Topic 1
Rosen-Roback

Gabriel M Ahlfeldt

Quantitative Spatial Economics

Humboldt University & Berlin School of Economics
Summer term 2024

Acknowledgements

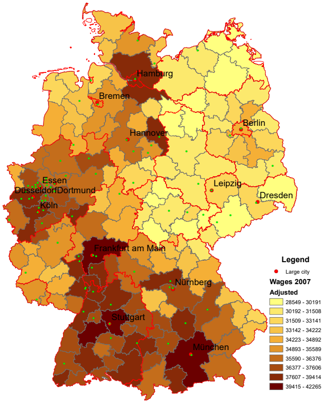
- This slide deck
  - builds on the lecture "spatial equilibrium across cities" by Tuukka Saarimaa (2021)
  - builds on the lecture "Rosen-Roback Framework" by Felipe Carozzi (2023)
  - uses material from Brueckner's textbook Chapter 11

Introduction

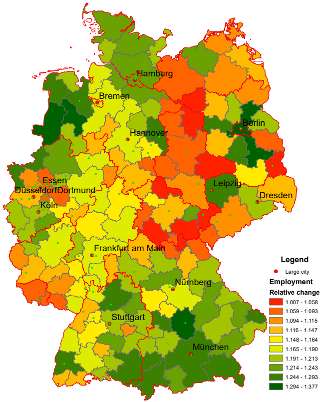
Wage and price differences in Germany

Does **migration equalize** wages and rents across regions?

