Joseph Sternberg, Dongpeng Xia
Appendix, Graphs, Tables, and Model Output:

Table 1.

Individual Variables and Statistics

Variable	Min	Q1	Median	Mean	Q3	Max
Median Home Sales Price (USD)	18,500	48,850	120,000	129,100	210,600	318,200
Households Estimate (thousands)	55,980	74,660	94,600	91,220	106,900	119,100
Real GDP Per Capita (USD)	19,230	27,040	35,630	36,050	46,670	52,680
Unemployment Rate (%)	3.200	4.850	5.700	6.065	7.100	11.200

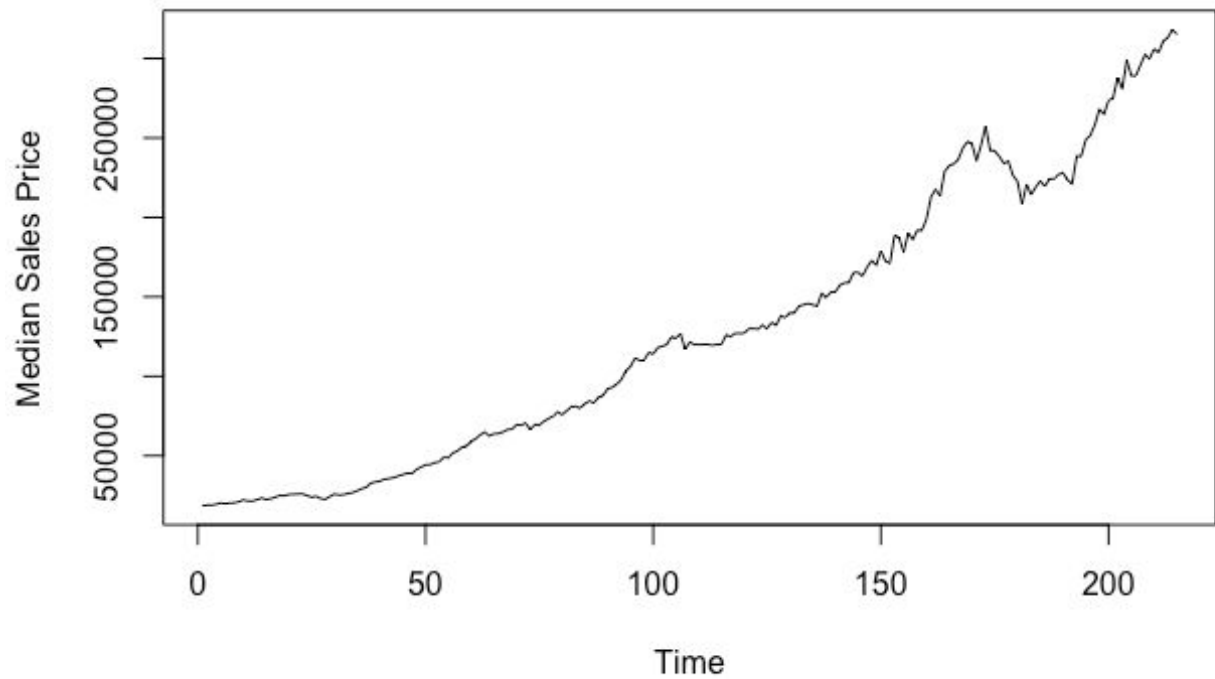*Figure 1:*

Median Home Sales Price over Time

Figure 2:
Household Estimates Over Time
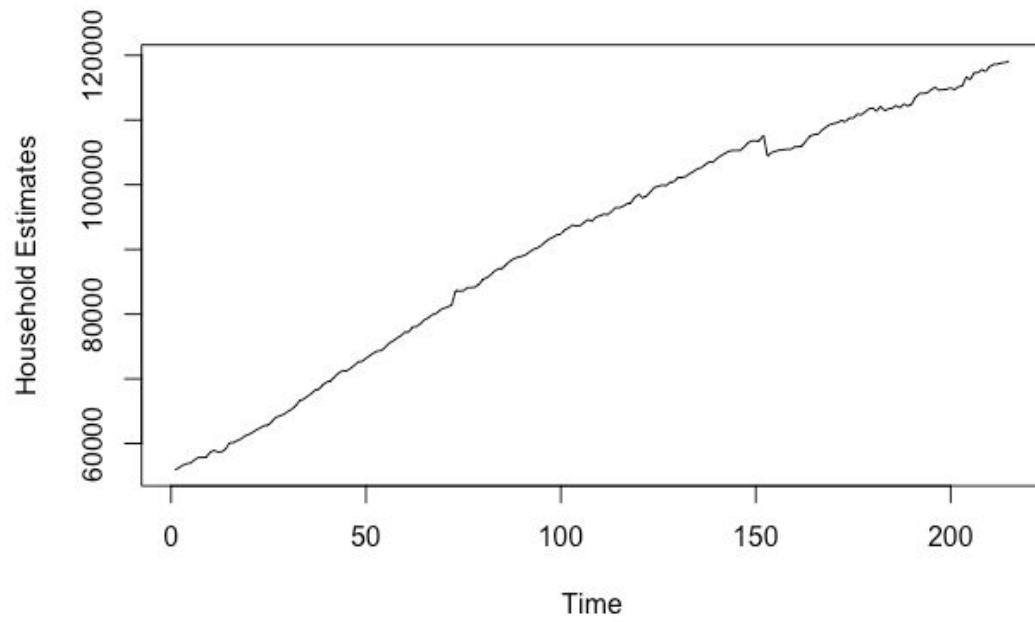

Figure 3:
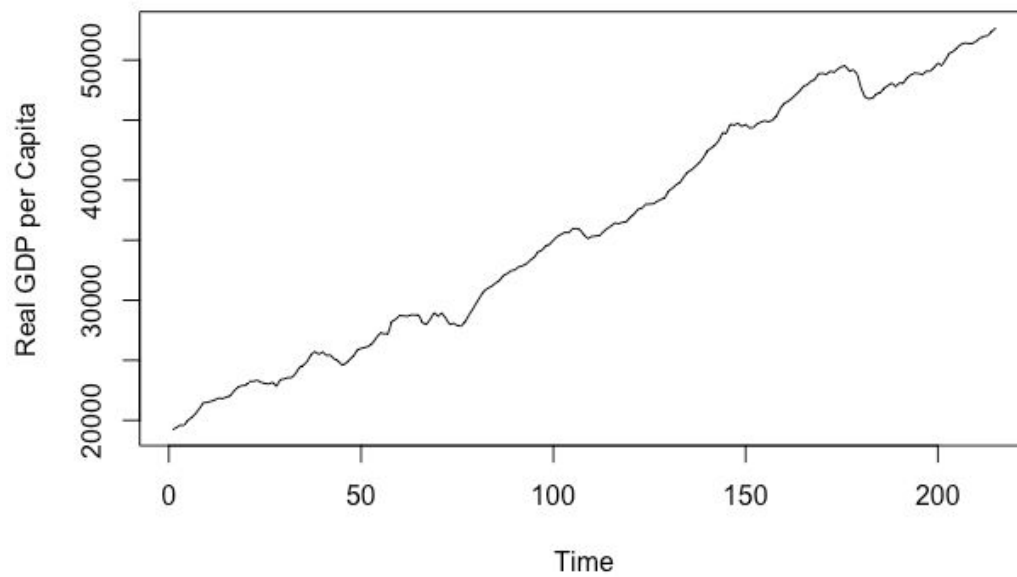Real GDP Per Capita Over Time

Figure 4:
Unemployment Rate Over Time
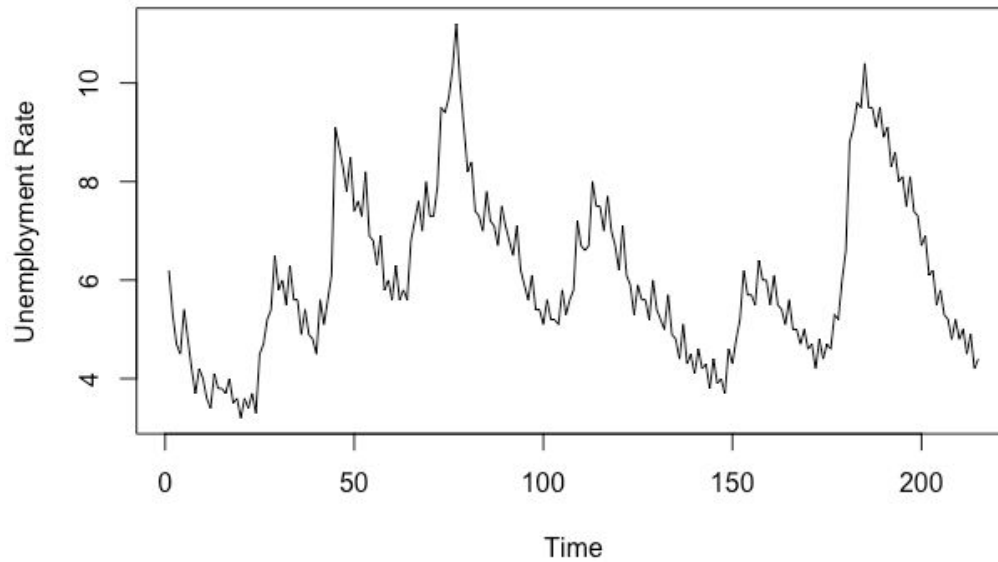
Figure 5:
Median Sales Price Regressed on Time

lm(formula = `Median Sales Price` ~ t)

Residuals:

Min	1Q	Median	3Q	Max
-23365	-13986	-5306	12308	43533

Coefficients:

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-19175.7	2416.8	-7.934	1.19e-13 ***
t (time)	1373.1	19.4	70.770	< 2e-16 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 17660 on 213 degrees of freedom
Multiple R-squared: 0.9592, Adjusted R-squared: 0.959
F-statistic: 5008 on 1 and 213 DF, p-value: < 2.2e-16

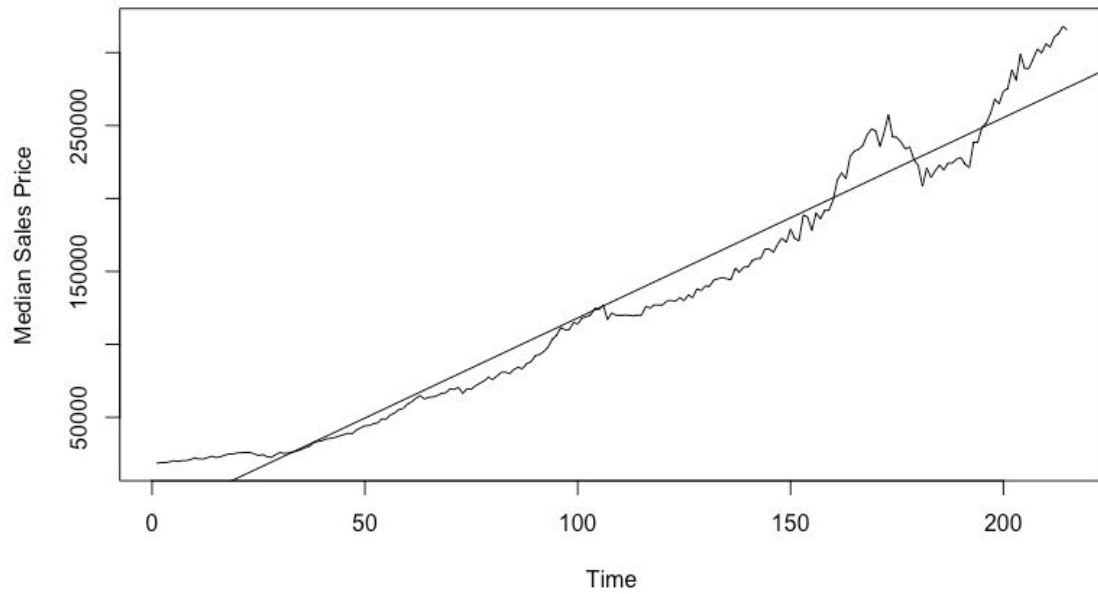*Figure 5 (Continued):*

Median Sales Price Regressed on Time
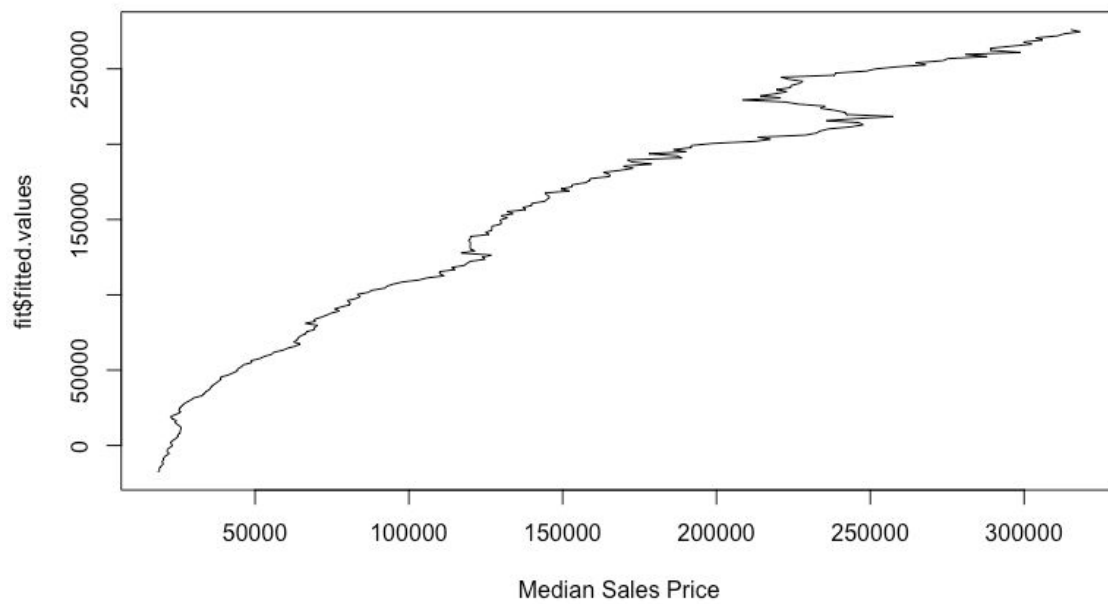

Figure 6:

Fitted Values vs Median Sales Price

Figure 7:
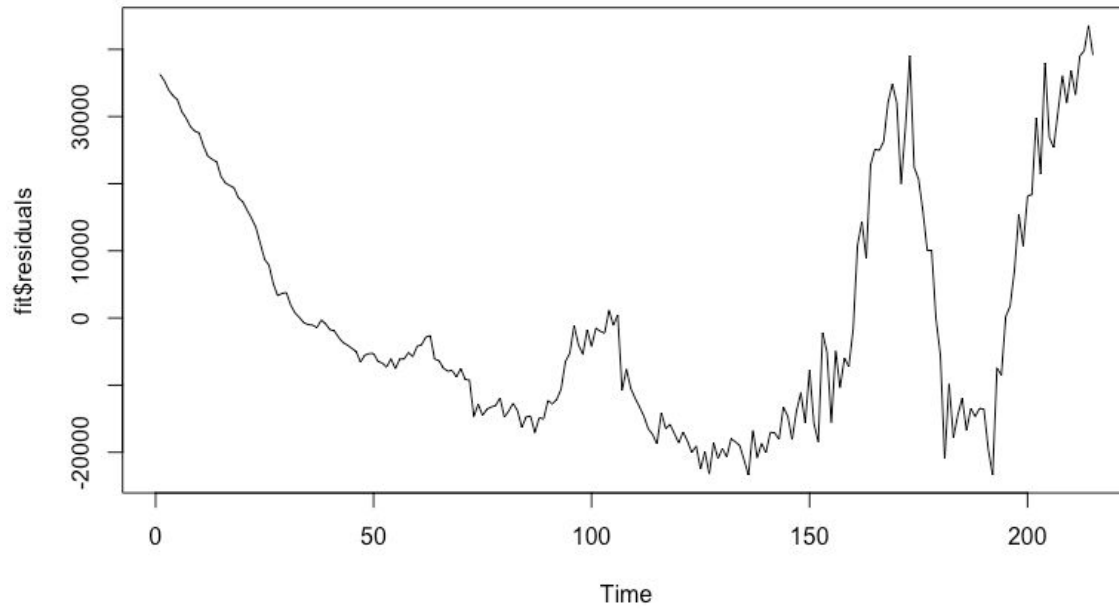Residuals over Time (Median Sales Price vs Time)

Figure 8:
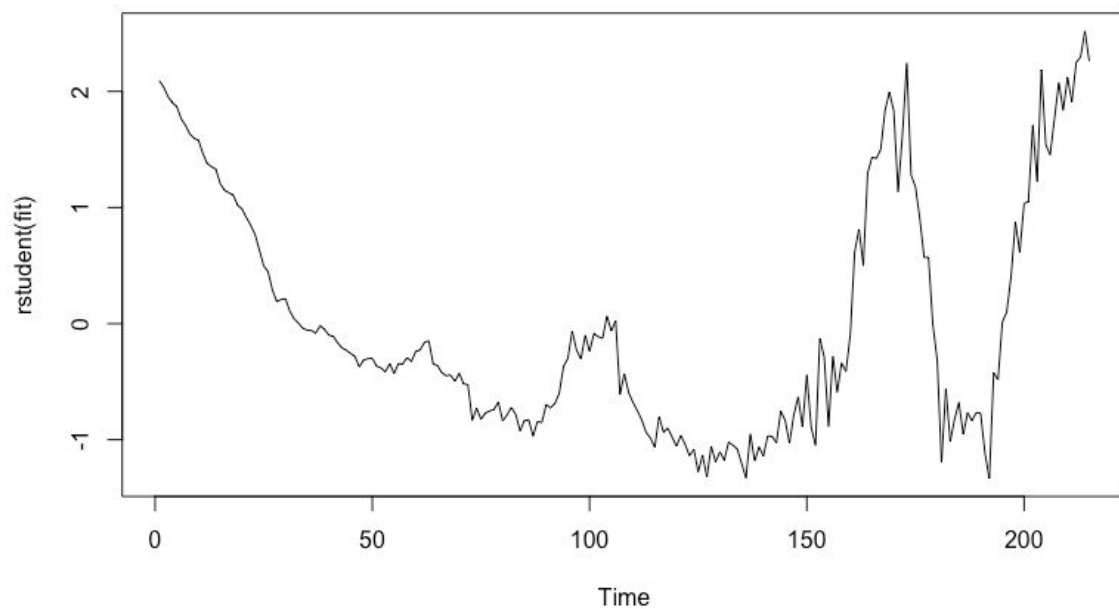Studentized Residuals over Time (Median Sales Price vs Time)

Figure 9:
ACF of Simple Linear Regression (Median Sales Price vs Time)

Series Median Sales Price

Figure 10:
PACF of Simple Linear Regression (Median Sales Price vs Time)
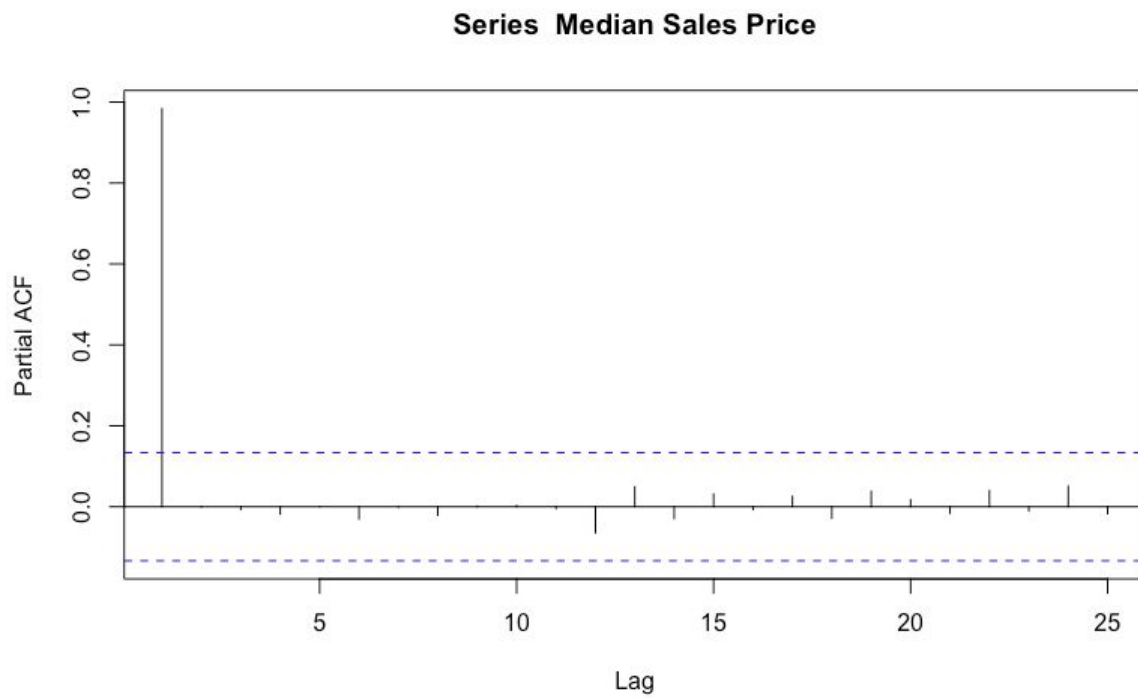

Series Median Sales Price

Figure 11, Seasonal Models:

Seasonal Model With Intercept:
lm(formula = mdHousePrice ~ mdqrtr)

Residuals:

Min	1Q	Median	3Q	Max
-111144	-80901	-9457	82114	188156

Coefficients:

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	128214.8	11952.4	10.727	<2e-16 ***
mdqrtr2Q	1829.6	16903.2	0.108	0.914
mdqrtr3Q	1242.6	16903.2	0.074	0.941
mdqrtr4Q	536.1	16982.8	0.032	0.975

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 87830 on 211 degrees of freedom
Multiple R-squared: 6.374e-05,   Adjusted R-squared: -0.01415
F-statistic: 0.004483 on 3 and 211 DF, p-value: 0.9996

Seasonal Model Without Intercept:
lm(formula = mdHousePrice ~ mdqrtr - 1)

Residuals:

Min	1Q	Median	3Q	Max
-111144	-80901	-9457	82114	188156

Coefficients:

	Estimate	Std. Error	t value	Pr(>|t|)
mdqrtr1Q	128215	11952	10.73	<2e-16 ***
mdqrtr2Q	130044	11952	10.88	<2e-16 ***
mdqrtr3Q	129457	11952	10.83	<2e-16 ***
mdqrtr4Q	128751	12065	10.67	<2e-16 ***

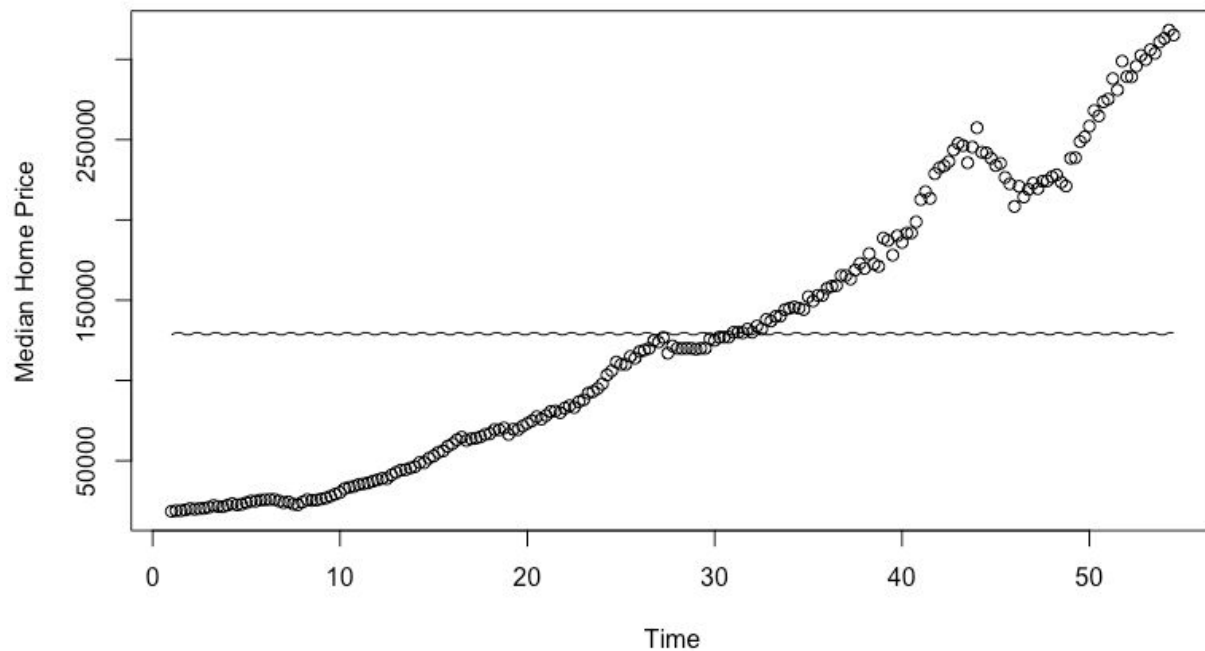Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 87830 on 211 degrees of freedom
Multiple R-squared: 0.6877, Adjusted R-squared: 0.6818
F-statistic: 116.2 on 4 and 211 DF, p-value: < 2.2e-16

Figure 12:
Harmonic Model (Without Removing Trend)

lm(formula = mdHousePrice ~ har)

Residuals:

Min	1Q	Median	3Q	Max
-110865	-80901	-9737	81834	188435

Coefficients:

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	129115.6	5976.1	21.605	<2e-16 ***
harcos(2*pi*t)	-621.3	8431.7	-0.074	0.941
harsin(2*pi*t)	649.4	8471.2	0.077	0.939

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 87620 on 212 degrees of freedom
Multiple R-squared: 5.333e-05,   Adjusted R-squared: -0.00938
F-statistic: 0.005653 on 2 and 212 DF, p-value: 0.9944

Figure 13:

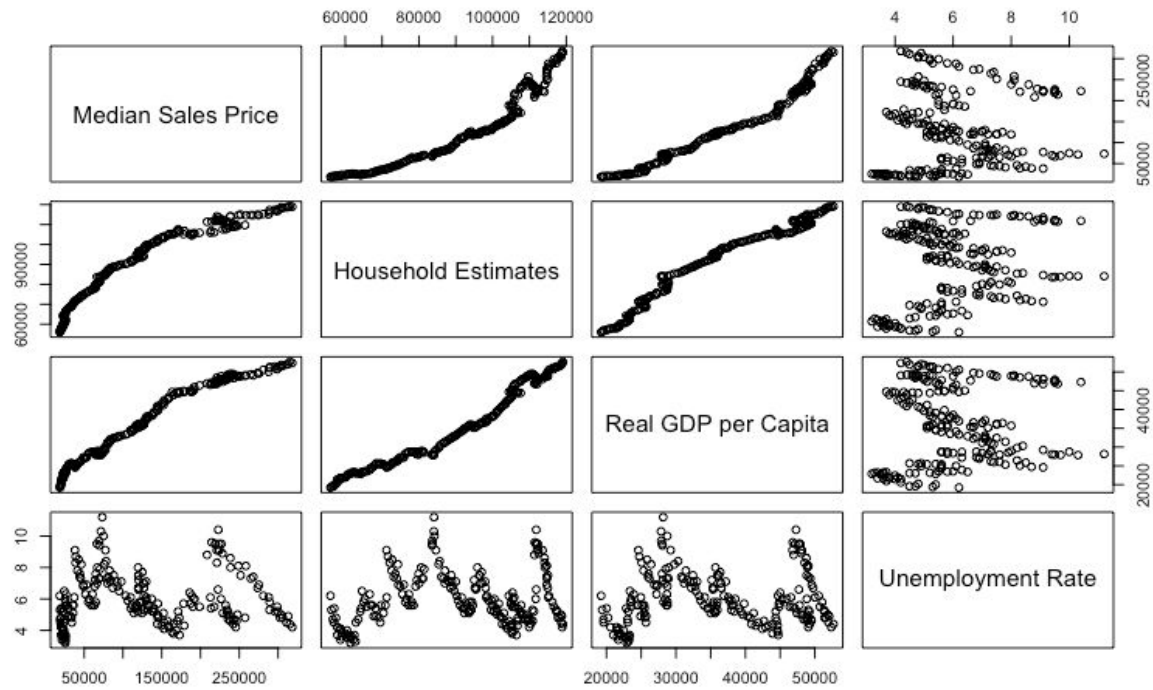Plots of Median Home Sales Price and (Household Estimates, GDP, Unemployment)

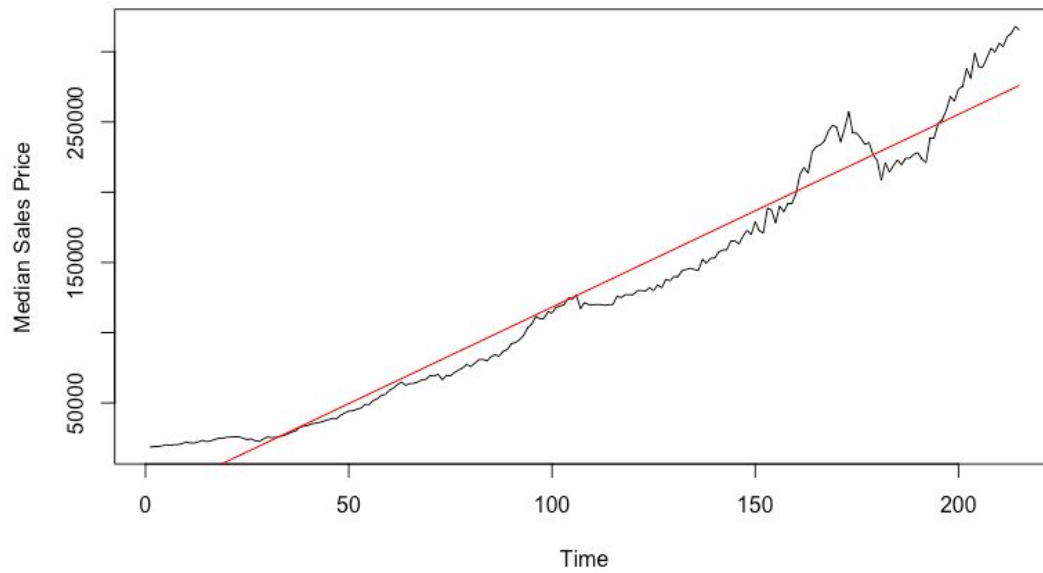*Table 2:*

AIC for each polynomial model:

n = order of polynomial

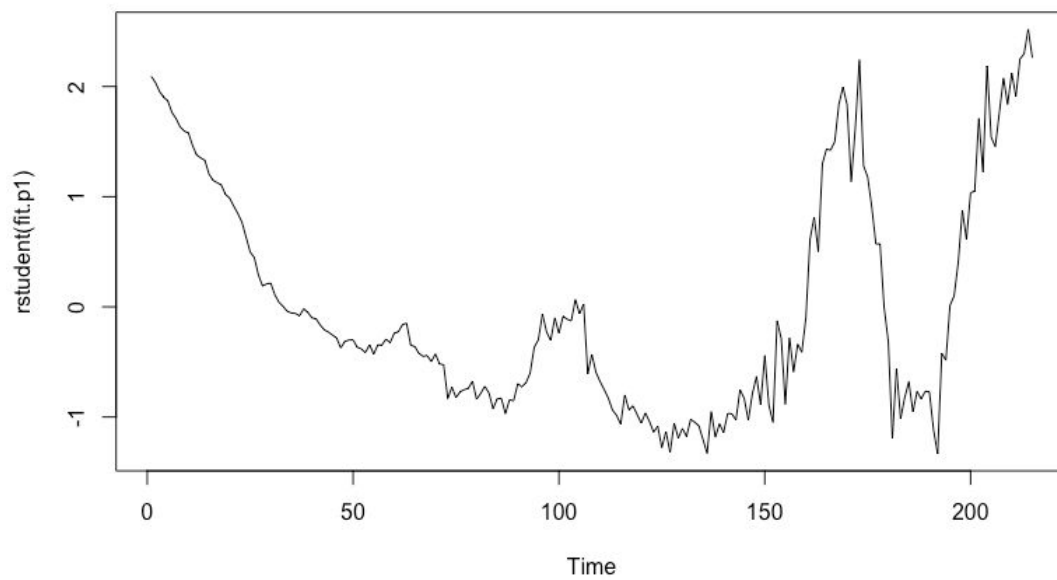| n	1	2	3	4	5
AIC	4819.051	4623.237	4624.662	4616.043	4617.563

n	6	7	8	9	10
AIC	4604.735	4568.420	4570.291	4537.316	4440.230

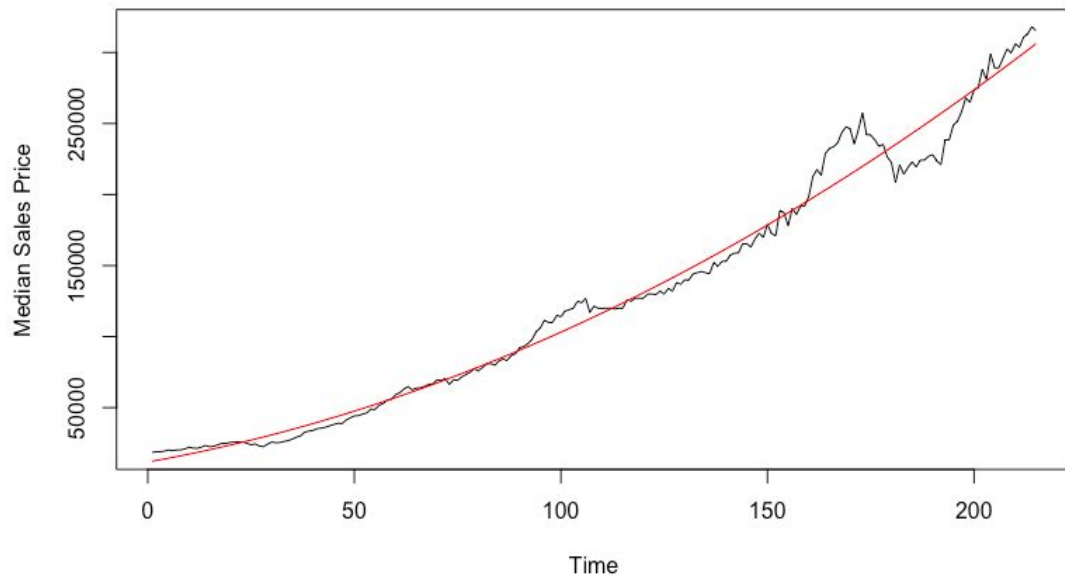*Figure 14, House Price vs Polynomials of Time*:
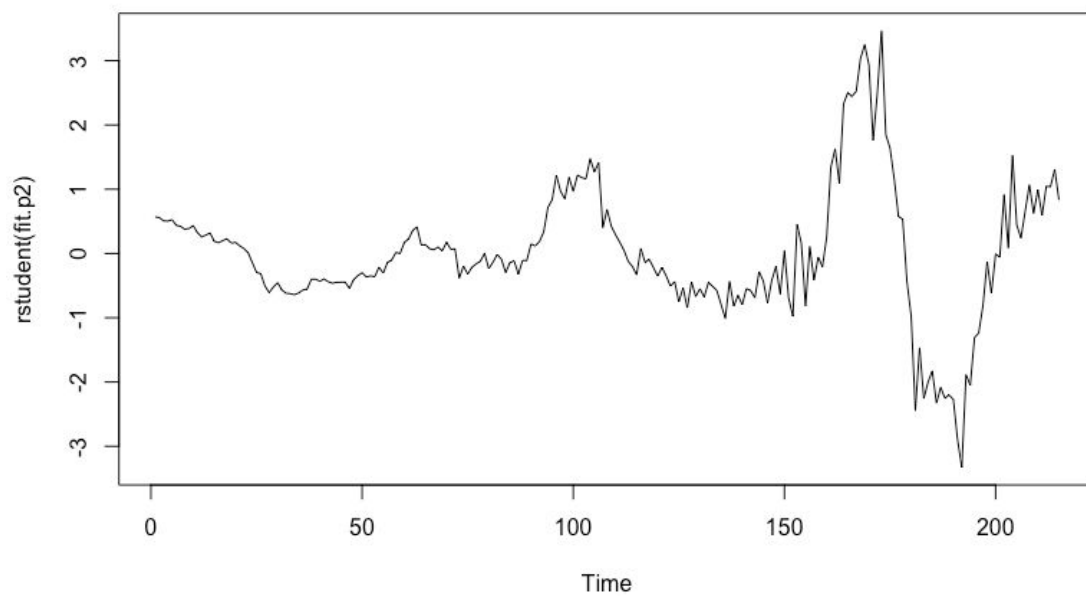
House Price vs Time (1st Order Polynomial)

