Topic 12
MRRH (2018): Counterfactuals

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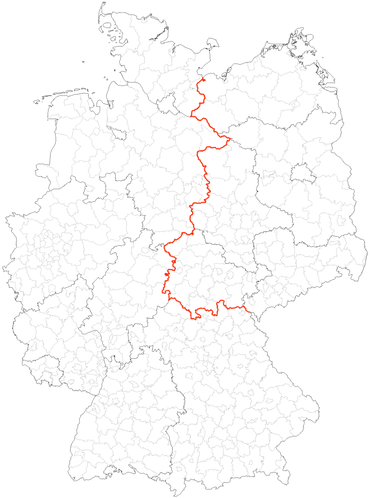
Introduction

Border effects

What is the effect of a **'Brexit'-like border**?

- Affects trade and commuting
- allows people to be mobile in terms of residence location?

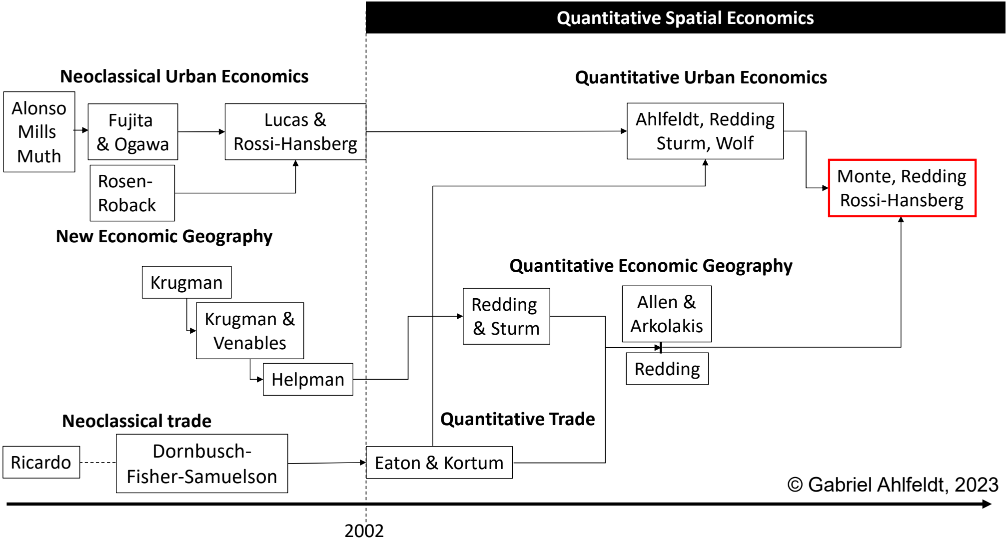
- Effects on
  - Tradable goods price, housing price, population
  - West vs. East



Monte, Redding, Rossi-Hansberg (2018) (MRRH)

- **Commuting and trade linkages in one model**
  - A 'true' **quantitative spatial model**?
- **Resolves tension** in quantitative economic geography models
  - If units are **too large** $\Rightarrow$ Many local effects are not detectable
    - E.g. localized effects of highways or subways
  - If units are **too small** $\Rightarrow$ Underestimate local labour supply
    - Miss that productive locations can draw workers from other locations
- **Spatial equilibrium**
  - Concentration force: Home market effect (like in RRH)
    - No productivity and amenity spillovers
  - Dispersion forces: Inelastically supplied land & idiosyncratic amenity (like in ARSW)
  - Zero profits and constant utility (like in ARSW & RRH)

History of thought



Roadmap

- **Monte, Redding, Rossi-Hansberg (2018)**
  - A canonical QSM
- **Topic 11**
  - Model
  - Equilibrium
  - Quantification
- **Topic 12** (today)
  - Seidel & Wickerath (2020) version of the model
    - Application to Germany
    - Toolkit written in MATLAB (instead of Mathematica) MRRH2015-toolkit
  - Counterfactuals

MRRH2018 toolkit

- `MRRH2018 toolkit` based on **Seidel & Wickerath (2020)** version of the model
  - All written in `MATLAB`
  - Quantification for **Germany**
  - Augmented with **endogenous housing supply and agglomeration economies**
    8. housing supply (7): $H_n = \bar{H}_n P_{H,n}^{\delta}$
    9. productivity: $A_i = \bar{A}_i L_i^{\nu}$,
    - **Two more endogenous variables** $\{H_n, A_i\}$**, two more equations** ✓
  - Calibrate **two more parameters**
    - (Short-run) Housing supply elasticity $\delta = 0.38$ (Lerbs, 2014)
    - Agglomeration elasticity $\nu = 0.05$ (Rosenthal & Strange 2004)
  - Some changes in notations
    - e.g. goods price index $P_n \rightarrow P_{Q,n}$, house price index $Q_n \rightarrow P_{H,n}$ Codebook

MRRH toolkit

- MRRH2018 and SW2020 quantify the model using **bilateral commuting flows**
  - Capture commuting cost non-parametrically
  - Rationalize **zero commuting flows** by setting **commuting costs to infinity**
- Approach not always feasible or desirable
  - Commuting flows may not be observable
  - May want to use **smooth transport cost measures**
    - allow for adjustments at extensive margin, from zero to positive flows
- Execute the `OwnData.m` script
  - Load your travel time matrix and replace trade and commuting distance measure
  - Let the `getBiTK.m` function **predict commuting flows**
  - `OwnData.m` will update other variables so that other solvers remain working
  - Add you measures of **relative changes** in trade ($\hat{d}_{ni}$) and commuting ($\hat{\tau}_{ni}$) distance

getBiTK.m

Algorithm 1: Solving workplace amenities and commuting probabities: `getBiTK.m`

Data: Given values for structural parameters $\{\mu, \epsilon\}$, observed wages, w_n, bilateral travel times, τ_{ni}, workplace employment L_n and residence employment R_n, and total population L

