

# **Topic 1**

## **Rosen-Roback**

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Quantitative Spatial Economics

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## Acknowledgements

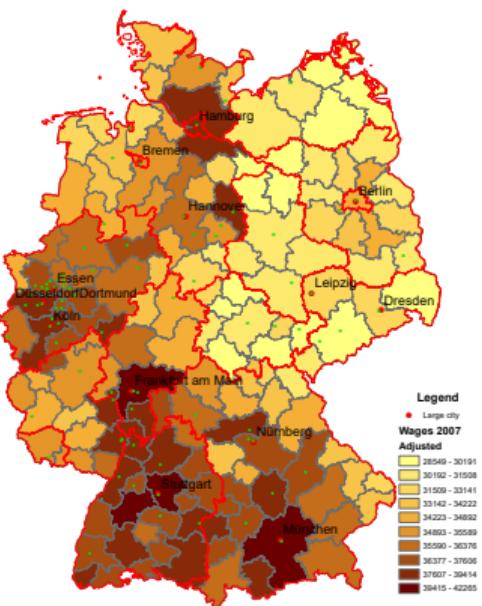
- This slide deck
    - builds on the lecture "spatial equilibrium across cities" by Tuukka Saarimaa (2021)
    - builds on the lecture "Rosen-Roback Framework" by Felipe Carozzi (2023)
    - uses material from Brueckner's textbook Chapter 11

## Introduction

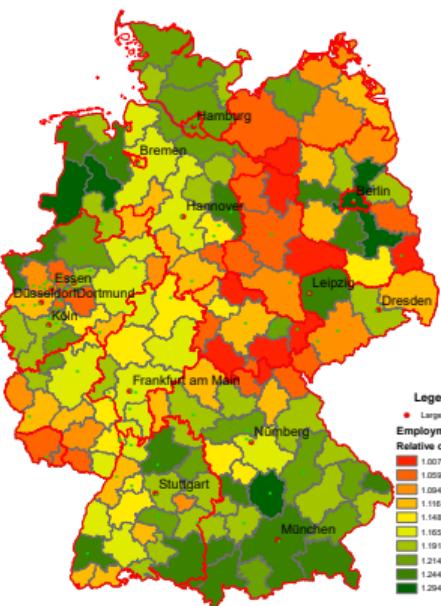
## Wage and price differences in Germany

Does **migration equalize** wages and rents across regions?