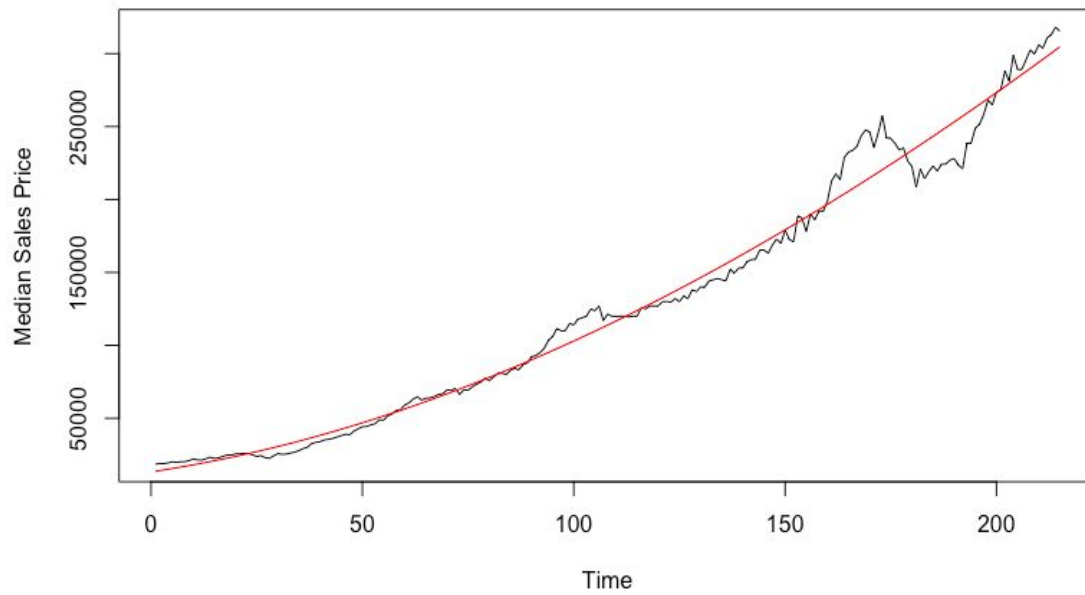Residual Plot of House Price vs Time (1st Order Polynomial)

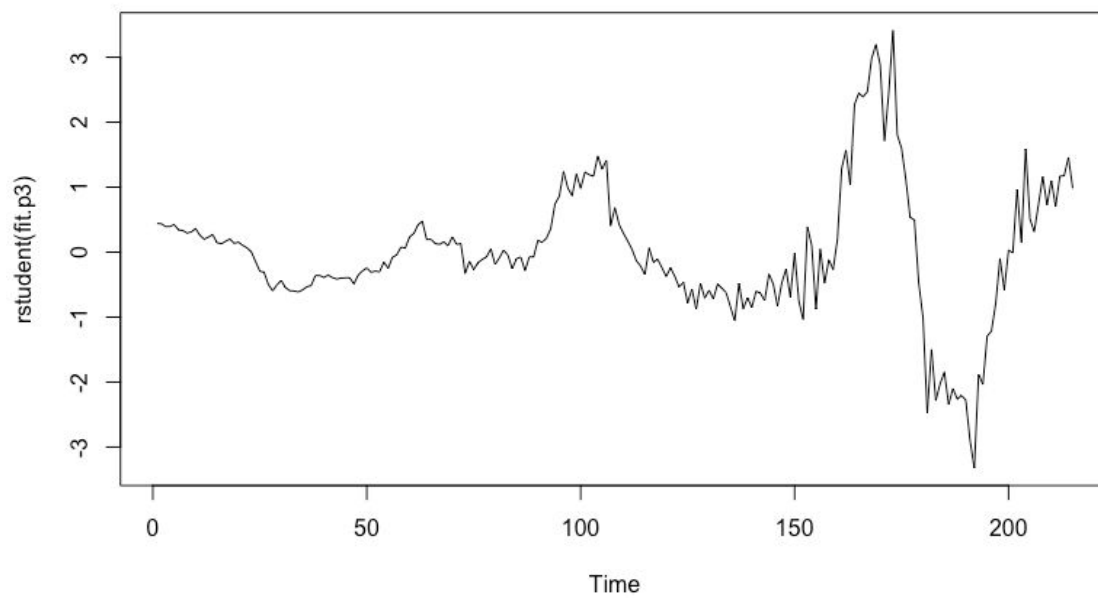House Price vs Time (2nd Order Polynomial)

Residual Plot of House Price vs Time (2nd Order Polynomial)
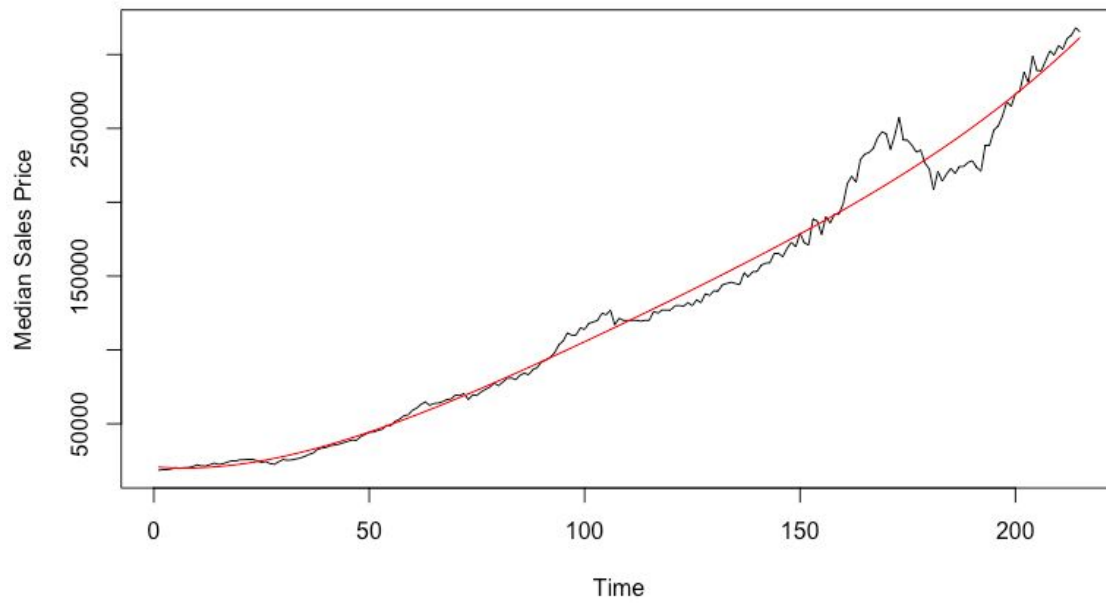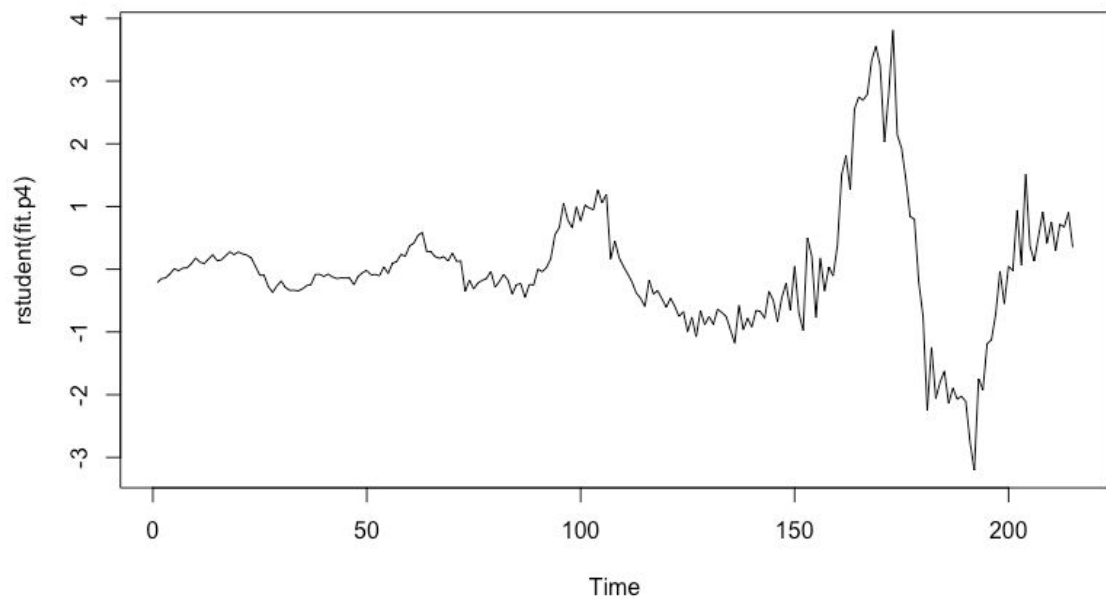
House Price vs Time (3rd Order Polynomial)

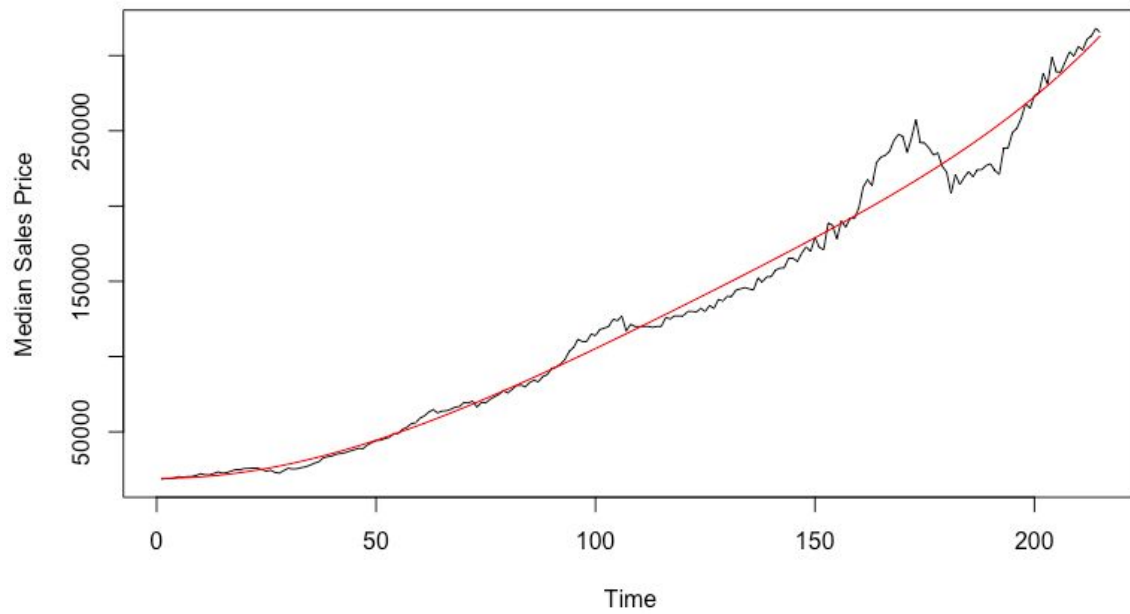Residual Plot of House Price vs Time (3rd Order Polynomial)

House Price vs Time (4th Order Polynomial)

Residual Plot of House Price vs Time (4th Order Polynomial)

House Price vs Time (5th Order Polynomial)

Residual Plot of House Price vs Time (5th Order Polynomial)

House Price vs Time (6th Order Polynomial)

Residual Plot of House Price vs Time (6th Order Polynomial)

House Price vs Time (7th Order Polynomial)

Residual Plot of House Price vs Time (7th Order Polynomial)

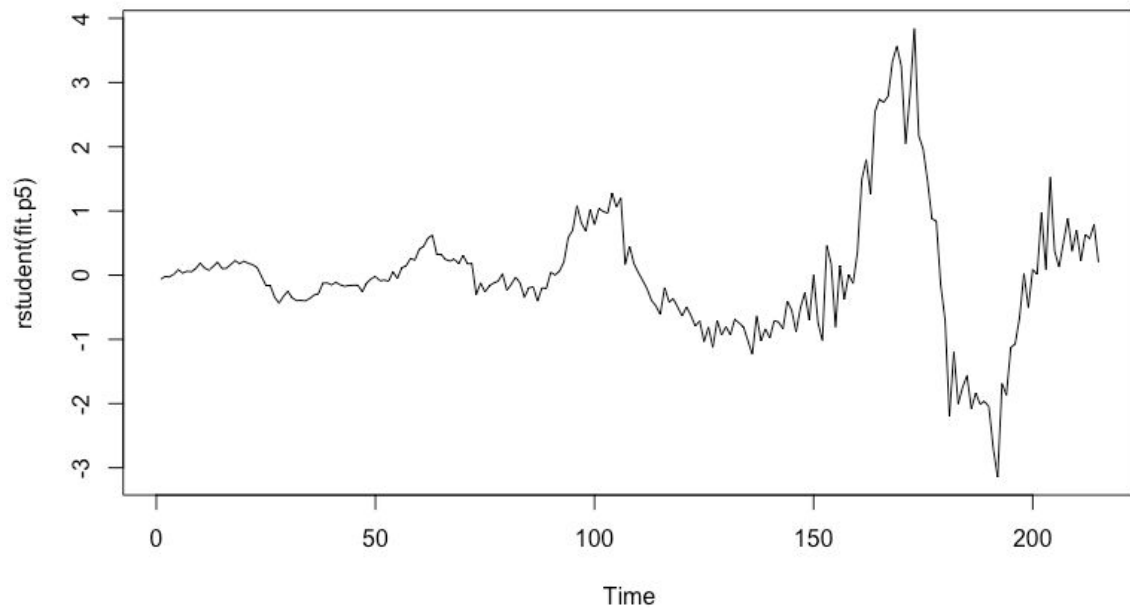House Price vs Time (8th Order Polynomial)

Residual Plot of House Price vs Time (8th Order Polynomial)
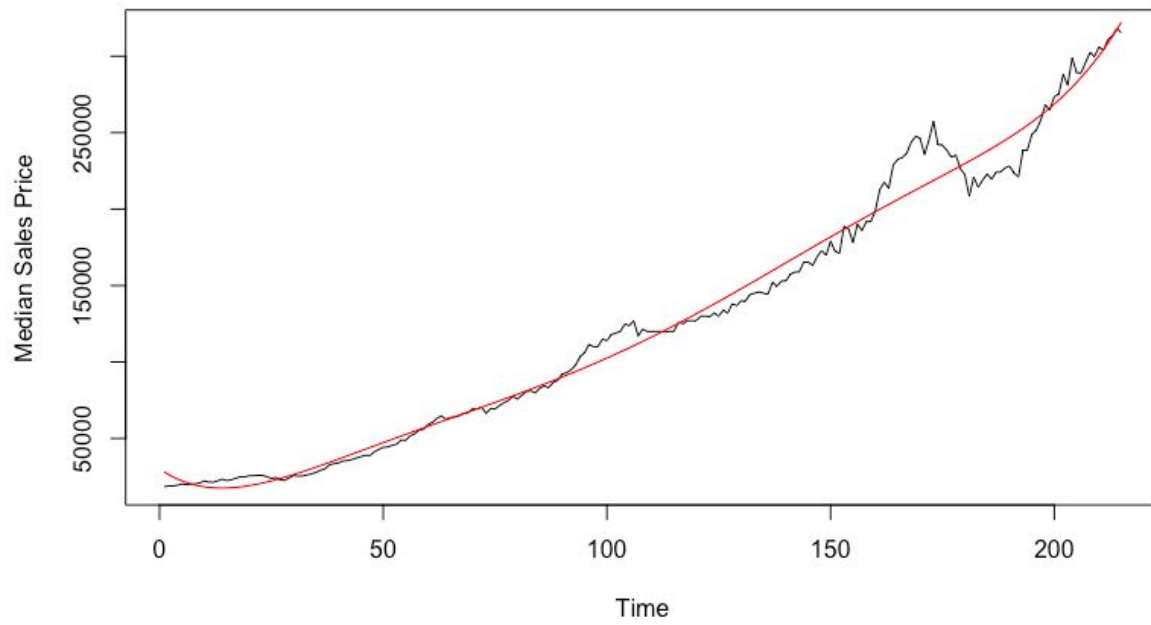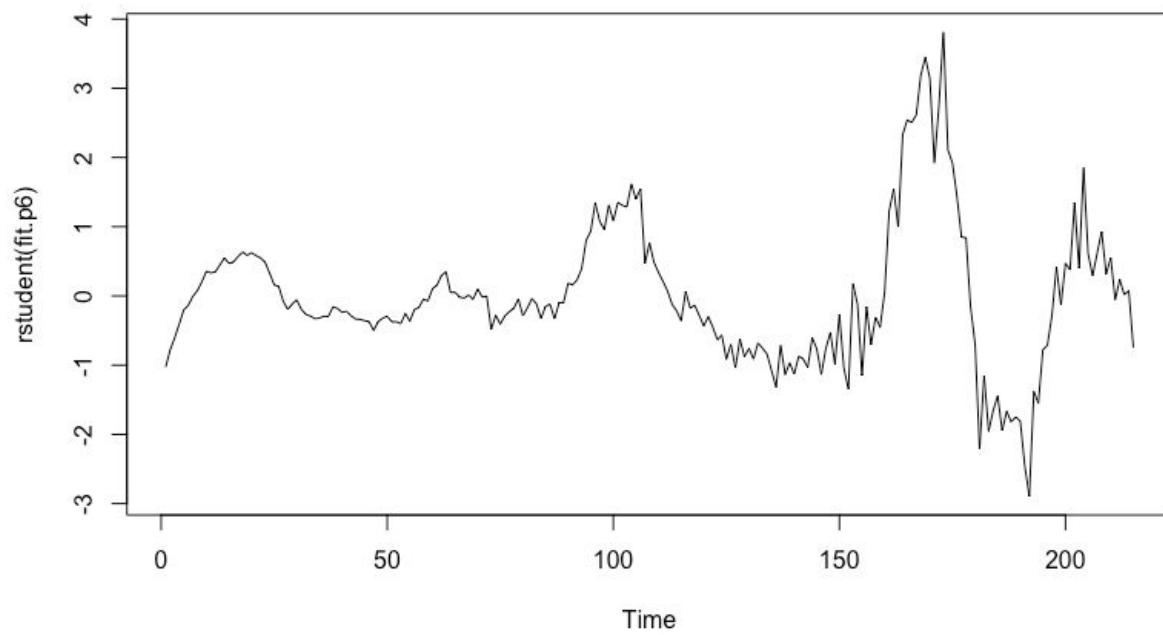
House Price vs Time (9th Order Polynomial)

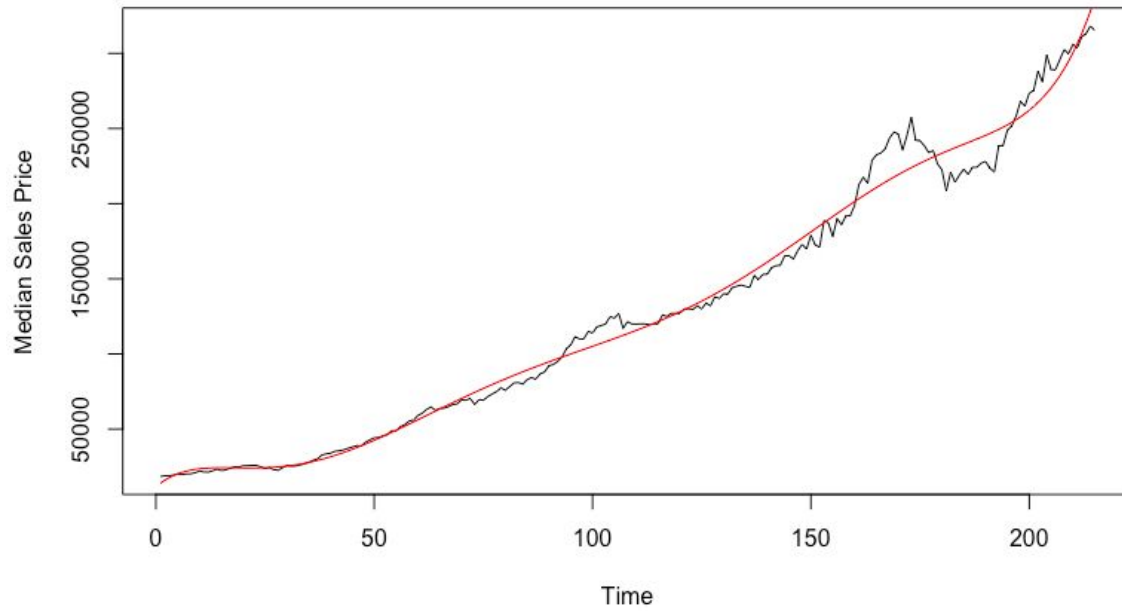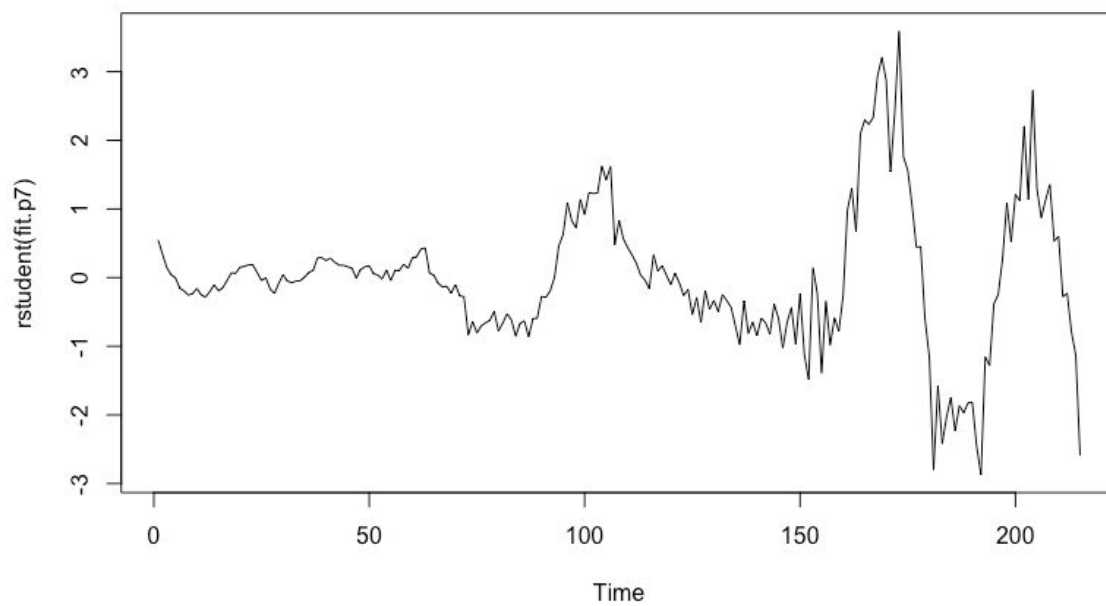Residual Plot of House Price vs Time (9th Order Polynomial)

House Price vs Time (10th Order Polynomial)

Residual Plot of House Price vs Time (10th Order Polynomial)

Figure 15:
Moving Average Smoother
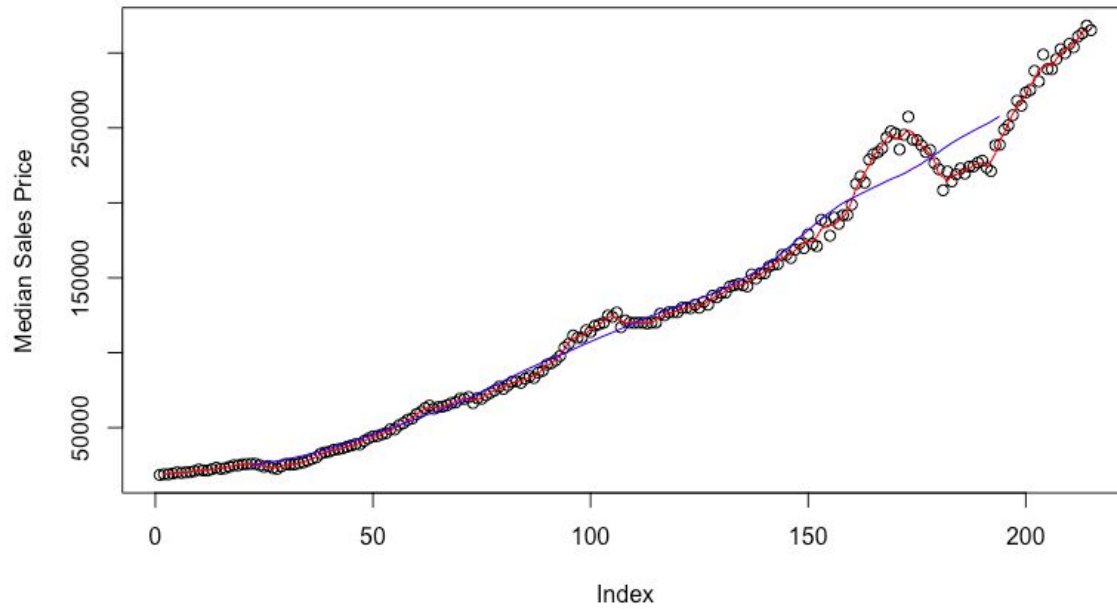
Figure 16:
Kernel Smoother
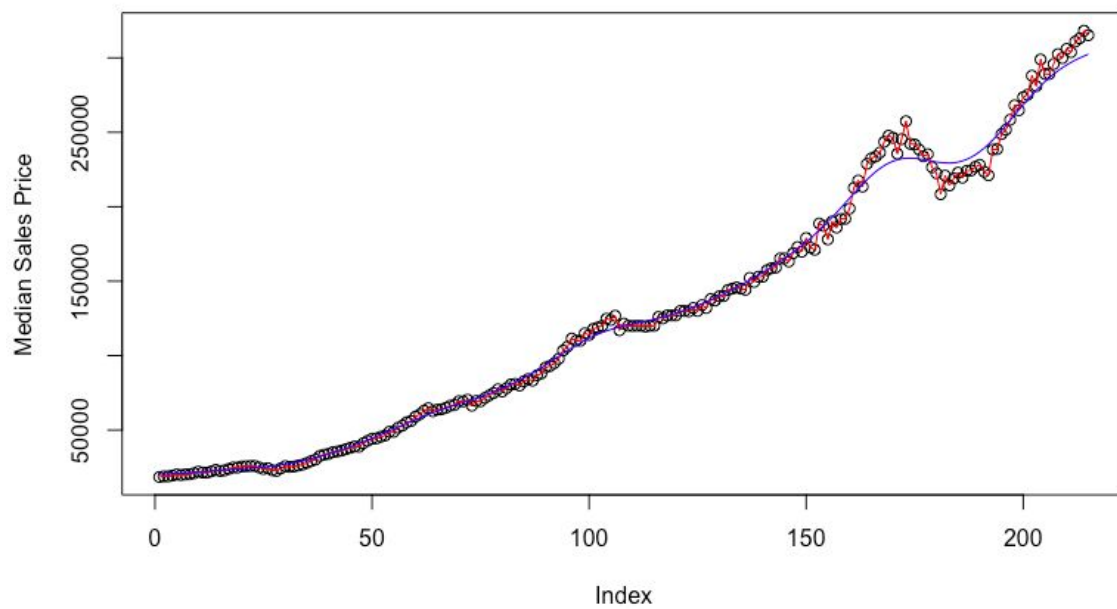
Figure 17:
Nearest Neighbor Smoother
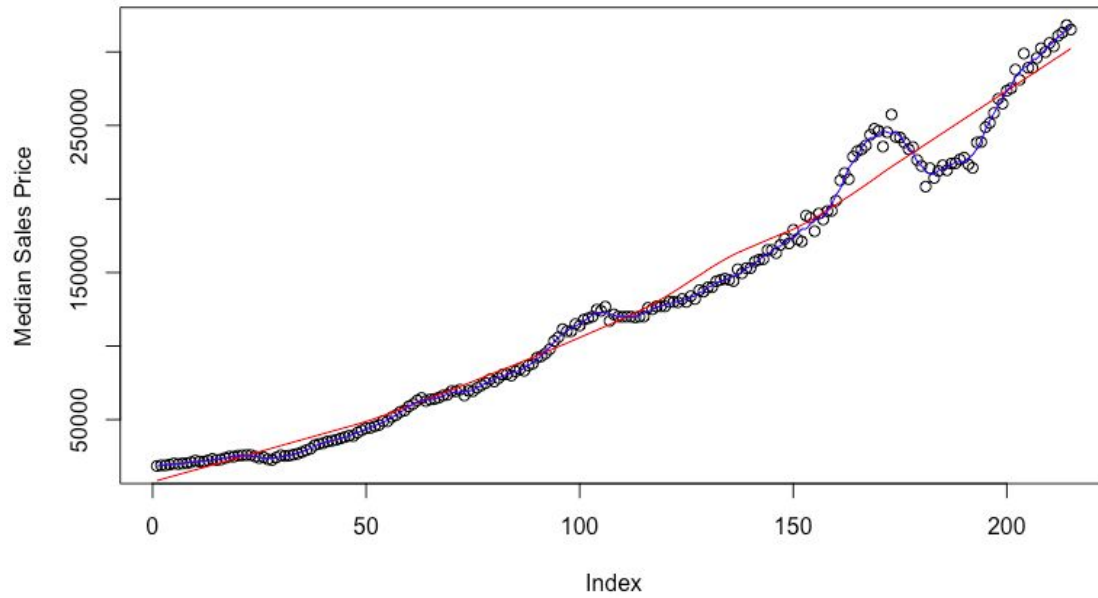
Figure 18:
Lowess Smoother
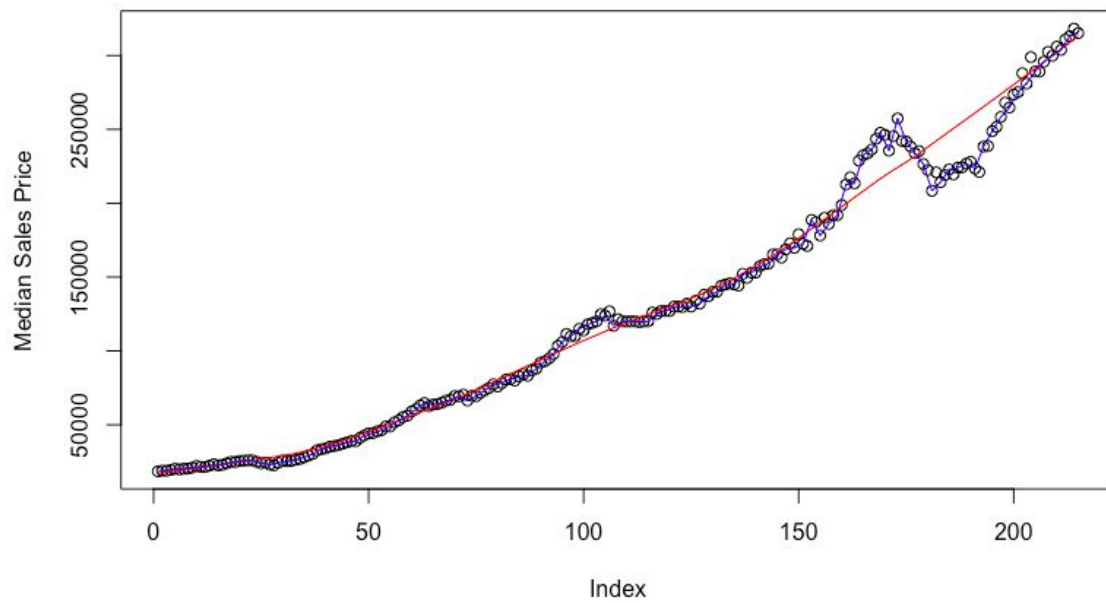
Figure 19:
Smoothing Splines
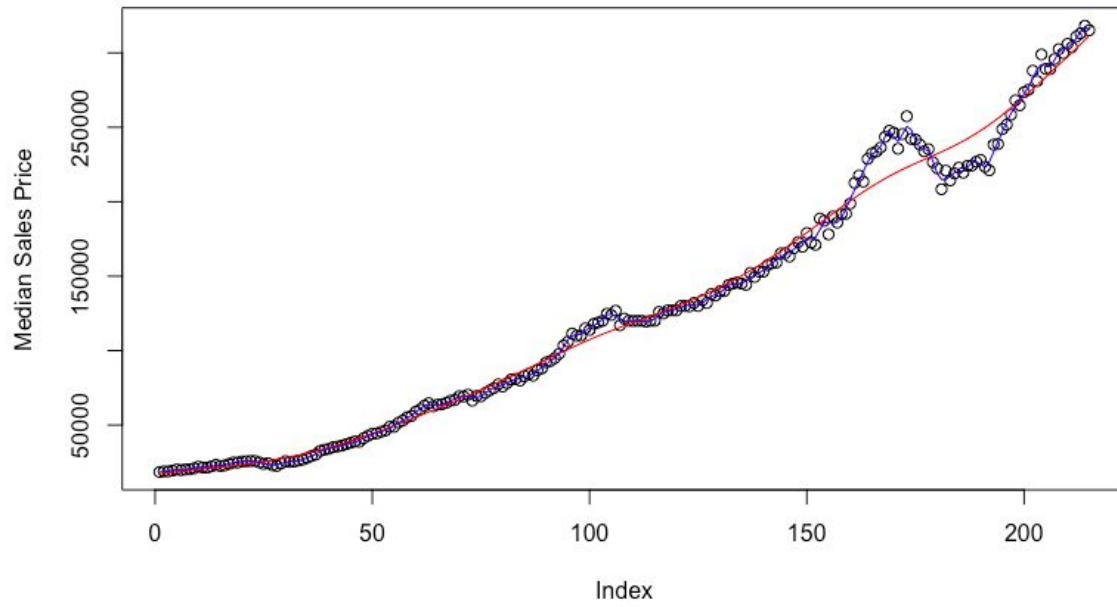
Figure 20:
Decomposition of House Prices (Additive)

