Topic 7
ARSW (2015): Quantification

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Introduction

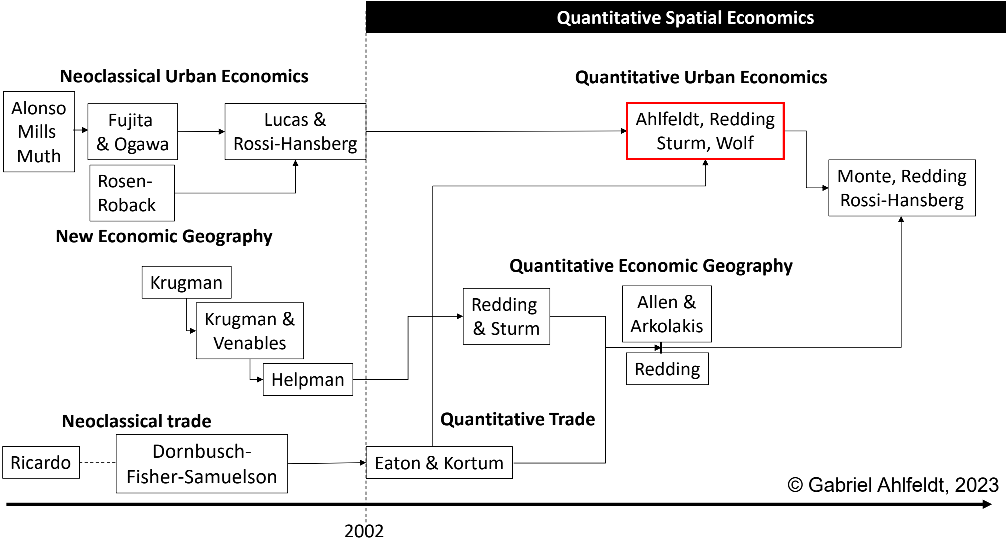
Recall the main components of a QSM

- **Parameters**
- **Fundamentals**
- **Endogenous variables**

Q: What is their role in?

- **Estimation**
- **Inversion**
- **Simulation**

History of thought



Roadmap

- **Topic 6**
  - Building blocks of the model
  - Reduced-form evidence
- **Topic 7** (today)
  - Estimation
  - Inversion
- **Topic 8**
  - Counterfactuals with exogenous fundamentals
  - Counterfactuals with agglomeration forces

Two variants of the model

- **Exogenous fundamentals**
  - Productivity, A_i, and amenity, B_i are exogenous
  - We can think about **estimation and inversion sequentially** (the standard case)
    - We will use solvers at the estimation stage
    - But with a good instrument one could theoretically estimate in "reduced-form"
- **Endogenous agglomeration forces**
  - $\{A_i, B_i\}$ depend on fundamentals and surrounding (endogenous) density
  - Can no longer think about estimation without inversion
    - Need to derive structural residuals to define moment conditions
    - Estimation nests the mapping from endogenous variables to fundamentals (inversion)
    - Need to impose the full structure of the model $\Rightarrow$ **structural estimation**
- This lecture is closely connected to the ARSW **toolkit**, use it! ARSW2015-toolkit

Exogenous fundamentals

Assumed parameters

- We (usually) begin the quantification by thinking about **parameter values**
  - We have the following structural parameters $\{\alpha, \beta, \mu, \varepsilon, \kappa\}$ see codebook
- Need to make a **decision**
  - **Which parameters to estimate, which parameters to borrow** from the literature
- Estimate parameters that are **central, novel and/or context-specific**

Assumed Parameter	Source	Value	
Residential floor space	$1-\beta$	Davis & Ortalo-Magné (2011)	0.25
Commercial floor space	$1-\alpha$	Valentinyi & Herrendorf (2008)	0.20
Share of land in property value	$1-\mu$	Combes et al. (2019)	0.25

- These parameter values are **'canonical'**, so safe to borrow from literature...

Estimation of $\varepsilon\kappa$

- Equation (4) in the Paper implies a **commuting gravity equation**:
  - $\pi_{ij} = \frac{T_i E_j (d_{ij} Q_i^{1-\beta})^{-\varepsilon} (B_i w_j)^{\varepsilon}}{\sum_{r=1}^{S} \sum_{s=1}^{S} T_r E_s (d_{rs} Q_r^{1-\beta})^{-\varepsilon} (B_r w_s)^{\varepsilon}}$, where $d_i j = \exp(\kappa * \tau_{ij})$
  - $\ln(\pi_{ij} H) = -(\kappa\epsilon)\tau_{ij} + \vartheta_i + \zeta_j + e_{ij}$ absorbing all i- and j-specific factors
    - where $(\pi_{ij} H)$ is the commuting flow (H added to turn probability into flow)
    - $\{\vartheta_i, \zeta_j\}$ are origin and destination fixed effects
- This gravity equation is **straightforward to estimate** (if you have bilateral flows)
  - Of course, there is a literature... (Head & Mayer, 2014)
    - PPML, distance IV for travel cost, etc. (often, it does not make a huge difference)
- So, we typically begin with estimating $(\kappa\epsilon)$
  - Then estimate ϵ, which implicitly determines κ

