Math 226A - HW1

Ahmed H. Mahmoud

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Problem No.1

Part (a) The following MATLAB code computes the exponential function $f(x) = e^x$ using the truncated series up to n terms.

```
function approx_exp = ApproxExpFunc(x, n)
%Approximate the exponential using truncated series
approx_exp = 1;  %first term is 1 (x^(0)/0!)
j_factorial = 1; %start with factorial of 0
for j=1:n
    j_factorial = j_factorial*(j);
    approx_exp = approx_exp + (x^(j))/j_factorial;
    end
end
```

With $n = 10$ and $x = \pm 5$, the relative error is of order 10^{-11}. However for $x = 30\pi$, it is ≈ 1 and for $x = -30\pi$, it is 10^{54}. Using higher value for n (i.e., $n = 40$), it helps decreasing the relative error for $x = \pm 5$ to be $< 10^{-16}$, it does not affect relative error of $x = 30\pi$ but it increases the relative error for $x = -30\pi$ to be 10^{71}.

The reason behind that is the ???????????????????????????

Part (b) Using remainder theorem for Taylor series, we can expect the error to be $< 10^{-13}$ only when $abs(x) < 0.33$. This is derived by solving the following:

$$E(x) = \frac{abs(e^x - R_{n,x})}{e^x} = \frac{e^x - \sum_{j=0}^{n}}{e^x} < 10^{13}$$

where $n = 10$. Using the fact that $e^x = e^{x/2+x/2} = e^{x/2}e^{x/2}$, we can designed an algorithm that runs recursively by splitting x into half and only runs the truncated series when $abs(x) < 0.33$. The following MATLAB code implements such an algorithm. As a safe guard, we used $abs(x) < 0.2$. This gives a relative error for inputs $-100 \leq x \leq 100$ to be always $< 10^{-13}$.

```
function accurate_exp = AccurateExpFunc(x,n)
    %More accurate approximation for the exp function
    %it exploites the fact that exp(x) = exp(x/2+x/2) = exp(x/2)*exp(x/2)
    %with in mind that ApproxExpFunc() give good approximation
```

```
    %(relative err < 10^-13) for x<0.2

    if abs(x) < 0.2
       accurate_exp = ApproxExpFunc(x, n);
    else
        accurate_exp = AccurateExpFunc(x/2, n);
        accurate_exp = accurate_exp*accurate_exp;
    end
end
```

Problem No.2

Part (b) $I(x)$ can be evaluated as follows:

$$I(x) = \int_x^{x+1} \frac{1}{1+s^2} ds = \int_x^{x+1} \frac{1}{s-i} \frac{1}{s+i} ds$$

$$I(x) = \frac{1}{2i} \left(\int_x^{x+1} \frac{1}{s-1} ds - \int_x^{x+1} \frac{1}{s+i} ds \right)$$

$$I(x) = \frac{1}{2i} \left(ln(s-i) - ln(s+i) \right)_x^{x+1}$$

$$I(x) = \frac{1}{2i} \left(ln\left(\frac{x+1-i}{x+1+i}\right) - ln\left(\frac{x-i}{x+i}\right) \right)$$

Problem No.3

Part 1:

Problem No.4

Part 1: