School of Engineering and Applied Science (SEAS), Ahmedabad University

CSE 400: Fundamentals of Probability in Computing

Project Scribe Submission

Group No.: S1 G17

Domain: ITS

Date: January 20, 2026

---

1 Bayes' Theorem

1.1 Weighted Average of Conditional Probabilities

Let A and B denote events in a sample space. Event A may be decomposed as

$$A = (A \cap B) \cup (A \cap B^c).$$

The events $A \cap B$ and $A \cap B^c$ are mutually exclusive. By Axiom 3,

$$\Pr(A) = \Pr(A \cap B) + \Pr(A \cap B^c).$$

Using the definition of conditional probability,

$$\Pr(A) = \Pr(A \mid B)\Pr(B) + \Pr(A \mid B^c)\big(1 - \Pr(B)\big).$$

The probability of event A is expressed as a weighted average of conditional probabilities, where the weights are the probabilities of the conditioning events.

1.2 Learning by Example

Example 3.1: Accident-Prone Insurance (Part 1)

The population is partitioned into accident-prone and non–accident-prone individuals. Let

- A denote the event that a policyholder is accident prone,
- A_1 denote the event that a policyholder has an accident within one year.

Given:

$$\Pr(A) = 0.3, \quad \Pr(A^c) = 0.7,$$
$$\Pr(A_1 \mid A) = 0.4, \quad \Pr(A_1 \mid A^c) = 0.2.$$

Conditioning on A,

$$\Pr(A_1) = \Pr(A_1 \mid A)\Pr(A) + \Pr(A_1 \mid A^c)\Pr(A^c).$$

Hence,

$$\Pr(A_1) = (0.4)(0.3) + (0.2)(0.7) = 0.26.$$

Example 3.1: Accident-Prone Insurance (Part 2)

Given that an accident has occurred within one year, the required probability is

$$\Pr(A \mid A_1) = \frac{\Pr(A \cap A_1)}{\Pr(A_1)}.$$

Using previously computed values,

$$\Pr(A \mid A_1) = \frac{\Pr(A)\Pr(A_1 \mid A)}{\Pr(A_1)} = \frac{(0.3)(0.4)}{0.26} = \frac{6}{13}.$$

1.3 Formal Introduction

1.3.1 Law of Total Probability (Formula 3.4)

Let $B_1, B_2, \ldots, B_n$ be mutually exclusive events with

$$\bigcup_{i=1}^{n} B_i = B.$$

For any event A,

$$A = \bigcup_{i=1}^{n} (A \cap B_i),$$

with mutually exclusive components. Therefore,

$$\Pr(A) = \sum_{i=1}^{n} \Pr(A \cap B_i) = \sum_{i=1}^{n} \Pr(A \mid B_i)\Pr(B_i).$$

1.3.2 Bayes Formula (Proposition 3.1)

Using $\Pr(A \cap B_i) = \Pr(B_i \mid A)\Pr(A)$, Bayes Formula is obtained:

$$\Pr(B_i \mid A) = \frac{\Pr(A \mid B_i)\Pr(B_i)}{\sum_{j=1}^{n} \Pr(A \mid B_j)\Pr(B_j)}.$$

Here,

- $\Pr(B_i)$ represents the **a priori probability**,
- $\Pr(B_i \mid A)$ represents the **a posteriori probability**.

1.4 Example 3.2: The Three Cards Problem

Three cards are available: one red–red (RR), one black–black (BB), and one red–black (RB). A card is selected uniformly and placed with one side up. Let R denote the event that the upper side is red.

The required probability is

$$\Pr(RB \mid R) = \frac{\Pr(R \mid RB)\Pr(RB)}{\Pr(R \mid RR)\Pr(RR) + \Pr(R \mid RB)\Pr(RB) + \Pr(R \mid BB)\Pr(BB)}.$$

Substituting values,

$$\Pr(RB \mid R) = \frac{(1/2)(1/3)}{(1)(1/3) + (1/2)(1/3) + (0)(1/3)} = \frac{1}{3}.$$

2 Random Variables

2.1 Motivation and Concept

A random variable is a real-valued function defined on a sample space Ω,

$$X : \Omega \to \mathbb{R}.$$

It assigns a numerical value to each outcome of an experiment. Attention is restricted to discrete random variables whose ranges are finite or countably infinite.

The distribution of a random variable is determined by

$$\Pr[X = a] = \Pr(\{\omega \in \Omega : X(\omega) = a\}).$$

2.2 Examples

Example 1: Tossing Three Fair Coins

Let Y denote the number of heads obtained. Then Y takes values $\{0, 1, 2, 3\}$ with

$$\Pr(Y = 0) = \frac{1}{8}, \quad \Pr(Y = 1) = \frac{3}{8}, \quad \Pr(Y = 2) = \frac{3}{8}, \quad \Pr(Y = 3) = \frac{1}{8}.$$

Since Y must assume one of these values,

$$\sum_{i=0}^{3} \Pr(Y = i) = 1.$$

3 Probability Mass Function

3.1 Definition

A random variable that takes at most a countable number of values is discrete. Let X be discrete with range

$$R_X = \{x_1, x_2, x_3, \dots\}.$$

The function

$$p_X(x_k) = \Pr(X = x_k)$$

is the Probability Mass Function (PMF) of X. It satisfies

$$\sum_k p_X(x_k) = 1.$$

3.2 Example: Two Independent Tosses of a Fair Coin

Let X denote the number of heads obtained. Then

$$p_X(x) = \begin{cases} \frac{1}{4}, & x = 0 \text{ or } x = 2, \\ \frac{1}{2}, & x = 1, \\ 0, & \text{otherwise.} \end{cases}$$

Hence,

$$\Pr(X > 0) = \Pr(X = 1) + \Pr(X = 2) = \frac{3}{4}.$$

3.3 Solved PMF Problem

Let the PMF be defined by

$$p(i) = c\frac{\lambda^i}{i!}, \quad i = 0, 1, 2, \ldots,$$

where $\lambda > 0$. Since

$$\sum_{i=0}^{\infty} p(i) = 1,$$

it follows that

$$c\sum_{i=0}^{\infty} \frac{\lambda^i}{i!} = 1.$$

Using the Taylor series expansion

$$e^{\lambda} = \sum_{i=0}^{\infty} \frac{\lambda^i}{i!},$$

we obtain

$$c = e^{-\lambda}.$$

Therefore,

$$\Pr(X = 0) = e^{-\lambda},$$

and

$$\Pr(X > 2) = 1 - \big[p(0) + p(1) + p(2)\big].$$