L2 S5

February 24, 2026 10:01 AM

Table of Contents:

1. Prelab
a. Viscosity calculations:
For calculating mixed viscosity, use Refutas method
Calculate VBN for each component
$VBN_i = 14.534 \times \ln\left(\ln\left(\frac{\eta}{density} + 0.8\right)\right) + 10.975$
$VBN_f = \sum x_i \times VBN_i$
$\frac{\eta}{density} = e^{e^{\frac{VBN_f - 10.975}{14.534}}} - 0.8$
Room temp: 19 deg.C as of 24-02-26
Water viscosity: 1.0266 mPa*s
Glycerol viscosity: 1551.5 mPa*s (from calculator)
Target viscosity glycerol 1: 2.0532 mPa*s
Target G2: 4.1064 mPa*s

Refutus method too complicated as it needs density variables which fluctuate with temperature: switching to Exponential Power-Form Model (Cheng, 2008) method with calculator online

From calculator: for 2.0533mPa*s, 79.87% water 20.13% glycerol
For 4.1075: 36.1% glycerol 63.9% water

Dilution calculation with glycerol 5um beads:

Conc. 0.5% weight by volume
5 beads per 1440x1080 pixels x 0.06845um per pixel
= 5 beads per 98.568 x 73.926 um x 5um
G of beads
2.5um rad per bead = 65.45 um^3
/1000^3= 65.45*10^-9 mm^3
/1000 = 65.45*10^-12 cm^3 (ml)
X1.05g/ml = 687.225*10^-13 g/bead
Volume of image = 36433um^3 /1000^3 /1000 = 3.6434*10^-8 ml
Beads per ml = 5/3.6434*10^-8 ml = 1.3723*10^8 beads/ml
g/bead * bead/ml = g/ml = 0.009431
0.005g/1ml initial
Initial/final conc. = 1.886X concentration
Solution: switch to 50X from 100X, 4X the image area

Dilution calculation with glycerol 3um beads:
1.5um rad = 14.137um^3
=...148.4385*10^-13 g/bead
Beads per ml = 10/3.6434*10^-8ml = 2.7447*10^8 beads/ml
g/bead * bead/ml = g/ml = 0.00407
Original = 0.005
Dilution: 1.2285X

Dilution calculation with glycerol 1um beads:
0.5um rad per bead = 0.5236 um^3
... = 5.4978*10^-13 g/bead
Beads per ml = 10/3.6434*10^-8 ml = 2.7447*10^8 beads/ml
g/bead * bead/ml = g/ml = 0.00015089
Original = 0.005
Initial/final conc. = 33.137X

Correction: FOV is 82.5um depth
Volume of image: 6.055*10^5um^3
/1000^4 = 6.055*10^-7ml
Beads/ml = 10/6.055*10^-7ml = 1.6515*10^7

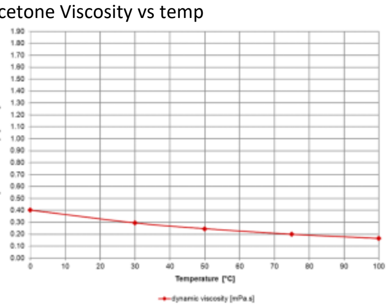
3um beads:
Beads/ml*148.4385*10^-13 g/bead= 0.00024515
Dilution Factor: 0.005/0.00024515 = 20.39567612X

1. Stokes-Einstein relation:
$\langle r^2 \rangle = 2dt \cdot D$

Theoretical Viscosity
$\eta_a = A \cdot e^{\frac{-E_a}{RT}}$
*these constants vary with temperature so a table must be used:

*Do acetone instead of methanol, 0.3mPa

Acetone Viscosity vs temp



Water Viscosity vs temp

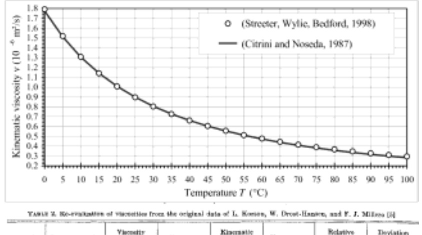


TABLE 2. Re-evaluation of viscosities from the original data of L. Korson, W. Drost-Hansen, and F. J. Millero [5]

Temp. °C	Density ρ, kg/m³	Viscosity μ, μPa s from [5]	Viscosity μ, μPa s	Kinematic viscosity ν, mm²/s	Uncertainty Δν	Relative uncertainty Δμ/μ or Δν/ν, %	Deviation from [5] %
10	999.7281	1306.9	1307	1.307	0.008	0.06	−0.02
15	999.1286	1138.2	1138	1.139	0.0015	0.13	−0.03
20	998.2336	1002.0	1002	1.003	0.0022	0.22	−0.01
25	997.0751	890.3	889	0.892	0.0031	0.35	−0.10
30	995.6783	797.5	796	0.800	0.0039	0.49	+0.18
35	994.0635	719.5	718	0.723	0.0047	0.65	+0.18
40	992.2473	653.3	652	0.658	0.0058	0.89	+0.15
45	990.24	596.3	595	0.601	0.0064	1.08	+0.16
50	988.07	547.1	547	0.553	0.0071	1.30	+0.11
55	985.73	504.2	504	0.511	0.0078	1.55	+0.03
60	983.24	466.4	467	0.475	0.0086	1.83	−0.09
65	980.59	433.4	434	0.443	0.0092	2.12	−0.17
70	977.81	405.9	405	0.414	0.0098	2.42	−0.26

A = (2.2724 ± 0.00071) × 10⁸ s²/m².
B = (6.7 ± 0.52) × 10⁻⁷ m².

