Lab 1 Session 6: Spatial Filtering (Fraunhofer Diffraction and Making Masks)

~~Date: 27 January 2026~~
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For Table of Contents, refer to session 4,
Pg 40

1. Session Goals
 i) Compare diffraction pattern to theory
 ii) Calibrate the Ronchi ruling (slit spacing a and slit width b)
 iii) Establish a quantitative baseline for spatial filtering 4. Complete
 iv) spatial filtering mask cases (a), (b), (d), (e), (f), (j), (k)

2. Reference: Pages 39-43 (Part A: Fraunhofer Diffraction, Part B: Spatial

3. Variables
Independent: Mask configuration (orders transmitted) **Dependent:** Real-space image intensity profile, Fourier-space diffraction pattern
Control: Camera settings (gain, exposure, gamma), LED wavelength, grating period, optical alignment

4. Apparatus:
In Addition to the Material listed on Pg 2-3. We need
 i) Additional Translational Stage
 ii) Wires, Black Tape, Allen Key of different Size
 iii) A rectangular Frame with a Post and Saddle

5. Additional Background

6. Procedure:

A: Fraunhofer Diffraction
Following procedure on Lab Script pg. 39-40 3.1 Procedure 1.
- Replaced variable diffraction grating with 10 lp/mm Ronchi ruling

microscope slide

- Set camera parameters: Exposure = 43000, Gamma = 1, Gain = 18
- Focused on "ZERO" lettering on Ronchi ruling for coarse focus
- Fine focused on blemishes (chipped metal on lettering) for optimal sharpness
- Inserted 10 nm bandpass filter in front of LED
- Verified Fourier image centro-symmetry
- Recorded real-space and Fourier-space images
- Extracted line profiles for calibration measurements

Observation: Despite turning the dial on the condenser lens, the focus appeared to change unexpectedly. Fine focusing required locating small defects (chipped metal) on the ruling surface rather than the grating lines themselves. This provides sharper focus targets than the periodic structure due to the Talbot effect creating multiple focal planes for periodic objects.

Observation: The Fourier image was centro-symmetrical, confirming that the optical elements are properly
aligned square to the optical axis. This symmetry verification is essential before quantitative measurements.

Calibiate the Real image:

10 lines on the camera is from 602px → 1150px

Pixel span 1150-602 = 548pixel for 10period

$$\text{Scal factor} = \frac{a}{\text{Pixel per period}} = \frac{100\,\mu m}{54.8\,px} = 1.82\,\mu m/\text{pixel}$$

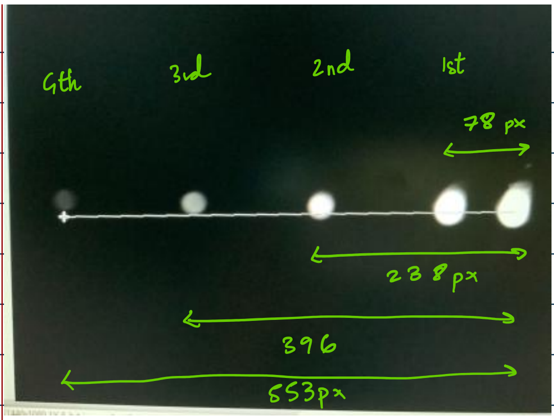
With the Real space camera having a pixel size $\Delta_{Pixel} = 3.45\,\mu m$.

$$M_{real} = \frac{\text{image scale}}{\text{pixel size}} = \frac{54.8 \times 3.45\,\mu m}{100\,\mu m} = \frac{189\,\mu m}{100\,\mu m} = 1.89\times$$

$$\text{Expected magnification} = \frac{f_5}{f_4} = \frac{300}{150} = 2.0\times$$

The values are close, but the difference is from the lens position not being exactly at focal length

Calebration for fourien Image:



Your label	Pixel distance	Ratio to 1st	Actual order m	Values
"1st"	78 px	1.00	m = 1	200 (1600, Exposure time, 1s, 18 Gain)
"2nd"	238 px	3.05	m = 3	100
"3rd"	396 px	5.08	m = 5	
"4th"	553 px	7.09	m = 7	

800 Exposure
120
50

142
10
5

We Note that we only see odd orders $m = 1, 3, 5, 7$.

