Topic 11
MRRH (2018): The Model

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Acknowledgements

- This slide deck uses material from the following lectures
  - Lecture 5 (Commuting, Migration, and Local Employment Elasticities) of Spatial Economics Lectures—by Stephen J. Redding

Introduction

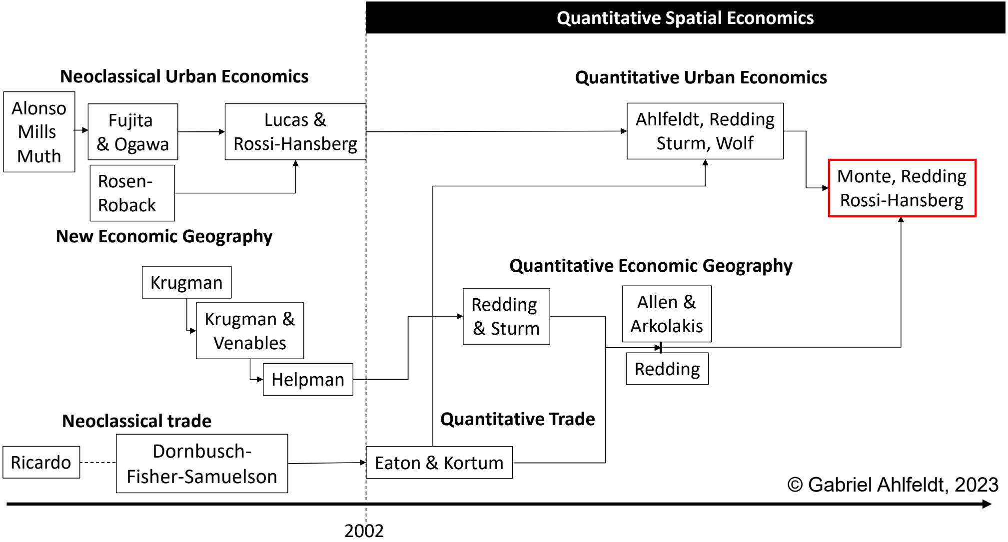
Recall

- **Quantitative urban models**
  - Focus on spatial linkages between locations in factor markets (commuting)
- **Quantitative economic geography models**
  - Focus on spatial linkages between locations in goods markets (trade)
- Many **similarities**
  - Realistic geography with discrete locations of arbitrary shape
  - Gravity equations & market access
  - Concentration and dispersion forces & spatial competition
  - Rationalize observed data via structural residuals (fundamentals)
  - Estimation, inversion, counterfactuals

Monte, Redding, Rossi-Hansberg (2018) (MRRH)

- **Commuting and trade linkages in one model**
  - A 'true' **quantitative spatial model**?
- **Resolves tension** in quantitative economic geography models
  - If units are **too large** ⇒ Many local effects are not detectable
    - E.g. localized effects of highways or subways
  - If units are **too small** ⇒ Underestimate local labour supply
    - Miss that productive locations can draw workers from other locations
- **Spatial equilibrium**
  - Concentration force: Home market effect (like in RRH)
    - No productivity and amenity spillovers
  - Dispersion forces: Inelastically supplied land & idiosyncratic amenity (like in ARSW)
  - Zero profits and constant utility (like in ARSW & RRH)

History of thought



Roadmap

- **Monte, Redding, Rossi-Hansberg (2018)**
  - A canonical QSM
- **Topic 11** (today)
  - Model
  - Equilibrium
  - Quantification
- **Topic 12**
  - Seidel & Wickerath (2020) version of the model
    - Application to Germany
    - Toolkit written in MATLAB (instead of Mathematica) MRRH2015-toolkit
  - Counterfactuals

Buidling blocks

RRH (2017) selection of building blocks

1. **Preferences:** Love of variety; Single traded sector; **workplace and residence amenities**; Residential land use; **idiosyncratic preferences for commuting routes**
2. **Production technology:** Increasing returns to scale; Exogenous productivity; No input-output linkages; No commercial land use
3. **Technology for the Movement of Goods:** Iceberg variable trade costs; **arbitrary** trade costs; Economic and Geographic Frictions; No non-traded goods besides residential land use
4. **Technology for the Movement of Ideas:** No knowledge externalities or diffusion; No innovation; No transferability of ideas
5. **Technology for the Movement of People:** Perfectly costless migration; **commuting**; Single worker type with no heterogeneity; No congestion in transportation
6. **Endowments:** Homogenous labor; Exogenous land endowments in regions within a single country; No capital
7. **Equilibrium:** Monopolistic competition; General equilibrium with a single country; **immobile landlord consuming income locally**; Trade is balanced in each location

