General Physics (1) For Engineering

Textbook:

Physics for Scientists and Engineers, seventh edition, Jewett / Serway

Dr. Awos Alsalman

Chapter 4

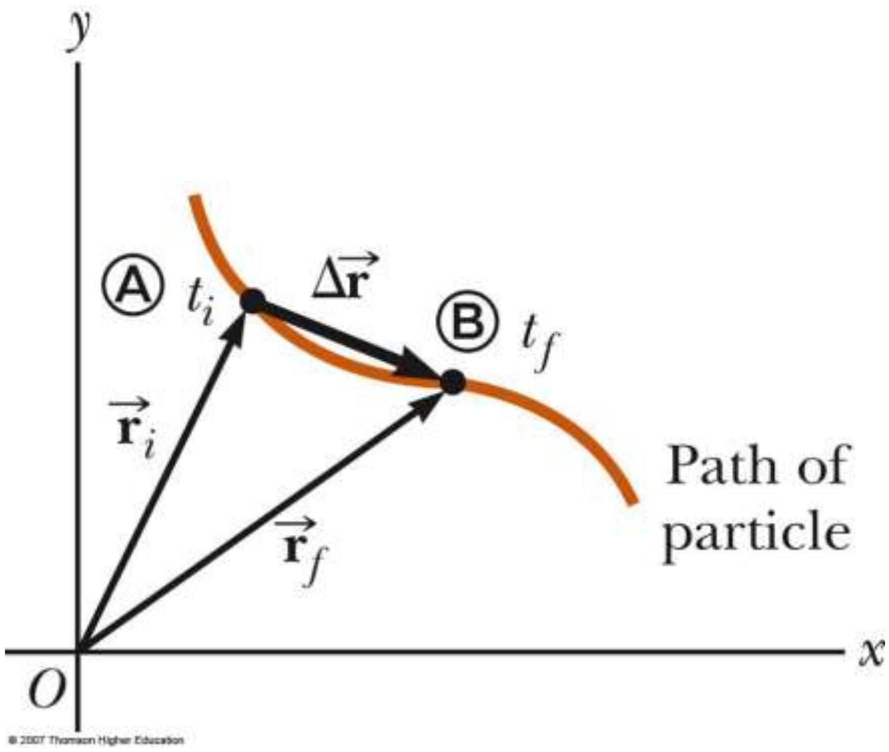
Motion in Two Dimensions

Motion in Two Dimensions

- Using + or – signs is not always sufficient to fully describe motion in more than one dimension
  - Vectors can be used to more fully describe motion
  - Will look at vector nature of quantities in more detail
- Still interested in displacement, velocity, and acceleration
- Will serve as the basis of multiple types of motion in future chapters

Position and Displacement

- The position of an object is described by its position vector, $\vec{\mathbf{r}}$
- The **displacement** of the object is defined as the ***change in its position***
- $\Delta\vec{\mathbf{r}} \equiv \vec{\mathbf{r}}_f - \vec{\mathbf{r}}_i$



General Motion Ideas

- In two- or three-dimensional kinematics, everything is the same as as in one-dimensional motion except that we must now use full vector notation
  - Positive and negative signs are no longer sufficient to determine the direction

Average Velocity

- The average velocity is the ratio of the displacement to the time interval for the displacement

