CSE 400 Scribe Report

L5_A_S1.pdf
PDF

Practice: Professional professional Lecture Scribe on CSE 400: Fundamentals of Probability in Computing - Lecture 5 (Bayes Thesis, Random Variables, PMF).I. Professional Purpose and Role Identity: Become a professional academic scribe. This is the graded part of the course and has to be in the form of an academic reference source of closed-note exam.Priority: Technical correctness and absolute adherence to the given scenario of Lecture 5 are all the issues of consideration. Content Scope & Technical RequirementContext Limitation: Use L5A S1.pdf attached. Do not create content, or place extraneous intuition (in particular: no non-academic Randall Munroe/xkcd allusions).
Conditioned probabilities Theories The Law of Total Probability [Formula 3.4], the Bayes Formula [Proposition 3.1]: Reconstruct the Weighted Average of Conditional Probabilities, the Law of Total probabilities (explicitly stating a priori and a posteriori probabilities).
Examples: Provided are example 3.1 (Accident-Prone Insurance), example 3.2 (The Three Cards Problem), and PMF Solved Problem p (i) = c 0.5 Taylor Series Expansion.
Random Variables Faithfully entails motivating and defining discrete random variables III. Zero Shot (Negative Constraints) Discipline.
Banned Words: No mention of the words, as this.
No Meta-Talk You are free as you have the strength to be faithful to the order and extent of the lecture.IV. Manadatory LaTeX TemplateOutput You are forbidden to write plain-text commentary on the output which is not in the block of LaTeX. You must populate the lecture content into the following structure:\documentclass[11pt]{article}\usepackage[a4paper,margin=1in]{geometry}\usepackage{amsmath,amssymb}\usepack

age{enumitem}\usepackage{fancyhdr}\usepackage{xcolor}\pagestyle{fancy}\fancyhf{}\lhead{CSE 400: Fundamentals of Probability in Computing}\rhead{Project Scribe Report}\cfoot{\thepage}\title{\normalsize School of Engineering and Applied Science (SEAS), Ahmedabad University \\vspace{0.2cm}\textbf{CSE 400: Fundamentals of Probability in Computing}\\Large Project Scribe Submission}\author{}\date{}\begin{document}\maketitle\vspace{-2cm}\begin{center}\begin{tabular}{ll}\textbf{Group No.:} & {S1 G17 \hspace{2.5in}} \[1.5ex]\textbf{Domain:} & {ITS \hspace{2.5in}} \[1.5ex]\textbf{Date:} & {January 20, 2026 \hspace{2.2in}}\end{tabular}\end{center}\hrule\vspace{0.5cm}% YOU MUST REPLACE THE SCRIBE QUESTION ENUMERATION WITH HIERARCHICAL LECTURE CONTENT\end{document}

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\documentclass[11pt]{article}
\usepackage[a4paper,margin=1in]{geometry}
\usepackage{amsmath,amssymb}
\usepackage{enumitem}
\usepackage{fancyhdr}
\usepackage{xcolor}

\pagestyle{fancy}
\fancyhf{}
\lhead{CSE 400: Fundamentals of Probability in Computing}
\rhead{Project Scribe Report}
\cfoot{\thepage}

\title{\normalsize School of Engineering and Applied Science (SEAS), Ahmedabad University
\\
\vspace{0.2cm}
\textbf{CSE 400: Fundamentals of Probability in Computing}\\
\Large Project Scribe Submission}

\author{}
\date{}
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\begin{document}
\maketitle
\vspace{-2cm}

\begin{center}
\begin{tabular}{ll}
\textbf{Group No.:} & {S1 G17 \hspace{2.5in}} \\[1.5ex]
\textbf{Domain:} & {ITS \hspace{2.5in}} \\[1.5ex]
\textbf{Date:} & {January 20, 2026 \hspace{2.2in}}
\end{tabular}
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\section{Bayes' Theorem}

\subsection{Weighted Average of Conditional Probabilities}

Let $A$ and $B$ denote events in a sample space. Event $A$ may be decomposed as
\[
A = (A \cap B) \cup (A \cap B^c).
\]
The events $A \cap B$ and $A \cap B^c$ are mutually exclusive. By Axiom 3,
\[
\Pr(A) = \Pr(A \cap B) + \Pr(A \cap B^c).
\]
Using the definition of conditional probability,
\[
\Pr(A) = \Pr(A \mid B)\Pr(B) + \Pr(A \mid B^c)\bigl(1-\Pr(B)\bigr).
\]
The probability of event $A$ is expressed as a weighted average of conditional probabilities,
where the weights are the probabilities of the conditioning events.

