

10

Statistics

10.1 STATISTICS is a branch of science dealing with the collection of data, organising, summarising, presenting and analysing data and drawing valid conclusions and thereafter making reasonable decisions on the basis of such analysis.

10.2 FREQUENCY DISTRIBUTION is the arranged data, summarised by distributing it into classes or categories with their frequencies.

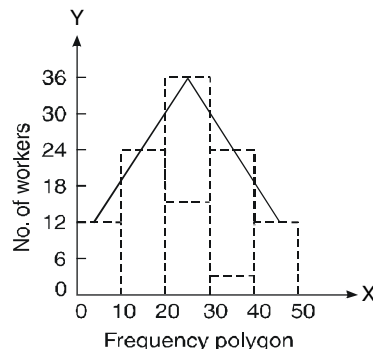
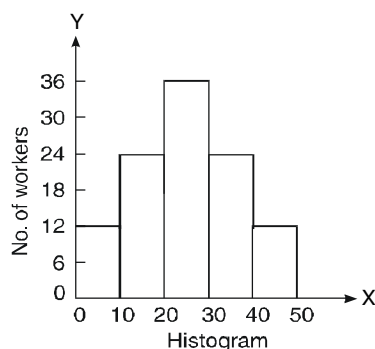
Wages of 100 workers

Wages in ₹	0-10	10-20	20-30	30-40	40-50
Numbers of workers	12	23	35	20	10

10.3 GRAPHICAL REPRESENTATION. It is often useful to represent frequency distribution by means of a diagram. The different types of diagrams are

1. Histogram
2. Frequency polygon
3. Frequency curve
4. Cumulative frequency curve or Ogive
5. Bar chart
6. Circles or Pie diagrams.

1. **Histogram** consists of a set of rectangles having their heights proportional to the class-frequencies, for equal class-intervals. For unequal class-interval, the areas of rectangles are proportional to the frequencies.



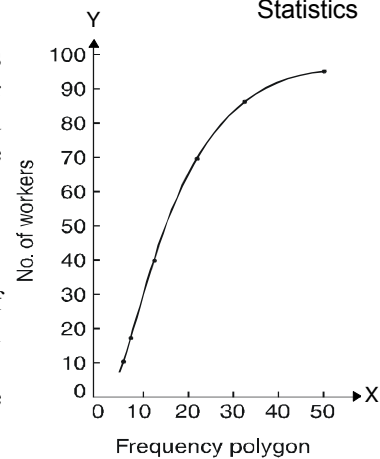
2. **Frequency Polygon** is a line graph of class-frequency plotted against class-mark. It can be obtained by connecting mid-points on the tops of the rectangles in the histogram.

3. Cumulative Frequency curve or the Ogive. If the various points are plotted according to the upper limit of the class as x co-ordinate and the cumulative frequency as y co-ordinate and these points are joined by a free hand smooth curve, the curve obtained is known as cumulative frequency curve or the Ogive.

10.4 AVERAGE OR MEASURES OF CENTRAL TENDENCY

An average is a value which is representative of a set of data. Average value may also be termed as measures of central tendency. There are five types of averages in common.

- (i) Arithmetic average or mean (ii) Median (iii) Mode
(iv) Geometric Mean (v) Harmonic Mean



10.5 ARITHMETIC MEAN

If $x_1, x_2, x_3, \dots, x_n$ are n numbers, then their arithmetic mean (A.M.) is defined by

$$AM = \frac{x_1 + x_2 + x_3 + \dots + x_n}{n} = \frac{\sum x}{n}$$

If the number x_1 occurs f_1 times, x_2 occurs f_2 times and so on, then

$$AM = \frac{f_1 x_1 + f_2 x_2 + \dots + f_n x_n}{f_1 + f_2 + \dots + f_n} = \frac{\sum fx}{\sum f}$$

This is known as direct method.

Example 1. Find the mean of 20, 22, 25, 28, 30.

Solution. $AM. = \frac{20 + 22 + 25 + 28 + 30}{5} = \frac{125}{5} = 25$

Ans.

Example 2. Find the mean of the following :

Numbers	8	10	15	20
Frequency	5	8	8	4

Solution. $\sum fx = 8 \times 5 + 10 \times 8 + 15 \times 8 + 20 \times 4 = 40 + 80 + 120 + 80 = 320$

$$\sum f = 5 + 8 + 8 + 4 = 25$$

$$AM. = \frac{\sum fx}{\sum f} = \frac{320}{25} = 12.8$$

Ans.

(b) Short cut method

Let a be the assumed mean, d the deviation of the variate x from a . Then

$$\frac{\sum fd}{\sum f} = \frac{\sum f(x-a)}{\sum f} = \frac{\sum fx}{\sum f} - \frac{\sum fa}{\sum f} = A.M. - \frac{a \sum f}{\sum f} = A.M. - a$$

$$\therefore A.M. = a + \frac{\sum fd}{\sum f}$$

Example 3. Find the arithmetic mean for the following distribution:

Class	0-10	10-20	20-30	30-40	40-50
Frequency	7	8	20	10	5

Solution. Let assumed mean (a) = 25.

Class	Mid-value x	Frequency f	$x - 25 = d$	fd
0 – 10	5	7	-20	-140
10 – 20	15	8	-10	-80
20 – 30	25	20	0	0
30 – 40	35	10	+ 10	+ 100
40 – 50	45	5	+ 20	+ 100
Total		50		- 20

$$A.M. = a + \frac{\sum fd}{\sum f} = 25 + \frac{-20}{50} = 24.6$$

Ans.

(c) Step deviation method

Let a be the assumed mean, i the width of the class interval and

$$D = \frac{x-a}{i}, A.M. = a + \frac{\sum fD}{\sum f} i$$

Example 4. Find the arithmetic mean of the data given in example 3 by step deviation method

Solution. Let $a = 25$

Class	Mid-value x	frequency f	$D = \frac{x-a}{i}$	$f \cdot D$
0 – 10	5	7	-2	-14
10 – 20	15	8	-1	-8
20 – 30	25	20	0	0
30 – 40	35	10	+ 1	+ 10
40 – 50	45	5	+ 2	+ 10
Total		50		- 2

$$A.M. = a + \frac{\sum fD}{\sum f} i = 25 + \frac{-2}{50} \times 10 = 24.6$$

Ans.

10.6 MEDIAN

Median is defined as the measure of the central item when they are arranged in ascending or descending order of magnitude.

When the total number of the items is odd and equal to say n , then the value of $\frac{1}{2}(n+1)$ th item gives the median.

When the total number of the frequencies is even, say n , then there are two middle items, and so

the mean of the values of $\frac{1}{2}n$ th and $\left(\frac{1}{2}n+1\right)$ th items is the median.

Example 5. Find the median of 6, 8, 9, 10, 11, 12, 13.

Solution. Total number of items = 7

$$\text{The middle item} = \frac{1}{2}(7+1)^{\text{th}} = 4^{\text{th}}$$

$$\text{Median} = \text{Value of the 4th item} = 10$$

Ans.

$$\text{For grouped data, Median} = l + \frac{\frac{1}{2}N - F}{f} \cdot i$$

where l is the lower limit of the median class, f is the frequency of the class, i is the width of the class-interval, F is the total of all the preceeding frequencies of the median-class and N is total frequency of the data.

Example 6. Find the value of Median from the following data:

No. of days for which absent (less than)	5	10	15	20	25	30	35	40	45
No. of students	29	224	465	582	634	644	650	653	655

Solution. The given cumulative frequency distribution will first be converted into ordinary frequency as under

Class- Interval	Cumulative frequency	Ordinary frequency
0 – 5	29	29 = 29
5 – 10	224	224 – 29 = 195
10 – 15	465	465 – 224 = 241
15 – 20	582	582 – 465 = 117
20 – 25	634	634 – 582 = 52
25 – 30	644	644 – 634 = 10
30 – 35	650	650 – 644 = 6
35 – 40	653	653 – 650 = 3
40 – 45	655	655 – 653 = 2

$$\text{Median} = \text{size of } \frac{655}{2} \text{ or } 327.5\text{th item}$$

327.5th item lies in 10-15 which is the median class.

$$M = l + \frac{\frac{N}{2} - C}{f} \cdot i$$

where l stands for lower limit of median class,

N stands for the total frequency,

C stands for the cumulative frequency just preceeding the median class,

i stands for class interval

f stands for frequency for the median class.

$$\begin{aligned} \text{Median} &= 10 + \frac{\frac{655}{2} - 224}{241} \times 5 \\ &= 10 + \frac{103.5 \times 5}{241} = 10 + 2.15 = 12.15 \end{aligned}$$

Ans.

10.7 MODE

Mode is defined to be the size of the variable which occurs most frequently.

Example 7. Find the mode of the following items :

0, 1, 6, 7, 2, 3, 7, 6, 6, 2, 6, 0, 5, 6, 0.

Solution. 6 occurs 5 times and no other item occurs 5 or more than 5 times, hence the mode is 6. **Ans.**

For grouped data, $Mode = l + \frac{f - f_{-1}}{2f - f_{-1} - f_1} \cdot i$

where l is the lower limit of the modal class, f is the frequency of the modal class, i is the width of the class, f_{-1} is the frequency before the modal class and f_1 is the frequency after the modal class.

Emperical formula

$$\text{Mean} - \text{Mode} = 3 [\text{Mean} - \text{Median}]$$

Example 8. Find the mode from the following data:

Age	0 – 6	6 – 12	12 – 18	18 – 24	24 – 30	30 – 36	36 – 42
Frequency	6	11	25	35	18	12	6

Solution.

Age	Frequency	Cumulative frequency
0 – 6	6	6
6 – 12	11	17
12 – 18	$25 = f_{-1}$	42
<u>18 – 24</u>	$35 = f$	77
24 – 30	$18 = f_1$	95
30 – 36	12	107
36 – 42	6	113

$$\begin{aligned}
 \text{Mode} &= l + \frac{f - f_{-1}}{2f - f_{-1} - f_1} \times i \\
 &= 18 + \frac{35 - 25}{70 - 25 - 18} \times 6 \\
 &= 18 + \frac{60}{27} = 18 + 2.22 = 20.22
 \end{aligned}$$

Ans.

10.8 GEOMETRIC MEAN

If $x_1, x_2, x_3, \dots, x_n$ be n values of variates x , then the geometric mean

$$G = (x_1 \times x_2 \times x_3 \times x_4 \times \dots \times x_n)^{\frac{1}{n}}$$

Example 9. Find the geometric mean of 4, 8, 16.

Solution.

$$G.M. = (4 \times 8 \times 16)^{1/3} = 8.$$

Ans.

10.9 HARMONIC MEAN

Harmonic mean of a series of values is defined as the reciprocal of the arithmetic mean of their reciprocals. Thus if H be the harmonic mean, then

$$\frac{1}{H} = \frac{1}{n} \left[\frac{1}{x_1} + \frac{1}{x_2} + \dots + \frac{1}{x_n} \right]$$

Example 10. Calculate the harmonic mean of 4, 8, 16.

Solution.
$$\frac{1}{H} = \frac{1}{3} \left[\frac{1}{4} + \frac{1}{8} + \frac{1}{16} \right] = \frac{7}{48}$$

$$H = \frac{48}{7} = 6.853$$

Ans.

10.10 AVERAGE DEVIATION OR MEAN DEVIATION

It is the mean of the absolute values of the deviations of a given set of numbers from their arithmetic mean.

If $x_1, x_2, x_3, \dots, x_n$ be a set of numbers with frequencies f_1, f_2, \dots, f_n respectively. Let \bar{x} be the arithmetic mean of the numbers x_1, x_2, \dots, x_n , then

$$\text{Mean deviation} = \frac{\sum f_i |x_i - \bar{x}|}{\sum f_i}$$

Example 11. Find the mean deviation of the following frequency distribution.

Class	0 – 6	6 – 12	12 – 18	18 – 24	24 – 30
Frequency	8	10	12	9	5

Solution. Let $a = 15$

Class	Mid- value x	Frequency f	$d = x - a$	fd	$ x - 14 $	$f x - 14 $
0–6	3	8	–12	–96	11	88
6–12	9	10	–6	–60	5	50
12–18	15	12	0	0	1	12
18–24	21	9	+6	54	7	63
24–30	27	5	+12	60	13	65
Total		44		–42		278

$$\text{Mean} = a + \frac{\sum fd}{\sum f} = 15 - \frac{42}{44} = 14 \text{ nearly}$$

$$\text{Average deviation} = \frac{\sum f |x - \bar{x}|}{\sum f} = \frac{278}{44} = 6.3$$

Ans.

10.11 STANDARD DEVIATION

Standard deviation is defined as the square root of the mean of the square of the deviation from the arithmetic mean.

$$S.D. = \sigma = \sqrt{\frac{\sum f(x - \bar{x})^2}{\sum f}}$$

Note. 1. The square of the standard deviation σ^2 is called variance.

2. σ^2 is called the second moment about the mean and is denoted by μ_2 .

10.12 SHORTEST METHOD FOR CALCULATING STANDARD DEVIATION

$$\begin{aligned} \text{We know that } \sigma^2 &= \frac{1}{N} \sum f(x - \bar{x})^2 = \frac{1}{N} \sum f(x - a - \overline{x - a})^2 \\ &= \frac{1}{N} \sum f(d - \overline{x - a})^2 \quad \text{Where } x - a = d \end{aligned}$$

$$\begin{aligned}
&= \frac{1}{N} \sum fd^2 - 2(\bar{x} - a) \frac{1}{N} \sum fd + (\bar{x} - a)^2 \frac{1}{N} \sum f \sum f = N \\
&= \frac{1}{N} \sum fd^2 - 2(\bar{x} - a) \frac{1}{N} \sum fd + (\bar{x} - a)^2 \\
\bar{x} &= a + \frac{\sum fd}{N} \quad \text{or} \quad \bar{x} - a = \frac{\sum fd}{N} \\
\sigma^2 &= \frac{1}{N} \sum fd^2 - 2 \left(\frac{\sum fd}{N} \right) \left(\frac{1}{N} \sum fd \right) + \left(\frac{\sum fd}{N} \right)^2 \\
&= \frac{1}{N} \sum fd^2 - \left(\frac{\sum fd}{N} \right)^2 \\
S.D. = \sigma &= \sqrt{\frac{\sum fd^2}{N} - \left(\frac{\sum fd}{N} \right)^2}
\end{aligned}$$

Note. Coefficient of variation = $\frac{\sigma}{\bar{x}} \times 100$

Example 12. Calculate the mean and standard deviation for the following data :

Size of item	6	7	8	9	10	11	12
Frequency	3	6	9	13	8	5	4

Solution. Assumed mean = 9

(A.M.I.E., Winter 2001)

x	f	$d = x - a$	$f.d.$	$f.d^2$
6	3	-3	-9	27
7	6	-2	-12	24
8	9	-1	-9	9
9	13	0	0	0
10	8	+1	8	8
11	5	+2	10	20
12	4	+3	12	36
	$\sum f = 48$		$\sum fd = 0$	$\sum fd^2 = 124$

$$\text{Mean} = a + \frac{\sum fd}{\sum f} = 9 + 0 = 9$$

$$S.D. = \sqrt{\frac{\sum f(x - \bar{x})^2}{\sum f}}$$

$$= \sqrt{\frac{\sum fd^2}{\sum f} - \left(\frac{\sum fd}{\sum f} \right)^2} = \sqrt{\frac{124}{48}} = 1.6$$

Ans.

