General Physics (1) For Engineering

Textbook:

Physics for Scientists and Engineers, seventh edition, Jewett / Serway

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Chapter 2

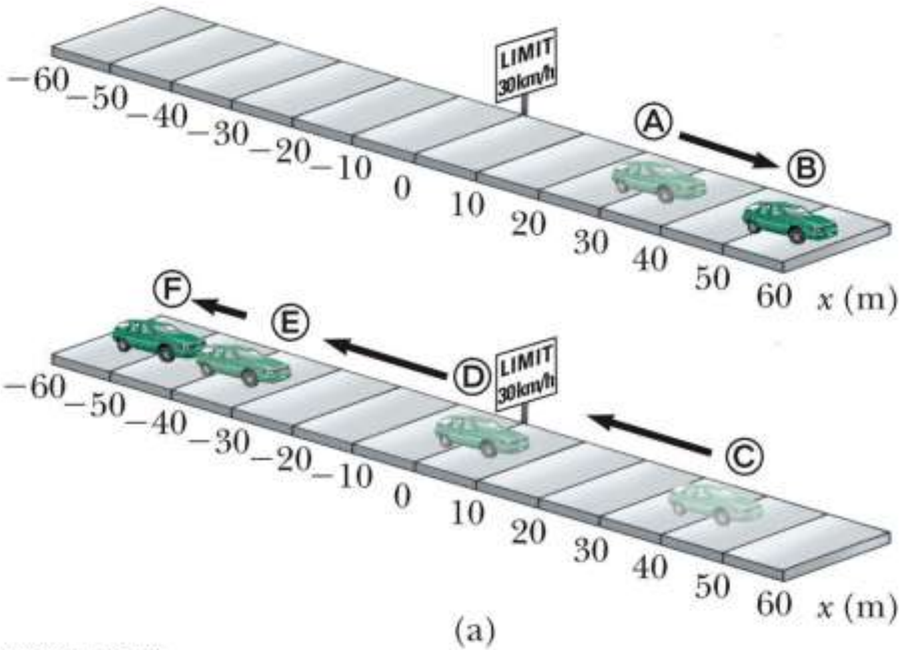
Motion in One Dimension

Kinematics

- Describes motion while ignoring the agents that caused the motion
- For now, will consider motion in one dimension
  - Along a straight line
- Will use the particle model
  - A particle is a point-like object, has mass but infinitesimal size

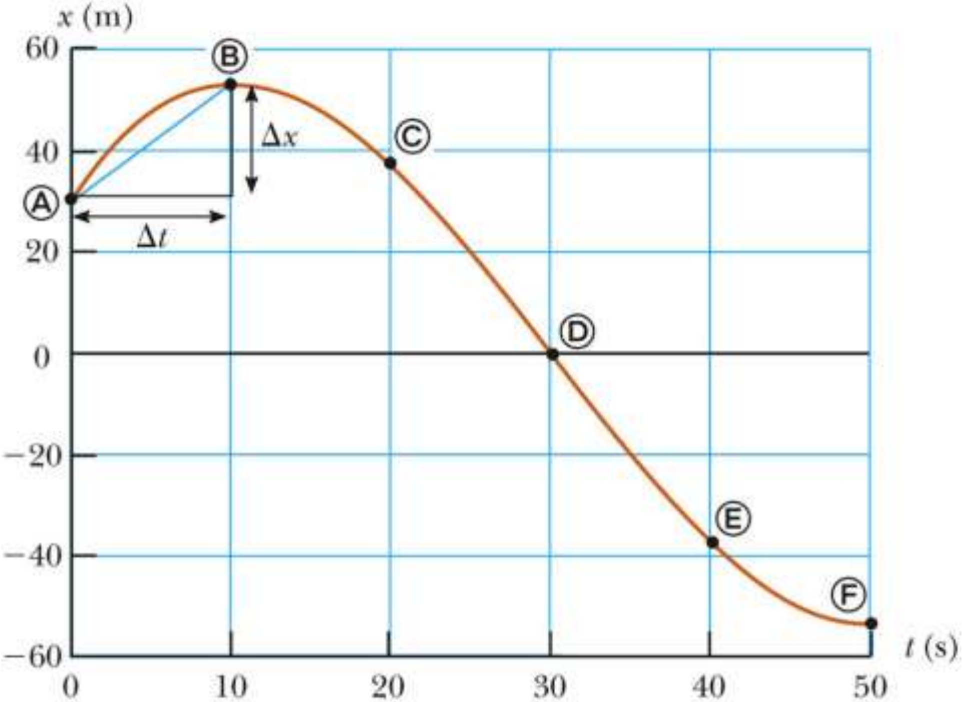
Position

- The object's **position** is its location with respect to a chosen reference point
  - Consider the point to be the origin of a coordinate system
- In the diagram, allow the road sign to be the reference point