Decomposition of additive time series
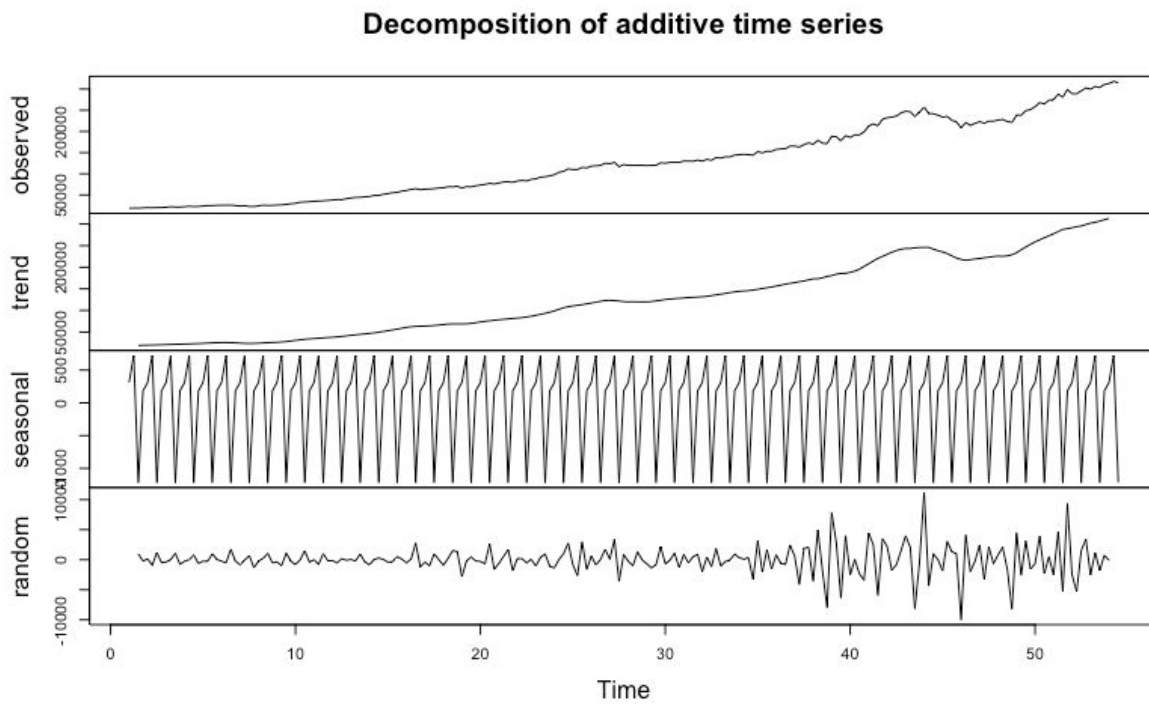
Figure 21:
Decomposition of House Prices (Multiplicative)

Decomposition of multiplicative time series
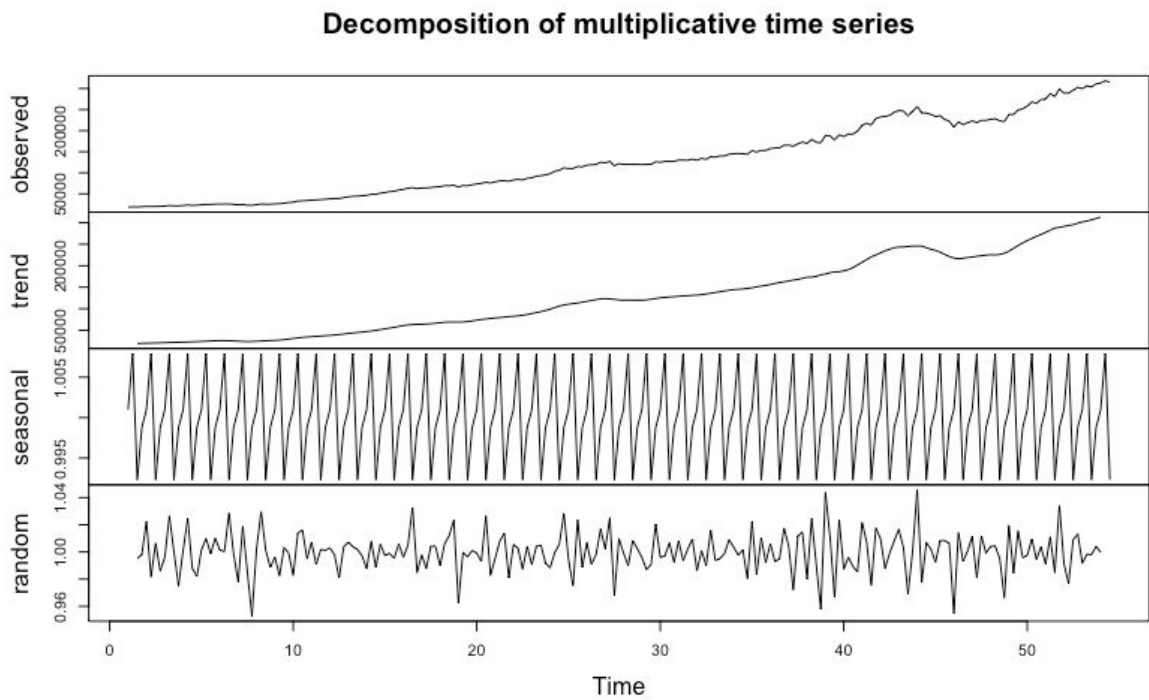
Figure 22:
House Prices, Additive Deseasonalized
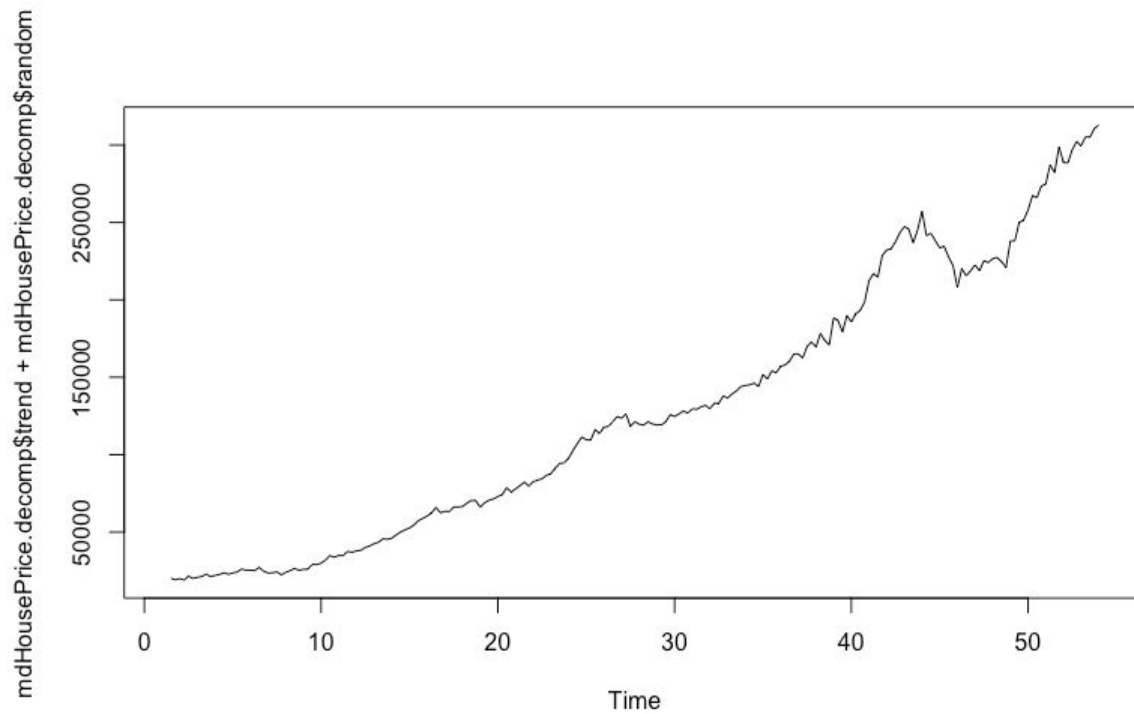
Figure 23:
House Prices, Multiplicative Deseasonalized
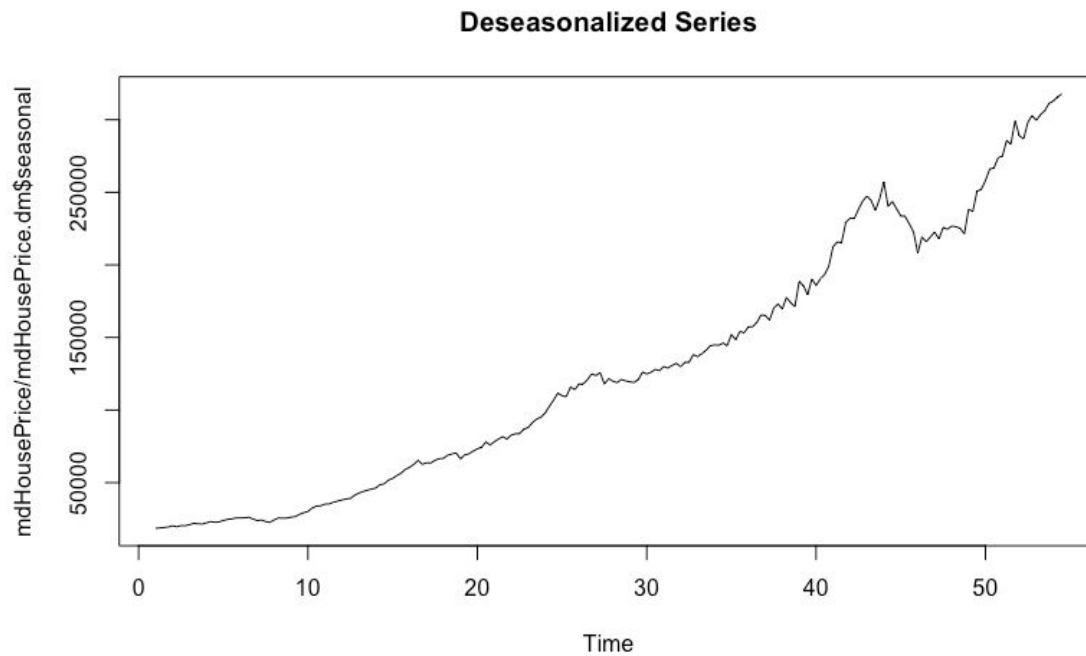
Figure 24:

House Prices, Additive Detrended
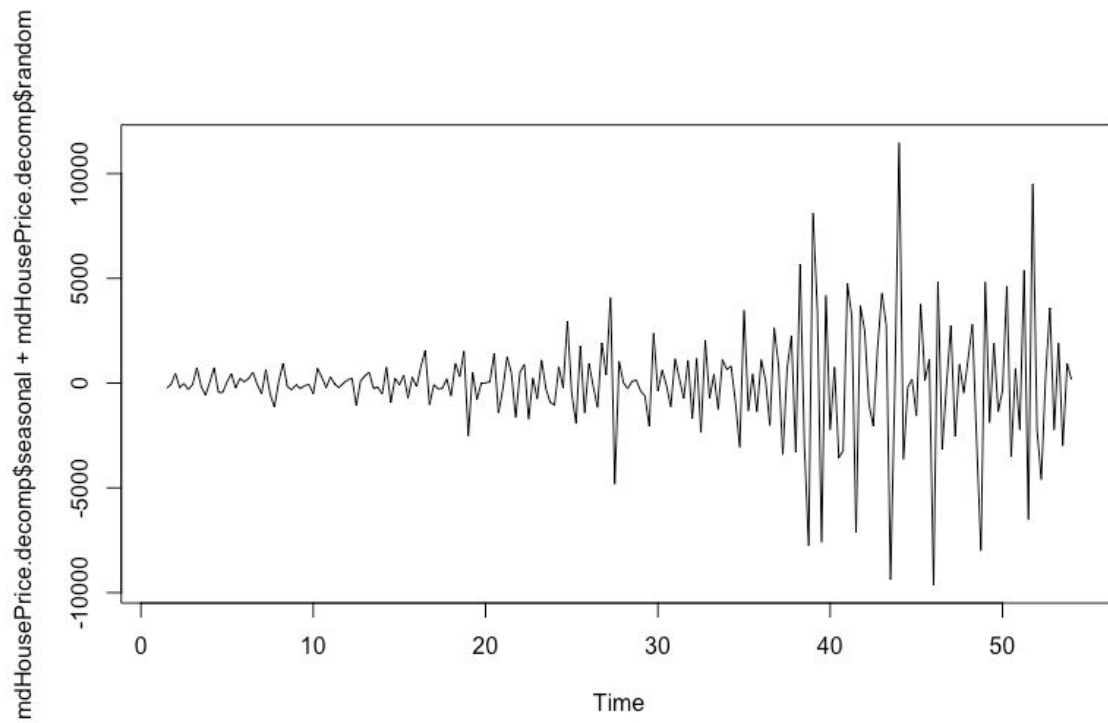
Figure 25:

House Prices, Multiplicative Detrended
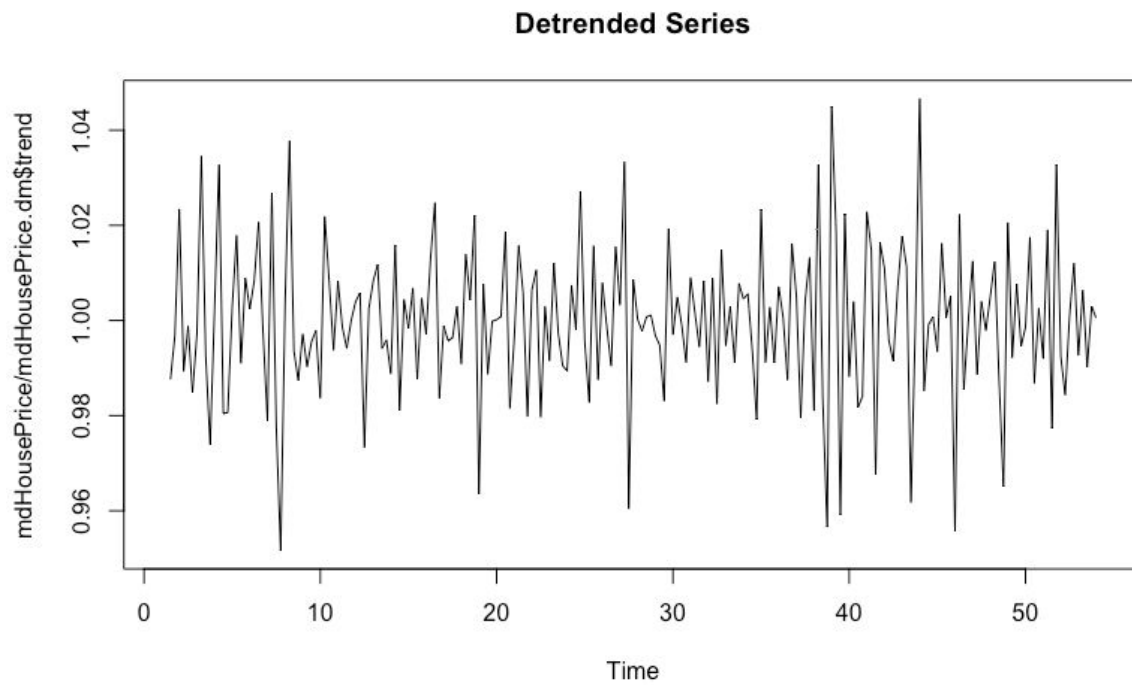
Figure 26:
House Prices, Additive Denoised
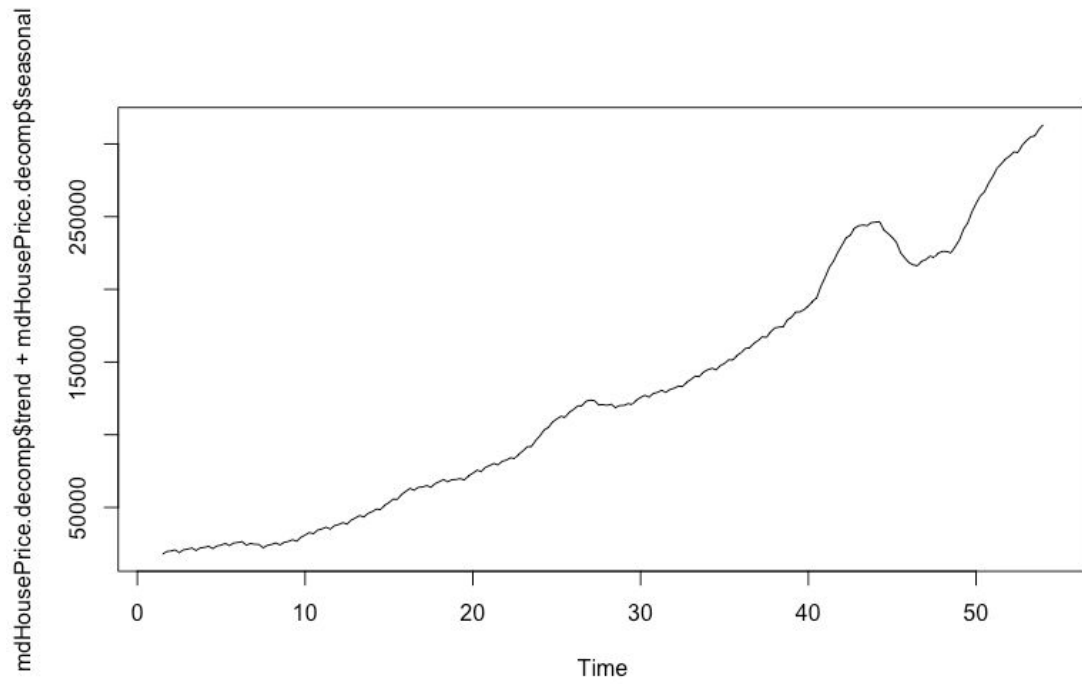
Figure 27:
House Prices, Multiplicative Denoised
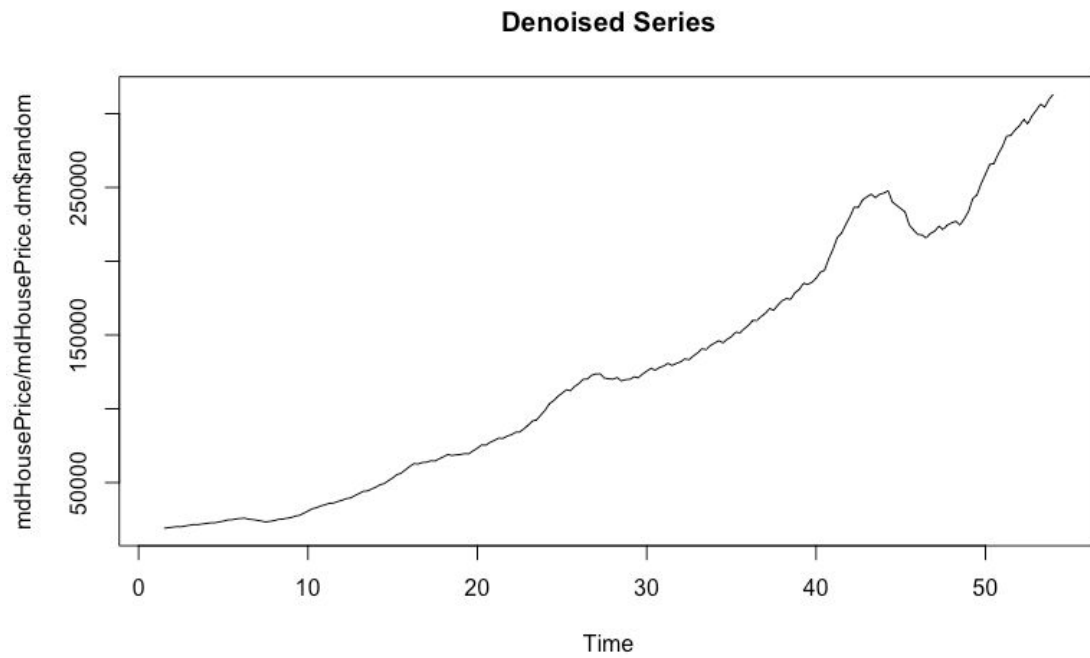
Table 3, Holt-Winters Last Quarter (2017 Q3):