This is what we observe from a 50% duty Cycle Ronchi ruling.

The even orders are suppressed because $m = 2, 4, 6$, because $\sin\left(\frac{m\pi b}{a}\right) = \sin\left(\frac{m\pi}{2}\right) = 0$ for even m when $\frac{b}{a} = \frac{1}{2}$

Fourier Camera Calibrate

$$\theta_m = \frac{m\lambda}{a} = \frac{m \times 525 \times 10^{-9}}{100 \times 10^{-6}} = m \times 5.2 \text{ mrad}$$

$$K_{cal} = \frac{\theta_1}{\Delta P_1} = \frac{5.25 \text{ mrad}}{78 \text{ px}} = 0.0673 \text{ mrad/pixel}$$

Order m	Measured (px)	Theory: m × 78 px	Discrepancy
1	78	78	0% (reference)
3	238	234	+1.7%
5	396	390	+1.5%
7	553	546	+1.3%

Excellent linearity! The small systematic offset (~1.5%) could be due to:

- Slight deviation from small-angle approximation at higher orders
- Minor lens aberrations
- Grating period slightly different from nominal 100 μm

⇒ Extracting Grating Period

$$\Delta p = \frac{78 + 238/3 + 396/5 + 553/7}{4} = \frac{78 + 79.3 + 79.2 + 79.0}{4}$$

$$= 78.9 \text{ px / order}$$

$$a = \frac{\lambda \cdot f_{obj}}{\Delta x} = \frac{\lambda}{\theta_1} = \frac{525 \text{nm}}{5.25 \text{ mrad}} = 100 \ \mu m$$

⇒ PixelValus

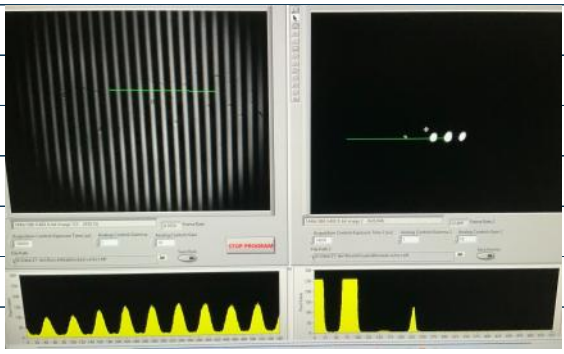
B. Spatial Filtering

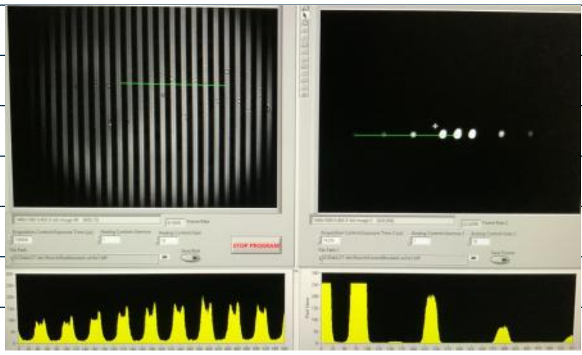
2:57 PM

Following procedure on Lab Script pg. 41-43

- 1. Verified Fourier patterns are symmetrical and well focused
- 2. Placed mask post at Fourier plane (after condenser lens, before beam splitter)
- 3. Mounted translational stage for precise mask positioning

