THE MONOCENTRIC CITY MODEL I

Urban Economics

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I COURSE COMPONENTS

the idea

- Block I
  - Introduction to Urban and Regional Economics and Course Overview
  - Topic I: Regional and urban concentration forces
  - Topic II: The empirics of agglomeration
  - Topic III: Costs and benefits of agglomeration
- Block 2
  - **Topic IV: Monocentric city I (household location choice)**
  - Topic V: Monocentric city II (household location choice)
  - Topic VI: Firm location choice
  - Topic VII: The urban economy in general equilibrium
- Block 3
  - Topic VIII: The vertical dimension of cities
  - Topic IX: Suburbanization and gentrification
  - Topic X: Hedonic analysis

I INTRODUCTION

roadmap

- **Last time: *Costs and benefits* of agglomeration**
  - **1) The economic effects of density**
    - **Density elasticities of economic outcomes**
    - **Net effect of density**
    - **Origins of rent effects of density**
  - **2) Equilibrium city size**
    - **Optimal city size**
    - **A system of cities**
  - **3) Compensating differentials**
    - **Real wages in spatial equilibrium**

I INTRODUCTION

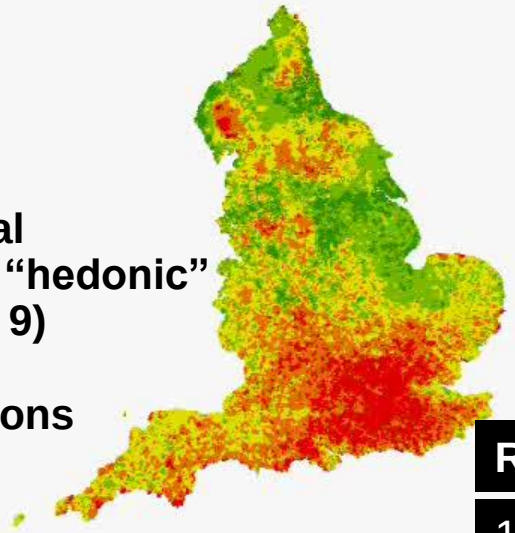
roadmap

- **This time: *The monocentric city model I***
  - **1) History of thought**
  - **2) The household bid-rent model**
    - **The utility maximization problem**
    - **Compensating differentials**
    - **Rent gradients**
    - **Substitution effects in consumption**
    - **Convexity of the rent gradient**
  - **3) Density gradients**
    - **Structural density**
    - **Population density**

I LAND VALUES IN ENGLAND

property prices

- **Property prices provided by Nationwide building society**
  - **1995-2010**
  - **Postcode reference**
  - **Adjusted for structural characteristics using "hedonic" regression (see week 9)**
  - **About 1 Mio transactions**



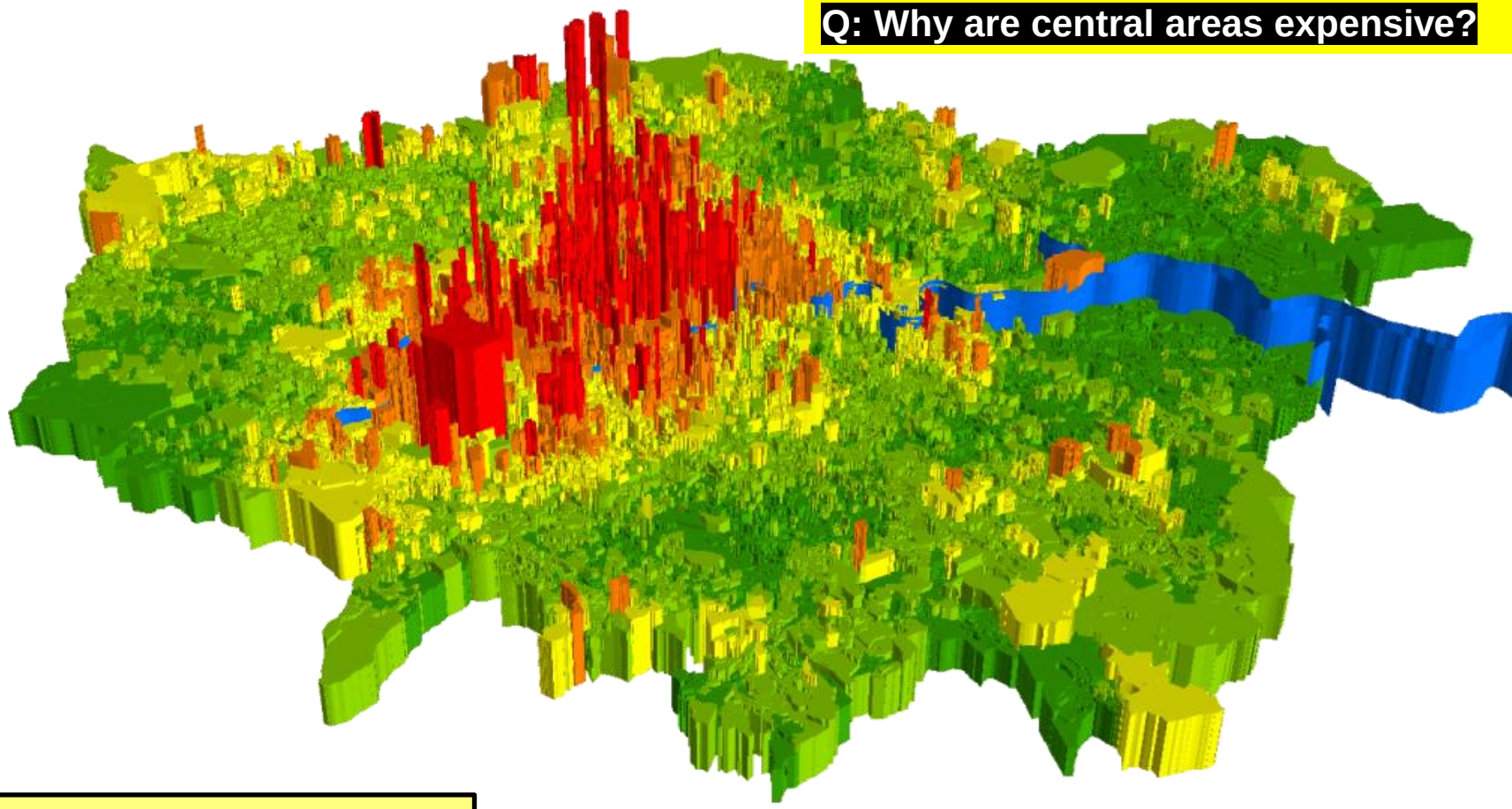
Residual land prices

1. $\log(P_i) = X_i b + OA_j + e_j$
2. Recover non-structural component
3. Spatial interpolation
4. Aggregate to 1km grid cells

I LONDON

stylized facts – land component in residential sales prices 2000-2008

Q: Why are central areas expensive?



Notes:

Regress: $\log(P_{it}) = Xb + \phi_t + \vartheta_j + \varepsilon_i$

Recover: $\hat{\vartheta}_j$

II HISTORY OF THOUGHT

roadmap

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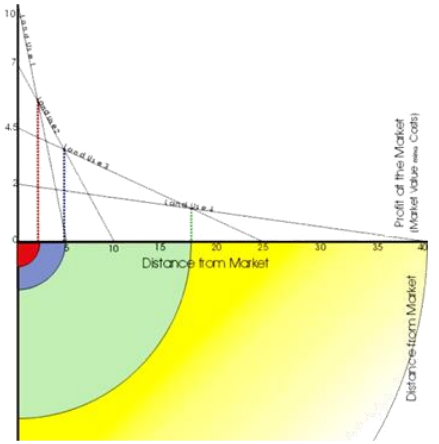
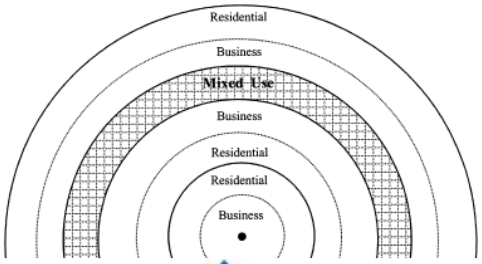
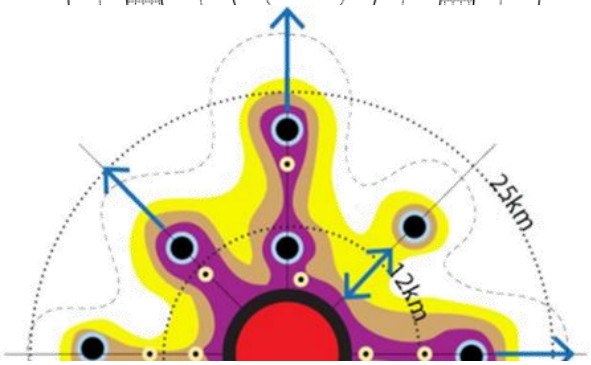
II HISTORY OF THOUGHT (SEE APPENDIX)

roadmap



- **Riccardo (1817):** *Compensating differential*
- **Von Thünen (1826):** *Distance from market place*
- **Alonso (1964):** *Distance from market place*
- **Mills (1967, 1972) and Muth (1969):** Add developers
- **Fujita & Ogawa (1982):** *Add **spillovers** to **linear** city*
- **Lucas & Rossi-Hansberg (2002):**
  - *Add **mixed land use,** City is a **disk***
- **Ahlfeldt, Redding, Sturm, Wolf (2015)**
  - No “awkward” geography
  - Empirically **tractable GE model**
  - First urban economics paper to win “Frisch medal”





III HOUSEHOLD BID-RENT MODEL

household bid-rent model

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III PLAN

household bid-rent model

- **Block II**
  - **Begin with a simple version of the Alonso model**
    - **Today**
  - **Add developers (Mills & Muth)**
    - **Today**
  - **Income segregation, comparative statics, open city**
    - **Topic V**
  - **Introduce firm location choice (elements of Fujita & Ogawa)**
    - **Topic VI**
  - **Outline the Ahlfeldt-Redding-Sturm-Wolf model**
    - **Topic VII**

III SIMPLE ALONSO BID-RENT WORLD

household bid-rent model

- **Pure economic rent**
  - Differential payment for an inelastically supplied good/service
    - E.g. productive **urban land** vs. **agricultural land**
  - The market value (V) reflects the future streams of rental income (R) over periods i discounted by on interest rate (r)

$$V = \sum_i \frac{R_i}{(1+r)^i}$$

Prices a good proxy for rent

FAMA (as opposed to Shiller) view

In reality, markets are inefficient (That's why we can make profits)

- **Simplistic monocentric bid-rent world**
  - A1 (costly) access to the (**exogenous**) CBD is the only locational advantage
  - A2 Plain featureless ground
  - A3 Identical households and firms
  - A4 No government
  - A5 No developers (households and firms occupy land)
  - A6 Population fixed

Simplification to derive testable hypotheses and counterfactuals

III RESIDENTIAL BID RENT WORLD

household bid-rent model

- Households maximize utility ***U***
- Households consume a composite consumption good ***c*** and land ***q***
  - $U = U(c, q); U' > 0, U'' < 0$
  - "Convex preferences" (concave utility function)
- Households must commute to the CBD to work
  - Transport costs (rate ***t*** over distance from the CBD ***x***) come net of an exogenous budget ***y***
  - Budget constraint
    - $y - tx = p_q q + c;\ p_c = 1$

Household spends budget on commuting and consumption of land and other goods

III SPATIAL EQUILIBRIUM

household bid-rent model

- **Spatial equilibrium condition**
  - Residents must be indifferent between locations
  - Utility fixed to $\bar{U}$
- **Land rent adjusts to compensate for transport cost savings**
  - Solve budget equation for p_q

$$p_q = \frac{y - tx - c}{q} \qquad \frac{\partial p_q}{\partial x} = -\frac{t}{q} < 0$$

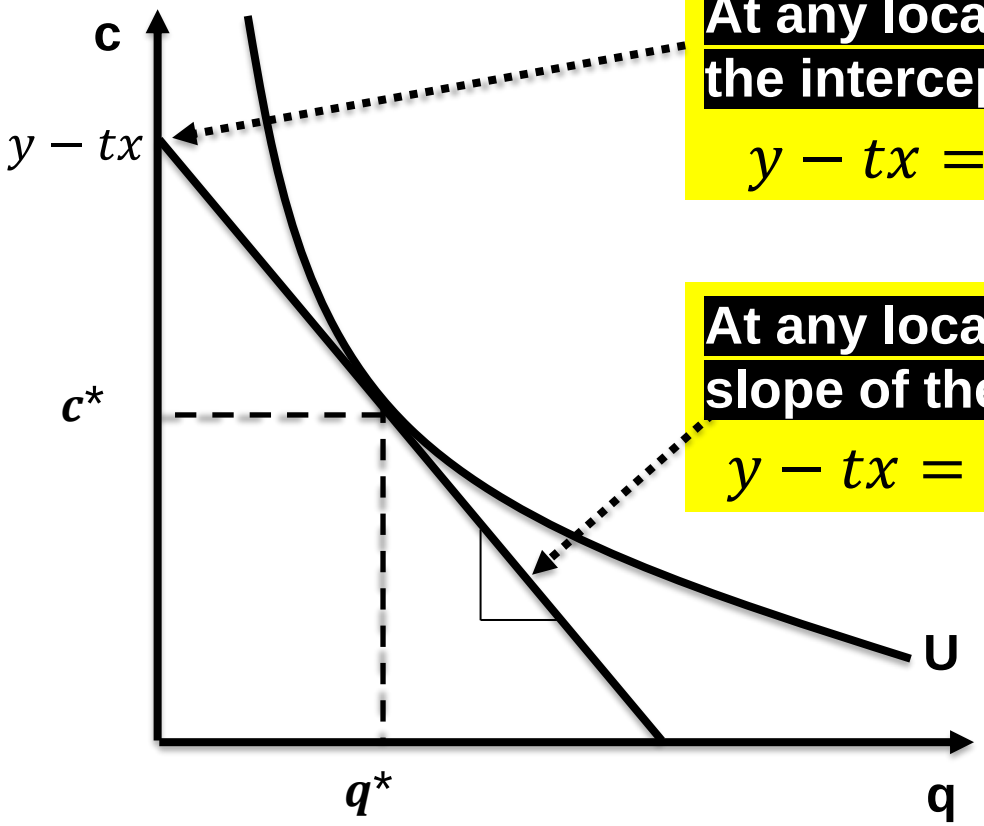
Rents p_q decrease in distance from the CBD

Q: What happens to housing consumption q as we move away from the CBD?

III ECON 101

household bid-rent model

- **From first-order conditions:** $\frac{U_q}{U_c} = MRS = -\frac{p_q}{P_c(=1)}$



At any location, commuting cost determine the intercept of the budget line

$$y - tx = p_q q + c \Rightarrow \boldsymbol{c|(q = 0) = y - tx}$$

At any location, rent p_q(x) determines the slope of the budget line

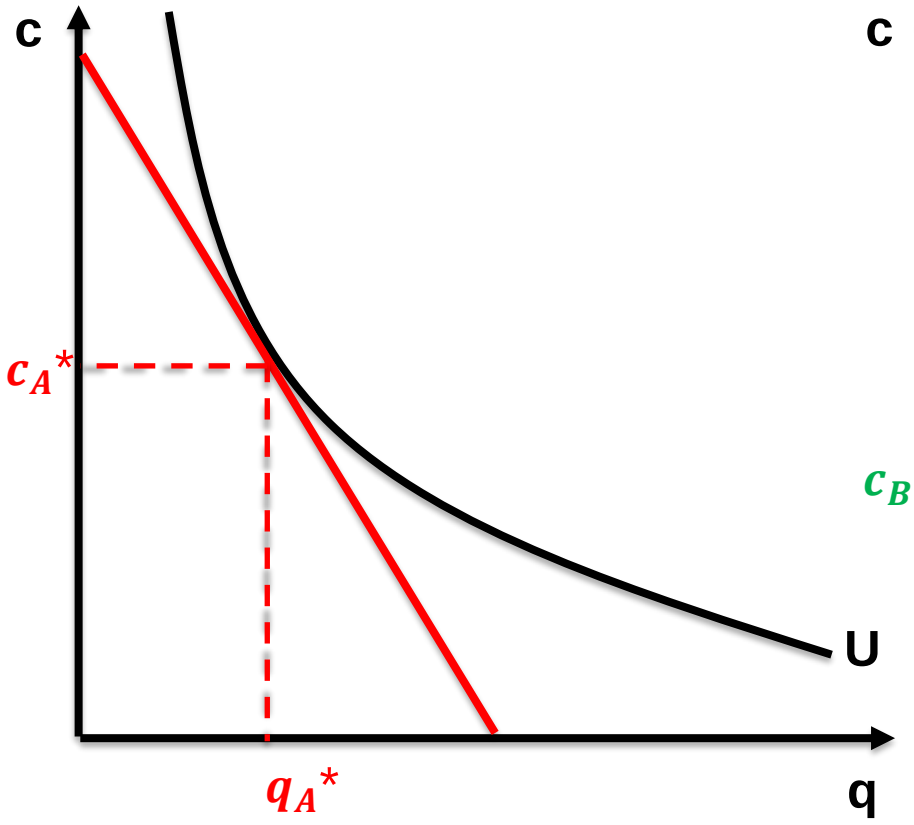
$$y - tx = p_q q + c \Rightarrow \boldsymbol{c = y - tx - p_q(x)q}$$

Spatial equilibrium implies that the budget line's shape depends on the distance from the CBD (*x*) !

