

# Lecture 11: Model free learning

**Radoslav Neychev**

1. Value-function and Q-function recap
2. Model-free and model-based learning
3. Q-learning
4. Temporal difference
5. SARSA and EV-SARSA
6. Experience replay

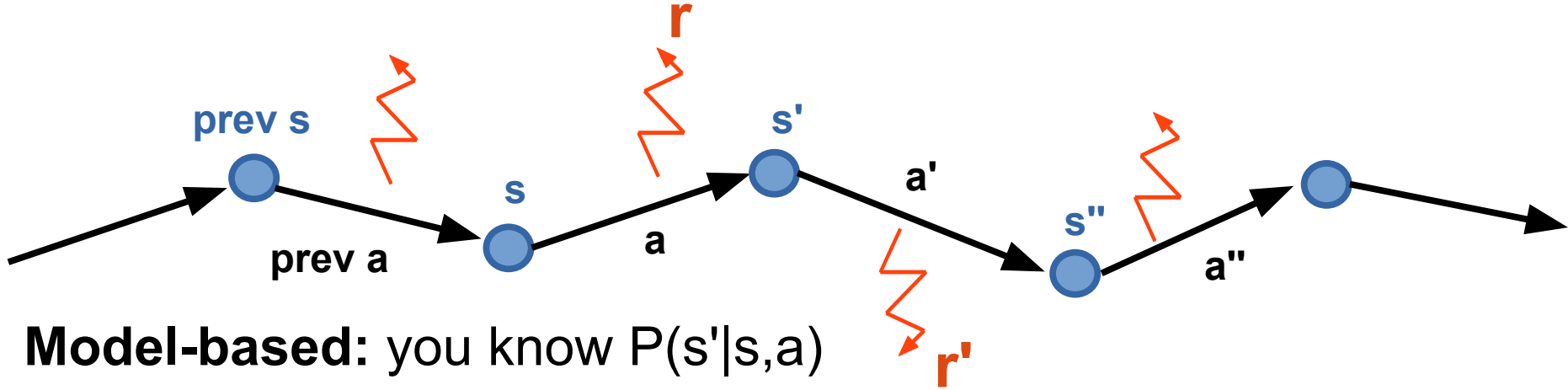
Based on: [https://github.com/yandexdataschool/Practical\\_RL/](https://github.com/yandexdataschool/Practical_RL/)

- $V_{\pi}(\mathbf{s})$  – expected G from state  $\mathbf{s}$  if you follow  $\pi$
- $V^*(\mathbf{s})$  – expected G from state  $\mathbf{s}$  if you follow  $\pi^*$  – optimal
- $Q_{\pi}(\mathbf{s}, \mathbf{a})$  – expected G from state  $\mathbf{s}$ 
  - if you start by taking action  $\mathbf{a}$
  - and follow  $\pi$  from next state on
- $Q^*(\mathbf{s}, \mathbf{a})$  – same as  $Q_{\pi}(\mathbf{s}, \mathbf{a})$  where  $\pi = \pi^*$  – optimal policy