Commuting gravity results

	(1)	(2)	(3)	(4)
	ln Bilateral Commuting Probability 2008			
Travel Time ($-\kappa\varepsilon$)	-0.0697***	-0.0702***	-0.0771***	-0.0706***
	(0.0056)	(0.0034)	(0.0025)	(0.0026)
Estimation	OLS	OLS	Poisson PML	Gamma PML
More than 10 Commuters	Yes	Yes	Yes	Yes
Fixed Effects	Yes	Yes	Yes	Yes
Observations	144	122	122	122
R-squared	0.8261	0.9059		

Note: Gravity equation estimates based on representative micro survey data on commuting for Greater Berlin for 2008. Observations are bilateral pairs of 12 workplace and residence districts (post 2001 Bezirke boundaries). Travel time is measured in minutes. Fixed effects are workplace district fixed effects and residence district fixed effects. The specifications labelled more than 10 commuters restrict attention to bilateral pairs with 10 or more commuters. Poisson PML is Poisson Pseudo Maximum Likelihood estimator. Gamma PML is Gamma Pseudo Maximum Likelihood Estimator. Standard errors in parentheses are heteroscedasticity robust. * significant at 10%; ** significant at 5%; *** significant at 1%.

Estimation of ε

- Most obvious way of estimating ε would be to use the gravity equation

$$\ln(\pi_{ij}H) = c - \underbrace{(\kappa\varepsilon)}_{\nu}\tau_{ij} - \underbrace{(\varepsilon(1-\beta))}_{\text{reduced-form parameter}} \ln Q_i + \varepsilon \ln w_j + \underbrace{\ln(T_i B_i E_j)}_{\text{structural residual}}$$

- We observe rents, so could infer ε from the reduced-form parameter of a given β
- Can also infer ϵ directly from $\ln w_j$ if we observe wages
  - ARSW do not observe wages, but w_j is observable in some data sets

What is the problem with this approach?

Spatial wage dispersion as a moment

- ARSW observe wages at the Bezirke (23) level
  - Mitte, Kreuzberg, Charlottenburg, etc.
- Choose ε so that **variance of wages** across Bezirke **in the model matches data**

$$H_{Mj} = \frac{\sum_{i=1}^{S} (w_j/d_{ij})^{\epsilon} H_{Ri}}{\sum_{s=1}^{S} (w_s/d_{is})^{\epsilon}}, \quad d_{ij} = e^{\kappa \tau_{ij}}.$$

- Given workplace employment (H_{Mj}), residence employment (H_{Ri}) and bilateral travel times (τ_{ij} and hence d_{ij}), **we can solve for** $\omega_j = E_j w_j^{\epsilon}$ (up to scale)
  - Problem is akin to finding wage in Ahlfeldt & Barr (2022), we just have $J-1$ wages

Transformed wage solver

Algorithm 1: for transformed wages (ω_j): `comegaopt0.m` (used in estimation)

Data: Given values for structural parameters $\{\alpha, \beta, \kappa\varepsilon\}$, bilateral travel times τ_{ij}, observed workplace employment H_{Mj} and residence employment H_{Ri}
Guesses of transformed wages $\tilde{\omega}_j^0$

while *Predicted workplace employment* $\hat{H}_{Mj} \neq H_{Mj}$ **do**
- Use guessed values of transformed wages $\tilde{\omega}_j^0$ in Eq. (S.44) to predict $\hat{H}_{Mj}$
- Generate new guesses $\tilde{\omega}_j^1 = \frac{H_{Mj}}{\hat{H}_{Mj}} \hat{\omega}_j^0$
- Update guesses to weighted combination of new and old guesses
- Normalize guesses by geometric mean

Result: Transformed wages $\tilde{\omega}_j$

- Straightforward intuition Matlab code
  - Increase wage if we predict lower supply than observed in data ($\frac{H_{Mj}}{\hat{H}_{Mj}} > 0$)
  - Reduce if we predict more supply than observed in data ($\frac{H_{Mj}}{\hat{H}_{Mj}} < 0$)

Moment condition for ε identification

- The log variance of wages across locations is
  - $\sigma^2_{\ln w} = \mathbb{E}\left[\left(\frac{1}{\varepsilon}\log(\omega_i) - \underbrace{\mathbb{E}\left[1/\varepsilon \log(\omega_i)\right]}_{=0}\right)^2\right] = \mathbb{E}\left[\left(\frac{1}{\varepsilon}\log(\omega_i)\right)^2\right]$
  - since $\mathbb{E}(\omega_i = 1)$ (due to normalization by geometric mean `incomegaopt0.m`)
- The moment condition that matches variance of wages in model and data is:
  - $\mathbb{E}\left[\left(\frac{1}{\varepsilon}\right)^2 \ln(\omega_j)^2 - \sigma^2_{\ln w_i}\right] = 0$, where $\sigma^2_{\ln w_i}$ is the variance in data
    - `cdensityoptren.m` computes the value in the objective function Matlab code
    - MATLAB `patternsearch` function takes care of the minimization Matlab code
- Objective function minimized at $\varepsilon = 6.83$

Inversion

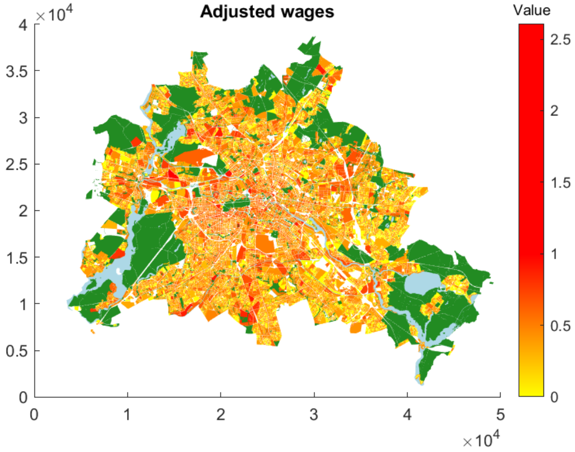
- **Inversion** means
  - rearranging the model's equation so that we can **solve for structural residuals**
  - for given values of structural parameters
  - and observed endogenous variables (and endowments)
- Here we have the following exogenous characteristics $\{T_i, E_i, A_i, B_i, \varphi_i, K_i, \xi_i, \tau_{ij}\}$
  - We observe K_i (land endowment) and τ_{ij}
- Some unobserved variables enter the model isomorphically, so **we invert**
  - Adusted productivity $\tilde{A}_i = A_i E_i^{\alpha/\varepsilon}$
  - Adjusted amenity $\tilde{B}_i = B_i T_i^{1/\varepsilon} \zeta_{Ri}^{1-\beta}$
  - Adjusted density of development $\tilde{\varphi}_i = \tilde{\varphi}_i(\varphi_i, {}_i^{1/\varepsilon}, \xi_i)$
- If we have parameters and these fundamentals we can solve the model (next week)

Inversion of productivity

- From profit-maximization and zero-profits (Eq. 12), we obtain Equation (23)
  - $\ln\left(\frac{\tilde{A}_i}{\bar{\tilde{A}}}\right) = (1-\alpha)\ln\left(\frac{\mathbb{Q}_{it}}{\bar{\mathbb{Q}}}\right) + \frac{\alpha}{\varepsilon}\ln\left(\frac{\omega_{it}}{\bar{\tilde{\omega}}}\right)$, where $\tilde{A}_i = A_i E_i^{\alpha/\varepsilon}$
  - Variables are normalized by the geometric mean since identification is up to scale
- **Productivity must be higher where firms can afford higher wages and rents**
  - Rosen-Roback within cities
- Straightforward to implement
  - We have a solver for transformed wages ω_i
  - We have an estimate of ϵ and set value of α
  - We observe rents, $\mathbb{Q}_i$
- Executed by `comegaoptC.m` $\Rightarrow$ minor extension of `comegaoptO.m` Matlab code

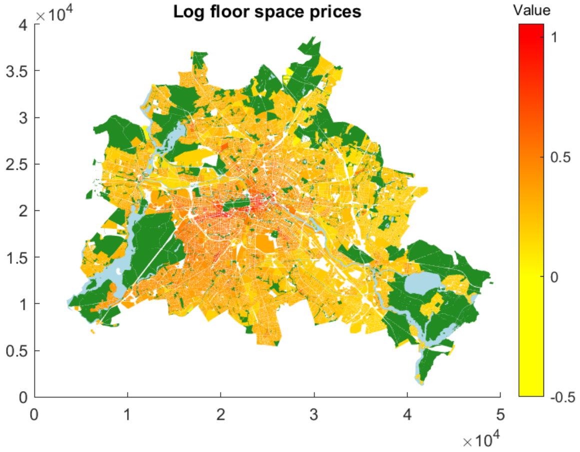
Productivity ingredient 1: Adjusted wages

- **High model-predicted wages in Mitte and Kudamm areas**



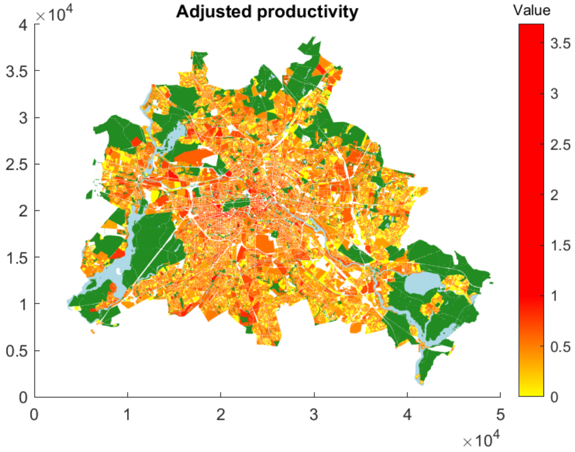
Productivity ingredient 2: Observed floor space