A Canonical Quantitative Spatial Model

- Consider an economy consisting of a set N **of regions indexed by** n
  - Each region is endowed with an **exogenous supply of land** (H_n)
- Economy is endowed with a **measure** $\bar{L}$ **of workers** who supply one unit of labor
- Workers are **perfectly geographically mobile**
- Regions are connected by a **bilateral transport network**
  - Can be used to **ship goods subject to iceberg trade costs**
    - $d_{ni} = d_{in} > 1$ units must be shipped from region i in order for one unit to arrive in region $n \neq i$, where $d_{nn} = 1$
  - Can be used to **commute** subject to an iceberg commuting costs
    - $\kappa_{ni} \in [1, \infty)$ commuting cost in terms of utility (proportionate to opportunity cost of time)

Preferences and Amenities (similar to ARSW)

- Utility of worker ω that lives in n and works in i is

$$U_{ni\omega} = \frac{b_{ni\omega}}{\kappa_{ni}} \left(\frac{C_{n\omega}}{\alpha}\right)^{\alpha} \left(\frac{H_{n\omega}}{1-\alpha}\right)^{1-\alpha}$$

  - where $C_{n\omega}$ is the CES consumption basket with elasticity of substitution σ, and $H_{n\omega}$ housing consumption
- Utility **cost of commuting** are given $\kappa_{ni} \in [1, \infty)$
  - Since it enters multiplicative in utility, it is proportionate to wage
- Amenities, $b_{ni\omega}$, drawn i.i.d. from Fréchet distribution $G_{ni}(b) = e^{-B_{ni} b^{-\epsilon}}$
  - where $B_{ni} > 0$ is the **average bilateral amenity**
  - $\epsilon > 1$ governs the dispersion of idiosyncratic tastes

Production (similar to RRH)

- Horizontally **differentiated varieties sold under monopolistic competition**
- **Only labor** required to produce $x_i(j)$ units of output in i is

$$l_i(j) = F + \frac{x_i(j)}{A_i}$$

- Prices (constant markup) at n are given by $p_{ni}(j) = \left(\frac{\sigma}{\sigma-1}\right)\frac{d_{ni}w_i}{A_i}$,
  - where $d_{ni} \geq 1$ denotes iceberg transport costs between i and n
- Constant equilibrium output $x_i(j) = A_i F(\sigma - 1)$ implies measure of varieties

$$M_i = \frac{L_i}{\sigma F}$$

Land Market (sort of similar to RRH)

- Price of land Q_n determined from land market clearing
  - There is an **inelastic supply** of housing at H_n (**imperfectly elastic in extension**)

$$\underbrace{H_n Q_n}_{\text{housing supply}} = \underbrace{(1-\alpha)v_n R_n}_{\text{housing demand}} \Rightarrow Q_n = (1-\alpha)\frac{v_n R_n}{H_n},$$

  - where v_n is expected income of residents at n and R_n is the total number of residents
- Land owned by **landlords**, who receive income from residents' expenditure on land
  - **Consume goods where they live**
- Total expenditure on goods is the sum of expenditures by residents and landlords

$$P_n C_n = a v_n R_n + (1-\alpha)v_n R_n = v_n R_n$$

Trade in Goods (as in RRH)

- Expenditure shares are given by

$$\pi_{ni} = \frac{L_i(d_{ni}\frac{w_i}{A_i})^{1-\sigma}}{\sum_{k\in N} L_k(d_{nk}\frac{w_k}{A_k})^{1-\sigma}}$$

  - Recall: This is the expenditure agents in n spend on goods produced in i
- The price of the consumption basket at n is given by

$$P_n = \left(\frac{\sigma}{\sigma-1}\right)\left(\frac{L_n}{\sigma F \pi_{nn}}\right)^{\frac{1}{1-\sigma}} \frac{w_n}{A_n}$$

Work-Residence Decision (as in ARSW)

- The indirect utility of an agent ω that lives in n and works in i is

$$U_{ni\omega} = \frac{b_{ni\omega} w_i}{\kappa_{ni} P_n^{\alpha} Q_n^{1-\alpha}}$$