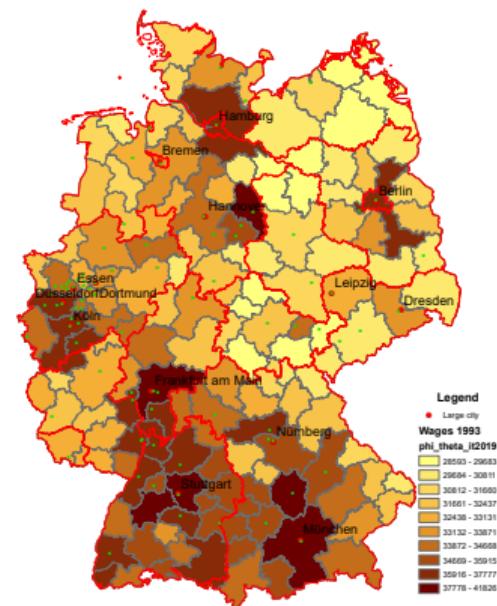
## Wage differences in Germany



(a) Wage  
2007

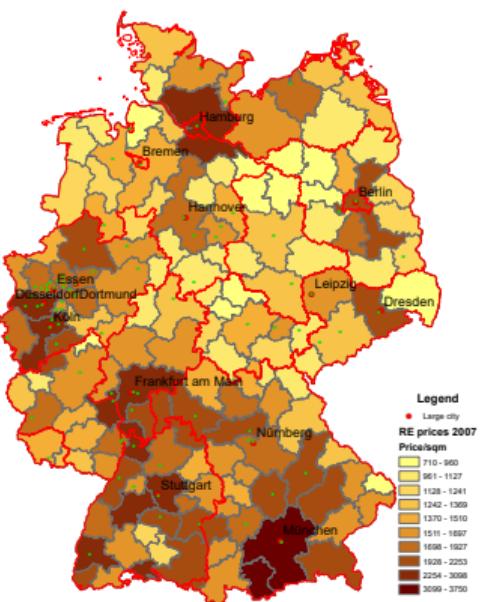


(b) Employment change  
2007-2019

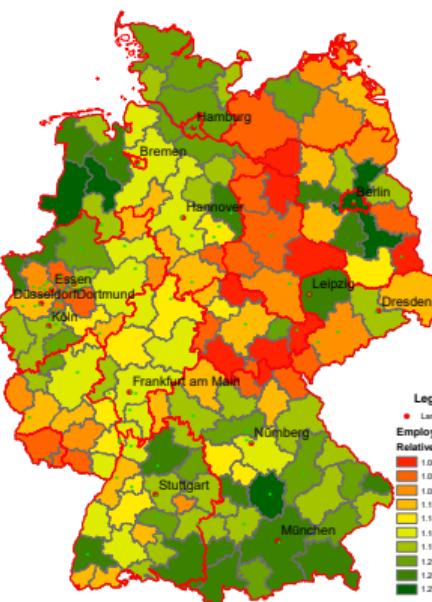


(c) Wage  
2019

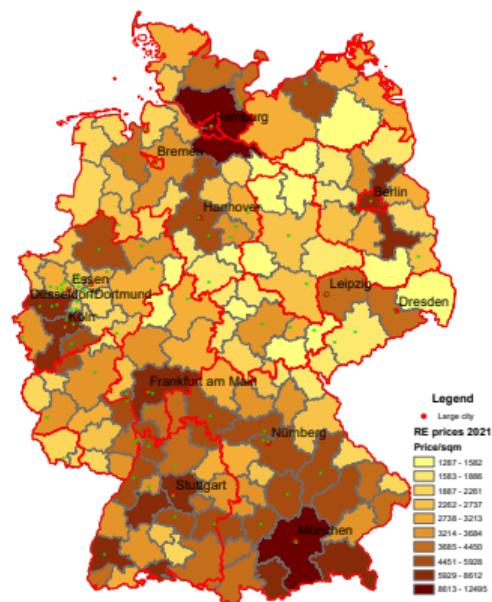
## Property price differences in Germany



(a) Price/sqm  
2007



(b) Employment change  
2007-2019



(c) Price/sqm  
2021

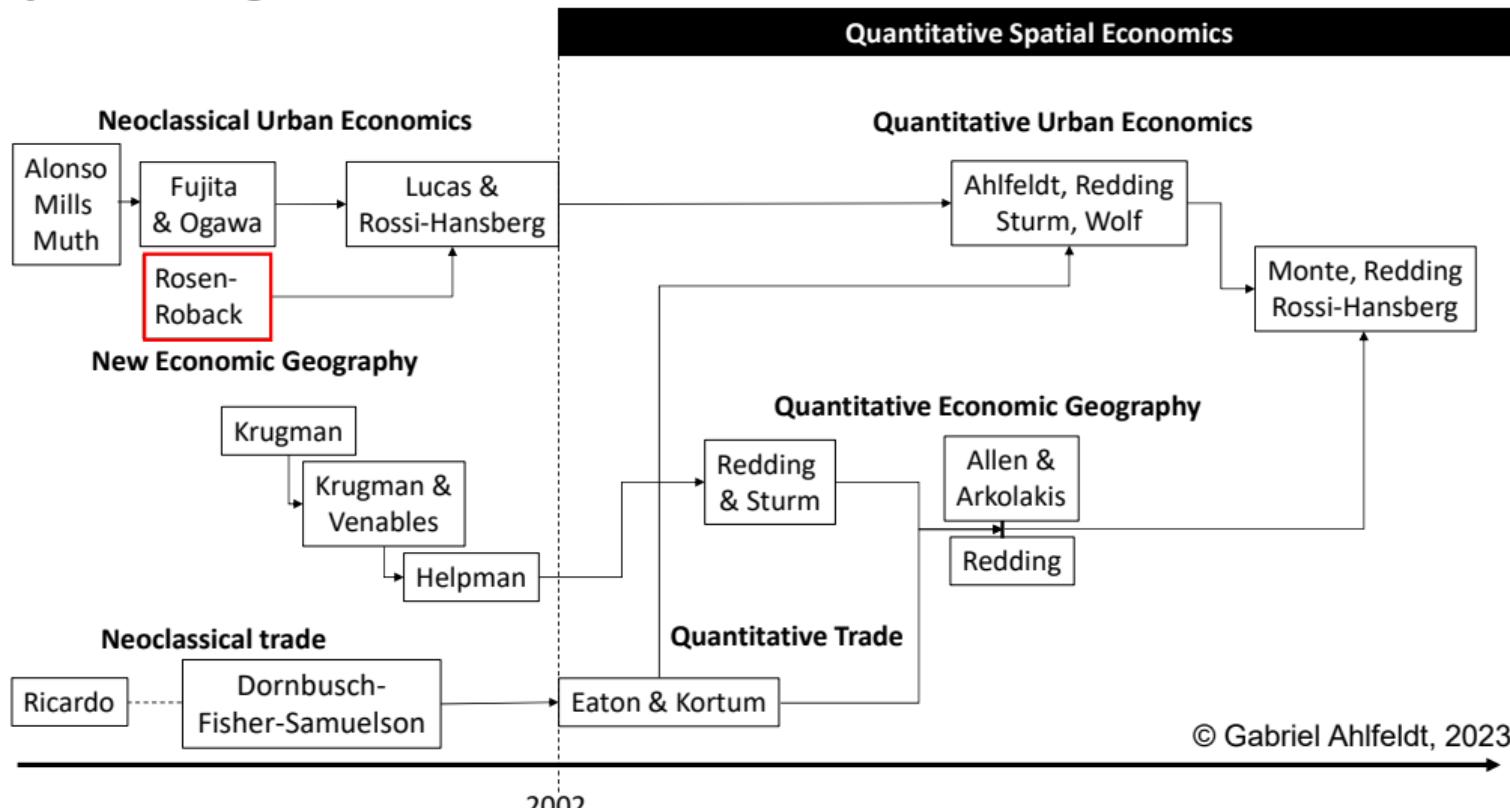
# Wage and price differences in Germany

If one city is an **attractive place to live**, will it have high or low real wages?

