CEGEP Linear Algebra Problems

An open source collection of
CEGEP level linear algebra problems

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http://obeymath.org/CEGEPLinearAlgebraProblems

Contents

Chapter 1

Systems of Linear Equations

1.1 Gaussian and Gauss-Jordan Elimination

1.1.1 (*by Jim Hefferon [JH]*) Use Gauss's Method to find the unique solution for each system.

a.

$$\begin{aligned} 2x &+ 3y = 13 \\ x &- y = -1 \end{aligned}$$

b.

$$\begin{aligned} x &\qquad - z = 0 \\ 3x &+ y \qquad = 1 \\ -x &+ y + z = 4 \end{aligned}$$

Chapter 2

Vector Spaces

2.1 Introduction to Vector Spaces

2.1.1 (*by Jim Hefferon [JH]*) Name the zero vector for each of these vector spaces.

a. The space of degree three polynomials under the natural operations.

b. The space of 2×3 matrices.

c. The space $\{f : [0,1] \to \mathbb{R} \mid f \text{ is continuous}\}$.

d. The space of real-valued functions of one natural number variable.

2.1.2 (*by Jim Hefferon [JH]*) Find the additive inverse, in the vector space, of the vector.

a. In $\mathcal{P}_3$, the vector $-3-2x+x^2$.

b. In the space $\mathcal{M}_{2\times 2}$,

$$\begin{bmatrix} 1 & -1 \\ 0 & 3 \end{bmatrix}.$$

c. In $\{ae^x + be^{-x} \mid a,b \in \mathbb{R}\}$, the space of functions of the real variable x under the natural operations, the vector $3e^x - 2e^{-x}$.

2.1.3 (*by Jim Hefferon [JH]*) For each, list three elements and then show it is a vector space.

a. The set of linear polynomials $\mathcal{P}_1 = \{a_0 + a_1 x \mid a_0, a_1 \in \mathbb{R}\}$ under the usual polynomial addition and scalar multiplication operations.

b. The set of linear polynomials $\{a_0 + a_1 x \mid a_0 - 2a_1 = 0\}$, under the usual polynomial addition and scalar multiplication operations.

2.1.4 (*by Jim Hefferon [JH]*) For each, list three elements and then show it is a vector space.

a. The set of 2×2 matrices with real entries under the usual matrix operations.

b. The set of 2×2 matrices with real entries where the $2,1$ entry is zero, under the usual matrix operations.

2.1.5 (*by Jim Hefferon [JH]*) For each, list three elements and then show it is a vector space.

a. The set of three-component row vectors with their usual operations.

b. The set

$$\{(x,y,z,w) \in \mathbb{R}^4 \mid x+y-z+w = 0\}$$

under the operations inherited from $\mathbb{R}^4$.

Appendix A

Answers to Exercises

1.1.1

a. $x = 2$, $y = 3$

b. $x = -1$, $y = 4$, and $z = -1$.

2.1.1

a. $0 + 0x + 0x^2 + 0x^3$

b. $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

c. The constant function $f(x) = 0$

d. The constant function $f(n) = 0$

2.1.2

a. $3 + 2x - x^2$

b. $\begin{bmatrix} -1 & +1 \\ 0 & -3 \end{bmatrix}$

c. $-3e^x + 2e^{-x}$

2.1.3

a. $1 + 2x$, $2 - 1x$, and x.

b. $2 + 1x$, $6 + 3x$, and $-4 - 2x$.

2.1.4

a.
$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \begin{bmatrix} -1 & -2 \\ -3 & -4 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

b.
$$\begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix}, \begin{bmatrix} -1 & -2 \\ 0 & -4 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

2.1.5

a. $(1, 2, 3)$, $(2, 1, 3)$, and $(0, 0, 0)$.

b. $(1, 1, 1, -1)$, $(1, 0, 1, 0)$ and $(0, 0, 0, 0)$.

References

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