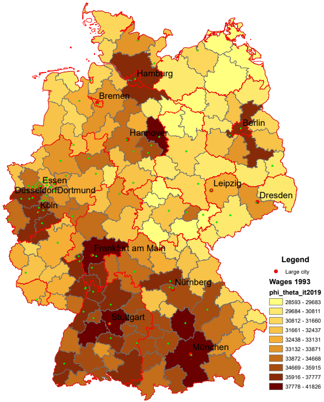
Wage differences in Germany



(a) Wage
2007

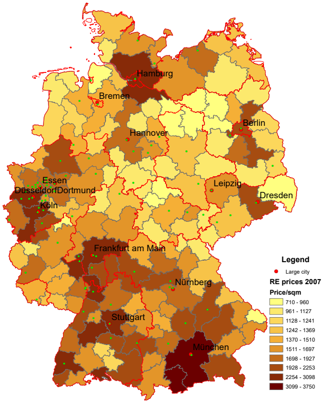


(b) Employment change
2007-2019

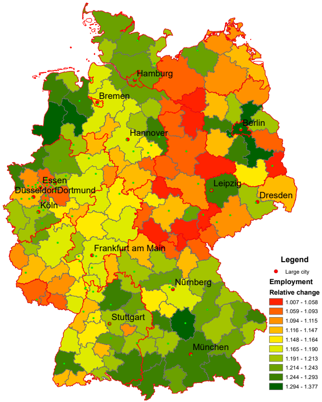


(c) Wage
2019

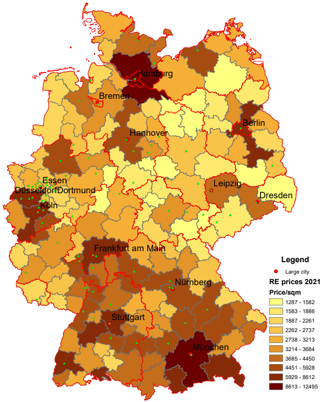
Property price differences in Germany



(a) Price/sqm 2007



(b) Employment change 2007-2019



(c) Price/sqm 2021

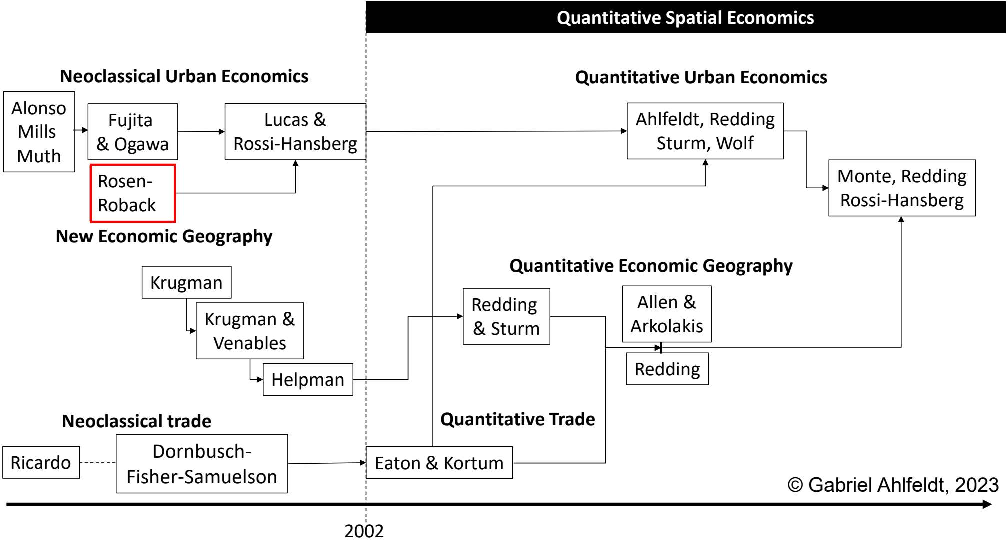
Wage and price differences in Germany

If one city is an **attractive place to live**, will it have high or low real wages?

Roadmap

- We analyze the **tradeoff** between income, amenities, and housing costs
  - across cities or regions
- We introduce the **spatial equilibrium** assumption
  - Identical utility levels across regions
  - Identical firm profits across regions
- **Rosen-Roback framework** is a core concept in urban economics
  - Name refers to Rosen (1979) and Roback (1982)
  - Refers to a family of models that build on the spatial equilibrium assumption
- Workhorse tool for valuing non-marketed goods

History of thought



Roback (1982)

- A famous paper by Jennifer Roback's 1980 PhD thesis
  - over 4700 Google Scholar citations (01/2024)
- **Not only important to urban economics**, but also to
  - Labor economics (local labor markets)
  - Environmental economics (pricing of environmental amenities)
  - Trade (markets and industry concentration)
  - Development economics (migration)
  - Local public finance

Basic assumptions

- The basic framework for the analysis is a simple **general equilibrium model**
  - capital and labor are assumed **completely mobile** across cities.
    - Complete mobility of labor means that the costs of changing residences are zero
    - Workers are homogenous, no idiosyncratic tastes for locations
  - **Intercity commuting costs** are assumed **prohibitive**
    - Workers live and work in the same local labour market
  - **Intracity commuting costs** are **zero**
    - focus attention on the across-city allocation of workers and firms
  - **Floor space supply is fixed** within cities
    - but is assumed interchangeable between uses within a city

Amenities

- **Consumer utility** depended on the consumption of
  - a composite commodity c
    - A basket of freely tradable goods
  - and housing q
- Consumers or workers also get utility from **urban amenities**
  - Different cities have different **exogenous amenities**
    - E.g., pleasant climate, nature, clean air, low crime
    - Amenities may be endogenous and depend on city's population
- **Quality of life** is an index of many different amenities (or disamenities)
  - we will denote it by a
  - akin to TFP shifter in production function

Utility function and dependence

- **Utility** depends on consumption of marketed (c, q) and non-marketed (a) goods

$$u = u(c, q, a)$$

- Utility is maximized subject to the **budget constraint** $y = c + pq$,
  - where the wage y is the sole source of income, the unit rent of housing is p and the commodity price is the numeraire (normalized to one)
- Using the first-order conditions, we obtain the **indirect utility**

$$V = V(y, p, a)$$

  - Naturally, we have $\frac{\partial V}{\partial y} > 0$, $\frac{\partial V}{\partial p} < 0$, $\frac{\partial V}{\partial a} > 0$
    - We view a as quality of life (a composite amenity)
    - a could also be a disamenity, in which case, of course, $\frac{\partial V}{\partial a} < 0$

Spatial equilibrium

- In equilibrium, workers must be **indifferent across locations**
  - If not, workers would **move to locations offering higher utility**
  - This will lead to **bidding up housing prices** or **pushing down wages**
  - The process will only stop when utilities are equalized everywhere
- Formally, this implies that the following condition must hold for all cities