Position-Time Graph

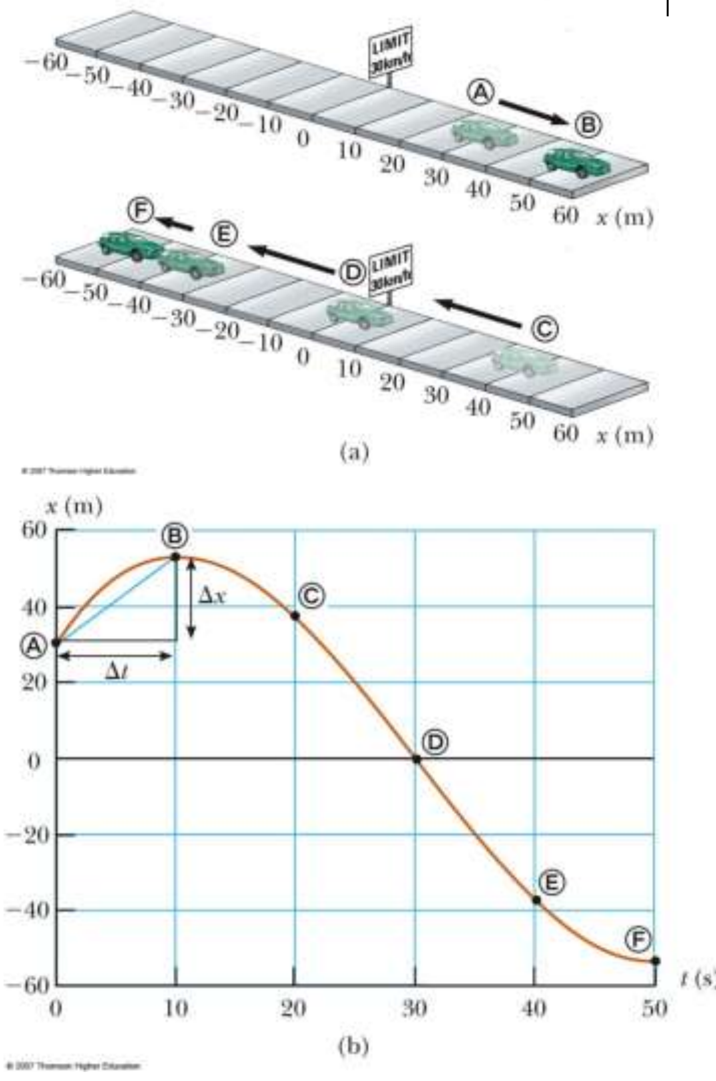
- The position-time graph shows the motion of the particle (car)
- The smooth curve is a guess as to what happened between the data points



(b)

Motion of Car

- Note the relationship between the position of the car and the points on the graph
- Compare the different representations of the motion



Data Table

- The table gives the actual data collected during the motion of the object (car)
- Positive is defined as being to the right

TABLE 2.1

Position of the Car at Various Times

Position	t (s)	x (m)
Ⓐ	0	30
Ⓑ	10	52
Ⓒ	20	38
Ⓓ	30	0
Ⓔ	40	-37
Ⓕ	50	-53

Displacement

- Defined as the change in position during some time interval
  - Represented as Δx

    $\Delta x \equiv x_f - x_i$
  - SI units are meters (m)
  - Δx can be positive or negative
- Different than distance – the length of a path followed by a particle, and the distance is always positive.

Distance vs. Displacement – An Example

- Assume a player moves from one end of the court to the other and back
- Distance is twice the length of the court
  - Distance is always positive
- Displacement is zero
  - $\Delta x = x_f - x_i = 0$ since
    $x_f = x_i$

Distance vs. Displacement – An Example

TABLE 2.2

Kinematic Equations for Motion of a Particle Under Constant Acceleration

Equation Number	Equation	Information Given by Equation
2.13	$v_{xf} = v_{xi} + a_x t$	Velocity as a function of time
2.15	$x_f = x_i + \frac{1}{2}(v_{xi} + v_{xf})t$	Position as a function of velocity and time
2.16	$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$	Position as a function of time
2.17	$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$	Velocity as a function of position

Note: Motion is along the x axis.

Vectors and Scalars

- Vector quantities need both magnitude (size or numerical value) and direction to completely describe them
  - Will use + and – signs to indicate vector directions
- Scalar quantities are completely described by magnitude only

Average Velocity

- **The average velocity is is rate at which the displacement occurs :**

$$v_{x,avg} \equiv \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{\Delta t}$$

  - **The x indicates motion along the x-axis**
- **The dimensions are length / time [L/T]**
- **The SI units are m/s**
- **It is a vector quantity**
- **Is also the slope of the line in the position – time graph**

Average Speed

- Speed is a scalar quantity
  - same units as velocity
  - total distance / total time: $v_{avg} \equiv \frac{d}{t}$
- The speed has no direction and is always expressed as a positive number
- Neither average velocity nor average speed gives details about the trip described

Example 2.1:

- **Find the displacement, average velocity, and average speed of the car in Figure 2.1a between position (A) and (F) .**

Example 2.1:Solution:

TABLE 2.1

Position of the Car at Various Times

Position	t (s)	x (m)
Ⓐ	0	30
Ⓑ	10	52
Ⓒ	20	38
Ⓓ	30	0
Ⓔ	40	−37
Ⓕ	50	−53

$$\text{Displacement}: \quad \Delta x = x_F - x_A = -53 - 30 = -83\text{m}$$

$$\text{Average velocity}: \quad v_{x,avg} = \frac{x_F - x_A}{t_F - t_A} = \frac{-53 - 30}{50 - 0} = -1.7\text{m/s}$$

$$\text{Average Speed}: \quad v_{avg} = \frac{127}{50} = 2.5\text{ m/s}$$

Instantaneous Velocity

- The limit of the average velocity as the time interval becomes infinitesimally short, or as the time interval approaches zero
- The instantaneous velocity indicates what is happening at every point of time
- Instantaneous velocity is a vector quantity.

