Topic 5
Preference heterogeneity

Gabriel M Ahlfeldt

Economic Geography

University of Barcelona
2025

Introduction

Motivation

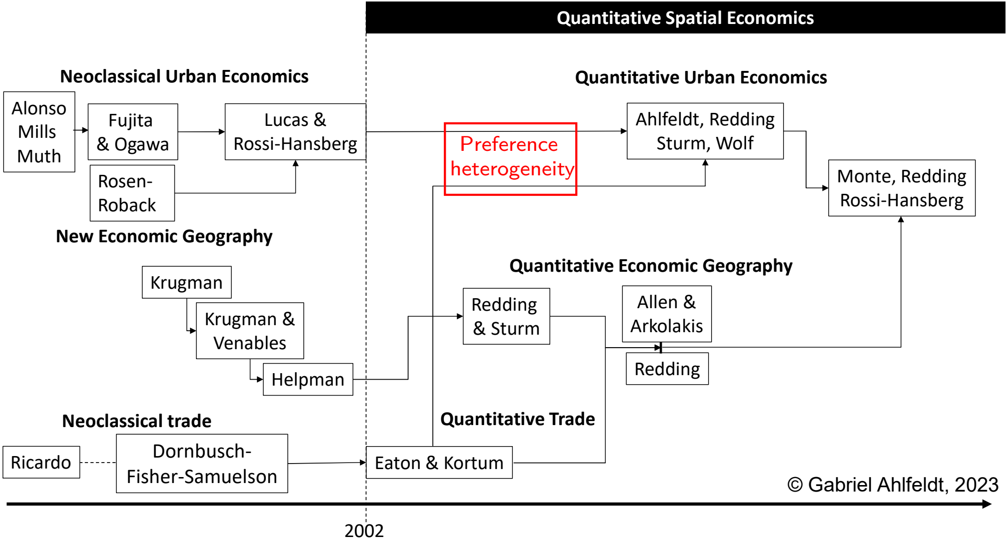
- So far, individuals have been **homogenous**
  - All workers value place-specific amenities the same
  - Infinite supply of workers to any location for a given combination of
    - Wages
    - Prices (including rents)
    - Place attributes
- In QSMs, we (usually) add **idiosyncratic tastes for location**
  - Drawn from extreme value distributions

What do idiosyncratic tastes capture and why do they matter?

Roadmap

- **Review extreme-value distributions**
  - Fréchet and Gumble
- **Derive location choice probabilities**
  - Binary case
  - Multiple locations
- **Role of preference heterogeneity in the spatial general equilibrium**
  - Dispersion of idiosyncratic tastes affects how
    - differences in fundamentals lead to
    - differences in prices and quantities on labour and land markets

History of thought



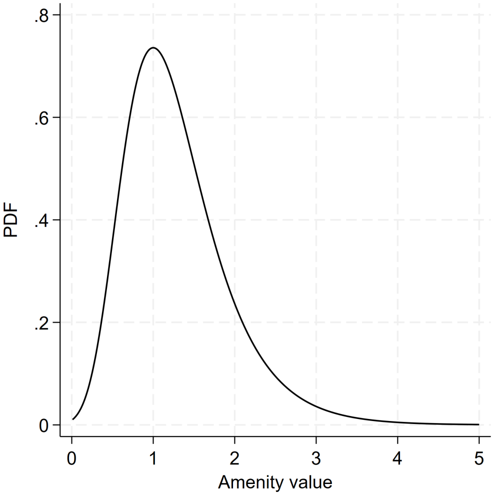
Extreme value distributions

Extreme value type I (Gumbel)

$$g(a) = \frac{1}{\beta}\exp\left(-\left[\frac{a-A}{\beta} + \exp\left(-\frac{a-A}{\beta}\right)\right]\right)$$

- We draw the value of the idiosyncratic amenity a from a distribution $g(a)$
  - First moment: $E(a) = A + \beta\Gamma = A + \left(\frac{1}{\varepsilon}\right)\Gamma$
  - Second moment: $\sigma_a^2 = \frac{\pi^2}{6}\beta^2 = \frac{\pi^2}{6}\left(\frac{1}{\varepsilon}\right)^2$
    - where $\varepsilon \equiv 1/\beta$ and Γ is the Euler–Mascheroni constant