Predict last quarter (2017 Q3) with multiplicative seasonal model:

95% Prediction Interval:

Quarter	Lower:	Fit:	Actual:	Upper:	SSE:
2017 Q3	311,504	315,886.5	315,200	320,269.1	3,644,496,176

Figure 28, Holt-Winters Last Quarter (2017 Q3):

Holt-Winters filtering
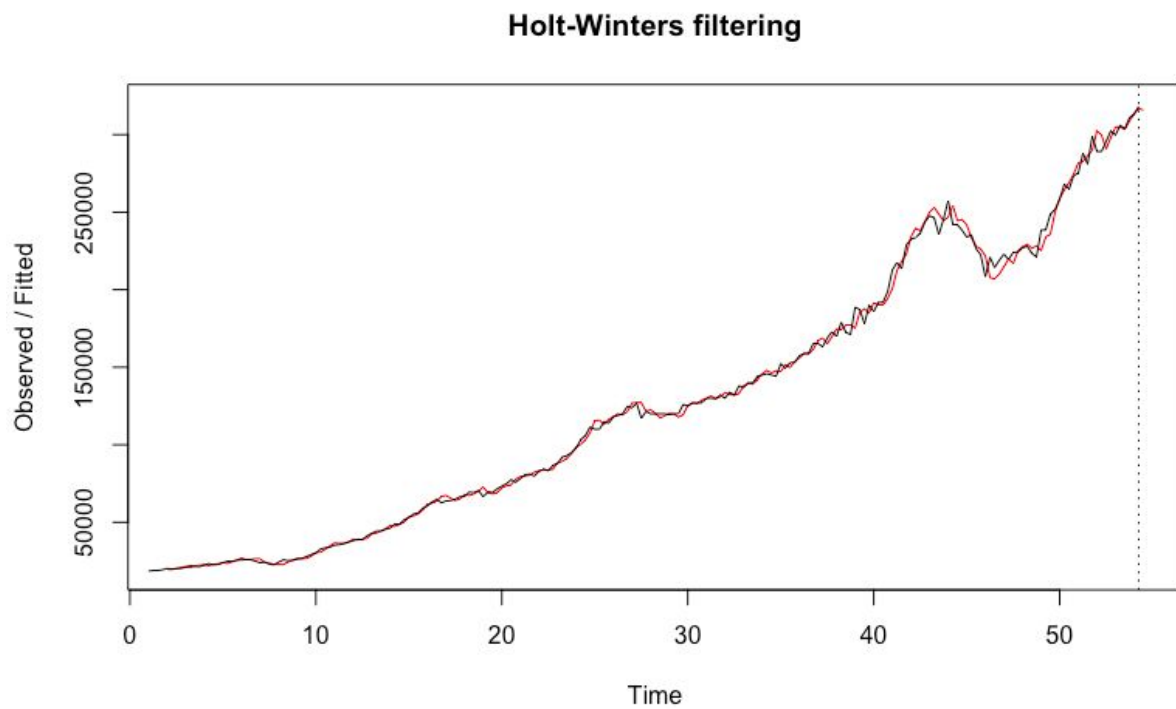
Table 4, Holt-Winters Last Two Quarters (2017 Q2, Q3):

Predict last two quarters (2017 Q2, Q3) with multiplicative seasonal model:

95% Prediction Interval:

Quarter	Lower:	Fit:	Actual:	Upper:	SSE:
2017 Q3	307,787.0	314,946.3	315,200	322,105.7	3,642,654,648
2017 Q2	312,452.8	316,841.2	318,200	321,229.6	3,642,654,648

Figure 29, Holt-Winters Last Two Quarters (2017 Q2, Q3):

Holt-Winters filtering
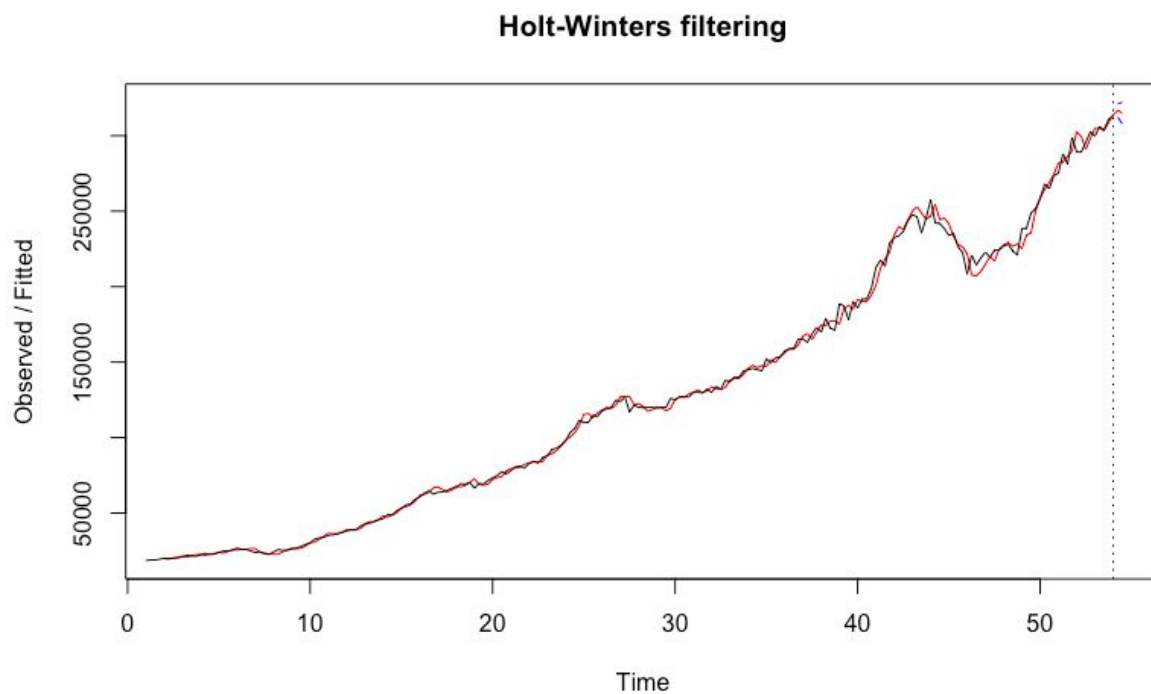
Table 5, Holt-Winters Last Three Quarters (2017 Q1, Q2, Q3):

Predict last three quarters (2017 Q1, Q2, Q3) with multiplicative seasonal model:

95% Prediction Interval:

Quarter	Lower:	Fit:	Actual:	Upper:	SSE:
2017 Q3	305,293.2	315,534.7	315,200	325,776.2	3,642,220,203
2017 Q2	310,052.1	317,306.6	318,200	324,561.2	3,642,220,203
2017 Q1	309,369.3	313,768.2	313,100	318,167.1	3,642,220,203

Figure 30, Holt-Winters Last Three Quarters (2017 Q1, Q2, Q3):

Holt-Winters filtering
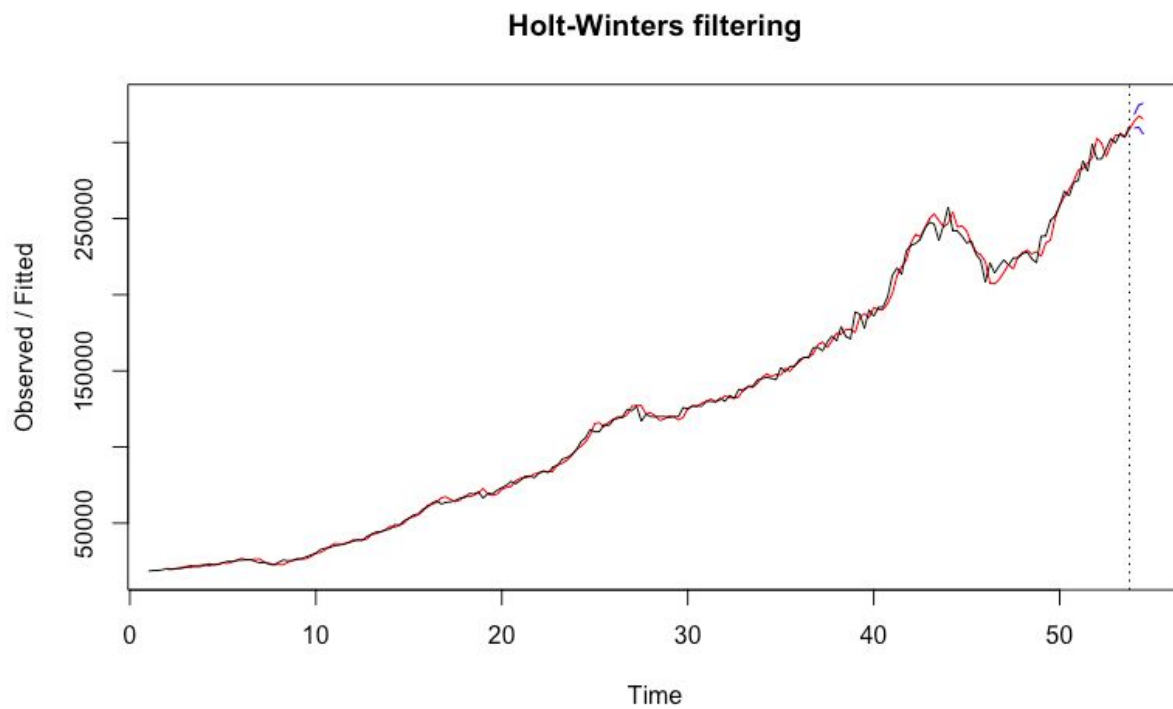
Table 6, Holt-Winters Last Four Quarters (2016 Q4, 2017 Q1, Q2, Q3):

Predict last four quarters (2016 Q4, 2017 Q1, Q2, Q3) with multiplicative seasonal model:

95% Prediction Interval:

Quarter	Lower:	Fit:	Actual:	Upper:	SSE:
2017 Q3	300,253.8	313,152.2	315,200	326,050.6	3,637,418,448
2017 Q2	304,924.6	315,326.8	318,200	325,729.0	3,637,418,448
2017 Q1	304,989.1	312,254.0	313,100	319,519.0	3,637,418,448
2016 Q4	304,302.3	308,697.7	310,900	313,093.1	3,637,418,448

Figure 31, Holt-Winters Last Four Quarters (2016 Q4, 2017 Q1, Q2, Q3):

Holt-Winters filtering
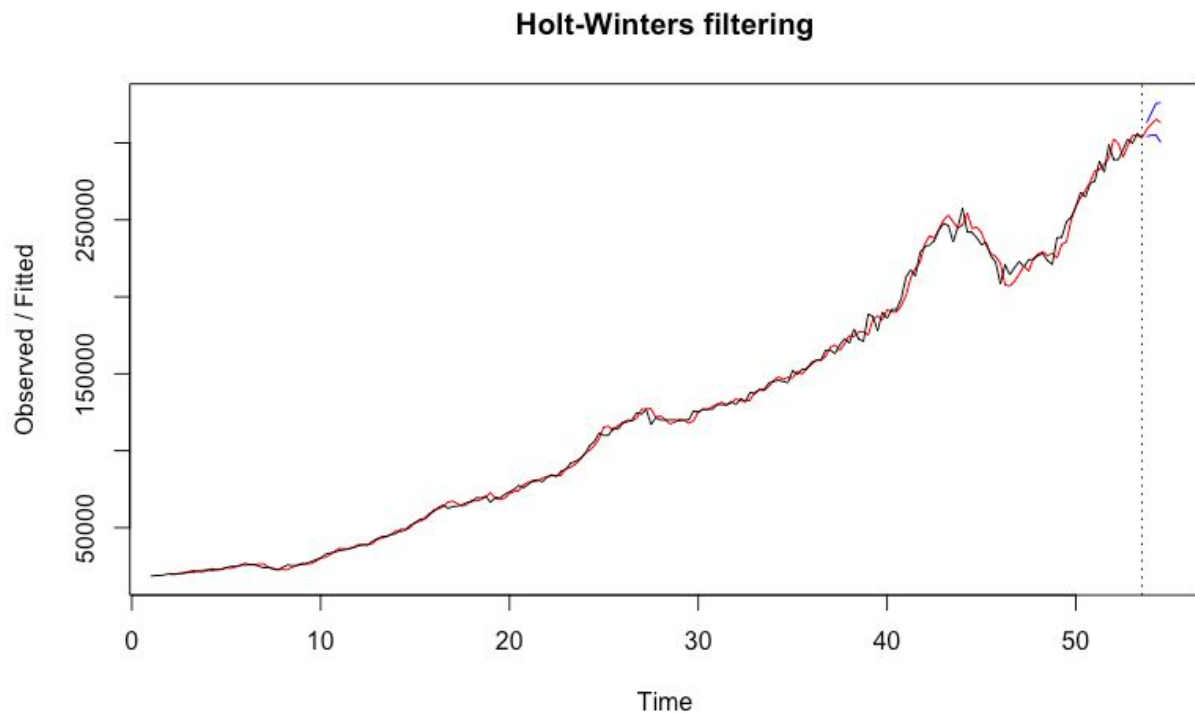
Table 7, Holt-Winters Next Four Quarters (2017 Q4, 2018 Q1, Q2, Q3):

Predict next four quarters (2017 Q4, 2018 Q1, Q2, Q3) with multiplicative seasonal model:

95% Prediction Interval:

Quarter	Lower:	Fit:	Actual:	Upper:	SSE:
2018 Q3	314,028.8	326,811.6	Unknown	339,594.5	3,644,966,412
2018 Q2	318,501.7	328,822.2	Unknown	339,142.6	3,644,966,412
2018 Q1	317,959.8	325,171.3	Unknown	332,382.9	3,644,966,412
2017 Q4	316,931.2	321,303.3	Unknown	325,675.3	3,644,966,412

Figure 32, Holt-Winters Next Four Quarters (2017 Q4, 2018 Q1, Q2, Q3):

Holt-Winters filtering
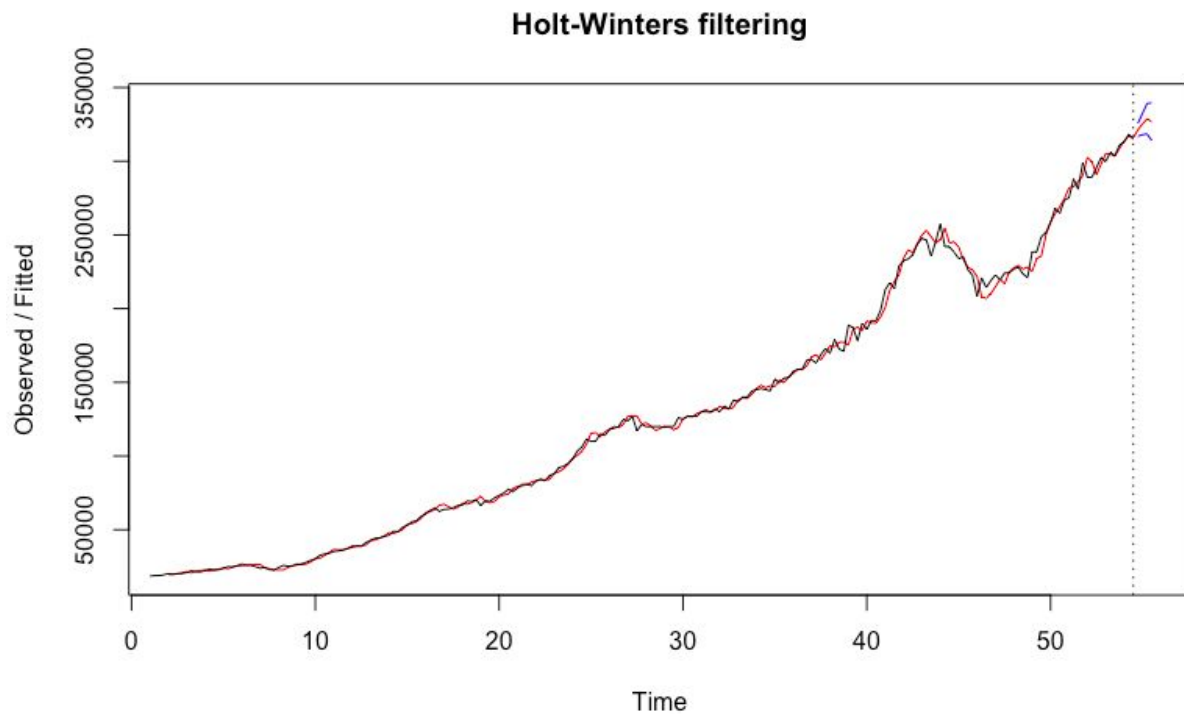
Figure 33:

Regression 1, Regression of Median Sales Price on Individual Economic Indicators

House Price vs Household Estimates
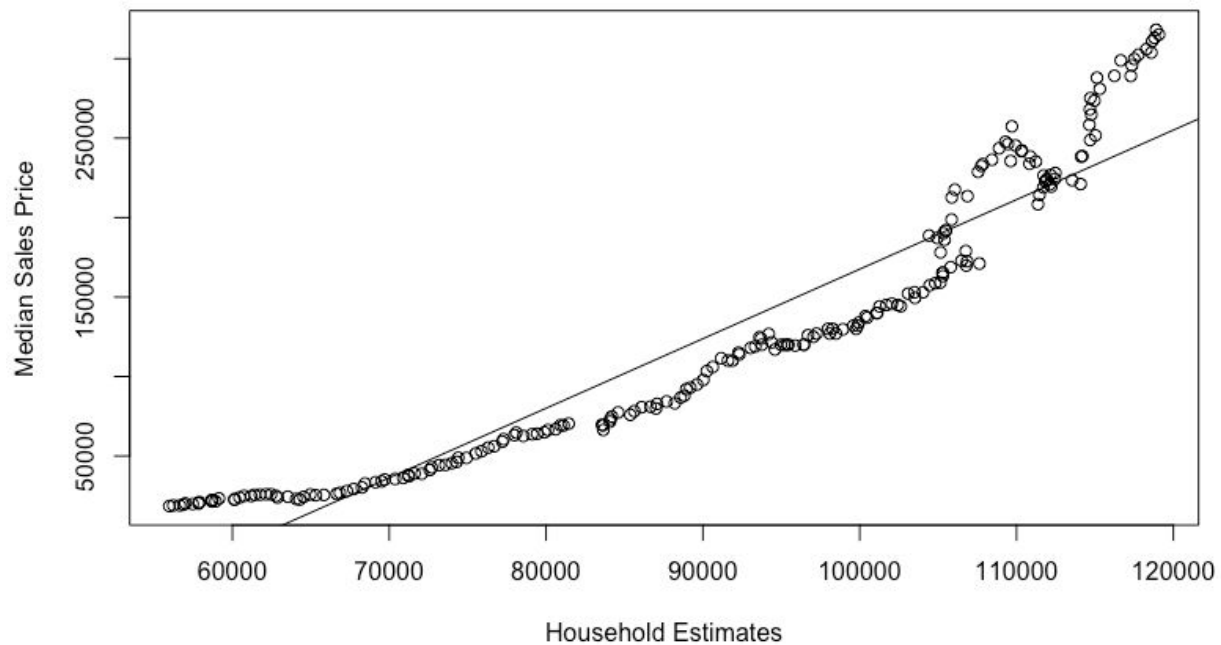
lm(formula = mdHousePrice ~ mdHouseholdEst

Residuals:

Min	1Q	Median	3Q	Max
-36550	-26022	-5968	23384	67961

Coefficients:

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-2.700e+05	9.315e+03	-28.98	<2e-16 ***
mdHouseholdEst	4.375e+00	1.000e-01	43.75	<2e-16 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 27660 on 213 degrees of freedom

Multiple R-squared: 0.8999, Adjusted R-squared: **0.8994**

F-statistic: 1914 on 1 and 213 DF, p-value: < 2.2e-16

House Price vs GDP Per Capita

lm(formula = mdHousePrice ~ mdGDP)

Residuals:

Min	1Q	Median	3Q	Max
-38581	-8461	-2588	5444	51358

Coefficients:

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-1.753e+05	4.283e+03	-40.93	<2e-16 ***
mdGDP	8.445e+00	1.144e-01	73.82	<2e-16 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 16960 on 213 degrees of freedom
Multiple R-squared: 0.9624, Adjusted R-squared: **0.9622**
F-statistic: 5450 on 1 and 213 DF, p-value: < 2.2e-16

House Price vs Unemployment Rate

lm(formula = mdHousePrice ~ mdUnemploy)

Residuals:

Min	1Q	Median	3Q	Max
-110962	-82261	-10477	74275	193813

Coefficients:

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	113729	22656	5.020	1.09e-06 ***
mdUnemploy	2538	3604	0.704	0.482

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 87320 on 213 degrees of freedom
Multiple R-squared: 0.002322,   Adjusted R-squared: **-0.002362**
F-statistic: 0.4956 on 1 and 213 DF, p-value: 0.4822

Figure 34:
Regression 3, House Price vs GDP, Household Estimates, and Unemployment

lm(formula = mdHousePrice ~ mdGDP + mdHouseholdEst + mdUnemploy)

Residuals:

Min	1Q	Median	3Q	Max
-36512	-8651	-1438	6189	51824

Coefficients:

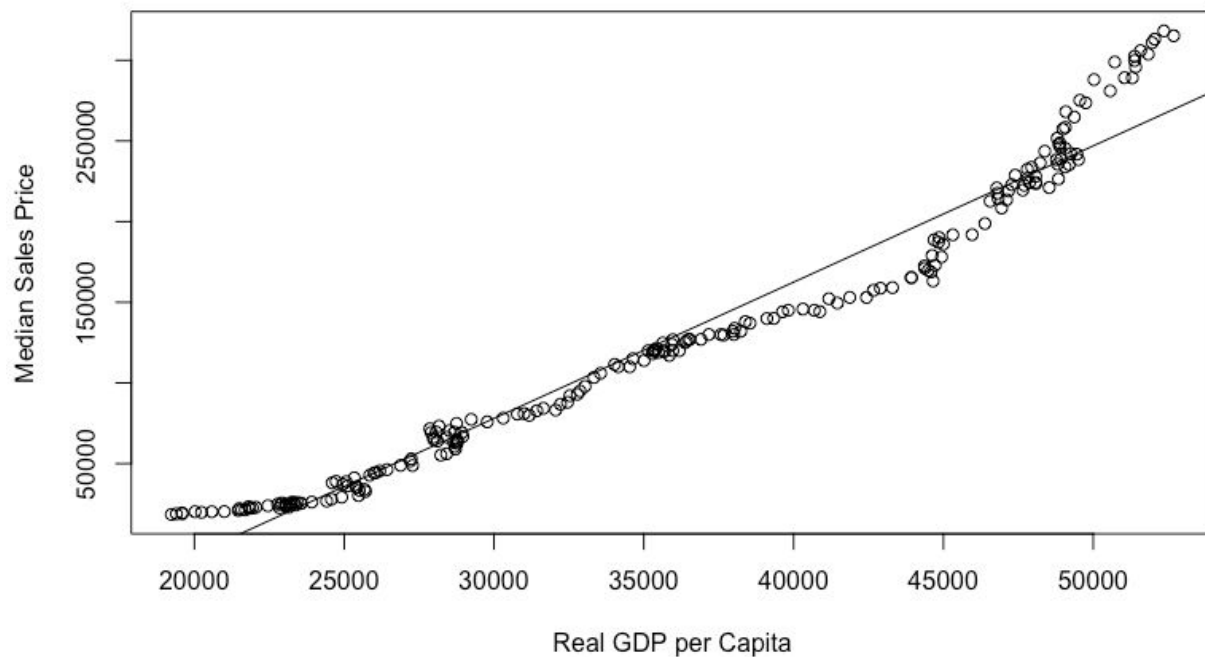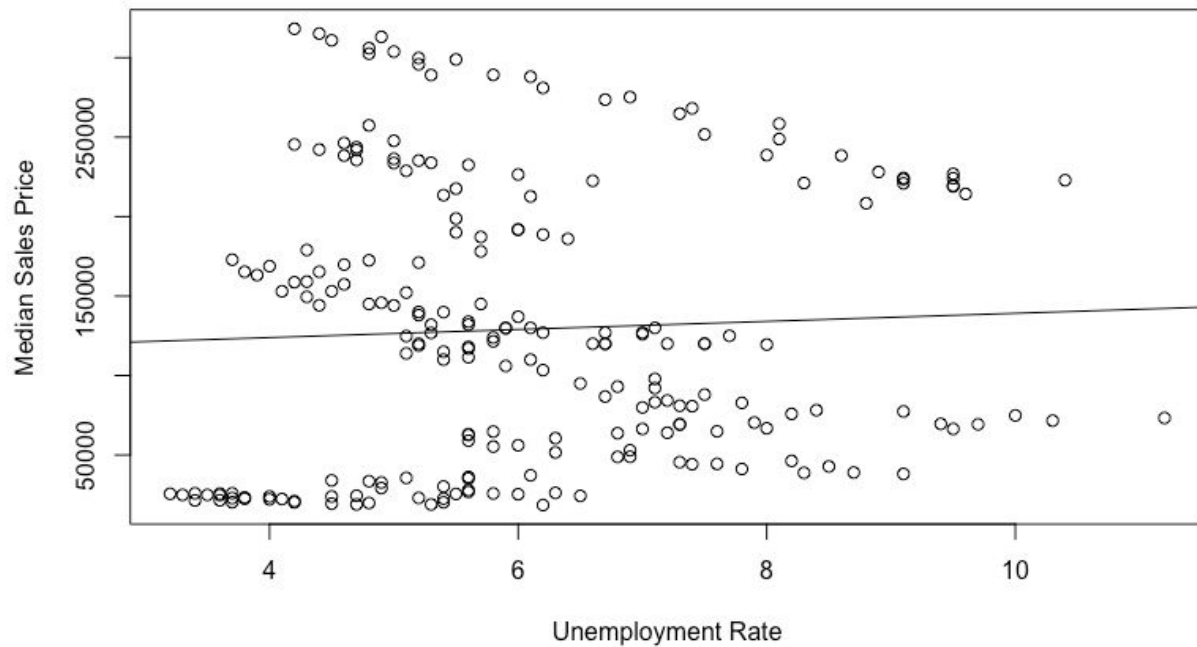|	Estimate	Std. Error	t value	Pr(>|t|)	
(Intercept)	-1.366e+05	8.238e+03	-16.585	< 2e-16	***
mdGDP	1.226e+01	6.324e-01	19.379	< 2e-16	***
mdHouseholdEst	-2.095e+00	3.429e-01	-6.110	4.73e-09	***
mdUnemploy	2.477e+03	7.798e+02	3.176	0.00171	**

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 15700 on 211 degrees of freedom
Multiple R-squared: 0.968,   Adjusted R-squared: 0.9676
F-statistic: 2131 on 3 and 211 DF, p-value: < 2.2e-16

Figure 35:
Regression 4, House Price vs GDP and Household Estimates

lm(formula = mdHousePrice ~ mdGDP + mdHouseholdEst)

Residuals:

Min	1Q	Median	3Q	Max
-41054	-8068	-1116	4796	48156

Coefficients:

	Estimate	Std. Error	t value	Pr(>|t|)	
(Intercept)	-1.376e+05	8.406e+03	-16.372	< 2e-16	***
mdGDP	1.117e+01	5.439e-01	20.543	< 2e-16	***
mdHouseholdEst	-1.491e+00	2.914e-01	-5.118	6.92e-07	***

Residual standard error: 16030 on 212 degrees of freedom
Multiple R-squared: 0.9665, Adjusted R-squared: 0.9662
F-statistic: 3060 on 2 and 212 DF, p-value: < 2.2e-16

Figure 36:

Graphs of Median Sales Price vs Household Estimates, Real GDP Per Capita
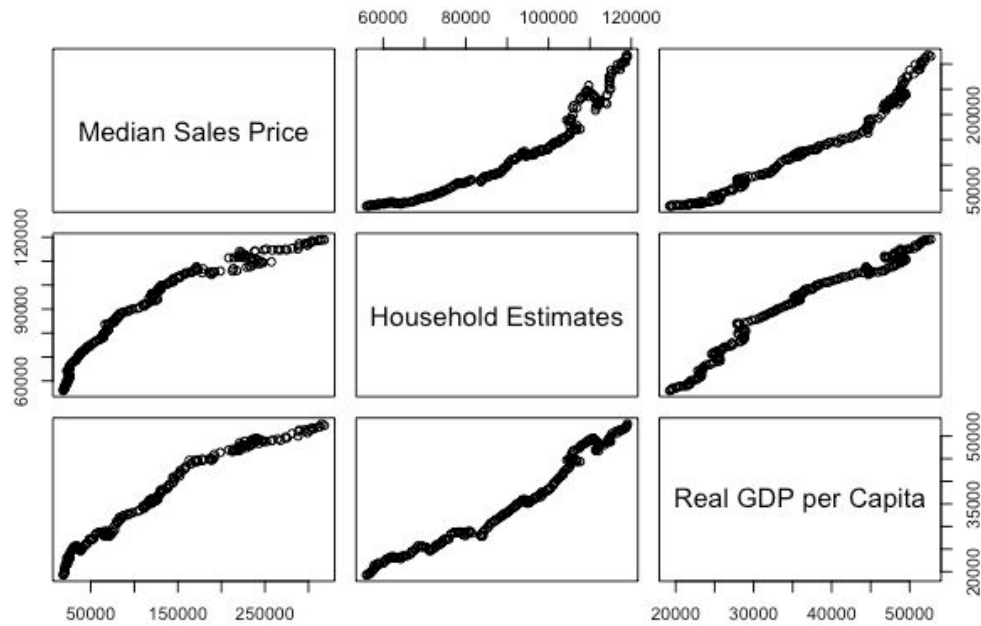
Figure 37:

Studentized Residuals of Median Sales Price vs Household Estimates and GDP
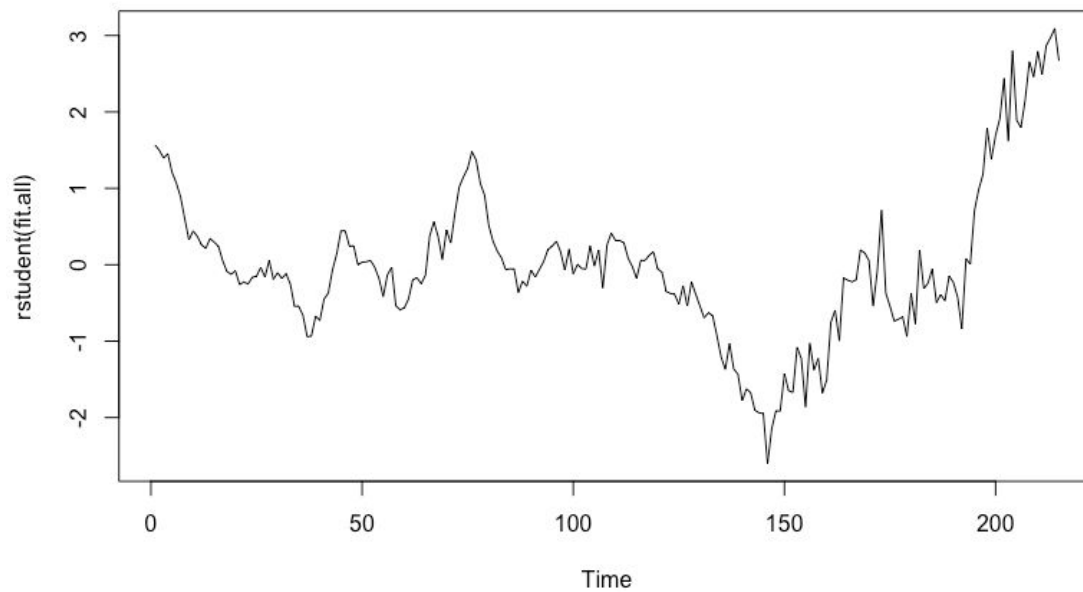
Figure 38:
Autocorrelation Function of Median Sales Price vs Household Estimates and GDP

Series rstudent(fit.all)
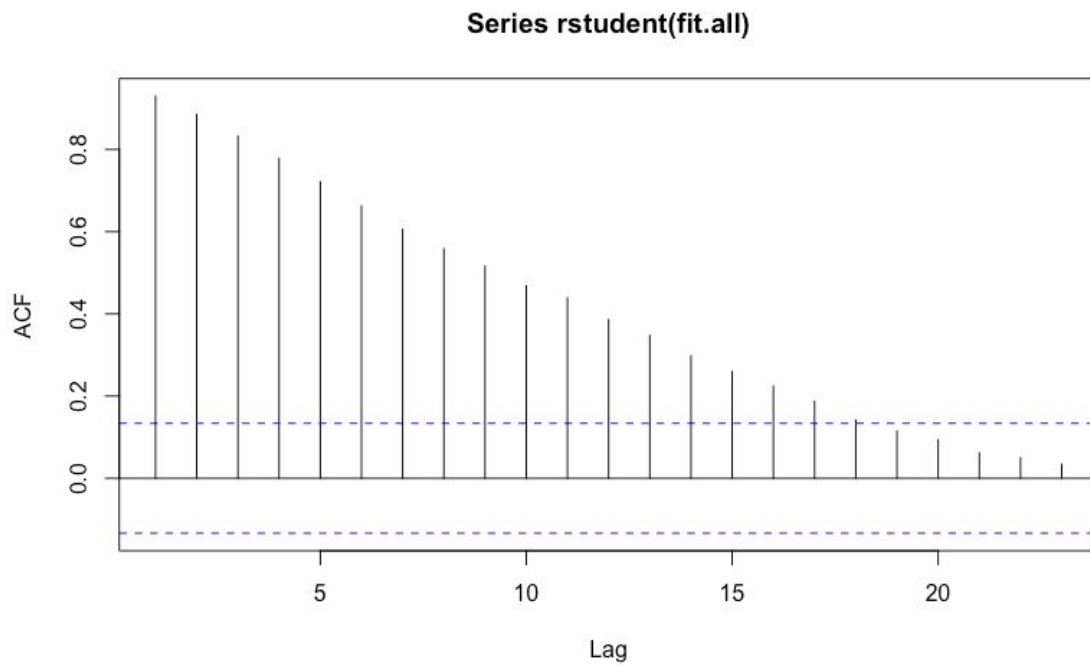
Figure 39:
Histogram of Studentized Residuals of Median Sales Price vs Household Estimates and GDP