# Roadmap

- ▶ We analyze the **tradeoff** between income, amenities, and housing costs
  - ▶ across cities or regions
- ▶ We introduce the **spatial equilibrium** assumption
  - ▶ Identical utility levels across regions
  - ▶ Identical firm profits across regions
- ▶ **Rosen-Roback framework** is a core concept in urban economics
  - ▶ Name refers to Rosen (1979) and Roback (1982)
  - ▶ Refers to a family of models that build on the spatial equilibrium assumption
- ▶ Workhorse tool for valuing non-marketed goods

# History of thought



Roback (1982)

- ▶ A famous paper by Jennifer Roback's 1980 PhD thesis
    - ▶ over 4700 Google Scholar citations (01/2024)
  - ▶ **Not only important to urban economics**, but also to
    - ▶ Labor economics (local labor markets)
    - ▶ Environmental economics (pricing of environmental amenities)
    - ▶ Trade (markets and industry concentration)
    - ▶ Development economics (migration)
    - ▶ Local public finance

## Basic assumptions

- The basic framework for the analysis is a simple **general equilibrium model**
    - capital and labor are assumed **completely mobile** across cities.
      - Complete mobility of labor means that the costs of changing residences are zero
      - Workers are homogenous, no idiosyncratic tastes for locations
    - **Intercity commuting costs** are assumed **prohibitive**
      - Workers live and work in the same local labour market
    - **Intracity commuting costs** are **zero**
      - focus attention on the across-city allocation of workers and firms
    - **Floor space supply is fixed** within cities
      - but is assumed interchangeable between uses within a city

## Amenities

- ▶ **Consumer utility** depended on the consumption of
    - ▶ a composite commodity  $c$ 
      - ▶ A basket of freely tradable goods
    - ▶ and housing  $q$
  - ▶ Consumers or workers also get utility from **urban amenities**
    - ▶ Different cities have different **exogenous amenities**
      - ▶ E.g., pleasant climate, nature, clean air, low crime
      - ▶ Amenities may be endogenous and depend on city's population
  - ▶ **Quality of life** is an index of many different amenities (or disamenities)
    - ▶ we will denote it by  $a$
    - ▶ akin to TFP shifter in production function

## Utility function and dependence

- Utility depends on consumption of marketed ( $c, q$ ) and non-marketed ( $a$ ) goods

$$u = u(c, q, a)$$

- ▶ Utility is maximized subject to the **budget constraint**  $y = c + pq$ ,
    - ▶ where the wage  $y$  is the sole source of income, the unit rent of housing is  $p$  and the commodity price is the numeraire (normalized to one)
  - ▶ Using the first-order conditions, we obtain the **indirect utility**

$$V = V(y, p, a)$$

- Naturally, we have  $\frac{\partial V}{\partial y} > 0$ ,  $\frac{\partial V}{\partial p} < 0$ ,  $\frac{\partial V}{\partial a} > 0$ 
    - We view  $a$  as quality of life (a composite amenity)
    - $a$  could also be a disamenity, in which case, of course,  $\frac{\partial V}{\partial a} < 0$

# Spatial equilibrium

- ▶ In equilibrium, workers must be **indifferent across locations**
  - ▶ If not, workers would **move to locations offering higher utility**
  - ▶ This will lead to **bidding up housing prices or pushing down wages**
  - ▶ The process will only stop when utilities are equalized everywhere
- ▶ Formally, this implies that the following condition must hold for all cities

$$V(y, p, a) = \bar{u}$$

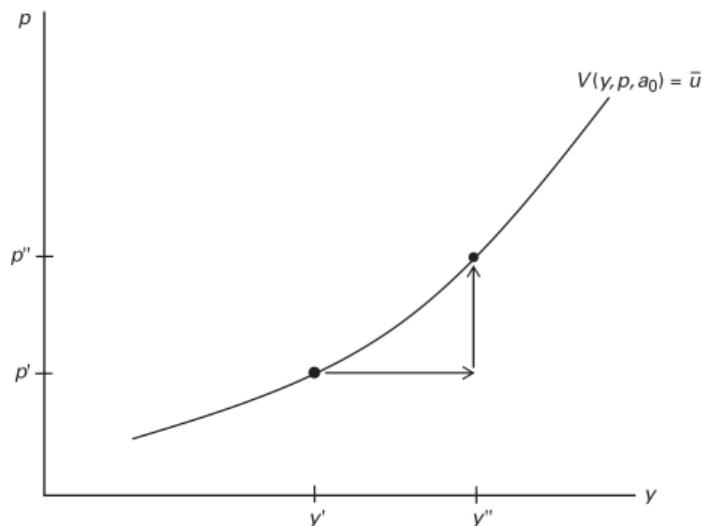
- ▶ For a given  $a$ , the indifference condition will only be satisfied if the effects of **differences in  $p$  and  $w$  across cities offset each other**

**Spatial equilibrium is a core concept of spatial economics**

Indifference curves are upward sloping

- ▶ Consider the point  $(p', y')$ 
    - ▶ The bottom-left point
  - ▶ Suppose that wage increases from  $y'$  to  $y''$ 
    - ▶ Moving to the right
    - ▶ This change would raise utility
  - ▶ Rent must increase
    - ▶ to keep utility constant
    - ▶ Moving up from  $p'$  to  $p''$