$$Q^*(s, a) = E_{s', r} [r(s, a) + \gamma \cdot V^*(s')]$$

$$V^*(s) = \max_a Q^*(s, a)$$

# Learning from trajectories



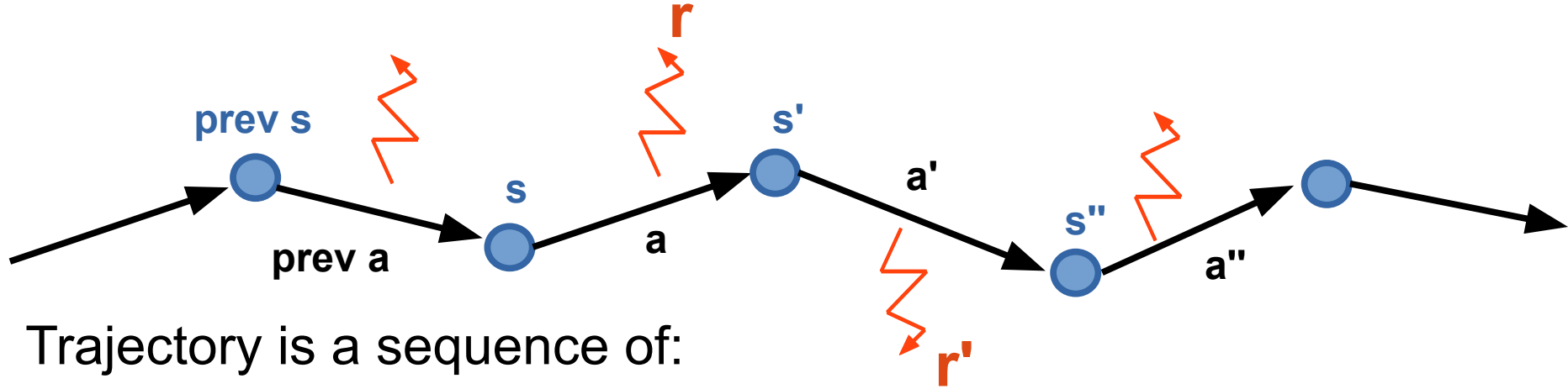
**Model-based:** you know  $P(s'|s,a)$

- can apply dynamic programming
- can plan ahead

**Model-free:** you can sample trajectories

- can try stuff out
- insurance not included

# Learning from trajectories



Trajectory is a sequence of:

- states ( $s$ )
- actions ( $a$ )
- rewards ( $r$ )

**Q:** What to learn?

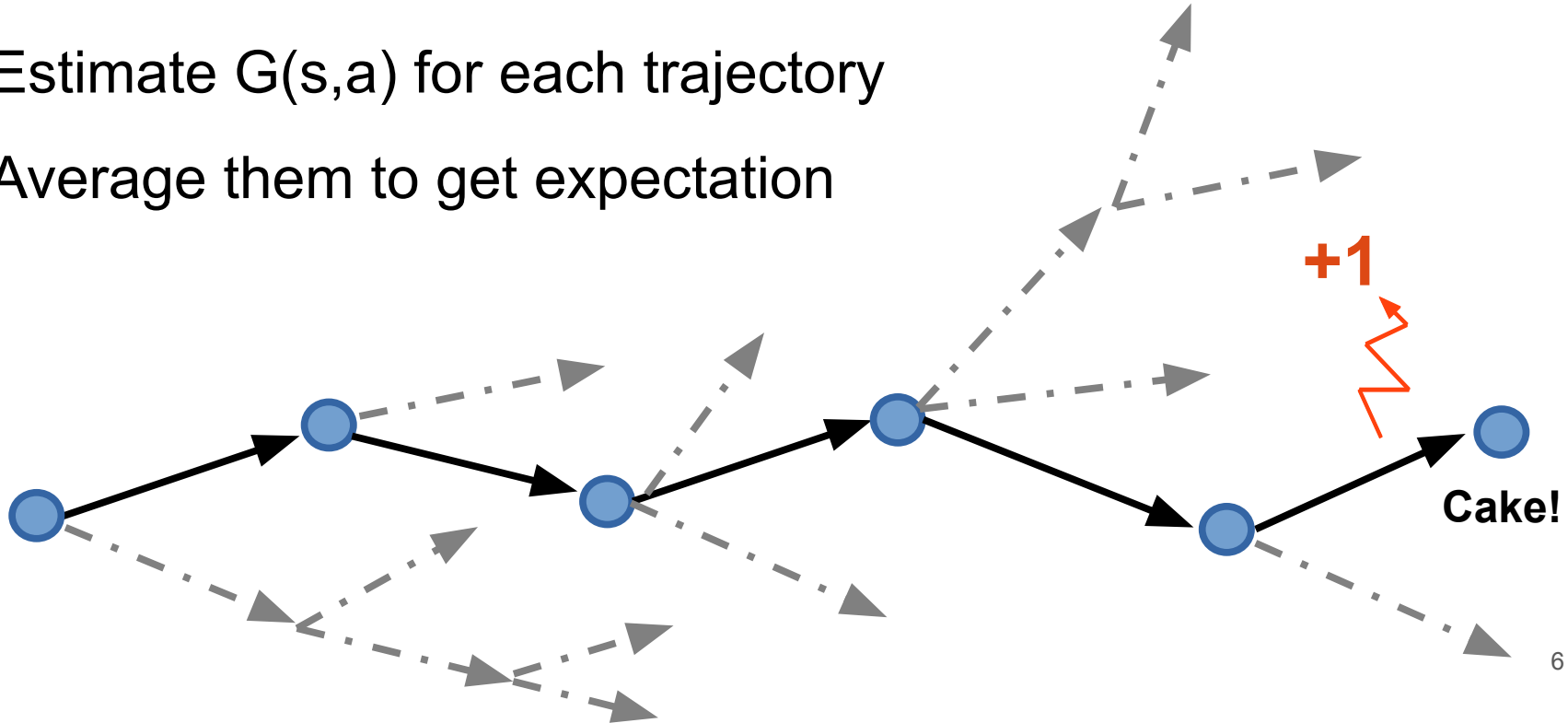
$V(s)$  or  $Q(s,a)$

We can only sample trajectories

$V(s)$  is useless  
without  $P(s'|s,a)$

# Idea 1: Monte-carlo

- Get all trajectories containing particular (s,a)
- Estimate  $G(s,a)$  for each trajectory
- Average them to get expectation



## Idea 2: Temporal difference

- $Q(s, a)$  can be improved iteratively!

$$Q(s_t, a_t) \leftarrow E_{r_t, s_{t+1}} r_t + \gamma \cdot \max_{a'} Q(s_{t+1}, a')$$

How to get the  
expected value?

## Idea 2: Temporal difference

- $Q(s, a)$  can be improved iteratively!

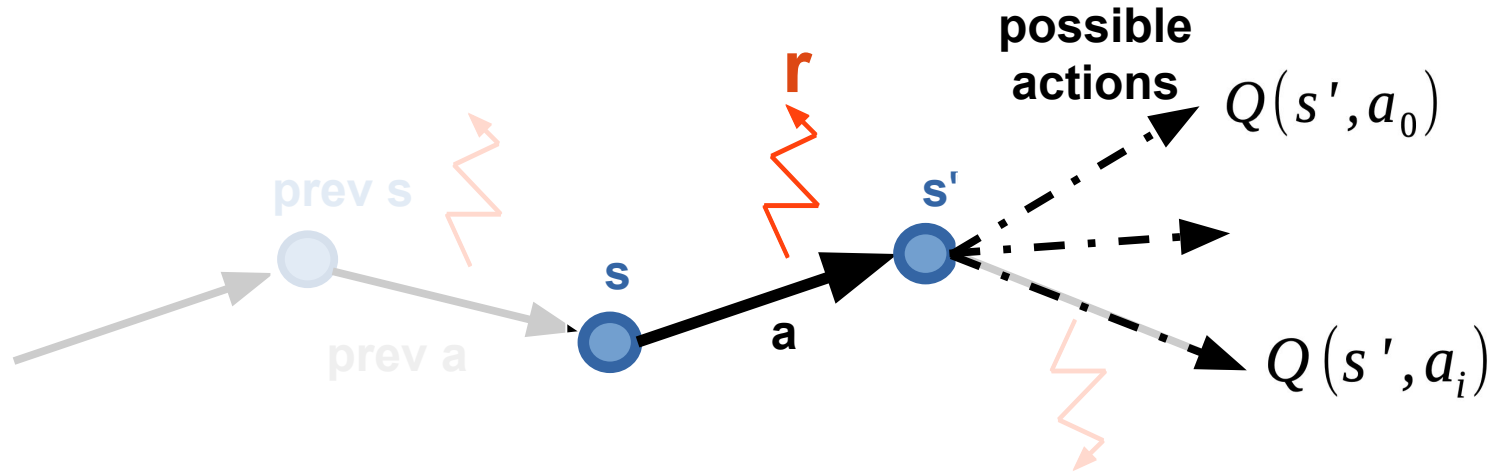
$$Q(s_t, a_t) \leftarrow E_{r_t, s_{t+1}} r_t + \gamma \cdot \max_{a'} Q(s_{t+1}, a')$$

$$E_{r_t, s_{t+1}} r_t + \gamma \cdot \max_{a'} Q(s_{t+1}, a') \approx \frac{1}{N} \sum_i r_i + \gamma \cdot \max_{a'} Q(s_i^{\text{next}}, a')$$

$$Q(s_t, a_t) \leftarrow \alpha \cdot (r_t + \gamma \cdot \max_{a'} Q(s_{t+1}, a')) + (1 - \alpha) Q(s_t, a_t)$$