Instantaneous Velocity, equations

- The general equation for instantaneous velocity is

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

- The instantaneous velocity can be positive, negative, or zero

Instantaneous Speed

- The instantaneous speed is the magnitude of the instantaneous velocity
- The instantaneous speed has no direction associated with it

Vocabulary Note

- “Velocity” and “speed” will indicate *instantaneous* values
- *Average* will be used when the average velocity or average speed is indicated

Example 2.3:

A particle moves along the x-axis. Its position varies with time according to the expression $x=-4t+2t^2$. where x is in meters and t is in seconds. The position-time graph for this motion is shown in figure 2.4. Notice that the particle moves in the negative x direction for the first second of motion, is momentarily at rest at the moment t=1 s, and moves in the positive x direction at times t>1 s.

(A)- Determine the displacement of the particle in the time intervals t=0 s to t=1 s and t=1 s to t=3 s.

(B)- Calculate the average velocity during these two time intervals.

(C)- Find the instantaneous velocity of the particle at t=2.5 s.

Example 2.3: Solution:

$$\Delta x_{A\to B} = x_B - x_A = [-4(1) + 2(1)^2] - [-4(0) + 2(0)^2] = -2\text{ m}$$

$$\Delta x_{B\to D} = x_D - x_B = [-4(3) + 2(3)^2] - [-4(1) + 2(1)^2] = +8\text{ m}$$

$$v_{x,avg(A\to B)} = \frac{\Delta x_{A\to B}}{\Delta t} = \frac{-2}{1} = -2\text{ m/s}$$

$$v_{x,avg(B\to D)} = \frac{\Delta x_{B\to D}}{\Delta t} = \frac{8}{2} = 4\text{ m/s}$$

$$v_x = \frac{dx}{dt} = -4 + 4t = -4 + 4(2.5) = +6\text{ m/s}$$

Particle Under Constant Velocity

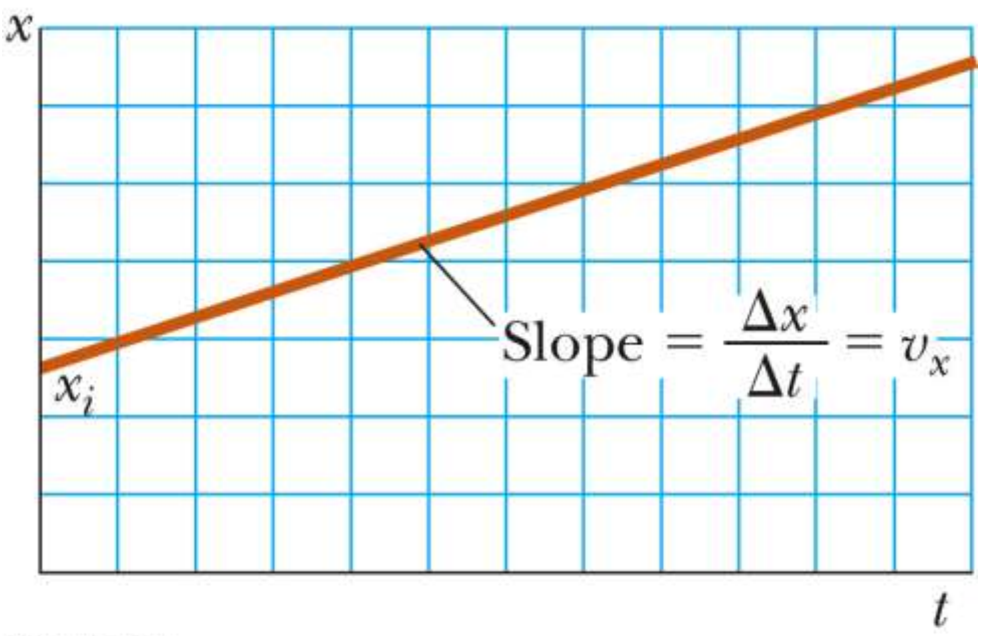
- **Constant velocity indicates the instantaneous velocity at any instant during a time interval is the same as the average velocity during that time interval**
  - $v_x = v_{x,\,avg}$
  - **The mathematical representation of this situation is the equation**

$$v_x = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{\Delta t} \quad or \quad x_f = x_i + v_x \Delta t$$

  - **Common practice is to let $t_i = 0$ and the equation becomes: $x_f = x_i + v_x\, t$ (for constant v_x)**

Particle Under Constant Velocity, Graph

- The graph represents the motion of a particle under constant velocity
- The slope of the graph is the value of the constant velocity
- The y-intercept is x_i



Example 2.4:

A scientist is studying the biomechanics of the human body. She determines the velocity of an experimental subject while he runs along a straight line at a constant rate. The scientist starts the stopwatch at the moment the runner passes a given point and stops it after the runner has passed another point 20 m away. The time interval indicated on the stopwatch is 4 s.

(A)- what is the runner's velocity?

(B)- If the runner continues his motion after the stopwatch is stopped, what is his position after 10 s has passed?