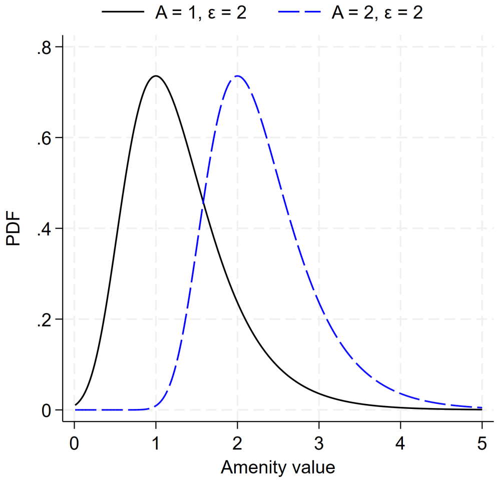
What happens if we increase A or ε?
What does this mean intuitively?



Extreme value type I (Gumbel)

$$g(a) = \frac{1}{\beta} \exp\left(-\left[\frac{a-A}{\beta} + \exp\left(-\frac{a-A}{\beta}\right)\right]\right)$$

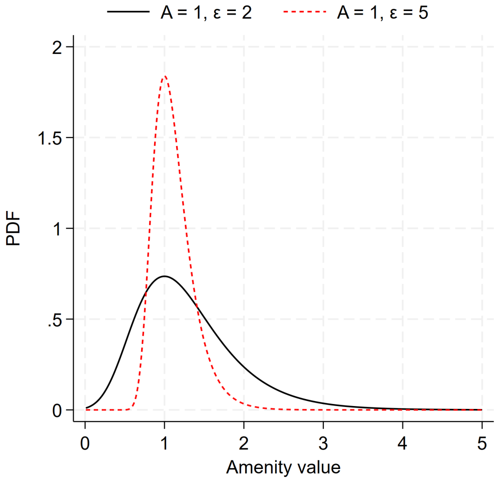
- A governs the first moment
  - E.g. the average idiosyncratic amenity at a location
- **Increasing A implies a higher amenity, on average**



Extreme value type I (Gumbel)

$$g(a) = \frac{1}{\beta} \exp\left(-\left[\frac{a-A}{\beta} + \exp\left(-\frac{a-A}{\beta}\right)\right]\right)$$

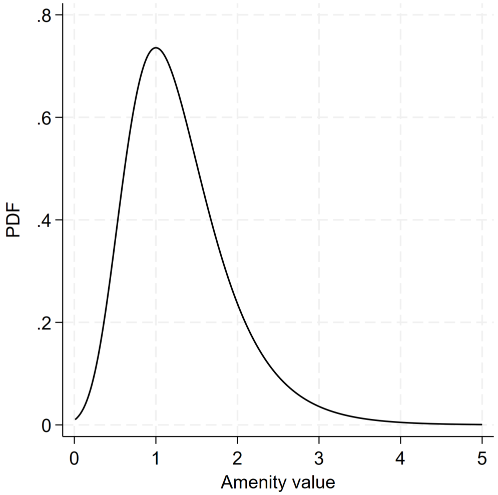
- ε affects first and second moments
  - the average idiosyncratic amenity
  - the dispersion
- **Increasing ε makes workers more similar in tastes**
  - less dispersion in tastes



Extreme value type II (Fréchet)

$$f(a) = \frac{\varepsilon}{A}\left(\frac{a}{A}\right)^{-1-\varepsilon} \exp\left(-\left(\frac{a}{A}\right)^{-\varepsilon}\right)$$

- We draw the value of the idiosyncratic amenity a from a distribution $f(a)$
  - First moment: $E(a) = A\Gamma\left(1 - \frac{1}{\varepsilon}\right)$
  - Second moment: $\sigma_a = A^2\Gamma\left(1 - \frac{2}{\varepsilon}\right) - \left(\Gamma\left(1 - \frac{1}{\varepsilon}\right)\right)^2$
    - where Γ is the Euler–Mascheroni constant
- Comparative statics similar to Gumbel
  - Larger $A \Rightarrow$ greater $E(a)$
  - Larger $\varepsilon \Rightarrow$ smaller σ_a
  - Though A and ε affect $E(a)$ and σ_a



Lessons for inversion

- **Recall**
  - QSM has parameters and fundamentals
  - For given parameters we invert first moment of fundamental (e.g. average amenity)
- **Notice**
  - First moment depends on structural paramater ε
  - Under Gumbel and Fréchet
- **Remember**
  - Fundamental amenity recovered is always specific to the chosen value of ϵ
  - Therefore, cannot change ϵ without changing A

