Spatial Filtering[1]

In this experiment, you will first review elementary Fraunhofer diffraction and then explore the relationship between Fourier transforms, diffraction, and image formation. In particular, you will learn about Abbe's theory of image formation and how to do "analog image processing" by studying and then manipulating the image of a periodic grating formed by an optical system.

Objectives

- To understand quantitative image formation in a system with several optical elements, and to think critically about optical system design (magnification, constraints on path length, signal-to-noise ratios, etc.).
- To better understand Fraunhofer diffraction theory, Fourier transforms, and the Convolution Theorem.
- To understand analog image processing—how modifying the electric field at the Fourier plane can change the image seen at the image plane.
- To learn to process 2-D images numerically and think about fitting periodic structures.
- To understand CMOS cameras, gain and shutter parameters, and to think about general signal-to-noise issues for digital imaging.

1 Introduction

Consider light from an object (for example, the print on this page) collected by a converging lens. According to basic ray-optics theory, when the object is properly located in front of the lens, a real-space image is formed behind the lens at a distance determined by the thin-lens formula.

A more interesting interpretation of the imaging process comes from the wave-optics theory of light. In the Abbe view, a lens forms an image in two steps: First, a far-field diffraction image is formed one focal length behind the lens. Then a second transform occurs between the diffraction plane and the image plane.

The diffraction image is, to a good approximation, the square of the Fourier transform of the object being imaged, or more accurately, of the *aperture function* $a(x, y)$ associated with the

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object. (The optical axis lies along the z-axis here, and the transverse co-ordinates are x and y). For this reason, we commonly refer to the plane lying one focal length behind the imaging lens as the *Fourier plane*. Each point in this plane corresponds to a Fourier component of the aperture function's two-dimensional transform. In analogy with Fourier-transform terminology, each component describes a particular *spatial frequency* in the decomposition of the aperture's transverse spatial profile.

The ability to directly access and modify the Fourier transform of an optical image opens up many possibilities for image processing. The basic idea of spatial filtering is that modifying the diffraction image alters the real-space image. Sometimes the changes in the image are fun, and sometimes they can be "useful" in that the image can be "improved" in some fashion. In this experiment, you will investigate this technique using a variety of test objects starting with a Ronchi ruling.

2 Background Summary

We will discuss spatial filtering at three increasing levels of sophistication:

1. *Fourier series*. In the simplest view, at the Fourier plane, an aperture function is "decomposed" into elements of its Fourier series expansion. Changing the Fourier coefficients changes the resulting image. We learn the *amplitude* of each order.

2. *Fraunhofer diffraction*. We analyze the electric field at the Fourier plane as the *Fourier transform* of the aperture function, assuming implicitly that it is very "far" from the aperture. The Ronchi ruling is modeled as an N-slit transmission function with finite-size source. We learn the *angles* (positions on a screen) for each order and understand their *width*.

3. *Fourier optics*. We learn the role of *lenses* and near-field, *Fresnel* diffraction. A lens can bring the far-field pattern "in from infinity" to one focal length behind the lens but adds unwanted phase-front curvature. Putting the object one focal length in front of the lens cancels the effect, leading to a true Fourier transform at the back focal plane.

The experiment can be done at all three levels of sophistication, but you will get more out of your results if you understand better the background.

2.1 Fourier Theory

We begin with a review of the Fourier series expansion of a periodic function, and the Fourier transform of a function.

2.1.1 The Fourier Series

Fourier's Theorem states that a periodic function $f(x)$ having a *spatial period* a can be synthesized by a sum of harmonic functions whose wavelengths are integral submultiples of a, i.e. $a, a/2, a/3,$ etc.[2] The Fourier-series representation of the function has the form

$$f(x) \;=\; \frac{A_0}{2} + \sum_{m=1}^{\infty} \left(A_m \cos mk_0x + B_m \sin mk_0x\right) \,, \tag{1}$$

where A_m and B_m are all constants known as (real) *Fourier coefficients* and where mk_0 are the *angular spatial frequencies*, where $k_0 = \frac{2\pi}{a}$ is the fundamental angular spatial frequency. The process of determining the coefficients A_m and B_m is known as *Fourier analysis*. The Fourier coefficients are found by integrating both sides of Eq. 1 over an interval equal to a:

$$A_0 = \frac{2}{a}\int_{-a/2}^{a/2} \mathrm{d}x\, f(x)\,, \tag{2}$$

$$A_m = \frac{2}{a}\int_{-a/2}^{a/2} \mathrm{d}x\, f(x) \cos mk_0x\,, \tag{3}$$

$$B_m = \frac{2}{a}\int_{-a/2}^{a/2} \mathrm{d}x\, f(x) \sin mk_0x\,. \tag{4}$$

Even functions contain only cosine terms ($B_m = 0$), while odd functions contain only sine terms (all $A_m = 0$)

With the help of Euler's equation ($\mathrm{e}^{\mathrm{i}\theta} = \cos\theta + \mathrm{i}\sin\theta$), the Fourier series given by Eq. 1 can also be expressed in complex notation using exponential functions:

$$f(x) = \sum_{m=-\infty}^{\infty} c_m\, \mathrm{e}^{-\,\mathrm{i}mk_0x}\,, \tag{5}$$

where $\mathrm{i}^2 = -1$ and where the *Fourier coefficient* c_m is given by

$$c_m = \frac{1}{a}\int_{-a/2}^{a/2} \mathrm{d}x\, f(x)\, \mathrm{e}^{\mathrm{i}mk_0x}\;. \tag{6}$$

Example

For example, consider the periodic function shown in Fig. 1. This could describe a grating, which is transparent when $f(x) = 0$. The aperture function is a square wave, with a period equal to

[2]Here a is the spatial period of $f(x)$ and has nothing to do with the aperture function $a(x, y)$ discussed below.

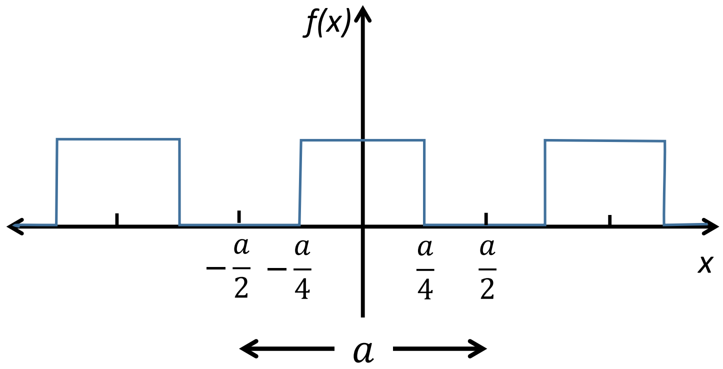

Figure 1: A periodic square-wave function of spatial period a.

twice the width of each slit. Applying Eq. 5, we find

$$c_0 = \frac{1}{2}\,, \tag{7}$$

$$c_1 = \frac{1}{\pi} \quad \text{and} \quad c_{-1} = \frac{1}{\pi}\,, \tag{8}$$

$$c_2 = 0\,, \tag{9}$$

$$c_3 = -\frac{1}{3\pi} \quad \text{and} \quad c_{-3} = -\frac{1}{3\pi} \quad . \tag{10}$$

Each amplitude corresponds to a distinct spatial frequency. When we display them at their corresponding frequencies, the result is called the *frequency spectrum*.

Using the amplitudes, we can express the function as

$$\begin{aligned} f(x) &= \frac{1}{2} + \frac{1}{\pi}\,\mathrm{e}^{\mathrm{i}k_0 x} + \frac{1}{\pi}\,\mathrm{e}^{-\,\mathrm{i}k_0 x} - \frac{1}{3\pi}\,\mathrm{e}^{3\,\mathrm{i}k_0 x} - \frac{1}{3\pi}\,\mathrm{e}^{-3\,\mathrm{i}k_0 x} \ldots \\ &= \frac{1}{2} + \frac{2}{\pi}\left(\cos k_0 x - \frac{1}{3}\cos 3k_0 x + \ldots\right)\,. \end{aligned} \tag{11}$$

The plots[3] in Fig. 2 show how the series converges to $f(x)$ as terms of higher spatial frequency are added. The fundamental angular spatial frequency ($m = 1$) roughly defines the outline of the original function. The finer features of the function $f(x)$, such as the corners, are accomplished by adding or subtracting harmonic functions of higher angular spatial frequencies. Finally, Fig. 3 shows the corresponding frequency spectrum for the amplitude.

When we image the Fourier plane, we record the intensity, not the amplitude. Thus, we expect to see peaks with an amplitude equal to the square of the calculated Fourier coefficients, as illustrated in Fig. 4.

[3]There are interactive graphs on the web that illustrate Fourier series, sometimes with animations.

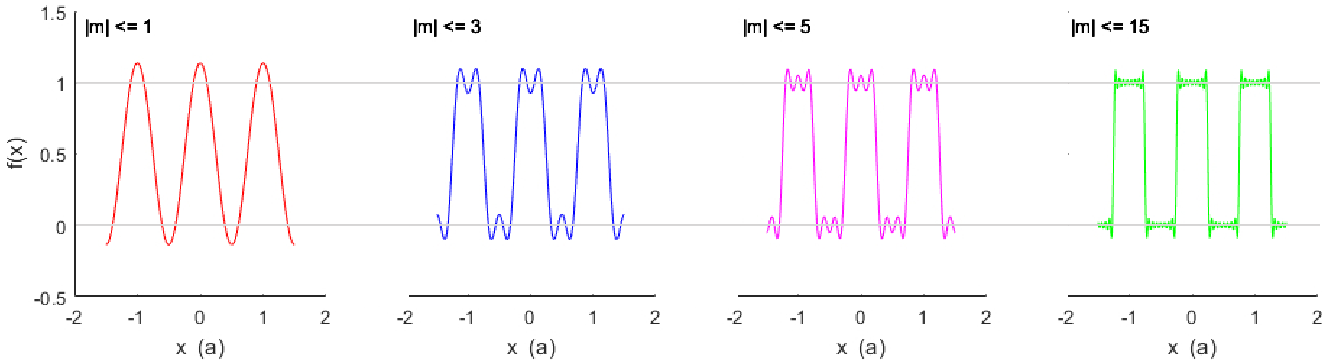

Figure 2: Increasing the number of of spatial frequencies of the Fourier series representing the function shown in Fig. 1 results in the sum converging to the original function.

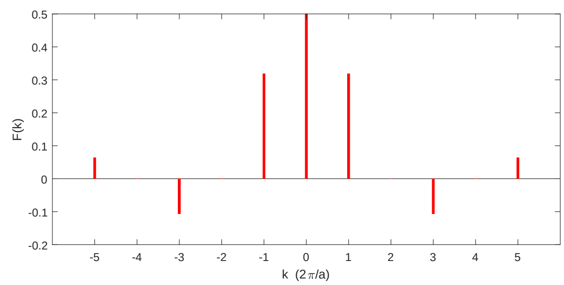

Figure 3: The frequency spectrum of the amplitude, for the function shown in Fig. 1 ($m \leq 5$).

2.1.2 The Fourier Transform

A non-periodic function $f(x)$ can also be represented by different angular spatial frequencies, but the representation will no longer just contain harmonics; it will contain a continuous distribution of angular spatial frequencies k. As any text on Fourier series will discuss, one route to the Fourier transform is to take the limit $a \to \infty$ for the period of a Fourier series. Then the harmonics mk_0 can be replaced by a continuous variable k and the sum over m by an integral over $\mathrm{d}k$. The amplitude of each frequency is then given by

$$F(k) = \mathcal{F}\{f\}(k) \equiv \int_{-\infty}^{\infty} \mathrm{d}x\, f(x)\, \mathrm{e}^{-\,\mathrm{i}kx}\ . \tag{12}$$

$F(k)$ is the Fourier transform of $f(x)$. On taking the inverse transform of the transform, we obtain the original function:

$$f(x) = \mathcal{F}^{-1}\{F\}(x) \equiv \int_{-\infty}^{\infty} \frac{\mathrm{d}k}{2\pi} F(k)\, \mathrm{e}^{\mathrm{i}kx}\ . \tag{13}$$

Notice that we write the integral in Eq. (13) in terms of $\frac{\mathrm{d}k}{2\pi}$ to remind ourselves of the idea that the "complementary" Fourier variables here are actually x and $\lambda^{-1} = k/2\pi$.[4]

[4]Some authors write Fourier-transform pairs as $F(\nu) = \int_{-\infty}^{\infty} \mathrm{d}x\, f(x)\, \mathrm{e}^{-\,\mathrm{i}2\pi\nu x}$ and $f(x) = \int_{+\infty}^{\infty} \mathrm{d}\nu\, F(\nu)\, \mathrm{e}^{\mathrm{i}2\pi\nu x}$, with $\nu \equiv \lambda^{-1}$, which puts the factors of 2π in the exponential terms. Others put $1/\sqrt{2\pi}$ symmetrically in front of *both* transform and inverse. As a further confusion, engineers usually write the formulas using $\mathrm{j} = -\,\mathrm{i}$.

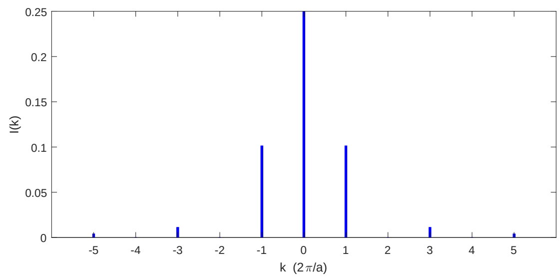

Figure 4: Idealized frequency spectrum of the intensity, for the function shown in Fig. 1 ($m \leq 5$). Compare Fig. 3.

2.1.3 Connection between Fourier series and Fourier transform

This section can be omitted in first reading without affecting understanding of subsequent sections.

For a periodic function, we can connect the Fourier series to the Fourier transform using the convolution and the comb function.

The comb function: The comb function is defined as a regular array of Dirac δ-functions located at $x = na$:

$$\mathrm{comb}_a(x) \equiv \sum_{n=-\infty}^{\infty} \delta(x - na). \tag{14}$$

The comb function is a periodic function with period a and in the context of optics represents an infinite series of infinitely narrow slits arranged along the x-axis.

The fourier transform of the comb function is again a comb function in fourier space (a non trivial result!):

$$\mathcal{F}\{\mathrm{comb_a}\}(k) = \frac{2\pi}{a} \sum_{m=-\infty}^{\infty} \delta(k - mk_0)\,, \qquad k_0 \equiv \frac{2\pi}{a}, \tag{15}$$

The convolution of two functions: Recall that the convolution of two real functions f and g is defined as

$$(f * g)(x) \equiv \int_{-\infty}^{\infty} f(x')\, g(x - x')\, dx'. \tag{16}$$

For example, the convolution of a function $f(x)$ with a δ-function at the location $x = a$, i.e. $g(x) = \delta(x - a)$ is:

$$(f * g)(x) = \int_{-\infty}^{\infty} f(x')\, \delta(x - a - x')\, dx' = f(x - a), \tag{17}$$

which simply shifts the origin of the original function $f(x)$: $f(x) \Rightarrow f(x - a)$.

An important property of the convolution of two functions is that its fourier transform is the product of the fourier transforms of the original functions:

$$h(x) = (f * g)(x) \Rightarrow H(k) = F(k)G(k) \tag{18}$$

Periodic function as a convolution: Now consider an arbitrary periodic function $f_p(x)$ with period a, and let $f(x)$ denote its restriction to a single period, e.g. on $[-a/2, a/2)$. The periodic extension of $f(x)$ can be written as a convolution with a comb (Eq. (14)):

$$f_p(x) = \big(f * \mathrm{comb_a}\big)(x), \tag{19}$$

Since the comb is a series of δ-functions, the convolution with the comb can be visualized as placing a copy of $f(x)$ at the shifted locations $x = na$.

Taking the Fourier transform and using the fact that convolution in space becomes multiplication in fourier space gives

$$\mathcal{F}\{f_p\}(k) = F(k)\, \mathcal{F}\{\mathrm{comb}_a\}(k). \tag{20}$$

so that

$$\mathcal{F}\{f_p\}(k) = \frac{2\pi}{a} \sum_{m=-\infty}^{\infty} F(mk_0)\, \delta(k - mk_0) \tag{21}$$

with $k_0 = 2\pi/a$. This shows that the Fourier transform of a periodic function consists of discrete spectral lines at harmonics of k_0, with weights given by the Fourier transform of a single period evaluated at the harmonic values $k = mk_0$.

Equivalently, writing the Fourier series in complex-exponential form,

$$f_p(x) = \sum_{m=-\infty}^{\infty} c_m e^{imk_0x}, \qquad c_m = \frac{1}{a} \int_{-a/2}^{a/2} f(x)\, e^{-imk_0x}\, dx, \tag{22}$$

we see that the coefficients can be expressed as the Fourier transform of a single period sampled at $k = mk_0$,

$$c_m = \frac{1}{a}\, F(mk_0). \tag{23}$$

Thus, the Fourier transform of the periodic extension consists of discrete components at the harmonics $k = mk_0$, whose weights are proportional to the Fourier series coefficients c_m.

In realistic measurements, these ideal δ-functions are broadened into peaks of finite width (e.g. due to finite resolution). In the case where the width is the same for all harmonics, the peak amplitude at $k = mk_0$ remains proportional to $|c_m|$, while the peak intensity scales as $|c_m|^2$. Therefore, a simple predictive model for the observed peak intensities can be obtained directly from the Fourier series coefficients.

Extensions of the Fourier transform and convolution: The Fourier transform and convolution ideas can be extended in several useful ways that are commonly used in Fourier optics (see the references for more detail). We mention two examples here for completeness; their physical meaning will become clearer in later sections:

- *Finite extent (truncation) of the diffracting object.* In many diffraction problems the object is not truly infinite (e.g. a grating with only a finite number of slits, or a limited field of view).

This can be modeled in 1D by multiplying the ideal infinite object $f(x)$ by an aperture (mask) function $m(x)$, such as a rectangular window. In Fourier space, multiplication becomes convolution:

$$\mathcal{F}\{m(x)f(x)\}(k) = \big(M * F\big)(k). \tag{24}$$

Thus truncation broadens and modulates the discrete diffraction orders by convolving them with the Fourier transform of the mask.

- *Extension to two dimensions.* Many diffraction masks are inherently two-dimensional, such as 2D lattices and apertures, leading to diffraction patterns that must be considered in the 2D Fourier plane, $F(k_\mathrm{x}, k_\mathrm{y})$. In this case the Dirac comb generalizes to a 2D comb (a regular lattice of δ-functions), and the Fourier series and transforms, and their correspondence, carries over directly. The 2D convolution theorem similarly enables treatment of a number of practical situations in diffraction and imaging, including the effect of a finite size of illumination source and finite detector resolution, via convolution with the appropriate *point-spread function.*

3 Fraunhofer Diffraction and the Fourier Transform

When light is incident on an obstacle or an aperture, the obstacle modifies the incoming wavefront. For example, a shadow is formed immediately behind the obstacle where light is blocked. However, because light is a wave, it does not propagate strictly in straight lines: it also bends around edges. As a result, the boundary of the shadow is not perfectly sharp.

Further downstream, the light emerging from different parts of the obstacle or aperture spreads out and overlaps. These overlapping waves interfere with one another, producing a spatial intensity pattern known as the *diffraction pattern.* In this section, we develop a model for diffraction and show that, in the far-field (Fraunhofer) limit, the diffraction pattern is closely related to the Fourier transform of the aperture.

3.1 Diffraction

We consider a monochromatic light wave incident on an aperture mask, and we study the intensity pattern observed on a screen a distance z away — the diffraction pattern — as illustrated in Fig. 5a. According to the Huygens principle, every point on a wavefront can be regarded as a source of a secondary spherical wavelet. As a result, the field observed at a point on the screen is the superposition of the wavelets emitted from all points within the aperture.

To be specific, consider a plane wave $e^{i(kz-\omega t)}$ propagating along the z-axis and illuminating an aperture located at $z = 0$. We describe the aperture by its *transmission function* $a(x', y')$, which gives the (possibly complex) field amplitude immediately after the mask. Here $k = 2\pi/\lambda$ is the wavenumber, $\omega = 2\pi\nu$ is the angular frequency, and $\nu = c/\lambda$ is the frequency of the light. The aperture could be a hole (as shown), a slit, or a *Ronchi ruling* (a pattern of alternating transparent and opaque stripes).

An infinitessimal area element $\mathrm{d}x'\,\mathrm{d}y'$ within the aperture acts as a point source that emits a spherical wave. We now calculate the contribution of this element to the field at a point (x, y) in the observation plane located a distance z downstream.

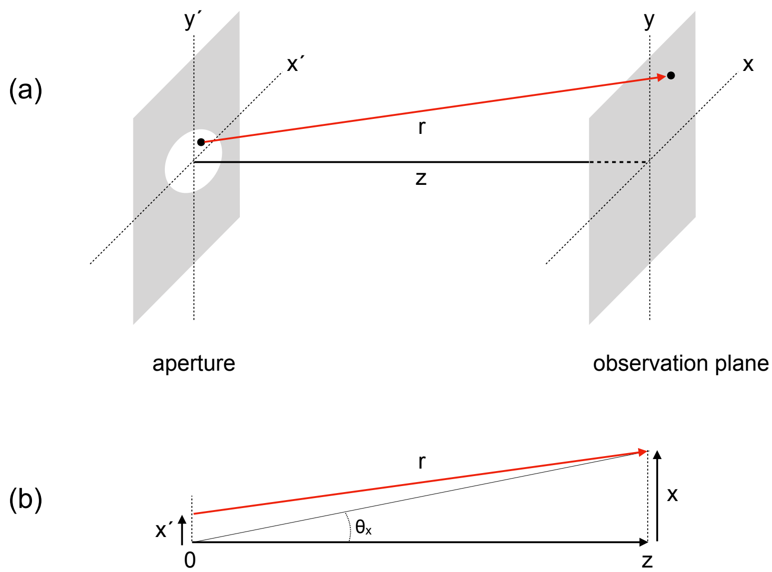

Figure 5: Geometry for Fraunhofer diffraction. (a) Aperture transmission function $a(x, y)$ at $z = 0$ diffracts light observed in a plane at distance z. (b) Simplified one-dimensional geometry.

Dropping the factor $e^{-i\omega t}$ (which will disappear when we compute intensities), we consider only the spatial dependence of the field. Let the vector $\mathbf{r}$ connect the point (x', y') in the aperture plane to the point (x, y) in the observation plane. Its magnitude is

$$r = \sqrt{z^2 + (x - x')^2 + (y - y')^2}. \tag{25}$$

Then the spherical wavelet produced by an aperture element with amplitude $a(x', y')\,\mathrm{d}x'\,\mathrm{d}y'$ has a complex amplitude at (x, y) given by

$$\mathrm{d}E\,(x, y) = a(x', y')\ \mathrm{d}x'\,\mathrm{d}y' \left(\frac{e^{ikr}}{r}\right). \tag{26}$$

Summing (integrating) over the entire aperture gives the total field in the observation plane:[5]

$$E(x, y) = \iint_{-\infty}^{\infty} \mathrm{d}x'\,\mathrm{d}y'\ a(x', y') \left(\frac{e^{ikr}}{r}\right), \tag{27}$$

where we recall, from Eq. (25) that r is a function of x' and y'. In general, this integral must be evaluated using approximations. The two most important regimes are *Fresnel diffraction* (near field) and *Fraunhofer diffraction* (far field), which we discuss next.

[5]The heuristic Huygens formula, Eq. (26), can be derived from a scalar approximation to Maxwell's equations. See the Appendix.

3.1.1 Fraunhofer Diffraction

If the observation plane is very far from the aperture, then we can expand $r(x, y)$ in a Taylor series. To simplify the equations, we drop the y-dependence and focus only on one-dimensional variations, as shown in Fig. 5b. Then, we write

$$r = z\left[1 + \frac{(x-x')^2}{z^2}\right]^{1/2} \approx z + \frac{1}{2z}\left[(x-x')^2\right] = z + \frac{1}{2z}\left(x^2 - 2xx' + x'^2\right) , \qquad (28)$$

and

$$\frac{\mathrm{e}^{\mathrm{i}kr}}{r} \approx \underbrace{\left(\frac{\mathrm{e}^{\mathrm{i}kz}}{z}\right)}_{\text{uniform amplitude}} \underbrace{\left(\mathrm{e}^{\mathrm{i}k\frac{x^2}{2z}}\right)}_{\text{phase factor}} \underbrace{\left(\mathrm{e}^{-\,\mathrm{i}k\left(\frac{x}{z}\right)x'}\right)}_{\text{important!}} \underbrace{\left(\mathrm{e}^{\mathrm{i}k\frac{x'^2}{2z}}\right)}_{\approx 1} . \qquad (29)$$

The first two terms are independent of x' and thus can be taken in front of the integral in Eq. (27). Note that we use the simplest approximation $r \approx z$ in the denominator because the effect on *amplitude* is negligible. The third term is the important one, whose significance is clearer when we realize that $x/z \approx \theta_x$ is the angle the diffracted ray makes with respect to the z-axis (Fig. 5b), in the small-angle approximation.

Finally, the fourth term is approximately unity for z large enough that spherical waves have an approximately planar wavefront. This corresponds to the *Fraunhofer (far-field) regime of diffraction*[6]. Retaining the final factor instead leads to *Fresnel diffraction*, which describes the intermediate (near-field) regime where wavefront curvature cannot be neglected. Fresnel diffraction therefore provides the smooth crossover between the geometric shadow close to the aperture and the Fraunhofer intensity pattern observed far from it.

This lab focuses on the Fraunhofer regime. Equation (27) then becomes (in one-dimensional form)

$$E(x) \approx \left(\frac{\mathrm{e}^{\mathrm{i}kz}}{z}\right)\left(\mathrm{e}^{\mathrm{i}k\frac{x^2}{2z}}\right)\int_{-\infty}^{\infty} \mathrm{d}x'\, a(x')\,\mathrm{e}^{-\,\mathrm{i}k\theta_x x'} \propto \int_{-\infty}^{\infty} \mathrm{d}x'\, a(x')\,\mathrm{e}^{-\,\mathrm{i}ux'} , \qquad (30)$$

where $u \equiv k\theta_x = kx/z$. In other words, up to phase factors that are one for the intensity $I \propto EE^*$, the field is proportional to the Fourier transform of the aperture function, with the Fourier variable u. Compare with Eq. (12).

Going back to the full two-dimensional expression, we have that the *field* at the observation plane is the 2D Fourier transform of $a(x', y')$. The camera (or your eye) records the *intensity*,

$$I(x, y) \propto EE^* \propto \left|\iint_{-\infty}^{\infty} \mathrm{d}x'\,\mathrm{d}y'\, a(x', y')\,\mathrm{e}^{-\,\mathrm{i}(ux'+vy')}\right|^2 , \qquad (31)$$

[6]More precisely, $kx'^2/z \ll \pi$, which implies that $z \gg x'^2/\lambda \sim (\text{aperture width})^2/\lambda$. This condition defines the *Fraunhofer limit* and determines just how far away (z) the observation plane needs to be. If $x \approx 1$ cm and $\lambda \approx 1$ µm, then we need $z \gg (10^{-2})^2/10^{-6} = 100$ m. That is pretty far! However, adding a lens allows the $z = \infty$ plane to be observed at a finite distance (one focal length behind the lens). See Sec. 4 and Fig. 7.

which is just the modulus squared of the two-dimensional Fourier transform in terms of variables $u = k\theta_x = kx/z$ and $v = k\theta_y = ky/z$. These variables are the transverse wavevector components (in the small-angle approximation) and are often referred to as the *spatial frequencies* in Fourier optics. For more details, see the Appendix or Lipson, Chs. 7 and 8.

Example: the Ronchi ruling In Sec. 2.1.1 we introduced a "first-level" picture of spatial filtering for a periodic 1D aperture function: the Fraunhofer diffraction pattern contains a spatial-frequency spectrum of the aperture, and for a periodic aperture, this spectrum consists of discrete diffraction orders, which can be represented using a Fourier series (Fig. 4). More generally, the Fraunhofer pattern is given by the squared magnitude of a Fourier transform, and is therefore a continuous function of diffraction angle.

As a concrete example of a periodic aperture function, consider a Ronchi ruling (a periodic array of slits) with N illuminated slits. The far-field intensity is (see Hecht Eq. 10.31 and Prelab Q1).

$$I(\theta) = I_0 \left(\frac{\sin\beta}{\beta}\right)^2 \left(\frac{\sin N\alpha}{N\sin\alpha}\right)^2, \tag{32}$$

where I_0 is the intensity in the forward direction ($\theta = 0$) due to any single slit, and

$$\alpha = \frac{ka}{2}\sin\theta, \qquad \beta = \frac{kb}{2}\sin\theta, \qquad k = \frac{2\pi}{\lambda}. \tag{33}$$

Here a is the grating period of the Ronchi ruling and b is the slit width (close to $b = a/2$, but it may differ slightly). The N-slit forward intensity is $I(0) = N^2 I_0$, and so the above expression is often normalized to $I(0)$ by writing $I_0 = I(0)/N^2$.

Interpreting Eq. (32): The first factor, $(\sin\beta/\beta)^2$, is the single-slit diffraction envelope of width b. The second factor produces a set of sharp interference peaks at angles corresponding to the grating orders. As N increases, these peaks become narrower and more closely resemble a comb of discrete lines, consistent with the Fourier-series picture introduced in Sec. 2.1.1. In the limit $N \to \infty$, the second factor approaches a Dirac-comb distribution (Eq. (15)) as a function of angle (or equivalently in transverse wavevector), with peak locations determined by the grating period a.

Figure 6 shows a sample diffraction pattern calculated using Eq. (32), together with the corresponding Fourier-series values of the discrete diffraction orders. To make the comparison more realistic, the plot also includes the effect of a finite angular source size, which broadens each ideal diffraction order into a peak of nonzero width. This broadening can be modeled by convolving the ideal diffraction pattern with a source function (here taken to be Gaussian for simplicity). Your source may have a different shape, but the qualitative effect — smoothing and broadening of the diffraction orders — is the same.

General aperture functions For arbitrary aperture functions in 1D or 2D, there may not be a simple analytical expression for the Fourier transform. In these cases, the Fraunhofer diffraction

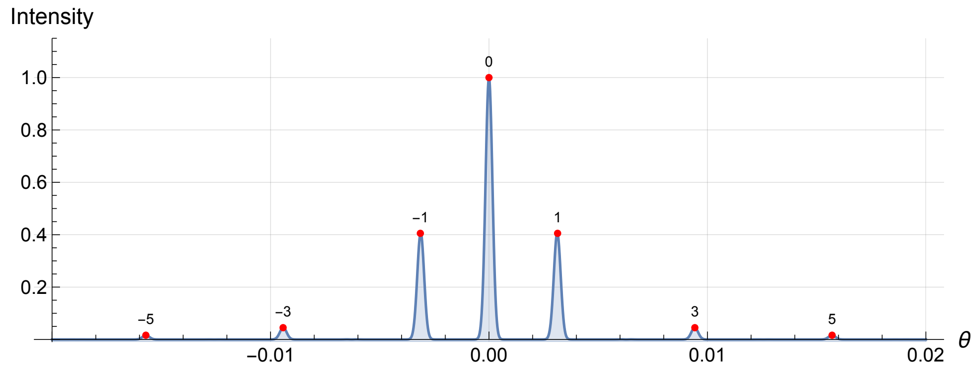

Figure 6: Diffraction pattern from an N-slit aperture, with N large and a finite angular source size. Calculated for $ka/2 = 60$, and $b = a/2$. Red markers denote the value of the corresponding Fourier series of order n (indicated above the marker).

pattern can be modeled numerically by evaluating the Fourier transform of the aperture transmission function using a discrete fast Fourier transform (FFT). In 1D, or where the y-dependence is irrelevant as for long slits, the field in the Fourier plane is proportional to the FFT of $a(x)$, and the intensity is proportional to the squared magnitude.

The following Python snippet computes the far-field intensity pattern for a 1D aperture $a(x)$. The FFT can be computed using either `numpy.fft` or `scipy.fft`. The code is in terms of `numpy.fft` for simplicity and portability.

