Topic 2
Monocentric city model

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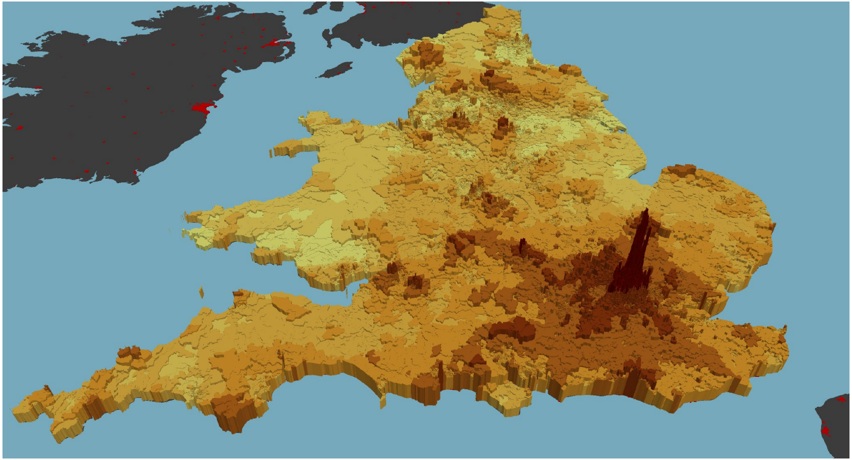
Acknowledgements

- This slide deck
  - uses material from Brueckner's textbook Chapter 2

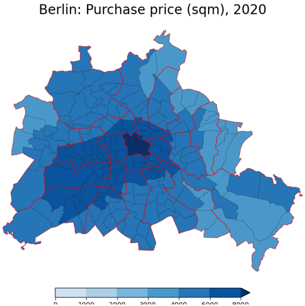
Introduction

Residential property prices in England and Wales

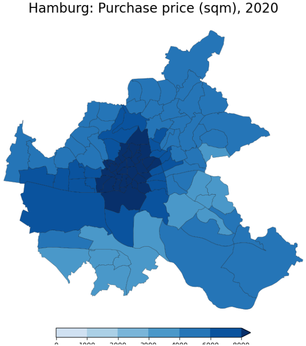


Source: LSE REEF Index powered by Ahlfeldt, Heblich, Seidel (2023). https://reef.res.lse.ac.uk/

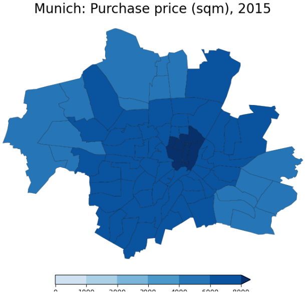
Residential property price in five largest cites in Germany



(a) Berlin 2020



(b) Hamburg 2020



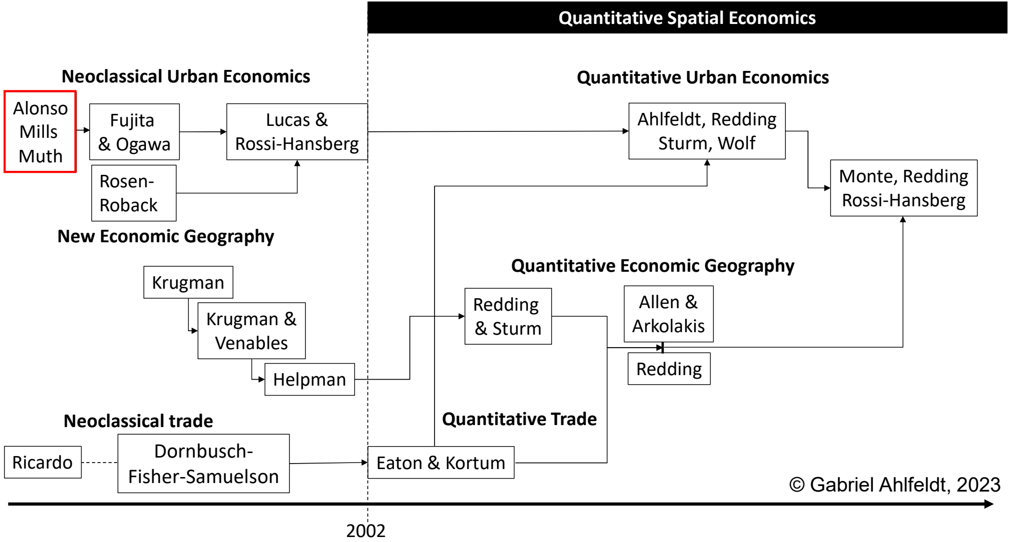
(c) Munich 2020

Why do property prices fall with distance from the urban core?

Roadmap

- In this lecture, we analyze the **monocentric city model**
  - **Brueckner (1987)**, a synthesis of Alonso (1964), Mills (1967), and Muth (1969)
- Explain **empirical regularities** observed in real-life cities
  - **Downward-sloping rent and density gradients**
  - Empirically observed in many cities around the world (Liotta et al. 2022)
- Trade-off of **commuting cost and housing rent** is the main mechanism
  - Bid rent leads to housing consumption and density effects
- A model of the **spatial equilibrium model *within* cities**
  - Identical utility levels and developer profits across space
- Workhorse tool for the analysis of internal city structure

History of thought



Assumptions

Basic assumptions I

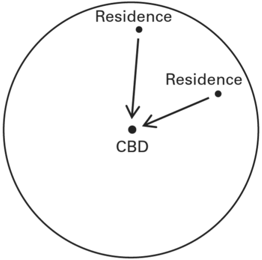
- Pure economic **rent**
  - Differential payment for an inelastically supplied good/service
    - E.g., productive urban land vs. agricultural land
- Market value V
  - future streams of rental income (R) over periods i discounted by interest rate (r)

$$V_L = \frac{R_{L1}}{(1+i)} + \frac{R_{L2}}{(1+i)^2} + \frac{R_{L3}}{(1+i)^3} + \frac{R_{L4}}{(1+i)^4} + \dots$$
$$= \sum_{t=1}^{\infty} \frac{R_{Lt}}{(1+i)^t} \approx \frac{R_L}{i} \quad \text{(Assumes } R_L \text{ is constant over time)}$$

- Observed prices a good proxy for market value (**efficient market hypothesis**)
  - **Terms 'rent', 'price', 'value' often used interchangeably**

Basic assumptions II

- Geography conforms to a simple stylized **monocentric city**
  - All **jobs** are in the central business district (**CBD**)
    - One-dimensional point (jobs don't occupy space)
  - access to the CBD is the only locational advantage
    - Plain featureless ground
  - Identical households and firms
  - No government (land use regulation)
  - Population fixed (for now)



Housing demand

Residential bid rent world

- Households maximize **utility** $U = U(c, q)$; $U' > 0, U'' < 0$
  - consume a **composite consumption good** c **and floor space** q
  - "Convex preferences" (concave utility function)
- Households must **commute to the CBD to work**
  - Transport costs reduce an exogenous budget y (wage)
  - Cost increases at rate t in distance x
- Budget constraint:

$$y - tx = p_q q + c$$

  - Household spends budget on commuting and consumption of land and other goods
  - Price of composite good normalized to $p_c = 1$

Spatial equilibrium

- **Spatial equilibrium condition**
  - Residents must be indifferent between locations $\Rightarrow$ Utility fixed to $\bar{U}$
- Floor space rent adjusts to compensate for transport cost savings
  - Solve budget equation for p_q

$$p_q = \frac{y - tx - c}{q}$$

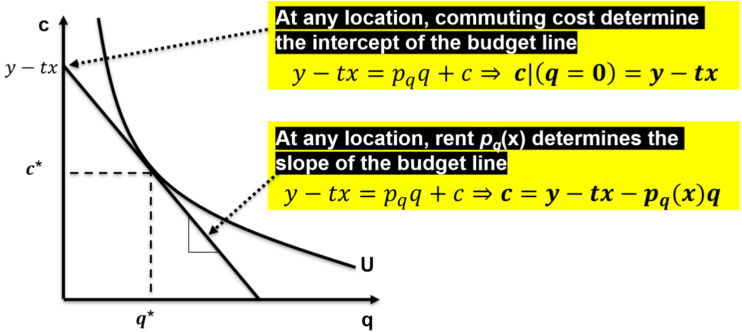
$$\frac{\partial p_q}{\partial x} = -\frac{t}{q} < 0$$

Rents p_q decrease in distance from the CBD

Q: What happens to housing consumption q as we move away from the CBD?