# Q-learning



Initialize  $Q(s, a)$  with zeros

- Sample  $\langle s, a, r, s' \rangle$  from the environment
- Compute new  $Q(s, a)$  estimation:

$$\hat{Q}(s, a) = r(s, a) + \gamma \max_{a_i} Q(s', a_i)$$

- Update  $Q(s, a)$ :

$$Q(s, a) \leftarrow \alpha \cdot \hat{Q}(s, a) + (1 - \alpha) Q(s, a)$$

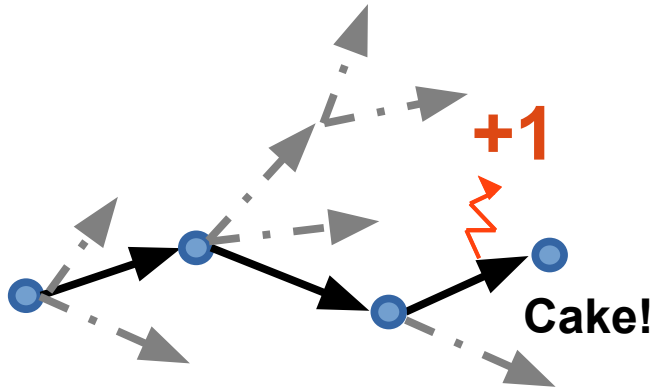
$$Q^*(s, a) = E_{s', r} [r(s, a) + \gamma \cdot V^*(s')]$$

$$Q(s_t, a_t) \leftarrow \alpha \cdot (r_t + \gamma \cdot \max_{a'} Q(s_{t+1}, a')) + (1 - \alpha) Q(s_t, a_t)$$

$$\pi(s) : \operatorname{argmax}_a Q(s, a)$$

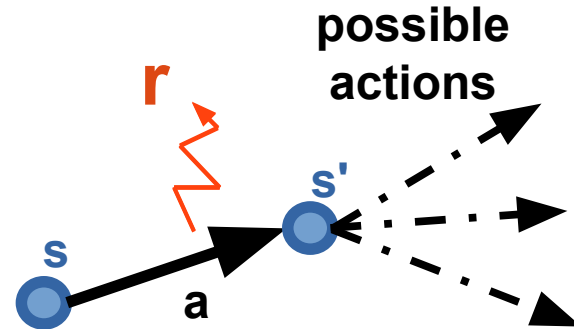
## Monte-carlo

- Averages Q over sampled paths



## Temporal Difference

- Uses recurrent formula for Q



## Monte-carlo

- Averages  $Q$  over sampled paths
- Needs full trajectory to learn
- Less reliant on Markov property

## Temporal Difference

- Uses recurrent formula for  $Q$
- Learns from partial trajectories
- Works with infinite MDP
- Requires less experience to learn

$$Q^*(s, a) = E_{s', r} [r(s, a) + \gamma \cdot V^*(s')]$$

$$Q(s_t, a_t) \leftarrow \alpha \cdot (r_t + \gamma \cdot \max_{a'} Q(s_{t+1}, a')) + (1 - \alpha) Q(s_t, a_t)$$

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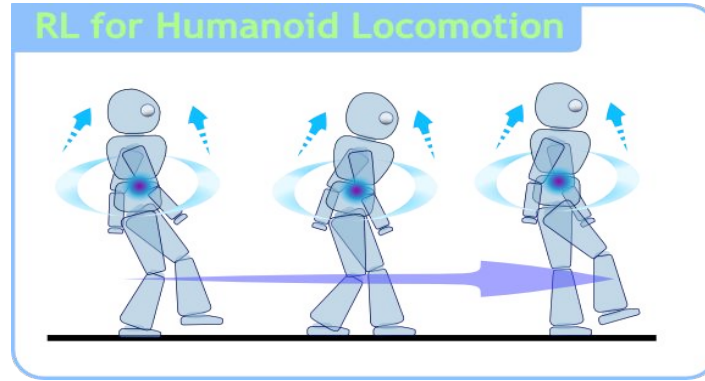
# Exploration and exploitation

What if agent is greedy and it always selects the “best” (according to the Q-value) action?

It will not be able to find better actions!