Listing 1: Numerical Fraunhofer diffraction from a 1D aperture $a(x)$ using an FFT

```
import numpy as np
import matplotlib.pyplot as plt

# --- Spatial grid ---
N = 2**14           # number of samples (power of 2 is fast)
L = 5e-3            # total spatial window size (m)
dx = L / N
x = (np.arange(N) - N/2) * dx

# --- Define an aperture transmission function a(x) ---
# Example: Ronchi ruling (period a, slit width b) with finite extent
a =100e-6          # period (m)
b = a/2             # slit width (m)
aperture = ((x % a) < b).astype(float)

# Optional: limit to a finite number of illuminated slits (window)
aperture *= (np.abs(x) < 0.5e-3)

# --- Fraunhofer field: Fourier transform of the aperture ---
E = np.fft.fftshift(np.fft.fft(np.fft.ifftshift(aperture))) * dx
I = np.abs(E)**2

# --- Fourier axis: u, equivalent to transverse wavevector k_x ---
u = 2*np.pi * np.fft.fftshift(np.fft.fftfreq(N, d=dx))

# --- Fourier axis: angle variable, scaled to natural units ---
lam = 514.0e-9 # in m
k = 2*np.pi / lam
theta_x = u / k
theta_x /= (lam / a) # scaled
```

```
# --- Plot ---
from matplotlib.ticker import MultipleLocator

plt.plot(theta_x, I / np.max(I))
plt.xlabel(r"$\theta_x / (\lambda/a)$")
plt.ylabel("Normalized intensity")
plt.title("Fraunhofer diffraction pattern (1D FFT)")
plt.grid(True)
plt.xlim(-3.5, 5.5)

ax = plt.gca()
ax.xaxis.set_major_locator(MultipleLocator(1))   # major ticks every 1
plt.show()
```

Extensions: Note that the code above does not include the effect of a finite angular source size (or partial spatial coherence), which broadens the diffraction orders as discussed earlier. Incorporating this effect — for example by convolving the ideal diffraction pattern with a source angular distribution — is left as an exercise. As a second extension, you can generalize the calculation to two dimensions by defining an aperture transmission function $a(x, y)$ on a 2D grid and using `np.fft.fft2`. The resulting 2D intensity pattern can be compared directly with camera images. This is particularly useful for more complicated 2D diffraction objects and the effect on them from spatial filtering that may be explored in advanced projects.

4 The Lens as a Fourier Transformer

We can now start to understand the role of the lenses. Figure 7 shows a slit (aperture function) illuminated by collimated light. In the Huygens picture, each point in the slit generates tiny wavelets that propagate away from the slit towards the lens. At $z \to \infty$, they will form the Fraunhofer diffraction pattern of the slit, but we can use a trick to view the far-field pattern without having to put a screen very far away.

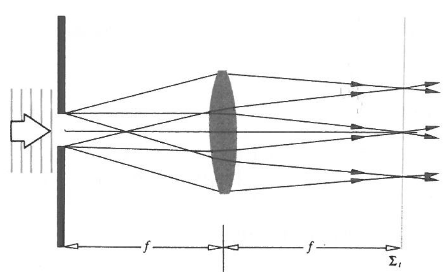

Figure 7: (Hecht Figure 11.5) The light diffracted by an aperture converges to form the far-field diffraction pattern at the back focal plane of the lens.

The trick is to insert a lens behind the aperture and then to put the observation screen a focal length behind the lens (the *back focal plane*), as illustrated in Fig. 7. Light propagating in the same direction, i.e., with the same $\mathbf{k}$, is focused by the lens onto the same point in the back focal plane of the lens. The lens causes the diffraction pattern to form at the back focal plane, which is thus called the transform or Fourier plane. If we place a screen at the Fourier plane, we can view the far-field diffraction pattern of the source spread across it.

A further subtle point illustrated in Fig. 7 is that the aperture (e.g., slit or Ronchi ruling) should be placed in the *front focal plane* of the lens (i.e., at a distance f in front of the lens). The full justification for this claim is given in the Appendix. Here, we can offer an intuitive argument: The light from the aperture diverges out as a spherical wave, whose curvature is just cancelled by the lens (remember that rays from one focal length in front of a lens are collimated and made parallel). The quadratic phase term $\mathrm{e}^{ik(x^2+y^2)/(2z)}$ coming from the 2D version of Eq. (30) is then canceled, making the field at the back focal plane an *exact* Fourier transform of the aperture function.

Note that if one is simply recording then intensity in the Fourier plane, the phase term becomes 1 and does not contribute. Thus, in the first part of this lab, it does not matter where the aperture is relative to the lens. But in the second part (spatial filtering), we will be attempting to alter the Fourier-transform *field*, and then it does matter.

4.1 Abbe theory of imaging and spatial filtering

We now extend our wave-based description of diffraction to *image formation by a lens*, following the classical theory developed by Abbe. A periodic object such as a Ronchi ruling makes the key ideas especially easy to visualize.

Two-stage process of image formation (Abbe theory). In Abbe's view, image formation by a lens occurs in two successive diffraction stages, as illustrated in Fig. 8. First, the lens produces the *Fraunhofer diffraction pattern* of the object in its back focal plane, often called the *Fourier plane*. As we learned in the previous section, for a periodic object like a Ronchi ruling this plane contains a set of diffraction orders, i.e. discrete spatial-frequency components of the field after transmission through the object. Second, these diffraction components propagate outward from the Fourier plane and interfere in the image plane to form the final image. Thus, from the wave point of view, imaging is a two-step process: the object field is decomposed into spatial frequencies at the Fourier plane and then recombined in the image plane. (A detailed discussion and derivation can be found in Lipson, Ch. 12 whiel a brief qulaititaive dicussion can be found in Hecht Ch. 13)

The 4-f imaging system. If the object is placed at the *front focal plane* of a single lens, the outgoing wave from a point on the object is collimated and the image is formed at infinity rather than at a finite distance. To create a real image at a convenient location while keeping the Fourier plane accessible, a two-lens system known as a *4-f setup* is used (Fig. 9a). It consists of an objective lens of focal length f_1 and an imaging (or "magnification") lens of focal length f_2, separated by $f_1 + f_2$. The total length from object plane to image plane is $2(f_1 + f_2)$, which is $4f$ when $f_1 = f_2 = f$.

In a 4-f system, the two-stage diffraction picture is especially clear. The objective lens produces the Fraunhofer diffraction pattern in its back focal plane. That plane is also the front focal plane

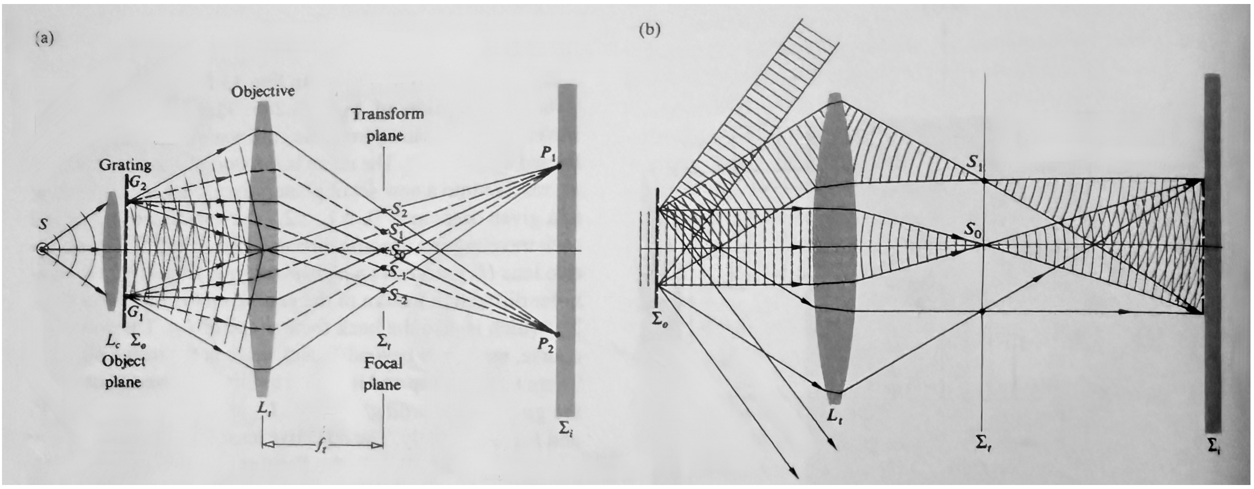

Figure 8: Abbe theory of imaging (Hecht Fig. 13.28). (a) Two-stage process of diffraction. A periodic object (e.g. a Ronchi ruling) is decomposed by the objective lens into spatial-frequency components in the back focal plane (the *Fourier plane*), where the diffraction orders appear. In the second stage of diffraction, the field from these orders then propagate to the image plane and interfere to form the final image. Thus, imaging can be viewed as a two-step diffraction process: Fourier decomposition followed by recombination. (b) The finite aperture of the lens truncates the highest orders of dffraction, which are responsible for the fine detail in the image, and results in a finite resolution limit

of the second lens, which performs another Fourier transform and produces the image in its back focal plane. Thus, the 4-f system performs a *double Fourier transform.* Up to an (irrelevant) scale factor, a double Fourier transform reproduces the original function, but with an inversion:

$$\mathcal{F}\left\{\mathcal{F}\{a(x)\}\right\}(x) = 2\pi\, a(-x), \tag{34}$$

and similarly in 2D. The minus sign corresponds to image inversion (Fig. 9b). If the lenses have different focal lengths, there is also a magnification factor $M = |f_2/f_1|$ associated with the image i.e. the image intensity is proportional to $\left|a\left(-\frac{f_2}{f_1}x\right)\right|^2$.

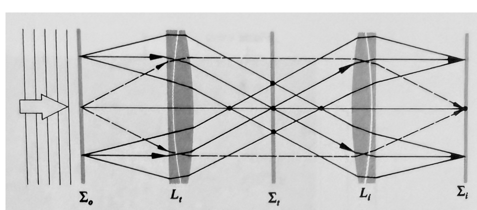

Figure 9: 4-f imaging system and spatial filtering (Hecht Fig. 13.30). (a) Two lenses with focal lengths f_1 and f_2 separated by $f_1 + f_2$ form a 4-f system. The figure shows the symmetric case of $f_1 = f_2$. The back focal plane of the objective lens is a Fourier plane where the Fraunhofer diffraction pattern of the object appears and where spatial filters can be inserted. (b) The second lens performs a second Fourier transform and forms an image at its back focal plane, with magnification $M = -f_2/f_1$.

Spatial filtering. A key advantage of the 4-f system is that it provides physical access to the Fourier plane (the back focal plane of the objective lens). By placing a filter mask in this

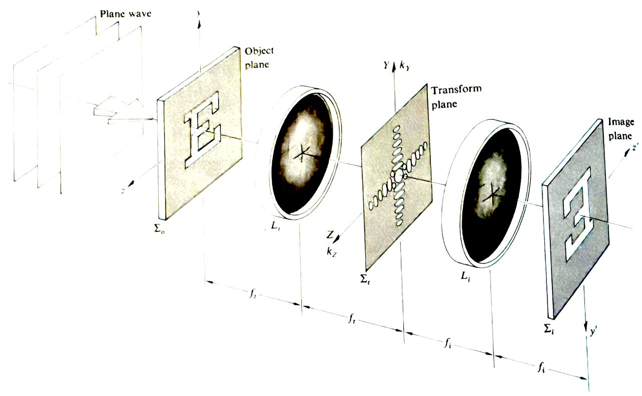

Figure 10: llustration of 4-f setup (Hecht Fig. 13.31. Incident plane wave on the object with a transmission function forming the letter “E”, leads to the diffraction pattern in the transform (Fourier) plane, and finally the image of eh object in the Image plane. Note the inverted image. Also note that the transverse coordinate labels in the figure differ from those used in this labscript.

plane, we can selectively transmit or block certain spatial frequencies of the object field — this is called *spatial filtering*. For a Ronchi ruling, the diffraction orders contain the information required to reconstruct the sharp square-wave image. Blocking higher diffraction orders removes high spatial frequencies, which smooths sharp edges and reduces image contrast. Conversely, removing the lower orders will accentuate the edges of the images. This idea underlies one of the main experiments in this lab: reconstructing a modified image using only selected diffraction orders. Figure 11 shows an example of modification of an image including selection of particular features using spatial filtering.

Diffraction-limited resolution (Abbe criterion). Abbe also used this Fourier interpretation to derive a fundamental limit on the resolution of an imaging system. A lens of finite diameter can only collect diffracted light up to a maximum angle, and therefore only a limited range of spatial frequencies can reach the image plane. High spatial frequencies correspond to fine details in the object; if these components are lost, the smallest features cannot be resolved.

For a grating object of period d illuminated by a plane wave at normal incidence, the first diffraction orders occur at

$$\sin\theta_1 \approx \frac{\lambda}{d}. \tag{35}$$

To form an image with contrast, at least the zeroth and one first order must pass through the objective lens. If the objective has *numerical aperture* $\mathrm{NA} = n\sin\theta_{\max}$ (with $n \approx 1$ in air), then the requirement $\sin\theta_1 \leq \sin\theta_{\max}$ leads to the Abbe resolution limit

$$d_{\min} = \frac{\lambda}{\mathrm{NA}}. \tag{36}$$

With suitable illumination (e.g. Köhler illumination, which effectively provides a range of incident illumination angles), one can improve the resolution by roughly a factor of two. In this case, the

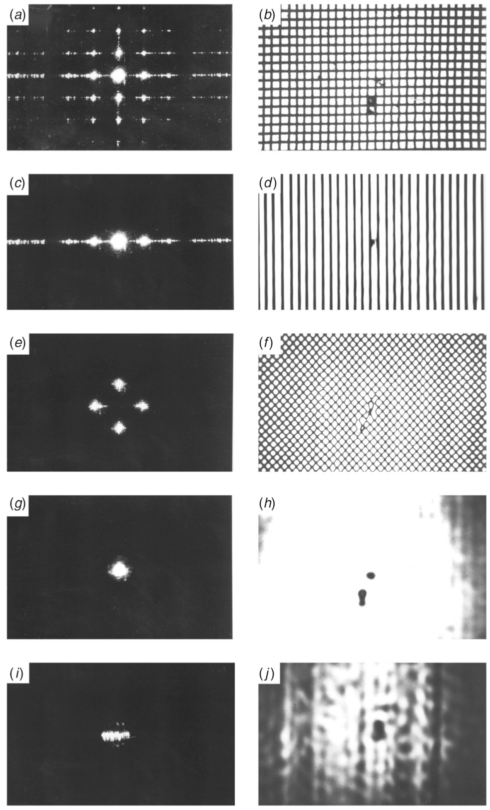

Figure 11: Effect of spatial filtering in the Fourier plane (Lipson Fig. 12.6). Left column shows the diffraction pattern in the Fourier plane while the right column shows the corresponding images. (a-b) Unmodified image of some dust overlaid with regular mesh. (d-j) Modified cases, for example (g,h) with mesh removed from the image.

standard Abbe criterion is

$$d_{\min} = \frac{\lambda}{2\,\mathrm{NA}}. \tag{37}$$

Modern *super-resolution* techniques can surpass this classical diffraction limit using additional information or nonlinear optical effects, but these methods are beyond the scope of this lab.

Rayleigh criterion (point sources). For resolving two nearby point sources, a closely related criterion is the *Rayleigh criterion.* For a circular aperture, the minimum resolvable separation in the image is approximately

$$\Delta x_{\min} \approx 0.61\,\frac{\lambda}{\mathrm{NA}}. \tag{38}$$

This is consistent in scale with Eq. (37), differing only by a numerical factor due to the specific shape of the diffraction pattern (Airy disk).

Practical limitations. In practice, many factors reduce resolution before the diffraction limit is reached, including lens aberrations, imperfect alignment, finite detector resolution, and stray light. High-quality optics can approach the diffraction limit, particularly near the center of the field of view. In this lab we use achromatic doublets, which perform significantly better than simple single-element lenses for the focal lengths used.

5 Köhler illumination

Illumination in optical imaging plays a much more important role than simply "lighting up" the sample. In the simplest presentation of Abbe imaging theory, the object is illuminated by a monochromatic plane wave at normal incidence. To perform spatial filtering and investigate Abbe imaging in this lab, we would therefore like illumination that is as close as possible to a plane wave.

In practice, our light source is an LED, which is an *extended* (spatially broad) source. If a *point* source is placed at the front focal plane of a lens, the lens produces a collimated output. However, because an LED has a finite chip size, it can only be *partially collimated*: different points on the LED produce outgoing plane waves at slightly different angles. The resulting illumination therefore consists of a *range of incident angles* rather than a single plane wave. In this lab, we want to be able to control that angular spread.