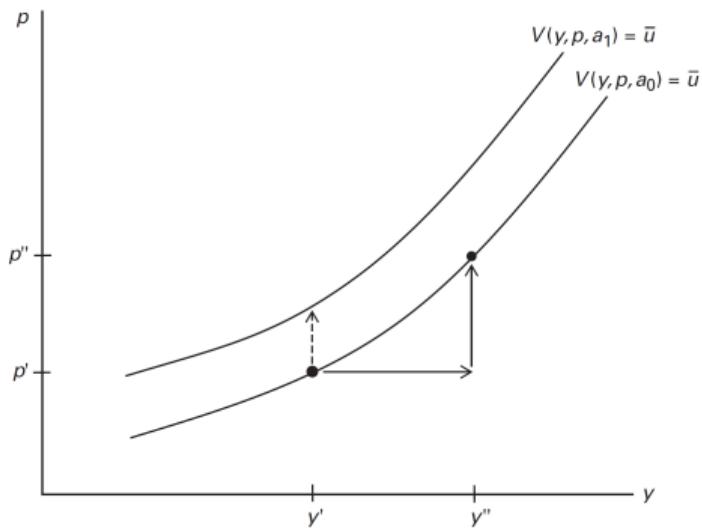
Q: How does the indifference curve shift if  $a$  increases?



# Indifference curves with different amenity levels

- ▶ Now consider another city with amenity level  $a_1 > a_0$
- ▶ Ceteris paribus,  $V$  would increase since  $\frac{\partial V}{\partial a} > 0$
- ▶ To ensure  $V = \bar{u}$  in both cities, we require that:
  - ▶  $p_1 > p_0$
  - ▶  $y_1 < y_0$
  - ▶ or a combination of both

Compensating differential



# Firms

- ▶ Firms produce the non-housing commodity  $c$  using the **inputs**
  - ▶ **Labour**, hired at wage  $y$
  - ▶ **Floor space** (of factories and offices), rented at rent  $p$ 
    - ▶ One integrated floor space market, no land use frictions
  - ▶ Other inputs can be ignored, as long as their price is the same in all regions (capital)
- ▶ Constant returns to scale
  - ▶ Cost per unit of output is the same no matter how much is being produced
- ▶ **Cost function**  $C = C(y, p, a)$  also **depends on**  $a$ , with  $\frac{\partial C}{\partial y} > 0$ ,  $\frac{\partial C}{\partial p} > 0$ ,  $\frac{\partial C}{\partial a} \leqslant 0$ 
  - ▶ An amenity to workers may be an amenity to firms (low crime)
  - ▶ May be a disamenity to firms (regulation of pollution)

# Spatial equilibrium (again)

- ▶ **Perfect competition and mobility** ensure that firms make **zero profits**
  - ▶ If a city offers greater profits it will attract firms
  - ▶ This will increase factor prices (real estate, labor)
  - ▶ Until **zero-profit equilibrium** is restored
- ▶ Since the price of  $c$  is the numeraire ( $=1$ ), the **spatial equilibrium condition** is:

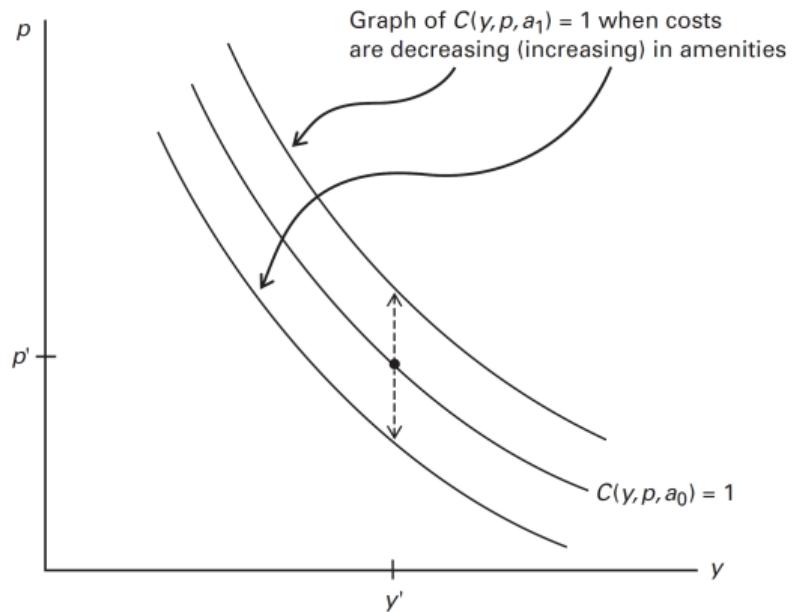
$$C(y, p, a) = 1$$

- ▶ Just saying that the unit cost  $C$  must equal the price of  $c$
- ▶ This condition generates **iso-profit curves** in the  $p$  and  $y$  space
  - ▶ Combinations of  $p$  and  $y$  that yield zero profits for a given level of  $a$

Does a shift iso-profit curve up or down?

