The chain *operators* corresponding to the incidence relations $\mathrm{VV} \subset V \times V$, $\mathrm{VE} \subset V \times E$, and $\mathrm{VF} \subset V \times F$ are given below:

$$
\begin{aligned}
&\mathcal{V}\mathcal{V} : C_0 \to C_0, \qquad && \mathcal{E}\mathcal{V} : C_0 \to C_1, \qquad && \mathcal{F}\mathcal{V} : C_0 \to C_2;\\
&\mathcal{V}\mathcal{E} : C_1 \to C_0, \qquad && \mathcal{E}\mathcal{E} : C_1 \to C_1, \qquad && \mathcal{F}\mathcal{E} : C_1 \to C_2;\\
&\mathcal{V}\mathcal{F} : C_2 \to C_0, \qquad && \mathcal{E}\mathcal{F} : C_2 \to C_1, \qquad && \mathcal{F}\mathcal{F} : C_2 \to C_2.
\end{aligned}
$$

The corresponding CSR matrices are readily computed:

$$
\begin{aligned}
&\mathcal{V}\mathcal{V} = \mathcal{V}\mathcal{E} \circ \mathcal{E}\mathcal{V} = \mathcal{E}\mathcal{V}^{\top} \circ \mathcal{E}\mathcal{V} \Rightarrow [\mathcal{V}\mathcal{V}] = M_1^t M_1\\
&\mathcal{V}\mathcal{E} = \mathcal{E}\mathcal{V}^{\top} \Rightarrow [\mathcal{V}\mathcal{E}] = M_1^t\\
&\mathcal{V}\mathcal{F} = \mathcal{F}\mathcal{V}^{\top} \Rightarrow [\mathcal{V}\mathcal{F}] = M_2^t\\
&\mathcal{E}\mathcal{V} \quad [\mathcal{E}\mathcal{V}] = M_1\\
&\mathcal{E}\mathcal{E} = \mathcal{E}\mathcal{V} \circ \mathcal{V}\mathcal{E} = \mathcal{E}\mathcal{V} \circ \mathcal{E}\mathcal{V}^{\top} \Rightarrow [\mathcal{E}\mathcal{E}] = M_1 M_1^t\\
&\mathcal{E}\mathcal{F} = \mathcal{E}\mathcal{V} \circ \mathcal{V}\mathcal{F} = \mathcal{E}\mathcal{V} \circ \mathcal{F}\mathcal{V}^{\top} \Rightarrow [\mathcal{E}\mathcal{F}] = M_1 M_2^t\\
&\mathcal{F}\mathcal{V} \quad [\mathcal{F}\mathcal{V}] = M_2\\
&\mathcal{F}\mathcal{E} = \mathcal{F}\mathcal{V} \circ \mathcal{V}\mathcal{E} = \mathcal{F}\mathcal{V} \circ \mathcal{E}\mathcal{V}^{\top} \Rightarrow [\mathcal{F}\mathcal{E}] = M_2 M_1^t\\
&\mathcal{F}\mathcal{F} = \mathcal{F}\mathcal{V} \circ \mathcal{V}\mathcal{F} = \mathcal{F}\mathcal{V} \circ \mathcal{F}\mathcal{V}^{\top} \Rightarrow [\mathcal{F}\mathcal{F}] = M_2 M_2^t.
\end{aligned}
$$