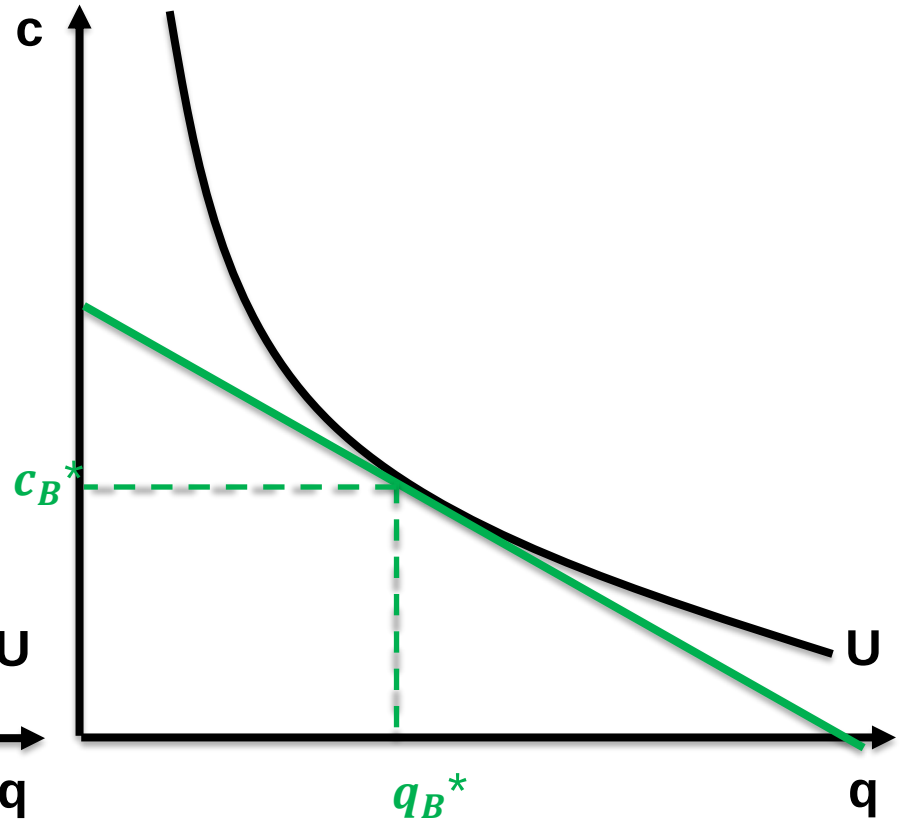
III CONSUMPTION

household bid-rent model

Central location A (high rent)



Peripheral location A (low rent)



III INITIAL PREDICTIONS

household bid-rent model

- **As distance from the CBD (x) increases**
  - **Rents decrease** $\frac{\partial p_q}{\partial x} = -\frac{t}{q} < 0$
  - **Land consumption increases** $\frac{\partial q}{\partial x} > 0$ $\left(\frac{\frac{d(q/c)}{q/c}}{\frac{d(1/p_q)}{1/p_q}} > 0\right)$
  - **Population density *D=1/q* decreases** $\frac{\partial D}{\partial x} < 0$
- **Further predictions: Rents**
  - **decrease in commuting cost** $\frac{\partial p_q}{\partial t} = -\frac{x}{q} < 0$
  - **increase in income** $\frac{\partial p_q}{\partial y} = \frac{1}{q} > 0$

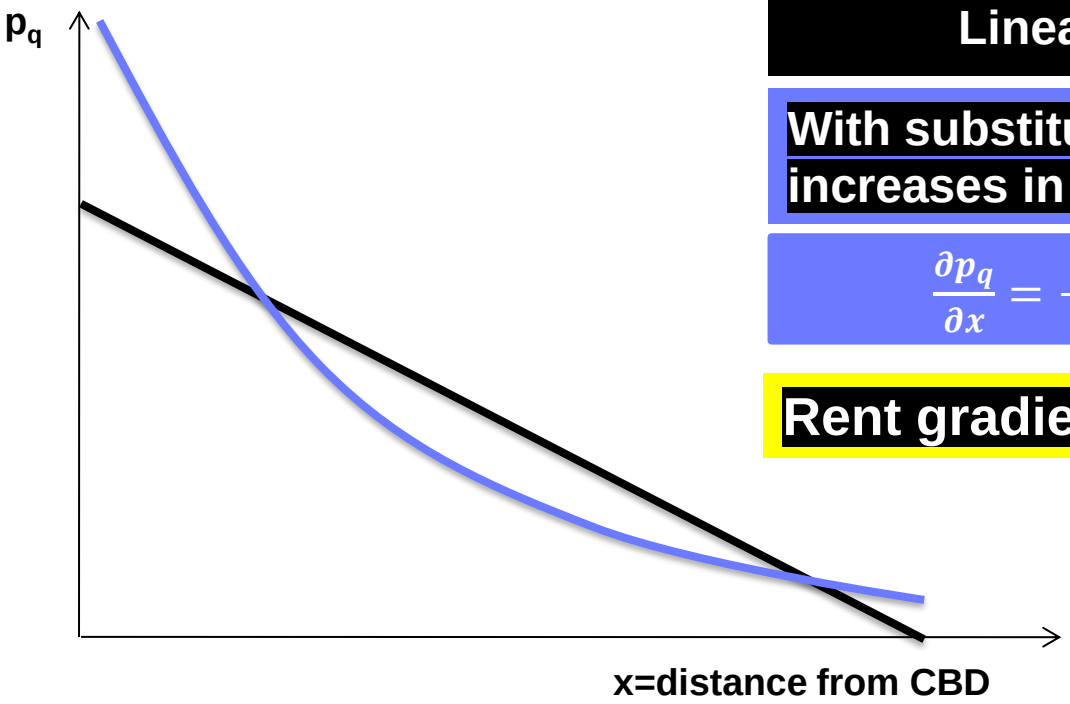
Rent gradient depends on *q* and *q* changes in *x*

Q: What is the implication of the shape of the rent gradient?

III INITIAL PREDICTIONS

household bid-rent model

- **Bid-rent negative function of distance from the CBD**



Without substitution in consumption: Fixed land consumption of $\overline{q}$

Linear gradient: $\frac{\partial p_q}{\partial x} = -\frac{t}{\overline{q}} < 0$

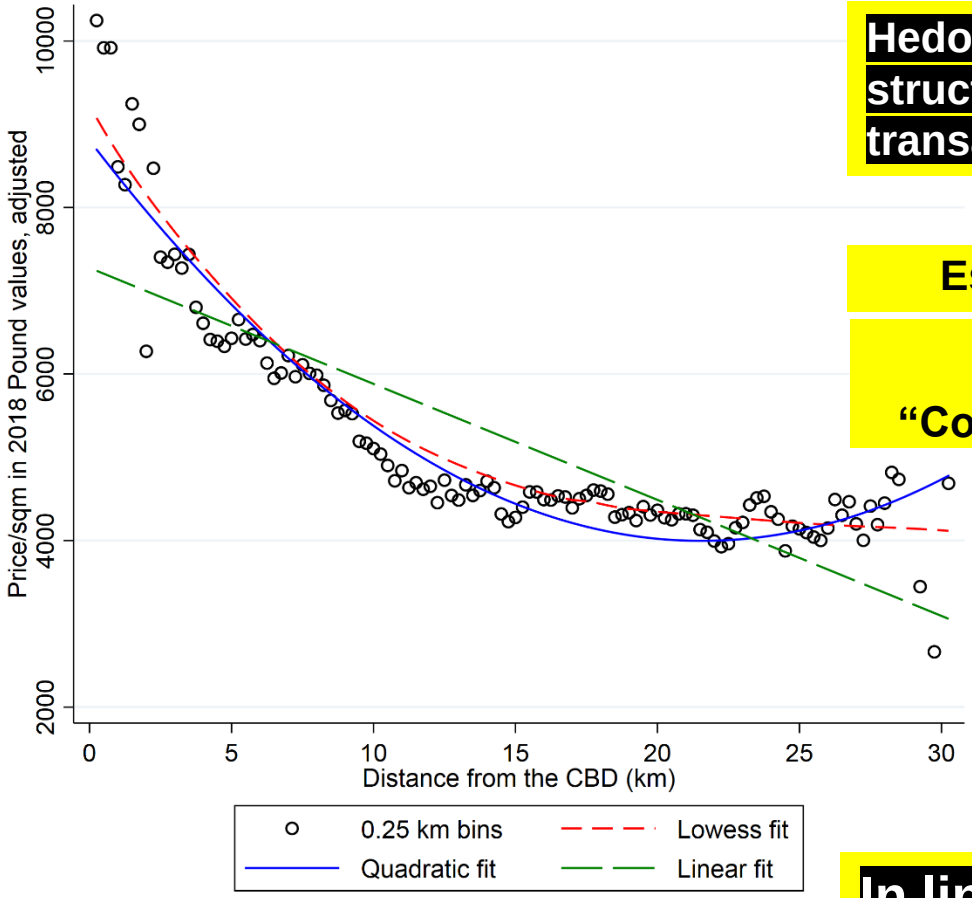
With substitution in consumption, q increases in x => Gradient "flattens out"

$$\frac{\partial p_q}{\partial x} = -\frac{t}{q(x)} < 0; \frac{dq}{dx} > 0 \Rightarrow \frac{\partial^2 p_q}{\partial x^2} > 0$$

Rent gradients in reality are convex!

III HOUSE PRICES PER SQM IN LONDON

household bid-rent model



Based on hedonic analysis of 1995-2008 transaction data.
Adjusted to 2018 prices using the Land Registry London house price index
(c) Ahlfeldt, 2018

Hedonic method to control for attributes such as structure age, heating system, garage, time of transaction, etc. Application in GY458 seminar

Estimate: $Ln\left(P^{sqm}\right)_{ict} = X_i b + \varphi_c + \omega_t + \varepsilon_{ict}$

Recover: $\exp\left[ln\left(\hat{P}^{sqm}\right)_{ict} - X_i\hat{b} - \hat{\omega}_t\right]$

"Collapse" (take the mean) to CBD distance bins

In line with theoretical prediction!