$$\vec{\mathbf{v}}_{avg} \equiv \frac{\Delta \vec{\mathbf{r}}}{\Delta t}$$

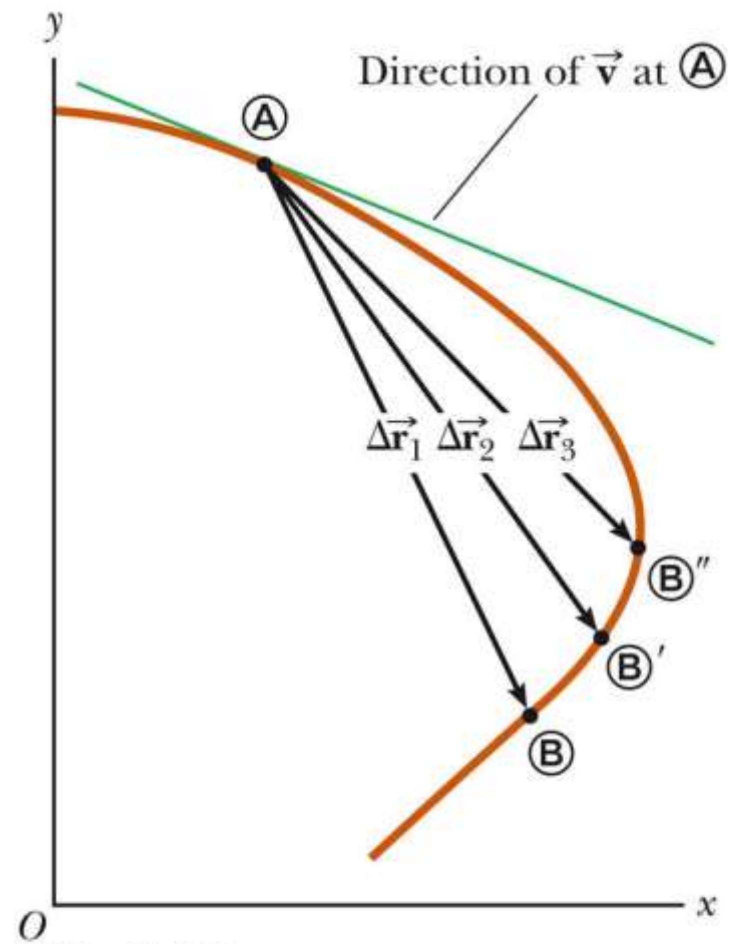
- The direction of the average velocity is the direction of the displacement vector
- The average velocity between points is *independent of the path* taken
  - This is because it is dependent on the displacement, also independent of the path

Instantaneous Velocity

- The instantaneous velocity is the limit of the average velocity as Δt approaches zero

$$\vec{\mathbf{v}} \equiv \lim_{\Delta t \to 0} \frac{\Delta \vec{\mathbf{r}}}{\Delta t} = \frac{d\vec{\mathbf{r}}}{dt}$$

  - As the time interval becomes smaller, the direction of the displacement approaches that of the line tangent to the curve



Instantaneous Velocity, cont

- The direction of the instantaneous velocity vector at any point in a particle's path is along a line tangent to the path at that point and in the direction of motion
- The magnitude of the instantaneous velocity vector is the speed
  - The speed is a scalar quantity

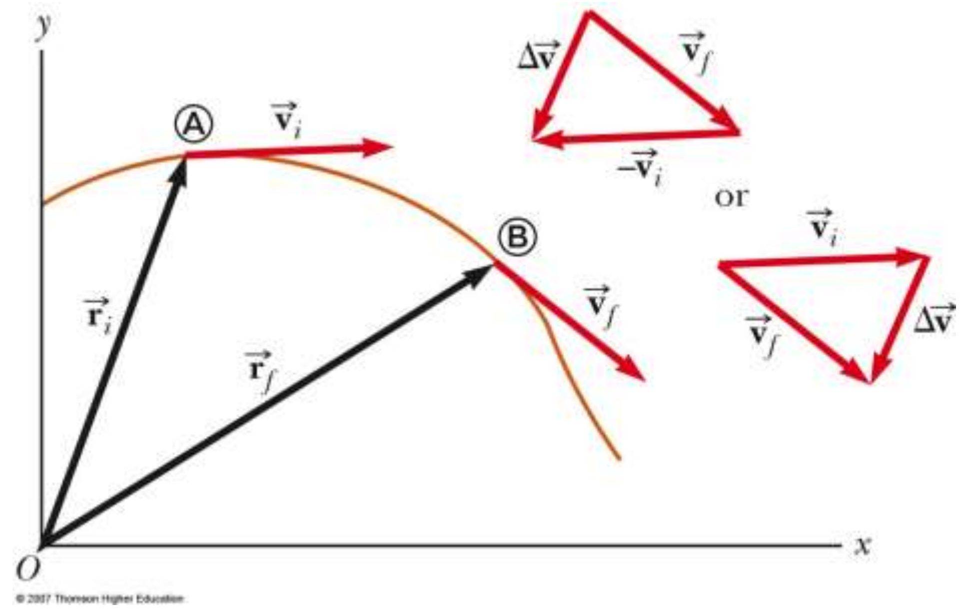
Average Acceleration

- The average acceleration of a particle as it moves is defined as the change in the instantaneous velocity vector divided by the time interval during which that change occurs.

$$\vec{\mathbf{a}}_{avg} \equiv \frac{\vec{\mathbf{v}}_f - \vec{\mathbf{v}}_i}{t_f - t_i} = \frac{\Delta \vec{\mathbf{v}}}{\Delta t}$$