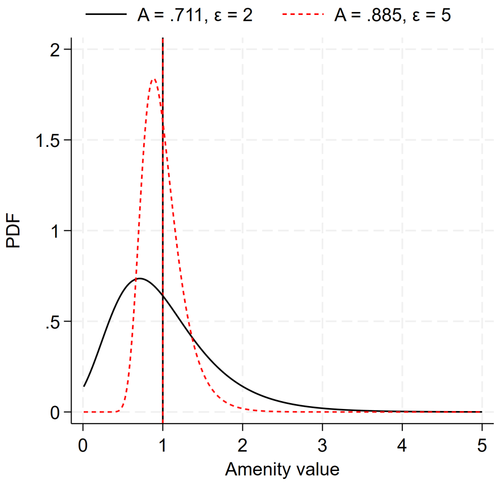
Can do counterfactuals under different values of ε.
But predicting what happens when ε changes is not straightforward

Inverting A (Gumbel)

$$A = E(a) - \frac{1}{\varepsilon}\Gamma$$

- We re-arrange first moment to find A
  - Can rationalize any given $E(a)$
- Say, we want $E(a) = 1$
  - We set $A = 0.711$ if $\varepsilon = 2$
  - We set $A = 0.885$ if $\varepsilon = 3$
- Principle is the same with Fréchet
  - Just with a different formula

$$A = \frac{E(a)}{\Gamma\left(1-\frac{1}{\epsilon}\right)}$$



Location choice probabilities

Discrete choice

- We want to move **beyond perfect mobility**
  - infinite supply of homogeneous workers
- Preference heterogeneity generates **well-behaved location choice probabilities**
  - Some people will **always choose a location**, even if real wage and amenity are low
    - But it could be a very small fraction of the population (with extreme preferences)
  - A **better location attracts a greater share** of the worker endowment, $\bar{L}$
    - Better in terms of higher wages, lower rents, lower prices, higher amenities
- Application of McFadden's (1974) seminal work
  - Nobel Prize winner in 2000

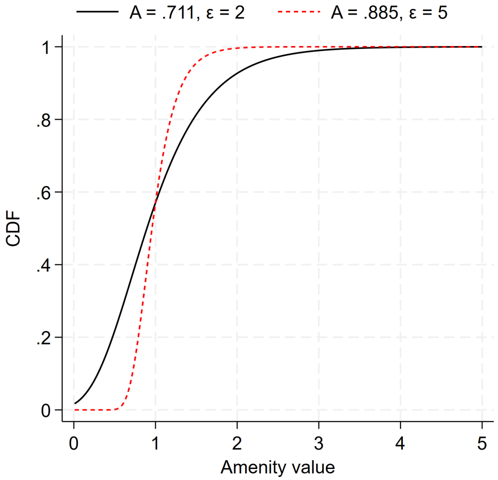
Let's develop the intuition in a simple binary choice setting: City vs. RoW

Cumulative density function (Gumbel)

$$G(a) = \exp\left(-\exp\left(-\left(a - A\right)\varepsilon\right)\right)$$

- CDF tells us the probability that a is equal or smaller than a certain value
  - Let a be idiosyncratic urban utility
  - About 60% chance that $a \leq 1$
- Outside option offering utility $\bar{U} = 1$
  - Probability of **not choosing city**: $Pr(a \leq \bar{U}) = G(\bar{U}) \approx 60\%$
  - Probability of **choosing city**: $Pr(a > \bar{U}) = 1 - G(\bar{U}) \approx 40\%$

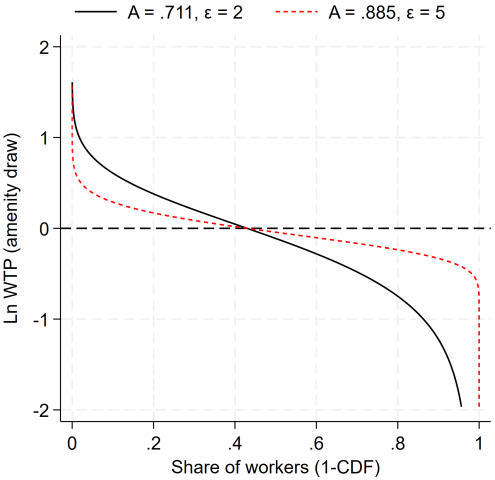
CDF gives us choice probabilities!