III EXCURSE: HEDONIC VALUE OF ATTRIBUTES

hedonic estimates

- Summary of Evidence (extended list from McDonald/McMillen)

School Quality	**+**	**Proximity to park**	**+**	**Proximity to public transit station**	**+**
Crime Rate	**-**	**Industrial noise**	**-**	**Adjacent to rail line, highway**	**-**
Property tax rate	**-**	**Heavy traffic on street**	**-**	**Proximity to highway interchange**	**+**
Neighbourhood wealth	**+**	**Location in floodplain**	**-**	**Proximity to church**	**-**
Air pollution	**-**	**Distance to employment**	**-**	**Zoning**	**+**
Airport noise	**-**	**Distance to shopping**	**-**	**Positive price trends**	**+**
Proximity to contaminated areas	**-**	**Distance to airport**	**-**	**Good architecture**	**+**
Proximity to nuclear plant	**-**	**Quality of nearby housing**	**+**		

Same theoretical and empirical modelling applies to other features
All locational features capitalize into rents/prices
Relevant for seminar on topic V, GY458, and GY485

IV DENSITY GRADIENTS

density gradients

- **This time: *The monocentric city model I***
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    - **Population density**

IV ADDING A SUPPLY SIDE

density gradients

- **Mills (1967)** and **Muth (1969)** add a **construction sector**
  - AMM model nicely summarized by **Brueckner (1987)**
- **Household consume *housing*** (rather than land)
- Profit-maximizing developers uses **land *l*** and **structural capital *N*** is inputs to produce housing ***H***

$$H = H(l, N), H' > 0, H'' < 0$$

- Profit function:

$$\pi = p_q H(N, l) - N - rl$$

  - ***r*** is the unit price of land, $p_n = 1$
- Capital / land ratio ***S = N/l***
  - ***Structural density***

Structural density related to building height, but not the same. Input rather than output of housing production function!

IV PROFIT-MAXIMIZING DEVELOPER

density gradients

- **Normalized per-land unit profit function.** $\pi^l = p_q h(S) - S - r$
  - **Where *h* = *H/l* is the *housing density***
- **Profit maximization defines *S* and *h* as a function of** p_q

$$\frac{\partial \pi^l}{\partial S} = 0 \Rightarrow p_q h'(S) = 1 \quad \Rightarrow S = S(p_q), S' > 0 \quad \Rightarrow h\left(S(p_q)\right), \frac{\partial h}{\partial p_q} > 0$$

- **Perfect competition implies zero profits**

$$\pi^l = p_q h\left(S(p_q)\right) - S(p_q) - r = 0$$

- **Land rent *r* is a function of** p_q

$$p_q \Rightarrow r(p_q), r' > 0$$

Capital / land, housing density, and land rents are positive functions of housing rent

III FURTHER PREDICTIONS

density gradients

- **Housing rent increases**
  - **Structural density increases** $\frac{\partial S}{\partial p_q} > 0$
  - **Housing density increases** $\frac{\partial h}{\partial p_q} > 0$
  - **Land rent increases** $\frac{\partial r}{\partial p_q} > 0$
- **As distance from the CBD increases**
  - **Structural density decreases** $\frac{\partial S}{\partial x} < 0$
  - **Housing density decreases** $\frac{\partial h}{\partial x} < 0$
  - **Land rent decreases** $\frac{\partial r}{\partial x} < 0$

Q: Implication for population density?

III BUILDING HEIGHTS

density gradients

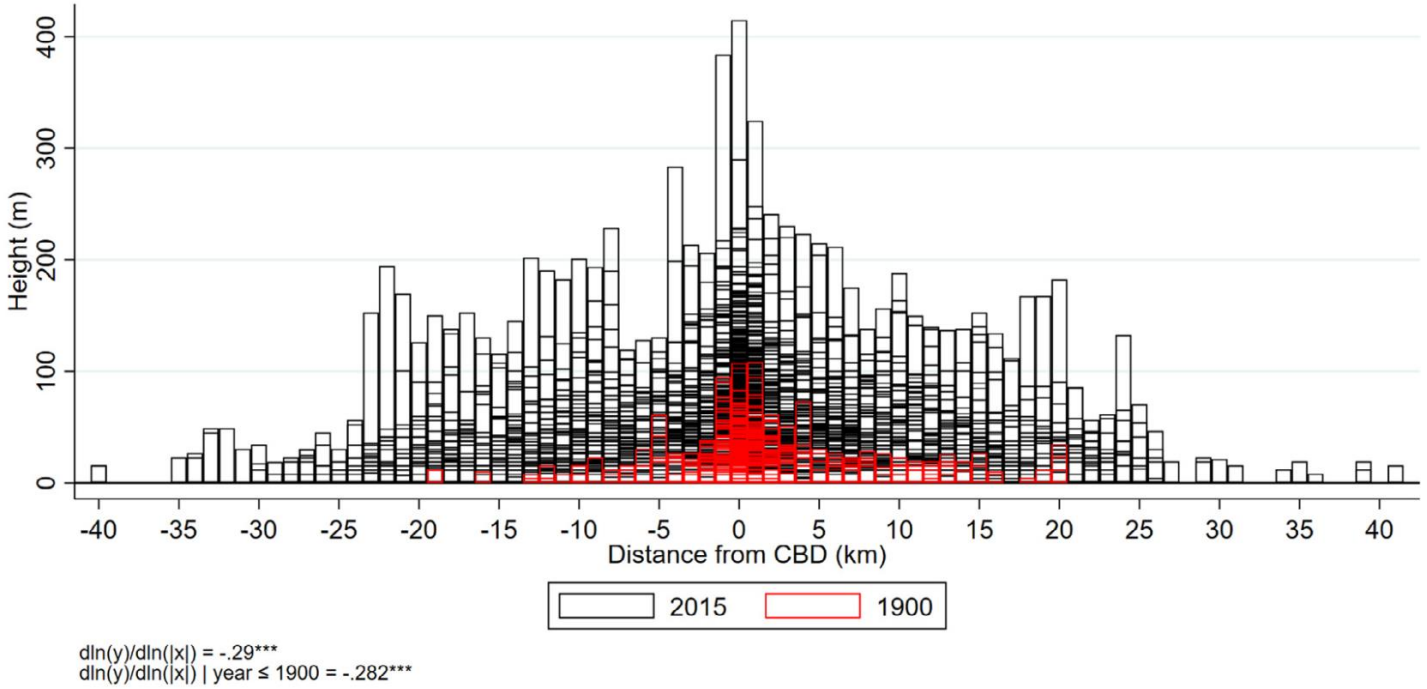
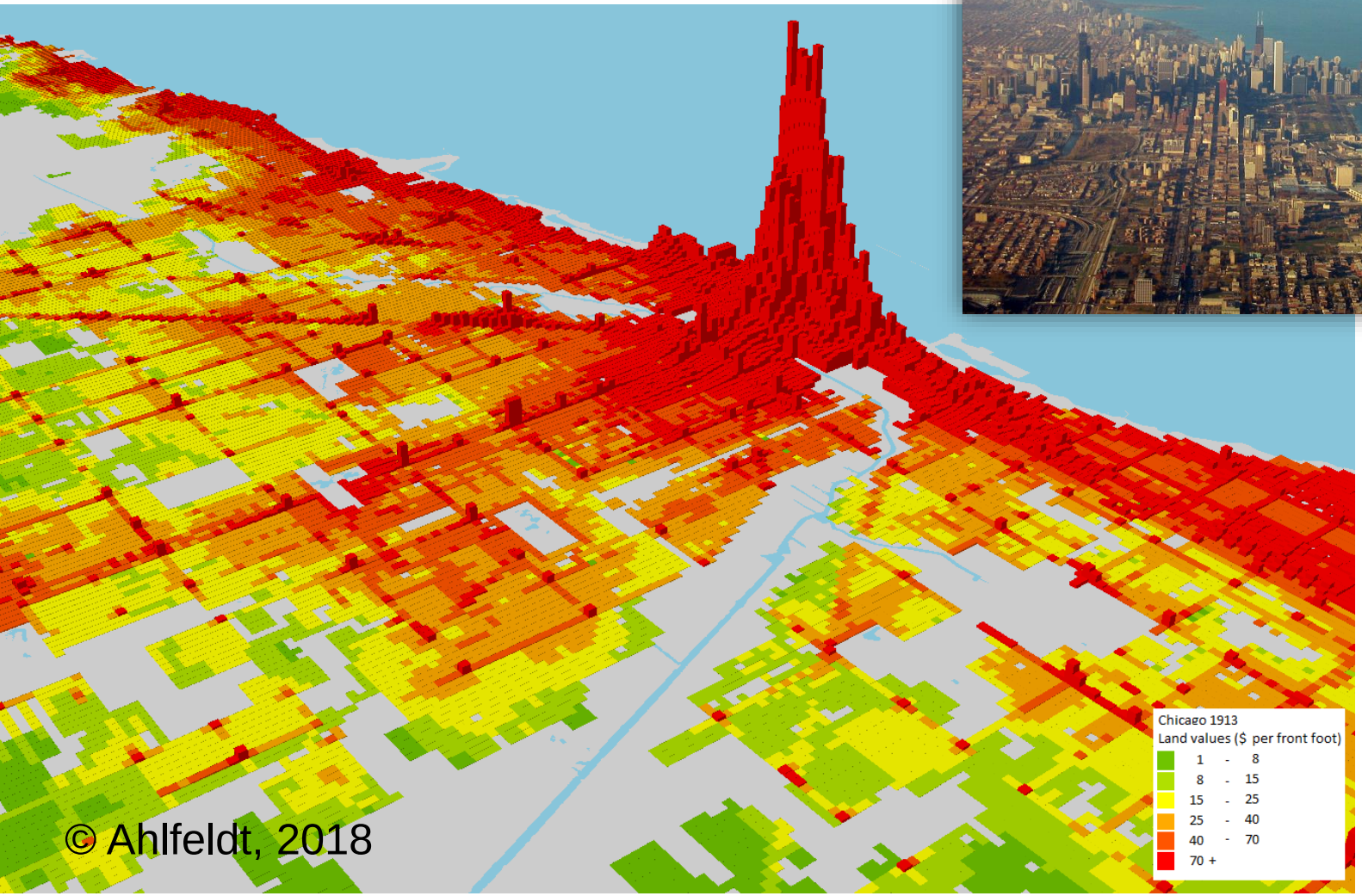


Fig. 11. Tallest buildings by North American city, and distance from the CBD Note: Each bar illustrates the height of the tallest building within a one-km bin to the west or the east of the CBD in one of 55 North American cities. Height data from Emporis. CBDs are the "global prime locations" identified by Ahlfeldt et al. (2020). Negative (positive) distance values indicate a location in the west (east) where the x-coordinate in the World Mercator projection is smaller (larger) than the x-coordinate of the CBD. Average height elasticities estimated conditional on city fixed effects. Data from https://www.emporis.com/ (see Ahlfeldt and McMillen (2018) for details).