At the same time, we learned above that high-resolution imaging can benefit from a large cone of illumination angles: in Abbe theory, suitable illumination can improve resolution by up to a factor of two. Thus, depending on the experiment, we may want either (i) nearly plane-wave illumination (small angular spread) for spatial filtering, or (ii) broad-angle illumination (large angular spread) for highest-resolution imaging.

It is also useful to control the *region of illumination* on the object (the illuminated field of view). For example, we may want to illuminate only the central part of the mask to improve optical performance, or to isolate a single feature on a complex aperture.

Key idea of Köhler illumination. Köhler illumination provides independent control of:

1. the *illumination field of view* (set by the *field iris*), and
2. the *illumination angular spread* (set by the *aperture iris*).

This independent control is achieved by imaging the field iris onto the plane of the object, while simultaneously imaging the source onto the aperture iris. A simplified schematic of the imaging path and illumination system used in this lab is shown in Fig. 12.

In the setup, the LED is approximately collimated by a collector lens and then passes through a *field iris* and a field lens, followed by a condenser lens that illuminates the object. The field and condenser lenses form a 4-f relay so that the field iris is imaged onto the object plane. Adjusting

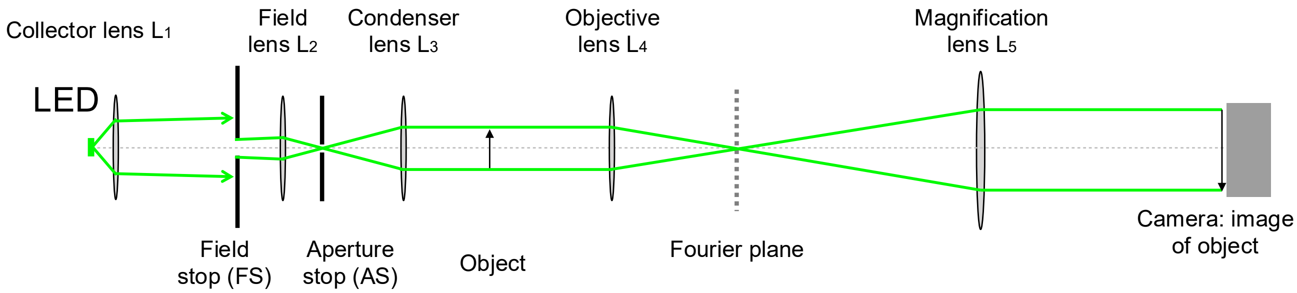

Figure 12: Imaging path for the experiment including the optics for Köhler illumination. See text for description.

the field iris therefore changes the illuminated region on the object, but does not significantly change the cone of illumination angles.

A second stop — the *aperture iris* (or, in some experiments, a pinhole) — is placed at the front focal plane of the condenser lens. The preceeding optics form an image of the LED chip near the aperture iris plane, so adjusting the aperture iris controls the effective size of the source. Because the stop lies in the front focal plane of the condenser, it controls the angular spread (the illumination NA) at the object, while leaving the illuminated field of view largely unchanged. The effects of the aperture and field irises are illustrated qualitatively in Figs. 13 –14.

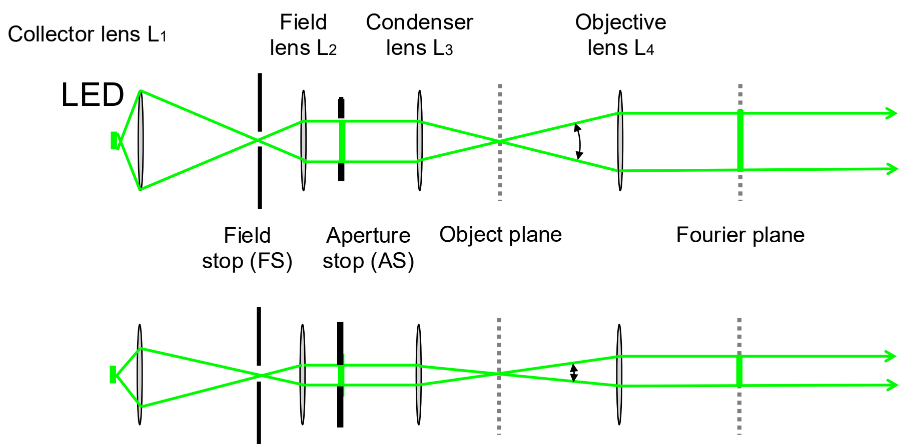

Figure 13: Aperture stop effect in Köhler illumination. The *aperture stop* iris approximately coincides with an image of the source (as indicated by a vertical green bar) and sets the angular cone of illumination on the object (illumination NA). Two cases for the aperture stop are shown: large diameter (top) and small diameter (bottom) with large and small illumination cone, respectively, at the object. With the aperture stop nearly closed (or replaced by a pinhole), the object is illuminated with a narrow cone of angles (quasi-plane-wave illumination). Note that the aperture stop, which is also imaged at the Fourier plane, also controls the effective size of the source spot size there: a sufficiently small spot size produces well-separated diffraction orders.

Finally, note that the aperture iris is imaged onto the back focal plane of the objective lens. Changing the aperture iris therefore affects the sharpness of the diffraction orders observed in the Fourier plane.

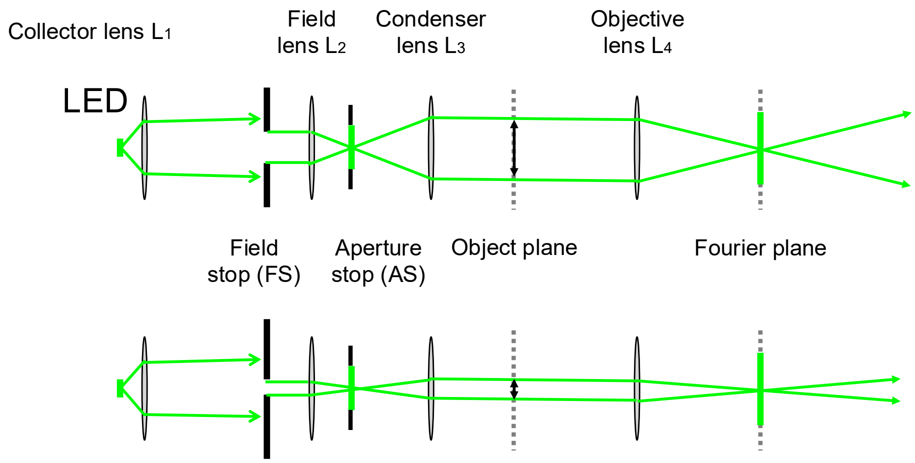

Figure 14: Field stop effect in Köhler illumination. The *field iris* is imaged onto the object plane and sets the illuminated region (field of view). Two cases for the field stop are shown: large diameter (top) and small diameter (bottom) with large and small illumination region, respectively, at the object. Note that the image of the LED source itself (green bars) do not coincide with the object, suppressing non-uniform illumination effects from the LED structure.

Illumination used for spatial filtering. For the spatial filtering experiments in this lab, we replace the aperture iris with a small pinhole to approximate a point source. This produces illumination with a small angular spread, i.e. a quasi-plane wave at the object. In practice, the pinhole diameter is chosen to be smaller than the diffraction-order spacing of the Ronchi ruling, so that the individual diffraction orders are cleanly separated in the Fourier plane.

One of the most valuable aspects of Köhler illumination is that the image of the LED source is *not* formed on the object plane. Instead, the source is imaged onto Fourier plane, which produces a more uniform illumination across the object and avoids projecting spatial structure of the LED chip onto the sample.

What you will do in this lab In this experiment you will use a 4-f optical system to observe both the Fourier-plane diffraction pattern and the image of one-dimensional aperture masks. For the Ronchi ruling, you will:

- record Fraunhofer diffraction patterns in the Fourier plane and identify the dominant spatial-frequency components (diffraction orders);
- perform spatial filtering by blocking selected spatial frequencies and observe how this modifies the reconstructed image;
- compare measured diffraction-order amplitudes to a Fourier-series (or Fourier-transform) model by extracting peak intensities and relating them to Fourier coefficients;
- reconstruct predicted images using truncated/filtered Fourier-series models and compare these predictions directly to measured images after spatial filtering;

- investigate edge ringing (Gibbs phenomenon) by varying the cutoff order $m_{\max}$ and quantitatively comparing the observed overshoot/ringing to the Fourier-series prediction;
- relate the highest transmitted spatial frequency (set by the aperture iris and by the objective NA) to the smallest resolvable feature size, and compare with the Abbe resolution criterion.

6 Experiments

Equipment List

Note the part numbers listed below are in most cases from Thorlabs and additional information on the parts can be retrieved from thorlabs.com.

- Two optical rails, one long and one short.
- Post holders on rail saddles. Most of the required postholder saddles are newer shiny black Thorlabs ones.
- Light source:
  - Light-emitting diode (LED) (λ = 525 nm, bandwidth 25 nm (LED525L) or 35 nm (LED528EHP)) combined with a $f = 20$ mm or 25 mm collector lens (L1) in an adjustable collimation tube.
  - Bandpass filter, center 515 nm, 10-nm bandpass FWHM.
  - Large mounted iris for blocking stray light.
- Kohler illumination optics:
  - Field stop: 0.5" diameter iris.
  - Field lens (L2) $f = 50$ mm plano-convex lens.
  - Aperture stop: 200 μm pinhole with fine X-Y adjustment (or 1" iris).
  - Condenser lens (L3): $f = 100$ mm achromatic doublet lens (AC254-100-A-ML) with fine X-Y adjustment via slip-ring mount.
- Objects:
  - Fixed orientation slit with adjustable width and rotatable slit with adjustable width.
  - X or X-Y adjustable sample holder
  - Test target microscope slide with 18 Ronchi ruling spacings ranging from 1.25 line pairs (one light line and one dark line) per millimeter (lp/mm) to 250 lp/mm (R1L3S6P)
  - Ronchi ruling test target, 10 lp/mm (R1L3S12N)
  - Various Ronchi rulings on 1" square thick glass
  - Birefringent resolution target, 2" x 2" (R2L2S1B)
  - Birefringent Ronchi ruling.
- Imaging and spatial filtering:
  - Objective lens (L4): $f = 150$ mm achromatic doublet lens (AC254-150-A-ML) with fine zoom focus adjustment.
  - Magnification lens (L5): 2" diameter $f = 300$ mm achromatic doublet lens (ACT508-300-A-ML)
  - Beamsplitter (BS).
  - Fourier imaging lens (L5): 2" diameter $f = 100$ mm achromatic doublet lens (AC508-100-A-ML)
  - Optional: 1 fixed diameter diaphrargm and 1 variable iris diaphragm
  - 3 translation stages: for transverse adjustment of Fourier camera and Fourier plane mask.
  - Various mounts for masks etc.
  - FLIR camera (please see *Protocol: Using the FLIR Blackfly Camera*)
  - Lens tube: screws onto camera front to reduce background room light on CCD.
  - Labview image acquisition program: `Dual_Camera_Line_Profile_v2.vi`
  - Image-processing software, e.g., ImageJ (please see *Protocol: Using ImageJ*)
- Other:
  - Mirror for autocollimation and other tasks.

Notes and optics handling precautions:

The full setup for spatial filtering with simultaneous imaging of the object and its diffraction pattern in the Fourier plane is illustrated in Fig. 15. A photograph of the setup is shown in Fig. 16.

Lenses and other optical elements are mounted on 1/2" posts 4"-6" long. The posts fit in post holders for height adjustment and the post holders are attached to saddles for placement on an optical rail. Most of the post holders that are required are newer shiny anodized black ones. There are some older components that may require the use of older post holder saddles.

Most of the lenses are AR coated and should be handled with care - they cost more than $100 per lens. Avoid touching the surfaces and only clean them after discussing with an instructor.

The objects such as Ronchi rulings on microscope slides (Figs. 17-18) are the most delicate and expensive. When mounting them in the microscope slide holder, always wear gloves and only hold them from their edges. Never touch the surfaces of the slide and do not attempt to clean the slide surfaces. Return them to their plastic protective storage cases when not in use. The coating side is closest side to you when the writing is the correct orientation.

The lenses are other elements are mostly mounted in threaded lens tubes, which allow the optics to be handled without touching the optical surfaces. The lens tubes can be temporarily unscrewed from the post holder mounts for alignment purposes, and the returned without loss of alignment.

Another convenient way to temporarily remove an element from the optical rail without loosing its position and alignment is to add an empty saddle adjacent to it. You can remove the optical element still attached to its saddle, and return it to its original position quite accurately using the surface of the adjacent saddle to index the position. Make sure when returning a saddle to the rail that it is well seated.

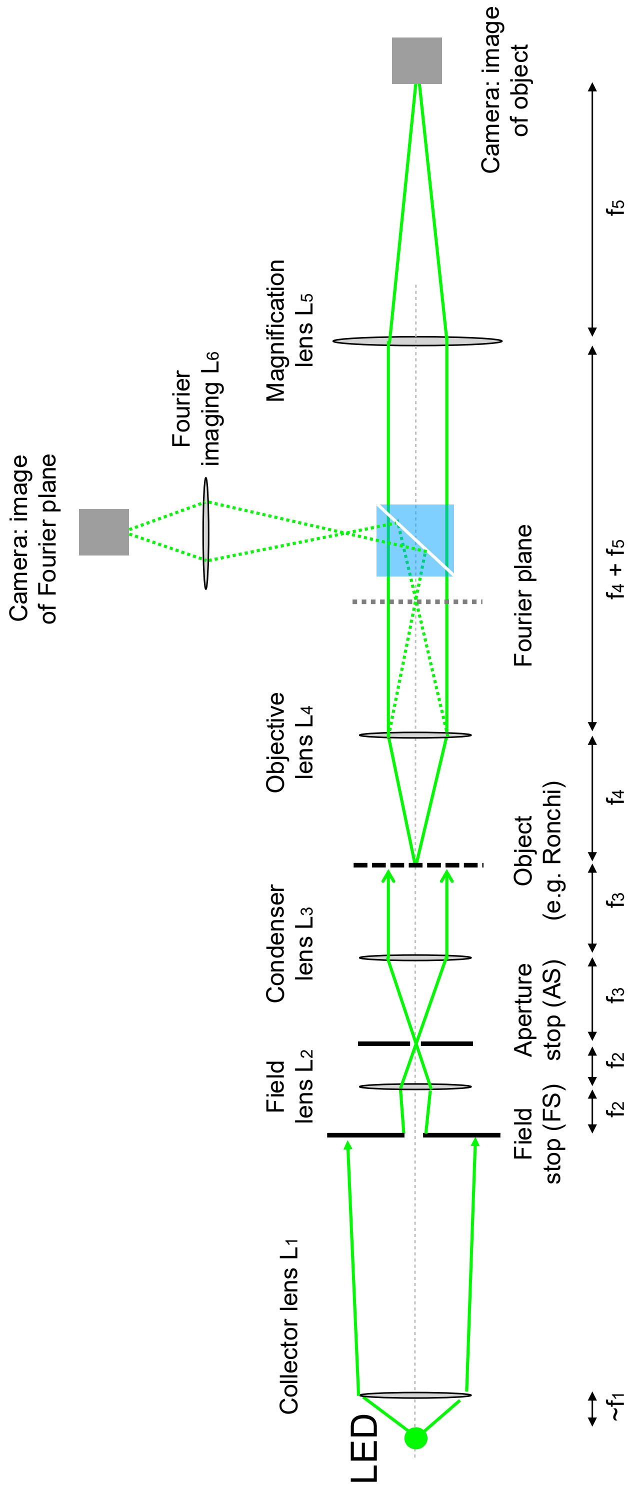

Figure 15: Setup for simultaneous imaging of an object and its diffraction pattern. Optics from LED to teh Object realized Köhler illumination setup. Focal lengths are listed in the equipment list and various distances are noted int he setup descriptions.

