

# **Topic 10**

## **RRH (2017): Counterfactuals**

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Summer term 2024

# Acknowledgements

- ▶ This slide deck uses material from the following lectures
  - ▶ Quantitative Spatial Economics lecture in Princeton University course EC552—by Esteban Rossi-Hansberg

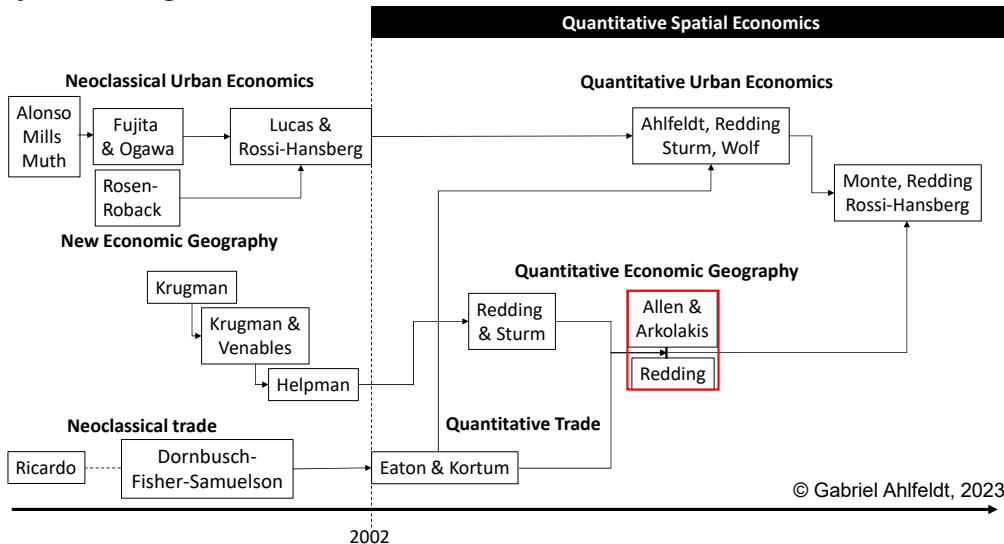
# Introduction

# Quantitative Spatial Models of Economic Geography

- ▶ Quantitative spatial models of economic geography emphasize **trade of goods**
  - ▶ Low trade costs to other locations  $\Rightarrow$  Greater market access
  - ▶ Greater market access  $\Rightarrow$  lower price index (and higher import/export shares)
  - ▶ Lower tradable prices  $\Rightarrow$  attract larger population
  - ▶ Larger population  $\Rightarrow$  higher non-tradable goods prices in **spatial equilibrium**
    - ▶ Inelastic supply of land generates dispersion force
    - ▶ Higher non-tradable prices compensate for lower tradable goods prices

**Q: How will the economic geography change if we reduce trade frictions?**

# History of thought



# Roadmap

- ▶ **Redding-Rossihansberg (2017) model**
  - ▶ A canonical QSM of Economic Geography
  - ▶ Multi-region version of the Helpman (1998) model
    - ▶ Also used in Redding & Sturm (2008)
- ▶ **Topic 9**
  - ▶ Model
  - ▶ Equilibrium
  - ▶ Quantification
- ▶ **Topic 10 (today)**
  - ▶ Counterfactuals

# Equilibrium solver

# Recall

- ▶ The equilibrium condition uses
  - ▶ the following equations trade share (9), price index (8), population mobility (13)
  - ▶ the assumption that trade costs are symmetric ( $d_{ni} = d_{in}$ )
- ▶ The equilibrium is referenced solely by  $L_n$ 
  - ▶ Can solve for  $L_n$  from Eq. (16) for given values of exogenous objects
    - ▶ structural parameters  $\{\alpha, \sigma\}$ , fundamentals  $\{A_n, H_n\}$ , trade costs  $d_{ni}$ , fixed cost,  $F$
  - ▶ We do not need to solve for any other endogenous variable simultaneously
    - ▶ there is a **recursive structure** to solve for the other endogenous objects
- ▶ Can treat  $L_n$  as our sole target variable and then solve for the rest

Sounds good in theory, in practice...



# In practice...

- ▶ ...MMH treat  $\{L_n, w_n\}$  as target variables
  - ▶ Guess them
  - ▶ Solve simultaneously
- ▶ Lesson for us:
  - ▶ Nice to show that the equilibrium can be referenced by one variable
  - ▶ If the solver is easier to write with two target variables, we may still do it...

```
Editor - /usr/net/ahlfeldg/Teaching/RRH2017-ARE/functions/solveHLwCtyOpen_E.m
calculateHHI.m GA_QSE_RRH2017_teaching.m solveHLwCtyOpen_E.m +
1  %%% Solve model
2
3  function [w_i, L_i, tradesh, dtradesh, converge, xtic] = solveHLwCtyOpen_E(fund, dist, bord, bordc, nobs)
4
5  global alpha sigma LL LLwest LLeast;
6
7  xtic = tic();
8
9  % Extract location characteristics from fundamentals matrix;
10 % fund(:,1)=a; fund(:,2)=H; fund(:,3)=Iwest; fund(:,4)=Ieast;
11 a=fund(:,1); H=fund(:,2); Iwest=fund(:,3); Ieast=fund(:,4);
12
13 % convergence indicator;
14 converge=0;
15
16 % Initialization based on a symmetric allocation;
17 L_i=double(ones(nobs,1)).*(LL./nobs);
18 w_i=double(ones(nobs,1));
19
20 % trade costs;
21 dd=double((dist.*bord.*bordc).^(1-sigma));
22
23 % *****;
24 % **** START LOOP TO SOLVE FOR WAGES AND POPULATION ****;
25 % *****;
```

## What seems noteworthy...

- ▶ Since MMH are targeting  $\{L_n, w_n\}$ , they need update rules for both
  - ▶  $L_n$  is relatively straightforward since we have a residential choice probability Eq. (15)
    - ▶ Will return  $\lambda \bar{L}$  for any combination of primitives and guesses of targets
    - ▶ Straightforward to compute  $L_n = \lambda_n \bar{L}$
    - ▶ Can use that to update guess of  $L_n$  until we converge
  - ▶ As for  $w_n$ , RRH mention:
    - ▶ Zero profits  $\Rightarrow$  wage bill equal to revenues
    - ▶ Redistribute rents to locals so that wage equals worker income
    - ▶ *income equals expenditure*
- ▶ **But how does this help with updating  $w_n$ ?**

Did you figure it out?

# Income equals expenditure

- ▶ Goods market clearing implies:
  - ▶ **Income** of workers producing goods at  $i$  must equal
  - ▶ **expenditure** of workers on goods produced at  $i$  in all  $n \in N$
- ▶ RRH compute expenditure using trade shares in solveHLwCtyOpen\_E.m
  - ▶ `income=double(w_i.*L_i)`
    - ▶ Simply the worker **income** corresponding to value of goods produced
  - ▶ `expend=double(tradesh*income)`
    - ▶ Sum of consumption of goods at all  $n$ , weighted by trade share  $\pi_{ni}$
    - ▶  $\pi_{ni}$  is the share of location  $n$ 's expenditure on goods produced in location  $i$  (in the MATLAB tradeshare matrix,  $n$  are columns,  $i$  are rows)
- ▶ If expenditure  $>$  ( $<$ ) income, we need to increase (reduce) the wage
  - ▶ `w_e=double(w_i.*(expend./income).^(1./(sigma-1)))`
    - ▶  $w$  up  $\Rightarrow$  income up, expenditure down (share in consumption of other regions will fall)
    - ▶ Higher prices  $\Rightarrow$  lower demand

# Exact hat algebra

# Conducting counterfactuals

- ▶ The standard approach to counterfactuals (for given parameters) in QSMs is
  - ▶ **Invert** unobserved **exogenous objects** using observed data
    - ▶ e.g. location fundamentals, transport costs
  - ▶ **Change** values of selected **primitives**
    - ▶ e.g. productivity in some regions, transport cost on some routes
  - ▶ **Solve** for the equilibrium
    - ▶ Under the new primitives
  - ▶ Compare new equilibrium values to initial values and **compute relative changes**

Can we skip inversion? We are using observed data for inversion anyways...