- **High observed rents in Mitte and Kudamm areas**



Inverted productivity

- **High inverted productivity in Mitte and Kudamm areas**

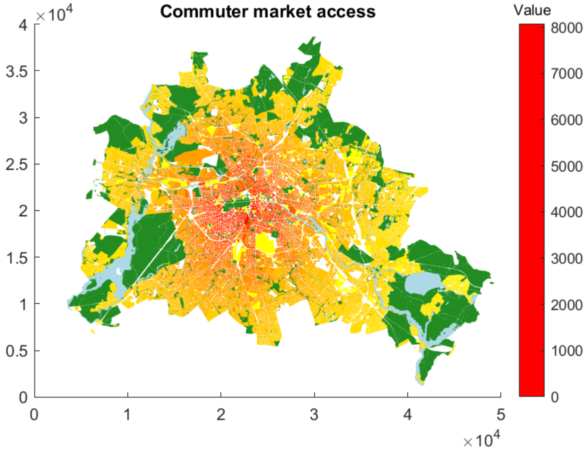


Inversion of amenity

- From the residential choice probabilities (5) and population mobility with the larger economy (9), we obtain Eq. (28)
  - $\ln\left(\frac{\tilde{B}_i}{\bar{\tilde{B}}}\right) = \frac{1}{\varepsilon}\ln\left(\frac{H_{Ri}}{\bar{H}_{Ri}}\right) + (1-\beta)\ln\left(\frac{\mathbb{Q}_i}{\bar{\mathbb{Q}}}\right) - \frac{1}{\varepsilon}\ln\left(\frac{W_i}{\bar{W}}\right)$, where $\tilde{B}_i = B_i T_i^{1/\varepsilon} {}_{Ri}^{1-\beta}$
  - where $W_{it} = \sum_{s=1}^{S} \frac{\omega_s}{e^{\nu T_{is}}}$ is commuting market access
  - Variables are normalized by the geometric mean since identification is up to scale
- **Amenity must be higher where:**
  - **Workers accept higher rents and lower** commuting-cost-weighted **wages**
  - **More workers choose to live**
    - Recall Topic 5: With heterogeneous workers inverse real wage does not recover QoL
- Straightforward to implement since we have $\{\omega_i, \epsilon, \beta, \mathbb{Q}_i\}$
- Executed by simple recursive algorithm `camen.m` Matlab code

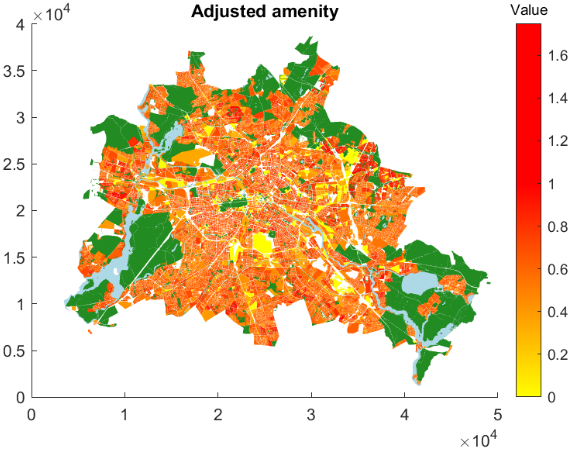
Amenity ingredient: Commuter market potential

- **High commuter market access in central areas (where rents are high)**



Inverted amenity

- **No obvious center-suburban pattern**
  - High rents and good CMA offset each other



Scale of $\{\tilde{A}_i, \tilde{B}_i\}$

- **Transparent recursive procedure identifies $\{\tilde{A}_i, \tilde{B}_i\}$ up to scale**
  - Works if all we care about are **relative differences** in $\{\tilde{A}_i, \tilde{B}_i\}$
    - E.g. if we want to explore what factors contribute to productivity and amenity
  - Used by ARSW in Section 6
- Relative $\{\tilde{A}_i, \tilde{B}_i\}$ should **not be used as a starting point for counterfactuals**
  - $\{\tilde{A}_i, \tilde{B}_i\}$ do not ensure that reservation utility $\bar{U}$ is met
  - Solving the model **will not generate the observed total employment**!
- ARSW use a different approach that matches observed H in counterfactuals
  - `cmodexog.m` solves simultaneously for $\{\tilde{A}_i, \tilde{B}_i\}$ targeting H Matlab code
- **Toolkit reconciles both approaches** $\Rightarrow$ code to rescale relative $\{\tilde{A}_i, \tilde{B}_i\}$

calcal_adj_TD.m

Algorithm 2: Rescaling adjusted amenities $\tilde{B}_i$ and productivities $\tilde{A}_i$ to rationalize observed population H: `calcal_adj_TD.m`

Data: Given values for structural parameters $\{\beta, \kappa\varepsilon, \varepsilon\}$, bilateral travel times τ_{ij}, adjusted productivities $\tilde{A}_i$, adjusted amenities $\tilde{B}_i$, observed floor space prices $\mathbb{Q}_j$

1. Normalize adjusted productivities $\tilde{A}_i$ by the geometric mean
2. Use rescaled adjusted productivities $\tilde{A}_i$ and observed floor space prices $\mathbb{Q}_j$ in Eq. (12) to compute rescaled adjusted wages $\tilde{w}_j$
3. Use bilateral travel times τ_{ij}, adjusted amenities $\tilde{B}_i$, observed floor space prices $\mathbb{Q}_j$, and rescaled adjusted wages $\tilde{w}_j$ to compute Φ (the denominator in Eq. (12) and the total employment in the model)
4. Rescale adjusted amenities $\tilde{B}_i$ by multiplying them by the adjustment factor $\left(\frac{H}{\Phi}\right)^{\frac{1}{\varepsilon}}$

Result: Rescaled adjusted productivity and amenity $\{\tilde{A}_i, \tilde{B}_i\}$

- Compute population in in model Φ and **rescale $\tilde{B}_i$ so that Φ matches H in data**
  - See supplement p. 18 to see that H scales in any spatially invariant component of B_i at an elasticity of ε (greater population implies higher house prices) Matlab code

Solving for $\{\tilde{A}_i, \tilde{B}_i\}$ simultaneously

Algorithm 3: Solving for adjusted wage $\tilde{w}_i$, adjusted productivity $\tilde{A}_i$ and adjusted amenity $(\tilde{B}_i)$: `cmodexog.m`