Structural density decreases in distance from the CBD

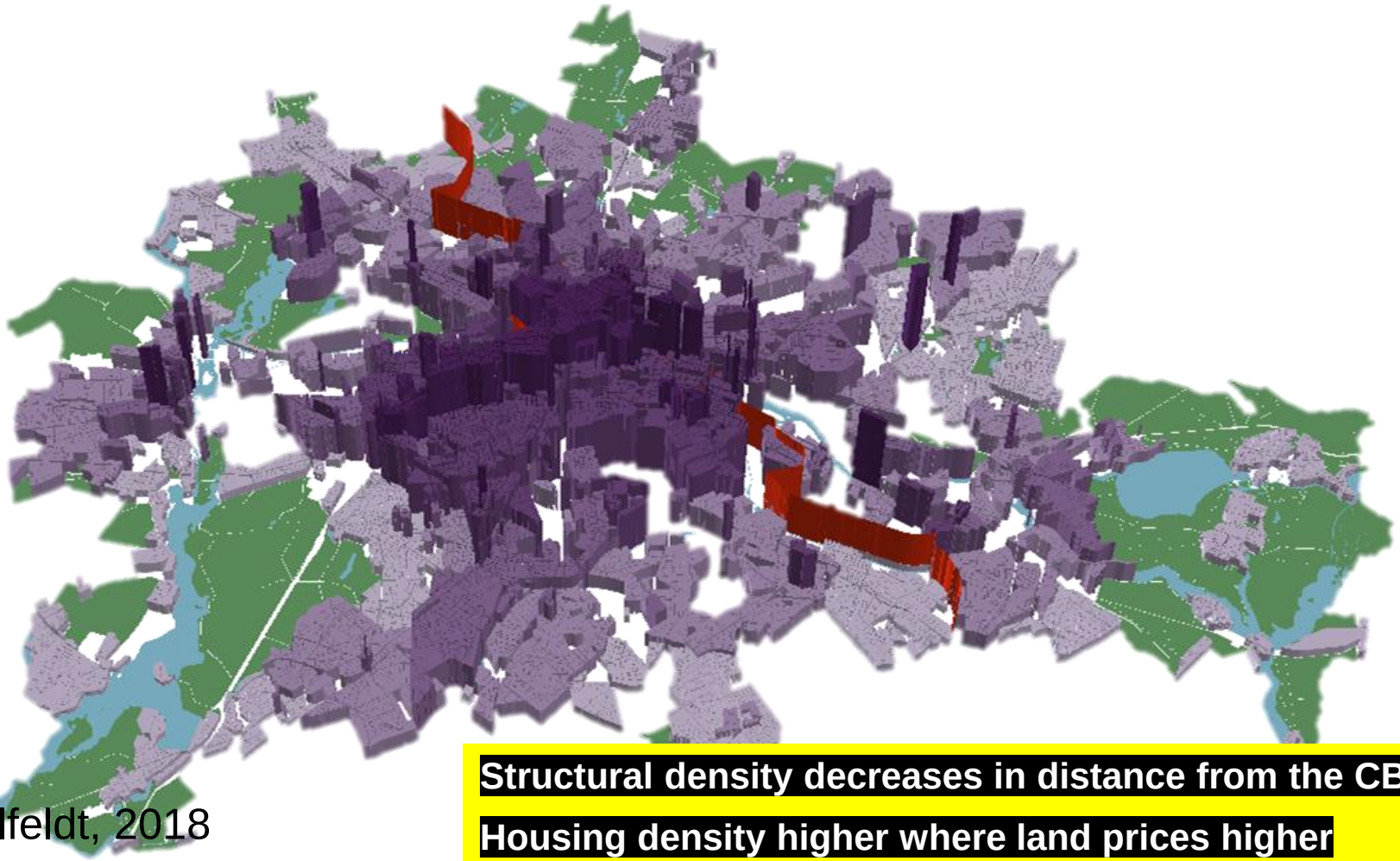
III LAND PRICES IN CHICAGO

density gradients



III FLOOR AREA RATIO IN BERLIN

density gradients



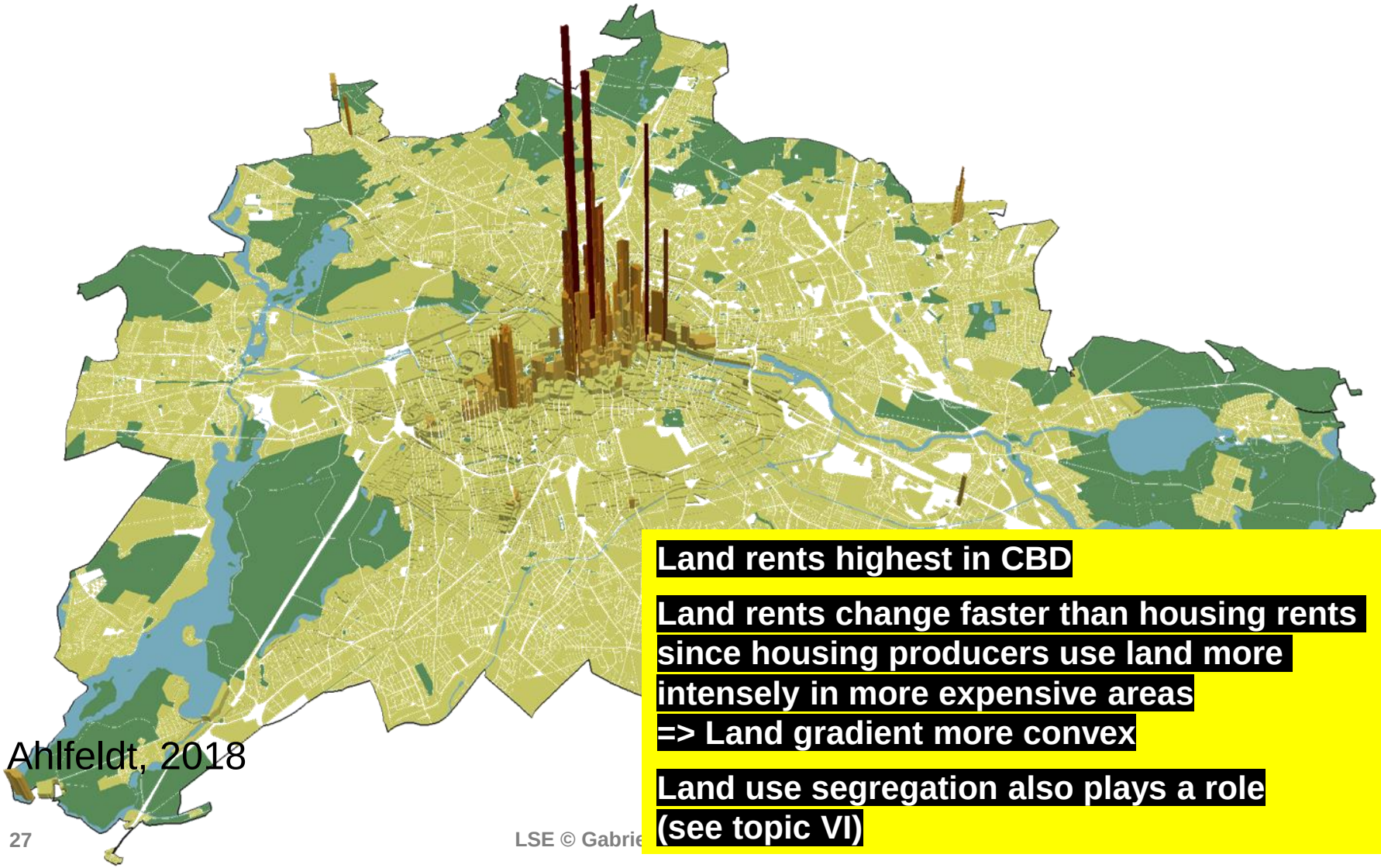
© Ahlfeldt, 2018

Structural density decreases in distance from the CBD

Housing density higher where land prices higher

III LAND PRICES IN BERLIN

density gradients



Land rents highest in CBD

Land rents change faster than housing rents since housing producers use land more intensely in more expensive areas
=> Land gradient more convex

Land use segregation also plays a role (see topic VI)

III POPULATION DENSITY

density gradients

- **Population density**
  - **In Alonso model:** $D(x) = \frac{1}{q(x)}$ ← **Space (land) per capita**
  - **In Mills-Muth model:** $D(x) = \frac{h(x)}{q(x)}$ ← **Housing density**; ← **Space (housing) per capita**