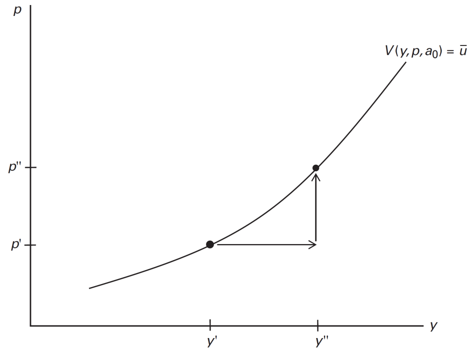
$$V(y, p, a) = \bar{u}$$

  - For a given a, the indifference condition will only be satisfied if the effects of **differences in p and w across cities offset each other**

Spatial equilibrium is a core concept of spatial economics

Indifference curves are upward sloping

- Consider the point (p', y')
  - The bottom-left point
- Suppose that wage increases from y' to y''
  - Moving to the right
  - This change would raise utility
- Rent must increase
  - to keep utility constant
  - Moving up from p' to p''

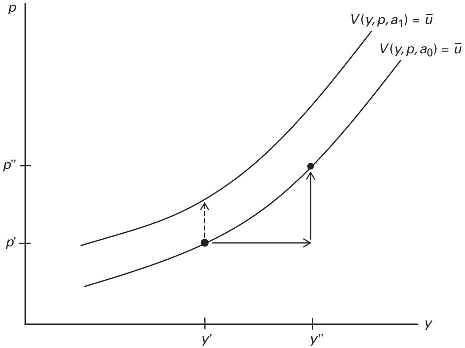


Q: How does the indifference curve shift if a increases?

Indifference curves with different amenity levels

- Now consider another city with amenity level $a_1 > a_0$
- Ceteris paribus, V would increase since $\frac{\partial V}{\partial a} > 0$
- To ensure $V = \bar{u}$ in both cities, we require that:
  - $p_1 > p_0$
  - $y_1 < y_0$
  - or a combination of both

Compensating differential



Firms

- Firms produce the non-housing commodity c using the **inputs**
  - **Labour**, hired at wage y
  - **Floor space** (of factories and offices), rented at rent p
    - One integrated floor space market, no land use frictions
  - Other inputs can be ignored, as long as their price is the same in all regions (capital)
- Constant returns to scale
  - Cost per unit of output is the same no matter how much is being produced
- **Cost function** $C = C(y, p, a)$ also **depends on** a, with $\frac{\partial C}{\partial y} > 0$, $\frac{\partial C}{\partial p} > 0$, $\frac{\partial C}{\partial a} \lesseqgtr 0$
  - An amenity to workers may be an amenity to firms (low crime)
  - May be a disamenity to firms (regulation of pollution)

Spatial equilibrium (again)

- **Perfect competition and mobility** ensure that firms make **zero profits**
  - If a city offers greater profits it will attract firms
  - This will increase factor prices (real estate, labor)
  - Until **zero-profit equilibrium** is restored
- Since the price of c is the numeraire ($=1$), the **spatial equilibrium condition** is:

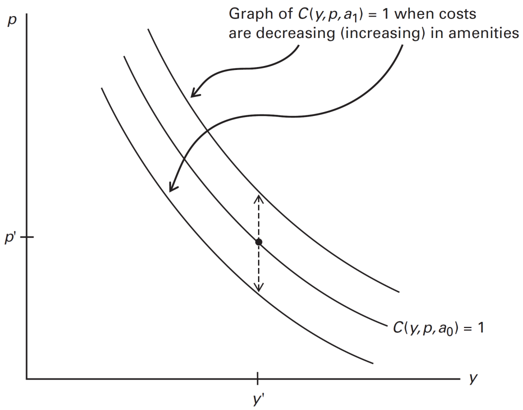
$$C(y, p, a) = 1$$

  - Just saying that the unit cost C must equal the price of c
- This condition generates **iso-profit curves** in the p and y space
  - Combinations of p and y that yield zero profits for a given level of a

Does a shift iso-profit curve up or down?

Iso-profit Curves

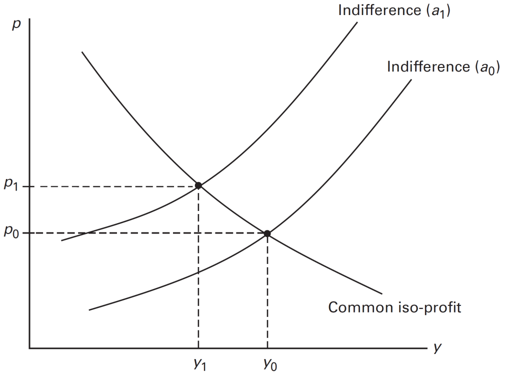
- Iso-profit curves are downward-sloping
  - For a given a, a higher y must be compensated by a lower p to keep unit cost constant
- If the amenity lowers costs, $a_1 > a_0$:
  - $p_1 > p_0$
  - $y_1 > y_0$
  - or a combination of both
- If the amenity increases costs
  - compensating differentials will point in opposite direction



Comparative statics: No amenity effect on firm cost

- **Equilibrium** must satisfy
  - indifference condition
  - iso-profit condition
- Holds only for the **intersection point**
  - Gives market-clearing p and y
- $a_1 > a_0$, shifts only indifference curve
  - No effect on firm costs
- Move uphill along the iso-profit curve

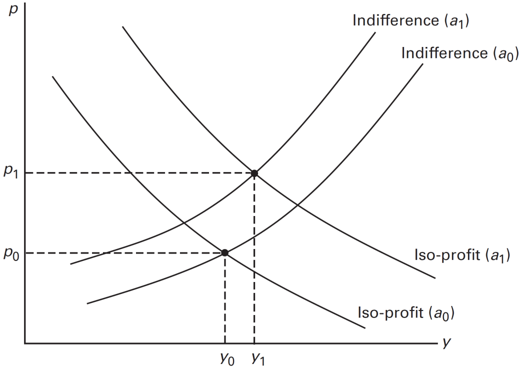
Workers accept **lower wage** and **higher rent** in more attractive city



Comparative statics: Firm costs decreasing in consumer amenity

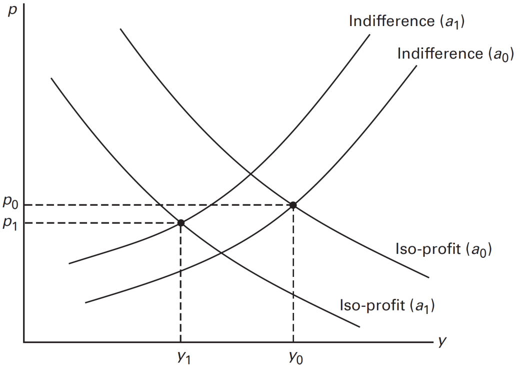
- $a_1 > a_0$ shifts iso-profit curve up
  - Firms can afford higher $\{y, p\}$
- Two forces pushing for higher rents
  - Firms accept higher rents
  - Residents accept higher rents
- Countervailing forces on wages
  - Firms can afford higher wages
  - Residents accept lower wages

Nominal wage may be higher in more attractive city. But workers still accept **lower real wage**



Comparative statics: Firm costs increasing in consumer amenity

- $a_1 > a_0$ shifts iso-profit curve down
  - Firms need to save on $\{y, p\}$
- Two forces pushing for lower wages
  - Firms need to save costs
  - Residents accept lower wages
- Countervailing forces on rents
  - Firms need to save costs
  - Residents accept higher rents



Rent may be lower in more attractive city. But workers still accept **lower real wage**

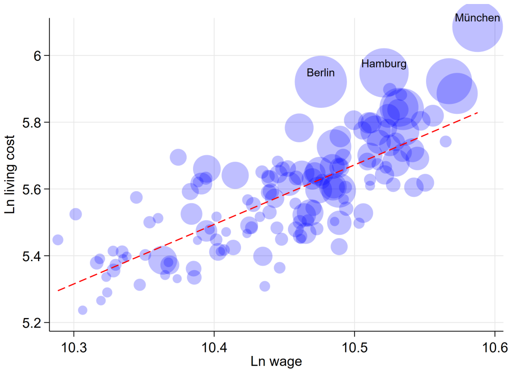
Equilibrium recap

- City with **high quality of life always has low real wage**
  - City with high quality of life and **high production amenity** (low cost)
    - High wage and even higher rent
  - City with high quality of life and **low production amenity** (high cost)
    - High rent, but not necessarily high wage

What type of city is Berlin?
High or low quality of life? high or low production amenity?