Example 2.4: Solution:

(A)- what is the runner's velocity?

$$v_x = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{\Delta t} = \frac{20-0}{4} = 5\, m/s$$

(B)- If the runner continues his motion after the stopwatch is stopped, what is his position after 10 s has passed?

$$x_f = x_i + v_x t = 0 + 5 \times 10 = 50\, m$$

Average Acceleration

- Acceleration is the rate of change of the velocity

$$a_{x,avg} \equiv \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{t_f - t_i}$$

- Dimensions are L/T^2
- SI units are m/s^2
- In one dimension, positive and negative can be used to indicate direction

Instantaneous Acceleration

- **The instantaneous acceleration is the limit of the average acceleration as Δt approaches 0**

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} = \frac{d^2x}{dt^2}$$

- **The term acceleration will mean instantaneous acceleration**
  - **If average acceleration is wanted, the word average will be included**

Example 2.6

The velocity of a particle moving along the x axis varies according to the expression $v_x=(40-5t^2)$ m/s, where t is in seconds.

(A)- Find the average acceleration in the time interval t=0 to t=2 s.

(B)- Determine the acceleration at t=2 s.

Example 2.6: Solution:

(A)- Find the average acceleration in the time interval t=0 to t=2 s.

$$v_{x(A)} = (40 - 5t^2) = (40 - 5(0)^2) = 40\ m/s$$

$$v_{x(B)} = (40 - 5t^2) = (40 - 5(2)^2) = 20\ m/s$$

$$a_{x,\,avg} = \frac{v_{x(B)} - v_{x(A)}}{t_B - t_A} = \frac{20 - 40}{2 - 0} = -10\ m/s^2$$

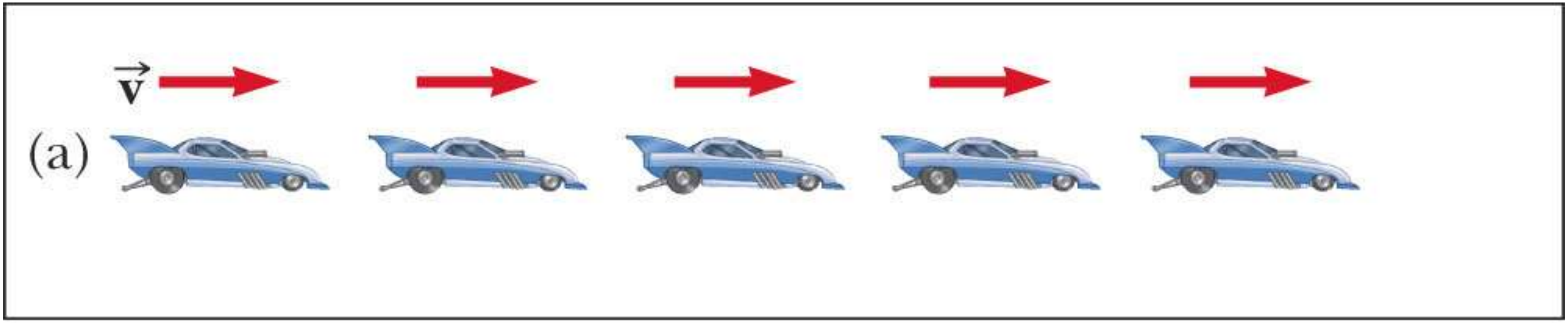
(B)- Determine the acceleration at t=2 s.

$$a_x = \frac{dv_x}{dt} = -5 \times 2t = -5 \times 2 \times 2 = -20\ m/s^2$$

Acceleration and Velocity, 1

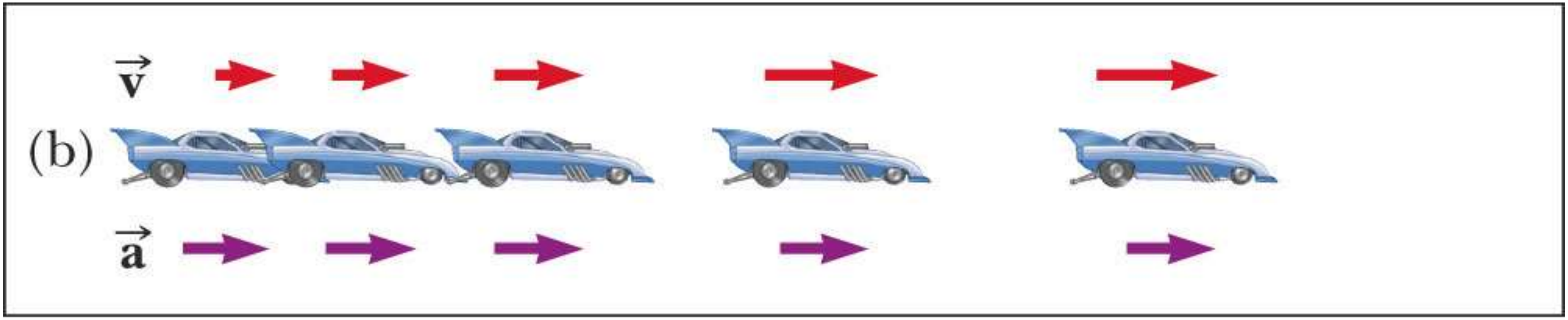
- When an object's velocity and acceleration are in the same direction, the object is speeding up
- When an object's velocity and acceleration are in the opposite direction, the object is slowing down

