

5_3

help me generate a lecture scribe for Lecture 5 of the course CSE 400 Fundamentals of Probability in Computing

OUTPUT: formal lecture scribe, intended strictly as a exam oriented reference material for reading-based or closed-book revision, I should be able to revise Lecture 5 using only this document, without referring to any external source

Context Restriction: (FOLLOW STRICTLY)

Use only the provided Lecture 5 PowerPoint slides

Do not use external knowledge, textbook material not explicitly shown, inferred intuition, reinterpretations, or additional explanations

Every definition, formula, example, and derivation must be directly traceable to the provided context

Required Content (Include only if present in the lecture):

Formal definitions and notation

Explicit assumptions and conditions

Clearly stated theorems, propositions, or named results

Proofs or proof sketches written step by step, exactly as presented

Worked examples with all intermediate steps shown

Structural Expectations: (COMPULSORY)

Follow the exact topic order and logical flow of Lecture 5

Organize the scribe using clear LaTeX sectioning (`\section`, `\subsection`)

Present theorems, propositions, and formulas in clearly identifiable mathematical environments

Clearly mark proofs and example solutions so they are easy to locate during revision

Exclude any content that is administrative or unrelated

to examinable material
Strict Exclusions:
Do not add:
New examples or problems
Intuition, motivation, or explanatory commentary
beyond what appears in the lecture
Simplifications, summaries, or textbook-style
elaborations
Any reasoning or explanation outside the lecture
material
Output Format (Non-Negotiable):
Output only complete LaTeX source code, enclosed
within a single
`\documentclass ... \end{document}` block
Do not include any plain text, commentary, or
explanations outside the LaTeX code

The LaTeX document must explicitly include:

Course name
Lecture number (Lecture 5)
Group No.: S1 G17
Date
The final output must be a faithful, well-structured,
exam-ready reconstruction of Lecture 5, with emphasis
on clarity, correctness, and revision usability.

I can do this **only after you provide the Lecture 5 material.**

Right now, the **required context is missing**. Your own constraints explicitly forbid me from using:

- external knowledge
- inferred structure
- textbook material
- reinterpretation or intuition

Without the **Lecture 5 PowerPoint slides**, generating *any* scribe would violate your “Context Restriction (STRICT)” clause.

What you need to do next

Please upload **one of the following**:

- The **Lecture 5 PowerPoint slides (PDF / PPT / images)**
or
- Clear **images/screenshots of all Lecture 5 slides**, in correct order

Once uploaded, I will:

- Reconstruct **only what appears in the slides**
- Preserve **exact topic order and notation**
- Include **definitions, theorems, proofs, and examples only if present**
- Produce **complete LaTeX source code** inside a single
`\documentclass ... \end{document}` block
- Explicitly include:
 - Course name
 - Lecture 5
 - Group No.: **S1 G17**
 - Date

No filler, no intuition, no additions. Just a clean, exam-ready scribe you can revise from in isolation.

 **Upload the Lecture 5 slides when ready.**



latex

```
\documentclass[11pt]{article}

\usepackage{amsmath,amssymb,amsthm}
\usepackage{geometry}
\usepackage{setspace}
\usepackage{hyperref}

\geometry{margin=1in}
\setstretch{1.15}

\newtheorem{proposition}{Proposition}
\newtheorem{definition}{Definition}
\newtheorem{example}{Example}
```

```
\begin{document}

\begin{center}
\textbf{CSE 400: Fundamentals of Probability in Computing}\n
\textbf{Lecture 5: Bayes' Theorem, Random Variables, and Probability Mass Function}\n
Group No.: S1 G17\n
Date: January 20, 2026
\end{center}
```

\section{Bayes' Theorem}

\subsection{Weighted Average of Conditional Probabilities}

Let A and B be events. We may express A as

$$\begin{bmatrix} A = AB \cup AB^c, \\ \end{bmatrix}$$

for, in order for an outcome to be in A , it must either be in both A and B or be in A but not in B .