# Iso-profit Curves

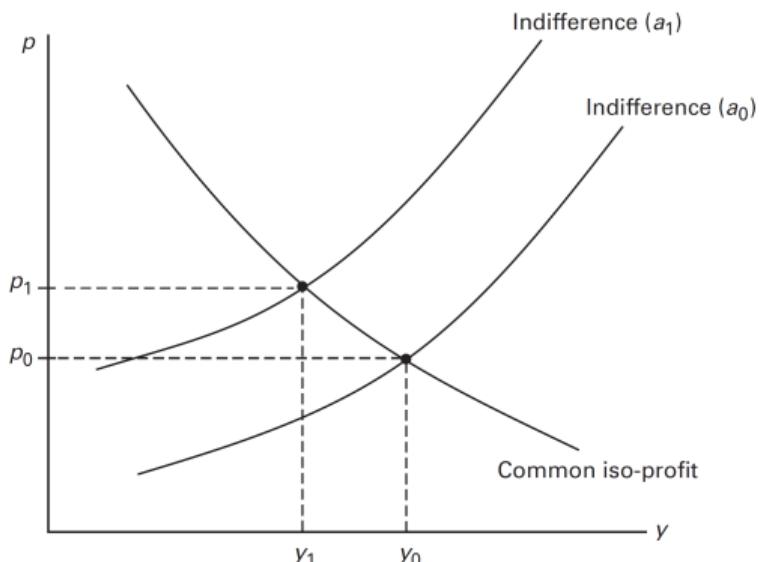
- ▶ Iso-profit curves are downward-sloping
  - ▶ For a given  $a$ , a higher  $y$  must be compensated by a lower  $p$  to keep unit cost constant
- ▶ If the amenity lowers costs,  $a_1 > a_0$ :
  - ▶  $p_1 > p_0$
  - ▶  $y_1 > y_0$
  - ▶ or a combination of both
- ▶ If the amenity increases costs
  - ▶ compensating differentials will point in opposite direction



# Comparative statics: No amenity effect on firm cost

- ▶ **Equilibrium** must satisfy
  - ▶ indifference condition
  - ▶ iso-profit condition
- ▶ Holds only for the **intersection point**
  - ▶ Gives market-clearing  $p$  and  $y$
- ▶  $a_1 > a_0$ , shifts only indifference curve
  - ▶ No effect on firm costs
- ▶ Move uphill along the iso-profit curve

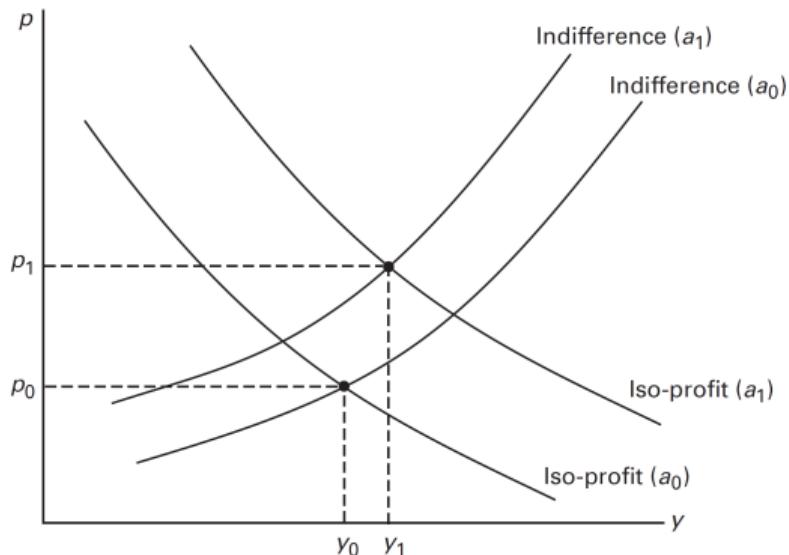
Workers accept **lower wage** and  
**higher rent** in more attractive city



# Comparative statics: Firm costs decreasing in consumer amenity

- ▶  $a_1 > a_0$  shifts iso-profit curve up
  - ▶ Firms can afford higher  $\{y, p\}$
- ▶ Two forces pushing for higher rents
  - ▶ Firms accept higher rents
  - ▶ Residents accept higher rents
- ▶ Countervailing forces on wages
  - ▶ Firms can afford higher wages
  - ▶ Residents accept lower wages

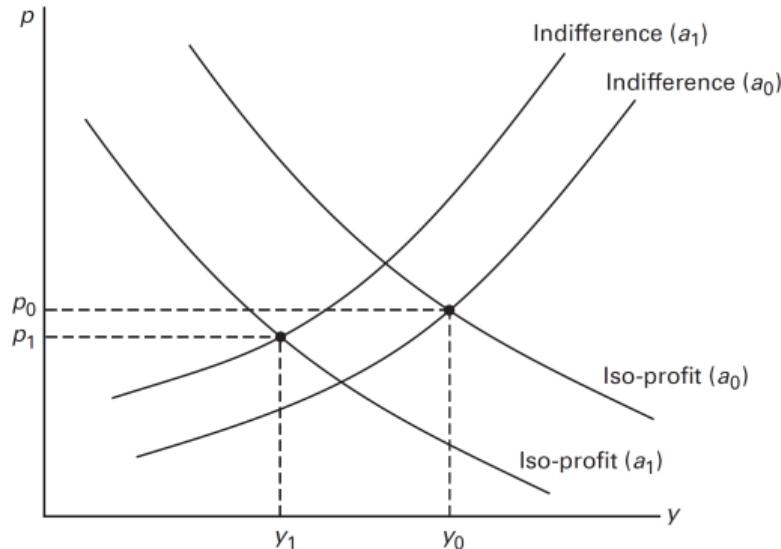
**Nominal wage** may be higher in more attractive city. But workers still accept **lower real wage**