Example 13. From the following frequency distribution, compute the standard deviation of 100 students :

Mass in kg	60 – 62	63 – 65	66 – 68	69 – 71	72 – 74
Number of students	5	18	42	27	8

Solution. Assumed mean = 67

Mass in kg	Number of students f	x	$d = x - 67$	$f \cdot d$	$f \cdot d^2$
60 – 62	5	61	– 6	– 30	180
63 – 65	18	64	– 3	– 54	162
66 – 68	42	67	0	0	0
69 – 71	27	70	3	81	243
72 – 74	8	73	6	48	288
	$\sum f = 100$			$\sum fd = 45$	$\sum f d^2 = 873$

$$S.D. = \sqrt{\frac{\sum fd^2}{\sum f} - \left(\frac{\sum fd}{\sum f}\right)^2} = \sqrt{\frac{873}{100} - \left(\frac{45}{100}\right)^2}$$

$$= \sqrt{8.73 - 0.2025} = \sqrt{8.5275} = 2.9202 \quad \text{Ans.}$$

Example 14. Compute the standard deviation for the following frequency distribution:

Class interval	0 – 4	4 – 8	8 – 12	12 – 16
Frequency	4	8	2	1

Solution. Assumed mean = 6

Class interval f	x	$d = x - 6$	fd	fd^2	
0 – 4	4	2	– 4	– 16	64
4 – 8	8	6	0	0	0
8 – 12	2	10	+ 4	8	32
12 – 16	1	14	+ 8	8	64
	$\sum f = 15$			$\sum fd = 0$	$\sum f d^2 = 160$

$$S.D. = \sqrt{\frac{\sum fd^2}{\sum f} - \left(\frac{\sum fd}{\sum f}\right)^2} = \sqrt{\frac{160}{15} - 0} = 3.266 \quad \text{Ans.}$$

10.13 MOMENTS

The r th moment of a variable x about the mean \bar{x} is usually denoted by μ_r is given by

$$\mu_r = \frac{1}{N} \sum f_i (x_i - \bar{x})^r, \quad \sum f_i = N$$

The r th moment of a variable x about any point a is defined by

$$\mu'_r = \frac{1}{N} \sum f_i (x_i - a)^r$$

In particular
$$\mu_0 = \frac{1}{N} \sum f_i (x - \bar{x})^0 = \frac{1}{N} \sum f_i = \frac{N}{N} = 1$$

$$\mu'_0 = \frac{1}{N} \sum f_i (x - a)^0 = \frac{1}{N} \sum f_i = \frac{N}{N} = 1$$

$$\mu_1 = \frac{1}{N} \sum f_i(x - \bar{x}) = 0, \quad \mu'_1 = \frac{1}{N} \sum f_i(x - a) = \bar{x} - a$$

$$\mu_2 = \frac{1}{N} \sum f_i(x - \bar{x})^2 = \sigma^2.$$

Relation between moments about mean and moment about any point.

$$\begin{aligned} \mu_r &= \frac{1}{N} \sum f_i(x - \bar{x})^r = \frac{1}{N} \sum f_i[(x - a) - (\bar{x} - a)]^r \\ &= \frac{1}{N} \sum f_i(X_i - d)^r \quad \text{where } X_i = x - a \text{ and } d = \bar{x} - a \\ &= \frac{1}{N} \left[\sum f_i X_i^r - {}^r C_1 \left(\sum f_i X_i^{r-1} \right) d + {}^r C_2 \left(\sum f_i X_i^{r-2} \right) d^2 - {}^r C_3 \left(\sum f_i X_i^{r-3} \right) d^3 + \dots \right] \\ &= \mu'_r - {}^r C_1 d \mu'_{r-1} + {}^r C_2 d^2 \mu'_{r-2} - {}^r C_3 d^3 \mu'_{r-3} + \dots \end{aligned}$$

In particular

$$\mu_2 = \mu'_2 - \mu_1^2$$

$$\mu_3 = \mu'_3 - 3\mu'_2 \mu'_1 + 2\mu_1^3$$

$$\mu_4 = \mu'_4 - 4\mu'_3 \mu'_1 + 6\mu'_2 \mu_1^2 - 3\mu_1^4$$

Note.1. The sum of the coefficients of the various terms on the right-hand side is zero.

2. The dimension of each term on right-hand side is the same as that of terms on the left.

10.14 MOMENT GENERATING FUNCTION

The moment generating function of the variate x about $x = a$ is defined as the expected value of $e^{t(x-a)}$ and is denoted by $M_a(t)$.

$$\begin{aligned} M_a(t) &= \sum P_i e^{t(x_i - a)} \\ &= \sum P_i + t \sum P_i(x_i - a) + \frac{t^2}{2!} \sum P_i(x_i - a)^2 + \dots + \frac{t^r}{r!} \sum P_i(x_i - a)^r + \dots \\ &= 1 + t\mu'_1 + \frac{t^2}{2!} \mu'_2 + \dots + \frac{t^r}{r!} \mu'_r + \dots \end{aligned}$$

where μ'_r is the moment of order r about a

$$\text{Hence} \quad \mu'_r = \text{coefficient of } \frac{t^r}{r!} \quad \text{or} \quad \mu'_r = \left[\frac{d^r}{dt^r} M_a(t) \right]_{t=0}$$

$$\text{again} \quad M_a(t) = \sum P_i e^{t(x_i - a)} = e^{-at} \sum P_i e^{tx_i} = e^{-at} M_0(t)$$

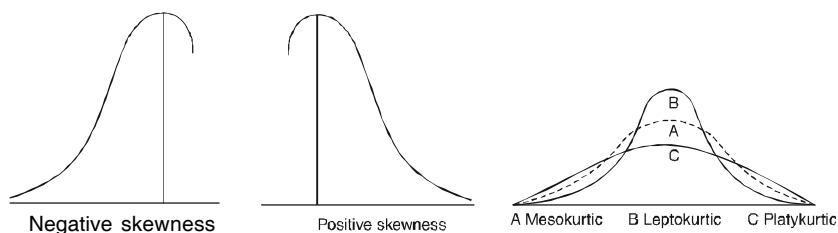
Thus the moment generating function about the point $a = e^{-at}$ moment generating function about the origin.

10.15 (1) SKEWNESS

Skewness denotes the opposite of symmetry. It is lack of symmetry. In a symmetrical series, the mode, the median, and the arithmetic average are identical.

$$\text{Coefficient of skewness} = \frac{\text{Mean} - \text{Mode}}{\text{standard deviation}}$$

(2) KURTOSIS. It measures the degree of peakedness of a distribution and is given by Measure of kurtosis



$$\beta_2 = \frac{\mu_4}{\mu_2^2}, \quad \mu_2 = \frac{\sum (x - \bar{x})^2}{N}, \quad \mu_4 = \frac{\sum (x - \bar{x})^4}{N},$$

If $\beta_2 = 3$, the curve is normal or mesokurtic.

If $\beta_2 > 3$, the curve is peaked or leptokurtic.

If $\beta_2 < 3$, the curve is flat topped or platykurtic.

Exercise 10.1

1. Marks obtained by 9 students in statistics are given below

52, 57, 40, 70, 43, 40, 65, 35, 48

Calculate the arithmetic mean.

Ans.

50.

2. Calculate the mean of the following:

Height in cm	65	66	67	68	69	70	71	72	73
Number of plants	1	4	5	7	11	10	6	4	2

Ans. 69.18.

3. Find the mean for the following distribution :

Marks	No. of students	Marks	No. of Students
0-10	3	50 – 60	15
10-20	5	60 – 70	12
20-30	7	70 – 80	6
30-40	10	80 – 90	2
40-50	12	90 – 100	8

Ans. 51.75

4. Determine the mode from the following figures:

25, 15, 23, 40, 27, 25, 23, 25, 20.

Ans. 25.

5. Find the median of the following :

20, 18, 22, 27, 25, 12, 15.

Ans. 20.

6. The Mean of 200 items was 50. Later on it was discovered that two items were misread as 92 and 8 instead of 192 and 88. Find the corrected mean

Ans. 53.6