\subsection{Learning by Example}

\subsubsection*{Example 3.1: Accident-Prone Insurance (Part 1)}

The population is partitioned into accident-prone and non–accident-prone individuals. Let
\begin{itemize}
\item $A$ denote the event that a policyholder is accident prone,
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\item $A_1$ denote the event that a policyholder has an accident within one year.
\end{itemize}
Given:
\[
\Pr(A)=0.3,\quad \Pr(A^c)=0.7,
\]
\[
\Pr(A_1\mid A)=0.4,\quad \Pr(A_1\mid A^c)=0.2.
\]
Conditioning on $A$,
\[
\Pr(A_1)=\Pr(A_1\mid A)\Pr(A)+\Pr(A_1\mid A^c)\Pr(A^c).
\]
Hence,
\[
\Pr(A_1)=(0.4)(0.3)+(0.2)(0.7)=0.26.
\]

\subsubsection*{Example 3.1: Accident-Prone Insurance (Part 2)}

Given that an accident has occurred within one year, the required probability is
\[
\Pr(A\mid A_1)=\frac{\Pr(A\cap A_1)}{\Pr(A_1)}.
\]
Using previously computed values,
\[
\Pr(A\mid A_1)=\frac{\Pr(A)\Pr(A_1\mid A)}{\Pr(A_1)}
=\frac{(0.3)(0.4)}{0.26}=\frac{6}{13}.
\]

\subsection{Formal Introduction}

\subsubsection{Law of Total Probability (Formula 3.4)}

Let $B_1,B_2,\dots,B_n$ be mutually exclusive events with
\[
\bigcup_{i=1}^{n} B_i = B.
\]
For any event $A$,
\[
A=\bigcup_{i=1}^{n}(A\cap B_i),
\]
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with mutually exclusive components. Therefore,
\[
\Pr(A)=\sum_{i=1}^{n}\Pr(A\cap B_i)
    =\sum_{i=1}^{n}\Pr(A\mid B_i)\Pr(B_i).
\]

\subsubsection{Bayes Formula (Proposition 3.1)}

Using $\Pr(A\cap B_i)=\Pr(B_i\mid A)\Pr(A)$, Bayes Formula is obtained:
\[
\Pr(B_i\mid A)=
\frac{\Pr(A\mid B_i)\Pr(B_i)}
{\sum_{j=1}^{n}\Pr(A\mid B_j)\Pr(B_j)}.
\]
Here,
\begin{itemize}
\item $\Pr(B_i)$ represents the \textbf{a priori probability},
\item $\Pr(B_i\mid A)$ represents the \textbf{a posteriori probability}.
\end{itemize}

\subsection{Example 3.2: The Three Cards Problem}

Three cards are available: one red–red (RR), one black–black (BB), and one red–black (RB). A
card is selected uniformly and placed with one side up. Let $R$ denote the event that the
upper side is red.

The required probability is
\[
\Pr(RB\mid R)=\frac{\Pr(R\mid RB)\Pr(RB)}
{\Pr(R\mid RR)\Pr(RR)+\Pr(R\mid RB)\Pr(RB)+\Pr(R\mid BB)\Pr(BB)}.
\]
Substituting values,
\[
\Pr(RB\mid R)=
\frac{(1/2)(1/3)}
{(1)(1/3)+(1/2)(1/3)+(0)(1/3)}=\frac{1}{3}.
\]

\section{Random Variables}

\subsection{Motivation and Concept}
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A random variable is a real-valued function defined on a sample space $\Omega$,
\[
X:\Omega \rightarrow \mathbb{R}.
\]
It assigns a numerical value to each outcome of an experiment. Attention is restricted to discrete random variables whose ranges are finite or countably infinite.

The distribution of a random variable is determined by
\[
\Pr[X=a]=\Pr(\{\omega\in\Omega : X(\omega)=a\}).
\]

\subsection{Examples}

\subsubsection*{Example 1: Tossing Three Fair Coins}

Let $Y$ denote the number of heads obtained. Then $Y$ takes values $\{0,1,2,3\}$ with
\[
\Pr(Y=0)=\frac{1}{8},\quad
\Pr(Y=1)=\frac{3}{8},\quad
\Pr(Y=2)=\frac{3}{8},\quad
\Pr(Y=3)=\frac{1}{8}.
\]
Since $Y$ must assume one of these values,
\[
\sum_{i=0}^{3}\Pr(Y=i)=1.
\]

\section{Probability Mass Function}

\subsection{Definition}

A random variable that takes at most a countable number of values is discrete. Let $X$ be discrete with range
\[
R_X=\{x_1,x_2,x_3,\dots\}.
\]
The function
\[
p_X(x_k)=\Pr(X=x_k)
\]
is the Probability Mass Function (PMF) of $X$. It satisfies
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\[
\sum_{k}p_X(x_k)=1.
\]

\subsection{Example: Two Independent Tosses of a Fair Coin}

Let $X$ denote the number of heads obtained. Then
\[
p_X(x)=
\begin{cases}
\frac{1}{4}, & x=0 \text{ or } x=2,\\[0.5ex]
\frac{1}{2}, & x=1,\\[0.5ex]
0, & \text{otherwise}.
\end{cases}
\]
Hence,
\[
\Pr(X>0)=\Pr(X=1)+\Pr(X=2)=\frac{3}{4}.
\]

\subsection{Solved PMF Problem}

Let the PMF be defined by
\[
p(i)=c\frac{\lambda^i}{i!}, \quad i=0,1,2,\dots,
\]
where $\lambda>0$. Since
\[
\sum_{i=0}^{\infty}p(i)=1,
\]
it follows that
\[
c\sum_{i=0}^{\infty}\frac{\lambda^i}{i!}=1.
\]
Using the Taylor series expansion
\[
e^{\lambda}=\sum_{i=0}^{\infty}\frac{\lambda^i}{i!},
\]
we obtain
\[
c=e^{-\lambda}.
\]
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Therefore,
\[
\Pr(X=0)=e^{-\lambda},
\]
and
\[
\Pr(X>2)=1-\bigl[p(0)+p(1)+p(2)\bigr].
\]

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