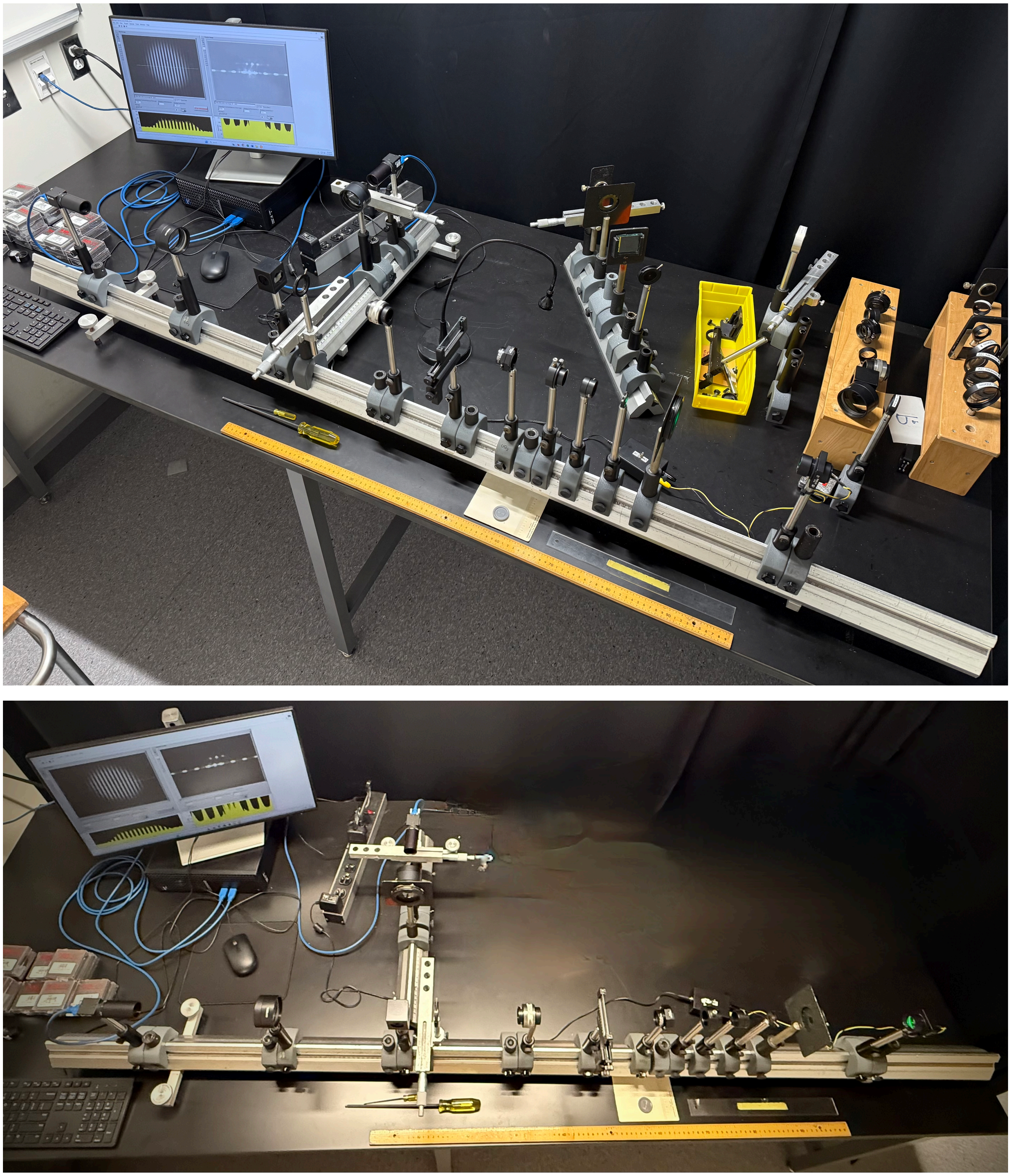

Figure 16: Photographs of the spatial filtering apparatus with simultaneous imaging of the object and its diffraction pattern.

Figure 17: Variable line grating test object. The variable-frequency gratings are 1.25, 1.67, 2.08, 2.50, 2.86, 3.33, 3.85, 4.17, 5.00, 6.67, 10.0, 12.5, 16.67, 26.0, 50.0, 100, 200, and 250 LP/mm.

Figure 18: Ronchi ruling test object with 10.0 LP/mm.

6.1 Setup Part 1

Short summary

1. Check collimation of LED.
2. Locate magnification lens position using a distant object.
3. Place objective lens.
4. Place object and focus.
5. Set up illumination: condenser lens and pinhole.
6. Place field lens and field aperture.
7. Calibrate the imaging with a known grating spacing.
8. Locate the Fourier plane with a viewing card.
9. Place a camera temporarily to the Fourier plane and verify diffraction order spacing with a known grating spacing.

Notes

- Use the rail saddles with the newer shiny black Thorlabs post holders. You have a few 3" and 4" post-holder heights for special tasks (see below). The only exceptions are translation stages and older large irises, which require saddles with the older post holders.
- It is normal to cycle back and redo parts of the alignment and setup. Do not expect to get it right the first time.
- Many alignment tolerances are at the 0.5–1 mm level.
- **Rule:** Do not compensate for misalignment of one element by readjusting optics that you have already aligned and fixed. Always correct the earliest misaligned element in the setup sequence.
- For more alignment tips, see the course handout *The Optical Systems Guide*.

1. **Verify collimation of LED source**

   Place the LED source (LED + collector lens) near the far end of the rail, about 20 cm from the rail end. Set the output beam height to 21.5 ± 0.5 cm above the top surface of the optical rail. Ensure the LED source is mounted over the center of the rail (saddles do not always sit squarely), and ensure the beam propagates approximately along the rail centerline.

Best collimation is obtained by adjusting the collector-lens position so that an image of the LED chip (e.g. shadows of bond wires) forms very far away (Fig. 19). Ideally, flip the LED source around and project across the room onto a wall (∼5 m away). A mirror near the LED can also redirect the beam toward an open wall space. Adjust the lens–LED spacing until the chip image is sharp at the far wall. If needed, ask an instructor how to change the collector lens–LED distance.

Success criterion: the beam will still diverge noticeably, but it should not form a clear waist (focus) anywhere along the rail. It is usually sufficient to verify this qualitatively without making adjustments; only adjust the collector lens–LED spacing if the beam is clearly converging to a focus within the setup. Do not spend excessive time trying to make the beam perfectly collimated; the LED source cannot produce a laser-like collimated beam.

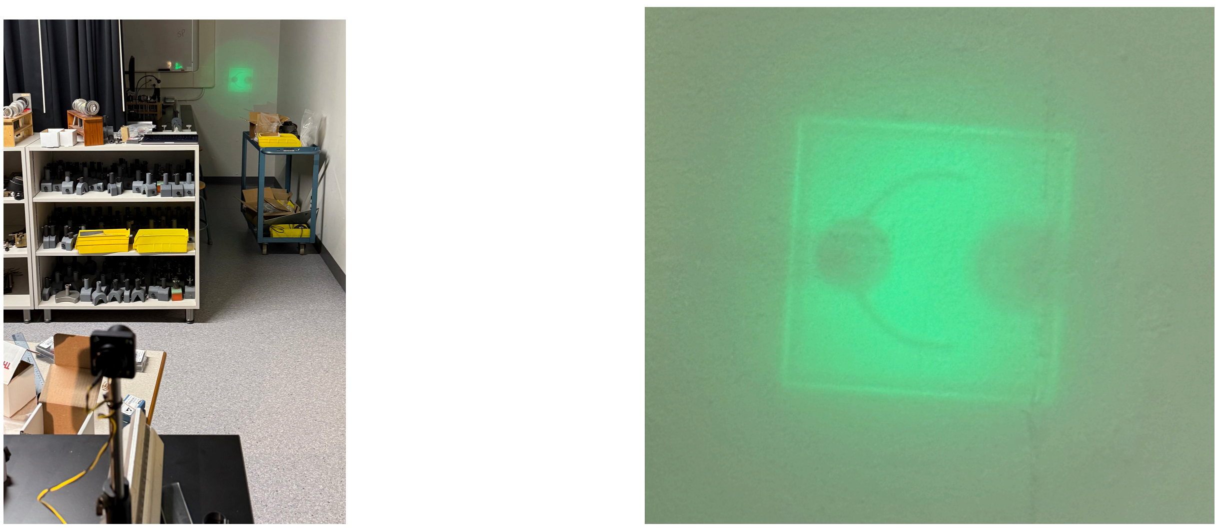

Figure 19: Verifying LED source collimation: Image fo LED source forms far away (∼ 5 m) with no intervening focus.

2. **Place camera and align LED** Identify the two cameras in the setup. The USB cameras should already be connected; avoid disconnecting the USB cables. Open the LabVIEW acquisition program `Dual_Camera_Line_Profile_v2.vi`, which displays two live camera streams. The real-space camera output is on the left panel. **This is the one you should use for now.** Ignore the Fourier camera output until Setup Round 2.

   Mount the camera on a 6” post (if not already done) and attach it to the optical rail using a saddle with a 3” post holder. Place the camera about 30 cm from the rail end closest to the computers (LED–camera distance ∼150 cm). Remove the lens tube (if installed) so that the CCD face is exposed and you can see where the beam lands.

   ~~Mount the camera on a 6” post (if not already done) and mount it to the optical rail using a saddle with a 3” post holder. Place the camera about 30 cm from the rail end closest to the computer (LED–camera distance ∼150 cm). Remove the lens tube (if installed) so that the CCD face is exposed and you can see where the beam lands.~~

Center the LED beam on the camera using the camera shadow cast on the wall behind: the shadow should be centered on the bright central spot of the LED illumination (Fig. 20). Rotate the LED source slightly in its post holder to center the shadow. Use a post collar to maintain height while rotating the post.

At this point: the LED beam travels down the rail centerline at a fixed height and is centered on the CCD, which is square to the optical rail (no tilt).

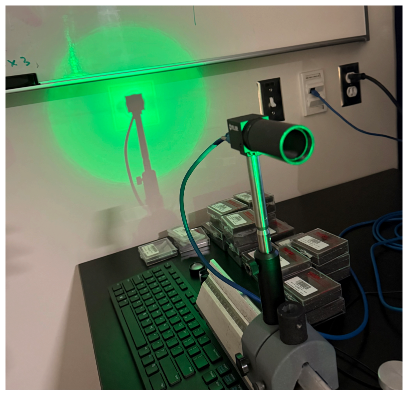

Figure 20: Centering the LED source on the camera using its cast shadow.

3. **Align magnification lens**

   Alignment proceeds in reverse order (mag lens back toward the LED) to enforce good system alignment.

   Place the magnification lens about 30 cm in front of the camera sensor plane (use a 4” post holder). Ensure correct lens orientation: the more curved surface should face the collimated side. The lens housing has an ∞ symbol pointing to the more collimated direction (away from the camera in this setup). Center the lens on the beam and ensure it is square to the rail/beam axis (not twisted).

   > Lens orientation: Plano-convex lenses minimize aberrations for infinite conjugate ratios when the curved side faces the object at infinity (“flat side to focus”). The same holds for achromats like the 300 mm magnification lens: the flatter side should face the focus on the CCD. The ∞ marking conveniently points to the side facing more collimated rays, and this marking can be found on other lenses in the setup as well. Symmetric biconvex lenses perform best near unit conjugate ratio. For more details, see *The Optical Systems Guide*.

   Mark the LED position with an adjacent empty saddle, then remove the entire LED saddle assembly from the rail and set it aside (do not remove the LED post from its post holder).

   Adjust the magnification lens position on the rail by focusing the image on the camera of:

   - first, a white card with small letters placed ∼1.5 m away at the opposite end of the rail and illuminated by a desk lamp, and

- second, a distant object near the lab entrance (> 5 m away). Use a mirror on the rail to point the imaging system toward the distant object.

The first step is a convenient coarse adjustment, but focusing on a distant object is **critical** for setting the lens at the correct distance from the camera.

Success criterion: you can focus sharply on the distant object, and the lens is approximately square to the optical axis and centered at the correct height.

Once the magnification lens position is correct, **do not adjust its position again**.

4. **Camera software and exposure settings**

   Use NI Vision Assistant to acquire and save images (e.g., for later analysis in ImageJ). Avoid saturation while using as much of the full dynamic range as possible. Suggested starting settings:

   - Always set `Gamma` = 1 (linear intensity response). Beware - the camera defaults to 0.8 value after restart.
   - Disable `Gain Auto`; start with gain = 18.
   - Disable `Exposure Auto`; choose exposure time as needed.

   See *Protocol: Using the FLIR Blackfly Camera* and NI Vision Assistant help: NI website.

   Also open the LabVIEW acquisition program (`Dual_Camera_Line_Profile_v2.vi`) and test doing the same (NI Vision Assistant needs to be closed first): The program has two screens for two cameras. Key camera settings can be set indenpedently for each camera. Ignore the Fourier Start with gain = 18 and vary exposure time first; adjust gain only if needed to keep exposures reasonable.

   Return the LED source to its marked position and verify alignment down the center of the rail.

5. **Objective lens**

   Place the $f = 150$ mm objective lens 45 cm from the magnification lens (center-to-center). Confirm correct objective orientation: The lens, which should already be attached to a silver fine zoom mount should face the camera side. Center the optic vertically and horizontally, and ensure it is square to the rail axis.

   > Alignment tips: Use the shadow cast by the objective mount to check centering. It is also useful to use a grey CCD camera cover disk with a black dot marked at the center as an alignment target. Hold it lightly against the lens tube or mount aperture.
   >
   > If the lens is not centered horizontally (i.e. left–right relative to the rail centerline) the post holder may not be centered on the saddle or aligned with the rail centerline. To make a small horizontal adjustment, loosen the screw on the underside of the saddle using the provided 1/4–20 ball driver and carefully shift the post-holder

position. Retighten the screw securely while ensuring the washer on the underside remains centered.

6. **Test object**

   Mount the variable line grating test object in the translating microscope stage (saddle with 3" post holder). Ensure the metallic coating faces the objective (so the image is not degraded by defects in the slide glass). The coating side faces you when the writing is correctly oriented (not mirrored).

   > Handling notes for the Thorlabs microscope slide test objects: wear gloves while handling and touch only the edges. Do not clean the slide. Ask an instructor for mounting assistance using the set screws on the translating microscope stage. Tighten the screws *lightly* to avoid flexing or breaking the microscope slide (cost $\sim$ \$800!).

   Irregular objects are often easier for assessing focus quality than periodic gratings.[7]

   Use part of the "THORLABS" lettering (or similar) on the test object to obtain a coarse focus on the camera. Achieve this initial focus by *adjusting the test object position*: slide the saddle along the rail until a reasonably focused image appears on the camera. Ensure the slide is approximately perpendicular to the optical axis.

   Finally, fine adjustment of the image focus can be achieved by rotating the objective lens zoom housing. Once you find the sharpest image, confirm that you have focus range in both directions.

   - When the image is slightly defocused, Fresnel fringes may be visible at sharp edges; best focus minimizes these fringes.
   - Small defects in the grating rulings also provide a sensitive focusing target.
   - Try to resolve the finest visible grating (e.g. > 10 lp/mm). Record the smallest resolvable line spacing (printed on the slide above each set of lines).

7. **Köhler illumination: condenser lens (illumination lens)**

   Place the condenser lens $\sim$100 mm from the object. Ensure the correct lens orientation (it will face the object and camera in its mount). Center it by ensuring the illuminated part of the object (e.g. the central letters of "THORLABS") remains centered in the camera image as you insert/remove the lens.

   The condenser lens mount is held in a slip ring that allows fine lateral adjustment. Use a small hex ball driver to loosen the set screws, shift the slip ring to center the lens, and retighten the screws to secure the position.

[7] Periodic gratings under approximately collimated illumination can exhibit the Talbot effect, producing multiple images at periodic distances. One may be mistaken for the true image. Irregular objects do not exhibit this ambiguity.

8. **Köhler illumination: aperture stop (pinhole)**

   Screw the 200 μm pinhole into the LM1XY translating mount, starting with the adjusters centered (white marker lines aligned). The XY mount compensates for any mount offset.

   Place the pinhole roughly 100 mm before the condenser lens to define the aperture stop and produce a clean illumination beam. Ensure the pinhole shadow is centered on the condenser lens (use the alignment cap with a dot). Adjust using the XY knobs or reposition the post holder in the saddle if needed.

   Finally, fine-adjust the illumination alignment using autocollimation. Place a mirror after the 100 mm condenser lens and **autocollimate** by observing the back reflection from the pinhole surface. The autocollimation technique is described in *The Optical Systems Guide*.

   Success criterion: the illumination beam remains centered on the chosen feature on the object and stays centered through the objective and onto the CCD. The illumination cone after the condenser lens should remain approximately constant in diameter until the objective lens.

9. **Köhler illumination: field lens (image the LED onto the aperture stop)**

   Insert the $f = 50$ mm field lens to image the LED source onto the pinhole (i.e. focus the LED image onto the aperture stop). Ensure the correct lens orientation.