Wages and rents in Germany

- Cities high above fitted line, have a high quality of life
  - See, Albouy and Lue (2015)
- Need to control for worker quality differences across regions
  - Wage index form between-city movers controlling for individual fixed effects
  - IAB data, 100% sample of employees
- Need to control for house quality differences across regions
  - Ahlfeldt, Heblich, Seidel (2022)



Note: Slope of linear fit: 1.78 (t-stat: 14.01), R2 =0.59

Quantitative framework

A quantitative framework

- Roback's (1982) theory is elegant and general
  - Avoids restrictions to specific functional forms
- For **quantitative analyses**, we need a parametric model
  - **Estimate** key parameters
  - **Rationalize** observed data (e.g. wages and prices) in levels and changes
  - **Conduct counterfactuals**—predict what happens if fundamentals change
- **Glaeser & Gottlieb (2009)** provide a quantitative spatial equilibrium framework
  - Very much in the tradition of Rosen-Roback
    - Competitive market in goods and factors
    - Free mobility and spatial equilibrium
    - Identical agents
  - Notice that cities are nested in a larger economy; no fixed worker endowment

A simple quantitative model following Glaeser & Gottlieb (2009)

- **Quantitative models** typically consist of
  - **Primitives** that are **exogenous**, including fundamentals, endowments, and parameters
  - **Endogenous objects** that are solved within the model
- Gottlieb and & Glaeser (2009) model cities, indexed by c, which differ in terms of

Exogenous Factors

- Productivities – A_c.
- Amenities – B_c.
- Land availability – M_c.

Endogenous Elements

- Population – L_c.
- Housing prices – P_c.
- Wages – W_c.

- Also a range of **parameters** which do not vary between cities

Labor Market I

- Competitive firm(s) produce a tradable good with $p_q = 1$ (Cobb-Douglas)

$$Q = A_c L^{\alpha} K^{\beta} \bar{Z}^{1-\alpha-\beta}$$

  - L is the labor factor, priced per city at w_c
  - K is tradable capital, common price $r = 1$.
  - $\bar{Z}$ is a fixed factor in the city (e.g., land allocated for production).
- Maximize profit:

$$\max_{L,K} Q(L, K, \bar{Z}) - wL - K$$

- **First-Order Conditions**
  - for K: $\frac{\partial \Pi}{\partial K} = \beta A_c L^{\alpha} \bar{Z}^{1-\alpha-\beta} = K^{1-\beta}$
  - for L: $\frac{\partial \Pi}{\partial L} = \alpha A_c K^{\beta} \bar{Z}^{1-\alpha-\beta} = L^{1-\alpha} w_c$

Labor Market II

- **Inverse demand function**:
  - Substituting K from FOC for capital into FOC for labor condition
  - taking logs

$$\log(w_c) = \Omega_1 + \frac{\log(A_c)}{1-\beta} - \frac{1-\alpha-\beta}{1-\beta}\log(L_c) + \frac{1-\alpha-\beta}{1-\beta}\log(\bar{Z})$$

  - wage w_c by fundamental productivity
  - w_c falls in population L_c
    - Downward slope generated by the fixed factor $\bar{Z}$
  - Notice that unlike in Roback (1982) **floor space is not an input** into production