Data: Given values of endogenous variables floor space prices, workplace employment, residence employment, $\{\mathbb{Q}_i, H_{Mj}, H_{Ri}\}$, structural parameters $\{\alpha, \beta, \kappa\varepsilon, \varepsilon\}$, bilateral travel times τ_{ij}, **guesses of adjusted productivity and adjusted amenity, $\{\tilde{A}_j^0, \tilde{B}_i^0\}$**

while *guesses of* $\{\tilde{A}_j^0, \tilde{B}_i^0\}$ *change* **do**

- Compute adjusted wages $\tilde{w}_j$ using guesses of $\tilde{A}_j^0$, observed $\mathbb{Q}_j$, and Eq. (12)
- Compute commuting probabilities π_{ij} using Eq. (4) using guesses of $\{\tilde{A}_j^0, \tilde{B}_i^0\}$ and $\tilde{w}_j$ using Eq. (4)
- Use π_{ij} to compute predicted workplace and residence employment $\{\hat{H}_{Mj}, \hat{H}_{Ri}\}$ using Eq. (5)
- Generate new guesses of $\tilde{A}_i^1$ by inflating old guesses by the ratio of observed over predicted workplace employment $H_{Mi}/\hat{H}_{Mi}$ (**we increase guesses if we underpredict employment**)
- Generate new guesses of $\tilde{B}_i^1$ by inflating old guess by the ratio of observed over predicted residence employment $H_{Ri}/\hat{H}_{Ri}$ (**we increase guesses if we underpredict employment**)
- Update guesses of $\tilde{A}_i^0$ to the weighted combination of new guess $\tilde{A}_i^1$ and old guesses $\tilde{A}_i^0$
- Update guesses of $\tilde{B}_i^0$ to the weighted combination of new guess $\tilde{B}_i^1$ and old guesses $\tilde{B}_i^0$
- Normalize guesses $\tilde{A}_i^0$ by the geometric mean to ensure a unit mean
- Inflate guesses of $\tilde{B}_i^0$ by the ratio of total employment in data over total predicted employment in model (**we make the city more attractive if we underpredict total employment**)

Use solved $\tilde{w}_j$, τ_{ij}, and $\kappa\epsilon$ to compute commuting market access (see p. 40 in supplement for equation)

Result: Adjusted productivities, adjusted amenities, adjusted wages, commuting probabilities, expected income, predicted workplace emplyoment, predicted residence employment, predicted total employment $\{\tilde{A}_j, \tilde{B}_j, \tilde{w}_j, \pi_{ij}, \mathbb{E}(\tilde{w}_s|i), \hat{H}_{Mi}, \hat{H}_{Ri}\}$

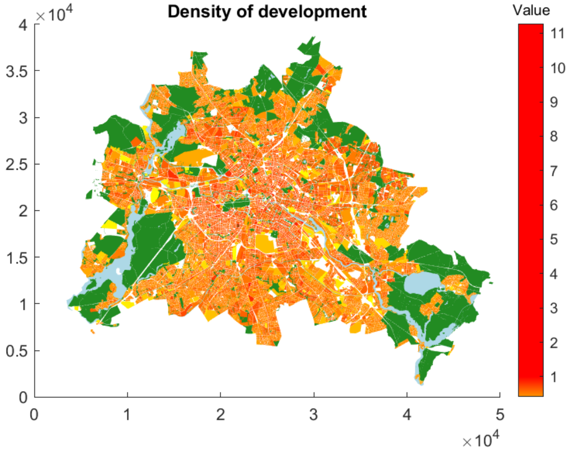
Matlab code

Density of development

- First-order condition and MRS give commercial floor space (Eq. S.49)
  - $\tilde{L}_{Mi} = \theta_i \tilde{L}_i = \left(\frac{\tilde{w}_i}{\alpha \tilde{A}_i}\right)^{\frac{1}{1-\alpha}} H_{Mi}$ ✓
- From Marshallian demand, we obtain
  - $\tilde{L}_{Ri} = (1-\theta_i)\tilde{L}_i = (1-\beta)\left[\sum_{s=1}^{S} \frac{(\tilde{w}_s/d_{is})^{\varepsilon}}{\sum_{r=1}^{S}(\tilde{w}_r/d_{ir})^{\varepsilon}} \tilde{w}_s\right] \frac{\tilde{H}_{Ri}}{Q_i}$
    - where expected income $\mathbb{E}(\tilde{w}_i) = \sum_{s=1}^{S} \frac{(\tilde{w}_s/d_{is})^{\varepsilon}}{\sum_{r=1}^{S}(\tilde{w}_r/d_{ir})^{\varepsilon}} \tilde{w}_s$ is computed by `expincome.m`
- From Eq. (19), we obtain the adjusted density of development
  - $\tilde{\varphi}_i = \frac{(1-\theta_i)\tilde{L}_i + \theta_i \tilde{L}_i}{K_i^{1-\mu}}$, computed by `cdenisty.m`
    - Equates total floor space demand to total floor space supply