   - Remove the pinhole temporarily (unscrew it from its mount, or remove the saddle after marking its position).
   - Adjust the field lens height to keep the beam centered at the object and objective.
   - Replace the pinhole and confirm that the LED image is focused onto the pinhole.

   If the focused LED spot shifts laterally on the pinhole due to the field lens, adjust the field lens laterally by shifting the post-holder position in its saddle, or try a different post holder. If absolutely necessary, slightly rotate the lens mount.

10. **Köhler illumination: field stop (field iris)**

    Place the 0.5” iris (ID12, small diameter) as an adjustable field stop. Center it on the beam. Temporarily remove the pinhole to focus the sharp edges of the iris on the camera image by sliding the iris along the rail.

    At minimum iris diameter, center the iris on the object and on the camera CCD. Horizontal centering may require slight adjustment of the post-holder position in the saddle.

    Reduce the iris to yield a $\sim$5 mm illuminated field at the test object. You should observe a flat field (uniform intensity) over this region. If not, consult an instructor; the LED spot may not be properly centered along the rail and earlier alignment steps may need revision.

    Reminder: the field stop controls the illuminated *area* on the object, while the aperture stop controls the effective size of the extended source and therefore the illumination *angular extent*, but not the field size.

11. **Final notes (setup part 1)**

    At this stage you have a complete imaging setup.

    - Add a large mounted iris in front of the field iris to reduce stray light from the LED source.
    - Add a lens tube in front of the camera to block ambient room light.

Testing

- Using a known line spacing on the variable grating test object, for example a grating of 10 lp/mm, calibrate the CCD image scale and determine the image magnification. Compare to the expected value.
- Locate the Fourier plane using a viewing card (use a grating of 10 lp/mm or lower (i.e. coarser)).
- Adjust the field stop iris so illumination covers only the central region of a single line spacing to avoid having two diffraction patterns from adjacent gratings. This corresponds to 1-1.5 mm diameter illumination region at the object plane.
- Place a second camera at the Fourier plane to record the diffraction images from one or two gratings (note: this blocks the path to the main camera).
- Determine the spacing between Fourier orders and convert to diffraction angle. Compare to expected values given the know line spacing.

6.2 Setup Part 2

In this part you set up **Fourier-plane imaging**. A camera placed directly at the Fourier plane blocks the beam, so we use a **beam splitter** to send a fraction of the light into a separate imaging arm on a 90° rail (see Fig. **??**).

1. **Beam splitter and 90° rail alignment**

   Insert a beam splitter (BS) into the main beam path, approximately 22 cm downstream of the objective lens (use a 4" post holder). This distance leaves enough room between the beamsplitter and the Fourier plane to insert an iris on a transverse translation stage at the Fourier plane (for later spatial filtering). Make sure the stage with iris can be mounted on a saddle and positioned correctly near the beamsplitter.

   - Center the main beam on the beamsplitter using the alignment cap.
   - Rotate the beamsplitter (use a post collar to hold the vertical position) so that the reflected beam propagates cleanly perpendicular to main rail and *away* from the table edge.

     > Alignment idea: Use the faint back reflection from the front surface of the beamsplitter to orient the beamsplitter square to the optical axis. If the back reflection is too dim to see well, temporarily remove the aperture-stop pinhole by unscrewing it to increase light while aligning the beamsplitter.

   - Adjust the short optical rail's feet so it lies horizontal and posts on it are vertical, and place the rail at 90° to the main rail.
   - Use a **pair of irises/apertures** along the short rail to guide the beam and verify it is centered and level.
   - If the beam is too dim to see well, temporarily remove the aperture pinhole to increase light while aligning the beamsplitter arm.

   Success criterion: the fourier imaging path passes through both alignment apertures on the 90° rail without walking off when you translate a card along the rail.

2. **Fourier imaging lens and Fourier camera**

   Place the Fourier imaging lens (50 mm diameter, $f = 100$ mm) in the 90° arm. Ensure correc orientation of the lens. The goal is to image the Fourier plane onto the Fourier camera with a suitable magnification so that several diffraction orders fit on the CCD, just up to 7th order at CCD edge for 10 LP/mm grating correspinding to about a magnification of about $1/2.5 - 1/3$.

   - Use a viewing card to locate where the Fourier pattern forms behind the imaging lens: near the back focal region you will first see a real-space image of the object and further beyond that the desired image of the diffraction pattern at the Fourier plane.

- Position the Fourier camera on a transverse translation stage so that the diffraction orders are sharply in focus with the aperture pinhole in place. The translation stage allows for centering the pattern on the CCD as well as exploring higher order by shifting laterally.
- Add a lens tube in front of the Fourier camera to block stray room light.
- Optional: add an iris/aperture in front of the imaging lens for later testing (leave it fully open for now).

Success criterion: the Fourier pattern is sharply focused, symmetric about the zero-order peak.

3. **Final notes for setup part 2**

   You should now have the full dual-camera spatial filtering setup. In the LabVIEW program, the left panel displays the real-space image (main camera), and the right panel displays the Fourier-space image (Fourier camera).

   If not already done, familiarize yourself with program operation:

   - Use the mouse to draw a cross-section on either image and view a live intensity profile (useful for alignment and quick measurements).
   - Save images for offline analysis and fitting (recommended for quantitative results).

Pre-lab Question 1 (Do before Lab Period 2): For the following, use the parameters of the experimental setup. The first two questions address Köhler illumination.

(a) **Field stop:** Given a desired illuminated field of view of 5 mm diameter at the object, what diameter should the iris at the field stop be set to? Note that the field stop is imaged onto the object plane under Köhler illumination.

(b) **Aperture stop:** In order to resolve diffraction orders clearly from the 10 lp/mm Ronchi ruling, what pinhole diameter at the aperture stop should be used so that the diameter (FWHM) of each diffraction-order peak is approximately 1/10 of the spacing between adjacent orders?

(c) **Spectral broadening:** Derive an expression for the FWHM of a diffraction order from a Ronchi ruling when the illumination has a finite spectral bandwidth $\Delta\lambda$. Assuming an LED bandwidth of 30 nm and an aperture-stop pinhole diameter of 200 μm, at what diffraction order will the FWHM of the order peak double compared to the zero-order peak? Comment on how this provides a rough criterion for the maximum usable order before spectral broadening significantly affects the diffraction pattern.

Due: Lab Period 2.

Testing (field stop vs. aperture stop)

The purpose of this testing is to understand the distinct roles of the **field stop** (field iris) and the **aperture stop** (pinhole / iris stop).

- Observe three things simultaneously with a grating object in place:
    1. the Fourier camera image,
    2. the real-space camera image,
    3. the illuminated region on the object (use a card at the object plane if needed).
- **Test 1: vary the field stop (field iris).** Adjust the field iris diameter while observing: This should change the **illuminated area on the object and on the camera CCD** (field of view).
    - Object and real-space camera: illuminated area on the object and on the camera CCD (field of view) should change.
    - Fourier camera: The spot size of diffracted orders should not change, however, but their intensity will.
- **Test 2: vary the aperture stop (using an iris at the aperture stop plane).** Mark the zero-order peak position on the Fourier camera display output. Mark the pinhole saddle location with an adjacent empty saddle and remove the entire pinhole saddle for easy return. Insert an iris at the aperture stop plane (so that the iris edges are in focus on the Fourier camera). Vary the iris diameter while observing:
    - Fourier camera: spot size of the diffraction orders should now change.
    - Object and real-space camera: illumination area should remain approximately constant but the intensity will change.
- After reinstalling the pinhole saddle, verify that alignment has not shifted: the zero-order peak should return to the same location and remain in focus on the camera display.

Testing and calibration: magnification Tests of magnification and diffraction orders with variable line grating.

Revisit the assessing the order spacing of diffraction for known gratings and use to extract magnification calibration of Fourier imaging. At this point you should notice that the peaks broaden with increasing order. From prelab, you estimate the effect of source bandwidth. To test introduce the 10 bandpass filter in front of the LED source. Intensity will be reduced. Does this clean up broadening - it should.

Resolution test What is finest grating period that can just fit on your CCD? This coudl be limitted by field of view but other optics limit the highest order, for example the edge of the objective lens.

Diffraction resolution effect of aperture in Fourier plane, which is similar to limitting lens diameter. Test introduction of aperture to limit max order at he fourier plane and effect on images. Choose a line pair and straddle bioth across the image. You shoudl see in Fourier plane overlapping diffractio papetresn for both. Reduce the aperture to point where the finer line spacing vanishes but the other is still viewable. In this case orders -1 0 +1 are still present for the coarser grating but not for the finer one. capture an image that demonstartes this resolution effect. This shows the effect of aperture and so NA on diffractive image resolution.

Testing and calibration: magnification and diffraction orders

At this stage your real-space and Fourier-space cameras should both be operational. Use the variable line grating test object to (i) calibrate magnification, and (ii) verify the expected diffraction order spacing on the Fourier camera.

1. **Observe order broadening and test the effect of source bandwidth.** Start with a 10 lp/mm grating. You should notice that diffraction peaks broaden with increasing order. From the prelab, estimate the expected effect of finite source bandwidth. To test this experimentally, place the 10 nm bandpass filter in front of the LED source and record the same Fourier pattern again (note: intensity will decrease).
   - Does the peak broadening decrease with the bandpass filter in place? It should.
   - Record a before/after comparison (with and without the filter).
2. **Record diffraction patterns for known line spacings.** Select two grating regions with known spatial frequency (e.g. 10 lp/mm and one other). For each one, record:
   - a real-space image (with the grating region clearly identified), and
   - a Fourier-space image showing at least the zero and ± 1 diffraction orders (more if visible).
3. **Measure diffraction order spacing and extract a calibration.** Using the Fourier-space image, measure the pixel spacing between the zero-order peak and the ± 1 peaks (and higher orders if visible). Compare the measured spacing to the expected diffraction angles from the grating period and the known source wavelength. Use these measurements to calibrate the Fourier camera scale (pixels per spatial frequency, or pixels per diffraction angle) and to verify the Fourier imaging magnification.

Resolution test: aperture-limited cutoff in the Fourier plane This test demonstrates how limiting the Fourier-plane aperture (equivalent to reducing the system numerical aperture) removes high spatial frequencies and reduces image resolution.

1. **Choose two grating regions in the same field of view.** Translate the object and adjust illumination region to place two different grating frequencies within the camera field simultaneously: one coarse spacing (easy to resolve) and one finer spacing (near the resolution limit), for example 26 and 50 lp/mm. Ideally, position them so that a single line profile across the real-space image crosses both gratings.

2. **Verify the corresponding Fourier patterns.** On the Fourier camera, you should see overlapping diffraction patterns from both gratings. The finer grating produces diffraction orders further from zero.

3. **Introduce a variable aperture at the Fourier plane and reduce its diameter.** Insert an iris (or aperture mask) at the Fourier plane. Slowly reduce the aperture diameter while monitoring both cameras:

   - In Fourier space: higher-order peaks are clipped first.
   - In real space: the finest grating contrast decreases and eventually the fine grating becomes unresolved, while the coarse grating remains visible.

4. **Record a clear demonstration of the cutoff.** Find an aperture diameter for which the coarse grating still shows visible contrast while the finer grating has essentially vanished. In this regime, the coarse grating still has at least the $-1, 0, +1$ orders transmitted, while the finer grating has lost its ± 1 orders (or they are strongly attenuated). Record paired real-space and Fourier-space images that demonstrate this effect.

5. **Interpretation.** Explain the observed resolution loss in terms of Fourier cutoff: limiting the Fourier-plane aperture removes high spatial frequencies, reducing the maximum diffraction order transmitted and therefore the smallest resolvable grating period.

Remove the iris aperture from the Fourier plane for the next section.

A. Fraunhofer diffraction

After completing the setup (Fig. **??**) and obtaining stable, simultaneous real-space and Fourier-space images, you are ready to observe Fraunhofer diffraction from periodic objects (Ronchi rulings) and to compare real-space and Fourier-space measurements quantitatively. In this configuration, the Fourier camera records the diffraction pattern in the back focal plane of the objective (or its re-imaged Fourier plane), while the real-space camera is used to position the object, verify focus, and measure the object geometry in real space.

In this section, you will focus on diffraction from a **Ronchi ruling** (10 lp/mm), which provides multiple diffraction orders and directly connects to spatial filtering and Fourier-series modeling.

The goals of this section are:

- to compare the diffraction pattern to theory,
- to calibrate the Ronchi ruling (slit spacing and slit width),
- and to establish a quantitative baseline for later spatial filtering.

Pre-lab Question 2 (Due Lab Period 2): Your Ronchi ruling consists of a series of long parallel slits (slit width b, period a). Monochromatic light incident on the object diffracts and forms a far-field (Fraunhofer) diffraction pattern.

(a) Derive Eq. (32) for the far-field diffraction intensity pattern of N slits.

(b) Make sample plots for different values of N and a, and describe qualitatively how each parameter affects:
   - the spacing between diffraction orders,
   - the width of diffraction peaks,
   - and the overall envelope of the pattern.

(c) Explore the effect of varying the duty cycle b/a. How does it affect the relative intensity of diffraction orders (including which orders may be suppressed)?

(d) In the experiment, diffraction orders will not be perfectly sharp and may broaden with order. Briefly explain at least two physical reasons why a real diffraction pattern may deviate from the ideal prediction of Eq. (32). (Hint: consider finite source size, finite spectral bandwidth, and imperfect alignment/aberrations.)

Replace the test object with the **10 lp/mm Ronchi ruling microscope slide**. The ruling spans most of the slide, allowing a $\sim$5 mm illuminated region and reducing alignment sensitivity.

- **Optimize focus and alignment.** The ruling in the real-space image must be in focus (see Talbot-image discussion in Footnote 7). Use lettering on the slide for coarse focusing and small defects on the ruling for fine focus.
- **Use the bandpass filter for quantitative diffraction.** Because you will analyze diffraction peak widths and intensities, insert the 10 nm bandpass filter in front of the LED source to reduce order broadening due to source bandwidth.
- **Record real-space and Fourier-space images.** Save images of both the Ronchi ruling and its diffraction pattern under the same conditions. Use the LabVIEW live line profile for alignment, then save images for careful analysis in Python or ImageJ.
- **Check Fourier symmetry.** The Fourier image should be symmetric about the zero order (see, e.g., Fig. 6). If it is not, ensure optical elements are square to the optical axis and minimize stray light.
- **Calibrate ruling parameters from real space.** Use the real-space image to estimate the slit spacing a and slit width b.
- **Calibrate ruling parameters from Fourier space.** Use the diffraction peak spacing to estimate a, and use the peak envelope and relative intensities to estimate b/a (duty cycle). Compare the two methods.
- **Compare to theory and comment on differences.** Quantitatively compare the measured diffraction pattern to the prediction for an ideal Ronchi ruling.[8]
- **Model limitations (discussion and optional modified fit)** Eq. (32) alone is typically insufficient to fit real data. Discuss why (e.g. finite source size, finite bandwidth, finite N, aberrations, background/stray light) and what must be added or modified to obtain a realistic model. Consider a modified model fit function that accounts for source-induced broadening, and assess the fit quality.

After calibrating the ruling and validating the diffraction model qualitatively and quantitatively, proceed to spatial filtering.

[8] *Ideal* Ronchi rulings are perfect square waves with 50% duty cycle. *Real* rulings may deviate due to non-ideal duty cycle, finite contrast, edge roughness, finite illuminated width, and partial coherence.

B. Spatial filtering

Spatial filtering modifies an image by selectively blocking or transmitting spatial-frequency components (diffraction orders) in the Fourier plane. In this section you will use Fourier-plane masks to modify the image of a Ronchi ruling and compare your observations to a Fourier-series model.

B1. Modifying the real-space image of a Ronchi ruling (qualitative)

First make a qualitative investigation of spatial filtering. You will need to construct Fourier masks of various types. Much of the filtering can be done using a post-mounted iris (25 mm diameter) on a transverse translation stage, which allows careful positioning in the Fourier plane. For cases where you need to block the zero order, you can construct a mask using wire or thin strips of tape attached to a square metal frame mounted on the translation stage.

- Begin with the reference case (all orders present) and verify that the Fourier pattern is symmetric and well focused. If it is not symmetric, check for any tilted optics. Ensure the real-space image is properly focused (this is essential).
- The source bandpass filter is not required for assessing the effects on the real-space image, but you can check its effect later (e.g., in the Gibbs ringing section).
- Try masking diffraction orders in the Fourier plane. For example, transmit only the $m = -1, 0, +1$ orders. Observe how the real-space image changes.
- As you make adjustments, record representative examples of:
  - the modified Fourier pattern, and
  - the resulting real-space image.
- Record a line profile of the real-space image and compare to the reference case (all orders present). Make sure you understand why the image changes when diffraction orders are removed.