Demand for residence

- CDF implies **downward-sloping demand for residence**
  - Number of workers in the city is $L = \mu \bar{L}$, where $\mu = 1 - G(\bar{U})$

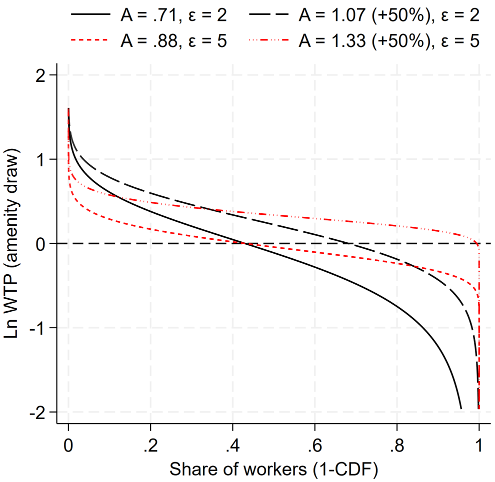
Valuation of the marginal resident falls the more workers enter the city!



Role of ε

- If we **increase** A, the residence demand curve shifts outwards
  - Average utility increases
  - **City attracts workers who enjoy urban life less relative to others**
- 50% increase in A leads to an increase in urbanization rate from ≈ to 40%
  - to ≈ 70% if $\epsilon = 2$
  - close to 100% if $\epsilon = 5$

ϵ governs population response in counterfactuals!



Multi-location setting I

- The logic extends to the setting with many locations (maths more involved)
  - **You will need to go through the intermediate steps at your own pace** (tutorial)
- Let's now assume workers indexed by o
  - can choose among many locations i
  - receive idiosyncratic utility according to Fréchet CDF $F(a_o) = \exp\left(-a_o^{-\epsilon}\right)$
  - obtain indirect utility $U = A_i a_{io} \frac{w_i}{p_i^{1-\alpha}}$
- The share of workers getting a utility of up to u at i is
  - $G_i(u) = Pr(U \leq u) = Pr\left(A_i a_{io} \frac{w_i}{p_i^{1-\alpha}} \leq u\right)$
- Using the CDF and the indirect utility function, we get
  - $Pr\left(a_{io} \leq \frac{p_i^{1-\alpha}}{w_i A_i} u\right) = F\left(\frac{p_i^{1-\alpha}}{w_i A_i} u\right) \Rightarrow G_i(u) = \exp\left(-\left(\frac{p^{1-\alpha}}{A_i w_i}\right)^{-\varepsilon} u^{-\varepsilon}\right)$

Multi-location setting II

- Want to know the **probability that workers choose** i, $\pi_i = Pr(u_i \geq \max\{u_r\} \forall i)$
  - Intuitively, this is the **probability that workers achieve a utility** u at i, $g_i(u)$, multiplied by the **probability that the maximum utility in any other location is less than or equal** to u, $\prod_{r \neq i} G_r(u)$, evaluated over all u:
  - $\pi_i = \int_0^\infty g_i(u) \left(\prod_{r \neq i} G_r(u) \right) du$
- Several steps later (using $G(u)$ from which we also derive $g(u)$...
  - **Location choice probability** $\mu_i = \frac{L_i}{L} = \frac{\left(\frac{A_i w_i}{p_i^{1-\alpha}} \right)^\epsilon}{\left(\sum_s \left(\frac{A_s w_s}{p_s^{1-\alpha}} \right)^\epsilon \right)}$

Eureka! ε is the labour supply elasticity to the city! (Moretti, 2010)

Role in general equilibrium

Implications for quality of life

- Recall **quality of life in Rosen-Roback framework** (Glaeser & Gottlieb, 2009)
  - $A_i = c\frac{p_i^\alpha}{w_i}$ (in GG2009 notations $\log(B_c) = \Omega_3 + \sigma \log(P_c) - \log(W_c)$)
- Solving the **location choice probability** equation for A_i, we get
  - $A_i = \underbrace{c\frac{p_i^\alpha}{w_i}}_{\text{RR-QoL}} L^{\frac{1}{\varepsilon}}$ Notice that if $\varepsilon \to \infty \Rightarrow L^{\frac{1}{\varepsilon}} \to 1$
- **QSM nests Rosen-Roback as special case** with $\varepsilon \to \infty$
  - By implication: If $\varepsilon < \infty$ and QSM is right, Rosen-Roback must give wrong QoL

Does RR over- or underestimate the urban QoL premium?