Histogram of rstudent(fit.all)
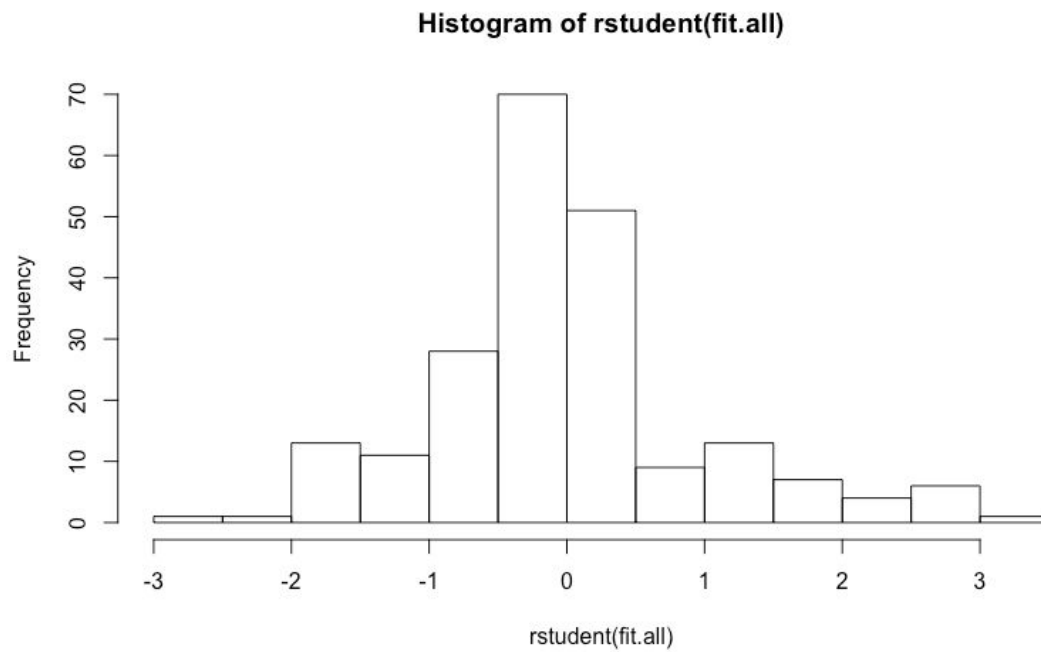
Figure 40:

QQ Plot of (Median Sales Price vs Household Estimates and GDP Model)

Normal Q-Q Plot
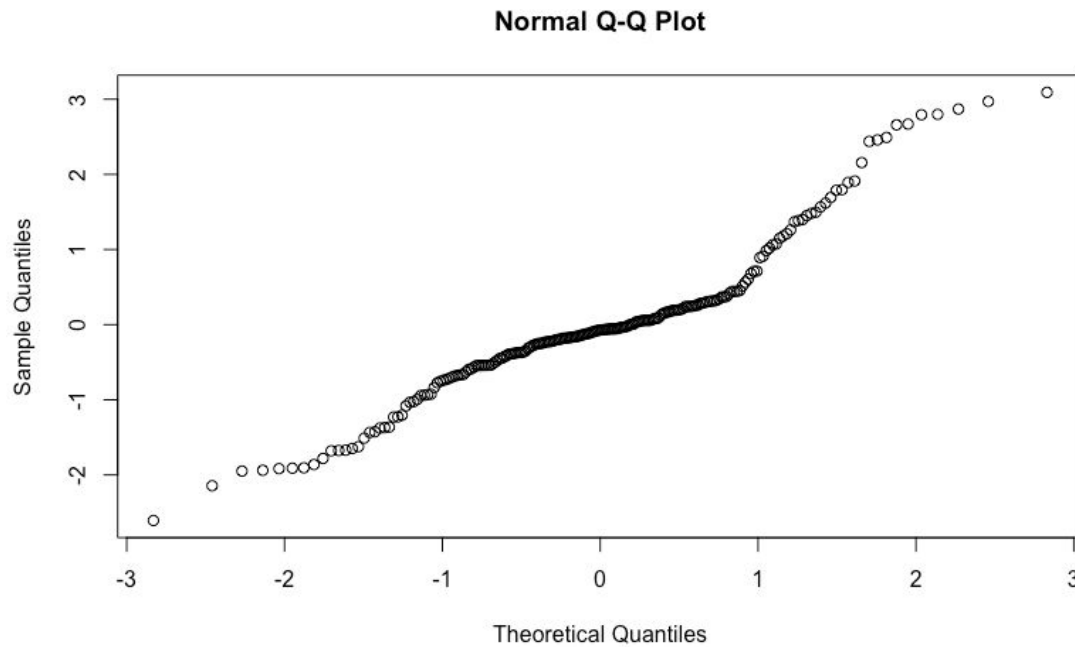
Figure 41:

Durbin Watson Test of (Median Sales Price vs Household Estimates, GDP, and Unemployment)

> dwtest(fit7)
          Durbin-Watson test
data: fit7
DW = 0.1117, p-value < 2.2e-16
alternative hypothesis: true autocorrelation is greater than 0

Figure 42, Regression 5, House Price vs GDP, Household, and Time

lm(formula = mdHousePrice ~ mdGDP + mdHouseholdEst + mdqrtr)

Residuals:

| Min	1Q	Median	3Q	Max
-41697	-7511	-1445	4905	47508

Coefficients:

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-1.371e+05	8.649e+03	-15.851	< 2e-16 ***
mdGDP	1.117e+01	5.473e-01	20.410	< 2e-16 ***
mdHouseholdEst	-1.490e+00	2.933e-01	-5.081	8.31e-07 ***
mdqrtr2Q	4.904e+01	3.105e+03	0.016	0.987
mdqrtr3Q	-1.736e+03	3.105e+03	-0.559	0.577
mdqrtr4Q	-6.738e+02	3.120e+03	-0.216	0.829

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 16130 on 209 degrees of freedom
Multiple R-squared: 0.9666, Adjusted R-squared: **0.9658**
F-statistic: 1209 on 5 and 209 DF, p-value: < 2.2e-16

Figure 43:

Cochrane-Orchutt Model (House Price vs GDP, Household, and Unemployment)

Durbin-Watson test

data: fit.trans

DW = 2.3456, p-value = 0.9934

alternative hypothesis: true autocorrelation is greater than 0

Pri^ce = -205023.8 + 0.71 (Household Est) - 61.27 (Unemployment) + 7.47 (GDP Per Capita)

Figure 44:
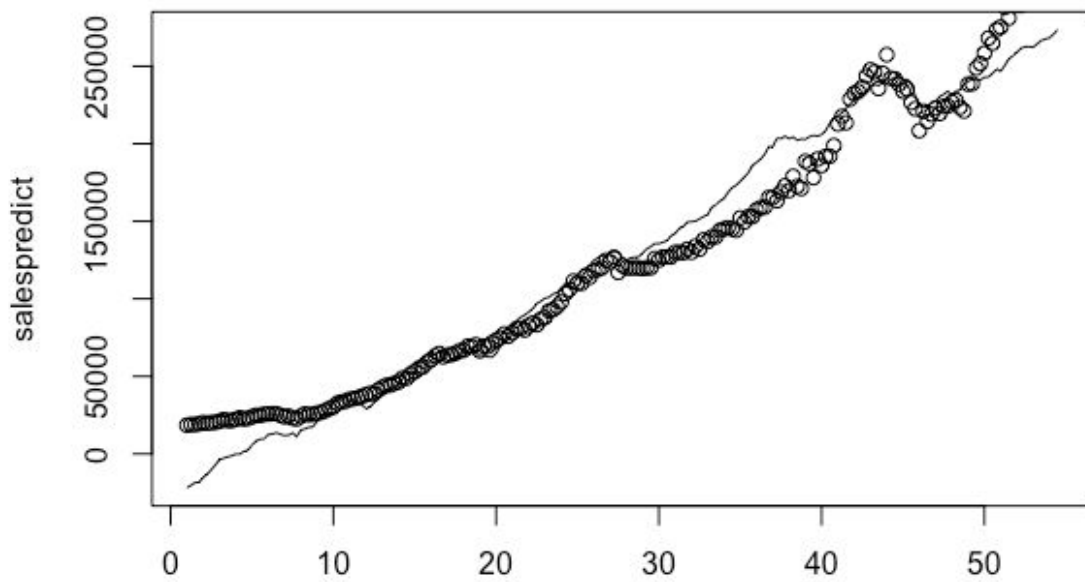
Cochrane-Orchutt Model (House Price vs GDP and Household Estimate)

Durbin-Watson test

data: fit.trans

DW = 2.2492, p-value = 0.959

alternative hypothesis: true autocorrelation is greater than 0

$\widehat{Pri}ce$ = -198300.4 + 0.49 (Household Est) + 7.85 (GDP Per Capita)

Figure 45:
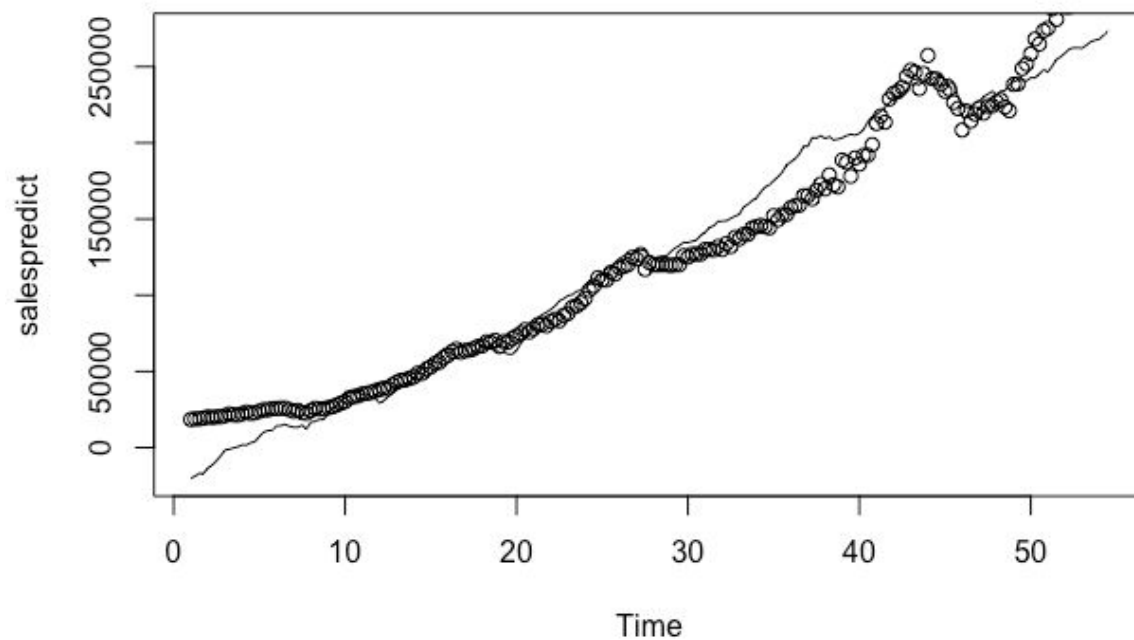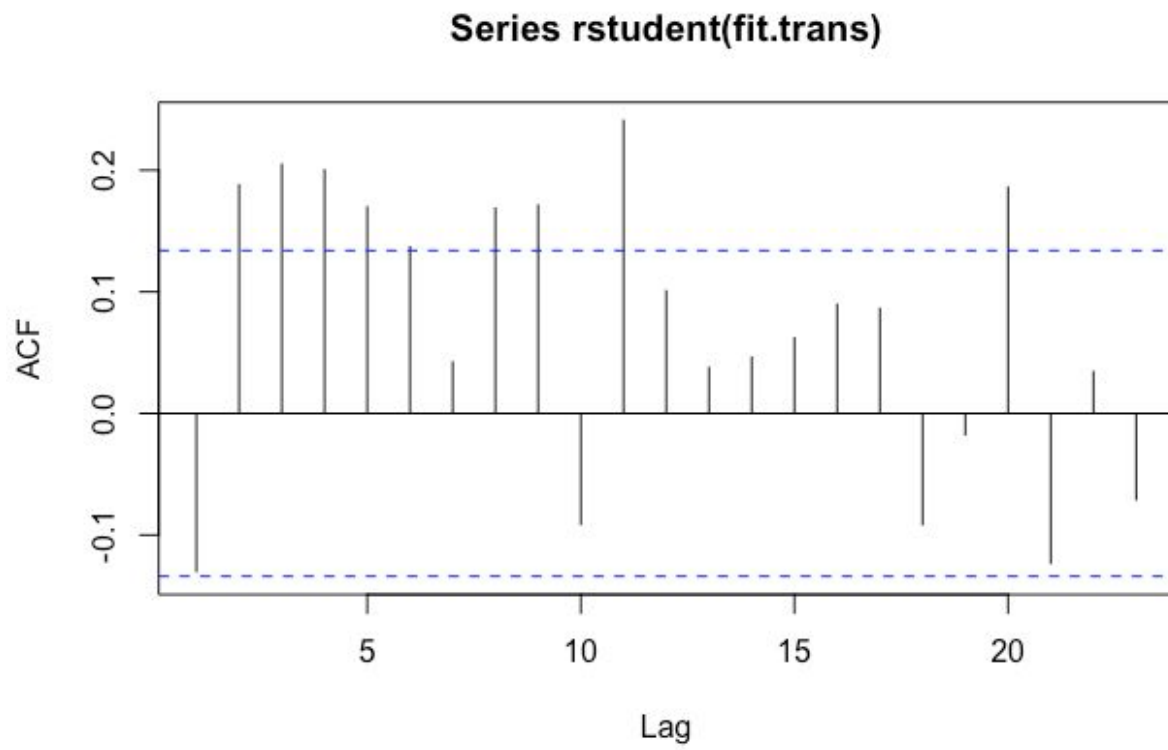Cochrane-Orchutt Model ACF (House Price vs GDP and Household Estimate)

Series rstudent(fit.trans)

Figure 46:

The Significance of Unemployment Rate after Removing Trend

> fit10<-lm(mdHousePrice~time(mdHousePrice))
> res2=residuals(fit10)
> fit11<-lm(res2~mdUnemploy)
> summary(fit11)

lm(formula = res2 ~ mdUnemploy)

Residuals:

Min	1Q	Median	3Q	Max
-31663	-8715	-2204	8757	36977

Coefficients:

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	30230.9	4047.1	7.470	2.04e-12 ***
mdUnemploy	-4984.8	643.9	-7.742	3.90e-13 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 15600 on 213 degrees of freedom
Multiple R-squared: 0.2196, Adjusted R-squared: 0.2159
F-statistic: 59.94 on 1 and 213 DF, p-value: 3.899e-13

Figure 47:

Augmented Dickey-Fuller Test

        Augmented Dickey-Fuller Test
data: mdHousePrice
Dickey-Fuller = -2.4078, Lag order = 5, p-value = 0.4049
alternative hypothesis: stationary

Figure 48, ARIMA (1,1,1):

arima(x = mdHousePrice, order = c(1, 1, 1))
Coefficients:

	ar1	ma1
	-0.4541	0.2749
s.e.	0.2143	0.2267

sigma^2 estimated as 21890311: log likelihood = -2112.14, aic = 4228.28

Figure 49, ARIMA(2,1,1):
arima(x = mdHousePrice, order = c(2, 1, 1))
Coefficients:

	ar1	ar2	ma1
	0.5340	0.3943	-0.7911
s.e.	0.0769	0.0642	0.0582

sigma^2 estimated as 18541821: log likelihood = -2094.66, aic = 4195.32

Figure 50, ARIMA(1,1,0):
arima(x = mdHousePrice, order = c(1, 1, 0))
Coefficients:

	ar1
	-0.1785
s.e.	0.0672

sigma^2 estimated as 22058381: log likelihood = -2112.95, aic = 4227.91

Figure 51, ARIMA(0,1,1):
arima(x = mdHousePrice, order = c(0, 1, 1))
Coefficients:

	ma1
	-0.1242
s.e.	0.0539

sigma^2 estimated as 22270348: log likelihood = -2113.97, aic = 4229.94

Figure 52, ARIMA(0,2,2):
arima(x = mdHousePrice, order = c(0, 2, 2))
Coefficients:

	ma1	ma2
	-1.2976	0.4972
s.e.	0.0559	0.0571

sigma^2 estimated as 18046605: log likelihood = -2082.67, aic = 4169.33
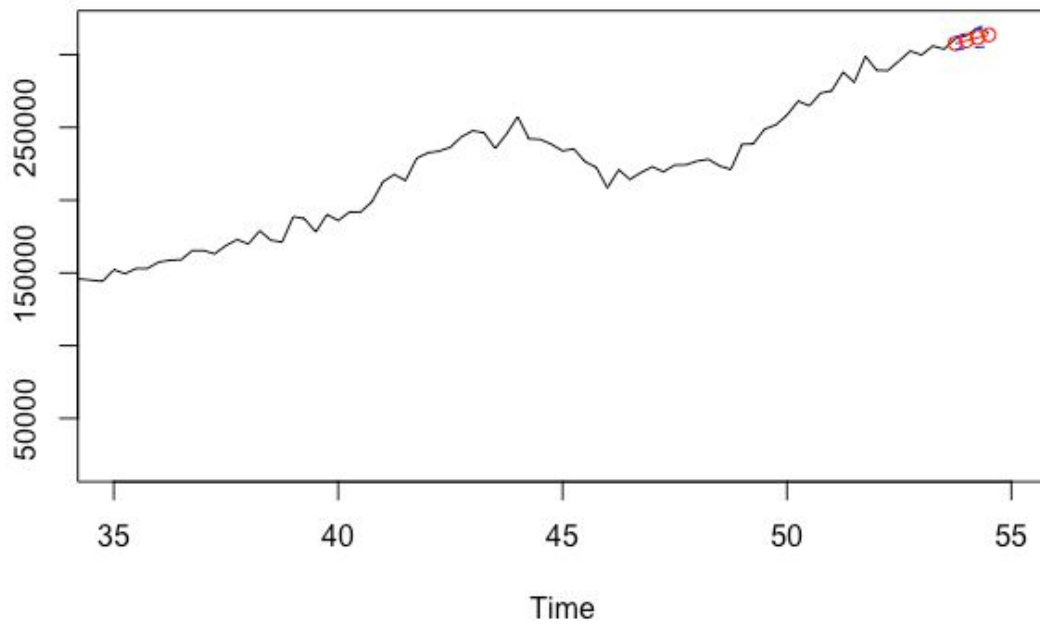
Figure 53, ARIMA(0,2,2) Forecasting:
Predictions for past values:

Q4 (2016)	Q1 (2017)	Q2 (2017)	Q3 (2017)
307646.5	309642.3	311638.0	313633.7

Standard Error:

4263.338	5214.790	6488.903	8024.082

Figure 54, ARIMA(0,2,2) Forecasting:

Source of Datasets:
FRED:
https://fred.stlouisfed.org/series/MSPUS#0
https://fred.stlouisfed.org/series/UNRATENSA#0
https://fred.stlouisfed.org/series/A939RX0Q048SBEA
https://fred.stlouisfed.org/series/TTLHHM156N#0