

# LaTeX Lecture Scribe

06/02/2026

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Below is a faithful LaTeX lecture scribe reconstructed strictly and only from the provided lecture slide...



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Generate a lecture scribe which can be used as a reference during the exams. It must be a perfect reconstruction of what was taught in class and should be written in such a way that a student can rely on it to revise the material. Use only the provided lecture slides and relevant textbook material as the context. Do not write any new content, explanations, interpretations, or examples that are not provided in the shared material. The scribe must include, only when present in the context, definitions and notation, assumptions and conditions, statements of theorems or results, proofs or derivations (written step by step exactly as given in the material no extra steps), theorems and results, visual/ tabular data and all worked examples including intermediate steps. Organize it with clear section headings and proper logical flow, strictly following the structure of the lecture, so that it can be used for exam revision. Provide me the latex format for this lecture scribe.



Below is a **faithful LaTeX lecture scribe** reconstructed **strictly and only** from the provided lecture slides and their embedded material.

No new explanations, interpretations, or examples have been added.

All definitions, notation, results, derivations, and worked examples follow the **exact order, wording, and logical flow of the lecture** and are suitable for **exam revision**.

## LaTeX Lecture Scribe

**Source:** CSE400 – Fundamentals of Probability in Computing

**Lecture 5:** Bayes' Theorem, Random Variables, and Probability Mass Function

L5\_A\_S1

latex

Copy code

```
\documentclass[12pt]{article}
\usepackage{amsmath, amssymb, amsthm}
\usepackage{geometry}
\geometry{margin=1in}
\usepackage{graphicx}

\begin{document}

\begin{center}
\Large \textbf{CSE400: Fundamentals of Probability in Computing} \\
\end{center}
\end{document}
```

```

\vspace{0.2cm}
\large \textbf{Lecture 5: Bayes' Theorem, Random Variables, and Probability Mass Function} \\
\vspace{0.2cm}
\normalsize Dhaval Patel, PhD \\
January 20, 2026
\end{center}

\tableofcontents
\newpage

%-----
\section{Bayes' Theorem}

\subsection{Weighted Average of Conditional Probabilities}

Let  $A$  and  $B$  be events. We may express  $A$  as
\[
A = AB \cup AB^c
\]
for, in order for an outcome to be in  $A$ , it must either be in both  $A$  and  $B$  or be in  $A$  but not in  $B$ .

As  $AB$  and  $AB^c$  are mutually exclusive, we have, by Axiom 3,
\[
\Pr(A) = \Pr(AB) + \Pr(AB^c)
\]

Using conditional probability,
\[
\Pr(A) = \Pr(A \mid B)\Pr(B) + \Pr(A \mid B^c)[1 - \Pr(B)]
\]

Hence, the probability of event  $A$  is a weighted average of the conditional probabilities with weights given by the probabilities of the conditioning events.

%-----
\subsection{Learning by Example}

\subsubsection*{Example 3.1 (Part 1)}

An insurance company believes that people can be divided into two classes: those who are accident prone and those who are not. The company's statistics show that an accident-prone person will have an accident within a fixed one-year period with probability  $0.4$ , whereas this probability is  $0.2$  for a person who is not accident prone. Assume that  $30\%$  of the population is accident prone.

\textbf{Find}: The probability that a new policyholder will have an accident within one year.

\textbf{Solution:}

```

Let

```
\[
A_1 = \{\text{policyholder has an accident within one year}\}
\]
\[
A = \{\text{policyholder is accident prone}\}
\]
```

Then,

```
\[
\Pr(A_1) = \Pr(A_1 \mid A)\Pr(A) + \Pr(A_1 \mid A^c)\Pr(A^c)
\]
\[
= (0.4)(0.3) + (0.2)(0.7) = 0.26
\]
```

%-----

**\subsubsection\***{Example 3.1 (Part 2)}

Suppose that a new policyholder has an accident within one year of purchasing a policy. What is the probability that he or she is accident prone?

**\textbf**{Solution:}

The desired probability is

```
\[
\Pr(A \mid A_1) = \frac{\Pr(AA_1)}{\Pr(A_1)}
\]
\[
= \frac{\Pr(A)\Pr(A_1 \mid A)}{\Pr(A_1)}
\]
\[
= \frac{(0.3)(0.4)}{0.26} = \frac{6}{13}
\]
```

%-----

**\subsection**{Formal Introduction: Law of Total Probability}

Suppose that  $B_1, B_2, \ldots, B_n$  are mutually exclusive events such that

```
\[
\bigcup_{i=1}^n B_i = B
\]
```

By writing

```
\[
A = \bigcup_{i=1}^n AB_i
\]
```

and using the fact that the events  $AB_i$  are mutually exclusive, we obtain

```
\[
\Pr(A) = \sum_{i=1}^n \Pr(AB_i) = \sum_{i=1}^n \Pr(A \mid B_i)\Pr(B_i)
\]
```

This is known as the **Law of Total Probability** (Formula 3.4).

```
%-----
```

```
\subsection{Bayes Formula}
```

Using

```
\[
\Pr(AB_i) = \Pr(B_i \mid A)\Pr(A)
\]
```

we get

```
\[
\Pr(B_i \mid A) = \frac{\Pr(A \mid B_i)\Pr(B_i)}{\sum_{j=1}^n \Pr(A \mid B_j)\Pr(B_j)}
\]
```

This is known as the **Bayes Formula** (Proposition 3.1).

Here,

```
\begin{itemize}
\item  $\Pr(B_i)$  is the a priori probability
\item  $\Pr(B_i \mid A)$  is the a posteriori probability
\end{itemize}
```

```
%-----
```

```
\subsection{Example 3.2 (Cards)}
```

Three cards are identical except:

```
\begin{itemize}
\item One card: red-red (RR)
\item One card: black-black (BB)
\item One card: red-black (RB)
\end{itemize}
```

One card is randomly selected and placed on the ground. If the upturned side is red, find the probability that the other side is black.

Let

```
\[
R = \{\text{upturned side is red}\}
\]
```

The desired probability is

```
\[
\Pr(RB \mid R) = \frac{\Pr(RB \cap R)}{\Pr(R)}
\]
```

```
\[
= \frac{\Pr(R \mid RB)\Pr(RB)}{\Pr(R \mid RR)\Pr(RR) + \Pr(R \mid RB)\Pr(RB) + \Pr(R \mid BB)\Pr(BB)}
\]
```

```
\[
= \frac{(1/2)(1/3)}{(1)(1/3) + (1/2)(1/3) + (0)(1/3)} = \frac{1}{3}
\]
```

```
%-----
\section{Random Variables}
```

```
\subsection{Motivation and Concept}
```

A random variable is a real-valued function defined on the sample space.

```
\begin{itemize}
\item Dice tossing: focus on the sum of dice
\item Coin tossing: focus on the number of heads
\end{itemize}
```

```
\textbf{Definition:}
```

A random variable  $X$  on a sample space  $\Omega$  is a function

```
\[
X: \Omega \rightarrow \mathbb{R}
\]
```

We restrict attention to discrete random variables unless stated otherwise.

```
%-----
\subsection{Distribution}
```

For a random variable  $X$ , let

```
\[
\{ \omega \in \Omega : X(\omega) = a \}
\]
```

denote an event, abbreviated as  $\{X = a\}$ .

The collection of probabilities

```
\[
\Pr(X = a)
\]
```

for all possible values of  $a$  is called the **distribution** of  $X$ .

These probabilities can be visualized using a bar diagram.

```
%-----
\subsection{Example}
```

**\textbf{Example 1:}** Tossing three fair coins.

Let  $Y$  be the number of heads.

$$\begin{aligned}\Pr(Y = 0) &= \frac{1}{8}, \quad \Pr(Y = 1) = \frac{3}{8}, \quad \Pr(Y = 2) = \frac{3}{8}, \quad \Pr(Y = 3) = \frac{1}{8}\end{aligned}$$

Since  $Y$  must take one of the values  $0$  through  $3$ ,

$$1 = \sum_{i=0}^3 \Pr(Y = i)$$

%-----

**\section{Probability Mass Function}**

**\subsection{Concept}**

A random variable that can take on at most a countable number of values is said to be **\textbf{discrete}**.

Let  $X$  be a discrete random variable with range

$$R_X = \{x_1, x_2, x_3, \dots\}$$

The function

$$p_X(x) = \Pr(X = x)$$

is called the **\textbf{Probability Mass Function (PMF)}** of  $X$ .

Since  $X$  must take one of the values  $x_k$ ,

$$\sum_k p_X(x_k) = 1$$

%-----

**\subsection{Example: Two Independent Tosses of a Fair Coin}**

Let  $X$  be the number of heads obtained.

$$\begin{aligned}p_X(x) &= \\ \begin{cases} \frac{1}{4}, & \text{if } x = 0 \text{ or } 2 \\ \frac{2}{4}, & \text{if } x = 1 \end{cases}\end{aligned}$$

```

\frac{1}{2}, & x = 1 \\
0, & \text{otherwise}
\end{cases}
\]

\[
\Pr(X > 0) = \Pr(X = 1) + \Pr(X = 2) = \frac{3}{4}
\]

%-----
\subsection{Example}

The PMF of a random variable  $X$  is given by
\[
p(i) = c \frac{\lambda^i}{i!}, \quad i = 0, 1, 2, \dots
\]
where  $\lambda > 0$ .

Since
\[
\sum_{i=0}^{\infty} p(i) = 1
\]
\[
c \sum_{i=0}^{\infty} \frac{\lambda^i}{i!} = 1
\]
and since
\[
e^{\lambda} = \sum_{i=0}^{\infty} \frac{\lambda^i}{i!}
\]
we obtain
\[
c = e^{-\lambda}
\]

\end{document}

```

If you want, I can next:

- split this into **exam-ready short notes**,
- verify it against a **specific textbook section**, or
- adjust formatting to **IEEE / university submission standards**.