

# Lab 1 Session 6: Spatial Filtering (Fraunhofer Diffraction and Making Masks)

Date: 27 January 2026 Lab Partner: Nathan Unruh

For Table of Contents, refer to session 4,  
Pg 40

## 1. Session Goals

- i) Compare diffraction pattern to theory
- ii) Calibrate the Ronchi ruling (slit spacing  $a$  and slit width  $b$ )
- iii) Establish a quantitative baseline for spatial filtering 4. Complete
- iv) spatial filtering mask cases (a), (b), (d), (e), (f), (j), (k)

## 2. Reference: Pages 39-43 (Part A: Fraunhofer Diffraction, Part B: Spatial)

## 3. Variables

**Independent:** Mask configuration (orders transmitted) **Dependent:**

Real-space image intensity profile, Fourier-space diffraction  
pattern

**Control:** Camera settings (gain, exposure, gamma), LED  
wavelength, grating period, optical alignment

## 4. Apparatus:

In Addition to the Material Listed on Pg 2-3. We need

- i) Additional Translational Stage
- ii) Wires, Black Tape, Allen Key of different size
- iii) A rectangular Frame with a Post and Saddle

## 5. Additional Background

## 6. Procedure:

### A: Fraunhofer Diffraction

Following procedure on Lab Script pg. 39-40 3.1 Procedure 1.

- Replaced variable diffraction grating with 10 lp/mm Ronchi ruling microscope slide
- Set camera parameters: Exposure = 43000, Gamma = 1, Gain = 18
- Focused on "ZERO" lettering on Ronchi ruling for coarse focus
- Fine focused on blemishes (chipped metal on lettering) for optimal sharpness
- Inserted 10 nm bandpass filter in front of LED
- Verified Fourier image centro-symmetry
- Recorded real-space and Fourier-space images
- Extracted line profiles for calibration measurements

Despite turning the dial on the condenser lens, the focus appeared to change unexpectedly. Fine focusing required locating small defects (chipped metal) on the ruling surface rather than the grating lines themselves. This provides sharper focus targets than the periodic structure due to the Talbot effect creating multiple focal planes for periodic objects.

Observation

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**Observation:** The Fourier image was centro-symmetrical, confirming that the optical elements are properly aligned square to the optical axis. This symmetry verification is essential before quantitative measurements.

Calibrate the Real image:

10 lines on the camera is from  $602 \text{ px} \rightarrow 1150 \text{ px}$

Pixel span  $1150 - 602 = 548 \text{ pixel}$  for 10 periods

$$\text{Scale factor} = \frac{a}{\text{Pixel per period}} = \frac{100 \mu\text{m}}{54.8 \text{ px}} = 1.82 \mu\text{m/pixel}$$

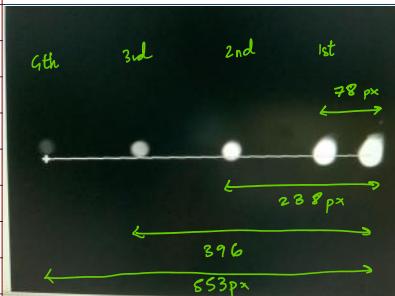
With the Real space camera having a pixel size  $\Delta_{\text{pixel}} = 3.45 \mu\text{m}$ .

$$M_{\text{real}} = \frac{\text{image scale}}{\text{pixel size}} = \frac{54.8 \times 3.45 \mu\text{m}}{100 \mu\text{m}} = 1.89 \times$$

$$\text{Expected magnification} = \frac{f_s}{f_u} = \frac{300}{150} = 2.0 \times$$

The values are close, but the difference is from the lens position not being exactly at focal length

Calibration for Fourier Image:



Your label	Pixel distance	Ratio to 1st	Actual order $m$	Values
"1st"	78 px	1.00	$m = 1$	100
"2nd"	238 px	3.05	$m = 3$	
"3rd"	396 px	5.08	$m = 5$	
"4th"	553 px	7.09	$m = 7$	

We Note that we only see odd orders  $m = 1, 3, 5, 7$ .

This is what we observe from a 50% duty cycle Raman grating.