Acceleration and Velocity, 2



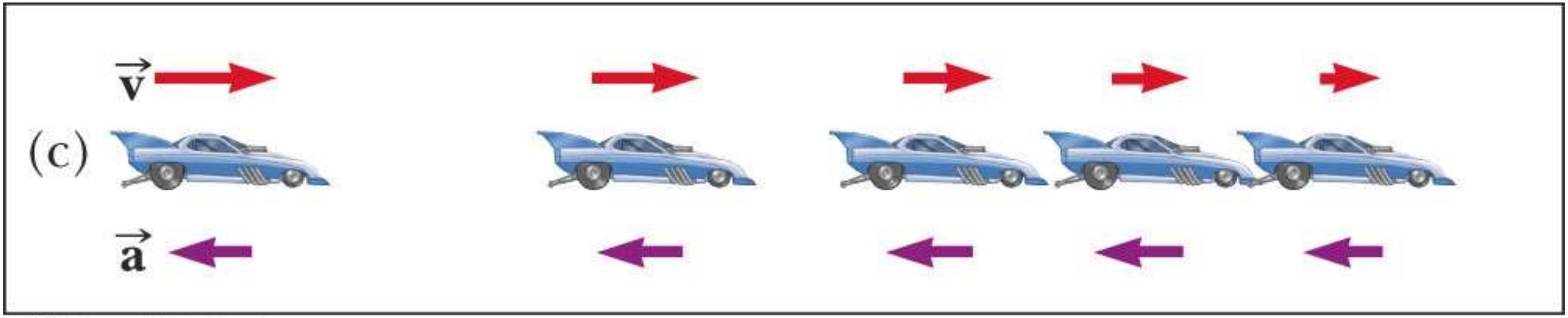
- **Images are equally spaced. The car is moving with constant positive velocity (shown by red arrows maintaining the same size)**
- **Acceleration equals zero**

Acceleration and Velocity, 3



- Images become farther apart as time increases
- Velocity and acceleration are in the same direction
- Acceleration is uniform (violet arrows maintain the same length)
- Velocity is increasing (red arrows are getting longer)
- This shows positive acceleration and positive velocity

Acceleration and Velocity, 4



- Images become closer together as time increases
- Acceleration and velocity are in opposite directions
- Acceleration is uniform (violet arrows maintain the same length)
- Velocity is decreasing (red arrows are getting shorter)
- Positive velocity and negative acceleration

Kinematic Equations – summary

TABLE 2.2

Kinematic Equations for Motion of a Particle Under Constant Acceleration

Equation Number	Equation	Information Given by Equation
2.13	$v_{xf} = v_{xi} + a_x t$	Velocity as a function of time
2.15	$x_f = x_i + \frac{1}{2}(v_{xi} + v_{xf})t$	Position as a function of velocity and time
2.16	$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$	Position as a function of time
2.17	$v_{xf}^{\ 2} = v_{xi}^{\ 2} + 2a_x (x_f - x_i)$	Velocity as a function of position

Note: Motion is along the x axis.

Kinematic Equations, specific

- For constant *a*, $v_{xf} = v_{xi} + a_x t$
- Can determine an object's velocity at any time *t* when we know its initial velocity and its acceleration
  - Assumes $t_i = 0$ and $t_f = t$
- Does not give any information about displacement

Kinematic Equations, specific

- For constant acceleration,

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

- Gives final position in terms of velocity and acceleration
- Doesn’t tell you about final velocity

Kinematic Equations, specific

- For constant ***a***,

$$v_{xf}^2 = v_{xi}^2 + 2a_x\left(x_f - x_i\right)$$

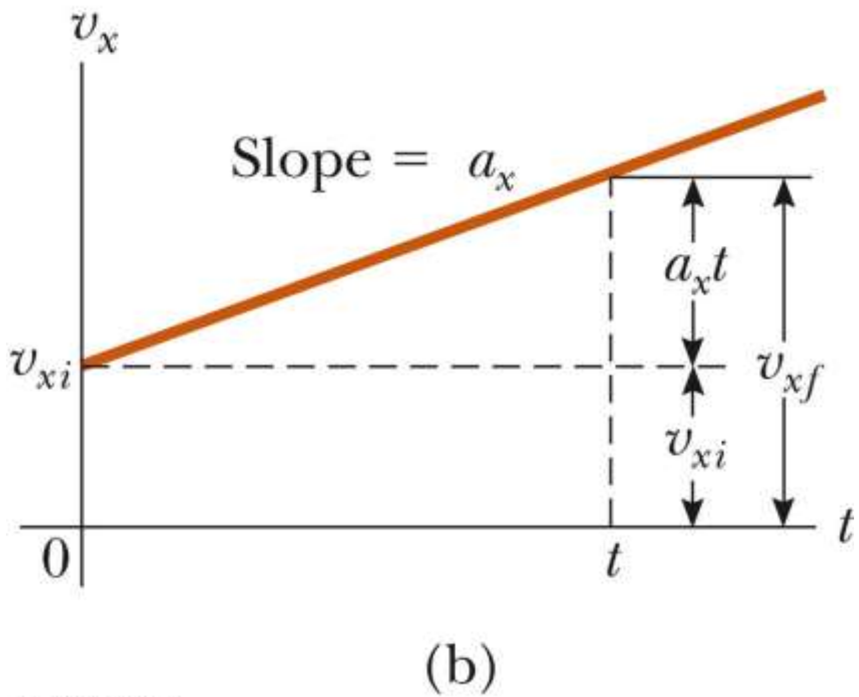
- Gives final velocity in terms of acceleration and displacement
- Does not give any information about the time