  - which is drawn from $G_{ni}(u) = e^{-\Psi_{ni} u^{-\epsilon}}$, with $\Psi_{ni} = B_{ni} \left(\kappa_{ni} P_n^{\alpha} Q_n^{1-\alpha}\right)^{-\epsilon} w_i^{\epsilon}$

- The unconditional probability that a worker lives in n and works in i is

$$\lambda_{ni} = \frac{B_{ni} \left(\kappa_{ni} P_n^{\alpha} Q_n^{1-\alpha}\right)^{-\epsilon} w_i^{\epsilon}}{\sum_{r \in N} \sum_{s \in N} B_{rs} \left(\kappa_{rs} P_r^{\alpha} Q_r^{1-\alpha}\right)^{-\epsilon} w_s^{\epsilon}}$$

- Free mobility implies that $\bar{U} = E[U_{ni\omega}]$ for all ni

Commuting (as in ARSW)

- Conditional probability that worker in n commutes to location i is

$$\lambda_{n|i} = \frac{B_{ni}\left(\frac{w_i}{\kappa_{ni}}\right)^{\epsilon}}{\sum_{s\in N} B_{ns}\left(\frac{w_s}{\kappa_{ns}}\right)^{\epsilon}}$$

- So labor market clearing implies that

$$L_i = \sum_{n\in N} \lambda_{n|i} R_n$$

- Expected residential income is then

$$v_n = \sum_{i\in N} \lambda_{n|i} w_i$$

Equilibrium

General Equilibrium

- The general equilibrium can be referenced by a vector of
  - six variables $\{w_n, v_n, Q_n, L_n, R_n, P_n\}$
  - and a scalar
- This vector solves
  - Six equations
    - Income equals expenditure (7): $w_i L_i = \sum_{n \in N} \pi_{ni} \bar{v}_n R_n$
    - average residential income (14): $\bar{v}_n = \sum_{i \in N} \lambda^R_{ni} w_i$
    - choice probabilities (2x 11): $\overline{v}_n = \sum_{i \in N} \lambda^R_{ni} w_i \ \lambda^n = \sum_{i \in N} \frac{\Phi_{ni}}{\Phi}, \quad \lambda_i \sum_{n \in N} \frac{\Phi_{ni}}{\Phi}$
    - price indices (8): $P_n = \frac{\sigma}{\sigma - 1} \left(\frac{L_n}{\sigma F \pi_{nn}} \right)^{\frac{1}{1-\sigma}} \frac{d_{nn} w_n}{A_n}$
  - Labour market clearing condition $\bar{L} = \sum_{n \in N} R_n = \sum_{n \in N} \bar{L}_n$
- **7 endogenous objects, 7 equations** ✓
  - MRRH provide sufficient conditions for equilibrium uniqueness and existence

Quantification

MRRH Data for Calibration

- Commodity Flow Survey (CFS)
  - Bilateral trade between 123 CFS regions
  - Bilateral distance shipped
- American Community Survey (ACS)
  - Commuting probabilities between counties
- Bureau of Economic Analysis
  - Employment by workplace county
  - Wages by workplace county
- GIS data
  - County maps
- Parameters from literature
  - Elasticity of substitution, $\sigma = 4$ (Broda and Weinstein, 2006),

Setting the Expenditure share on housing

- The **non-housing expenditure share**:
  - In RRH2017: $\alpha = 0.75$ (Davis and Ortalo-Magne, 2011)
  - In MRRH2018: $\alpha = 0.6$ (Bureau of Economic Analysis)
- Recall: Sufficient condition for uniqueness from RRH (exogenous housing supply)
  - $\sigma(1-\alpha) > 1$
- MRRH have **endogenous housing supply in model extensions**
  - Congestion force is smaller $\Rightarrow$ Risk of **multiple equilibria**
  - Good that sufficient statistic is higher

	σ	α	$\sigma(1-\alpha) > 1$
RRH	5	0.75	1.25
MRRH	4	0.6	1.6

Estimating distance elasticity of trade

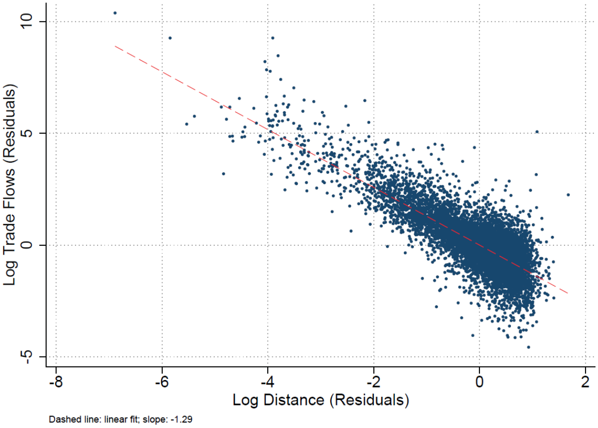
- MRRH **quantify the model for US counties**
  - Only observe **trade for CFS regions, which are larger than counties**
- Trade data still helps **estimating distance elasticity of trade** $\psi(1-\sigma)$