Ahlfeldt, Bald, Roth, Seidel toolkit

- **Toy version** of the full quantitative spatial model
  - Abstracts from trade costs and non-tradable sector
  - But offers **analytical solutions**!
- Get toolkit
  - GitHub repository: ▸ Open ABRS-toolkit
    - Contains the description of the toy model and intuition, **worth reading**
  - Install Stata ado file: `ssc install ABRS`
    - Creates the four-quadrant diagram
    - For the syntax, type: `help ABRS`
  - Javascript version: ▸ Open ABRS-toolkit (web-version)
    - Only bar chart showing effects on equilibrium outcomes

Preferences

- Worker living in location i derives utility from the consumption of goods ($Q_{i\omega}$) and floor space ($h_{i\omega}$) according to

$$U_{i\omega} = \left(\frac{Q_{i\omega}}{\alpha}\right)^{\alpha} \left(\frac{h_{i\omega}}{1-\alpha}\right)^{1-\alpha} \exp[a_{i\omega}], \tag{1}$$

  - where $Q_{i\omega}$ represents a final good (Armington, 1969)
    - locally assembled from tradable intermediate goods $q_{ji\omega}$
    - according to the CES-aggregator
    - at zero cost under perfect competition
    - shipped from origin j to destination i,

$$Q_{i\omega} = \left[\sum_{j \in J} (q_{ji\omega})^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{2}$$

Mobility frictions

- Idiosyncratic preference for location $\exp[a_{i\omega}^{\theta}]$ (Gumbel)

$$F_i^{\theta}(a) = \exp\left(-(A_i)^{\gamma^{\theta}} \exp\left\{-\left[\gamma a + \Gamma\right]\right\}\right) \text{ with } \gamma > 0, \tag{3}$$

- A_i (average preference) serves as an exogenous measure of local quality of life
  - Notice that **Gumbel CDF has been engineered** to give $A_i = \mathbb{E}(a|i)$
- γ governs the dispersion of individual amenity shocks
  - Introduces imperfect spatial arbitrage $\Rightarrow$
  - inverse measure of mobility frictions

Technology: Housing

- Supplied under **perfect competition** combining a share of the globally available capital stock, K_i (available at unit prices), with **fixed land supply**, $\bar{T}_i$:

$$H_i^S = \eta_i \left(\frac{\bar{T}_i}{\delta}\right)^{\delta} \left(\frac{K_i}{1-\delta}\right)^{1-\delta}. \tag{4}$$

- Total factor productivity η_i
  - captures the role of regulatory (e.g. height regulations) and physical (e.g. a rugged surface) **constraints** (Saiz 2010)
- δ governs the housing supply elasticity $\frac{\partial \ln H_i^S}{\partial \ln p_i^H} = \frac{1-\delta}{\delta}$ (inverse relationship)
  - Use first-order conditions in profit function

Technology: Non-housing goods

- **Each location produces a unique variety** of a tradable intermediate good
  - using labour L_i as the only production input
  - under perfect competition according to
- $q_i = \varphi_i L_i$
  - Endogenous labour productivity $\varphi_i = \bar{\varphi}_i L_i^{\zeta}$
    - increases in local employment according to the **agglomeration elasticity** ζ.
- We get price $p_{ji} = w_j/\varphi_j$ and trade share $\chi_{ji} = \frac{(w_j/\varphi_j)^{1-\sigma}}{\sum_k (w_k/\varphi_k)^{1-\sigma}}$
  - Trade in intermediate goods is free
  - Perfect competition equates prices to marginal costs

Location choice

- Probability λ_i^θ that a worker lives in location i:

$$\lambda_i = \frac{(A_i w_i / \mathcal{P}_i)^\gamma}{\sum_{j \in J} \left(A_j w_j^\theta / \mathcal{P}_j \right)^\gamma}, \tag{5}$$

  - where $\mathcal{P}_i \equiv (P_i)^\alpha (p_i^H)^{1-\alpha}$ is the aggregate consumer price index.
  - Mobility of workers equalizes expected utility in equilibrium

This is what we have derived before...