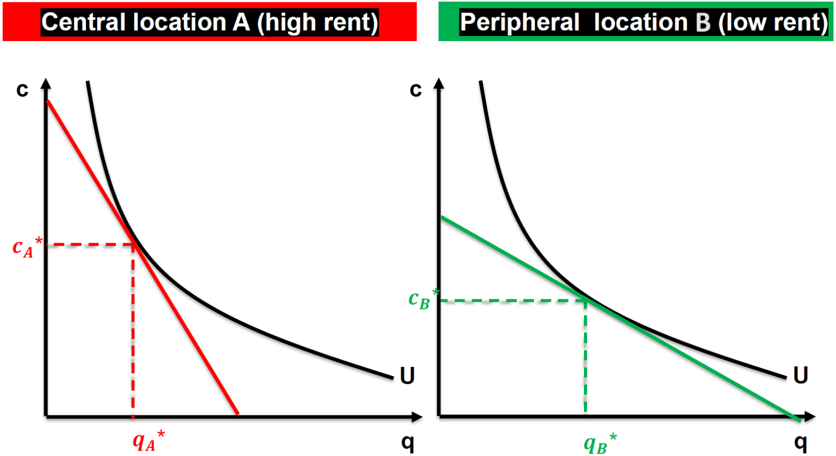
Econ 101

- From first-order conditions: $\frac{U_q}{U_c} = MRS = -\frac{p_q}{P_c(=1)}$



As distance x increases $\Rightarrow$ flatter budget line $\Rightarrow$ More housing consumption

Housing consumtion in center vs. suburb

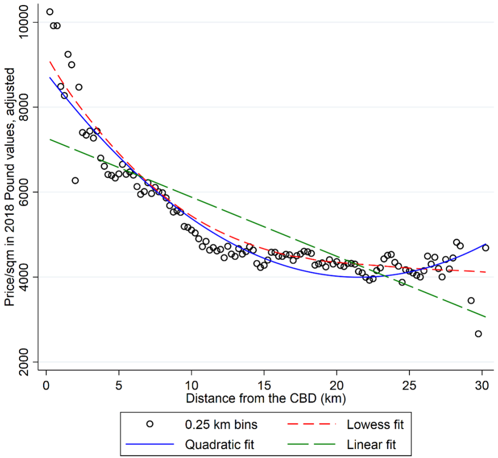


Initial predictions

- As distance from the CBD (x) increases
  - Rents decrease $\frac{\partial p_q}{\partial x} = -\frac{t}{q} < 0$
    Land consumption increases $\frac{\partial q}{\partial x} > 0$
- **Rent gradients are convex**!
  - $\frac{\partial p_q}{\partial x} = -\frac{t}{q(x)} < 0$; $\frac{dq}{dx} > 0 \Rightarrow \frac{\partial^2 p_q}{\partial x^2} > 0$
  - Due to consumption substitution effect
- Further predictions: Rents
  - decrease in commuting cost $\frac{\partial p_q}{\partial t} = -\frac{x}{q} < 0$
    increase in income $\frac{\partial p_q}{\partial y} = \frac{1}{q} > 0$

Convex house price gradient in London

- Use 'hedonic' approach (Rosen, 1974) to control attributes
  - Structure age, heating system, garage, time of transaction, etc.
- Estimate
  - $\ln(P_{ict}^{sqm}) = X_i b + \phi_c + \omega_t + \varepsilon_{ict}$
  - b are hedonic implicit prices used to value amenities (Greenstone, 2017)
- Recover
  - $\exp\left[\ln(\hat{P}_{ict}^{sqm}) - X_i \hat{b} - \hat{\omega}_t\right]$
- "Collapse" to CBD distance bins
  - Compute means within bin



Based on hedonic analysis of 1995-2008 transaction data.
Adjusted to 2018 prices using the Land Registry London house price index
(c) Ahlfeldt, 2018