$$\ln \pi_{ni} = [(1-\sigma)\psi] \ln dist_{ni} = O_i + D_n + [(1-\sigma) \ln e_{ni})],$$

  - where ψ is the distance elasticity of trade cost
  - and $\ln e_{ni}$ captures unobserved trade costs
  - Standard empirical trade gravity equation (see Topic 9)
- RRH set $[(1-\sigma)\psi] = -1.5$
  - Ahlfeldt, Roth, Seidel (2023) estimate $[(1-\sigma)\psi] = -3$ for Germany (see Topic 9)

Estimating distance elasticity of trade

- Estimated slope: $[(1-\sigma)\psi] = -1.29$
  - After controlling for original and destination fixed effects
  - Implies $\psi = 0.43$ given $1-\sigma = 3$
  - Log-log approximation not too bad
- **Can predict** $d_{ni} = dist_{ni}^{\psi}$ **at county level**



Inversion of Productivities

- County trade balance implies

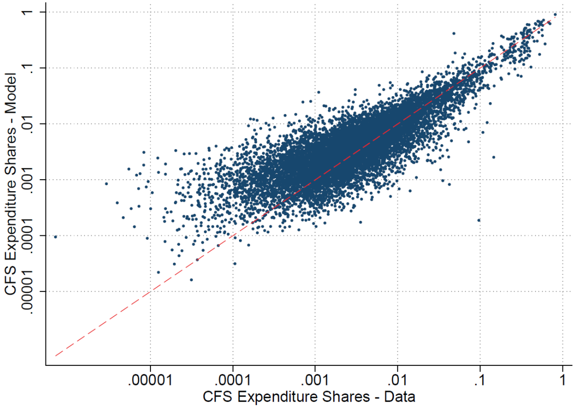
$$w_i L_i = \sum_{n \in N} \pi_{ni} v_n R_n = \sum_{n \in N} \frac{L_i (d_{ni} w_i)^{1-\sigma} A_i^{\sigma-1}}{\sum_{k \in N} L_k (d_{nk} w_k)^{1-\sigma} A_k^{\sigma-1}} v_n R_n.$$

  - We observe (or can compute) $\{w_i, L_i, R_i, v_i\}$
  - We have predicted $d_{ni}^{1-\sigma} = (\text{dist}_{ni})^{-1.29}$
- We can (numerically) solve uniquely for productivities, A_i (up to a constant)
  - Intuitively, firms in i pay for wage bill with expenditures they receive from n
    - If $w_i L_i > \sum_{n \in N} \pi_{ni} v_n R_n$, **not enough expenditure on goods produced in** i
    - **Need to increase** $A_i \Rightarrow$ productivity increases, price falls and production expands
    - Expenditure shares on goods from i increase $\Rightarrow$ Revenues of firms in i increase
  - Keep adjusting A_i until revenues $\sum_{n \in N} \pi_{ni} v_n R_n$ exactly match $w_i L_i$
- More next next week... MRRH2015-toolkit

Overidentifying trade shares

- How well do we do on bilateral shares?
  - Model generates county-shares
    $$\pi_{ni} = \frac{L_i (d_{ni} \frac{w_i}{A_i})^{1-\sigma}}{\sum_{k \in N} L_k (d_{nk} \frac{w_k}{A_k})^{1-\sigma}}$$
  - Aggregate model shares to CFS level
  - Model shares **positively correlated** with observed CFS shares

Does it look alright?



