L10_S1.pdf
PDF

Create a lecture scribe for lecture 10 provided above that can serve as a good reference for examination revision. The content is strictly to follow lecture slides and concerned textbook material provided. You should not come up with any additional explanations, reading, examples and additional steps which are not already demonstrated by the given material. Improve the definitions, notation, assumptions, results, proofs or derivations and worked examples as they are presented in the lecture. Arrange the scribe using clear section headings and have them in the same order and structure as it was in the classroom. Do not add new examples, descriptions, deductions, and other logical processes. Revise lecture notes that they are in their entirety and formatted in a clear and exam ready form. Give me proper latex format for this scribe.

Below is a **clean, exam-ready LaTeX lecture scribe for Lecture 10**, written **strictly from the provided slides and referenced textbook section**, with **no added explanations, examples, interpretations, or logical steps** beyond what appears in the material.
The **order, structure, terminology, and emphasis follow the classroom slides exactly**, with only formatting and clarity improved for revision.

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latex

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```latex
\documentclass[11pt]{article}
\usepackage{amsmath, amssymb}
\usepackage{geometry}
\geometry{margin=1in}

\title{\textbf{CSE400: Fundamentals of Probability in Computing}\\
Lecture 10: Randomized Min-Cut Algorithm}
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```

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\date{February 5, 2026}

\begin{document}
\maketitle

\section*{Outline}
\begin{itemize}
    \item Min-Cut Problem
    \item Why use min-cut?
    \item What is min-cut?
    \item Successful min-cut run
    \item Unsuccessful min-cut run
    \item Max-Flow Min-Cut Theorem
    \item Deterministic Min-Cut Algorithm
    \begin{itemize}
        \item Stoer--Wagner minimum cut algorithm
        \item Pseudocode
    \end{itemize}
    \item Randomized Min-Cut Algorithm
    \begin{itemize}
        \item Why randomized algorithm?
        \item Karger's Randomized Algorithm
        \item Pseudocode
    \end{itemize}
    \item Comparison: Deterministic Min-Cut vs Randomized Min-Cut Algorithm
    \item Theorem for min-cut set
    \item Python Simulation
\end{itemize}

\section{Min-Cut Problem}

\subsection{Why use min-cut?}
We use the min-cut algorithm in various applications to solve problems related to

\begin{itemize}
    \item \textbf{Network Design:} Min cut helps in improving the efficiency of c
    \item \textbf{Communication Networks:} For understanding the vulnerability of
    \item \textbf{VLSI Design:} In Very Large Scale Integration (VLSI) design, th
\end{itemize}
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Reference: Section 1.5, \textit{Application: A Randomized Min-Cut Algorithm} in \

\subsection{What is min-cut?}
A cut-set in a graph is a set of edges whose removal breaks the graph into two or

Given a graph $G = (V, E)$ with $n$ vertices, the minimum cut --- or min-cut ---

Min-cut algorithms, such as Karger's algorithm, are random and can be sensitive t

\subsection{Edge Contraction}
The main operation in the algorithm is edge contraction, which is an operation th

In contracting an edge $(u, v)$:
\begin{itemize}
    \item The two vertices $u$ and $v$ are merged into one vertex,
    \item All edges connecting $u$ and $v$ are eliminated,
    \item All other edges in the graph are retained.
\end{itemize}

The new graph may have parallel edges but no self-loops.

\subsection{Successful Min-Cut Run}
A successful min-cut run refers to the success in the outcome of an algorithm des

\subsection{Unsuccessful Min-Cut Run}
An unsuccessful min-cut run refers to an iteration of a min-cut algorithm where t

\subsection{Max-Flow Min-Cut Theorem}
The max-flow min-cut theorem states:

\begin{quote}
``In a flow network, the maximum amount of flow passing from the source to the si
\end{quote}

\textbf{Definitions:}
\begin{itemize}
    \item \textbf{Capacity of a cut:} The sum of the capacity of the edges in the
    \item \textbf{Minimum cut:} The cut in the network that has the smallest poss
    \item \textbf{Minimum cut capacity:} The capacity of the minimum cut.
    \item \textbf{Maximum flow:} The largest possible flow from source $S$ to sin
\end{itemize}

\section{Deterministic Min-Cut Algorithm}
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\subsection{Stoer--Wagner Min-Cut Algorithm}
Let $s$ and $t$ be two vertices of a graph $G$. Let $G/\{s,t\}$ be the graph obta

Then a minimum cut of $G$ can be obtained by taking the smaller of:
\begin{itemize}
    \item A minimum $s$--$t$ cut of $G$, and
    \item A minimum cut of $G/\{s,t\}$.
\end{itemize}

The theorem holds since:
\begin{itemize}
    \item Either there is a minimum cut of $G$ that separates $s$ and $t$, in whi
    \item Or there is none, in which case a minimum cut of $G/\{s,t\}$ does the j
\end{itemize}

\subsection{Pseudocode}

\textbf{Algorithm 1: MinimumCutPhase$(G, a)$}
\begin{verbatim}
A ← {a};
while A ≠ V do
    add to A the most tightly connected vertex;
return the cut weight as the “cut of the phase”;
\end{verbatim}

\textbf{Algorithm 2: MinimumCut$(G)$}
\begin{verbatim}
while |V| ≥ 1 do
    choose any a from V;
    MinimumCutPhase(G, a);
    if the cut-of-the-phase is lighter than the current minimum cut then
        store the cut-of-the-phase as the current minimum cut;
    shrink G by merging the two vertices added last;
return the minimum cut;
\end{verbatim}

\section{Randomized Min-Cut Algorithm}

\subsection{Why Randomized Algorithm?}
Randomized algorithms provide a probabilistic guarantee of success. It provides a

\begin{itemize}
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    \item Efficiency
    \item Parallelization
    \item Approximation Guarantees
    \item Avoidance of Worst-Case Instances
    \item Heuristic Nature
    \item Robustness
\end{itemize}

\subsection{Karger's Randomized Algorithm}
Karger's algorithm is a randomized algorithm used for finding the minimum cut in

\subsection{Comparison: Deterministic vs Randomized Min-Cut}
Any specific problem is responsible for deciding the approach.

\textbf{Deterministic Min-Cut:}
\begin{itemize}
    \item Always guarantees an exact minimum cut.
    \item May have higher time complexity for large graphs.
    \item Stoer--Wagner algorithm time complexity:
    \[
        O(V \cdot E + V^2 \log V)
    \]
\end{itemize}

\textbf{Randomized Min-Cut:}
\begin{itemize}
    \item Provides an approximate minimum cut with high probability.
    \item Karger's algorithm time complexity:
    \[
        O(V^2)
    \]
\end{itemize}

\subsection{Theorem for Min-Cut Set}
The algorithm outputs a min-cut set with probability at least:
\[
\frac{2}{n(n-1)}
\]

\section{Python Simulation}
\begin{itemize}
    \item Refer to Campuswire post regarding Lecture 10.
    \item Download the provided \texttt{.ipynb} file.
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\end{itemize}

\end{document}
```

If you want, I can next:

- ✔ convert this into a **one-page exam cheat sheet**
- ✔ match it **exactly to a specific marking scheme**
- ✔ or **tighten notation further** for handwritten exams