1. Stokes-Einstein relation:
   $\langle r^2 \rangle = 2dt \cdot D$
   $D = \dfrac{k_B \cdot T}{6\pi \cdot \eta \cdot R}$, $m^3 s^{-3}$
   $k_B = Boltz. Cst. = 1.380649 \cdot 10^{-23} m^2 kg \cdot s^{-2} K^{-1}$
   $\eta = viscosity, Pa \cdot s \text{ or } kg \cdot s \cdot m^{-2}$
   $R = bead\ radius, m$

   Concentration effects:
   - Increased effective fluid viscosity
   - Fused particles have effectively increased radius, more drag and therefore less brownian motion
   - Wall effects increase viscosity from liquid shearing effects

   Einstines Viscosity Equation (for suspension)
   $\eta = \eta_0 (1 + 2.5\phi)$
   $\eta = apparent\ viscosity, \eta_0 = original\ viscosity\ (of\ mixed\ fluid)$
   $\phi = partical\ fraction\ volume = \left(\dfrac{V_{bead} \cdot n_{bead}}{V_{solution}} \right)$
   For higher concentrations, use Batchelor equation:
   $\eta = \eta_0 (1 + 2.5\phi + 6.2\phi^2)$

   Concentration should be chosen where particles do not clump or fuse, so increased radius effects can be ignored.

   Wall Interactions:
   Use Faxén's Laws.
   D_‖ = D_0 · [1 - 9/16 · (R/h) + 1/8 · (R/h)^3 - 45/256 · (R/h)^4 - 1/16 · (R/h)^5 ...]
   Volume of slide: est. 50um
   Max particle size: 5um
   Minimum wall effect for 5um: D=D_0*(1-0.1138)

2. Objectives:
   - Make different bead samples, 7 different slides varying concentration, viscosity, particle size 3 times.
   - Record 3 avi videos of beads for each slide (21 videos)
   - Repeat samples to get consistent data (14 mixtures/slides, 42 videos)
   - Data analysis during spare time
   - Uncertainties in data if we have sufficient lab time

3. Procedure
   *actual framerate 66.019fps

   Target slides:

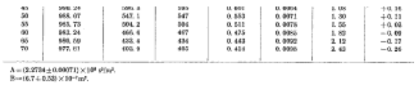
Slide	Mix/Visc. (% glycerol)	Bead Size (um)	Conc (beads/FOV)
1	0	3um	10
2	20	1um	10
3	20	3um	10
4	20	5um	10
5	20	3um	5
6	20	3um	20
7	36	3um	10

   Dilution factors:

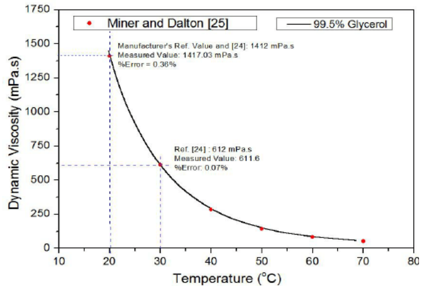
Slide	Factor	Percent Stock
1	20.3956	4.90%
2		
3		
4		
5		
6		
7		

   Notes:
   - Once we used a field depth of 82.5micrometers (tape depth), the amount of beads improved a lot (previously there were 200 beads in the field of view
   - Probably 2X as diluted as above calculation

   To Do
   - Lookup actual FOV of microscope
   - Try using nailpolish instead of tape
   - Make sure slide is symetrical so particles don't drift to escape
   - Find a spot on slide where concentration of beads is correct so we can analyze the other effects



Glycerol Viscosity vs temp



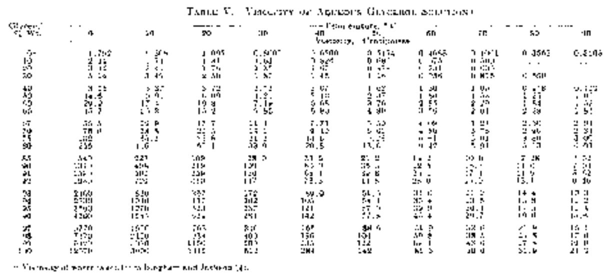


Table V. Viscosity of Aqueous Glycerol Solutions

- Finish collecting rest of slides, analyzing motion, comparing results and macthing with formula