Average Acceleration, cont

- As a particle moves, the direction of the change in velocity is found by vector subtraction
  $$\Delta\vec{\mathbf{v}} = \vec{\mathbf{v}}_f - \vec{\mathbf{v}}_i$$
- The average acceleration is a vector quantity directed along
  $$\Delta\vec{\mathbf{v}}$$



Instantaneous Acceleration

- The instantaneous acceleration is the limiting value of the ratio $\Delta \vec{\mathbf{v}}/\Delta t$ as Δt approaches zero

$$\vec{\mathbf{a}} \equiv \lim_{\Delta t \to 0} \frac{\Delta \vec{\mathbf{v}}}{\Delta t} = \frac{d\vec{\mathbf{v}}}{dt}$$

  - The instantaneous equals the derivative of the velocity vector with respect to time

Kinematic Equations for Two-Dimensional Motion

- When the two-dimensional motion has a constant acceleration, a series of equations can be developed that describe the motion
- These equations will be similar to those of one-dimensional kinematics
- Motion in two dimensions can be modeled as two *independent* motions in each of the two perpendicular directions associated with the x and y axes
  - Any influence in the y direction does not affect the motion in the x direction

Kinematic Equations, 2

- Position vector for a particle moving in the xy plane $\vec{\mathbf{r}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$
- The velocity vector can be found from the position vector

$$\vec{\mathbf{v}} = \frac{d\vec{\mathbf{r}}}{dt} = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}}$$

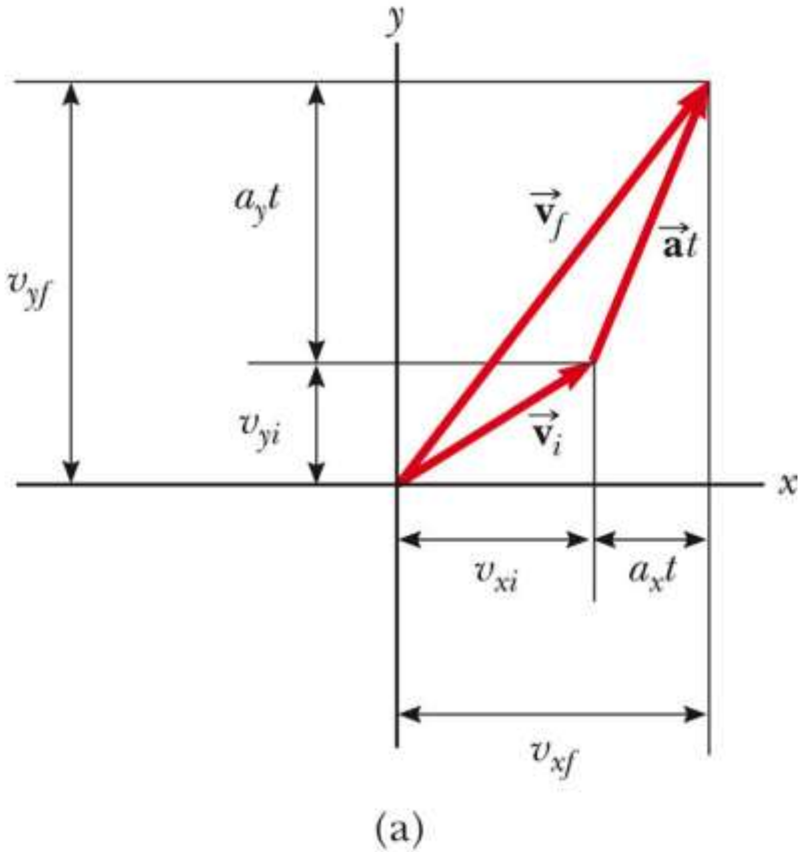
  - Since acceleration is constant, we can also find an expression for the velocity as a function of time: $\vec{\mathbf{v}}_f = \vec{\mathbf{v}}_i + \vec{\mathbf{a}}t$

Kinematic Equations, 3

- The position vector can also be expressed as a function of time:
  - $\vec{\mathbf{r}}_f = \vec{\mathbf{r}}_i + \vec{\mathbf{v}}_i t + \frac{1}{2}\vec{\mathbf{a}}t^2$
  - This indicates that the position vector is the sum of three other vectors:
    - The initial position vector
    - The displacement resulting from the initial velocity
    - The displacement resulting from the acceleration