Housing Market

- **Housing** demand generated by the resident population
  - Preferences $u(x,h) = B_c x^{1-\sigma} h^{\sigma}$
  - Use F.O.C. to derive Marshallian demand $h = \frac{\sigma w_c}{P_c}$
  - Housing demand: $H_D = h_c L_c = \frac{\sigma w_c}{P_c} L_c$
- **Supply of housing** produced by developers using one unit of land
  - Convex construction costs: $c(D) = dD^{\delta}$, where D is the building height & $\delta > 1$
  - Abstracting from land costs, max. profits $\Pi = (P_c D - \delta_0 D^{\delta})$ yields $D_c^* = \left(\frac{P_c}{d_0 \delta}\right)^{\frac{1}{\delta - 1}}$
  - Housing supply: $H_S = M_c \times D_c^* = M_c \left(\frac{P_c}{d_0 \delta}\right)^{\frac{1}{\delta - 1}}$
- Housing market clears in **equilibrium**: $H_D = H_S$

$$\Rightarrow \log(P_c) = \Omega_2 + \frac{\delta - 1}{\delta} \left[\log(w_c) + \log(L_c) - \log(M_c)\right]$$

Spatial equilibrium

- Cities have the same level of utility
  - Just like in Roback (1982)
- Max utility: $u(x, h) = B_c x^{1-\sigma} h^{\sigma}$ s. t. budget constraint: $P_c + x = W_c$
  - Indirect utility: $V(W_c, P_c, B_c) = (1-\sigma)^{-1-\sigma} \sigma^{\sigma} \frac{B_c W_c}{P_c^{\sigma}}$
- **Equilibrium**
  - Constant utility in urban system: $V(W_c, P_c, B_c) = \bar{V}$
- **Indifference condition** (in logs):

$$\log(W_c) = \Omega_3 - \log(B_c) + \sigma \log(P_c)$$

  - Upward-sloping just like in Roback (1982)

General equilibrium I

- We have derived a system of **three** equations:

$$\log(w_c) = \Omega_1 + \frac{\log(A_c)}{1-\beta} - \frac{1-\alpha-\beta}{1-\beta}\log(L_c) + \frac{1-\alpha-\beta}{1-\beta}\log(Z)$$

$$\log(P_c) = \Omega_2 + \frac{\delta-1}{\delta}[\log(w_c) + \log(L_c) - \log(M_c)]$$

$$\log(w_c) = \Omega_3 - \log(B_c) + \sigma\log(P_c)$$

  - Has **three** endogenous objects: Population, wages, housing prices
  - All the other objects are exogenous, (A_c), (M_c), (B_c), and parameters

As many equations as unkonwns ⇒ Solve for the general equilibrium ✓

General equilibrium II

- Solve for the endogenous objects and express them in terms of exogenous objects

$$\log(L_c) = \Omega_4 + \eta(\delta + \sigma - \sigma\delta)\log(\hat{A}_c) + \eta(1-\beta)(\delta\log(B_c) + \sigma(\delta-1)\log(M_c))$$
$$\log(w_c) = \Omega_5 + \sigma\eta(\delta-1)\log(\hat{A}_c) - \eta(1-\alpha-\beta)(\delta\log(B_c) + \sigma(\delta-1)\log(M_c))$$
$$\log(P_c) = \Omega_6 + (\delta-1)\eta(\log(\hat{A}_c) + \alpha\log(B_c) - (1-\alpha-\beta)\log(M_c))$$

  - with $\eta = [(1-\beta)\delta - \alpha(\sigma + \delta - \sigma\delta)]^{-1}$, $\log(\hat{A}_c) = \log(A_c) + (1-\alpha-\beta)\log(Z_c)$
    - **Useful homework:** Rearrange eqs. from previous slide to obtain above eqs.!
- Can now solve for endogenous objects for any given values of primitives

Ready for counterfactuals statics ✓

Counterfactuals

- Can **quantify the effect of a change in an exogenous location characteristic**:

$$\ln Y_c^1 = \frac{\partial \ln Y_c}{\partial \ln X_c} d \ln X_c + \ln Y_c^0$$

  - where superscript 1 denotes the counterfactual and 0 denotes the initial value
- **Play around with the `GG2009-toolkit` to strengthen your intuition!**
  - Will deliver $\Delta \ln Y$ for user-specified $\Delta \ln X$ ▸ Open GG2009-toolkit
- Qualitatively, we obtain the following derivatives