When a = 0

- **When the acceleration is zero,**
  - $v_{xf} = v_{xi} = v_x$
  - $x_f = x_i + v_x t$
- **The constant acceleration model reduces to the constant velocity model**

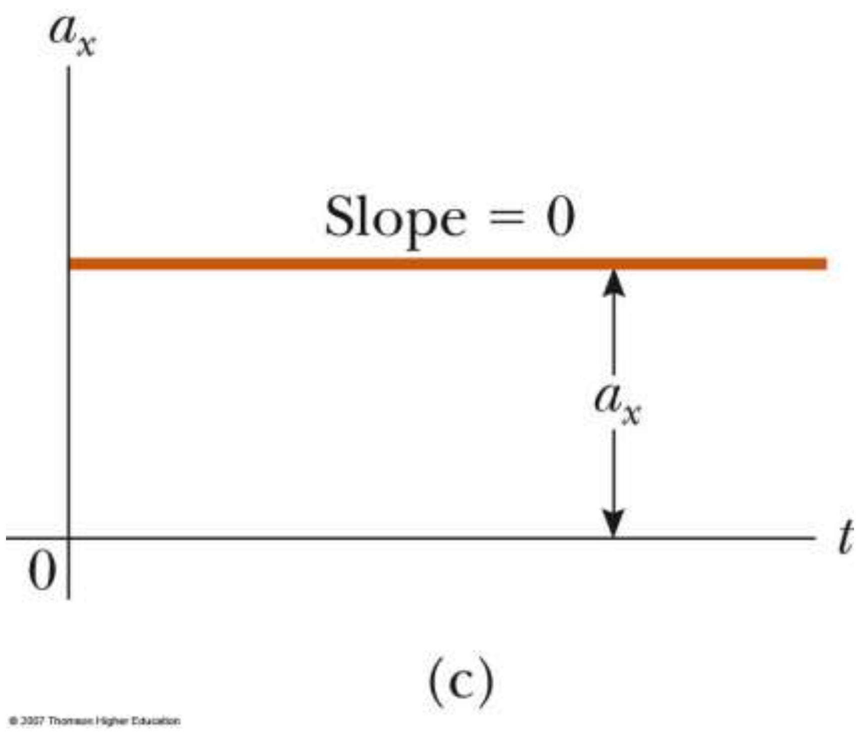
Graphical Look at Motion: velocity – time curve

- **The slope gives the acceleration**
- **The straight line indicates a constant acceleration**



Graphical Look at Motion: acceleration – time curve

- The zero slope indicates a constant acceleration



Example 2.7:

A jet lands on an aircraft carrier at 140 mi/h = 63m/s.

(A)- what is its acceleration (assumed constant) if it stops in 2 s due to an arresting cable that snags the jet and brings it to a stop?

(B)- If the jet touches down at position x_i=0, what is its final position?

Example 2.7: Solution:

(A)- what is its acceleration (assumed constant) if it stops in 2 s due to an arresting cable that snags the jet and brings it to a stop?

$$a_x = \frac{v_{xf} - v_{xi}}{t} \approx \frac{0-63}{2} = -31.5\,\mathrm{m/s^2}$$

(B)- If the jet touches down at position x_i=0, what is its final position?

$$x_f = x_i + \frac{1}{2}(v_{xi} + v_{xf})t = 0 + \frac{1}{2}(63+0)2 = 63\,\mathrm{m}$$

$$x_f = x_i + v_i t + \frac{1}{2}a_x t^2 = 0 + 63 \times 2 + \frac{1}{2}(-31.5)2^2 = 63\,\mathrm{m}$$

Example 2.8:

A car travelling at a constant speed of 45 m/s passes a trooper on a motorcycle hidden behind a billboard. One second after the speeding car passes the billboard, the trooper sets out from the billboard to catch the car, accelerating at a constant rate of 3 m/s^2.

How long does it take her to overtake the car?

Example 2.8: Solution:

How long does it take her to overtake the car?

See figure 2.13

$$x_{car} = x_B + v_{x\,car} t = 45 + 45t$$

$$x_f = x_i + v_{xi} t + \frac{1}{2} a_x t^2 = 0 + (0)t + \frac{1}{2} \times 3t^2$$

$$\text{to overtake it must be:} \qquad x_{trooper} = x_{car}$$

$$\frac{1}{2} \times 3t^2 = 45 + 45t \quad \Rightarrow \quad t = 31\,s$$

Example

- 3- A person walks first at a constant speed of 5.00 m/s along a straight line from point A to point B and then back along the line from B to A at a constant speed of 3.00 m/s.
- (a) What is her average speed over the entire trip?
- (b) What is her average velocity over the entire trip?