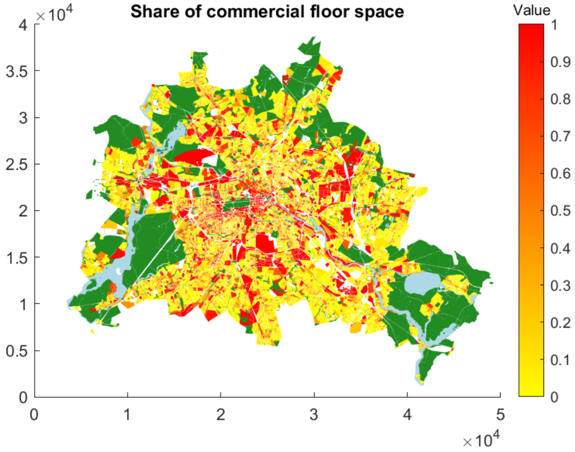
Inverted density of development

- **Density of development falls with distance from CBD as predicted by MCM and seen in data (Topic 3)**



Commercial floor space share

- Since we have residential and commercial floor space we can compute **commercial floor space share**
- **Reflects commercial areas (Mitte, Kudamm), industrial areas, and other non-residential uses (e.g. airports) as expected**



Endogenous agglomeration forces

Introducing endogenous productivity and amenity

- With endogenous agglomeration forces, $\{A_i, B_i\}$ become endogenous objects
  - $A_j = a_j \times \Upsilon_j^{\lambda}, \quad \Upsilon_j = \left[\sum_{s=1}^{S} e^{-\delta\tau_{is}} \left(\frac{H_{Ms}}{K_s}\right)\right]$
  - $B_i = b_i \times \Omega_i^{\eta}, \quad \Omega_i = \left[\sum_{s=1}^{S} e^{-\rho\tau_{is}} \left(\frac{H_{Rs}}{K_s}\right)\right]$
- Four more parameters to estimate
  - $\{\lambda, \delta, \eta, \rho\}$
- $\{a_i, b_i\}$ are now the **structural residuals**
  - on which we impose orthogonality in **estimation**
  - which we recover in inversion
  - Straightforward to invert, given parameters, $\{A_i, B_i\}$, and $\{H_{Mi}, H_{Ri}\}$
    - Computed by simple algorithms `cprod.m` `cres.m`

Estimation of spillover parameters

- Estimation of spillover parameters $\{\lambda, \delta, \eta, \rho\}$ is demanding
  - Need exogenous variation to estimate endogenous agglomeration effects
  - Non-linear system of equation
- **Structural GMM estimation** exploiting impact of division and unification
  - Iterative procedure that nets computation of structural residuals
    - For any combination of parameter values evaluated, structural residuals are computed
  - $\Delta \ln\left(\frac{\tilde{a}_{it}}{\tilde{a}_t}\right) = (1-\alpha)\Delta \ln\left(\frac{\mathbb{Q}_{it}}{\mathbb{Q}_t}\right) + \frac{\alpha}{\varepsilon}\Delta \ln\left(\frac{\tilde{\omega}_{it}}{\tilde{\omega}_t}\right) - \lambda \Delta \ln\left(\frac{\Upsilon_{it}}{\Upsilon}{}_t\right)$
  - $\Delta \ln\left(\frac{\tilde{b}_{it}}{\tilde{b}_t}\right) = \frac{1}{\varepsilon}\Delta \ln\left(\frac{H_{Rit}}{H_{Rt}}\right) + (1-\beta)\Delta \ln\left(\frac{\mathbb{Q}_{it}}{\mathbb{Q}_t}\right) - \frac{1}{\varepsilon}\Delta \ln\left(\frac{\tilde{\omega}_{it}}{\tilde{\omega}_t}\right) - \eta \Delta \ln\left(\frac{\Omega_{it}}{\Omega_t}\right)$

Δ indicates long differences pre/post division or pre/post unification

Moment Conditions

- Changes in adjusted fundamentals uncorrelated with exogenous change in surrounding economic activity from division/reunification
  - $\mathbb{E}\left[\mathbb{I}_k \times \Delta \ln\left(\frac{\tilde{a}_{it}}{\tilde{a}_t}\right)\right] = 0$, for $k \in \{1, \ldots, K\}$
  - $\mathbb{E}\left[\mathbb{I}_k \times \Delta \ln\left(\frac{\tilde{b}_{it}}{\tilde{b}_t}\right)\right] = 0$, for $k \in \{1, \ldots, K\}$
    - where $\mathbb{I}_k$ are indicators for distance-from-CBD grid cells
- Fraction of workers that commute less than 30 minutes matches value in data, ϑ
  - $\mathbb{E}\left[\vartheta H_{Mi} - \sum_{j \in_j} \frac{\omega_j / e^{\nu T ij}}{\sum_{s=1}^{S} \omega_s / e^{\nu T is}} H_{Ri}\right] = 0$
- Wage dispersion across Bezirke (as in the case with exogenous fundamentals)
  - $\mathbb{E}\left[\left(\frac{1}{\varepsilon}\right)^2 \ln(\omega_j)^2 - \sigma^2_{\ln w_i}\right] = 0$