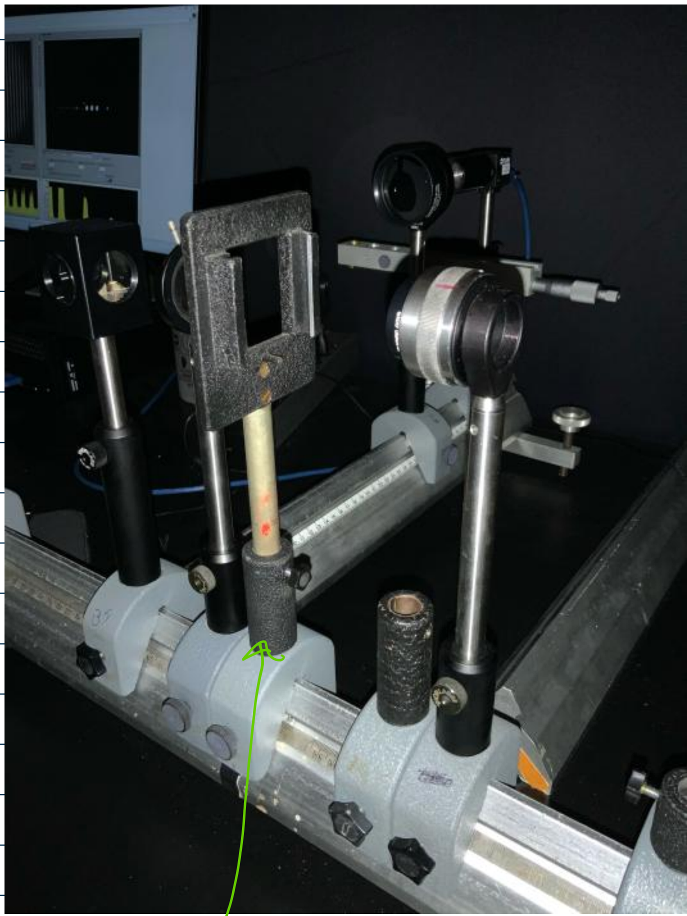
Saved as file: Defualt-and-blocked-ve1to1.tiff





At (

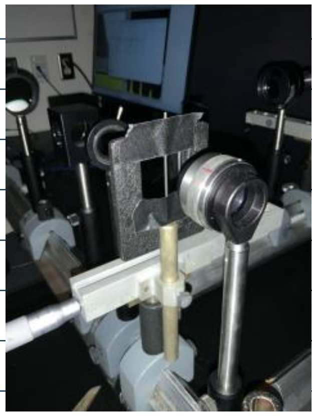
At the Fourier Plane, after that condenser Lense, before the Beam Splitter. We placed a post, on this post we are making masks to block different diffraction patterns.



Placing the fourier mask after the condenser lense. at the fourier plane

TIP: We are using an hellen screw with a translational stage. This works very very well!

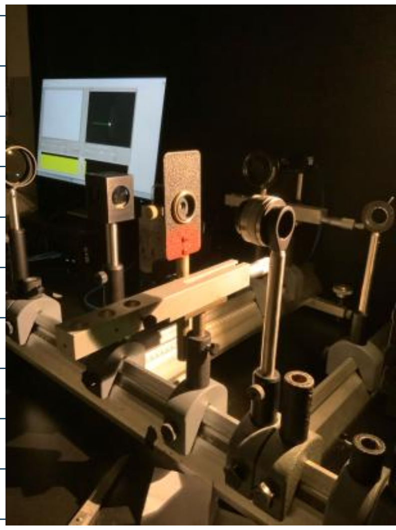
Case (a) All orders (reference case)



Saved Images as "RonchiReal-A.tiff"
"RonchiFourier-A.tiff"
Default image. Looks bright and the real image is all well illuminated

We accidently increased the Iris so new images as

Saved Images as "RonchiReal-A2.tiff"
"RonchiFourier-A2.tiff"



Observation: Image is bright and well illuminated. Real-space image shows sharp, well-defined grating
lines with high contrast. This serves as the reference for all subsequent filtering cases.

Case(b) Zero order only:

Orders transmitted: $m = 0$ only
Orders blocked: All $m \neq 0$

Observation: Intensity significantly reduced. Real-space image shows uniform illumination
with NO spatial variation — the grating lines completely disappear.