Housing supply

Developers

- Profit-maximizing **developers** produce housing $H = H(l, N), H' > 0, H'' < 0$
  - Use land l and structural capital N as inputs to
- Profit function:

$$\pi = p_q H(N, l) - N - rl$$

  - r is the unit price of land, price of capital normalized to $p_n = 1$
- **In per-land unit terms**, the **profit function** becomes

$$\pi^l = p_q h(S) - S - r$$

  - where $S = \frac{N}{l}$ is the capital-land ratio aka **'structural density'**

Profit maximization

- Profit maximization implies the **first-order condition** $p_q h'(S) = 1$
- We have a well-behaved solution for $S = S(p_q), S' > 0$
  - Since $h' > 0$ and $h'' < 0$
- Using S in $h(S)$, implies that $h(S(p_q)), \frac{\partial h}{\partial p_q} > 0$
  - Profitable to **build taller when rent is higher** since building tall is expensive
- Perfect competition implies zero profits
  - $\pi^l = p_q h(S(p_q)) - S(p_q) - r = 0$
- Land rent r is a function of p_q
  - $p_q \Rightarrow r(p_q), r' > 0$

Land price is high because rent is high, not the other way round

Further Predictions

- **Housing rent** increases
  - Structural density increases $\frac{\partial S}{\partial p_q} > 0$
    Housing density increases $\frac{\partial h}{\partial p_q} > 0$
    Land rent increases $\frac{\partial r}{\partial p_q} > 0$
    Population density increases $\frac{\partial D}{\partial p_q} > 0$ since $\frac{\partial h}{\partial p_q} > 0, \frac{\partial q}{\partial p_q} > 0$
- As **distance from the CBD** increases
  - Structural density decreases $\frac{\partial S}{\partial x} < 0$
    Housing density decreases $\frac{\partial h}{\partial x} < 0$
    Land rent decreases $\frac{\partial r}{\partial x} < 0$
    Population density decreases $\frac{\partial D}{\partial x} < 0$ since $\frac{\partial h}{\partial x} < 0, \frac{\partial q}{\partial x} < 0$

Heights in North American cities

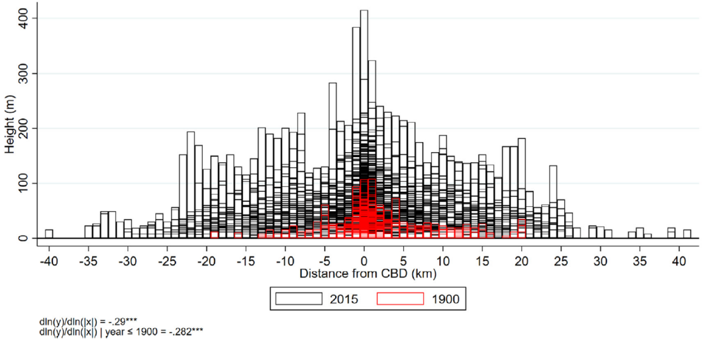
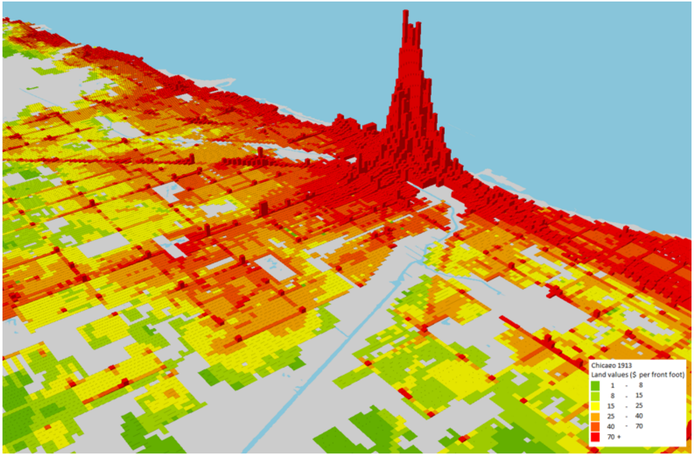


Fig. 11. Tallest buildings by North American city, and distance from the CBD Note: Each bar illustrates the height of the tallest building within a one-km bin to the west or the east of the CBD in one of 55 North American cities. Height data from Emporis. CBDs are the "global prime locations" identified by Ahlfeldt et al. (2020). Negative (positive) distance values indicate a location in the west (east) where the x-coordinate in the World Mercator projection is smaller (larger) than the x-coordinate of the CBD. Average height elasticities estimated conditional on city fixed effects. Data from https://www.emporis.com/ (see Ahlfeldt and McMillen (2018) for details).