Estimating commuting decay

- The unconditional probability that a worker lives in n and works in i is

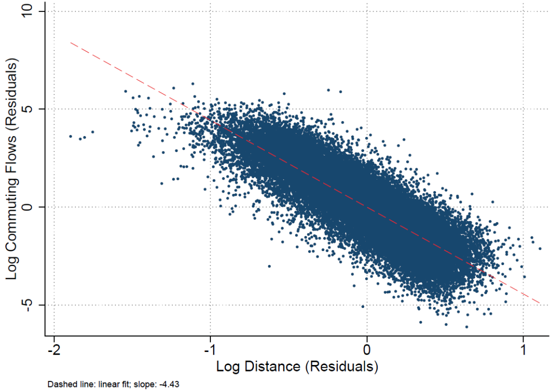
$$\lambda_{ni} = \frac{B_{ni}\left(\kappa_{ni} P_n^{\alpha} Q_n^{1-\alpha}\right)^{-\epsilon} w_i^{\epsilon}}{\sum_{r \in N}\sum_{s \in N} B_{rs}\left(\kappa_{rs} P_r^{\alpha} Q_r^{1-\alpha}\right)^{-\epsilon} w_s^{\epsilon}}$$

  - Let $\log \mathcal{B}_{ni} = \log(B_{ni}\kappa_{ni}^{-\epsilon}) = \log \mathbb{B}_n + \log \mathbb{B}_i - \phi(\log(\text{dist}_{ni})) + \log \mathbb{B}_{ni}$,
    - where implicitly $\kappa_{ni} = dist_{ni}^{\tau}$ so that $-\phi \equiv -\epsilon\tau$
- We can estimate ϕ in a reduced-form regression

$$\log \lambda_{ni} = g_0 + \eta_n + \mu_i - \phi \log \text{dist}_{ni} + \log \mathbb{B}_{ni},$$

Commuting decay

- Estimated commuting decay $-\phi = -4.43$
- Decay is much steeper than in trade (-1.29)
- Expected, since **moving people is more expensive than moving goods**



Estimating preference heterogeneity

- To estimate the preference heterogeneity ϵ, remove w_i from destination FE

$$[\log \lambda_{ni} + \phi \log \text{dist}_{ni}] = g_0 + \eta_n + \epsilon \log w_i + \log u_{ni},$$

  - where $\log u_{ni} = \log \mathbb{B}_i + \log \mathbb{B}_{ni}$ and ϕ taken from previous regression
- w_i **correlated with error** component $\mathbb{B}_i$, which is a labour supply shifter
    - Workers accept lower wage if there is a positive workplace amenity
- Need an **instrument** for wage that is uncorrelated with labour supply
  - Use fundamental productivity A_i as a labour demand shifter $\Rightarrow \epsilon = 3.3$
  - Have inverted A_i using *income* $=$ *expenditure* and observed $\{w_i, L_i, d_{ni}\}$

Q: Sounds good?

Inversion of amenities

- We observe or can solve for all elements of bilateral commuting probabilities

$$\lambda_{ni} = \frac{B_{ni}\left(\kappa_{ni} P_n^{\alpha} Q_n^{1-\alpha}\right)^{-\epsilon} w_i^{\epsilon}}{\sum_{r \in N} \sum_{s \in N} B_{rs}\left(\kappa_{rs} P_r^{\alpha} Q_r^{1-\alpha}\right)^{-\epsilon} w_s^{\epsilon}}$$

- except 'ease of commuting' $\mathcal{B}_{ni} = (B_{ni}\kappa_{ni}^{-\epsilon})$
  - Straightforward to recover $\mathcal{B}_{ni}$ as the structural residual
- $\mathcal{B}_{ni}$ **is all we need to run counterfactuals**
  - Since B_{ni} and κ_{ni} are isomorphic in the model,

Counterfactuals

MRRH counterfactuals

- MRRH use the model to perform different counterfactuals
  - Local employment elasticities
  - Changes in commuting costs
- Derived in terms of exact hat algebra

We discuss the MRRH counterfactuals briefly and then move on to the `MRRH2018-toolkit` and its didactic counterfactuals due to accessibility of the MATLAB code MRRH2015-toolkit