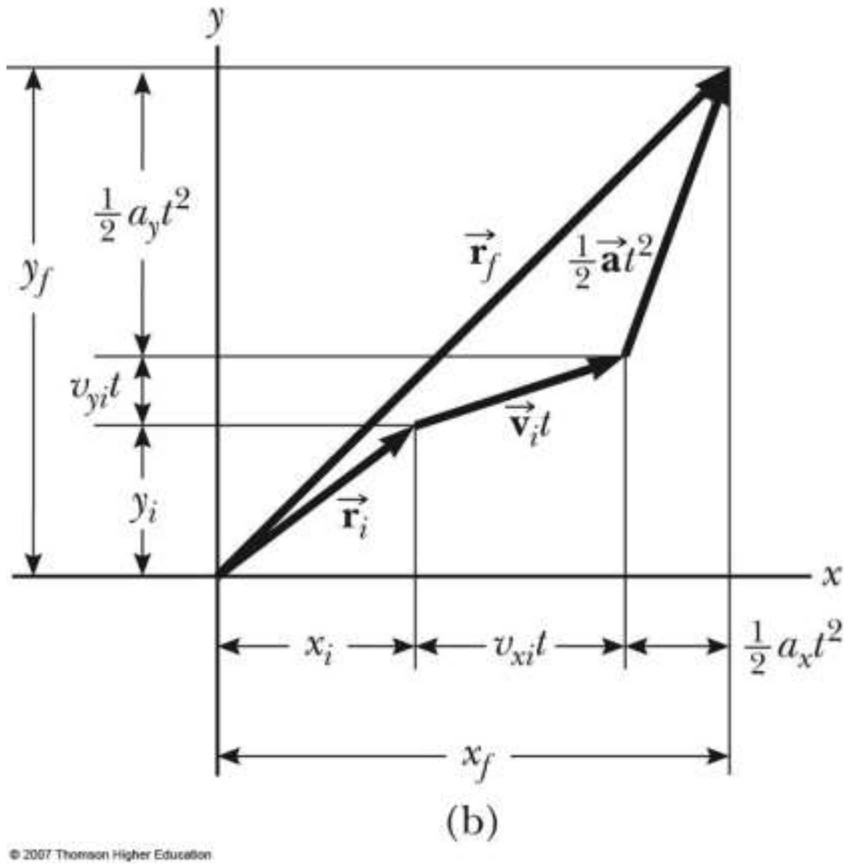
Kinematic Equations, Graphical Representation of Final Velocity

- The velocity vector can be represented by its components
- $\vec{\mathbf{v}}_f$ is generally not along the direction of either $\vec{\mathbf{v}}_i$ or $\vec{\mathbf{a}}$



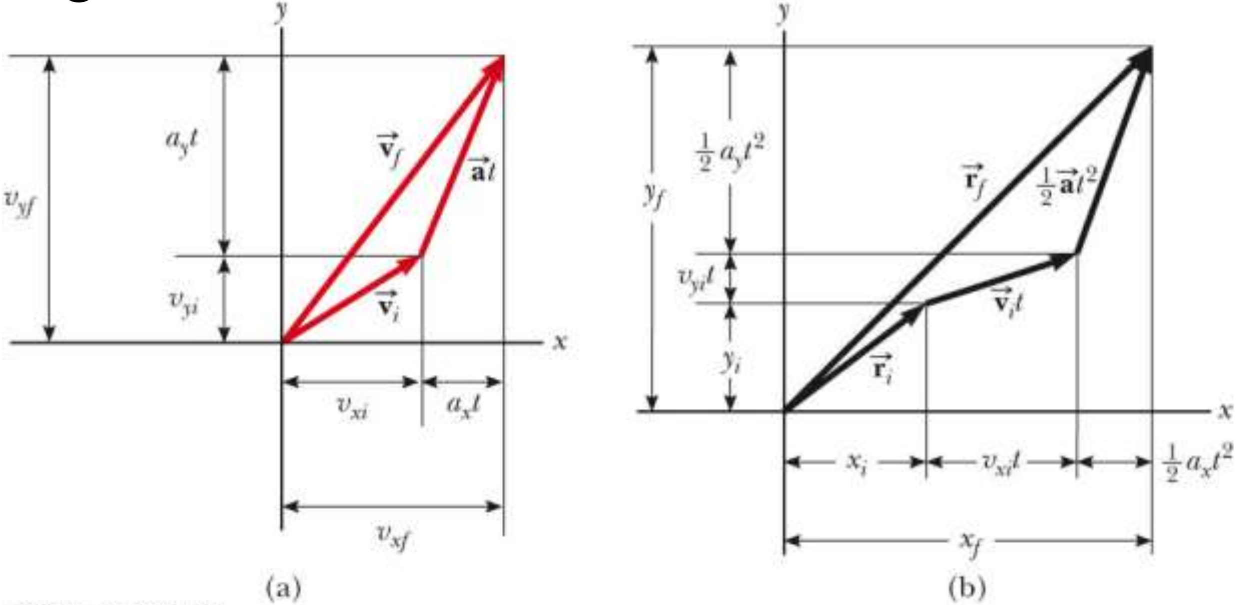
Kinematic Equations, Graphical Representation of Final Position