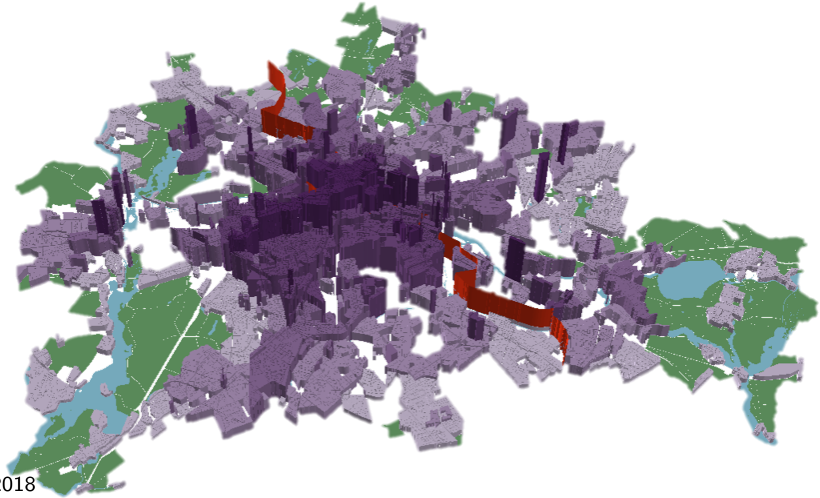
Source: Ahlfeldt & Barr (2022): The economics of skyscrapers: A synthesis. Journal of Urban Economics.

Land values in Chicago 1913



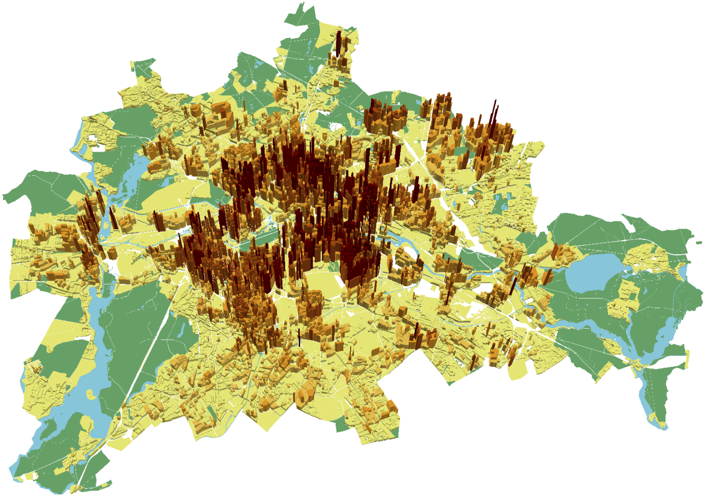
(c) Ahlfeldt, 2018

Floor area ratio in Berlin 2006



(c) Ahlfeldt, 2018

Population density in Berlin 2006



(c) Ahlfeldt, 2024

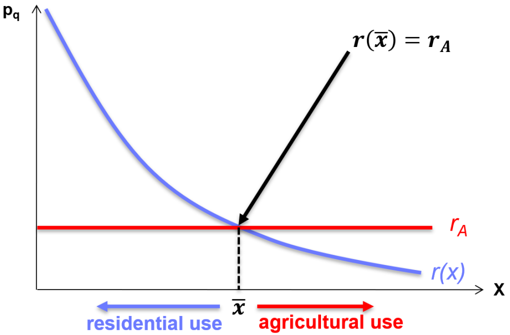
Land prices in Berlin 2016

(c) Ahlfeldt, 2018

Equilibrium & Counterfactual

Agricultural rent

- Agricultural rent, r_A is the opportunity cost of urban land use
  - Developers turn land into urban use IFF rent exceeds r_A
- Intersection of land bid rent curve $r(x)$ and r_A determines **where city ends**