Pre-lab Question 3 (Due Lab Period 3): Determine the Fourier series for a periodic grating using the parameters from your calibration of the Ronchi ruling. Write a program to generate the predicted *intensity* profile from a truncated Fourier series. Recall that intensity is proportional to the product of the field and its complex conjugate (see Eq. 31). Your program should also allow modification of Fourier coefficients to mimic spatial filtering. Make example predictions for several of the filtering cases listed below.

B2. Modifying the real-space image of a Ronchi ruling (quantitative)

Set up the configuration in Fig. 15 so that you can observe simultaneously the real-space image and the Fourier-space (diffraction) image. Prepare and insert masks in the Fourier plane to select combinations of diffraction orders listed below.

For each investigated case:

- Record the real-space image and the Fourier-space image.
- Save a line profile for each (use the same profile direction and scaling when comparing cases).
- Use your Fourier-series model to predict the real-space intensity profile for that set of transmitted orders, and compare prediction to measurement (overlay or quantitative comparison).
- Compare the *relative* intensities of transmitted diffraction orders to the squared magnitudes of the corresponding Fourier coefficients. Comment on discrepancies.

  Note: quantitative comparison requires that camera intensity levels be comparable from image to image. Record (and if possible hold constant) camera gain, exposure time, and gamma. Avoid saturation.

Mask cases: record real-space and Fourier-space intensity profiles for the cases below.

Minimum required set: (a), (b), (d), (e), (f), (j), and the low-pass series (k) for at least two values of $m_{\max}$ in addition to $m_{\max} = 1$. *Extensions:* any additional cases of your choice (including high-pass filtering, (l)).

(a) All orders (reference case);

(b) Zero order only;

(c) $+1$ only;

(d) $0, +1$ (Abbe minimal criterion);

(e) $+1, 0, -1$;

(f) $+1, -1$;

(g) $+2, 0, -2$ and also $+2, -2$ (note: for a perfect square wave, even harmonics such as ± 2 would be suppressed);

(h) All positive orders (oblique dark field);

(i) All positive orders and the zero order (half shadow);

(j) All orders except zero (dark-field / dark-ground method);

(k) $-m_{\max}, \ldots, -1, 0, +1, \ldots, +m_{\max}$ (low-pass filtering; choose at least two values of $m_{\max}$ in addtiton to 1 e.g. $1, 3, 5$);

(l) All orders $> |3|$ (high-pass filtering).

B3. Order cutoff and Gibbs ringing (required analysis)

A Ronchi ruling has sharp edges, so its Fourier-series coefficients decay slowly with order. Truncating the series (or blocking high orders with a Fourier-plane aperture) produces ringing and overshoot near edges (Gibbs-like effects, which are in fact false detail in the image). In this analysis you will quantify how the image changes as you vary the maximum transmitted order.

- Use the low-pass filtering case ((k)) to transmit only orders with $|m| \le m_{\max}$. Repeat this measurement for $m_{\max} = 1$ and for at least two larger values (e.g. $m_{\max} = 1, 3, 5$ or $1, 3, 7$, depending on which orders are visible).
- For each $m_{\max}$, record the real-space image and a line profile that crosses a few grating periods, with particular attention to the behaviour near sharp transitions (edges).
- Compare the measured line profile to your Fourier-series prediction truncated at the same $m_{\max}$. If the data quality is sufficient, this comparison may be done using a fit to the model.
- Quantify the ringing by reporting (for at least one edge):
  - the overshoot amplitude relative to the step height, and
  - the spatial period (or approximate width) of the ringing.
- Explain why increasing $m_{\max}$ sharpens the edge but does not eliminate ringing until very high order.

Required discussion questions

- **Abbe minimal criterion:** Explain why case (d) $(0, +1)$ represents a minimal criterion for resolving a periodic structure (Abbe theory). Why does transmitting only the zero order produce no spatial variation?
- **Dark-field filtering:** Compare cases (a) and (j). Explain why blocking the zero order produces edge-enhanced or "dark-field" contrast.
- **Order cutoff / ringing:** Using your low-pass series, discuss how increasing $m_{\max}$ affects edge sharpness and Gibbs-like ringing.

Optional extension question

- **High-pass filtering (case (l)):** Explain why transmitting only $|m| > 3$ emphasizes edges and produces ringing, and comment on why this case is often dim and noise-sensitive.

Presentation note: In your notebook, record data as you take it. For presentation (technical brief or report), build a compact figure with multiple subpanels of real-space images, Fourier-space images, and line profiles so that the effects of masking can be compared easily (ideally on one page).

C. Short projects

Choose **one** short project below. For your Lab 2 write-up, consider completing one project that is primarily **qualitative** and one that is primarily **quantitative** (if time permits).

1. *Phase object* (e.g. air above a heated wire, or a phase grating). The Lipson reference contains a helpful discussion of why phase objects are normally invisible and how to observe them. **Hint:** a pure phase object changes the phase but not the intensity of a uniform plane wave, so it is normally invisible unless phase variations are converted into intensity variations through interference (e.g. via spatial filtering or phase contrast). Use the camera to observe the image formed by either:

   - a phase grating ($1'' \times 1''$ on thick glass substrate), or
   - a Birefringent Resolution Target ($2'' \times 2''$ slide, Thorlabs R2L2S1B),

   using the following Fourier-plane filtering techniques:

   (a) *bright field*: diffraction pattern unmodified;
   (b) *phase contrast*: place the $\lambda/4$ retardation plate on the zero order;
   (c) *dark field*: block the zero order;
   (d) *Schlieren*: block all orders on one side of the zero order.

   Describe and explain what you see in all cases. Which technique is best for observing phase objects, and why? Interpret your observations using the discussion in Lipson.

   Note: Even without any spatial filtering, a phase object can become visible when slightly defocused. In bright-field imaging, the contrast of a phase object is typically **lowest at best focus** and increases when you defocus (often with a contrast sign change on either side of focus).

   A useful model for a 1D phase grating is:

   $$f(x) = \mathrm{e}^{\mathrm{i}\phi\,\mathrm{square}(x)} = 1 + \left(\mathrm{e}^{\mathrm{i}\phi} - 1\right)\mathrm{square}(x), \tag{39}$$

   where square(x) is a periodic square wave alternating between 0 and 1. Use this to calculate what you will observe in the Fourier (transform) plane. (Hint: ensure the zero-order term matches the average of $f(x)$ over one period for all ϕ.)

2. *Two-dimensional object.* Create a 2D object by stacking *two* Ronchi rulings face-to-face so the patterned regions are in (nearly) the same plane. Choose an angle (e.g. 90°, 45°, etc.). Alternatively, try a printed photograph on a transparency (semi-transparent) as the object. Use spatial filtering to improve image quality, and compare optical filtering to digital processing in ImageJ.

   (a) Investigate the impact of placing horizontal and vertical slits in the Fourier plane.

(b) Add low-frequency "noise" in real space by placing small objects on the mesh (paint, chalk dust, threads, etc.). Can you design a Fourier-plane mask that removes these features while preserving the desired image?

Note: For this project, use the 100 μm or 200 μm pinhole to limit the effective source size.

3. *Free the monkey?* We provide an image of a cute but sad monkey behind regular cage bars. Your goal is to remove the bars and "free the monkey". Try both:

   - **optical filtering**: make a transparency of the image and use a Fourier-plane mask to suppress the bar frequencies, and
   - **digital filtering**: use tools in ImageJ (e.g. Fourier filtering, notch filters).

   You may also try a different image containing a mix of regular and irregular patterns (e.g. dust on a mesh). **Beware:** this is a challenging project.

   An easier variant is to "clean up" the man in the half-tone image described in A. P. Buckman and R. A. Woolley, *J. Phys. E: Sci. Instrum.* **12**, 95–97 (1979), available on Canvas. You may also choose your own image.

   Note: For this project, try a 100 μm or 200 μm pinhole to limit effective source size.

4. *Effect of extended source on image quality and resolution.* Investigate how imaging quality and resolution depend on the aperture stop size (effective source size) and on the Fourier-plane aperture (effective objective diameter). Use the variable line grating test object to access a range of spatial frequencies.

   Available source-limiting pinholes include 100 μm, 200 μm, and 300 μm, and you may also use an aperture iris. Consider both:

   - resolution changes as the aperture stop is opened/closed, and
   - the appearance of false detail / Gibbs ringing as you vary the Fourier-plane aperture and the aperture stop.

7 Suggested Timeline

Lab Period 1:

- Answer Pre-lab Question 1 before coming to the lab: Basic optics design calculations. Imaging questions for the field stop and aperture stop.
- Complete setup except for the Fourier imaging arm. Explore real-space imaging using the variable line grating test object, and observe Fourier-space features qualitatively in the back focal plane of the objective lens (using a viewing card).

Lab Period 2:

- Answer Pre-lab Question 1 before coming to the lab: Imaging questions for the field stop and aperture stop, and effect of source spectral bandwidth.
- Answer Pre-lab Question 2 before coming to the lab: Ronchi ruling diffraction pattern + design calculations for the Fourier imaging setup.
- Complete the setup including the beamsplitter and Fourier-plane imaging arm.
- Verify Köhler illumination stop behaviour (field stop vs aperture stop) using both cameras and a viewing card.
- Using the variable line grating test object:
  - Verify magnification and diffraction order spacing on the Fourier camera.
  - Baseline imaging resolution vs aperture stop / Fourier-plane aperture (or effective lens diameter).
- Investigate Fraunhofer diffraction for the 10 lp/mm Ronchi ruling.

Lab Period 3:

- Answer Pre-lab Question 3 before starting quantitative spatial filtering: Fourier-series model for images and spatial filtering.
- Fraunhofer diffraction should be complete (and at least partly analyzed) before starting spatial filtering.
- Start spatial filtering — test as many masks as possible. Analyze at least one mask in detail during the lab to ensure you are on track.
- Complete further mask analysis at home before Period 4 (to verify data quality and fitting pipeline).

Lab Period 4:

- Finish masks, if needed. Consult with the TA on how many are sufficient if pressed for time.

Lab Periods 5–8:

- Complete remaining masks and refine the setup. You should have a fit function based on a Fourier-series model and assess sensitivity to experimental variables (e.g. focus, illumination uniformity).
- Use the variable line grating test object to explore imaging resolution and Gibbs ringing (false detail) when varying Fourier-plane aperture and illumination aperture stop.
- Complete a short project of your choice. For the Lab 2 write-up, consider one quantitative and one qualitative extension.

8 Items to include in your documentation

In addition to a record of day-to-day progress, calculations, diagrams, essential equations, lists of equipment, filenames, and settings, please ensure you include at least the following items in either your lab notebook or your analysis notebook.

General

- A brief project plan (goals, milestones, and how you will allocate work across lab periods).
- A record of key experimental settings for each dataset (camera gain/exposure/gamma, field stop diameter, aperture stop diameter, and Fourier-plane mask/aperture used).

Figures

- Real-space and Fourier-space images of a single slit.
- Real-space and Fourier-space images of a Ronchi ruling.
- Intensity profile(s) for the Ronchi diffraction pattern, including an assessment of whether a fit is appropriate.
- For each situation specified in Part B:
  - Real-space image and corresponding intensity profile.
  - Fourier-space image and corresponding intensity profile.
- Comparison of real-space intensity profiles to a Fourier-series model (fit or overlay), including the effect of maximum order cutoff.
- Figures supporting your chosen project extension.

Analysis

- Pixel-to-distance conversion (magnification calibration).
- Single-slit diffraction analysis, including an assessment of fit quality and uncertainty in fitted parameters (e.g. slit width).
- Ronchi ruling parameters: slit width and slit spacing, extracted from both real-space and Fourier-space measurements where possible.
- Ronchi diffraction analysis: peak intensities vs. diffraction order, and assessment of agreement with the expected Fourier-series / grating model.
- Model functions for real-space intensity profiles using Fourier-series components, and assessment of fit quality or overlay of model on data.
- Analysis supporting your chosen project extension.

Appendix

Here, we outline the proof that if you place an aperture $a(x, y)$ in the *front focal plane* of a lens, then the *back focal plane* will have a field proportional to the Fourier transform of the aperture function, $A(u, v)$, where, in the small-angle approximation, $u = kx/f$ and $v = ky/f$. Here, $k = 2\pi/\lambda$ is the wavenumber, f is the focal length of the (thin) lens, and x and y are lateral positions in the plane a distance f behind the lens. In the main text, we gave a heuristic (handwaving) argument for the result. Here, we go through the argument in detail, for those who want to understand the result more deeply. The derivation follows that given in the book *Introduction to Fourier Optics*, Joseph Goodman (McGraw-Hill, 2nd ed., 1996), Section 5.2. It assumes basic knowledge of Maxwell's equations and some comfort with Fourier transforms.

We will prove our statement by considering the slightly more general situation shown in Fig. S1, where the aperture is a distance d in front of the lens and is illuminated by a plane wave.

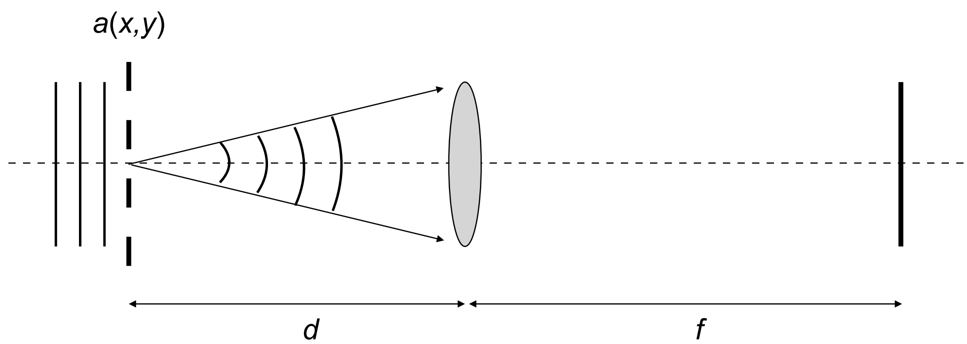

Figure S1: Aperture $a(x, y)$ a distance d in front of a lens of focal length f. A sample Huygens wavelet showing the creation of a spherical wavefront after the aperture is included.

If the aperture (at $z = 0$) is illuminated by a plane wave of unit amplitude, the field just behind the aperture is $a(x, y)$. Our goal is to determine the field at the back focal plane, a distance $z = d + f$ from the aperture. The calculation proceeds in three steps: (1) We put the aperture $a(x, y)$ just behind the lens and ask what field is produced in the back focal plane. (2) We move the aperture just before the lens. (3) We move it finally to a distance d in front of the lens. But before we start, we discuss how to propagate fields along a beam axis (z-axis).

(0) *Propagation of a paraxial light beam.* From Maxwell's equations, it is easy to show that electromagnetic fields having a harmonic time dependence $\sim e^{-i\omega t}$ obey the *Helmholtz equation,*

$$(\nabla^2 + k^2)E(x, y, z) = 0\,, \tag{S1}$$

with $k = 2\pi/\lambda$ and $\nabla^2 = \partial_{xx} + \partial_{yy} + \partial_{zz}$, the 3D Laplace operator. Here, E can be any one of the electric-field components, as we neglect effects linked to wave polarization.