# Q-learning: problems

Imagine a robot learning to walk



Initial  $Q(s,a)$  are zeros

Robot uses  $\operatorname{argmax} Q(s,a)$

It has just learned to crawl with positive reward

**Now it has no chance to learn how to walk**

# Exploration-exploitation tradeoff

Two potential approaches (for now):

- $\epsilon$ -greedy policy:
  - Select random action with  $\epsilon$  probability, otherwise select best according to the current Q-function
- Softmax:
  - Sample action from a distribution generated by softmax normalization of Q-values:

$$\pi(a|s) = \text{softmax}\left(\frac{Q(s, a)}{\tau}\right)$$



# Example: Cliff-world

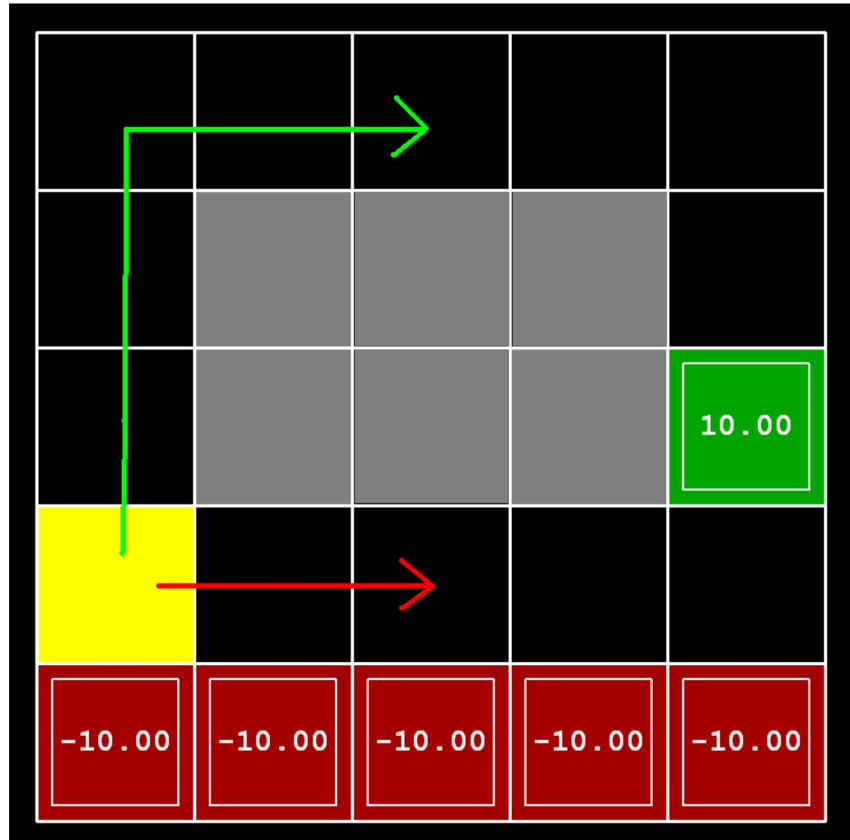
No stochasticity in the environment,  $\epsilon$ -greedy policy

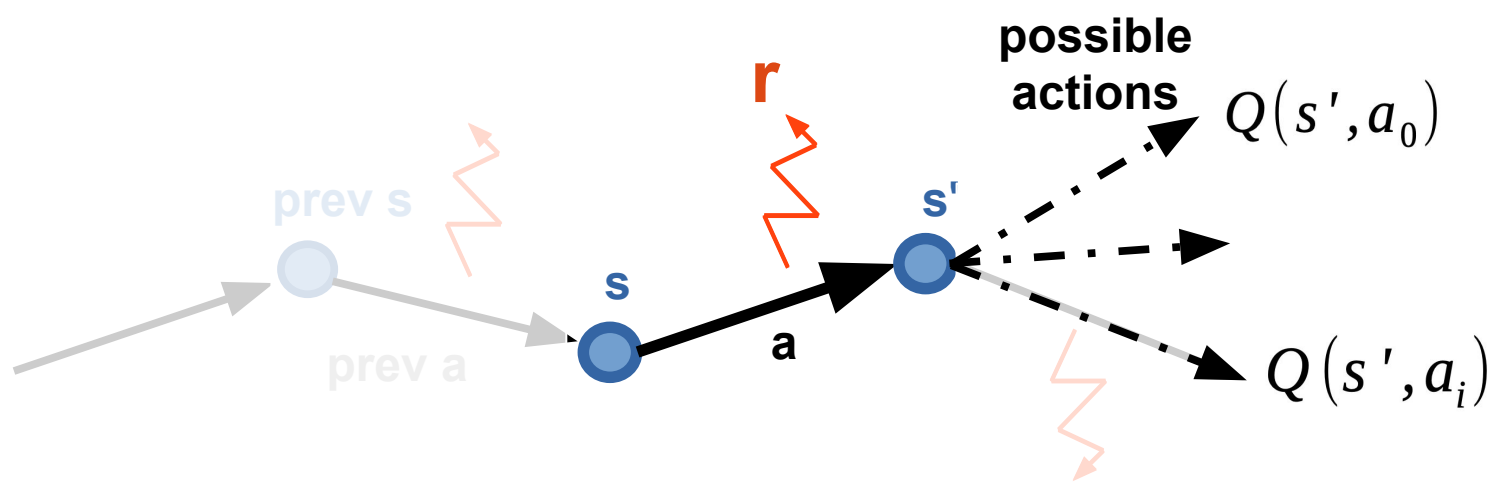
Let  $\epsilon = 0.15$ ,  $\gamma = 0.99$