The even orders are suppressed because  $n = 2, 4, 6$ , because  $\sin\left(\frac{m\pi b}{a}\right) = \sin\left(\frac{m\pi}{2}\right) = 0$  for even  $m$  when  $\frac{b}{a} = \frac{1}{2}$

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### Fresnel Camera Calibration

$$\Theta_m = \frac{m\lambda}{a} = \frac{m \times 525 \times 10^{-9}}{100 \times 10^{-6}} = m \times 5.2 \text{ mrad}$$

$$k_{\text{cal}} = \frac{\theta_1}{\Delta p_1} = \frac{5.2 \text{ mrad}}{78 \text{ px}} = 0.0673 \text{ mrad/px}$$

Verification: Measured vs Theory			
Order $m$	Measured (px)	Theory: $m \times 78 \text{ px}$	Discrepancy
1	78	78	0% (reference)
3	238	234	+1.7%
5	396	390	+1.5%
7	553	546	+1.3%

Excellent linearity! The small systematic offset ( $\sim 1.5\%$ ) could be due to:

- Slight deviation from small-angle approximation at higher orders
- Minor lens aberrations
- Grating period slightly different from nominal 100  $\mu\text{m}$

### ⇒ Extracting Grating Period

$$\Delta p = \frac{78 + 238/3 + 396/5 + 553/7}{4} = \frac{78 + 79.3 + 79.2 + 79.0}{4}$$

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$$\Delta p = \frac{78 + 238/3 + 396/5 + 553/7}{4} = \frac{78 + 79.3 + 79.2 + 79.0}{4}$$

$$= 78.9 \text{ px/order}$$

$$\alpha = \frac{\lambda \cdot f_{obj}}{\Delta x} = \frac{\lambda}{D_i} = \frac{525 \text{ nm}}{5.25 \text{ mic}} = 100 \text{ pm}$$

#### Summary: Both Calibrations

Method	Parameter	Value
Real-space	Scale	1.82 μm/pixel
	Magnification	1.89×
	Period $a$	100 μm (known)
Fourier-space	Calibration	0.0673 mrad/pixel
	Period $a$ (verified)	100 μm ✓
	Duty cycle $b/a$	~0.5 (even orders absent)

⇒ Pix elValue

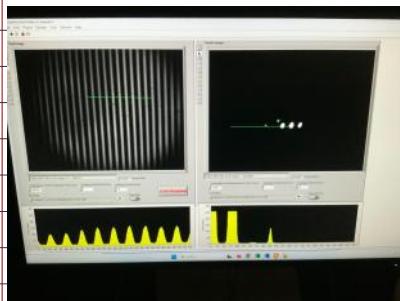
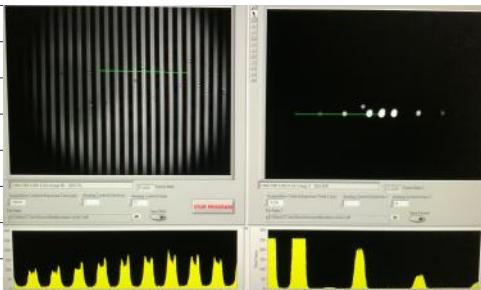
#### B. Spatial Filtering

2:57 PM

Following procedure on Lab Script pg. 41-43

- 1. Verified Fourier patterns are symmetrical and well focused
- 2. Placed mask post at Fourier plane (after condenser lens, before beam splitter)
- 3. Mounted translational stage for precise mask positioning

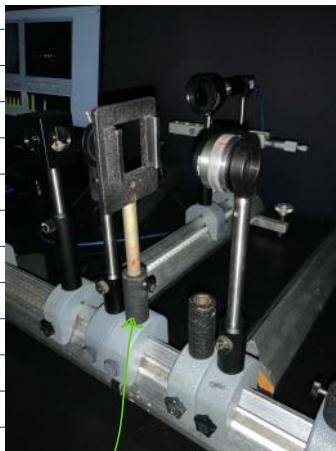
Saved as file: Default-and-blocked-vert01.tif



At [redacted]

At

At the Fourier Plane, after the condenser lens, before the Beam Splitter. We placed a post, on this post we are making masks to block different diffraction patterns.



TIP: We are using an helliead screw with a translational stage. This works very very well!

placing the four mask  
after the condenser lens.  
at the fourier plane

#### Case (a) All orders (reference case)

Saved Images as "RonchiReal-A.tiff"  
"RonchiFourier-A.tiff"  
Default image. Looks bright and the real image is all well illuminated

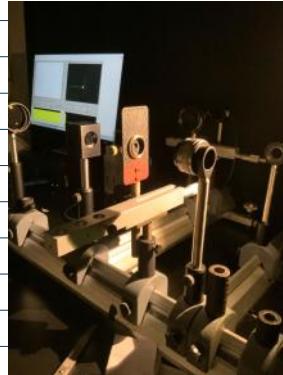
We accidentally increased the Iris so new images as



Saved Images as "RonchiReal-A2.tiff"  
"RonchiFourier-A2.tiff"

#### Observation:

Image is bright and well illuminated.  
Real-space image shows sharp, well-defined grating lines with high contrast. This serves as the reference for all subsequent filtering cases.



#### Case (b) Zero order only:

Orders transmitted:  $m = 0$  only

Orders blocked: All  $m \neq 0$

Observation: Intensity significantly reduced. Real-space image shows uniform illumination with NO spatial variation — the grating lines completely disappear.

File Name: "RonchiReal-B.tiff" and "RonchiFourier-B.tiff"

Physical Interpretation: Zero Order Only

The zeroth order carries only the DC (average) component of the image.

Blocking all other orders

removes all spatial frequency information, resulting in a uniform gray

$m_{\max}$  Orders Transmitted

1 -1, 0, +1

3 -3, -1, 0, +1, +3

5 -5 to +5

7 -7 to +7

All Reference

Iris Setting

Smallest (just fits  $\pm 1$ )

Medium (just fits  $\pm 3$ )

Larger (just fits  $\pm 5$ )

Even larger (just fits  $\pm 7$ )

Fully open

The zeroth order carries only the DC (average) component of the image.

Blocking all other orders

removes all spatial frequency information, resulting in a uniform gray field:

$$I(x) = |C_0|^2 = \left(\frac{1}{2}\right)^2 = 0.25$$

This demonstrates why at least one non-zero order is required to resolve any periodic structure.

Case(d) 0, +1 (Abbe minimal criterion):

Orders transmitted:  $m = 0, +1$

Orders blocked: All negative orders,  $m \geq 2$

Mask adjustment: Moved mask forward to make effective aperture larger, then slid on translational stage to select 0 and +1 only.

Filename: "RonchiReal-D.tif" and "RonchiFourier-D.tif"

**Observation:** More noticeable lines appear in between each solid white line.

The image shows periodic structure but with reduced contrast and asymmetric appearance.

Explanation: Abbe Minimal Criterion

This is the minimum requirement to resolve a periodic structure. With only 0 and +1:

$$E(x) = C_0 + C_1 e^{iK_0x} = \frac{1}{2} + \frac{1}{\pi} e^{iK_0x}$$

The intensity shows periodicity at the fundamental frequency, but the asymmetric order selection

produces a "traveling wave" appearance with reduced contrast compared to symmetric filtering.

More noticeable lines in between each solid white line

Case(e) +1, 0, -1:

Orders transmitted:  $m = -1, 0, +1$

Orders blocked:  $|m| \geq 2$

Mask: Larger Allen key taped to translational stage. (Smart Move :)

**Observation:** Intensity further reduced compared to reference. Image shows sinusoidal variation —

the sharp square-wave edges are replaced by smooth, rounded transitions.

Focus and sharpness are

lost; peaks on line profile become rounded.

Physical Interpretation: Low-Pass with  $m_{max}$

With symmetric  $\pm 1$  and 0:

$$E(x) = \frac{1}{2} + \frac{2}{\pi} \cos(K_0 x)$$

This produces a pure sinusoidal intensity modulation. All high-frequency content (sharp edges) is removed, demonstrating that edge sharpness requires higher-order Fourier components.

#### Case (f): Orders $\pm 1, -1$ Only (Zero Blocked)

Orders transmitted:  $m = +1, -1$

Orders blocked:  $m = 0$  and  $|m| \geq 3$

Mask: Thinnest Allen key + similar-sized screwdriver

Files: RonchiReal-F.tiff, RonchiFourier-F.tiff

**Observation:** Very low intensity. Real-space image shows frequency doubling — the apparent period is half the original grating period.

**Explanation:** Frequency Doubling Effect

Blocking the zero order while transmitting  $\pm 1$ :

#### INSET EQUATION

The intensity varies at frequency  $2K_0$ , producing an image with twice the spatial frequency (half the period) of the original grating. This is a classic demonstration of how phase information affects image reconstruction.

#### Case (j): All Orders Except Zero (Dark-Field)

Orders transmitted: All  $m \neq 0$

Orders blocked:  $m = 0$  only

Mask: Slit mask blocking central maximum only

**Observation:** Real-space image appears as a uniform dark screen with very faint edge features barely visible.

**Dark Field Imaging**

Files: RonchiReal-J.tiff, RonchiFourier-J.tiff

**Explanation:** Blocking only the zero order removes the DC background:

#### INSERT

For a square wave, this enhances edges while suppressing uniform regions. However, for a well-focused grating, the result appears very dark because most energy is in the zero order. Dark-field imaging is more effective for phase objects or objects with defects.

### Case (k): Low-Pass Filtering ( $m_{\text{max}} = 2$ )

Orders transmitted:  $m = -2, -1, 0, +1, +2$  (effectively  $-1, 0, +1$  since even orders absent)

Orders blocked:  $|m| \geq 3$

Files: RonchiReal-K2.tif, RonchiFourier-K2.tif

**Observation:** Similar to case (e) since  $m = \pm 2$  orders are naturally suppressed for 50% duty cycle grating. Image shows sinusoidal profile with rounded edges.

## Summary of Spatial Filtering Results

Case	Orders Transmitted	Trans-	Contrast	Period	Key Observation
(a)	All (reference)	High	$a$		Sharp square wave, well-defined edges
(b)	0 only	None	N/A		Uniform gray — no spatial variation
(d)	0, $\pm 1$	Medium	$a$		Asymmetric, extra lines between peaks
(e)	$-1, 0, +1$	Medium	$a$		Sinusoidal, rounded edges, reduced sharpness
(f)	$+1, -1$	Low	$a/2$		<b>Frequency doubling</b> — period halved
(j)	All except 0	Very low	$a$		Uniform dark screen, faint edges
(k)	$ m  \leq 2$	Medium	$a$		Same as (e) due to absent even orders

## F. Discussion

### i) Abbe Minimal Criterion

Case (d) demonstrates that transmitting orders 0 and  $\pm 1$  is the minimum requirement to resolve periodicity. The zeroth order alone (case b) produces no spatial variation because it carries only the average intensity. At least one interference between orders 0 and  $\pm 1$  is needed to create intensity modulation at the grating frequency.

### ii) Dark-Field Filtering

Comparing cases (a) and (j): blocking the zero order removes the DC background, which should enhance edges. However, for a well-focused amplitude grating, most light energy is in the zero order, so blocking it results in a very dim image. Dark-field imaging is more effective for:

- Phase objects (where zero order doesn't dominate)
- Detecting small defects or particles
- Edge enhancement in low-contrast samples

### iii) Frequency Doubling (Case f)

The most striking result is case (f), where blocking the zero order while transmitting  $\pm 1$  produces apparent frequency doubling. This occurs because:

## EQUATION

The squared cosine has twice the frequency of the original field. This demonstrates that image reconstruction depends on phase relationships between Fourier components, not just their

amplitudes.

## 8. Uncertainties (Derivation)

Source	Type	Magnitude
Pixel position measurement	Random	$\pm 2 \text{ pixels}$
LED wavelength (with filter)	Systematic	$525 \pm 5 \text{ nm}$
Grating period tolerance	Systematic	$100 \pm 1 \mu\text{m} (1\%)$
Lens focal length	Systematic	$\pm 1 \text{ mm}$
Mask positioning	Random	$\pm 0.5 \text{ mm}$

The dominant uncertainty is probably the pixel position measurement ( $\pm 2 \text{ px}$ ) contributes:

## EQUATION

### 9. Session Reflection

#### i) Goals Status

- ✓ Compared diffraction pattern to theory — excellent agreement (< 2% discrepancy)
- ✓ Calibrated Ronchi ruling:  $a = 100 \mu\text{m}$ ,  $b/a = 0.5$  (confirmed by absent even orders)
- ✓ Established quantitative baseline: scale =  $1.82 \mu\text{m}/\text{pixel}$ ,  $M = 1.89 \times$
- ✓ completed spatial filtering cases (a), (b), (d), (e), (f), (j), (k)

#### ii) Key Learnings

1. Odd-order-only pattern confirms 50% duty cycle
2. Allen key on translational stage is excellent for precise Fourier-plane masking
3. Zero order alone produces no spatial variation (Abbe criterion)
4. Blocking zero order causes frequency doubling when only  $\pm 1$  transmitted

#### iii) To Do Next Session

- Complete Gibbs ringing analysis (B3) with  $m_{\max} = 1, 3, 5$
- Measure overshoot amplitude and ringing period quantitatively
- Extract line profiles and compare to Fourier series model
- Record intensity values for  $I_m/I_1$  comparison to theory