.897191625334762e−02	1.199299336099394e+01

Figure 6: Approximate Eigenvalues using $K = 400$

Problem 4:

(a)

Smallest Eigenvalues	Largest Eigenvalues
7.006639006040011e−03	1.197198980830589e+01
7.006639006045063e−03	1.197434170555307e+01
1.400782323539451e−02	1.197434170623140e+01
1.400782323540438e−02	1.197899099253525e+01
2.100900746475027e−02	1.197899099253527e+01
2.100900746475296e−02	1.198599217676461e+01
2.565829371816093e−02	1.198599217676461e+01
2.565829376859185e−02	1.199293916949044e+01
2.801018851772900e−02	1.199299336099396e+01
2.801019169410766e−02	1.199299336099397e+01

Figure 7: Approximate Eigenvalues using $K = 800$

Smallest Eigenvalues	Largest Eigenvalues
7.006639006034983e−03	1.197899099253524e+01
7.006639006042774e−03	1.197899099253525e+01
7.006639006050910e−03	1.197899099253525e+01
7.006642433917978e−03	1.198599217676460e+01
1.400782323538701e−02	1.198599217676461e+01
1.400782323539747e−02	1.198599217676462e+01
1.400782323539999e−02	1.199299336099394e+01
2.100900746475157e−02	1.199299336099396e+01
2.100900746475692e−02	1.199299336099397e+01
2.100900746476310e−02	1.199299336099399e+01

Figure 8: Approximate Eigenvalues using $K = 1200$

Smallest Eigenvalues	Largest Eigenvalues
7.006639006034724e−03	1.197899099253525e+01
7.006639006037083e−03	1.198599217676456e+01
7.006639006037985e−03	1.198599217676460e+01
7.006639006044512e−03	1.198599217676460e+01
7.006639006047808e−03	1.198599217676461e+01
1.400782323539137e−02	1.199299336099392e+01
1.400782323539370e−02	1.199299336099395e+01
1.400782323539474e−02	1.199299336099396e+01
1.400782323540451e−02	1.199299336099397e+01
1.412781215190661e−02	1.199299336099398e+01

Figure 9: Approximate Eigenvalues using $K = 1600$

Smallest Eigenvalues	Largest Eigenvalues
7.006639006032009e−03	1.198599217676460e+01
7.006639006034958e−03	1.198599217676460e+01
7.006639006035550e−03	1.198599217676461e+01
7.006639006037528e−03	1.198599260901220e+01
7.006639006037725e−03	1.199299336099394e+01
7.006639006042897e−03	1.199299336099395e+01
1.400782323536894e−02	1.199299336099396e+01
1.400782323539092e−02	1.199299336099396e+01
1.400782323539872e−02	1.199299336099396e+01
1.400782323540259e−02	1.199299336099397e+01

Figure 10: Approximate Eigenvalues using $K = 2000$

Smallest Eigenvalues	Largest Eigenvalues
7.006639006040594e−03	1.197434170623141e+01
1.400782323539662e−02	1.197434170623141e+01
1.400782323539662e−02	1.197434170623141e+01
1.400782323539662e−02	1.197899099253525e+01
2.100900746475265e−02	1.197899099253525e+01
2.100900746475265e−02	1.197899099253525e+01
2.100900746475265e−02	1.198599217676460e+01
2.565829376859075e−02	1.198599217676460e+01
2.565829376859163e−02	1.198599217676460e+01
2.565829376859163e−02	1.199299336099396e+01

Figure 11: Exact Eigenvalues