General equilibrium I

- **Goods market clearing** (GMC): $w_i L_i = \sum_{j \in J} \frac{(w_i/\varphi_i)^{1-\sigma}}{\sum_k (w_k/\varphi_k)^{1-\sigma}} \left(\alpha w_j L_j + r_j \bar{T}_j \right)$
  - Wage bill at i must equate to the revenues from all locations j
- **Labor market clearing** (LMC): $L_i = \lambda_i \bar{L}$
  - Labour input L_i must equate labour supply
    - product of the share of workers living in i, λ_i, and the national labour endowment $\bar{L}$.
- **Floor-space market clearing** (FMC): $p_i^H = \left(\frac{\tilde{\alpha} \delta w_i L_i}{\eta_i^{\frac{1}{\delta}} \bar{T}_i} \right)^{\delta}$,
    - where $\tilde{\alpha} \equiv (1-\alpha) + \alpha(1-\beta)(1-\delta)$ is a constant.
  - Set simply set $H_i^S = H_i^D$
    - H^D is Marshallian housing demand
    - $H_i^S = \eta^{\frac{1}{\delta}} \bar{T}_i (p_i^H)^{\frac{1-\delta}{\delta}} / \delta$ from first-order conditions in profit function

General equilibrium II

- For the toolkit, we consider **relative differences in a two-region case**
  - $\hat{x} = \frac{x_i}{x_j}$
- System of equations simplifies to
  - GMC: $\hat{L} = \hat{\bar{\varphi}}^{\frac{\sigma-1}{\Lambda}} \hat{w}^{-\frac{\sigma}{\Lambda}}$
  - LMC: $\hat{L} = \left(\hat{A}\hat{w}(\hat{p}^H)^{\alpha-1}\right)^{\gamma}$
  - FMC: $\hat{p}^H = \hat{k}\left(\hat{w}\hat{L}\right)^{\delta}$
    - where $\Lambda \equiv 1 - \zeta(\sigma - 1)$ and $k_i \equiv \frac{1}{\eta_i}\left(\frac{\tilde{\alpha}\delta}{\bar{T}_i}\right)^{\delta}$ collect various primitives

3 log-linear equations with 3 unkonwns $\{\hat{L}, \hat{w}, \hat{p}^H\} \Rightarrow$ **closed-form solution** ✓

Mapping

- We have **direct mapping from primitives to endogenous objects**

$$\hat{L} = \hat{k}^{-\frac{(1-\alpha)\sigma}{\Delta}} \hat{\bar{\varphi}}^{\frac{(\sigma-1)(1-(1-\alpha)\delta)}{\Delta}} \hat{A}^{\frac{\sigma}{\Delta}}$$

$$\hat{w} = \hat{k}^{\frac{(1-\alpha)\Lambda}{\Delta}} \hat{\bar{\varphi}}^{\frac{(\sigma-1)(1/\gamma+(1-\alpha)\delta)}{\Delta}} \hat{A}^{-\frac{\Lambda}{\Delta}}$$

$$\hat{p}^H = \hat{k}^{\frac{\Lambda+\sigma/\gamma}{\Delta}} \hat{\bar{\varphi}}^{\frac{(\sigma-1)(1+1/\gamma)\delta}{\Delta}} \hat{A}^{\frac{\delta(\sigma-\Lambda)}{\Delta}}$$

  - where $\Delta \equiv \Lambda[1-(1-\alpha)\delta] + \sigma[1/\gamma + (1-\alpha)\delta]$
- This mapping is all we need to do counterfactuals ▸ Open ABRS-toolkit (web-version)
  - change fundamentals $\{A, \varphi, \eta\}$

GE in four quadrants

- For **didactic purposes** it is useful to combine GMC, LMC, FMC to give

$$\hat{L} = \hat{k}^{-\frac{1-\alpha}{\frac{1}{\gamma}+(1-\alpha)\delta}} \cdot \hat{A}_1^{\frac{1}{\frac{1}{\gamma}+\delta(1-\alpha)}} \cdot \hat{w}^{\frac{1-\delta(1-\alpha)}{\frac{1}{\gamma}+\delta(1-\alpha)}}$$

