# Codebook for:

# Toolkit for measuring quality of life under spatial frictions [*]

Gabriel M. Ahlfeldt [†] Fabian Bald [‡] Duncan H.W. Roth [§] Tobias Seidel [¶]

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**Abstract**

This codebook is part of the toolkit for measuring quality of life under spatial frictions (`ABRSQOL`). The toolkit implements a numerical solution algorithm to invert unobserved quality of life from observed data based on Ahlfeldt, Bald, Roth, Seidel (2024): Measuring quality of life under spatial frictions. This codebook summarizes the relevant primitives and endogenous objects of the models and introduces the algorithm in pseudo-code. The focus of the toolkit is on the measurement of quality of life, exclusively.

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[†]Humboldt University Berlin, CEPR & CESifo. Email: g.ahlfeldt@hu-berlin.de

[‡]European University Viadrina Frankfurt (Oder), CRED. Email: Bald@europa-uni.de

[§]Institute for Employment Research (IAB) & IZA. Email: duncan.roth@iab.de

[¶]University of Duisburg-Essen, CESifo & CRED. Email: tobias.seidel@uni-due.de

# 1 Objects

Table 1 introduces the objects used in the description of the the numerical solution algorithm in Section 2. We use the same notations as in Ahlfeldt et al. (2024), but only cover a subset of objects relevant to the algorithmic implementation.

Table 1: Relevant objects

| Object | Description |
|---|---|
| | Structural parameters |
| $\alpha$ | Expenditure share on land |
| $\beta$ | Share of tradables in non-housing consumption expenditure |
| $\gamma$ | Idiosyncratic taste parameter (inverse labour supply elasticity) |
| $\xi$ | Hometown valuation |
| | Exogenous characteristics |
| $A_i$ | Quality of life |
| $\bar{L}_i^b$ | Hometown population |
| $\bar{L}$ | Aggregate labour endowment across all regions |
| | Endogenous variables and scalars |
| $L_i$ | Residence population (and employment) |
| $\mathcal{L}_i$ | Adjustment factor capturing effect of hometown population on measurement of $A_i$ |
| $\Psi_i^b$ | Utility discount associated with having left the hometown introduced in the main paper in the context of location choice probabilities |
| $P_i^t$ | Tradable goods price |
| $p_i^n$ | local services price |
| $p_i^H$ | Floor space price |
| $w$ | Wage |

# 2 Algorithm

In the quantitative spatial model by (Ahlfeldt et al., 2024), the economy consists of a set of cities $i, j \in J$ and is populated by $\bar{L} = \sum_m \bar{L}_m^b$ mobile workers who grew up in different hometowns $m \in J$. The model can be used to invert quality of life in the presence of spatial frictions. Relative QoL $\hat{A}$ is perfectly identified up to a normalisation (or numéraire location) and given by:

$$\hat{A}_i = \frac{(\hat{P}_i^t)^{\alpha\beta}(\hat{p}_i^n)^{\alpha(1-\beta)}(\hat{p}_i^H)^{1-\alpha}}{\hat{w}_i}\left(\hat{L}_i/\hat{\mathcal{L}}_i\right)^{\frac{1}{\gamma}}, \tag{1}$$

where we define $\mathcal{L}_i \equiv (\exp[\xi] - 1)\, \Psi_i^b \bar{L}_i^b + \sum_{m \in J} \Psi_m^b \bar{L}_m^b$.

In Algorithm 1 we use pseudo-code to outline how a dataset with observed endogenous variables, $\{L_i, p_i^H, p_i^n, P_i^t, w_i\}$, the exogenous variable, $\bar{L}_i^b$, as well as structural parameters $\{\alpha, \beta, \gamma, \xi\}$ can be used to identify QoL under mobility (local ties, idiosyncratic tastes) and trade frictions (variation in tradable goods and non-tradable service prices).