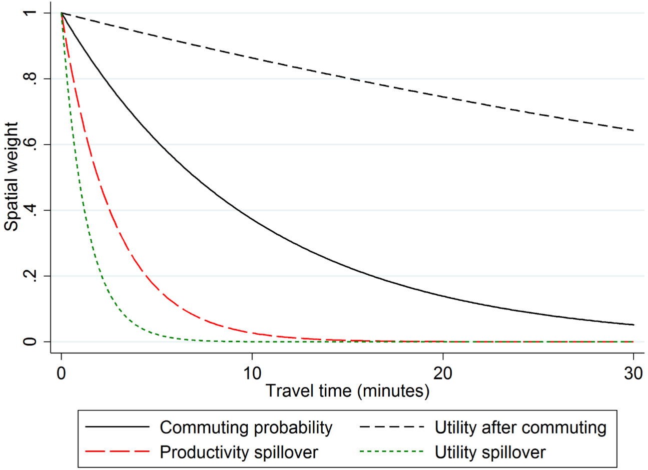
Parameter estimates

	(1) Division Efficient GMM	(2) Reunification Efficient GMM	(3) Division and Reunification Efficient GMM
Commuting Travel Time Elasticity ($\kappa\epsilon$)	0.0951***	0.1011***	0.0987***
	(0.0016)	(0.0016)	(0.0016)
Commuting Heterogeneity (ϵ)	7.6278***	7.7926***	7.7143***
	(0.1085)	(0.1152)	(0.1049)
Productivity Elasticity (λ)	0.0738***	0.0449***	0.0657***
	(0.0056)	(0.0071)	(0.0048)
Productivity Decay (δ)	0.3576***	0.8896***	0.3594***
	(0.0945)	(0.3339)	(0.0724)
Residential Elasticity (η)	0.1441***	0.0740***	0.1444***
	(0.0080)	(0.0287)	(0.0073)
Residential Decay (ρ)	0.8872***	0.5532***	0.7376***
	(0.2774)	(0.3699)	(0.1622)

Note: Generalized Method of Moments (GMM) estimates. Heteroscedasticity and Autocorrelation Consistent (HAC) standard errors in parentheses (Conley 1999). * significant at 10%; ** significant at 5%; *** significant at 1%.

Localized spillovers

- Spillovers highly localized within a few hundred meters, consistent with reduced-form evidence (Anzarghi and Henderson 2008)
- Structural estimation approach is non-standard and computationally demanding
  - Not covered in the toolkit
  - Explore replication directory

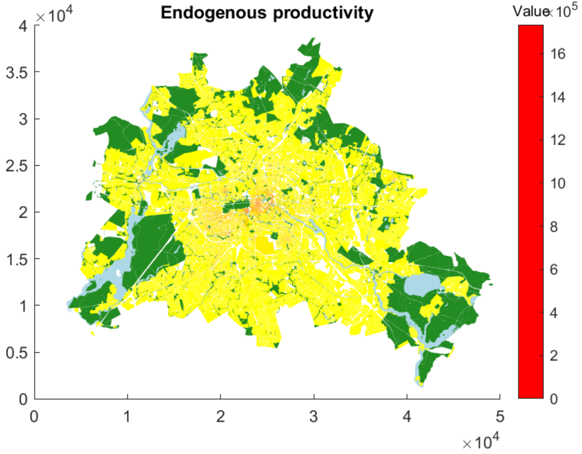


Inversion

- For given for $\{\lambda, \delta, \eta, \rho\}$, inversion of $\{a_i, b_i\}$ is straightforward
  - We observe $\{H_{Mi}, H_{Ri}, K_i, \tau_{ij}\}$
  - Can compute $\Upsilon_j = \left[\sum_{s=1}^{S} e^{-\delta\tau_{is}} \left(\frac{H_{Ms}}{K_s}\right)\right]$ and $\Omega_i = \left[\sum_{s=1}^{S} e^{-\rho\tau_{is}} \left(\frac{H_{Rs}}{K_s}\right)\right]$
  - Then solve for $a_i = \frac{\tilde{A}_i}{\Upsilon_i^{\lambda}}$ and $b_i = \frac{\tilde{B}_i}{\Omega_i^{\eta}}$
- Follow the same steps as in the case with exogenous fundamentals
  - Decompose $\{\tilde{A}_i, \tilde{B}_i\}$ using algorithms `cprod.m` `cres.m`
  - ARSW nest these algorithms within the simultaneous inversion procedure
  - Decompose $\{A_i, B_i\}$ using algorithms `smodendog.m`
  - Of course, the approach would also work with sequential procedure
    - As long as $\{\tilde{A}_i, \tilde{B}_i\}$ are rescaled using Matlab code

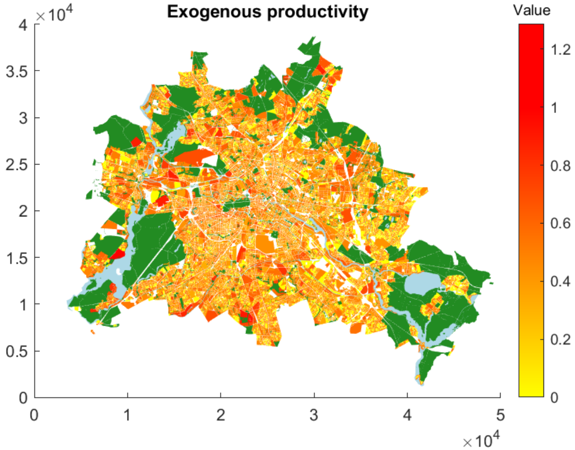
Endogenous productivity

- Endogenous productivity gains naturally **largest in employment centers in Mitte and Kudamm**