- The vector representation of the position vector
- $\vec{\mathbf{r}}_f$ is generally not along the same direction as $\vec{\mathbf{v}}_i$ or as $\vec{\mathbf{a}}$
- $\vec{\mathbf{v}}_f$ and $\vec{\mathbf{r}}_f$ are generally not in the same direction



Graphical Representation Summary

- Various starting positions and initial velocities can be chosen
- Note the relationships between changes made in either the position or velocity and the resulting effect on the other



Example 4.1 Motion in a Plane

A particle starts from the origin at t=0 with an initial velocity having an x component of 20 m/s and a y component of -15 m/s. The particle moves in the xy plane with an x component of acceleration only, given by $a_x = 4.0\ m/s^2$.

(A) Determine the total velocity at any time.

(B) Calculate the velocity and speed of the particle at t=5.0 s.

(C) Determine the x and y coordinates of the particle at any time t and its position vector at this time

Solution

(A) Determine the total velocity at any time.

$$\vec{v}_f = \vec{v}_i + \vec{a}t = (v_{xi} + a_x t)\hat{i} + (v_{yi} + a_y t)\hat{j}$$

$$\vec{v}_f = [20 m/s + (4.0 m/s^2)t]\hat{i} + [-15 m/s + (0)t]\hat{j}$$

$$\vec{v}_f = [(20 + 4.0t)\hat{i} - 15\hat{j}] \quad m/s$$

Example 4.1 Motion in a Plane

(B) Calculate the velocity and speed of the particle at t=5.0 s.

$$\vec{v}_f = [(20 + 4.0(5.0))\hat{i} - 15\hat{j}]\ \mathrm{m/s} = (40\hat{i} - 15\hat{j})\ \ \mathrm{m/s}$$

$$\theta = \tan^{-1}\left(\frac{v_{yf}}{v_{xf}}\right) = \tan^{-1}\left(\frac{-15}{40}\right) = -21^\circ$$

$$v_f = \left|\vec{v}_f\right| = \sqrt{v_{xf}^2 + v_{yf}^2} = \sqrt{(40)^2 + (-15)^2} = 43\ \ \mathrm{m/s}$$

Example 4.1 Motion in a Plane

(C) Determine the x and y coordinates of the particle at any time t and its position vector at this time

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2 = (20t + 2.0t^2) \quad \text{m}$$

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2 = (-15t) \quad \text{m}$$

$$\vec{r}_f = x_f\hat{i} + y_f\hat{j} = [(20t + 2.0t^2)\hat{i} - 15t\,\hat{j}] \quad \text{m}$$

Example

A fish swimming in a horizontal plane has velocity $\mathbf{v}_i = (4.00\hat{\mathbf{i}} + 1.00\hat{\mathbf{j}})$ m/s at a point in the ocean where the position relative to a certain rock is $\mathbf{r}_i = (10.0\hat{\mathbf{i}} - 4.00\hat{\mathbf{j}})$ m. After the fish swims with constant acceleration for 20.0 s, its velocity is $\mathbf{v} = (20.0\hat{\mathbf{i}} - 5.00\hat{\mathbf{j}})$ m/s. (a) What are the components of the acceleration? (b) What is the direction of the acceleration with respect to unit vector $\hat{\mathbf{i}}$? (c) If the fish maintains constant acceleration, where is it at $t = 25.0$ s, and in what direction is it moving?

solution

$$\vec{v}_i = \left(4.00\hat{i} + 1.00\hat{j}\right) \text{ m/s} \quad \text{and} \quad \vec{v}(20.0) = \left(20.0\hat{i} - 5.00\hat{j}\right) \text{ m/s}$$