In the *paraxial approximation*, $E(x, y, z)$ is approximately a plane wave $\sim e^{i(kz-\omega t)}$ propagating along the z-axis, with only small-angle deviations allowed. We thus write

$$E(x, y, z) = \psi(x, y, z)\, e^{ikz}\,, \tag{S2}$$

where the field $\psi(x, y, z)$ is assumed to vary slowly along z, since the fast $\mathrm{e}^{\mathrm{i}kz}$ dependence in z has been factored out. Substituting Eq. (S2) into Eq. (S1) and using the slow z-dependence of ψ to write $\partial_{zz}\psi \ll \{\partial_{xx}\psi, \partial_{yy}\psi\}$ leads to the *paraxial Helmoltz equation*,

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + 2\,\mathrm{i}\,k\frac{\partial}{\partial z}\right)\psi(x, y, z) = 0\,. \tag{S3}$$

To find out how fields propagate along z, we Fourier transform Eq. (S3) in x and y:

$$\left(-u^2 - v^2 + 2\,\mathrm{i}\,k\frac{\partial}{\partial z}\right)\Psi(u, v, z) = 0\,, \tag{S4}$$

where the new function Ψ has two variables (u, v) in the Fourier domain, but z is still in the space domain. From the definition of the Fourier transform (Eq. 12),

$$\Psi(u, v, z) = \iint_{-\infty}^{\infty} \mathrm{d}x\,\mathrm{d}y\,\psi(x, y, z)\,\mathrm{e}^{-\,\mathrm{i}(ux+vy)}\,, \tag{S5}$$

Now Eq. (S4) is just a simple, first-order, linear, ordinary differential equation for Ψ as a function of z (with u and v constant). Thus,

$$\frac{\mathrm{d}\Psi}{\mathrm{d}z} = \left(\frac{u^2 + v^2}{2\,\mathrm{i}k}\right)\Psi\,. \tag{S6}$$

Solving Eq. (S6) and recalling that $1/\,\mathrm{i} = -\,\mathrm{i}$ gives

$$\Psi(u, v, z) = \Psi(u, v, 0)\,\mathrm{e}^{\frac{-\,\mathrm{i}\left(u^2+v^2\right)z}{2k}}\,, \tag{S7}$$

with $\Psi(u, v, 0)$ the Fourier transform of the field in the $z = 0$ plane. In other words, in Fourier space, we can *propagate* a field a distance z “down” the z-axis, $\Psi(u, v, 0) \to \Psi(u, v, z)$, by simply multiplying by $\mathrm{e}^{\frac{-\,\mathrm{i}\left(u^2+v^2\right)z}{2k}}$.

To find the field at z in real space, we apply the inverse Fourier transform $\mathcal{F}^{-1}$,

$$\begin{aligned}
\psi(x, y, z) &= \mathcal{F}^{-1}\left[\Psi(u, v, 0)\,\mathrm{e}^{\frac{-\,\mathrm{i}\left(u^2+v^2\right)z}{2k}}\right] \\
&= \mathcal{F}^{-1}\left[\Psi(u, v, 0)\right] * \mathcal{F}^{-1}\left[\mathrm{e}^{\frac{-\,\mathrm{i}\left(u^2+v^2\right)z}{2k}}\right] \\
&= \psi(x, y, 0) * \mathcal{F}^{-1}\left[\mathrm{e}^{\frac{-\,\mathrm{i}\left(u^2+v^2\right)z}{2k}}\right]\,,
\end{aligned} \tag{S8}$$

where $*$ denotes convolution and $\mathcal{F}^{-1}\left[\mathrm{e}^{\frac{-\,\mathrm{i}\left(u^2+v^2\right)z}{2k}}\right] = -\frac{\mathrm{i}k}{2\pi z}\,\mathrm{e}^{\frac{\mathrm{i}k}{2z}(x^2+y^2)}$. Thus,

$$\psi(x, y, z) = -\frac{\mathrm{i}k}{2\pi z}\iint \mathrm{d}x'\,\mathrm{d}y'\,\psi(x', y', 0)\,\mathrm{e}^{\frac{\mathrm{i}k}{2z}\left[(x-x')^2+(y-y')^2\right]}\,. \tag{S9}$$

Multiplying $\psi(x, y, z)$ by $\mathrm{e}^{\mathrm{i}kz}$ then gives $E(x, y, z)$ in a form that is equivalent to Eq. (27) when we use the paraxial form of r given in Eq. (28). Explicitly, we have

$$E(x,y,z) = -\frac{\mathrm{i}k}{2\pi}\frac{\mathrm{e}^{\mathrm{i}kz}}{z}\iint \mathrm{d}x'\,\mathrm{d}y'\,\psi(x',y',0)\,\mathrm{e}^{\frac{\mathrm{i}k}{2z}\left[(x-x')^2+(y-y')^2\right]}\,. \tag{S10}$$

Equation (S10) describes the real-space propagator for the electric field, with $E(x, y, 0) \to E(x, y, z)$.

As an application of the above formalism, we find the field in the back focal plane of a thin lens (i.e., a distance f behind a lens of focal length f), due to an aperture placed a distance d in front of the lens. We break the calculation into three steps: (1) aperture to lens; (2) through the lens; (3) lens to back focal plane. We do these in reverse:

Lens to back focal plane. Consider a field $E(x, y, 0^+)$ just after the lens (i.e., at $z = 0^+$). From Eq. (S10), the field at the back focal plane is

$$E(x,y,f) = -\frac{\mathrm{i}k}{2\pi}\frac{\mathrm{e}^{\mathrm{i}kf}}{f}\iint \mathrm{d}x'\,\mathrm{d}y'\,E(x',y',0^+)\,\mathrm{e}^{\frac{\mathrm{i}k}{2f}\left[(x-x')^2+(y-y')^2\right]}\,. \tag{S11}$$

(2) *Through the thin lens.* A thin lens (approximately) converts plane waves to spherical waves that converge to a distance f behind the lens, as illustrated in Fig. S2a. The effect of the lens can be modeled as creating a phase delay,

$$\mathrm{e}^{-\frac{\mathrm{i}k}{2f}\left(x^2+y^2\right)}\,. \tag{S12}$$

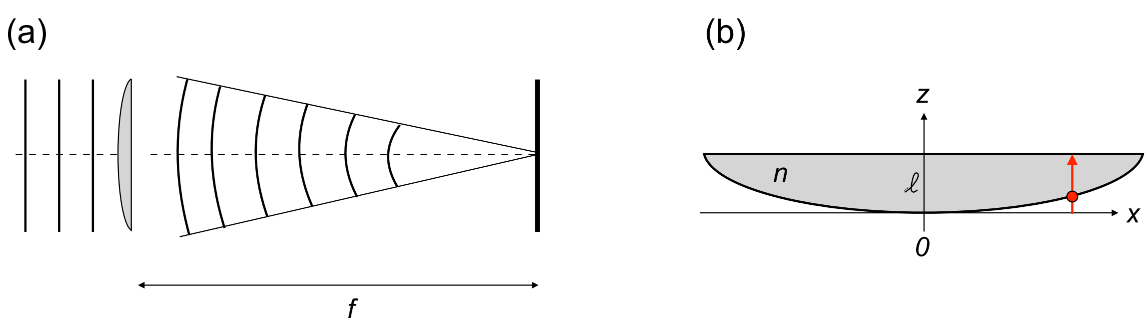

Figure S2: (a) Thin lens of focal length f converts plane waves to converging spherical waves. (b) Detail of plano-convex geometry for lens of index n and thickness ℓ (at the centre). The red arrow is a distance x from the optical axis. The red marker is at $z = x^2/(2R)$, along the optical axis.

To verify this claim, consider the geometry in Fig. S2b, which shows a plano-convex lens (for simplicity). A plane wave propagating along z (going up in the figure) first traverses a distance

$x^2/(2R^2)$ in air (index = 1) and then a distance $\ell - x^2/(2R^2)$ in glass (index = n). The optical path length OPL $\equiv \int \mathrm{d}z\, n(x,z)$ is then

$$\mathrm{OPL} = \underbrace{(1)\frac{x^2}{2R}}_{\text{air}} + \underbrace{n\left[\ell - \frac{x^2}{2R}\right]}_{\text{glass}} = n\ell - (n-1)\frac{x^2}{2R} \approx -(n-1)\frac{x^2}{2R} = -\frac{x^2}{2f}\,. \tag{S13}$$

In the next-to-last step, we apply the *thin-lens* approximation $\ell \to 0$ and neglect the overall (uniform) phase $n\ell$. In the last step, we use the *lens-maker's formula* of elementary optics to identify

$$\frac{1}{f} = (n-1)\left(\frac{1}{R} - \frac{1}{\infty}\right) = \frac{n-1}{R}\,. \tag{S14}$$

Going back to three dimensions (x, y, z) and remembering that an optical path length OPL leads to a phase factor $e^{\mathrm{i}k\,(\mathrm{OPL})}$, we find Eq. (S12).

Now consider an aperture $a(x, y)$ placed just *before* the lens. As we have seen, the effect of the lens is simply to multiply the aperture function by the phase factor in Eq. (S12). At the back focal plane, we have, from Eq. (S11),

$$E(x,y,f) = -\frac{\mathrm{i}k}{2\pi}\left(\frac{\mathrm{e}^{\mathrm{i}kf}}{f}\right)\iint \mathrm{d}x'\,\mathrm{d}y'\,\underbrace{a(x',y')\,\mathrm{e}^{-\frac{\mathrm{i}k}{2f}\left(x'^2+y'^2\right)}}_{E(x',y',0^+)}\,\mathrm{e}^{\frac{\mathrm{i}k}{2f}\left[(x-x')^2+(y-y')^2\right]} \tag{S15a}$$

$$= -\frac{\mathrm{i}k}{2\pi}\left(\frac{\mathrm{e}^{\mathrm{i}kf}}{f}\right)\mathrm{e}^{\frac{\mathrm{i}k}{2f}\left(x^2+y^2\right)}\iint \mathrm{d}x'\,\mathrm{d}y'\,a(x',y')\;\mathrm{e}^{\frac{-\mathrm{i}k}{f}(xx'+yy')} \tag{S15b}$$

$$= -\frac{\mathrm{i}k}{2\pi}\left(\frac{\mathrm{e}^{\mathrm{i}kf}}{f}\right)\mathrm{e}^{\frac{\mathrm{i}k}{2f}\left(x^2+y^2\right)}\,\mathcal{F}\left[a(x,y)\right]_{u=kx/f,\,v=ky/f}\,, \tag{S15c}$$

$$= -\frac{\mathrm{i}k}{2\pi}\left(\frac{\mathrm{e}^{\mathrm{i}kf}}{f}\right)\mathrm{e}^{\frac{\mathrm{i}k}{2f}\left(x^2+y^2\right)}\,A(u,v)\big|_{u=kx/f,\,v=ky/f}\,. \tag{S15d}$$

In passing from Eq. (S15a) to (S15b), we expand the squared terms in the exponent, to cancel the x'^2 and y'^2 factors. Physically, the Huygens wavelets produce an expanding spherical wavefront while the lens adds a converging factor. At $z = f$ (back focal plane), the two factors just cancel. Notice that in the heuristic discussion given in the main text, we argued (1) that $\mathrm{e}^{\frac{\mathrm{i}k}{2z}\left(x'^2+y'^2\right)} \approx 1$ for z far enough behind the aperture (Fraunhofer limit); and (2) that adding a lens brings "infinity" back to one focal length behind the lens, where we can more conveniently view the diffraction pattern. Here, we see exactly why this heuristic argument works.

(3) *Aperture a distance d in front of the lens.* Equation (S15d) is almost the result we want: The field in the back focal plane is a Fourier transform of the aperture function, *except* for that pesky quadratic phase factor, $\mathrm{e}^{\frac{\mathrm{i}k}{2f}\left(x^2+y^2\right)}$. If we are interested only in imaging the diffraction pattern,

the phase factor does not matter, as it multiplies the intensity by a factor of unity. But if our goal is to modify the Fourier transform to perform image processing, then we should get rid of it. Fortunately, there is a simple trick to do this: We place the aperture a distance f in front of the lens. That is, we put the aperture in the *front focal plane* of the lens. We will see the result more clearly if we first consider the effects of placing the aperture a distance d in front, as illustrated in Fig. S1.

From Eq. (S8), propagating a field from the aperture at a distance d (defined now as the $z = 0$ plane) behind the lens implies that the field just before the lens, at $z = d$, is given by

$$E(x, y, d) = \mathrm{e}^{\mathrm{i}kd}\,\mathcal{F}^{-1}\left[A(u, v)\,\mathrm{e}^{\frac{-\mathrm{i}\left(u^2+v^2\right)d}{2k}}\right]. \tag{S16}$$

This field $E(x', y', d)$ replaces $a(x', y')$ in the integral in Eq. (S15b), giving

$$\begin{aligned}
E(x, y, d+f) &= -\frac{\mathrm{i}k}{2\pi}\left(\frac{\mathrm{e}^{\mathrm{i}kf}}{f}\right)\mathrm{e}^{\frac{\mathrm{i}k}{2f}\left(x^2+y^2\right)}\,\mathrm{e}^{\mathrm{i}kd}\,\mathcal{F}\left\{\mathcal{F}^{-1}\left[A(u, v)\,\mathrm{e}^{\frac{-\mathrm{i}\left(u^2+v^2\right)d}{2k}}\right]\right\}_{u=kx/f,\,v=ky/f} \\
&= -\frac{\mathrm{i}k}{2\pi}\left(\frac{\mathrm{e}^{\mathrm{i}k(d+f)}}{f}\right)\mathrm{e}^{\frac{\mathrm{i}k}{2f}\left(x^2+y^2\right)}\left[A(u, v)\,\mathrm{e}^{\frac{-\mathrm{i}\left(u^2+v^2\right)d}{2k}}\right]_{u=kx/f,\,v=ky/f} \\
&= -\frac{\mathrm{i}k}{2\pi}\left(\frac{\mathrm{e}^{\mathrm{i}k(d+f)}}{f}\right)\mathrm{e}^{\frac{\mathrm{i}k}{2f}\left(x^2+y^2\right)}\,\mathrm{e}^{\frac{-\mathrm{i}\left((kx/f)^2+(ky/f)^2\right)d}{2k}}\,A(u, v)\big|_{u=kx/f,\,v=ky/f} \\
&= -\frac{\mathrm{i}k}{2\pi}\left(\frac{\mathrm{e}^{\mathrm{i}k(d+f)}}{f}\right)\mathrm{e}^{\frac{\mathrm{i}k}{2f}\left(x^2+y^2\right)}\,\mathrm{e}^{\frac{-\mathrm{i}k}{2f}\left(x^2+y^2\right)\frac{d}{f}}\,A(u, v)\big|_{u=kx/f,\,v=ky/f} \\
&= -\frac{\mathrm{i}k}{2\pi}\left(\frac{\mathrm{e}^{\mathrm{i}k(d+f)}}{f}\right)\mathrm{e}^{\frac{\mathrm{i}k}{2f}\left(x^2+y^2\right)(1-d/f)}\,A(u, v)\big|_{u=kx/f,\,v=ky/f}\,.
\end{aligned} \tag{S17}$$

Finally, we see that if $d = f$ (i.e., the aperture is in the front focal plane of the lens), the exponential term with $(1 - d/f)$ is just unity, and we have

$$E(x, y, 2f) = -\frac{\mathrm{i}k}{2\pi}\left(\frac{\mathrm{e}^{2\,\mathrm{i}kf}}{f}\right)A(u, v)\big|_{u=kx/f,\,v=ky/f}\,, \tag{S18}$$

which is the claim made in the main text. Physically, putting the aperture f in front of the lens creates a diverging phase that just cancels the converging phase created by the plano-convex lens.

Supplemental Reading

Although we have tried to make this write-up relatively self-contained, you may benefit from seeing other treatments—particularly if you write a formal report on this lab.

- J. R. Meyer-Arendt, *Classical and Modern Optics*, Prentice-Hall, 1972, Section 4.3 (on Canvas). This excerpt is at a mostly descriptive level – simpler than the treatment given here – which can be useful if you are confused about the "big picture."
- Eugene Hecht, *Optics*, 4th ed., Addison-Wesley, 2002, particularly
  - Section 7.3, Review of Fourier Series,
  - Section 10.2, Review of Fraunhofer Diffraction,
  - Chapter 11, Fourier Optics, and
  - Section 13.2, Abbe's Theory of Image Formation and Spatial Filtering.

  There are hard copies of this book in the Optics Laboratory (editions may vary). A simple version of the theory is presented. The accompanying presentation is very wordy, which is an advantage, or not, depending on your taste. There is a 5th edition, too, from 2016, which the library currently does not own. The sections differ slightly.
- A. Lipson, S. G. Lipson, and H. Lipson, *Optical Physics*, 4th ed., Cambridge Univ. Press, 2011. Read Chapters 7.1, 8.1–8.5, and 12.1–12.4. In Chapter 12, Section 12.4 is the most directly relevant. Available online through SFU Library. This is the current text for Physics 455 and is more advanced than Hecht.
- J. Goodman, *Introduction to Fourier Optics*, 2nd ed., McGraw-Hill, 1996. The details of the latter part of the derivation in the Appendix are largely drawn from this reference, which is a graduate-level text.
- Digital Imaging Handout (on Canvas)
- Optical Systems Guide (on Canvas)