Guesses of workplace amenity B_i^0

while *Predicted workplace amenity* $B_i^1 \neq B_i^0$ **do**

- Use guesses B_i^0 in MRRH2018 Eq. (12) to compute conditional commuting probability $\lambda^1_{ni|n}$
- Use $\lambda^1_{ni|n}$ to predict workplace employment L_n^1 using MRRH2018 Eq. (13)
- Compute new guesses of workplace amenity as $B_i^1 = B_i^0 \frac{L_n}{L_n^1}$; increase workplace amenity to attract more workers if observed employment exceeds predicted employment
- Update guesses B_i^0 to weighted combination of old and new guesses: $\zeta B_i^1 + (1 - \zeta) B_i^0$

Compute residential choice probability $\lambda_n^R = \frac{R_n}{\sum_n R_n}$

Compute unconditional location choice probabilities $\lambda_{ni} = \lambda_{ni|n} \lambda_n^R$ using MRRH2018 Eq. (12)

Compute commuting flows $L_{ni} = \lambda_{ni} L$

Result: Workplace amenity, B_i, conditional choice probabilities, $\lambda_{ni|n}$, unconditional choice probabilities, λ_{ni}, commuting flows L_{ni}

- getBiTK.m finds the workplace amenity B_i that rationalizes $\{L_n, R_n\}$ under a commuting distance matrix
- Then predicts flows (similar to wage solver in ARSW2015) $\Rightarrow$ B_n can be interpreted as transformed wage

Quantification

Quantification

- Covered estimation of Ψ and ϵ in Topic 11
  - MRRH toolkit sets values to SW2020 choices `MRRH2018_toolkit.m`
- We can abstract from some fundamentals using **exact hat algebra**
- **Need initial values of trade shares π_{ni}, which requires A_i**
- Use a solver that targets A_i and recovers π_{ni}
  - `solveProductTradeTK.m` finds A_i values that satisfy **income equals expenditure**
    - $w_i L_i = \sum_n \pi_{ni} \bar{v}_n R_n$
  - Increase A_i if firms in i do not attract enough expenditure from workers in n
    - Greater productivity $\Rightarrow$ expansion of production $\Rightarrow$ Larger expenditure shares $\sum_n \pi_{ni}$
  - Returns $\{A_i, \pi_{ni}, \pi_{nn}, P_{Q,n}\}$

solveProductTK.m

Algorithm 2: Solving for exogenous productivity: `solveProductTK.m`

Data: Given values for structural parameters $\{\sigma, \nu\}$, observed workplace employment L_n and residence employment R_n, wages, w_n, residential wages, $\bar{v}_n$, and bilateral trade cost, d_{ni}

Guesses of exogenous productivity $\bar{A}_i^0$

while *Income not equal expenditure* $w_i L_i \neq \sum_n \pi_{ni} \bar{v}_n R_n$ **do**

- Use guesses $\bar{A}_i^0$ in SW2020 Eq. (10) to compute trade shares π_{ni}
- Compute worker income $w_i L_i$, the left-hand side in SW2020 Eq. (12)
- Use π_{ni} to compute worker expenditure $\sum_n \pi_{ni} \bar{v}_n R_n$, the right-hand side of SW2020 Eq. (12)
- Compute new guess of productivity $\bar{A}_i^1 = \bar{A}_i^0 \frac{\sum_n \pi_{ni}}{\sum_n \pi_{ni} \bar{v}_n R_n}$; increase productivity if firms do not attract enough expenditure
- Update guesses $\bar{A}_i^0$ to weighted combination of old and new guesses: $\zeta \bar{A}_i^1 + (1 - \zeta) \bar{A}_i^0$

Use $\bar{A}_n$ and π_{ni} to compute tradable goods price index $P_{Q,n}$ using SW2020 Eq. (11)

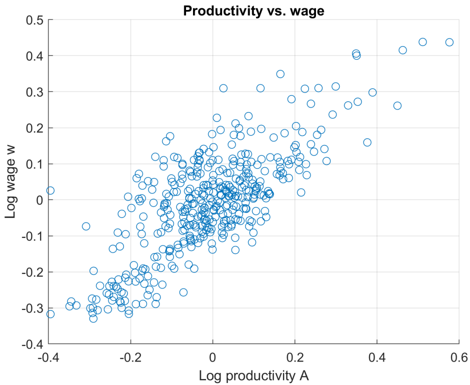
Result: Exogenous productivity $\bar{A}_i$, trade shares π_{ni}, tradable goods price index $P_{Q,n}$

`solveProductTradeTK.m`

Productivity

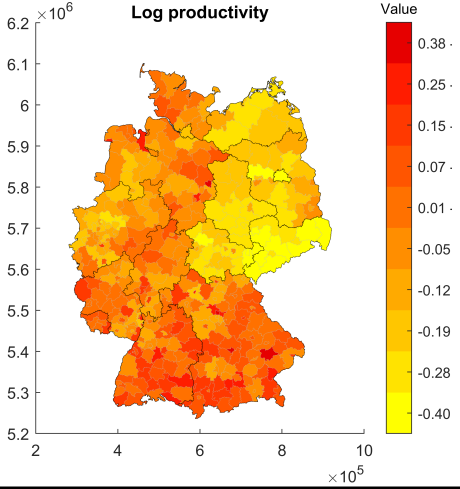
- Fundamental productivity $\bar{A}_n$ positively related to wages w_n
- But there is some dispersion
- Notice that SW2020 do not use $\bar{A}_n$ as an instrument for w_n when estimating the preference heterogeneity parameter ϵ
  - Use a Bartik-IV instead

What do you think?