# Comparative statics: Firm costs increasing in consumer amenity

- ▶  $a_1 > a_0$  shifts iso-profit curve down
  - ▶ Firms need to save on  $\{y, p\}$
- ▶ Two forces pushing for lower wages
  - ▶ Firms need to save costs
  - ▶ Residents accept lower wages
- ▶ Countervailing forces on rents
  - ▶ Firms need to save costs
  - ▶ Residents accept higher rents

Rent may be lower in more attractive city. But workers still accept **lower real wage**



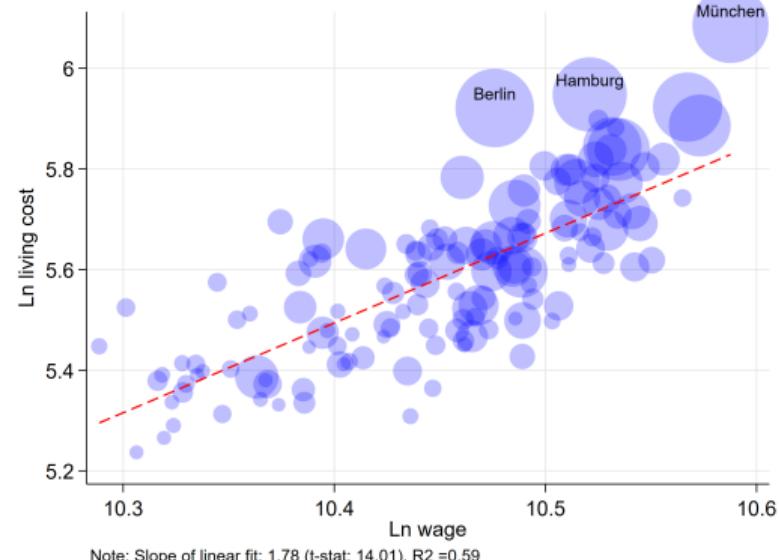
# Equilibrium recap

- ▶ City with **high quality of life always has low real wage**
  - ▶ City with high quality of life and **high production amenity** (low cost)
    - ▶ High wage and even higher rent
  - ▶ City with high quality of life and **low production amenity** (high cost)
    - ▶ High rent, but not necessarily high wage

What type of city is Berlin?  
High or low quality of life? high or low production amenity?

# Wages and rents in Germany

- ▶ Cities high above fitted line, have a high quality of life
  - ▶ See, Albouy and Lue (2015)
- ▶ Need to control for worker quality differences across regions
  - ▶ Wage index form between-city movers controlling for individual fixed effects
  - ▶ IAB data, 100% sample of employees
- ▶ Need to control for house quality differences across regions
  - ▶ Ahlfeldt, Heblich, Seidel (2022)



## Quantitative framework

## A quantitative framework

- ▶ Roback's (1982) theory is elegant and general
    - ▶ Avoids restrictions to specific functional forms
  - ▶ For **quantitative analyses**, we need a parametric model
    - ▶ **Estimate** key parameters
    - ▶ **Rationalize** observed data (e.g. wages and prices) in levels and changes
    - ▶ **Conduct counterfactuals**—predict what happens if fundamentals change
  - ▶ **Glaeser & Gottlieb (2009)** provide a quantitative spatial equilibrium framework
    - ▶ Very much in the tradition of Rosen-Roback
      - ▶ Competitive market in goods and factors
      - ▶ Free mobility and spatial equilibrium
      - ▶ Identical agents
    - ▶ Notice that cities are nested in a larger economy; no fixed worker endowment

## A simple quantitative model following Glaeser & Gottlieb (2009)

- **Quantitative models** typically consist of
    - Primitives that are **exogenous**, including fundamentals, endowments, and parameters
    - **Endogenous objects** that are solved within the model
  - Gottlieb and & Glaeser (2009) model cities, indexed by  $c$ , which differ in terms of

## **Exogenous Factors**

- ▶ Productivities –  $A_c$ .
  - ▶ Amenities –  $B_c$ .
  - ▶ Land availability –  $M_c$ .
  - ▶ Also a range of **para**

## Endogenous Elements

- ▶ Population –  $L_c$ .
  - ▶ Housing prices –  $P_c$ .
  - ▶ Wages –  $W_c$ .