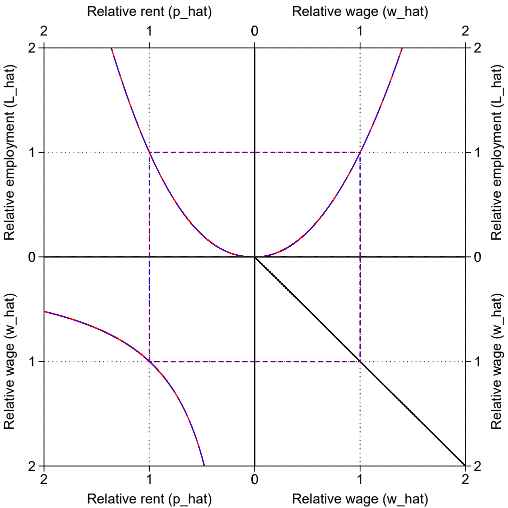
$$\hat{p}^H = \hat{k}^{\frac{1}{1-(1-\alpha)\delta}} \cdot \hat{A}_1^{-\frac{\delta}{1-\delta(1-\alpha)}} \cdot \hat{L}^{\frac{\delta\left(\frac{1}{\gamma}+1\right)}{1-(1-\alpha)\delta}}$$

$$\hat{w} = \left(\frac{\hat{p}^H}{\hat{k}}\right)^{\frac{\zeta(\sigma-1)-1}{\delta(\sigma-1)(\zeta+1)}} \cdot \hat{\varphi}^{\frac{1}{\zeta+1}}$$

  - Notice that we map $\hat{w}$ into $\hat{L}$, $\hat{L}$ into $\hat{p}^H$, and then $\hat{p}^H$ back into $\hat{w}$
- **A $\hat{w}$ that gets us back to the same $\hat{w}$ satisfies all equilibrium conditions**

ABRS "A_hat_1=0"

- Quadrant 1
  - Higher worker wage ⇒ greater employment due to **upward-sloping labour supply**
- Quadrant 2
  - Greater employment ⇒ higher rent due to greater housing demand & imperfectly elastic supply
- Quadrant 3
  - Higher rent implies larger employment ⇒ Firm wage must be lower given **downward-sloping labour demand**
- Quadrant 4
  - Projects firm wage onto worker wage



Questions I

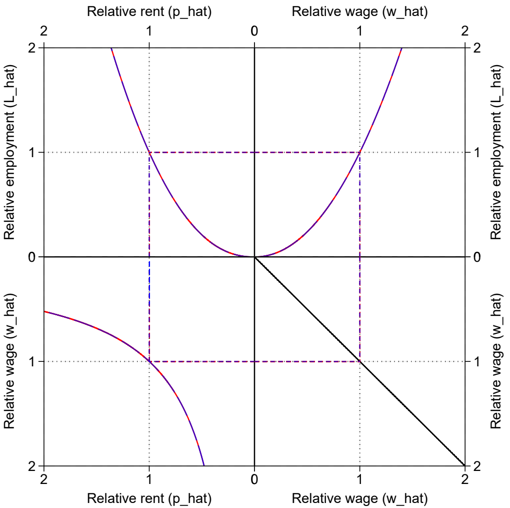
Q1: What happens if we set $\gamma = 1$ (instead of 3)

```
ABRS "A_hat_1=0" "gamma=1"
```

Q2: What happens if we set $\ln \hat{A} = 0.2$

```
ABRS
```

Q3: What happens if we set $\ln \hat{A} = 0.2$ and increase γ? ▸ ABRS-toolkit (web)



Questions II

Q4: What happens if we set $\ln \hat{A} = 0.2$ alone vs. $\ln \hat{A} = 0.2$ and $\ln \hat{\eta} = 0.8$?

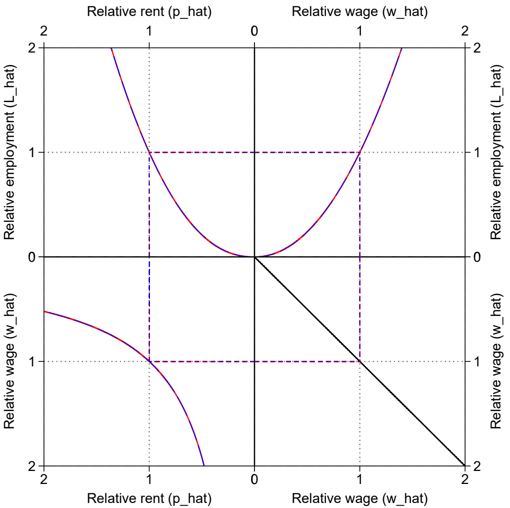
```
ABRS "A_hat_2=0.2" "eta_hat_2=0.8"
```