Counterfactual

- Let's use the **closed-city** model for comparative static analysis (counterfactuals)
  - Model consists of exogenous $\{r_A, y, t, L\}$ and endogenous objects $\{\bar{x}, p_q, r, S, D, U\}$
  - Can evaluate the effects of exogenous on endogenous objects
- Let's focus on the **effect of a decrease in transport cost** t
  - Assume that we build a radial transport system (a network of radial subways)

Q: What happens to rents/density in the center and the suburbs?

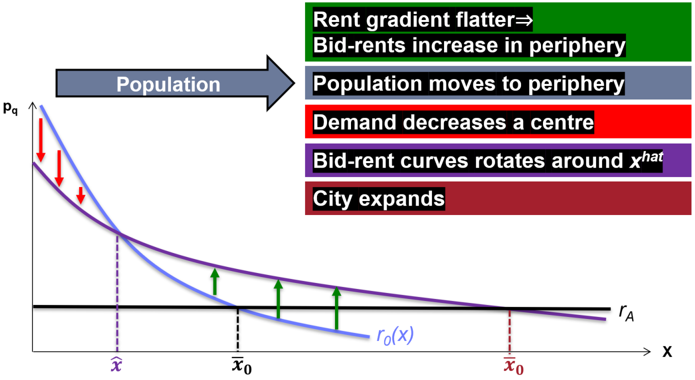
Decrease in transport costs in closed city

- Bid rent curve slope

$$\frac{\partial p_q}{\partial x} = -\frac{t}{q}$$

- Negative change in t flattens bid-rent curve

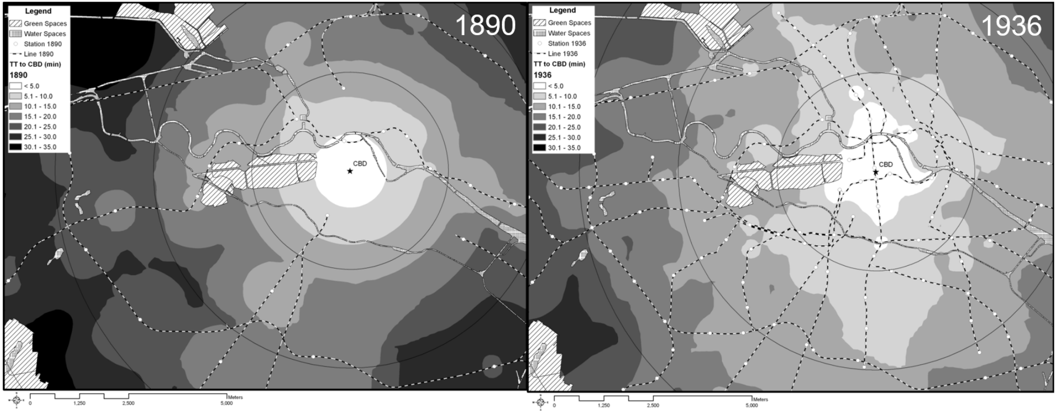
$$\frac{d\left(\frac{\partial p_q}{\partial x}\right)}{dt} = -\frac{1}{q} < 0$$