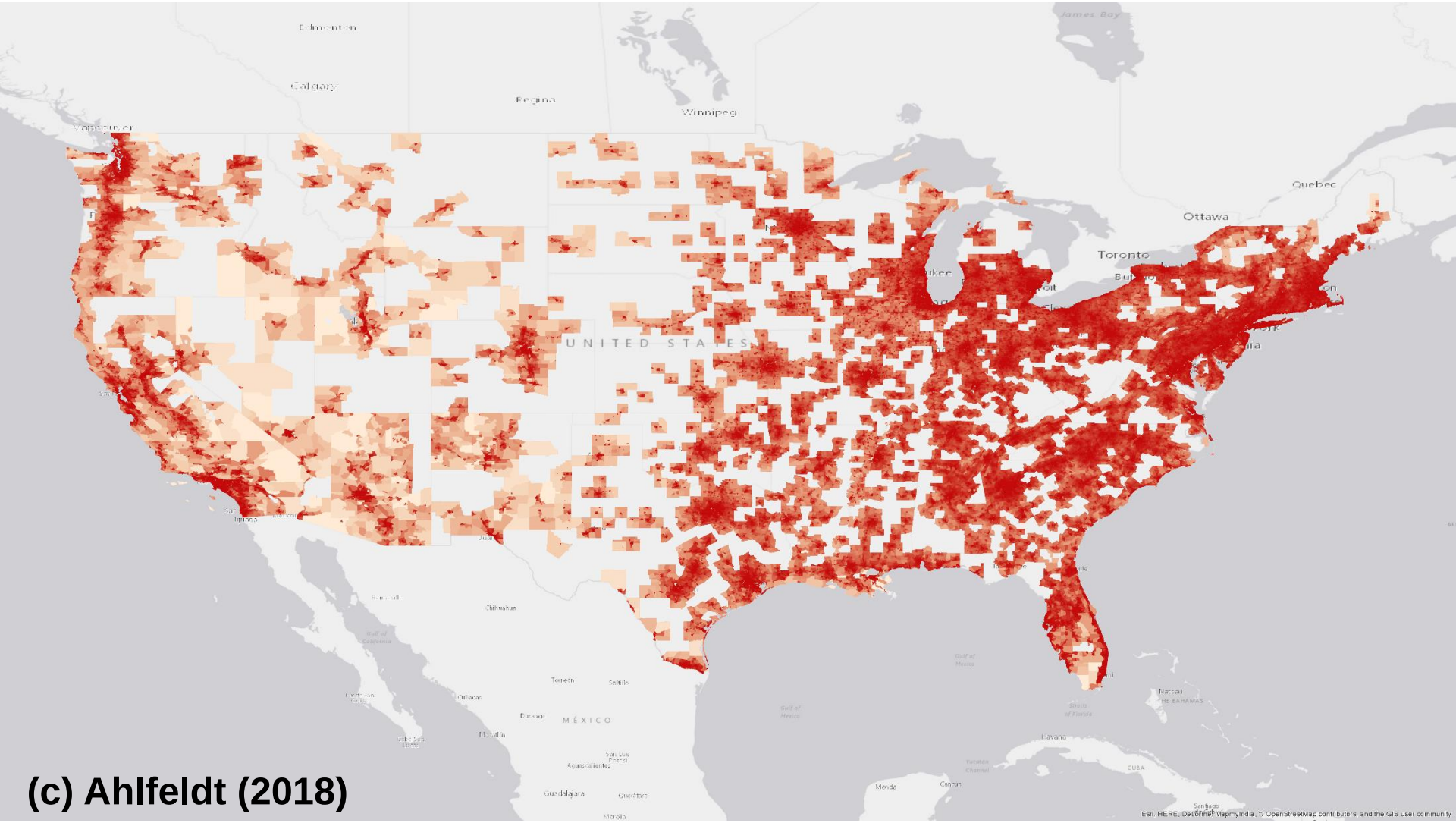
- **As distance from the CBD increases**
  - **Housing density decreases**
  - **Space consumption increases**

$$\frac{dD(x)}{dx} = \frac{\overbrace{h'(x)}^{<0}\,q(x) - h(x)\overbrace{q'(x)}^{>0}}{\underbrace{[q(x)]^2}_{>0}} < 0$$

Population density decreases in distance from the CBD

III POPULATION DENSITY IN THE US

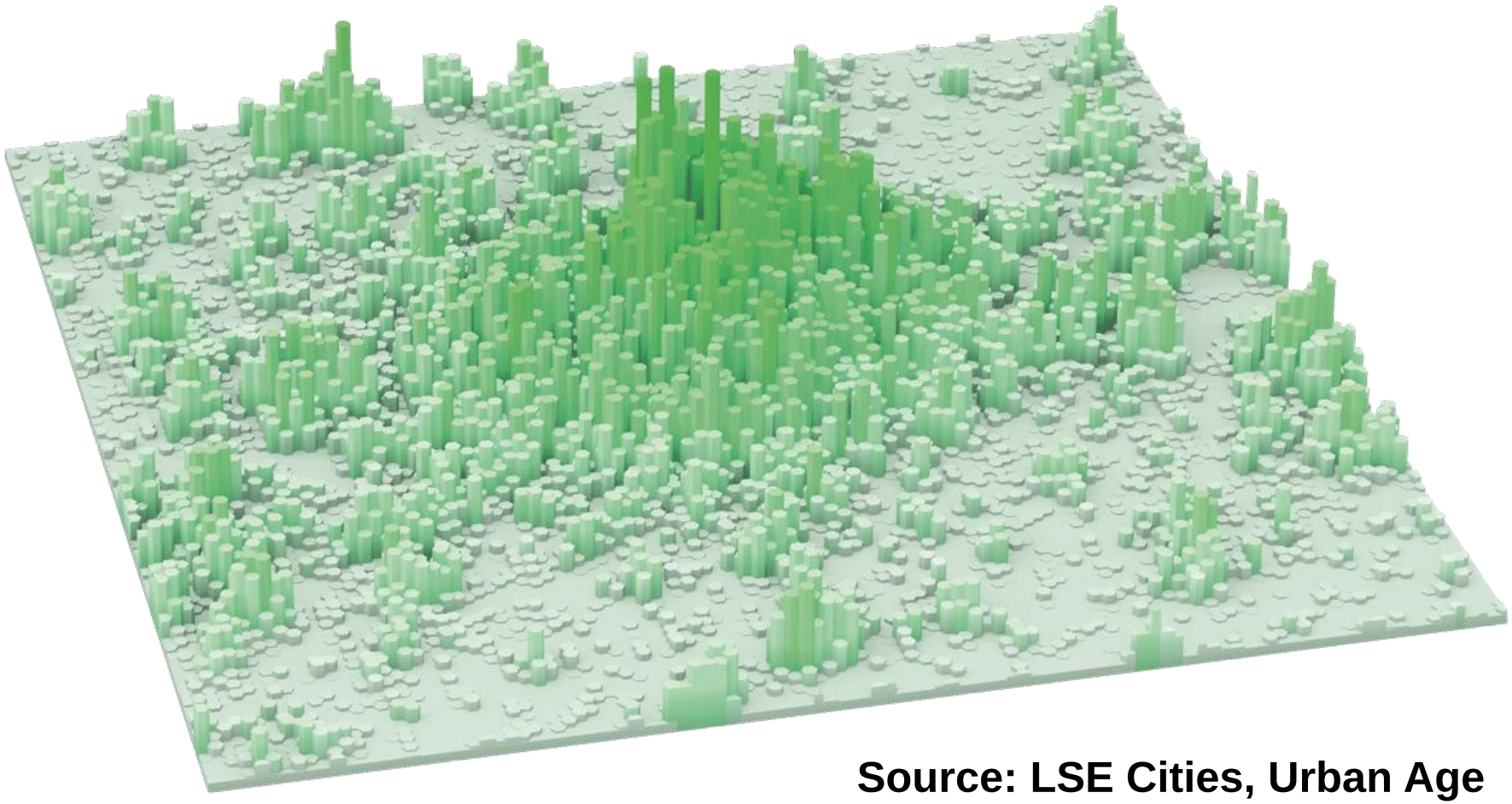
density gradients



(c) Ahlfeldt (2018)