# Exact hat algebra

- ▶ Dekle et al. (2007) denote the
  - ▶ known value of a variable in the initial equilibrium by  $x$
  - ▶ unknown value of a variable in the counterfactual equilibrium by  $x'$  (with a prime)
  - ▶ relative change in the variable by  $\hat{x} = \frac{x'}{x}$  (with a hat)
- ▶ We can compute the counterfactual value  $x' = \hat{x}x$ 
  - ▶ We only need the relative changes and initial levels of a variable
- ▶ Aim of **exact hat algebra** is
  - ▶ express **relative changes in endogenous variables** as functions of
    - ▶ **relative changes in primitives**
    - ▶ **initial values of endogenous variables**

We can derive system of equations that avoid levels of fundamentals altogether

# Example

- ▶ RRH assume that researcher
  - ▶ observes  $\{L_n, w_n, \pi_{ni}\}$
  - ▶ obtained estimates of the values of structural parameters  $\alpha, \sigma$
  - ▶ have computed a measure of relative change in bilateral trade costs  $\hat{d}_{ni}$
- ▶ They derive a system of equations to evaluate the effects of changes in trade cost
  - ▶ combining Eqs. (9), (10), (11), (12), (13)
- ▶ System of Eqs. (18), (19), (20) has **three equations and three unknowns** ✓
  - ▶ Can use it to solve for  $\{\hat{w}_n, \hat{\lambda}_n, \hat{\pi}_{ni}\}$  using only  $\{\lambda_n, w_n, \pi_{ni}, \hat{d}_{ni}\}$
  - ▶ Counterfactual values in levels are  $w'_n = w_n \hat{w}_n, \lambda'_n = \lambda_n \hat{\lambda}_n, \pi_{ni} = \pi_{ni} \hat{\pi}_{ni}$
  - ▶ Recall that  $L_n = \lambda_n \bar{L}$

# Exact hat system of equations

- ▶  $\hat{w}_i \hat{\lambda}_i (w_i \lambda_i) = \sum_{n \in N} \hat{\pi}_{ni} \hat{w}_n \hat{\lambda}_n \pi_{ni} (w_n \lambda_n)$ 
  - ▶ 'hat' variant of **income equals expenditure**  $w_i L_i = \sum_n \pi_{ni} w_n L_n$ 
    - ▶ where  $L_n = \lambda_n \bar{L}$  and  $\bar{L}$  drops out in ratios
- ▶  $\hat{\pi}_{ni} \pi_{ni} = \frac{(\hat{d}_{ni} \hat{w}_i)^{1-\sigma} \hat{L}_i \pi_{ni}}{\sum_{k \in N} (\hat{d}_{nk} \hat{w}_k)^{1-\sigma} \hat{L}_k \pi_{nk}}$ 
  - ▶ 'hat' variant of **trade shares**
    - ▶ Notice that  $\hat{L}_i$  could be written as  $\hat{\lambda}_i$  and  $A_i$  drops out in ratios
- ▶  $\hat{\lambda}_n \lambda_n = \frac{\hat{\pi}_{nn}^{-\frac{\alpha}{\sigma(1-\alpha)-1}} \lambda_n}{\sum_{k \in N} \hat{\pi}_{kk}^{-\frac{\alpha}{\sigma(1-\alpha)-1}} \lambda_k}$ 
  - ▶ 'hat' variant of of residential choice probability equation
    - ▶ Notice that  $\{A_n, H_n\}$  drop out in ratios



# Welfare

- ▶ Welfare is given by the population mobility condition:  $V_n = \frac{v_n}{P_n^\alpha r_n^{1-\alpha}} = \bar{V}$  (Eq. 13)
  - ▶  $v_n = w_n / \alpha$  (Eq. 11)
  - ▶  $P_n = \frac{\sigma}{\sigma-1} \left( \frac{L_n}{\sigma F \pi_{nn}} \right)^{\frac{1}{1-\sigma}} \frac{w_n}{A_n}$  (Eq. 10)
  - ▶  $r_n = \frac{1-\alpha}{\alpha} \frac{w_n L_n}{H_n}$  (Eq. 12)
- ▶ Plug into Eq. (13) and observe the magic of 'exact hat algebra'
  - ▶  $\hat{\bar{V}} = \frac{\bar{V}'}{\bar{V}} = \left( \frac{1}{\hat{\pi}_{nn}} \right)^{\frac{\alpha}{\sigma-1}} \left( \frac{1}{\hat{\lambda}_n} \right)^{\frac{\sigma(1-\alpha)-1}{\sigma-1}}$  (Eq. 21)
    - ▶ Various primitives  $A_n, H_n, F$  and multiplicative constants cancel out in ratios

# Merits of exact hat algebra

- ▶ Exact hat algebra allows expressing counterfactual values with **fewer primitives**
  - ▶ **Simplifies notations**
    - ▶ Can drop exogenous objects  $\Rightarrow$  leaner equations
  - ▶ **Counterfactuals more transparent**
    - ▶ Link between forcing variable and outcome variables in relative differences
  - ▶ Can **save a space** in the paper as we can skip inversion
    - ▶ QSE papers tend to be too large  $\Rightarrow$  Issue in publication process

Q: How do RRH use exact hat algebra in their code?

# Counterfactuals

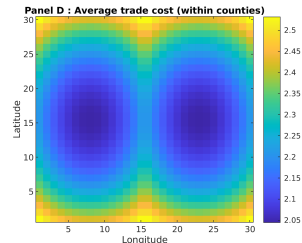
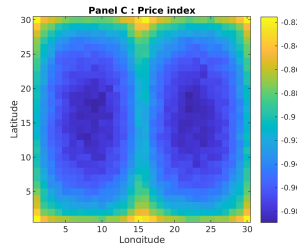
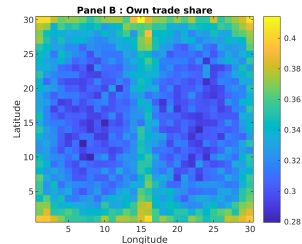
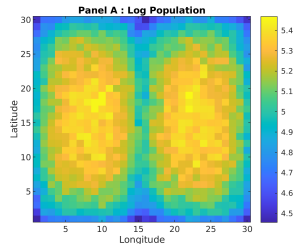
# Replication directory

**You can replicate the following counterfactuals using the RRH replication directory with GA edits available from [Moodle](#)**  
Follow the GA\_QSE\_RRH2017\_teaching.m script

# Recall

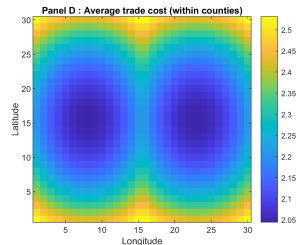
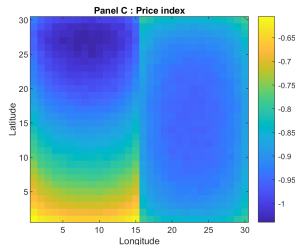
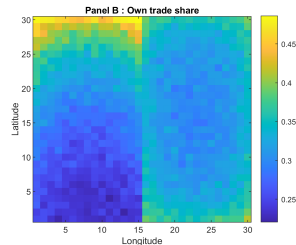
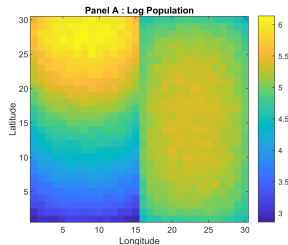
- ▶ RRH model with GA's parameterization of  $A_n$
- ▶ Central locations have lower average trade costs (within countries)
- ▶ Results in more trade (lower own share) and a lower price index
- ▶ And, indeed, a greater population

Q: What will happen if we increase productivity in the north of the western country?



# With productivity gradient

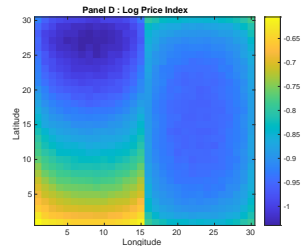
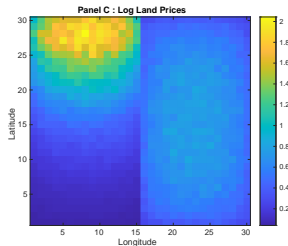
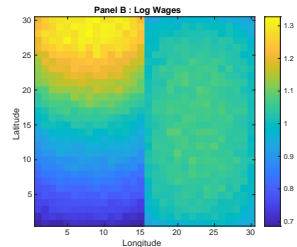
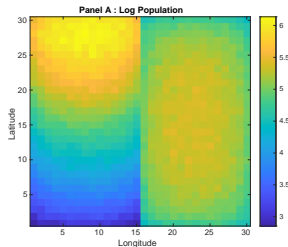
- ▶ Higher productivity in the north of west, shifts labour demand outwards
- ▶ Larger population in the north of west
- ▶ Own trade share increases in the north of west
  - ▶ not because places are more remote
  - ▶ they are larger and sell more to themselves  $\Rightarrow$  'home market effect'



# Initial equilibrium

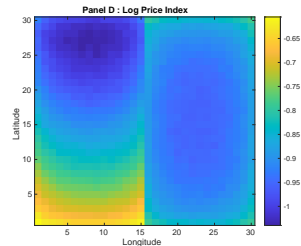
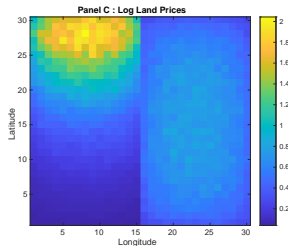
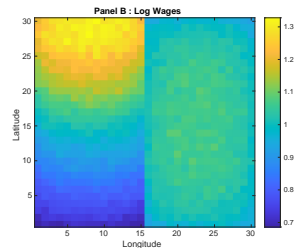
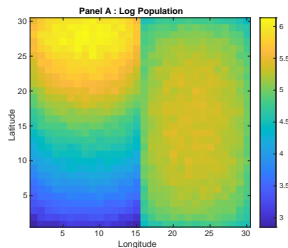
- ▶ Higher productivity in the north of west:
  - ▶ higher wages
  - ▶ higher rents
- ▶ West has **more unequal** distribution of productivity
  - ▶ Also a **more unequal** distribution of pop.
    - ▶ HHI West: 34.7%
    - ▶ HHI East: 23%

Q: What happens when removing border frictions **wit-  
hin** countries?



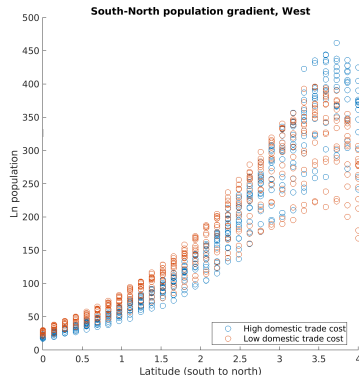
# No domestic borders

- ▶ Iceberg border friction between grid points now 1 instead of 2
  - ▶ Within East and West
    - ▶ Friction between East and West persists
- ▶ Agglomeration force weaker  
⇒ less inequality in West
  - ▶ HHI West: 31.1% (-3.6pp)
- ▶ Hardly any change in East
  - ▶ HHI East: 23.4% (+0.4pp)

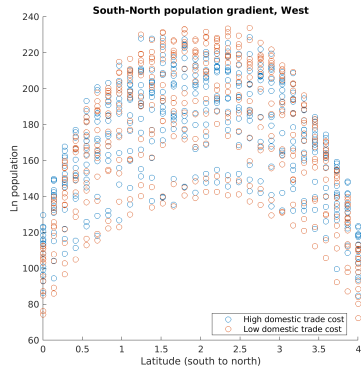




# Gradients with and without domestic borders



Agglomerated north loses pop. in **west**  
Weaker agglo force  $\Rightarrow$  less inequality



No big changes in the **east**  
Perhaps a bit more dispersion

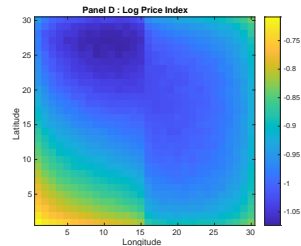
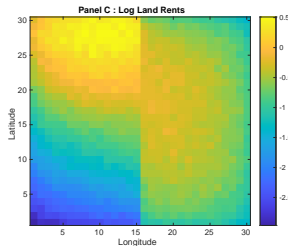
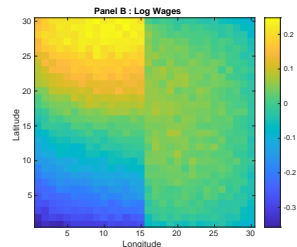
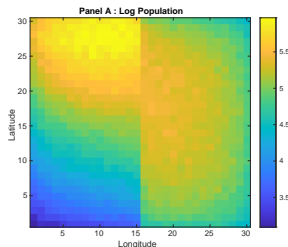
# Trade costs and agglomeration

- ▶ **High trade costs reinforce fundamental productivity advantages**
  - ▶ Market access generates an agglomeration force
  - ▶ Consumers benefit from access to products
  - ▶ Firms benefit from access to consumers
  - ▶ With higher trade costs, stronger agglomeration effect

Q: What happens when we remove border frictions **between** countries?

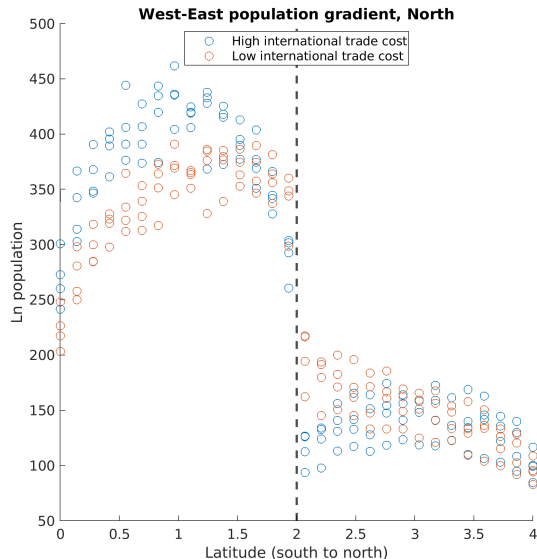
# No international borders

- ▶ Economic activity shifts towards the international border
  - ▶ Formerly remote areas now have the greatest market access
- ▶ Positive welfare effect: 3.4%
  - ▶ All regions benefit from **greater market access**
  - ▶ Great market access  $\Rightarrow$  smaller own trade share  
 $\pi_{nn} \Rightarrow$  greater  $\bar{V}$  (see Eq. 21)



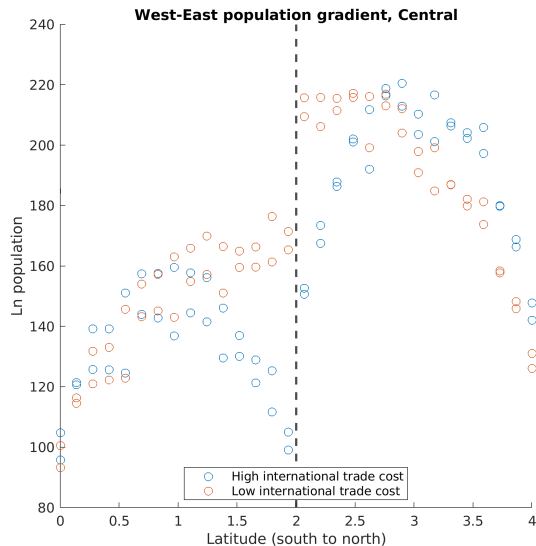
# No international borders: North

- ▶ Economic activity shifts towards the international border
- ▶ **'Dip' disappears** on both sides of the border
- ▶ In the less productive east, population density increases close to the border
  - ▶ Locations in the north-east benefit from market access to the productive north-west