Results

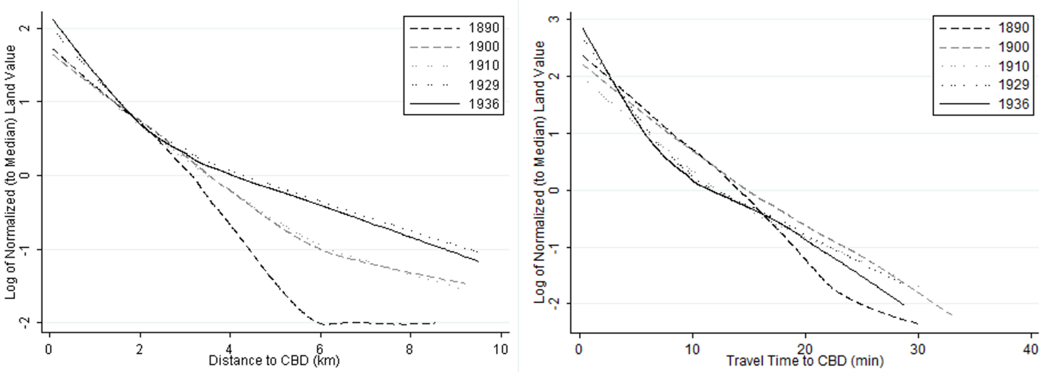
- Housing rents decrease in centre, increase in periphery
  - Structural & housing density and land rent behave similarly
- Housing consumption q
  - Rise in centre (where p falls)
  - Fall in periphery (where p increases)
- Population density
  - Falls in centre, rises in periphery
- Utility increases since cost of commuting decreases
  - Similar to a positive income shock

Evidence from Berlin (Ahlfeldt & Wendland, 2011)



- **Travel times to CBD fell as subway network developed**

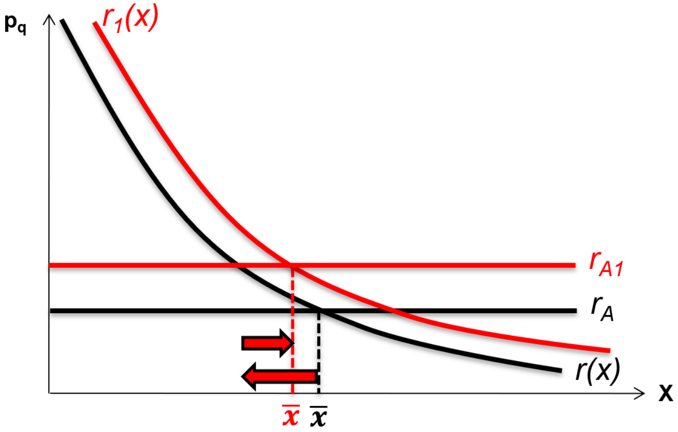
Evidence from Berlin (Ahlfeldt & Wendland, 2011)



- **Distance gradients flatten; travel time gradients don't ✓**
  - International evidence: Baum-Snow (2007) and Gonzalez-Navarro & Turner (2018)

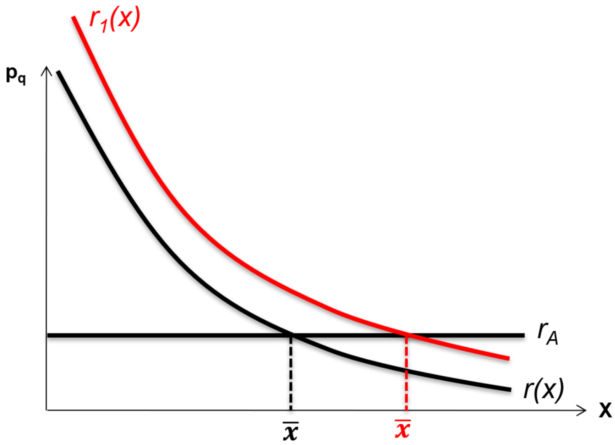
Increase in agricultural rent

- **Urban area shrinks**
  - Some land converted into agricultural use
- Scarcity of urban land **increases bid-rent**
  - And densities
  - Causes a rebound effect



Increase in urban population

- **Housing demand increases**
  - Since more households
- **Rents increase**
  - And densities
- **City expands**
  - Developers convert agricultural land into urban land