## Labor Market I

- Competitive firm(s) produce a tradable good with  $p_q = 1$  (Cobb-Douglas)

$$Q = A_c L^\alpha K^\beta \bar{Z}^{1-\alpha-\beta}$$

- ▶  $L$  is the labor factor, priced per city at  $w_c$
  - ▶  $K$  is tradable capital, common price  $r = 1$ .
  - ▶  $\bar{Z}$  is a fixed factor in the city (e.g., land allocated for production)

- Maximize profit:

$$\max_{L,K} Q(L, K, \bar{Z}) - wL - K$$

- ## ► First-Order Conditions

- for  $K$ :  $\frac{\partial \Pi}{\partial K} = \beta A_c L^\alpha \bar{Z}^{1-\alpha-\beta} = K^{1-\beta}$
  - for  $L$ :  $\frac{\partial \Pi}{\partial L} = \alpha A_c K^\beta \bar{Z}^{1-\alpha-\beta} = L^{1-\alpha} w_c$

## Labor Market II

### ► Inverse demand function:

- ▶ Substituting  $K$  from FOC for capital into FOC for labor condition
  - ▶ taking logs

$$\log(w_c) = \Omega_1 + \frac{\log(A_c)}{1-\beta} - \frac{1-\alpha-\beta}{1-\beta} \log(L_c) + \frac{1-\alpha-\beta}{1-\beta} \log(\bar{Z})$$

- ▶ wage  $w_c$  by fundamental productivity
  - ▶  $w_c$  falls in population  $L_c$ 
    - ▶ Downward slope generated by the fixed factor  $\bar{Z}$
  - ▶ Notice that unlike in Roback (1982) **floor space is not an input** into production

## Housing Market

- ▶ **Housing** demand generated by the resident population
    - ▶ Preferences  $u(x, h) = B_c x^{1-\sigma} h^\sigma$
    - ▶ Use F.O.C. to derive Marshallian demand  $h = \frac{\sigma w_c}{P_c}$
    - ▶ Housing demand:  $H_D = h_c L_c = \frac{\sigma w_c}{P_c} L_c$
  - ▶ **Supply of housing** produced by builders developers using one unit of land
    - ▶ Convex construction costs:  $c(D) = dD^\delta$ , where  $D$  is the building height &  $\delta > 1$
    - ▶ Abstracting from land costs, max. profits  $\Pi = (P_c D - \delta_0 D^\delta)$  yields  $D_c^* = \left(\frac{P_c}{d_0 \delta}\right)^{\frac{1}{\delta-1}}$
    - ▶ Housing supply:  $H_S = M_c \times D_c^* = M_c \left(\frac{P_c}{d_0 \delta}\right)^{\frac{1}{\delta-1}}$
  - ▶ Housing market clears in **equilibrium**:  $H_D = H_S$ 

$$\Rightarrow \log(P_c) = \Omega_2 + \frac{\delta - 1}{\delta} [\log(w_c) + \log(L_c) - \log(M_c)]$$

## Spatial equilibrium

- ▶ Cities have the same level of utility
    - ▶ Just like in Roback (1982)
  - ▶ Max utility:  $u(x, h) = B_c x^{1-\sigma} h^\sigma$  s. t. budget constraint:  $P_c + x = W_c$ 
    - ▶ Indirect utility:  $V(W_c, P_c, B_c) = (1 - \sigma)^{-1-\sigma} \sigma^\sigma \frac{B_c W_c}{P_c^\sigma}$
  - ▶ **Equilibrium**
    - ▶ Constant utility in urban system:  $V(W_c, P_c, B_c) = \bar{V}$
  - ▶ **Indifference condition** (in logs):
 
$$\log(W_c) = \Omega_3 - \log(B_c) + \sigma \log(P_c)$$
    - ▶ Upward-sloping just like in Roback (1982)

## General equilibrium I

- We have derived a system of **three** equations:

$$\log(w_c) = \Omega_1 + \frac{\log(A_c)}{1-\beta} - \frac{1-\alpha-\beta}{1-\beta} \log(L_c) + \frac{1-\alpha-\beta}{1-\beta} \log(Z)$$

$$\log(P_c) = \Omega_2 + \frac{\delta - 1}{\delta} [\log(w_c) + \log(L_c) - \log(M_c)]$$

$$\log(w_c) = \Omega_3 - \log(B_c) + \sigma \log(P_c)$$

- ▶ Has **three** endogenous objects: Population, wages, housing prices
  - ▶ All the other objects are exogenous,  $(A_c)$ ,  $(M_c)$ ,  $(B_c)$ , and parameters

As many equations as unknowns  $\Rightarrow$  Solve for the general equilibrium ✓

## General equilibrium II

- Solve for the endogenous objects and express them in terms of exogenous objects

$$\log(L_c) = \Omega_4 + \eta(\delta + \sigma - \sigma\delta) \log(\hat{A}_c) + \eta(1 - \beta)(\delta \log(B_c) + \sigma(\delta - 1) \log(M_c))$$

$$\log(w_c) = \Omega_5 + \sigma\eta(\delta - 1)\log(\hat{A}_c) - \eta(1 - \alpha - \beta)(\delta\log(B_c) + \sigma(\delta - 1)\log(M_c))$$

$$\log(P_c) = \Omega_6 + (\delta - 1)\eta(\log(\hat{A}_c) + \alpha \log(B_c) - (1 - \alpha - \beta) \log(M_c)))$$

- with  $\eta = [(1 - \beta)\delta - \alpha(\sigma + \delta - \sigma\delta)]^{-1}$ ,  $\log(\hat{A}_c) = \log(A_c) + (1 - \alpha - \beta) \log(Z_c)$

► **Useful homework:** Rearrange eqs. from previous slide to obtain above eqs.!