Since AB and AB^c are mutually exclusive, by Axiom 3,

$$\begin{bmatrix} \Pr(A) = \Pr(AB) + \Pr(AB^c). \\ \end{bmatrix}$$

Using conditional probability,

$$\begin{bmatrix} \Pr(A) = \Pr(A \mid B)\Pr(B) + \Pr(A \mid B^c)[1 - \Pr(B)]. \\ \end{bmatrix}$$

Hence, the probability of event A is a weighted average of the conditional probabilities, with weights given by the probabilities of the conditioning events.

\subsection{Learning by Example}

\begin{example}[Example 3.1, Part 1]

An insurance company divides people into two classes: accident-prone and not accident-prone. The probability that an accident-prone person has an accident within one year is \$0.4\$, while for a non-accident-prone person it is \$0.2\$. If \$30\%\$ of the population is accident-prone, find the probability that a new policyholder has an accident within one year.

\end{example}

\textbf{Solution.}

Let A_1 denote the event that the policyholder has an accident within one year, and let A denote the event that the policyholder is accident-prone. Then,

\[

$$\Pr(A_1) = \Pr(A_1 | A)\Pr(A) + \Pr(A_1 | A^c)\Pr(A^c).$$

\]

Substituting values,

\[

$$\Pr(A_1) = (0.4)(0.3) + (0.2)(0.7) = 0.26.$$

\]

\begin{example}[Example 3.1, Part 2]

Suppose that a new policyholder has an accident within one year. What is the probability that the policyholder is accident-prone?

\end{example}

\textbf{Solution.}

The desired probability is

\[

$$\Pr(A | A_1) = \frac{\Pr(A \cap A_1)}{\Pr(A_1)} = \frac{\Pr(A)\Pr(A_1 | A)}{\Pr(A_1)}.$$

\]

Substituting values,

\[

$$\Pr(A | A_1) = \frac{(0.3)(0.4)}{0.26} = \frac{6}{13}.$$

\]

\subsection{Formal Introduction: Law of Total Probability and Bayes Formula}

Suppose that B_1, B_2, \dots, B_n are mutually exclusive events such that

\[

$$\bigcup_{i=1}^n B_i = B.$$

\]

Then, exactly one of the events B_1, B_2, \dots, B_n must occur. Writing

\[

$$A = \bigcup_{i=1}^n AB_i,$$

\]

and using the fact that the events AB_i are mutually exclusive, we obtain

\[

$$\Pr(A) = \sum_{i=1}^n \Pr(AB_i) = \sum_{i=1}^n \Pr(A | B_i)\Pr(B_i).$$

\]

This is known as the \textbf{Law of Total Probability} (Formula 3.4).

\begin{proposition}[Bayes Formula]

For events A and B_1, B_2, \dots, B_n as above,

\[

$$\Pr(B_i | A) = \frac{\Pr(A | B_i) \Pr(B_i)}{\sum_{j=1}^n \Pr(A | B_j) \Pr(B_j)}.$$

\]

\end{proposition}

Here, $\Pr(B_i)$ is the *a priori* probability, and $\Pr(B_i | A)$ is the *posteriori* probability of B_i given A .

\subsection{Learning by Example}

\begin{example}[Example 3.2]

Three cards are identical in form: one has both sides red (RR), one has both sides black (BB), and one has one side red and one side black (RB). A card is randomly selected and placed on the ground. If the upper side is red, find the probability that the other side is black.

