Curves, surfaces and splines with LAR *

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Abstract

In this module we implement above LAR most of the parametric methods for polynomial and rational curves, surfaces and splines discussed in the book [Pao03], and implemented in the PLaSM language and in the python package pyplasm.

Contents

1 Introduction

2 Transfinite Bézier

⟨ Multidimensional transfinite Bézier 1 ⟩ ≡

*This document is part of the *Linear Algebraic Representation with CoChains* (LAR-CC) framework [CL13]. April 24, 2014

```
""" Multidimensional transfinite Bezier """
def larBezier(U,d=3):
   def BEZIER0(controldata_fn):
      N = len(controldata_fn)-1
      def map_fn(point):
         t = U(point)
         controldata = [fun(point) if callable(fun) else fun
            for fun in controldata_fn]
         out = [0.0 for i in range(len(controldata[0]))]
         for I in range(N+1):
            weight = CHOOSE([N,I])*math.pow(1-t,N-I)*math.pow(t,I)
            for K in range(len(out)):  out[K] += weight*(controldata[I][K])
         return out
      return (COMP([AA(COMP),DISTR]))([AA(SEL)(range(d)), map_fn])
   return BEZIER0

def larBezierCurve(controlpoints):
   dim = len(controlpoints[0])
   return larBezier(S1,dim)(controlpoints)
◇
```

Macro referenced in 3a.

3 Coons patches

⟨ Transfinite Coons patches 2 ⟩ ≡

```
""" Transfinite Coons patches """
def larCoonsPatch (args):
   su0_fn , su1_fn , s0v_fn , s1v_fn = args
   def map_fn(point):
      u,v=point
      su0 = su0_fn(point) if callable(su0_fn) else su0_fn
      su1 = su1_fn(point) if callable(su1_fn) else su1_fn
      s0v = s0v_fn(point) if callable(s0v_fn) else s0v_fn
      s1v = s1v_fn(point) if callable(s1v_fn) else s1v_fn
      ret=[0.0 for i in range(len(su0))]
      for K in range(len(ret)):
         ret[K] = ((1-u)*s0v[K] + u*s1v[K]+(1-v)*su0[K] + v*su1[K] +
         (1-u)*(1-v)*s0v[K] + (1-u)*v*s0v[K] + u*(1-v)*s1v[K] + u*v*s1v[K])
      return ret
   return (COMP([AA(COMP),DISTR]))([[S1,S2,S3], map_fn])
◇
```

Macro referenced in 3a.

4 Computational framework

4.1 Exporting the library

"lib/py/splines.py" 3a ≡

```
""" Mapping functions and primitive objects """
⟨ Initial import of modules 4b ⟩
⟨ Multidimensional transfinite Bézier 1 ⟩
⟨ Transfinite Coons patches 2 ⟩
◇
```

5 Examples

Some examples of curves

"test/py/splines/test01.py" 3b ≡

```
""" Example of Bezier curve """
import sys
""" import modules from larcc/lib """
sys.path.insert(0, 'lib/py/')
from splines import *

controlpoints = [[-0,0],[1,0],[1,1],[2,1],[3,1]]
dom = larDomain([32],'simplex')
obj = larMap(larBezierCurve(controlpoints))(dom)
VIEW(STRUCT(MKPOLS(obj)))

obj = larMap(larBezier(S1,2)(controlpoints))(dom)
VIEW(STRUCT(MKPOLS(obj)))
◇
```

Transfinite cubic surface

"test/py/splines/test02.py" 3c ≡

```
""" Example of transfinite surface """
import sys
""" import modules from larcc/lib """
sys.path.insert(0, 'lib/py/')
from splines import *

dom = larDomain([20],'simplex')
C0 = larBezier(S1,3)([[0,0,0],[10,0,0]])
C1 = larBezier(S1,3)([[0,2,0],[8,3,0],[9,2,0]])
C2 = larBezier(S1,3)([[0,4,1],[7,5,-1],[8,5,1],[12,4,0]])
C3 = larBezier(S1,3)([[0,6,0],[9,6,3],[10,6,-1]])
dom2D = larExtrude1(dom,20*[1./20])
```

```
obj = larMap(larBezier(S2,3)(AA(CONS)([C0,C1,C2,C3])))(dom2D)
VIEW(STRUCT(MKPOLS(obj)))
◇
```

Coons patch interpolating 4 boundary curves

"test/py/splines/test03.py" 4a ≡

```
""" Example of transfinite Coons surface """
import sys
""" import modules from larcc/lib """
sys.path.insert(0, 'lib/py/')
from splines import *
Su0 = larBezier(S1,3)([[0,0,0],[10,0,0]])
Su1 = larBezier(S1,3)([[0,10,0],[2.5,10,3],[5,10,-3],[7.5,10,3],[10,10,0]])
Sv0 = larBezier(S2,3)([[0,0,0],[0,0,3],[0,10,3],[0,10,0]])
Sv1 = larBezier(S2,3)([[10,0,0],[10,5,3],[10,10,0]])
dom = larDomain([20],'simplex')
dom2D = larExtrude1(dom,20*[1./20])
out = larMap(larCoonsPatch(AA(CONS)([Su0,Su1,Sv0,Sv1])))(dom2D)
VIEW(STRUCT(MKPOLS(out)))
◇
```

A Utility functions

Initial import of modules

⟨ Initial import of modules 4b ⟩ ≡

```
from pyplasm import *
from scipy import *
import os,sys
""" import modules from larcc/lib """
sys.path.insert(0, 'lib/py/')
from lar2psm import *
from simplexn import *
from larcc import *
from largrid import *
from mapper import *
◇
```

Macro referenced in 3a.

References

[CL13] CVD-Lab, *Linear algebraic representation*, Tech. Report 13-00, Roma Tre University, October 2013.

[Pao03] A. Paoluzzi, *Geometric programming for computer aided design*, John Wiley & Sons, Chichester, UK, 2003.