(a) $$a_x = \frac{\Delta v_x}{\Delta t} = \frac{20.0 - 4.00}{20.0} \text{ m/s}^2 = \boxed{0.800 \text{ m/s}^2}$$

$$a_y = \frac{\Delta v_y}{\Delta t} = \frac{-5.00 - 1.00}{20.0} \text{ m/s}^2 = \boxed{-0.300 \text{ m/s}^2}$$

(b) $$\theta = \tan^{-1}\left(\frac{-0.300}{0.800}\right) = -20.6° = \boxed{339° \text{ from } +x \text{ axis}}$$

solution

(c) At $t = 25.0$ s its position is specified by its coordinates and the direction of its motion is specified by the direction angle of its velocity:

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2 = 10.0 + 4.00(25.0) + \frac{1}{2}(0.800)(25.0)^2 = \boxed{360 \text{ m}}$$

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2 = -4.00 + 1.00(25.0) + \frac{1}{2}(-0.300)(25.0)^2 = \boxed{-72.7 \text{ m}}$$

$$v_{xf} = v_{xi} + a_x t = 4 + 0.8(25) = 24 \text{ m/s}$$

$$v_{yf} = v_{yi} + a_y t = 1 - 0.3(25) = -6.5 \text{ m/s}$$

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-6.50}{24.0}\right) = \boxed{-15.2^\circ}$$

- A car moving in *xy*-plane. This car has *x*- and *y*-coordinates that vary with time (t) according to: $x(t)= 4t-0.75t^3$
- and ; $y(t)=-t^2-0.03t^3$
- where *x* and y in meter and *t* in second.
- Find the car **position** (r) at t=2 s.
- Find the Displacement of the car during time intervals from t=0 s to t=1 s.
- Find the average velocity of the car during time intervals from t=0 s to t=1 s.
- Find the average acceleration of the car during time intervals from t=0s to t=1s.
- Find the magnitude of the **instantaneous velocity** of the car at t=2 *s*.
- Find the magnitude of the **instantaneous acceleration** of the car at t=2 *s*.

Example

- A particle initially located at the origin has an acceleration of **a=3.00j** m/s^2
- and an initial velocity of $\mathbf{v_i}$**=5.00i** ***m/s.***
- Find (a) the vector position of the particle at any time *t,*
- (b) the velocity of the particle at any time *t,*
- *(c) the coordinates* of the particle at *t = 2.00 s, and*
- *(d) the speed of the* particle at *t = 2.00 s.*

$$\vec{\mathbf{a}} = 3.00\hat{\mathbf{j}}\ \mathrm{m/s^2};\ \vec{\mathbf{v}}_i = 5.00\hat{\mathbf{i}}\ \mathrm{m/s};\ \vec{\mathbf{r}}_i = 0\hat{\mathbf{i}} + 0\hat{\mathbf{j}}$$

(a) $$\vec{\mathbf{r}}_f = \vec{\mathbf{r}}_i + \vec{\mathbf{v}}_i t + \frac{1}{2}\vec{\mathbf{a}}t^2 = \boxed{\left[5.00t\hat{\mathbf{i}} + \frac{1}{2}3.00t^2\hat{\mathbf{j}}\right]\ \mathrm{m}}$$

$$\vec{\mathbf{v}}_f = \vec{\mathbf{v}}_i + \vec{\mathbf{a}}t = \boxed{\left(5.00\hat{\mathbf{i}} + 3.00t\hat{\mathbf{j}}\right)\ \mathrm{m/s}}$$

(b) $$t = 2.00\ \mathrm{s},\ \vec{\mathbf{r}}_f = 5.00(2.00)\hat{\mathbf{i}} + \frac{1}{2}(3.00)(2.00)^2\hat{\mathbf{j}} = \left(10.0\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}}\right)\ \mathrm{m}$$

so $x_f = \boxed{10.0\ \mathrm{m}}$, $y_f = \boxed{6.00\ \mathrm{m}}$

$$\vec{\mathbf{v}}_f = 5.00\hat{\mathbf{i}} + 3.00(2.00)\hat{\mathbf{j}} = \left(5.00\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}}\right)\ \mathrm{m/s}$$

$$v_f = \left|\vec{\mathbf{v}}_f\right| = \sqrt{v_{xf}^2 + v_{yf}^2} = \sqrt{(5.00)^2 + (6.00)^2} = \boxed{7.81\ \mathrm{m/s}}$$