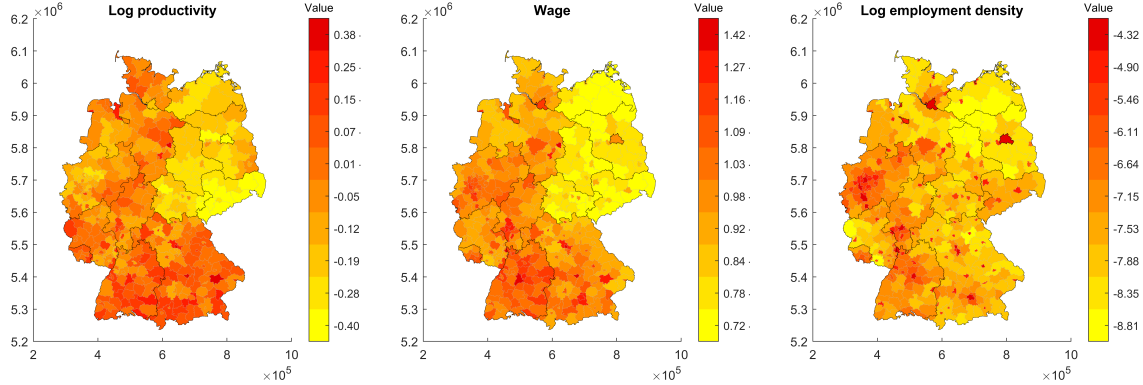


Productivity

- Plausible **East-West heterogeneity**
  - Productivity in east still recovering from division period
- **Not much of a big-city effect**
  - (over?)-attributed to endogenous component L_n^{ν}
  - There is an urban productivity premium in $A_n = \bar{A} L_n^{\nu}$

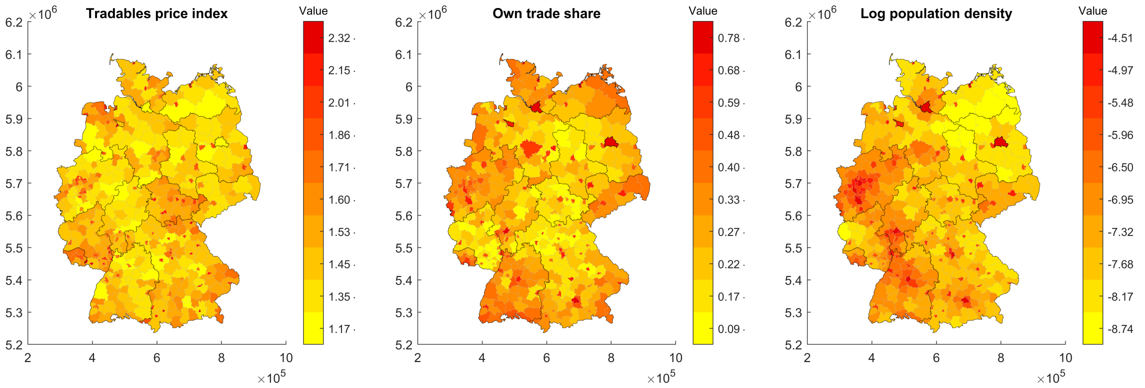


Employment, wage, and productivity



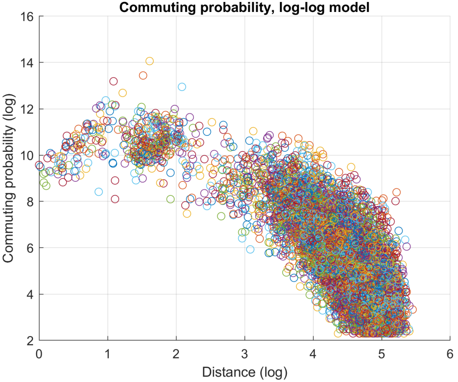
- Larger regions have high wages without high fundamental productivity (agglomeration effect)

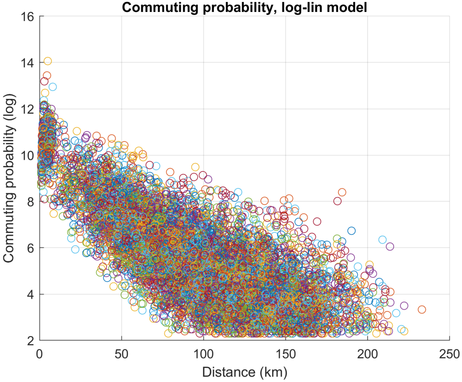
Population, own trade share, and tradable goods price



- Larger regions have greater own trade share (inverse market access) but no higher prices

Parameterizing commuting cost





- MRRH and SW parameterize $\kappa_{ni} = \tau_{ni}^{-\epsilon\mu}$ whereas ARSW parameterize $\kappa_{ni} = \exp\left(-\epsilon\mu\tau_{ni}\right)$

Counterfactuals

Recall: Exact hat algebra

- Dekle et al. (2007) denote the
  - known value of a variable in the initial equilibrium by x
  - unknown value of a variable in the counterfactual equilibrium by x' (with a prime)
  - relative change in the variable by $\hat{x} = \frac{x'}{x}$ (with a hat)
- We can compute the counterfactual value $x' = \hat{x}x$
  - We need the relative changes and initial levels of a variable
- Aim of **exact hat algebra** is
  - express **relative changes in endogenous variables** as functions of
    - **relative changes in primitives**
    - **initial values of endogenous variables**

We can derive system of equations that avoid levels of fundamentals altogether

Writing a numerical solver using exact hat algebra I

- Recall the **standard procedure within fixed-point algorithms**
  - We start with **guesses** of the **target objects**
  - Use guesses of target objects to **predict non-target referencing objects**
  - Use predictions of non-target referencing objects to **predict target objects**
  - **Update guesses** and iterate until we converge
- With exact hat algebra, we apply the **same procedure** to **relative changes** $\hat{x}$
  - Write down endogenous outcomes for counterfactual $x' = \hat{x}x$
  - Divide by x to obtain expressions for $\hat{x}$
  - Arrange $\hat{x}$ expressions to implement the above algorithmic procedure