Which trajectory will Q-learning prefer?

**The red one!**

Despite it will not maximize the reward (due to  $\epsilon$ -greedy behaviour)





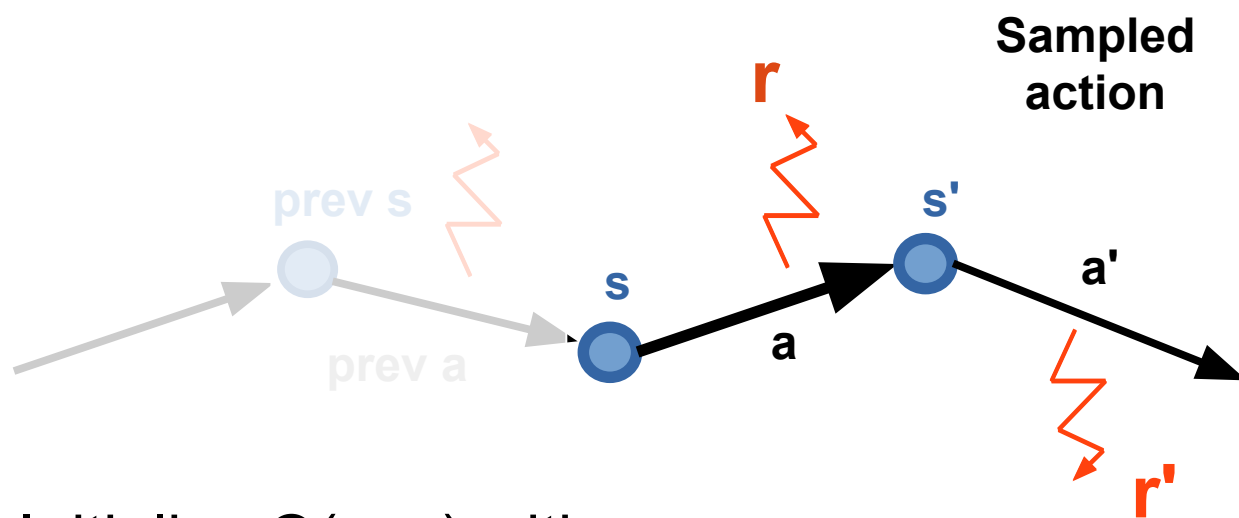
Initialize  $Q(s, a)$  with zeros

- Sample  $\langle s, a, r, s' \rangle$  from the environment
- Compute new  $Q(s, a)$  estimation:

$$\hat{Q}(s, a) = r(s, a) + \gamma \max_{a_i} Q(s', a_i)$$

- Update  $Q(s, a)$ :