- Can now solve for endogenous objects for any given values of primitives

Ready for counterfactuals statics ✓

## Counterfactuals

- Can quantify the effect of a change in an exogenous location characteristic:

$$\ln Y_c^1 = \frac{\partial \ln Y_c}{\partial \ln X_c} d \ln X_c + \ln Y_c^0$$

- ▶ where superscript 1 denotes the counterfactual and 0 denotes the initial value
  - ▶ **Play around with the GG2009-toolkit to strengthen your intuition!**
    - ▶ Will deliver  $\Delta \ln Y$  for user-specified  $\Delta \ln X$  [▶ Open GG2009-toolkit](#)
  - ▶ Qualitatively, we obtain the following derivatives

	$\Delta A_c$	$\Delta B_c$	$\Delta M_c$
$\Delta L_c$	+	+	+
$\Delta w_c$	+	-	-
$\Delta P_c$	+	+	-

## Estimation

- Need to **quantify the model** before we can work quantitatively
    - First step is to settle on **parameter values**, which we can **set or estimate**
  - For example, solve the inverse labour supply equation for  $\log(L_c)$

$$\log(L_c) = \underbrace{\Omega_1 \times \frac{1-\beta}{1-\alpha-\beta} + \log(\bar{Z})}_{\text{constant}} - \underbrace{\frac{1-\beta}{1-\alpha-\beta}}_{\text{labour demand elasticity}} \log(w_c) + \underbrace{\frac{1}{1-\alpha-\beta} \log(A_c)}_{\text{structural residual}}$$

- Can estimate using labour demand elasticity in **reduced-form**
    - Can infer  $\beta$  if we know  $\alpha$  and vice versa
    - Identifying variation in  $\log(w_c)$  must be uncorrelated with demand shifter,  $A_c$ 
      - **Excludable instrument** shifts labour supply, exclusively, e.g.  $B_c$

### **Structural model helps developing transparent identification strategy**

## Inversion

- To fully quantify the model, we also need to observe or **invert fundamentals**
    - Using observed values of endogenous variables and given parameter values
  - To recover quality of life,  $B_c$ , we can rearrange the indifference condition.

$$\log(B_c) = \Omega_3 + \sigma \log(P_c) - \log(W_c)$$

- To identify  $B_c$  up to a multiplicative constant we just need:
    - Need wages,  $w_c$ , rents (or prices)  $P_c$  and the expenditure share of housing,  $\delta = 0.33$
    - Typical value used in the literature (see Ahlfeldt & Pietrostefani, 2019)

**Have fun with the inversion in the tutorial**

## Valuing amenities

## Valuing non-marketed goods

- ▶ Recovering  $B_c$  is not only useful to measure quality of life
    - ▶ And compute quality-of-life rankings
  - ▶ We use the 'compensating differential' to value non-marketed goods

$$\ln B_c = \sum_j \gamma^j \ln X_c^j + \varepsilon_c,$$

- ▶ where  $X_c^j$  is one of  $j \in J$  (dis)amenities
  - ▶ It is straightforward (in principle) to estimate 'hedonic' price  $\gamma^j = \frac{\partial \ln B_c}{\partial \ln X_c^j}$
  - ▶ Straightforward to monetize since  $\frac{\partial \ln V}{\partial \ln w_c} = \frac{\partial \ln V}{\partial \ln B_c}$

**This how we value clean air, crime, public schools... (Greenstone, 2017)**

## Conclusion

- ▶ **Rosen-Roback framework is a core concept in spatial economics**
    - ▶ A family of models that share the idea of a frictionless spatial equilibrium
    - ▶ Perfect mobility implies indifference, zero-profits and perfect spatial arbitrage
  - ▶ Framework can be used to **recover quality of life**
    - ▶ From spatial variation in wages and rents
  - ▶ Workhorse tool for '**revealed-preference**' **valuation non-marketed goods**
    - ▶ Very important for social cost-benefit analysis and public investment decisions

## Next week: Monocentric city model

# Literature

## Core readings

- ▶ Brueckner, J. (2011): **Wage-Based Indexes of Urban Quality of Life.** Lectures on urban economics. Chapter 11. MIT Press.
- ▶ Glaeser, E., J. Gottlieb (2009): **The Wealth of Cities: Agglomeration Economies and Spatial Equilibrium in the United States.** Journal of Economic Literature 2009, 47(4).

## Other readings

- ▶ Ahlfeldt, G., Pietrostefani, E. (2019): The economic effects of the density: A synthesis. Journal of Urban Economics, 111.
- ▶ Ahlfeldt, G., S. Heblich, T. Seidel (2023): Micro-geographic property price and rent indices. Regional Science and Urban Economics, 98.
- ▶ Albouy, D. and B. Lue (2015): Driving to Opportunity: Local Rents, Wages, Commuting, and Sub-Metropolitan Quality-of-Life. Journal of Urban Economics, September 2015, 89, 74-92.
- ▶ Greenstone, M. (2017): The Continuing Impact of Sherwin Rosen's "Hedonic Prices and Implicit Markets: Product Differentiation in Pure Competition". Journal of Political Economy, 126(6).
- ▶ Rosen, S. (1979). Wage-Based Indexes of Urban Quality of Life. In *Current Issues in Urban Economics*, edited by P. Mieszkowski and M. Straszheim. Baltimore: Johns Hopkins University Press.
- ▶ Roback, J. (1982). Wages, Rents, and the Quality of Life. Journal of Political Economy 90(4), 1257-1278.