Counterfactual outcomes and counterfactual relative changes

1. $w_i \hat{w}_i L_i \hat{L}_i = \sum_{n \in N} \pi_{ni} \hat{\pi}_{ni} \bar{v}_n \hat{\bar{v}}_n R_n \hat{R}_n$ $\Rightarrow \hat{w}_i = \frac{1}{w_i L_i \hat{L}_i} \sum_{n \in N} \pi_{ni} \hat{\pi}_{ni} \bar{v}_n \hat{\bar{v}}_n R_n \hat{R}_n$
2. $\hat{\bar{v}}_n \bar{v}_n = \sum_{i \in N} \frac{\lambda_{ni} \hat{B}_{ni} (\hat{w}_i / \hat{\kappa}_{ni})^{\epsilon}}{\sum_{s \in N} \lambda_{ns} \hat{B}_{ns} (\hat{w}_s / \hat{\kappa}_{ns})^{\epsilon}} \hat{w}_i w_i$ $\Rightarrow \hat{\bar{v}}_n = \frac{1}{\bar{v}_n} \sum_{i \in N} \frac{\lambda_{ni} \hat{B}_{ni} (\hat{w}_i / \hat{\kappa}_{ni})^{\epsilon}}{\sum_{s \in N} \lambda_{ns} \hat{B}_{ns} (\hat{w}_s / \hat{\kappa}_{ns})^{\epsilon}} \hat{w}_i w_i$
3. $\hat{P}_{H,n} P_{H,n} = P_{H,n} \left(\hat{\bar{v}}_n \hat{R}_n \right)^{\frac{1}{1+\delta}}$ $\Rightarrow \hat{P}_{H,n} = \left(\hat{\bar{v}}_n \hat{R}_n \right)^{\frac{1}{1+\delta}}$
4. $\hat{\pi}_{ni} \pi_{ni} = \frac{\pi_{ni} \left(\hat{L}_i \right)^{1-(1-\sigma)\nu} \left(\hat{d}_{ni} \hat{w}_i / \hat{A}_i \right)^{1-\sigma}}{\sum_{k \in N} \hat{\pi}_{nk} \left(\hat{L}_k \right)^{1-(1-\sigma)\nu} \left(\hat{d}_{ni} \hat{w}_k / \hat{A}_k \right)^{1-\sigma}}$ $\Rightarrow \hat{\pi}_{ni} = \frac{\left(\hat{L}_i \right)^{1-(1-\sigma)\nu} \left(\hat{d}_{ni} \hat{w}_i / \hat{A}_i \right)^{1-\sigma}}{\sum_{k \in N} \hat{\pi}_{nk} \left(\hat{L}_k \right)^{1-(1-\sigma)\nu} \left(\hat{d}_{ni} \hat{w}_k / \hat{A}_k \right)^{1-\sigma}}$
5. $\hat{\lambda}_{ni} \lambda_{ni} = \frac{\lambda_{ni} \hat{B}_{ni} \left(\left(\hat{P}_{Q,n} \right)^{\alpha} \left(\hat{P}_{H,n} \right)^{(1-\alpha)} \right)^{-\epsilon} (\hat{w}_i \hat{\kappa}_{ni})^{\epsilon}}{\sum_{r \in N} \sum_{s \in N} \lambda_{rs} \hat{B}_{rs} \left(\left(\hat{P}_{Q,r} \right)^{\alpha} \left(\hat{P}_{H,r} \right)^{(1-\alpha)} \right)^{-\epsilon} (\hat{w}_s \hat{\kappa}_{rs})^{\epsilon}}$ $\Rightarrow \hat{\lambda}_{ni} = \frac{\hat{B}_{ni} \left(\left(\hat{P}_{Q,n} \right)^{\alpha} \left(\hat{P}_{H,n} \right)^{(1-\alpha)} \right)^{-\epsilon} (\hat{w}_i \hat{\kappa}_{ni})^{\epsilon}}{\sum_{r \in N} \sum_{s \in N} \lambda_{rs} \hat{\Phi}_{rs}}$
6. $\hat{P}_{Q,n} P_{Q,n} = P_{Q,n} \left(\frac{\left(\hat{L}^{(t)} \right)^{1-(1-\sigma)\nu}}{\hat{\pi}_{nn}} \right)^{\frac{1}{1-\sigma}} \frac{\hat{d}_{nn} \hat{w}_n}{\hat{A}_n}$ $\Rightarrow \hat{P}_{Q,n} = \left(\frac{\left(\hat{L}^{(t)} \right)^{1-(1-\sigma)\nu}}{\hat{\pi}_{nn}} \right)^{\frac{1}{1-\sigma}} \frac{\hat{d}_{nn} \hat{w}_n}{\hat{A}_n}$
7. $\hat{R}_i R_i = \bar{L} \sum_{i \in N} \lambda_{ni} \hat{\lambda}_{ni}$ $\Rightarrow \hat{R}_n = \frac{\bar{L}}{R_n} \sum_{i \in N} \lambda_{ni} \hat{\lambda}_{ni}$
8. $\hat{L}_n L_n = \bar{L} \sum_{n \in N} \lambda_{ni} \hat{\lambda}_{ni}$ $\Rightarrow \hat{L}_n = \frac{\bar{L}}{L_n} \sum_{n \in N} \lambda_{ni} \hat{\lambda}_{ni}$

Writing a numerical solver using exact hat algebra II

- Conditional on
  - known parameter values of $\{\alpha, \delta, \epsilon, \nu, \mu, \psi, \sigma\}$
  - observed or solved initial levels of $\{w_n, L_n, R_n \lambda_{ni}, \pi_{ni}\}$
  - assumed changes in fundamentals $\{\hat{A}_n, \hat{B}_n, \hat{d}_{ni}, \hat{\kappa}_{ni}\}$
- there is
  - a recursive structure that solves for $\{\hat{\bar{v}}, \hat{L}_n, \hat{R}_n, \hat{P}_{Q,n}, \hat{P}_{H,n}, \hat{\pi}_{ni}\}$ for given $\{\hat{w}_n, \hat{\lambda}_i\}$
  - a recursive structure that solves for $\{\hat{w}_n, \hat{\lambda}_i\}$ for given $\{\hat{\bar{v}}, \hat{L}_n, \hat{R}_n, \hat{P}_{Q,n}, \hat{P}_{H,n}, \hat{\pi}_{ni}\}$
- We can **guess, predict, update** $\{\hat{w}_n, \hat{\lambda}_i\}$

Let $\{\hat{w}_n, \hat{\lambda}_i\}$ be our target variables

Algorithm 3: Solving for relative counterfactual changes: `counterFactsTK.m`

Data: Changes in exogenous productivities $\hat{\hat{A}}_n$, amenities $\hat{B}_{ni}$, commuting costs $\hat{\kappa}_{ni}$, trade costs $\hat{d}_{ni}$. Initial levels of wages w_n, residential wages $\bar{v}_n$, workplace employment L_n, residence population R_n, trade shares π_{ni}. Structural parameters $\{\alpha, \sigma, \epsilon, \delta, \nu\}$.

Initialize guesses of changes in wages, $\hat{w}_n^0$, and unconditional commuting probabilities, $\lambda_{ni}^{\hat{0}}$

while $\hat{w}_n^0 \neq \hat{w}_n^1$ *or* $\lambda_{ni}^{\hat{0}} \neq \lambda_{ni}^{\hat{1}}$ **do**

- Use Algorithm 4 `updateResWageTK` and $\hat{w}_n^0$ to compute change in residential wage $\hat{\bar{v}}_n^0$
- Use Algorithm 5 `updateEmplTK` and $\hat{\lambda}_{ni}^0$ to compute change in workplace employment $\hat{L}_n^0$
- Use Algorithm 6 `updateResidentsTK` and $\hat{\lambda}_{ni}^0$ to compute change in residence population $\hat{R}_n^0$
- Use Algorithm 7 `updateHousePriceTK` and $\{\hat{\bar{v}}_n^0, \hat{R}_n^0\}$ to compute change in housing price $\hat{P}_{H,n}^0$
- Use Algorithm 8 `updateTradeshTK` and $\{\hat{w}_n^0, \hat{L}_n^0\}$ to compute change in trade shares $\hat{\pi}_{ni}^0$
- Use Algorithm 9 `updatePricesTK` and $\{\hat{w}_n^0, \hat{L}_n^0, \pi_{ni}^0\}$ to compute change in tradable goods price $\hat{P}_{Q,n}^0$
- Use Algorithm 10 `updateWageTK` and $\{\hat{L}_n^0, \hat{\pi}_{ni}^0, \hat{\bar{v}}_n^0, \hat{R}_n\}$ to predict change in wage $\hat{w}_n^1$
- Normalize $\hat{w}_n^1$ to ensure that w_n' has a unit mean
- Use Algorithm 11 `updateLamTK` and $\{\hat{P}_{Q,n}^0, \hat{P}_{H,n}^0, \hat{w}_n^0\}$ to predict change in unconditional commuting probabilities $\hat{\lambda}_{ni}^1$
- Update guesses of $\hat{w}_n^0$ to convex combination of old guesses and predictions $\zeta \hat{w}_n^1 + (1-\zeta)\hat{w}_n^0$
- Update guesses of $\hat{\lambda}_{ni}^0$ to convex combination of old guesses and predictions $\zeta \hat{\lambda}_{ni}^1 + (1-\zeta)\hat{\lambda}_{ni}^0$

Compute change in expected utility (see below)

Result: Changes in wages $\hat{w}_n$, residential wages $\hat{\bar{v}}_{ni}$, tradable goods price $\hat{P}_{Q,n}$, trade shares $\hat{\pi}_{ni}$, unconditional commuting probabilities $\hat{\lambda}_{ni}$, housing price $\hat{P}_{H,n}$, residence population $\hat{R}_n$, workplace employment $\hat{L}_n$, expected utility $\hat{\bar{U}}$

`counterFactsTK.m` Nested algorithms

Welfare I

- Expected utility is es defined in MRRH2018 Eq. (15):

$$\bar{U} = \Gamma\left(\frac{\epsilon-1}{\epsilon}\right)\left[\sum_{r\in N}\sum_{s\in N} B_{rs}\left(\left(\hat{P}_{Q,r}P_{Q,r}\right)^{\alpha}\left(P_{H,r}\right)^{1-\alpha}\right)^{-\epsilon}\left(w_s\right)^{-\epsilon}\right]^{\frac{1}{\epsilon}}$$

- We want to compute relative changes without taking a stance on fundamental B_{ni}
- Write down expected utility in counterfactual

$$\hat{\bar{U}}\bar{U} = \Gamma\left(\frac{\epsilon-1}{\epsilon}\right)\left[\sum_{r\in N}\sum_{s\in N} \hat{B}_{rs}B_{rs}\left(\hat{\kappa}_{rs}\kappa_{rs}\left(\hat{P}_{Q,r}P_{Q,r}\right)^{\alpha}\left(\hat{P}_{H,r}P_{H,r}\right)^{1-\alpha}\right)^{-\epsilon}\left(\hat{w}_s w_s\right)^{-\epsilon}\right]^{\frac{1}{\epsilon}}$$

- which we can rewrite as

$$\hat{\bar{U}} = \frac{\left[\sum_{r\in N}\sum_{s\in N} \hat{B}_{rs}B_{rs}\left(\hat{\kappa}_{rs}\kappa_{rs}\left(\hat{P}_{Q,r}P_{Q,r}\right)^{\alpha}\left(\hat{P}_{H,r}P_{H,r}\right)^{1-\alpha}\right)^{-\epsilon}\left(\hat{w}_s w_s\right)^{-\epsilon}\right]^{\frac{1}{\epsilon}}}{\left[\sum_{r\in N}\sum_{s\in N} B_{rs}\left(\kappa_{rs}\left(P_{Q,r}\right)^{\alpha}\left(P_{H,r}\right)^{1-\alpha}\right)^{-\epsilon}\left(w_s\right)^{-\epsilon}\right]^{\frac{1}{\epsilon}}}.$$

Welfare II

- Terms in brackets correspond to the multilateral resistance terms in initial and counterfactual unconditional commuting probabilities in MRRH Eq. (10), hence: $\sum_r \sum_s B_{rs}\kappa_{ni}\left(P_{Q,r}^{\alpha}P_{H,r}^{1-\alpha}\right)^{-\epsilon} w_s^{\epsilon} = \frac{1}{\lambda_{ni}} B_{ni}\kappa_{ni}\left(P_{Q,n}^{\alpha}P_{H,n}^{1-\alpha}\right)^{-\epsilon} w_i^{\epsilon}$
  - and accordingly for the counterfactual
- Substitute in and observe the magic of exact hat algebra...

$$\hat{\bar{U}} = \frac{\left[\frac{1}{\hat{\lambda}_{ni}\lambda_{ni}}\hat{B}_{ni}B_{ni}\left(\hat{\kappa}_{ni}\kappa_{ni}\left(\hat{P}_{Q,n}P_{Q,n}\right)^{\alpha}\left(\hat{P}_{H,n}P_{H,n}\right)^{1-\alpha}\right)^{-\epsilon}(\hat{w}_i w_i)^{-\epsilon}\right]^{\frac{1}{\epsilon}}}{\left[\frac{1}{\lambda_{ni}}B_{ni}\left(\kappa_{ni}\left(P_{Q,n}\right)^{\alpha}\left(P_{H,n}\right)^{1-\alpha}\right)^{-\epsilon}(w_i)^{-\epsilon}\right]^{\frac{1}{\epsilon}}} = \frac{\hat{B}_{ni}^{\frac{1}{\epsilon}}\hat{\kappa}_{ni}^{-\frac{1}{\epsilon}}}{\hat{\kappa}_{ni}\left(\hat{P}_{Q,n}\right)^{\alpha}\left(\hat{P}_{H,n}\right)^{1-\alpha}\hat{w}_i}$$

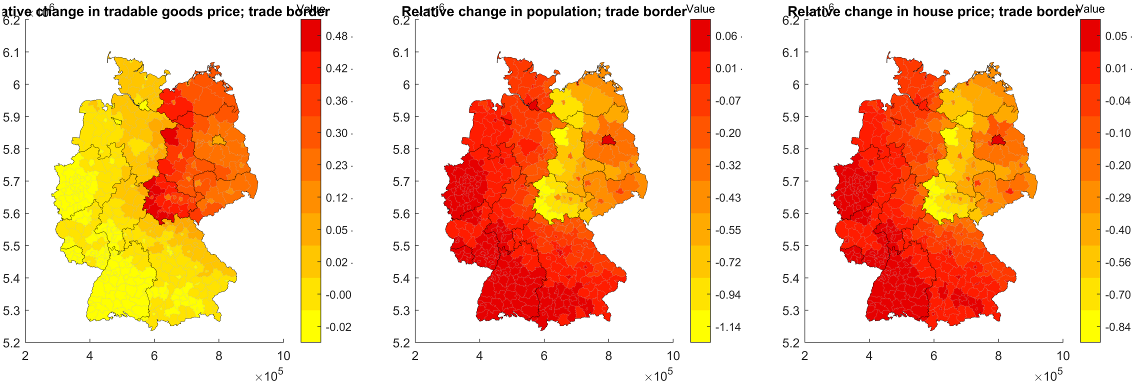
- `counterFactsTK.m` computes all we need...

Border example

Illustrative counterfactuals

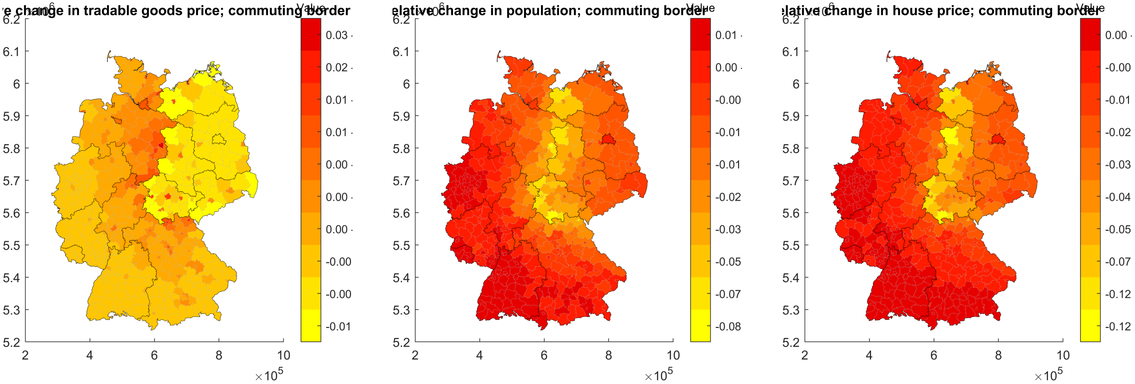
- Forcing variables for counterfactuals in the MRRH2018 model $\{\hat{A}_n, \hat{B}_n, \hat{d}_{ni}, \hat{\kappa}_{ni}\}$
  - `MRRH2018_toolkit.m` offers simple syntax that support manipulation
- Let's focus on spatial frictions
  - commuting cost κ_{ni}
  - trade cost d_{ni}
  - We parameterize both as a function of straight-line distance (using `OwnData.m`)
  - Set either $\hat{\kappa}_{ni} = \infty$ of $\hat{d}_{ni} = \infty$ or both, to simulate border effect
    - Border affects trade and commuting, but allows for migration across border
- Of course, many other counterfactuals are possible
- Use changes in travel times computed in GIS
  - Effect may differ on commuting cost and trade cost, e.g. in case of high-speed rail
  - Need to take a stance on how costs scale in the distance measure
    - not necessarily at a unit elasticity