III POPULATION DENSITY IN LONDON

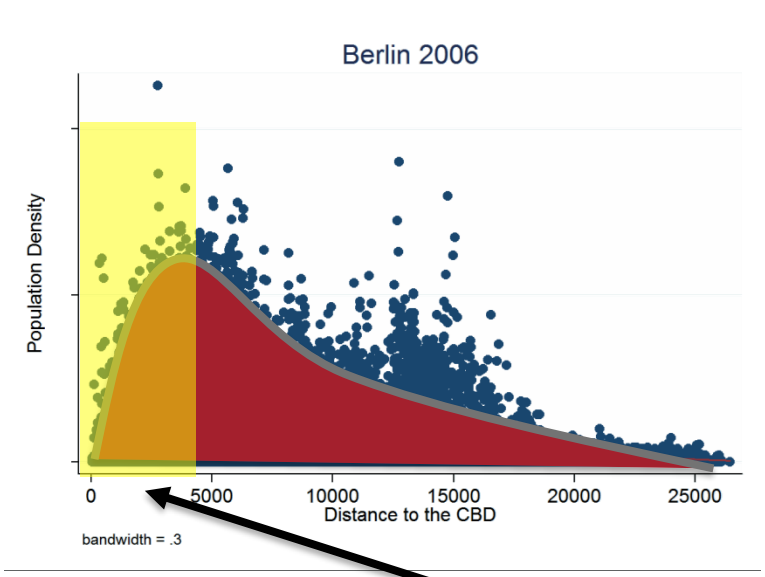
density gradients

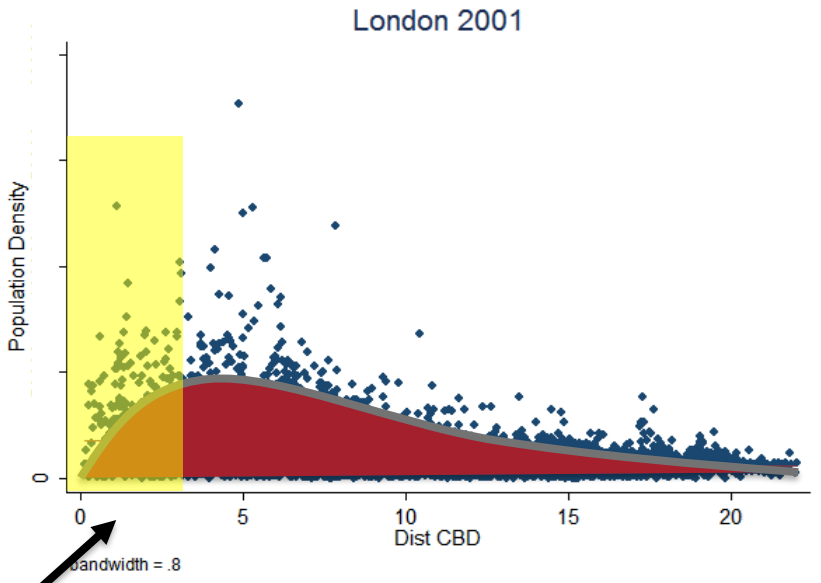


Source: LSE Cities, Urban Age

III POPULATION DENSITY

Population density



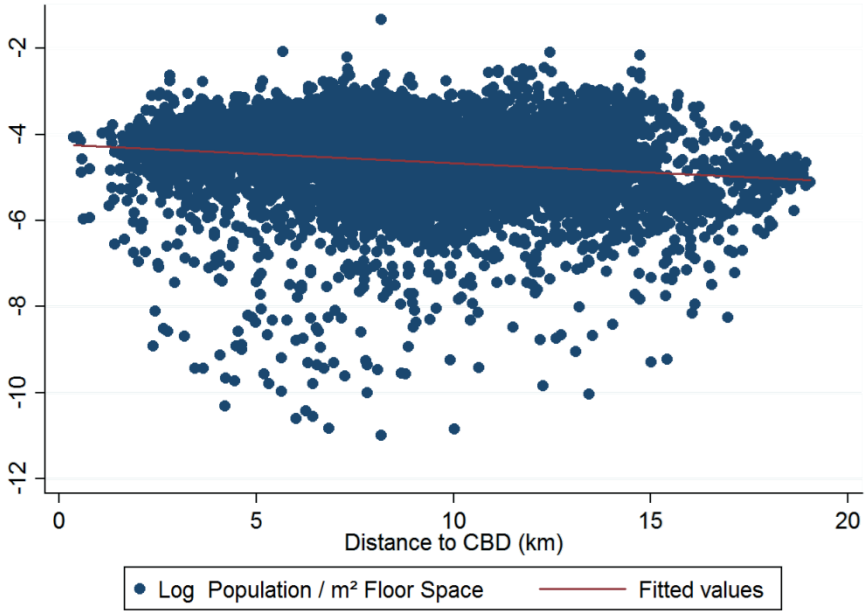


Q: What is going on here?

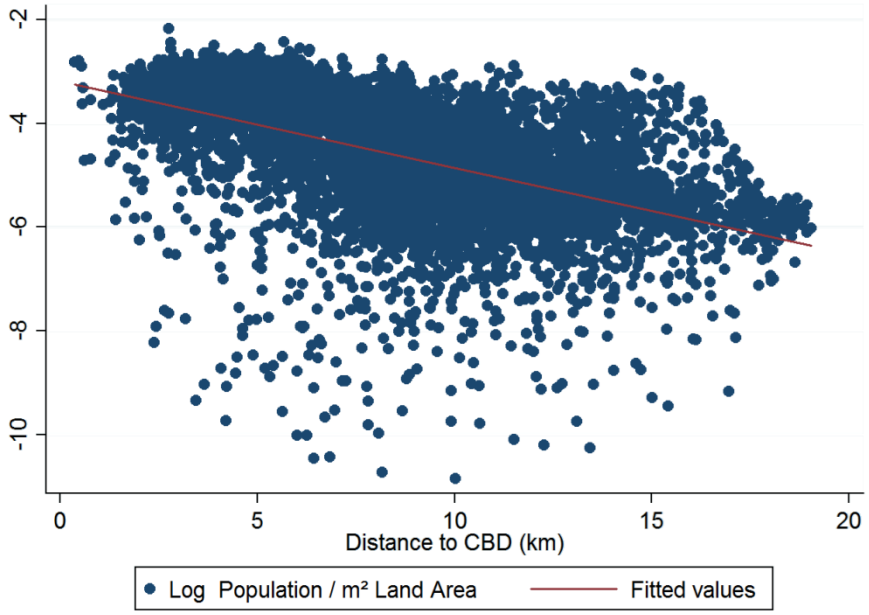
IV POPULATION PER SPACE UNIT (BERLIN)

Population density

Ln (population / floor space)



Ln (population / land area)



- **Residential blocks only**

Q: In line with model predictions?

III HOW RELEVANT IS THE AMM MODEL?

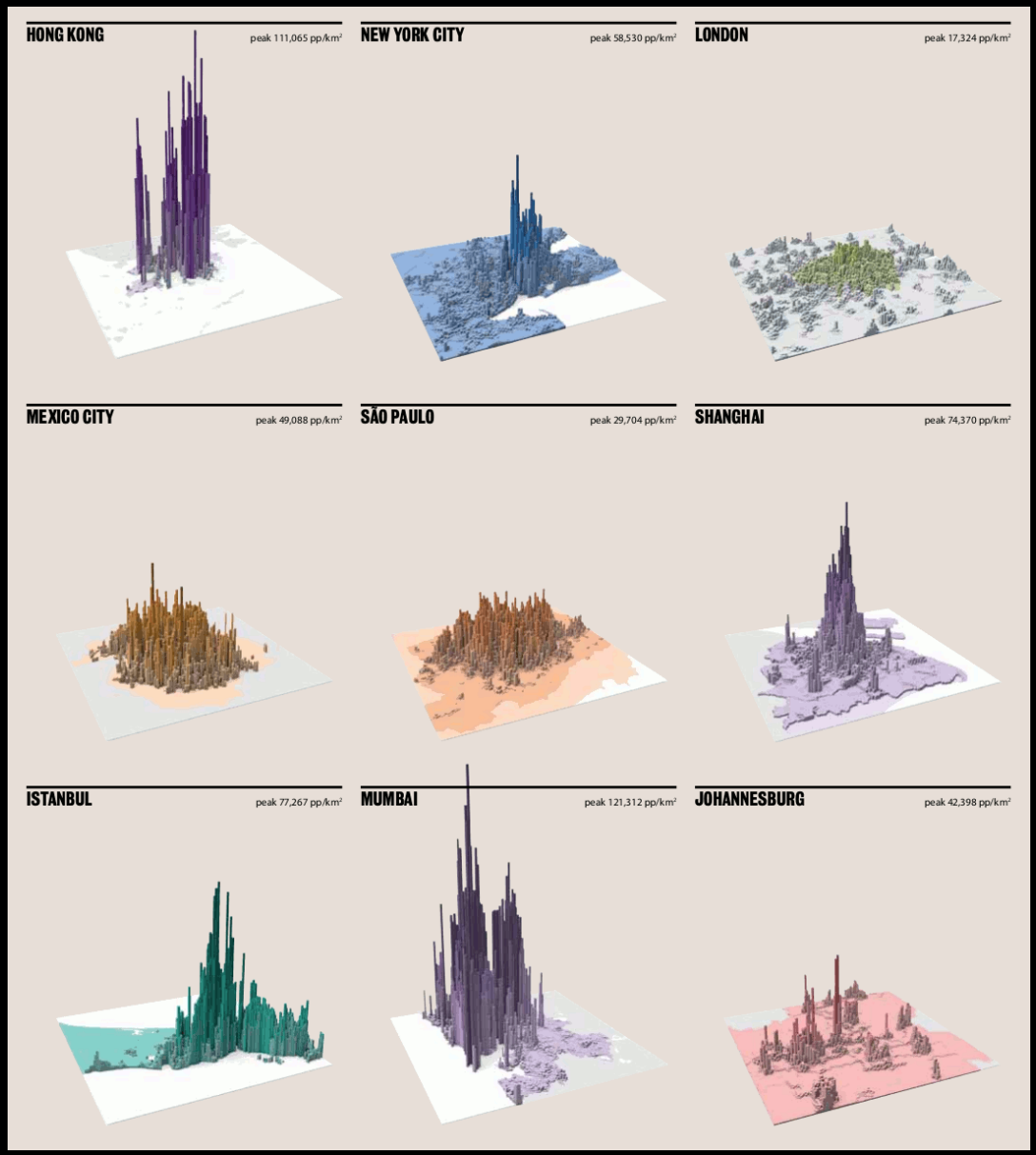
Density gradients

Every model is a simplification of reality

A good model can be simple, but should explain relevant features of the real world

Q1 How widely applicable is the AMM model. How monocentric/centralized are world-cities today?

Q2 Which type of cities tend to be more centralized?