	ΔA_c	ΔB_c	ΔM_c
ΔL_c	$+$	$+$	$+$
Δw_c	$+$	-	-
ΔP_c	$+$	$+$	-

Estimation

- Need to **quantify the model** before we can work quantitatively
  - First step is to settle on **parameter values**, which we can **set or estimate**
- For example, solve the inverse labour supply equation for $log(L_c)$

$$\log(L_c) = \underbrace{\Omega_1 \times \frac{1-\beta}{1-\alpha-\beta} + \log(\bar{Z})}_{\text{constant}} \quad \underbrace{-\frac{1-\beta}{1-\alpha-\beta}}_{\text{labour demand elasticity}} \quad \log(w_c) + \underbrace{\frac{1}{1-\alpha-\beta}\log(A_c)}_{\text{structural residual}}$$

- Can estimate using labour demand elasticity in **reduced-form**
  - Can infer β if we know α and vice versa
  - Identifying variation in $\log(w_c)$ must be uncorrelated with demand shifter, A_c
    - **Excludable instrument** shifts labour supply, exclusively, e.g. B_c

Structural model helps developing transparent identification strategy

Inversion

- To fully quantify the model, we also need to observe or **invert fundamentals**
  - Using observed values of endogenous variables and given parameter values
- To recover quality of life, B_c, we can rearrange the indifference condition.

$$\log(B_c) = \Omega_3 + \sigma \log(P_c) - \log(W_c)$$

- To identify B_c up to a multiplicative constant we just need:
  - Need wages, w_c, rents (or prices) P_c and the expenditure share of housing, $\delta = 0.33$
    - Typical value used in the literature (see Ahlfeldt & Pietrostefani, 2019)

Have fun with the inversion in the tutorial

Valuing amenities

Valuing non-marketed goods

- Recovering B_c is not only useful to measure quality of life
  - And compute quality-of-life rankings
- We use the 'compensating differential' to value non-marketed goods

$$\ln B_c = \sum_j \gamma^j \ln X_c^j + \varepsilon_c,$$

  - where X_c^j is one of $j \in J$ (dis)amenities
- It is straightforward (in principle) to estimate 'hedonic' price $\gamma^j = \frac{\partial \ln B_c}{\partial \ln X_c^j}$
  - Straightforward to monetize since $\frac{\partial \ln V}{\partial \ln w_c} = \frac{\partial \ln V}{\partial \ln B_c}$

This how we value clean air, crime, public schools... (Greenstone, 2017)

Conclusion

- **Rosen-Roback framework is a core concept in spatial economics**
  - A family of models that share the idea of a fricitonless spatial equilibrium
  - Perfect mobility implies indifference, zero-profits and perfect spatial arbitrage
- Framework can be used to **recover quality of life**
  - From spatial variation in wages and rents
- Workhorse tool for **'revealed-preference' valuation non-marketed goods**
  - Very important for social cost-benefit analysis and public investment decisions

Next week: **Monocentric city model**

Literature

Core readings

- **Brueckner, J. (2011): Wage-Based Indexes of Urban Quality of Life. Lectures on urban economics. Chapter 11. MIT Press.**
- **Glaeser, E., J. Gottlieb (2009): The Wealth of Cities: Agglomeration Economies and Spatial Equilibrium in the United States. Journal of Economic Literature 2009, 47(4).**

Other readings

- Ahlfeldt, G., Pietrostefani, E. (2019): The economic effects of the density: A synthesis. Journal of Urban Economics, 111.
- Ahlfeldt, G., S. Heblich, T. Seidel (2023): Micro-geographic property price and rent indices. Regional Science and Urban Economics, 98.
- Albouy, D. and B. Lue (2015): Driving to Opportunity: Local Rents, Wages, Commuting, and Sub-Metropolitan Quality-of-Life. Journal of Urban Economics, September 2015, 89, 74-92.
- Greenstone, M. (2017): The Continuing Impact of Sherwin Rosen's "Hedonic Prices and Implicit Markets: Product Differentiation in Pure Competition". Journal of Political Economy, 126(6).
- Rosen, S. (1979). Wage-Based Indexes of Urban Quality of Life. In *Current Issues in Urban Economics*, edited by P. Mieszkowski and M. Straszheim. Baltimore: Johns Hopkins University Press.
- Roback, J. (1982). Wages, Rents, and the Quality of Life. Journal of Political Economy 90(4), 1257–1278.