In the algorithm, we refer to the following residential choice probability:

$$\lambda_i = \sum_m \lambda_{im} = \frac{(A_i w_i/\mathcal{P}_i)^\gamma}{\sum_{j \in J} (A_j w_j/\mathcal{P}_j)^\gamma} \left( \sum_{m \neq i} \Psi_m^b \bar{L}_m^b + \Psi_i^b \cdot \exp[\xi] \bar{L}_i^b \right) / \bar{L}, \tag{2}$$

with $\Psi_m^b = \left(1 + \frac{(\exp[\xi]-1)(A_m w_m/\mathcal{P}_m)^\gamma}{\sum_{j \in J}(A_j w_j/\mathcal{P}_j)^\gamma}\right)^{-1} < 1$ the utility discount associated with having left the hometown. The probability of residing in $i$ increases in QoL and nominal wages and declines in the aggregate consumer price index. Due to local ties, the residential choice probability also depends on the distribution of the hometown population. For given hometown population, the probability of living in $i$ increases in $\xi$, owing to an increasing value of local ties, $\Psi_i^b \cdot \exp[\xi] > 1$. The number of workers residing in $i$ is $L_i = \lambda_i \bar{L}$.[1]

[1] Note that if the utility gain from living in one's hometown is negligible (e.g. $\exp(\xi) \to 1$), there is no discount associated with living elsewhere, such that $\left(\sum_{m \neq i} \Psi_k^b L_m^b + \left(\Psi_i^b \cdot \exp[\xi]\right) \bar{L}_i^b\right) \to \bar{L}$.

**Algorithm 1:** Numerical solution algorithm to invert QoL

Start with values for variables $\{L_i, \bar{L}_i^b, p_i^H, p_i^n, P_i^t, w_i\}$ and structural parameters $\{\alpha, \beta, \gamma, \xi\}$

Normalise employment and birth-place population data, such that $\bar{L} = \sum_{i\in J} L_i = \sum_{i\in J} \bar{L}_i^b$

Calculate relative values of all variables with respect to a numéraire location (e.g. in "hat-algebra"): $\hat{\mathcal{V}} = \{\hat{L}, \hat{\bar{L}}^b, \hat{p}^H, \hat{p}^n, \hat{P}^t, \hat{w}\}$

Solve for the aggregate consumer price index $\mathcal{P}_i \equiv \left(P_i^t\right)^{\alpha\beta} (p_i^n)^{\alpha(1-\beta)} (p_i^H)^{1-\alpha}$.

Set convergence parameter $\kappa \in (0,1)$

Set precision rule to govern deviation between guesses and model solution.

Set $count = 1$

Guess values of $A_i$ and normalise relative to numéraire location

**while** $count < maxiter$ **do**

  Solve for $\Psi_i^b$ by using its definition below Eq. (2), such that $\Psi_i^b = \left(1 + \frac{(\exp[\xi]-1)(A_i w_i/\mathcal{P}_i)^\gamma}{\sum_{j\in J}(A_j w_j/\mathcal{P}_j)^\gamma}\right)^{-1}$

  Solve for $\mathcal{L}_i$ by using its definition below Eq. (1), such that $\mathcal{L}_i \equiv (\exp[\xi] - 1)\, \Psi_i^b \bar{L}_i^b + \sum_{m\in J} \Psi_m^b \bar{L}_m^b$

  Solve for $\hat{\mathcal{L}}$.

  Solve for relative QoL, $\hat{A}^{\text{new}}$, by using Eq. (1).

  Check deviation between guesses and model solution

$$target = round(abs(\hat{A}^{\text{new}} - \hat{A}), precision)$$

  **if** $target == 0$ **then**

    break;

  **else**

    Update initial guesses or updated values of $\hat{A}$:

$$\hat{A}^{\text{up}} = A^{\text{up}} = \kappa \cdot \hat{A}^{\text{new}} + (1-\kappa) \cdot \hat{A}$$

    Use updated values and re-iterate

$$A_i = \hat{A} = \hat{A}^{\text{up}}$$

**Result:** Equilibrium values of $\hat{A}$

## References

**Ahlfeldt, Gabriel M., Fabian Bald, Duncan Roth, and Tobias Seidel**, “Measuring quality of life under spatial frictions,” *WP*, 2024.