\end{example}

\textbf{Solution.}

Let RR , BB , and RB denote the events that the chosen card is all red, all black, or red-black, respectively. Let R denote the event that the upturned side is red. Then,

\[

$$\Pr(RB | R) = \frac{\Pr(R | RB) \Pr(RB)}{\Pr(R | RR) \Pr(RR) + \Pr(R | RB) \Pr(RB) + \Pr(R | BB) \Pr(BB)}.$$

\]

Substituting values,

\[

$$\Pr(RB | R) = \frac{(1/2)(1/3)}{(1)(1/3) + (1/2)(1/3) + (0)(1/3)} = \frac{1}{3}.$$

\]

\section{Random Variables}

\subsection{Motivation and Concept}

When an experiment is performed, interest often lies in a function of the outcome rather than the outcome itself. These real-valued functions defined on the sample space are called **random variables**.

\begin{definition}

A random variable X on a sample space Ω is a function

\[

$X : \Omega \rightarrow \mathbb{R}$

]

that assigns a real number $X(\omega)$ to each sample point ω in Ω .

\end{definition}

Unless otherwise stated, attention is restricted to discrete random variables, which take values in a finite or countably infinite subset of \mathbb{R} .

\subsection{Distribution of a Random Variable}

Let a be any number in the range of a random variable X . The event

[

$\{\omega \in \Omega : X(\omega) = a\}$

]

is denoted by $X = a$. The probability $\Pr(X = a)$ is defined as the probability of this event.

The collection of probabilities $\Pr(X = a)$ for all possible values of a is called the

\textbf{distribution} of X .

\subsection{Example}

\begin{example}

Suppose an experiment consists of tossing three fair coins. Let Y denote the number of heads that appear.

\end{example}

Then Y takes values $0, 1, 2, 3$ with probabilities

[

$\Pr(Y=0)=\frac{1}{8}, \quad$

$\Pr(Y=1)=\frac{3}{8}, \quad$

$\Pr(Y=2)=\frac{3}{8}, \quad$

$\Pr(Y=3)=\frac{1}{8}.$

]

Since Y must take one of these values,

[

$1 = \sum_{i=0}^3 \Pr(Y=i).$

]

\section{Probability Mass Function}

\subsection{Concept}

\begin{definition}

A random variable that can take on at most a countable number of possible values is said to be **discrete**.

\end{definition}

Let X be a discrete random variable with range $R_X = \{x_1, x_2, x_3, \dots\}$. The function

$$\begin{aligned} p_X(x_k) &= \Pr(X = x_k) \\ \end{aligned}$$

is called the **probability mass function (PMF)** of X . Since X must take one of the values in R_X ,

$$\begin{aligned} \sum_k p_X(x_k) &= 1. \\ \end{aligned}$$

\subsection{Example: Two Independent Tosses of a Fair Coin}

Let X denote the number of heads obtained when a fair coin is tossed twice. Then the PMF of X is

$$\begin{aligned} p_X(x) &= \\ \begin{cases} \frac{1}{4}, & x = 0 \text{ or } x = 2, \\ \frac{1}{2}, & x = 1, \\ 0, & \text{otherwise}. \end{cases} \\ \end{aligned}$$

\end{cases}

Hence,

$$\begin{aligned} \Pr(X > 0) &= \Pr(X=1) + \Pr(X=2) = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}. \\ \end{aligned}$$

\subsection{Example}

The probability mass function of a random variable X is given by

$$\begin{aligned} p(i) &= c \frac{\lambda^i}{i!}, \quad i = 0, 1, 2, \dots, \\ \end{aligned}$$

where $\lambda > 0$.

Since

\[

$\sum_{i=0}^{\infty} p(i) = 1,$
we have
 $c \sum_{i=0}^{\infty} \frac{\lambda^i}{i!} = 1.$

Using

$$\sum_{i=0}^{\infty} \frac{\lambda^i}{i!} = e^{\lambda},$$

it follows that

$$c = e^{-\lambda}.$$

Therefore,

$$\Pr(X=0) = p(0) = e^{-\lambda},$$

and

$$\Pr(X>2) = 1 - [p(0) + p(1) + p(2)].$$

`\end{document}`

% Source: Lecture 5 slides (L5_A_S1.pdf) :contentReference[oaicite:0]{index=0}