Q5: What happens if we set $\ln \hat{A} = 0.2$ alone vs. $\ln \hat{A} = 0.2$ and $\ln \hat{\varphi} = 0.8$?

```
ABRS "A_hat_2=0.2" "phibar_hat_2=0.8" "max_L_hat=4"
```

Q6: What happens if we keep $\ln \hat{A} = 0$ and set $\ln \hat{\eta} = 0.8$ and $\ln \hat{\varphi} = 0.8$?

```
ABRS "A_hat_1=0" "A_hat_2=0" "eta_hat_2=0.8" "phibar_hat_2=0.8"
"max_L_hat=4"
```



Conclusion

Summary

- With **extreme-value distributed location preferences**
  - Allows deriving discrete choice probabilities
  - Upward-sloping labor demand and downward-sloping housing demand
    - **More preference heterogeneity, less mobility** and steeper curves
- Important effects on general equilibrium See further comments on Q1-Q6 in Appendix
  - **Real wages no longer reflect QoL differences!** (unless γ is large)
    - Housing productivity increases real wages
    - Labour productivity increases real wages
  - **High real wages wrongly attributed to low QoL in RR**!

Next week: **The ARSW model**

Literature I

Core readings

- **Ahlfeldt, G., Bald, F., Roth, D., Seidel, T. (2024)**: Measuring quality of life under spatial frictions: Toy version of the model. https://github.com/Ahlfeldt/ABRS-toolkit
- **Ahlfedt, G., Redding, S., Sturm, D., Wolf, N. (2015): "Supplement to: The economics of Density: Evidence From the Berlin Wall" (Econometrica, 83(6))** , Sections S.2.1, S.2.2, S.2.3.
- **Moretti, E. (2010): Local labour markets. Handbook of Labor Economics, Volume 4b.**

Other readings

- Armington, P. (1969). A Theory of Demand for Products Distinguished by Place of Production. IMF Staff Papers, 16(1), 159-178.
- McFadden, D. (1974): Conditional Logit Analysis of Qualitative Choice Behavior. In P. Zarembka (Ed.), Frontiers in Econometrics (pp. 105-142). Academic Press
- Saiz, A. (2010): The Geographic Determinants of Housing Supply. The Quarterly Journal of Economics, 125(3), 1253–1296

Appendix

Comments on Q1-Q6

- Q1
  - Curve in Quadrant 1 flatter since labor supply is less elastic
    - Notice that axes are reversed relative to the standard demand-supply diagram
    - Flatter curve implies **less** elastic labour supply
- Q2
  - Curve in Quadrant 1 is steeper since greater labor supply for the same wage
    - Consequentially, wage lower
  - Curve in Quadrant 2 steeper since for the same employment, wage is lower $\Rightarrow$ housing demand lower
    - Rent higher since the positive employment effect dominates the negative wage effect on housing demand
  - Real wage lower by 0.09 log units, less than half of the QoL difference (0.2)
    - Rosen-Roback framework delivers wrong measurement of QoL

Comments on Q1-Q6

- Q3
  - The larger γ, the closer the inverse real wage difference gets to the QoL difference
    - At $\gamma = 10$, real wage is 0.15 lower; at $\gamma = 10$, real wage is 0.175 lower; at $\gamma = 65$, real wage is 0.19 lower; at $\gamma = 500$, real wage is 0.199 lower
- Q4
  - Curve in the second Quadrant is steeper since lower rent due to greater housing supply
  - Curve in the first Quadrant is steeper since lower rent implies that workers accept lower wage
    - For a given employment level
  - Curve in the third Quadrant shifts inwards since at the same rent there are more workers $\Rightarrow$ lower wage clears the market

Comments on Q1-Q6

- Q5
  - Curve in Quadrant 5 shifts outwards since
    - labor demand increases as workers are more productive
    - at same rent (employment), firms pay higher wages (perfect competition)
  - RR **qualitatively wrong** since real wage difference positive
    - (implying negative QoL differential)
- Q6
  - Effects in first and second Quadrants like in Q4 ($\hat{\eta}$ effect)
  - Shift effects of $\hat{\eta}$ and $\hat{\varphi}$ cancel out each other in Quadrant 3
    - Curve does not shift
  - RR **qualitatively wrong** since real wage difference positive
    - (implying negative QoL differential)