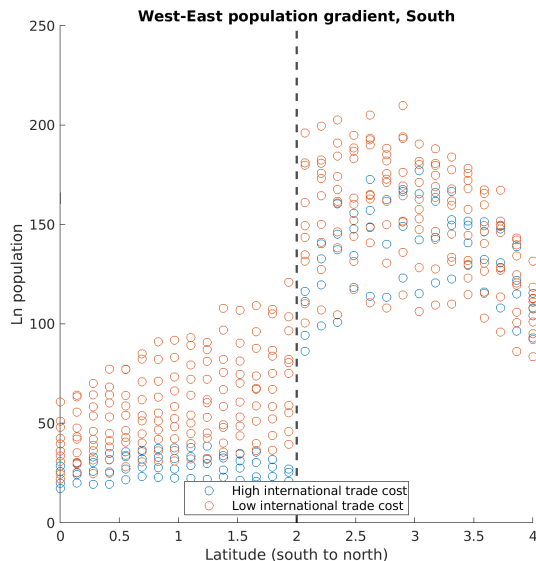
# No international borders: Central

- ▶ Productivity is larger in the west, but central parts of east are more populated
  - ▶ In the west, fundamental productivity advantage attracts workers to north
  - ▶ In the east, market access advantage attracts workers to central parts
- ▶ Without border friction, **border dip disappears**



# No international borders: South

- ▶ Productivity is similar on both sides of the border
- ▶ But east has a greater market access since much of the population in the west is in the north)
- ▶ Without border friction
  - ▶ **border dip disappears**
  - ▶ Population **density gradient emerges in the west** since areas close to east benefit from market access



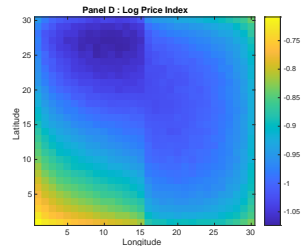
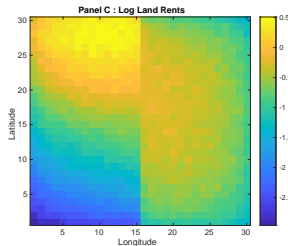
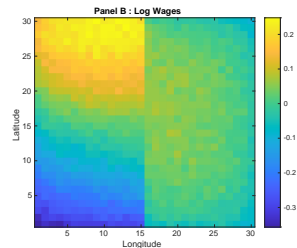
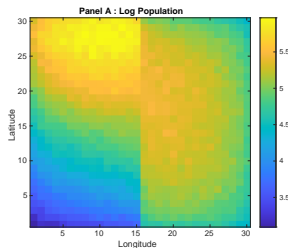
# Market access and border effects

- ▶ **Removal of border frictions turns remote areas into central areas**
  - ▶ Increase in market access on both sides of the border  $\Rightarrow$  Local economic development
  - ▶ Effect particularly large on the side of the border with initially lower market access
- ▶ **Compelling evidence** supports the role of market access
  - ▶ Evidence from German Division and Reunification: Redding & Sturm (2008)
  - ▶ Evidence from EU integration: Caliendo et al. (2021)

Q: What's the international border effect under lower domestic trade frictions?

# No international borders

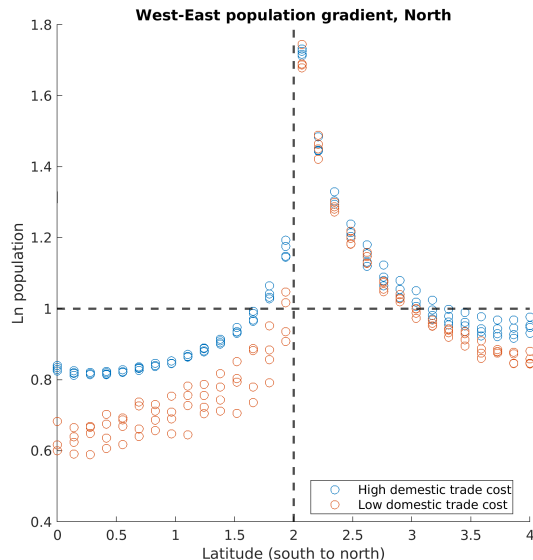
- ▶ Same experiment as before, but **without border frictions between grid cells**
  - ▶ domestic border friction in initial equilibrium and counterfactual
  - ▶ Only international border friction changes
- ▶ Similar spatial adjustments
  - ▶ Economic activity shifts towards the international border
- ▶ **Larger positive welfare effect: 5% (instead of 3.4%)**