Open city

Counterfactual in open city

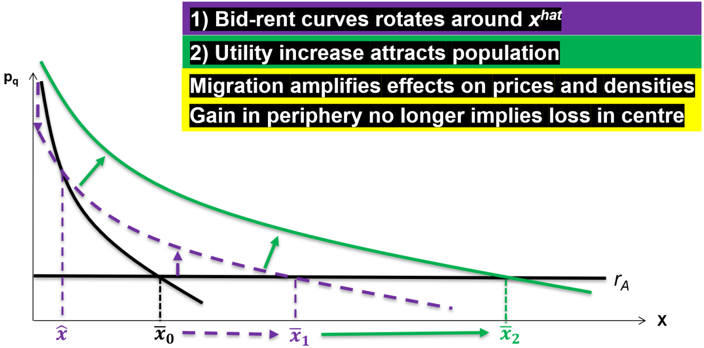
- "Open city" case: Free between-city migration
  - **Population L is endogenous**
  - Utility U is exogenous
  - Between-city spatial equilibrium a la **Rosen-Roback**
- Predictions in open-city case are similar, but require an **additional step**
  - Step 1: Hold L constant and evaluate the change using closed-city model
    - Same as before in closed-city case
  - Step 2: **Adjust population** to cancel the utility change
    - Larger $L \Rightarrow$ higher prices and lower housing consumption

Q: What does the radial subway mean for the centre?

Decrease in transport costs in open city

- Bid rent-curve flattens
  - like in closed-city model
- Bid-rent curve shifts outwards
  - Subway makes the city more attractive
  - Attracts workers and increases demand
    - population CF

Impact in city center ambiguous



Summary of counterfactuals

Closed-city case	Effect on	Housing rent		Housing cons.		Str. density		Pop. density		Land rent		
Increase in	City area	C	P	C	P	C	P	C	P	C	P	Utility
Income	Increase	Decrease	Increase	Increase	?	Decrease	Increase	Decrease	Increase	Decrease	Increase	Increase
Commuting cost	Decrease	Increase	Decrease	Decrease	Increase	Increase	Decrease	Increase	Decrease	Increase	Decrease	Decrease
Agricultural land rent	Decrease	Increase	Increase	Decrease	Decrease	Increase	Increase	Increase	Increase	Increase	Increase	Decrease
Population	Increase	Increase	Increase	Decrease	Decrease	Increase	Increase	Increase	Increase	Increase	Increase	Decrease

Open-city case	Effect on	Housing rent		Housing cons.		Str. density		Pop. density		Land rent		
Increase in	City area	C	P	C	P	C	P	C	P	C	P	Population
Income	Increase	Increase	Increase	Decrease	Decrease	Increase	Increase	Increase	Increase	Increase	Increase	Increase
Commuting cost	Decrease	Decrease	Decrease	Increase	Increase	Decrease	Decrease	Decrease	Decrease	Decrease	Decrease	Decrease
Agricultural land rent	Decrease	No effect	No effect	No effect	No effect	No effect	No effect	No effect	No effect	No effect	No effect	Decrease

Legend: Increase; Decrease; No effect; ? = Ambiguous

C=Centre
P=Periphery

Own summary of Brueckner (1987)

Summary

- **Monocentric city model is a workhorse tool in urban economics**
  - Simplified view of the world, but with empirical support
- Useful for comparative statics, ask the question:
  - Do we expect cross-city migration?
    - If all cities change in the same way $\Rightarrow$ no migration
    - If just one city changes $\Rightarrow$ migration incentive
- **Absence of an endogenous CBD with employment is a significant limitation**

Next week: **A quantitative MCM with endogenous land use**

Appendix I: Codebook

Variable/parameter	Description
$D(x)$	Population density at x
$h(x)$	Housing density at x
L	Total city population
$p_q(x)$	Housing rent at x
$q(x)$	Housing consumption at x
$r(x)$	Land rent at x
rA	Agricultural rent
$S(x)$	Structural density at x
t	Transport cost per unit of x
U	City-level utility
x	Distance from the CBD
y	City-level income per capita

Literature I

Core readings

- **Brueckner J. (1987): The Structure of Urban Equilibria, Handbook of Regional and Urban Economics, Vol II, 821-846.**
- **Brueckner, J. (2011): Wage-Based Indexes of Urban Quality of Life. Lectures on urban economics. Chapter 2. MIT Press.**

Other readings

- Ahlfeldt, G., S. Heblich, T. Seidel (2023): Micro-geographic property price and rent indices. Regional Science and Urban Economics, 98.
- Ahlfeldt. G., N. Wendland (2011): Fifty Years of Urban Accessibility: The Impact of the Urban Railway Network on the Land Gradient in Berlin 1890-1936. Regional Science and Urban Economics, 41, (2), 77-88
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