File Name: "RonchiReal-B.tiff" and
"RonchiFourier-B.tiff"

Physical Interpretation: Zero Order Only
The zeroth order carries only the DC (average) component of the image. Blocking all other orders
removes all spatial frequency information, resulting in a uniform gray field:

field:

$$I(x) = |C_0|^2 = \left(\frac{1}{2}\right)^2 = 0.25$$

This demonstrates why at least one non-zero order is required to resolve any periodic structure.

Case(d) 0, +1 (Abbe minimal criterion):

Orders transmitted: $m = 0, +1$
Orders blocked: All negative orders, $m \geq 2$

Mask adjustment: Moved mask forward to make effective aperture larger, then slid on translational
stage to select 0 and +1 only.

FileName: "RonchiReal-D.tiff" and "RonchiFourier-D.tiff"

Observation: More noticeable lines appear in between each solid white line. The image
shows periodic structure but with reduced contrast and asymmetric appearance.

Explanation: Abbe Minimal Criterion
This is the minimum requirement to resolve a periodic structure. With only 0 and +1:

$$E(x) = C_0 + C_1 e^{ik_0 x} = \frac{1}{2} + \frac{1}{\pi} e^{ik_0 x}$$

The intensity shows periodicity at the fundamental frequency, but the asymmetric order selection
produces a "traveling wave" appearance with reduced contrast compared to symmetric filtering.

More noticeable lines in between each solid white line

Case(e) +1, 0, −1:

Orders transmitted: $m = -1, 0, +1$
Orders blocked: $|m| \geq 3$

Mask: Larger Allen key taped to translational stage. (Smart Move :)

Observation: Intensity further reduced compared to reference. Image shows sinusoidal variation —
the sharp square-wave edges are replaced by smooth, rounded transitions. Focus and sharpness are
lost; peaks on line profile become rounded.

Physical Interpretation: Low-Pass with mmax
With symmetric ±1 and 0:

$$E(x) = \frac{1}{2} + \frac{2}{\pi} \cos(K_0 x)$$

This produces a pure sinusoidal intensity modulation. All high-frequency content (sharp edges) is
removed, demonstrating that edge sharpness requires higher-order Fourier components.

Case (f): Orders +1, −1 Only (Zero Blocked)

Orders transmitted: $m = +1, -1$
Orders blocked: $m = 0$ and $|m| \geq 3$
Mask: Thinnest Allen key + similar-sized screwdriver

Files: RonchiReal-F.tiff, RonchiFourier-F.tiff

Observation: Very low intensity. Real-space image shows frequency doubling — the apparent period is half the original grating period.

Explanation: Frequency Doubling Effect
Blocking the zero order while transmitting ±1:

$$E(x) = \frac{1}{\pi}\left(e^{iK_0 x} + e^{-iK_0 x}\right) = \frac{2}{\pi} \cos(K_0 x$$

$$I(x) = \frac{4}{\pi} \cos^2(K_0 x) = \frac{2}{\pi^2}\left(1 + \cos 2K_0 x\right)$$

The intensity varies at frequency 2k0, producing an image with twice the spatial frequency (half
the period) of the original grating. This is a classic demonstration of how phase information affects
image reconstruction.

Case (j): All Orders Except Zero (Dark-Field)

Orders transmitted: All m /= 0
Orders blocked: m = 0 only
Mask: Slit mask blocking central maximum only

Observation: Real-space image appears as a uniform dark screen with very faint edge features
barely visible.
Dark-Field Imaging

Files: RonchiReal-J.tiff, RonchiFourier-J.tiff

Explanation: Blocking only the zero order removes the DC background:

$$I(x) = \left|\sum_{m=0} C_m \, e^{imK_0x}\right|^2 = |f(x) - C_0|^2$$

For a square wave, this enhances edges while suppressing uniform regions. However, for a wellfocused
grating, the result appears very dark because most energy is in the zero order. Dark-field
imaging is more effective for phase objects or objects with defects.