Border affects trade only



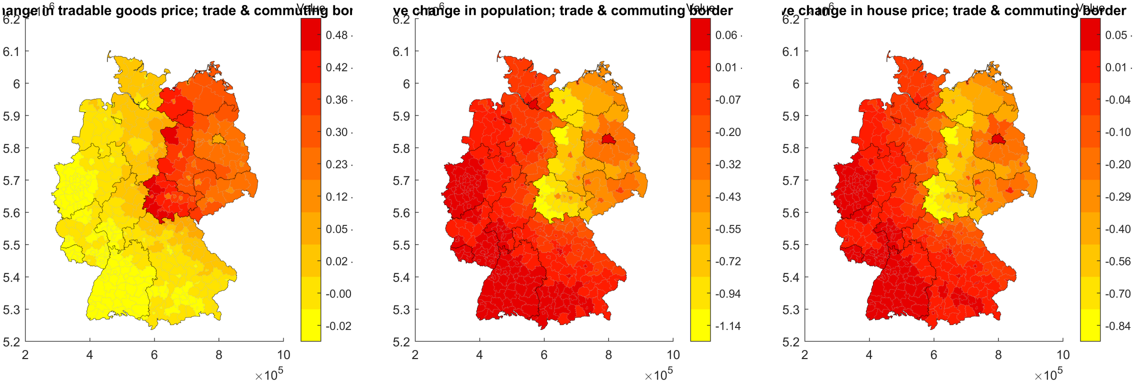
- Tradable goods price increase close to border; stronger on the eastern (smaller home market)
- Population adjusts until lower housing cost offsets for higher tradable goods prices
- Expected utility effect: -2.15%

Border affects commuting only



- Housing price falls close to border; stronger on the eastern side
- Population adjusts until lower housing cost offsets for loss of CMA
- Expected utility effect: -0.08%

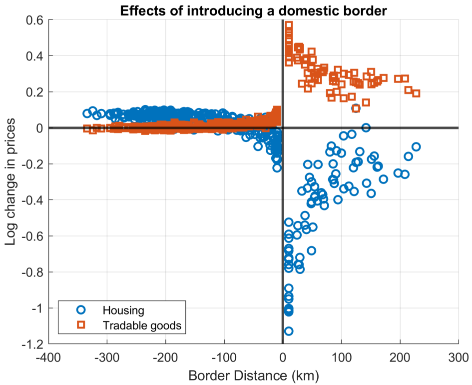
Border affects trade and commuting



- Effects resemble trade border effect; commuting border effect is much smaller and more localized
- Expected utility effect: -2.16%

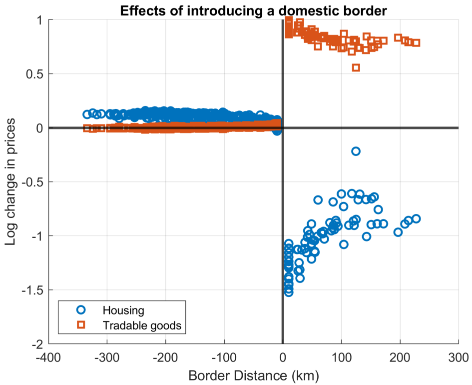
Border affects trade and commuting

- Tradable goods price increases close to border
  - stronger on the eastern (smaller home market)
- House prices adjust more strongly
  - Compensate for tradable goods prices
  - Lower expenditure share $\alpha > 0.5$



Border affects trade and commuting: under 50% lower trade cost

- Price effects more uniform within west and east
  - lower spatial decay, trade less localized
- East hit even harder by higher prices
  - More reliant on non-domestic market due to lower trade cost
- Border effect has larger welfare effects if trade costs are low
  - -5.17% instead of -2.16%



Conclusion

Summary

- **MRRH** integrate spatial linkages via **commuting and trade** into one model
- Powerful tool to quantitatively evaluate the effect of spatial frictions
- `MRRH2018_toolkit.m` is an easy-to-use tool to run counterfactuals

fun with your own counterfactuals

Literature I

Core readings

- **Monte, F., Redding, S., Rossi-Hansberg, E. (2018): Commuting, Migration, and Local Employment Elasticities. American Economic Review, 108(12), 3855-90.**

Other readings

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Appendix

Other algorithms I

Algorithm 4: Update residential wages: `updateResWageTK.m`

Data: Changes in amenities $\hat{B}_{ni}$, wages $\hat{w}_n$, commuting cost $\hat{\kappa}_{ni}$. Initial levels of unconditional commuting probabilities λ_{ni}, residential wages $\bar{v}_n$, wages w_n. Structural parameter ϵ

Compute changes in residential wages $\hat{v}_n^{(t)} = \frac{1}{\bar{v}_n} \sum_{i \in N} \frac{\lambda_{ni} \hat{B}_{ni} \left(\hat{w}_i^{(t)} / \hat{\kappa}_{ni} \right)^{\epsilon}}{\sum_{s \in N} \lambda_{ns} \hat{B}_{ns} \left(\hat{w}_s^{(t)} / \hat{\kappa}_{ns} \right)^{\epsilon}} \hat{w}_i^{(t)} w_i$ (exact hat version of MRRH Eq. (14), see MRRH2018 Appendix Eq. B.17)

Result: Changes in residential wages $\hat{v}_n^{(t)}$

`updateResWageTK.m` `counterFactsTK.m`

Other algorithms II

Algorithm 5: Update workplace employment: `updateEmplTK.m`

Data: Changes in unconditional commuting probabilities $\hat{\lambda}_{ni}$. Initial levels of unconditional commuting probabilities λ_{ni}, workplace employment L_n, total employment $\bar{L}$

Compute changes in workplace employment $\hat{L}_i^{(t)} = \frac{\bar{L}}{L_i} \sum_{n \in N} \lambda_{ni} \hat{\lambda}_{ni}^{(t)}$ (exact hat algebra version of MRRH2018 Eq. 11, see MRRH2018 Appendix Eq. B.23)

Result: Changes in workplace employment $\hat{L}_i^{(t)}$

`updateEmplTK.m` `counterFactsTK.m`

Other algorithms III

Algorithm 6: Update residence population: `updateResidentsTK.m`

Data: Changes in unconditional commuting probabilities $\hat{\lambda}_{ni}$. Initial levels of unconditional commuting probabilities λ_{ni}, residence population R_n, total employment $\bar{L}$

1 Compute changes in residential population $\hat{R}_i^{(t)} = \frac{\bar{L}}{R_n} \sum_{i \in N} \lambda_{ni} \hat{\lambda}_{ni}^{(t)}$ (exact hat algebra version of MRRH2018 Eq. 11, see MRRH2018 Appendix Eq. B.22)

Result: Changes in residence population $\hat{R}_n^{(t)}$

`updateResidentsTK.m` `counterFactsTK.m`

Other algorithms IV

Algorithm 7: Update housing price: `updateHousePriceTK.m`

Data: Changes in residential wages $\hat{\bar{w}}_n$ and changes in residence population $\hat{R}_n^{(t)}$. Structural parameter δ

1 Compute changes in housing price $\hat{P}_{H,n}^{(t)} = \left(\hat{\bar{v}}_n^{(t)} \hat{R}_n^{(t)}\right)^{\frac{1}{1+\delta}}$ (SW2020 Eq. 9 in ratios)

Result: Changes in residence population $\hat{L}_i^{(t)}$

`updateTradeshTK.m` `counterFactsTK.m`

Other algorithms V

Algorithm 8: Update trade shares: `updateTradeshTK.m`

Data: Changes in employment $\hat{L}_n$, wages $\hat{w}_n$, exogenous productivities $\hat{\bar{A}}_n$, trade cost $\hat{d}_{ni}$. Initial levels of trade shares π_{ni}. Structural parameters $\{\sigma, \nu\}$

1 Compute changes in trade shares $\hat{\pi}_{ni}^{(t)} = \frac{\left(\hat{L}_i^{(t)}\right)^{1-(1-\sigma)\nu}\left(\hat{d}_{ni}\hat{w}_i^{(t)}/\hat{A}_i\right)^{1-\sigma}}{\sum_{k\in N}\hat{\pi}_{nk}\left(\hat{L}_k^{(t)}\right)^{1-(1-\sigma)\nu}\left(\hat{d}_{ni}\hat{w}_k^{(t)}/\hat{A}_k\right)^{1-\sigma}}$ (exact hat algebra version of SW2020 Eq. 10; see MRRH2018 Appendix Eq. B.20, augmented by agglomeration economies introduced by SW2020)

Result: Changes in trade shares $\hat{\pi}_{ni}^{(t)}$

`updateTradeshTK.m` `counterFactsTK.m`

Other algorithms VI

Algorithm 9: Update tradable goods price: `updatePricesTK.m`

Data: Changes in employment $\hat{L}_n$, wages $\hat{w}_n$, trade shares $\hat{\pi}_{ni}$, trade costs $\hat{d}_{ni}$, exogenous productivities $\hat{\hat{A}}_n$. Structural parameters $\{\sigma, \nu\}$

Compute changes in trade shares $\hat{P}^{(t)}_{Q,n} = \left(\frac{\left(\hat{L}^{(t)}\right)^{1-(1-\sigma)\nu}}{\hat{\pi}^{(t)}_{nn}} \right)^{\frac{1}{1-\sigma}} \frac{\hat{d}_{nn}\hat{w}^{(t)}_n}{\hat{A}_n}$ (SW2020 Eq. 11 in ratios)

Result: Changes in tradable goods price $\hat{P}^{(t)}_{n,Q}$

Note: Notice that $\hat{d}_{nn}$ is missing in the description of the algorithms in MRRH2018 and SW2020. This is inconsequential in most cases since trade costs parameterized based on geographic distance won't change within regions. In the toolkit, the equation is generalized in case more sophisticated measures are available in potential applications.

`updatePricesTK.m` `counterFactsTK.m`

Other algorithms VII

Algorithm 10: Update wage: `updateWageTK.m`

Data: Changes in employment $\hat{L}_n$, trade shares $\hat{\pi}_{ni}$, residential wages $\hat{\bar{v}}_n$, resident population $\hat{R}_n$. Initial levels of workplace employment L_n, workplace wages w_n, trade shares π_{ni}, residential wages $\bar{v}_n$, residential population R_n.

1 Compute changes in trade shares $\tilde{w}_i^{(t+1)} = \frac{1}{w_i L_i \hat{L}_i^{(t)}} \sum_{n \in N} \pi_{ni} \hat{\pi}_{ni}^{(t)} \bar{v}_n \hat{\bar{v}}_n^{(t)} R_n \hat{R}_n^{(t)}$ (exact hat algebra version of MRRH Eq. 7; see MRRH2018 Appendix Eq. B.16)

Result: Changes in wages $\hat{w}_n^{(t)}$

`updateWageTK.m` `counterFactsTK.m`

Other algorithms VIII

Algorithm 11: Update unconditional commuting probabilities: `updateLamTK.m`

Data: Changes in amenities $\hat{B}_{ni}$, tradable goods price $\hat{P}_{Q,n}$, housing price $\hat{P}_{H,n}$, workplace wage $\hat{w}_n$, commuting costs $\hat{\kappa}_{ni}$. Initial levels of unconditional commuting probabilities $\hat{\lambda}_{ni}$.

1 Compute changes in unconditional commuting probabilities

$$\tilde{\lambda}_{ni}^{(t+1)} = \frac{\hat{B}_{ni}\left(\left(\hat{P}_{Q,n}^{(t)}\right)^{\alpha}\left(\hat{P}_{H,n}^{(t)}\right)^{(1-\alpha)}\right)^{-\epsilon}\left(\hat{w}_i^{(t)}\hat{\kappa}_{ni}\right)^{\epsilon}}{\sum_{r\in N}\sum_{s\in N}\lambda_{rs}\hat{B}_{rs}\left(\left(\hat{P}_{Q,r}^{(t)}\right)^{\alpha}\left(\hat{P}_{H,r}^{(t)}\right)^{(1-\alpha)}\right)^{-\epsilon}\left(\hat{w}_s^{(t)}\hat{\kappa}_{rs}\right)^{\epsilon}}$$

(exact hat algebra version of MRRH Eq. 10; see MRRH2018 Appendix Eq. B.20)

Result: Changes in unconditional commuting probabilities $\hat{\lambda}_{ni}^{(t)}$

`updateLamTK.m` `counterFactsTK.m`