$$Q(s, a) \leftarrow \alpha \cdot \hat{Q}(s, a) + (1 - \alpha) Q(s, a)$$



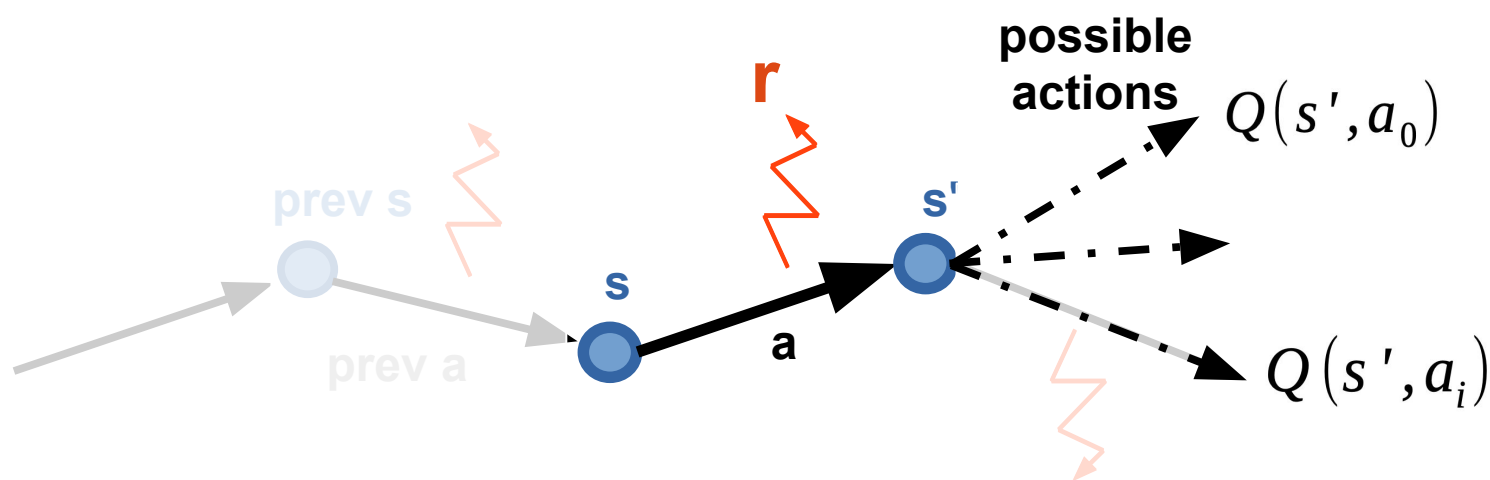
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Initialize  $Q(s, a)$  with zeros

- Sample  $\langle s, a, r, s' \rangle$  from the environment
- Compute new  $Q(s, a)$  estimation:

- Update  $Q(s, a)$ :
 
$$\hat{Q}(s, a) = r(s, a) + \gamma \underset{a_i \sim \pi(a|s')}{E} Q(s', a_i)$$

$$Q(s, a) \leftarrow \alpha \cdot \hat{Q}(s, a) + (1 - \alpha) Q(s, a)$$

## On-policy

- Agent trains on experience generated with its own policy
- Can't learn off-policy

Examples:

- Cross-entropy method
- SARSA

## Off-policy

- Agent trains on any kind of experience
- Can still learn on-policy

Examples:

- Q-learning
- EV-SARSA

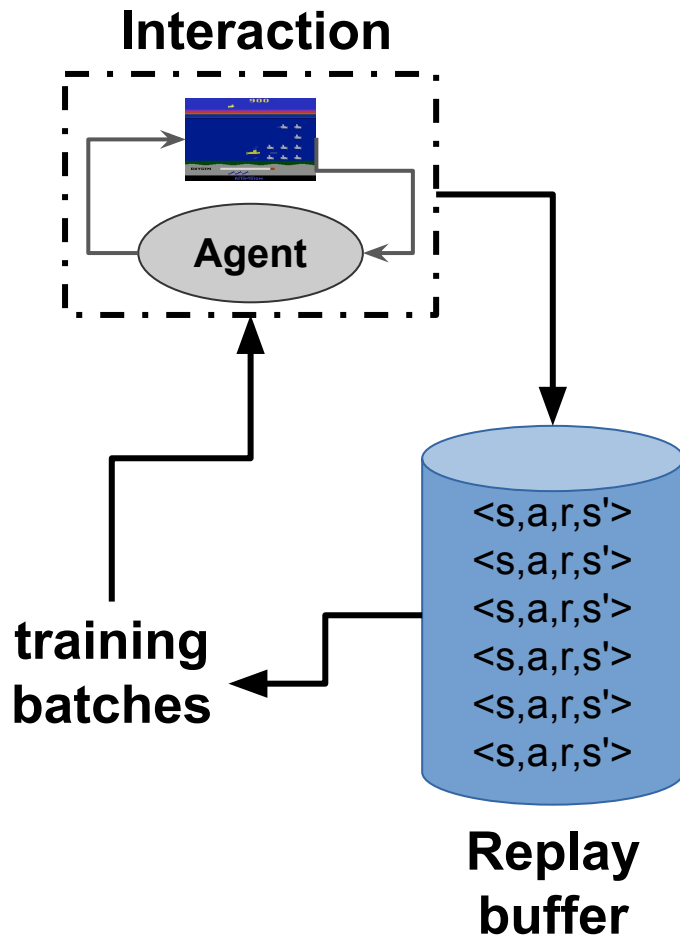
## Idea:

Store several past  
interactions

$\langle s, a, r, s' \rangle$

Train on random  
subsamples

## Experience replay



## Idea:

Store several past interactions

$\langle s, a, r, s' \rangle$

Train on random subsamples

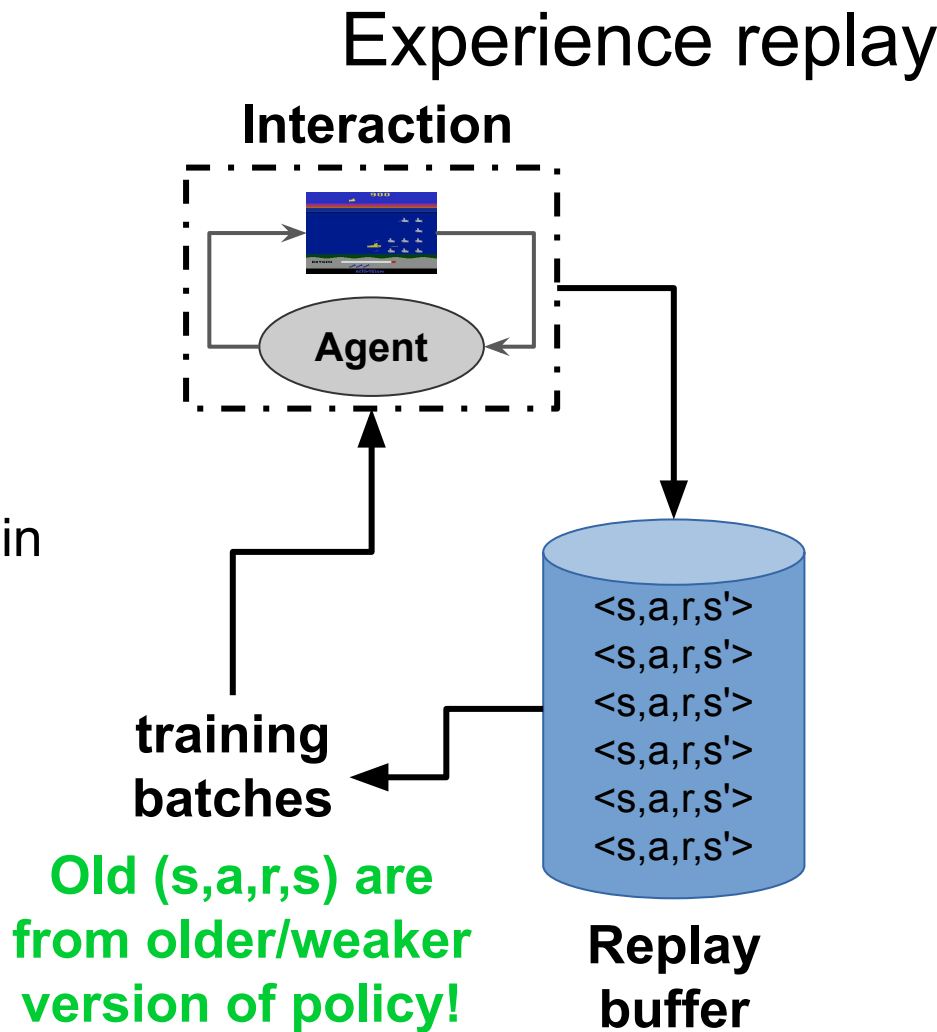
## Training curriculum:

- Play 1 step and record it
- Pick N random transitions to train

## Profit:

you don't need to revisit same  $(s, a)$   
many times to learn it.

**Only works with  
off-policy algorithms!**



- Q-learning allows to learn some approximation of the reward function and the environment model
  - So we can use it to solve the desired problem
- Remember what  $Q(s, a)$  and  $V(s)$  functions do
- Remember both about exploration and exploitation
  - At least using greedy policy or softmax smoothing
- Remember the difference between on-policy and off-policy algorithms!