Exact hat algebra

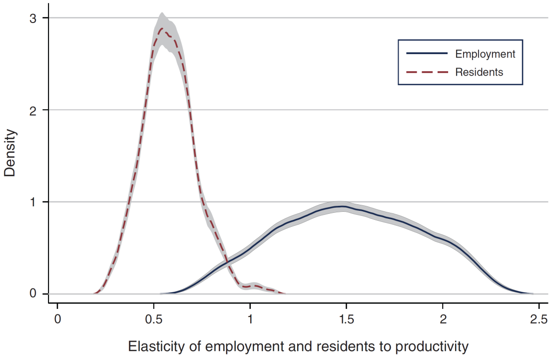
- MRRH show how to use exact hat algebra in numerical solver to obtain changes in endogenous variables for given
  - changes in exogenous variables $\{\hat{A}_n, \hat{B}_n, \hat{\kappa}_{ni}, \hat{d}_{ni}\}$
  - Values structural parameters $\{\alpha, \sigma, \epsilon, \delta, \kappa\}$
  - Observed data $\{L_{ni}, w_i, \pi_{ni}\}$
- Treat $\{\hat{\lambda}_{ni}, \hat{w}_i\}$ as target variables
  - Start with guesses
  - Use recursive structure to solve for values of other referencing objects
  - Use values of other referencing objects to predict $\{\hat{\lambda}_{ni}, \hat{w}_i\}$
  - Update $\{\hat{\lambda}_{ni}, \hat{w}_i\}$ until convergence
- More next week... MRRH2015-toolkit

Local employment elasticities

- Find out how strongly **employment reponds to labour demand shocks**
  - Local employment elasticity $\equiv$ Elasticity of employment with respect to productivity
- Key point is that extant literature underestimates this elasticity
  - By focusing on migration and **ignoring commuting**
  - Commuting makes local labor supply much more elastic
- **Conduct 3,111 counterfactuals** (one for each county)
  - Shock a county by 5%
  - Use the model to compute by how much local employment changes
  - Compute the local employment elasticity
- Compare to **residents elasticity** (how does resident pop. respond)

Local employment elasticities

- Employment elasticity ranges from 0.5 to 2.5 with mean of 1.52
- Resident elasticity much lower (0.2-1.2)
- Employment elasticity is larger in "open" commuting markets
  - Substantiated by reduced-form evidence



Commuting cost

- What are the effects of empirically observed **reductions in commuting cost**?
  - Due to improvements in rolling stock and infrastructure
- Measure ease of commuting in relative terms: $\tilde{\mathcal{B}}_{ni} \equiv \left(\frac{\mathcal{B}_{ni}\mathcal{B}_{in}}{\mathcal{B}_{nn}\mathcal{B}_{ii}}\right)^{\frac{1}{2}} = \left(\frac{L_{ni}L_{in}}{L_{nn}L_{ii}}\right)^{\frac{1}{2}}$
  - **Normalize bilateral 'ease' by within 'ease'** at origin and destination
    - Ease of commuting only identified up to a constant
  - Geometric mean of ease in both directions (assuming symmetry of infrastructure?)
- Can express 'ease of commuting' solely in terms of observed commuting flows L_{ni}
  - All other terms in the commuting probability equation cancel out after normalization
  - Recall that $L_{ni} = \lambda_{ni}\bar{L}$

Commuting cost counterfactual

- Relative ease of commuting increased from 1990 to 2010
  - by 4 (25^{th} pct) to 12 (50^{th} pct) and 21 (75^{th} pct) percent
- Counterfactuals where $B_{ni}\kappa_{ni}^{-\epsilon}$ change on all routes

$$\hat{\bar{U}} = \left(\frac{1}{\hat{\lambda}_{ii}}\right)^{\frac{1}{\epsilon}} \left(\frac{1}{\hat{\pi}_{ii}}\right)^{\frac{\alpha}{\sigma-1}} \left(\frac{\hat{w}_i}{\hat{\bar{v}}_i}\right)^{1-\alpha} \frac{\hat{L}_i^{\frac{\alpha}{\sigma-1}}}{\hat{R}_i^{1-\alpha}}$$

- Commuting **cost reductions have increased utility** by 0.9 to 6.9 percent
  - Assuming observed changes in ease of commuting are driven by κ (not B)
  - No change in internal ease of commuting...

Conclusion

Summary

- **Quantitative urban models** emphasize the trade between regions
  - Follow in the tradition of neoclassical urban models
- **Quantitative economic geography** emphasize the trade between regions
  - Follow in the tradition of New Economic Geography models
- **MRRH** integrate spatial linkages via **commuting and trade** into one model
  - Use the model to show that commuting makes local labour supply more elastic
  - Also show that improvements in commuting technology have generated welfare

Next week: **Counterfactuals for Germany**

Literature I

Core readings

- **Monte, F., Redding, S., Rossi-Hansberg, E. (2018): Commuting, Migration, and Local Employment Elasticities. American Economic Review, 108(12), 3855-90.**

Other readings

- Ahlfedt, G., Redding, S., Sturm, D., Wolf, N. (2015): The economics of Density: Evidence From the Berlin Wall. Econometrica, 83(6), 2127–2189.
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