MAT 226B Large Scale Matrix Computation
Homework 1

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Problem 1:

Let $A = [a_{j,k}] \in \mathbb{R}^{n \times n} \succ 0$ and $L = [l_{j,k}]$ be its Cholesky factor.

Algorithm 1: Cholesky Factorization

Input: $A = [a_{j,k}] \in \mathbb{R}^{n \times n} \succ 0$
Output: $L = [l_{j,k}]$ such that $A = LL^T$
$l_{j,k} = a_{j,k}, \forall j \geq k, j, k = 1, 2, \ldots, n$;
for $k = 1, 2, \ldots, n$ **do**
    $l_{k,k} = \sqrt{l_{k,k}}$;
    $l_{k+1:n,k} = \frac{1}{l_{k,k}} l_{k+1:n,k}$;
    if *condition* **then**
        instructions1;
        instructions2;
    else
        instructions3;
    end
end

Problem 2:

Problem 3:

Problem 4:

Figure 1 shows the associated graph $G(A)$ of matrix A along with the steps of the minimum degree algorithm. From these steps, the reordering of the nodes will be $2, 4, 5, 3, 6, 7, 1, 8, 9$

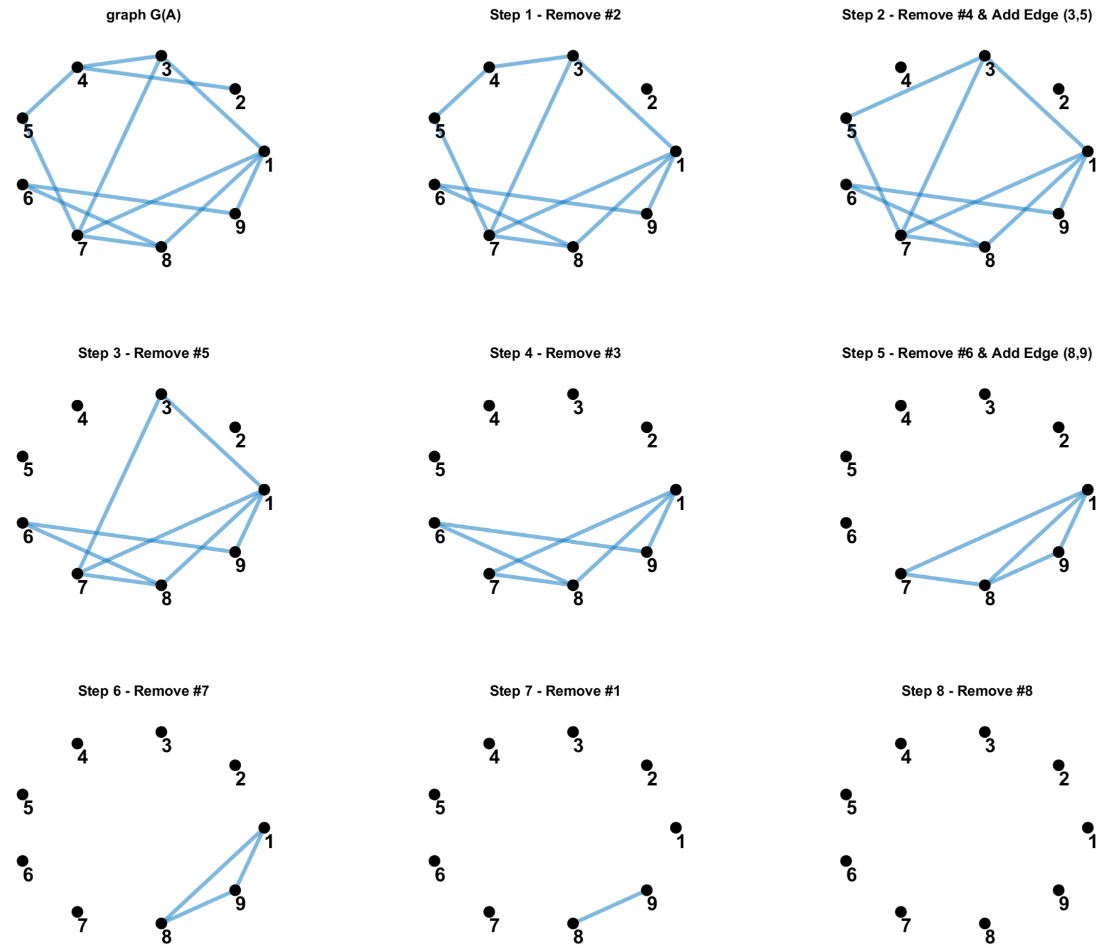

Figure 1: Graph $G(A)$ along with the 8 steps of the minimum degree algorithm applied on it.

From the reordering above, the permutation matrix can be constructed such that

$$P = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

From which, we can compute $P^T AP$ to be

$$P^T AP = \begin{bmatrix} * & * & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 & 0 & 0 & 0 \\ 0 & * & * & 0 & 0 & * & 0 & 0 & 0 \\ 0 & * & 0 & * & 0 & * & * & 0 & 0 \\ 0 & 0 & 0 & 0 & * & 0 & 0 & * & * \\ 0 & 0 & * & * & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & 0 & * & * & * & * \\ 0 & 0 & 0 & 0 & * & * & * & * & 0 \\ 0 & 0 & 0 & 0 & * & 0 & * & 0 & * \end{bmatrix}$$

Applying Cholesky factorization to $P^T AP$ we get the following lower triangular matrix where the fill-in elements are shown with $+$

$$L = \begin{bmatrix} * & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & * & * & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & * & + & * & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & * & 0 & 0 & 0 & 0 \\ 0 & 0 & * & * & 0 & * & 0 & 0 & 0 \\ 0 & 0 & 0 & * & 0 & * & * & 0 & 0 \\ 0 & 0 & 0 & 0 & * & * & * & * & 0 \\ 0 & 0 & 0 & 0 & * & 0 & * & + & * \end{bmatrix}$$

Problem 5: