Physics 332 Analysis Assignment 2
Due Tuesday, Feb 10, 2026 on Crowdmark

Instructions: Please submit scanned/photo-ed solutions to each question separately into the respective Crowdmark slot.

For questions parts (b)–(f), provide the responses in a IPython notebook and submit it as a pdf. Where a brief explanation is required, you can type it as a comment field in the notebook.

1. Quadratic fit through zero (20 points)

You measure some data y as a function of x and expect (on some prior theoretical grounds) to find a quadratic relation of the form $y = ax^2$. The independent variable is x, the measurement y, and the unknown quadratic coefficient a. Through previous experiments, you have established that each measurement has roughly Gaussian distributed errors with a standard deviation σ. Each measurement is also independent of all others.

(a) Write down the likelihood function $\mathcal{L}$ for the set of measurements and model. Working with $-\ln \mathcal{L}$ and differentiating once and once again, find general expressions (in terms of the $\{x_k\}$ and $\{y_k\}$) for the maximum likelihood estimate $\hat{a}$ for the quadratic coefficient and its uncertainty σ_a.

(b) Next, you measure the $N = 10$ data points shown below, which are generated using $\sigma = 1$. Use the curve-fit routine in Python[1] to find the best estimate $\hat{a} \pm \sigma_a$.

x values	0	1	2	3	4	5	6	7	8	9
y data	0.7	-0.5	0.4	1.6	7.6	13.7	18.0	25.0	33.0	39.6

(c) What is the χ^2? Recall that if the fit is good, then $P(\chi^2)$ should have mean ν and variance 2ν. Here $\nu = N - m$ is the degrees of freedom where N is the number of data points and m the number of fit parameters. In other words, we expect χ^2 to be approximately in the range $\nu \pm \sqrt{2\nu}$. Is this fit "good" by this standard?

(d) Setting a significance of 5%, does the fit meet the χ^2 goodness of fit test?

(e) Use the explicit expressions for $\hat{a}$ and σ_a that you derived in part (a) and evaluate on the data set above. Show that you get the same result as given by your least-squares fit program in part (b).

(f) Redo the linear fit in part (b) with a model including an intercept, $ax + b$, and letting both the slope a and the intercept b be free parameters. Show that the optimal value of χ^2 is lower. It might then seem that it is better to fit the two parameters to the data, instead of one. Explain why this is *not* a good idea, in this case.

[1] The Python notebook `Template-Fit` on Canvas can be a good starting point.