Case (k): Low-Pass Filtering (m max = 2)

Orders transmitted: m = −2, −1, 0, +1, +2 (effectively −1, 0, +1 since even orders absent)
Orders blocked: |m| ≥ 3
Files: RonchiReal-K2.tiff, RonchiFourier-K2.tiff

Observation: Similar to case (e) since m = ±2 orders are naturally suppressed for 50% duty cycle
grating. Image shows sinusoidal profile with rounded edges.

Summary of Spatial Filtering Results

Summary of Spatial Filtering Results

Case	Orders Transmitted	Contrast	Period	Key Observation
(a)	All (reference)	High	a	Sharp square wave, well-defined edges
(b)	0 only	None	N/A	Uniform gray — no spatial variation
(d)	0, +1	Medium	a	Asymmetric, extra lines between peaks
(e)	−1, 0, +1	Medium	a	Sinusoidal, rounded edges, reduced sharpness
(f)	+1, −1	Low	$a/2$	**Frequency doubling** — period halved
(j)	All except 0	Very low	a	Uniform dark screen, faint edges
(k)	$\|m\| \leq 2$	Medium	a	Same as (e) due to absent even orders

7. Discussion

i) Abbe Minimal Criterion

Case (d) demonstrates that transmitting orders 0 and +1 is the minimum requirement to resolve periodicity. The zeroth order alone (case b) produces no spatial variation because it carries only the average intensity. At least one interference between orders 0 and ± 1 is needed to create intensity modulation at the grating frequency.

ii) Dark-Field Filtering

Comparing cases (a) and (j): blocking the zero order removes the DC background, which should enhance edges. However, for a well-focused amplitude grating, most light energy is in the zero order, so blocking it results in a very dim image. Dark-field imaging is more effective for:

- Phase objects (where zero order doesn't dominate)
- Detecting small defects or particles
- Edge enhancement in low-contrast samples

iii) Frequency Doubling (Case f)

The most striking result is case (f), where blocking the zero order while transmitting ± 1 produces apparent frequency doubling. This occurs because:

frequency doubling. This occurs because:

$$I(x) \propto \cos^2(K_0 x) = \frac{1}{2}\left(1+\cos(2K_0 x)\right)$$

The squared cosine has twice the frequency of the original field. This demonstrates that image reconstruction
depends on phase relationships between Fourier components, not just their amplitudes.

8. Uncertainties (Derivation)

Source	Type	Magnitude
Pixel position measurement	Random	±2 pixels
LED wavelength (with filter)	Systematic	525 ± 5 nm
Grating period tolerance	Systematic	100 ± 1 μm (1%)
Lens focal length	Systematic	±1 mm
Mask positioning	Random	±0.5 mm

The Dominant Uncertainty is probably the Pixel Position measurement (±2 px) contributes:

$$\frac{\delta\theta}{\theta} = \frac{\delta p}{p} = \frac{2}{78} = 2.6\,\%$$

9. Session Reflection

i) Goals Status

✓ Compared diffraction pattern to theory — excellent agreement (< 2% discrepancy)

✓ Calibrated Ronchi ruling: $a = 100$ μm, $b/a = 0.5$ (confirmed by absent even orders)

✓ Established quantitative baseline: scale = 1.82 μm/pixel, $M = 1.89\times$

✓ Completed spatial filtering cases (a), (b), (d), (e), (f), (j), (k)

ii) Key Learnings

1. Odd-order-only pattern confirms 50% duty cycle
2. Allen key on translational stage is excellent for precise Fourier-plane masking
3. Zero order alone produces no spatial variation (Abbe criterion)
4. Blocking zero order causes frequency doubling when only ±1 transmitted

iii) To Do Next Session

- Complete Gibbs ringing analysis (B3) with mmax = 1, 3, 5
- Measure overshoot amplitude and ringing period quantitatively
- Extract line profiles and compare to Fourier series model
- Record intensity values for Im/I1 comparison to theory