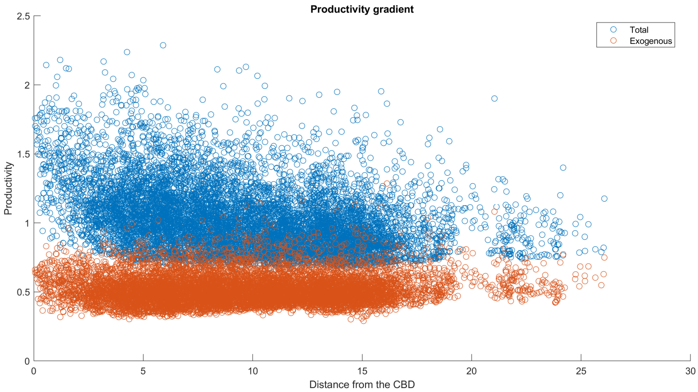


Exogenous productivity

- Exogenous productivity looks spatially random



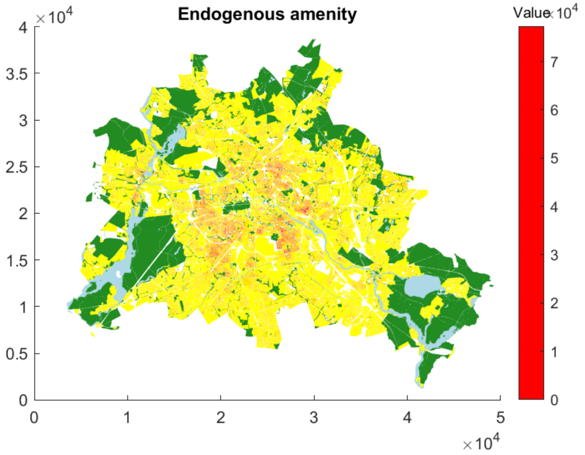
Exogenous vs. total productivity



CBD gradient in productivity explained by endogenous agglomeration forces

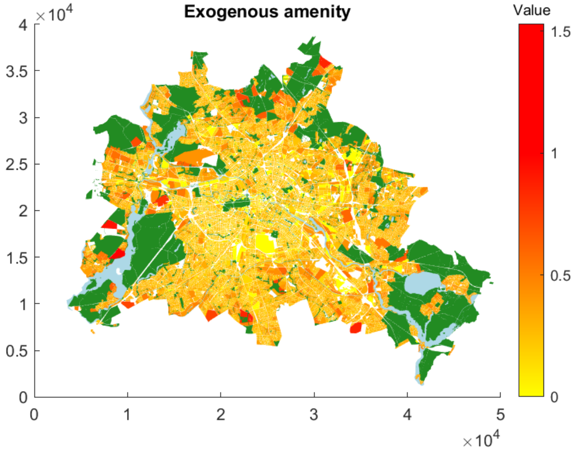
Endogenous amenity

- Endogenous amenity naturally **largest in Wilhelmine Ring**

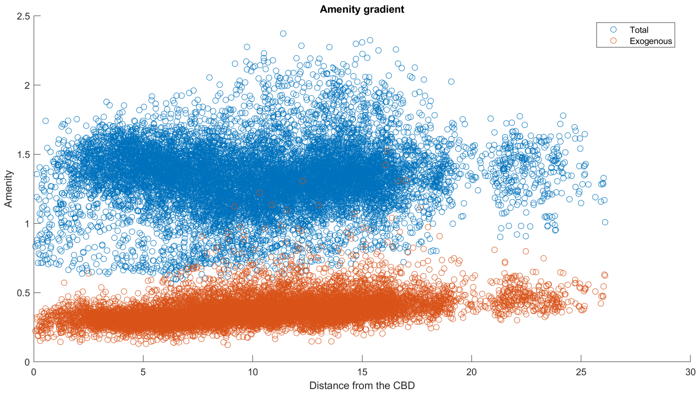


Exogenous amenity

- Exogenous amenity generally larger in suburban areas close to natural amenities



Exogenous vs. total amenity



Endogenous amenity in centre, exogenous amenity in suburbs

Conclusion

Summary

- Fundamental productivity and amenity are **structural residuals** in the model
  - Solve for them to derive moment conditions for estimation
  - Recover them in inversion
- Can allow for an endogenous component that depends on **surrounding density**
  - Then exogenous components become structural residuals
- **Endogenous agglomeration forces are quantitatively important**
  - Explain much of the CBD gradients in productivity and amenity
    - Exogenous productivity appears spatially random ✓
    - Exogenous amenity higher close to natural amenities ✓

Next week: **counterfactuals**

Literature I

Core readings

- **Ahlfedt, G., Redding, S., Sturm, D., Wolf, N. (2015): The economics of Density: Evidence From the Berlin Wall. Econometrica, 83(6), 2127–2189.**

Other readings

- Arzaghi, M, V. Henderson (2008): Networking off Madison Avenue. Review of Economic Studies. 75(4), 1011–1038.
- Davis, M., F. Ortalo-Magné (2011): Household expenditures, wages, rents, Review of economic dynamics, 14(2), 248-261.
- Combes, P.P., G. Duranton, L. Gobillon (2019): The Costs of Agglomeration: House and Land Prices in French Cities, The Review of Economic Studies, Volume 86(4), 1556–1589.
- Head, K., Mayer, T. (2014): Gravity Equations: Workhorse, Toolkit, and Cookbook. In Handbook of International Economics (Vol. 4), pp. 131-195
- Valentinyi, Á., B. Herrendorf (2008): "Measuring Factor Income Shares at the Sectoral Level," Review of Economic Dynamics, 11(4), 820–835.