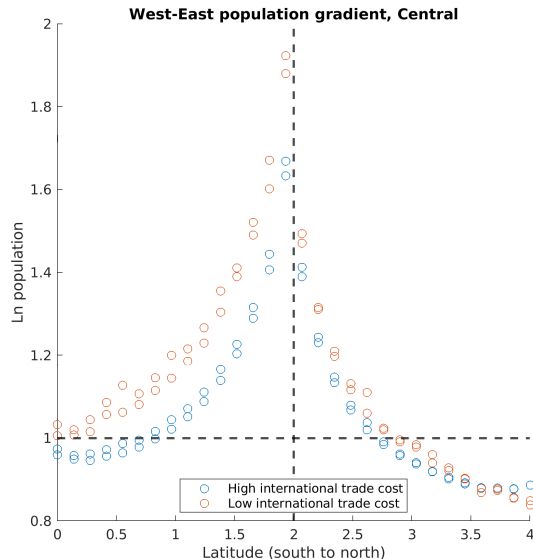
# No international borders: North

- ▶ Economic activity shifts towards the international border
- ▶ **Stronger adjustments** under low domestic trade costs
  - ▶ Loss of economic activity in west is GE effect stronger gains in MA in central and southern parts of west



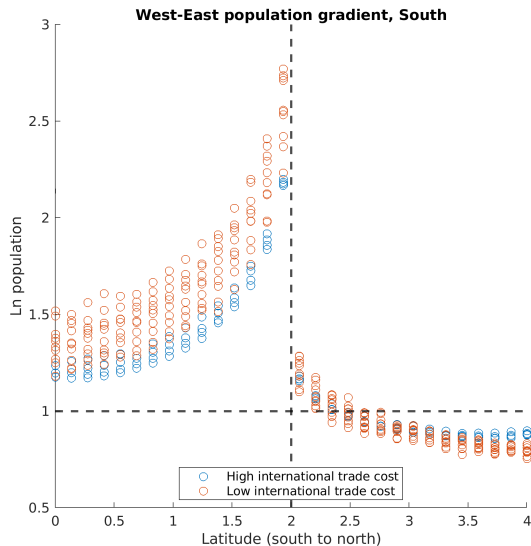
## No international borders: Central

- ▶ Economic activity shifts towards the international border
- ▶ **Stronger adjustments** under low domestic trade costs



## No international borders: South

- ▶ Economic activity shifts towards the international border
- ▶ **Stronger adjustments** under low domestic trade costs



# Conclusion

# Summary

- ▶ **QSMs of Economic Geography** emphasize the trade between regions
  - ▶ Market access (MA) plays a crucial role
    - ▶ Reduces tradable goods price index and attracts workers
  - ▶ Prediction that market access promotes economic development has empirical support
- ▶ In reduced-form, **trade MA works similar to commuting MA**
  - ▶ At a small geographic (within cities) commuting MA clearly more important
  - ▶ At a large geographic scale (between labour markets) trade MA more important
  - ▶ At intermediate geographic scale, both could matter...

Next week: **Commuting and trade in one model**

# Literature I

## Core readings

- ▶ Redding, S., Rossi-Hansberg, E. (2017): Quantitative Spatial Economics. *Annual Review of Economics*, 9, 21–58.

## Other readings

- ▶ Caliendo, L., Oromolla L. D., Parro, F., Sforza, A. (2021): Goods and Factor Market Integration: A Quantitative Assessment of the EU Enlargement. *Journal of Political Economy*, 129(12)
- ▶ Dekle R., Eaton J., Kortum S. (2007): Unbalanced trade. *American Economic Review*, 97(2):351–55
- ▶ Redding, S., Sturm, D. (2008): The Costs of Remoteness: Evidence from German Division and Reunification. *American Economic Review*, 98(5), 1766–97