Solution

(a) Let d represent the distance between A and B. Let t_1 be the time for which the walker has the higher speed in $5.00\ \text{m/s} = \frac{d}{t_1}$. Let t_2 represent the longer time for the return trip in $-3.00\ \text{m/s} = -\frac{d}{t_2}$. Then the times are $t_1 = \frac{d}{(5.00\ \text{m/s})}$ and $t_2 = \frac{d}{(3.00\ \text{m/s})}$. The average speed is:

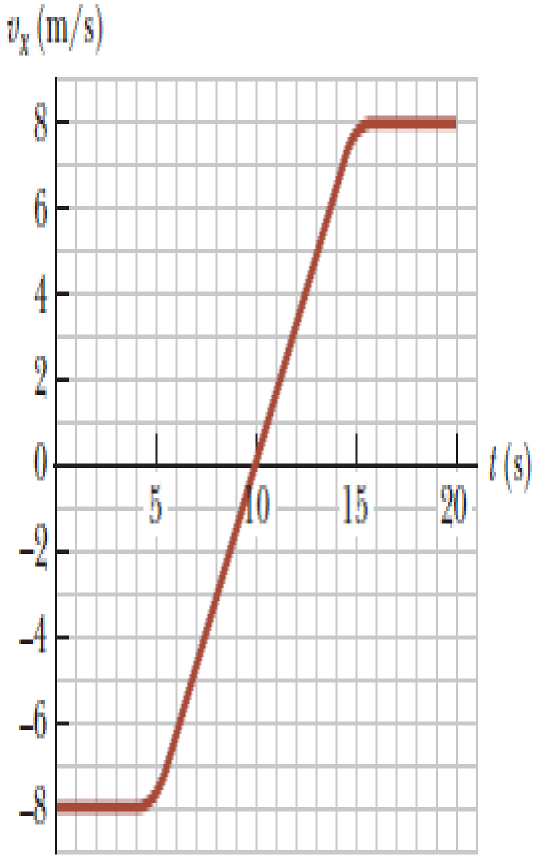
$$v_{avg} = \frac{\text{Total distance}}{\text{Total time}} = \frac{d+d}{d/(5.00\ \text{m/s}) + d/(3.00\ \text{m/s})} = \frac{2d}{(8.00\ \text{m/s})d/(15.0\ \text{m}^2/\text{s}^2)}$$

$$v_{avg} = \frac{2(15.0\ \text{m}^2/\text{s}^2)}{8.00\ \text{m/s}} = \boxed{3.75\ \text{m/s}}$$

(b) She starts and finishes at the same point A. With total displacement = 0, average velocity $= \boxed{0}$.

Example

- 13- A velocity–time graph for an object moving along the *x axis* is shown in NEXT Figure
- a) Plot a graph of the acceleration versus time.
- Determine the average acceleration of the Object
- (b) in the time interval *t = 5.00 s to t = 15.0 s and*
- (c) in the time interval *t = 0 to t = 20.0 s.*



(a) Acceleration is the slope of the graph of v versus t.

For $0 < t < 5.00$ s, $a = 0$.

For 15.0 s $< t < 20.0$ s, $a = 0$.

For 5.0 s $< t < 15.0$ s, $a = \frac{v_f - v_i}{t_f - t_i}$.

$$a = \frac{8.00 - (-8.00)}{15.0 - 5.00} = 1.60\ \text{m/s}^2$$

We can plot $a(t)$ as shown.

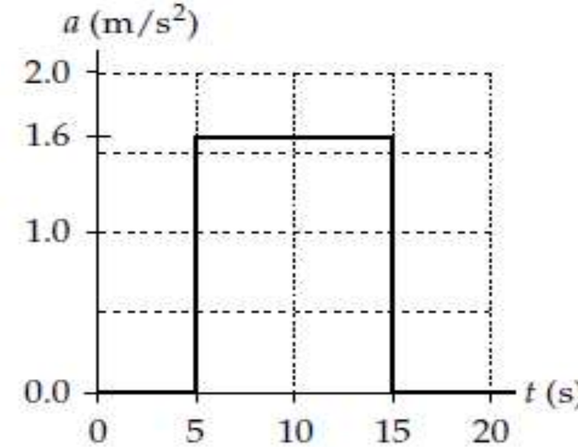


FIG. P2.12

(b) $a = \frac{v_f - v_i}{t_f - t_i}$

(i) For 5.00 s $< t < 15.0$ s, $t_i = 5.00$ s, $v_i = -8.00$ m/s,

$$t_f = 15.0\ \text{s}$$
$$v_f = 8.00\ \text{m/s}$$
$$a = \frac{v_f - v_i}{t_f - t_i} = \frac{8.00 - (-8.00)}{15.0 - 5.00} = \boxed{1.60\ \text{m/s}^2}.$$

(ii) $t_i = 0$, $v_i = -8.00$ m/s, $t_f = 20.0$ s, $v_f = 8.00$ m/s

$$a = \frac{v_f - v_i}{t_f - t_i} = \frac{8.00 - (-8.00)}{20.0 - 0} = \boxed{0.800\ \text{m/s}^2}$$

Example

- 18- An object moves along the *x axis according to the equation* $x = 3.00t^2 - 2.00t + 3.00$,
- *where x is in meters and t* is in seconds. Determine
- (a) the average speed between *t = 2.00 s and t = 3.00 s, (b) the instantaneous speed at t = 2.00 s and at t = 3.00 s,*
- *(c) the average acceleration* between *t = 2.00 s and t = 3.00 s, and*
- *(d) the instantaneous* acceleration at *t = 2.00 s and t = 3.00 s.*

(a) At $t = 2.00$ s, $x = \left[3.00(2.00)^2 - 2.00(2.00) + 3.00\right]$ m $= 11.0$ m.

At $t = 3.00$ s, $x = \left[3.00(9.00)^2 - 2.00(3.00) + 3.00\right]$ m $= 24.0$ m

so

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{24.0 \text{ m} - 11.0 \text{ m}}{3.00 \text{ s} - 2.00 \text{ s}} = \boxed{13.0 \text{ m/s}}.$$

(b) At all times the instantaneous velocity is

$$v = \frac{d}{dt}\left(3.00t^2 - 2.00t + 3.00\right) = (6.00t - 2.00) \text{ m/s}$$

At $t = 2.00$ s, $v = \left[6.00(2.00) - 2.00\right]$ m/s $= \boxed{10.0 \text{ m/s}}$.

At $t = 3.00$ s, $v = \left[6.00(3.00) - 2.00\right]$ m/s $= \boxed{16.0 \text{ m/s}}$.

(c) $a_{avg} = \dfrac{\Delta v}{\Delta t} = \dfrac{16.0 \text{ m/s} - 10.0 \text{ m/s}}{3.00 \text{ s} - 2.00 \text{ s}} = \boxed{6.00 \text{ m/s}^2}$

(d) At all times $a = \dfrac{d}{dt}(6.00t - 2.00) = \boxed{6.00 \text{ m/s}^2}$. This includes both $t = 2.00$ s and $t = 3.00$ s.

Example

- 32- A particle moves along the *x axis. Its position is given by* the equation
- $$x = 2 + 3t - 4t^{2},$$
- *with x in meters and t in seconds.* Determine (a) its position when it changes direction and
- (b) its velocity when it returns to the position it had at $t = 0$.

(a) Compare the position equation $x = 2.00 + 3.00t - 4.00t^2$ to the general form

$$x_f = x_i + v_i t + \frac{1}{2}at^2$$

to recognize that $x_i = 2.00$ m, $v_i = 3.00$ m/s, and $a = -8.00\ \text{m/s}^2$. The velocity equation, $v_f = v_i + at$, is then

$$v_f = 3.00\ \text{m/s} - \left(8.00\ \text{m/s}^2\right)t.$$

The particle changes direction when $v_f = 0$, which occurs at $t = \frac{3}{8}$ s. The position at this time is

$$x = 2.00\ \text{m} + (3.00\ \text{m/s})\left(\frac{3}{8}\ \text{s}\right) - \left(4.00\ \text{m/s}^2\right)\left(\frac{3}{8}\ \text{s}\right)^2 = \boxed{2.56\ \text{m}}.$$

(b) From $x_f = x_i + v_i t + \frac{1}{2}at^2$, observe that when $x_f = x_i$, the time is given by $t = -\frac{2v_i}{a}$. Thus, when the particle returns to its initial position, the time is

$$t = \frac{-2(3.00\ \text{m/s})}{-8.00\ \text{m/s}^2} = \frac{3}{4}\ \text{s}$$

and the velocity is $v_f = 3.00\ \text{m/s} - \left(8.00\ \text{m/s}^2\right)\left(\frac{3}{4}\ \text{s}\right) = \boxed{-3.00\ \text{m/s}}$.

Example

- Speedy Sue, driving at 30.0 m/s, enters a one-lane tunnel. She then observes a slow-moving van 155 m ahead traveling at 5.00 m/s. Sue applies her brakes but can accelerate only at -2.00 m/s^2 because the road is wet. Will there be a collision? If yes,
- determine how far into the tunnel and at what time the collision occurs. If no,
- determine the distance of closest approach between Sue's car and the van.

Take the original point to be when Sue notices the van. Choose the origin of the x-axis at Sue's car. For her we have $x_{is} = 0$, $v_{is} = 30.0\ \text{m/s}$, $a_s = -2.00\ \text{m/s}^2$ so her position is given by

$$x_s(t) = x_{is} + v_{is}t + \frac{1}{2}a_s t^2 = (30.0\ \text{m/s})t + \frac{1}{2}(-2.00\ \text{m/s}^2)t^2.$$

For the van, $x_{iv} = 155\ \text{m}$, $v_{iv} = 5.00\ \text{m/s}$, $a_v = 0$ and

$$x_v(t) = x_{iv} + v_{iv}t + \frac{1}{2}a_v t^2 = 155 + (5.00\ \text{m/s})t + 0.$$

To test for a collision, we look for an instant t_c when both are at the same place:

$$30.0t_c - t_c^2 = 155 + 5.00t_c$$
$$0 = t_c^2 - 25.0t_c + 155.$$

From the quadratic formula

$$t_c = \frac{25.0 \pm \sqrt{(25.0)^2 - 4(155)}}{2} = 13.6\ \text{s or } \boxed{11.4\ \text{s}}.$$

The roots are real, not imaginary, so $\boxed{\text{there is a collision}}$. The smaller value is the collision time. (The larger value tells when the van would pull ahead again if the vehicles could move through each other). The wreck happens at position

$$155\ \text{m} + (5.00\ \text{m/s})(11.4\ \text{s}) = \boxed{212\ \text{m}}.$$