LIOTTA ET AL (2022)

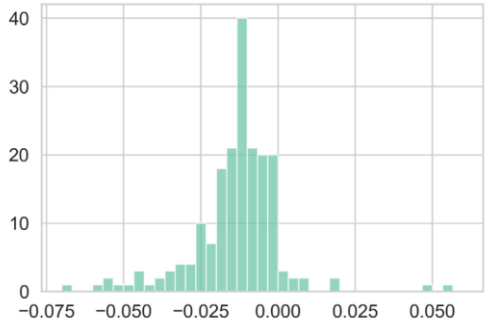
Testing the monocentric standard urban model in a global sample of cities

- **Provide a variant of the monocentric standard urban model (MCM)**
  - Work with a measure of income – transport cost (distance in appendix)
- **Collect a large data set with within-city variation for 192 cities**
  - Pop. densities, rents, dwelling sizes, transportation times, land cover
  - At 1 km^2 grid cells
- **Estimate gradients for a GLOBAL sample of cities**
  - Rents
  - Population density
- **Analyze determinants of heterogeneity**
  - Rent gradients
  - Population density gradients
  - Urban land cover

RENT GRADIENTS

Testing the monocentric standard urban model in a global sample of cities

Most cites have rent gradients consistent with the MCM



(a) Histogram - distribution of parameter f'_c.

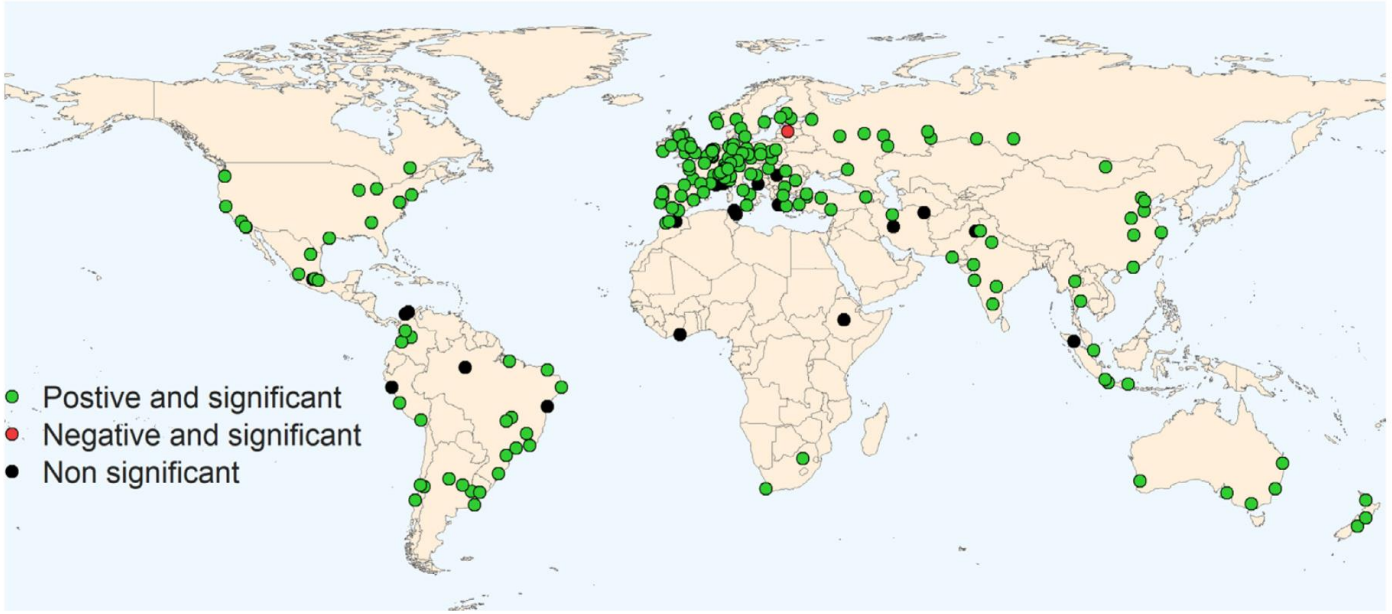


Fig. 2. Map of the direction and significance of parameter f_c (rent elasticity with respect to income net of transportation costs), estimated with 2SLS. *This parameter is positive and significant in 167 cities, negative and significant in one city, and non-significant in 23 cities.*

On average, rent decreases by about 1.5% per 1-km distance from the CBD

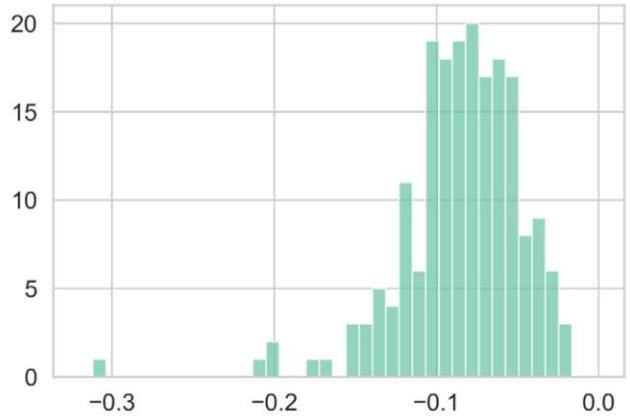
But there is heterogeneity

	Mean	Min.	Q1	Med.	Q3	Max.	Nb. of obs.
OLS	-0.014	-0.070	-0.019	-0.012	-0.006	0.056	191

(b) Table - distribution of parameter f'_c.

POPULATION DENSITY GRADIENTS

Testing the monocentric standard urban model in a global sample of cities



(a) Histogram - distribution of parameter h'_c.

	Mean	Min.	Q1	Med.	Q3	Max.	Nb. of obs.
OLS	-0.085	-0.309	-0.102	-0.081	-0.060	-0.017	192

(b) Table - distribution of parameter h'_c.

. B.6. Distribution of the estimates of parameter h'_c (density gradient estimated with Robustness check 1) in the 192 cities.

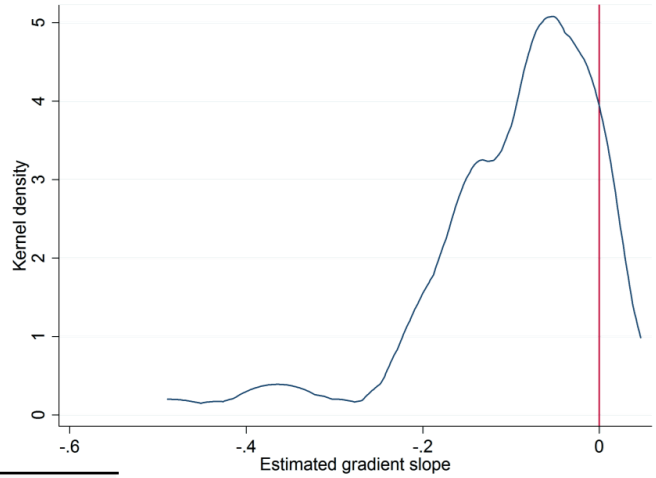
On average, rent decreases by about 9% per 1-km distance from the CBD, but there is heterogeneity

	Main specification
log(population)	−5.428***
	(0.856)
log(income)	
Monocentricity index	4.346*** ?
	(1.591)
Gini index	−0.104
	(0.147)
Percentage of informal housing	−0.176**
	(0.070)
Coastal city	1.583
	(1.508)
Regulatory environment	3.119*
	(1.749)
Asia	2.602
	(3.939)
Africa	−8.157***
	(2.968)
Oceania	−9.958***
	(2.442)
North America	2.598
	(3.429)
South America	−3.264
	(3.801)
Constant	95.918***
	(12.958)
Observations	192
R^2	0.508
Adjusted R^2	0.478
F Statistic	29.49***

IV BERTAUD & MAELPEZZI (2003)

population gradients in 50 cities

- **Majority of cities centralized**
- **But degree of centralization varies significantly in observable attributes**



	(1)		(2)	
	Estimated gradient slope			
Constrained geography (dummy)	0.018	(0.026)	-0.019	(0.032)
(Former) non-Chinese socialist economy	0.007	(0.021)	0.034	(0.020)
Mixed market / non-market planning	0.095***	(0.034)	0.050*	(0.026)
Non-market planning	0.153***	(0.036)	0.099***	(0.029)
Log population	0.068***	(0.015)	0.077***	(0.012)
Log GDP per capita	0.025***	(0.009)	0.001	(0.007)
Log denisty			-0.052***	(0.012)
Log dispearsion			0.119***	(0.032)
Constant	-0.164***	(0.024)	-0.138***	(0.018)
Observations	50		50	
R^2	0.601		0.761	
AIC	-118.0		-139.7	

Notes: Standard errors in parentheses. Meta-analysis of Bertaud & Maelpezzi (2003)
*** $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$**

Non-market planning cities more decentralized

Larger cities more decentralized

Richer cities more decentralized

Denser cities more centralized

On average density decreases by 17.8% per km distance from CBD

Monocentric urban model „remarkably good“ in predicting real-world outcomes

SUMMARY

Conclusion

- **Alonso-Mills-Muth model is a workhorse tool in urban economics**
  - Assumes all employment is concentrated in CBD
  - Spatial equilibrium implies lower rents at larger commuting distances
  - Housing rent decreases in distance from the CBD
  - Housing consumption increases in distance from the CBD
- **Developers respond to changes in housing rent**
  - Structural density decreases in distance from the CBD
  - Land rents decrease in distance from the CBD
- **Most cities tend to be centralized, but there are exceptions**
  - Planned cities
- **Next: Monocentric city model II**
  - Open-city model, income segregation, comparative statics

THANKS

READING

- Core readings:
  - Brueckner J., 1987, “The Structure of Urban Equilibria”, Handbook of Regional and Urban Economics, Vol II, 821-846.
  - DiPasquale-Wheaton, chapter 3: The urban land market: Rents and prices
  - McDonald & McMillen, chapter 6 & 7
- Complementary readings and references:
  - Ahlfeldt G., S.J. Redding, D.M. Sturm, and N. Wolf, 2015, “The Economics of Density: Evidence from the Berlin Wall”, Econometrica 83(6), 2127-2189.
  - Ahlfeldt G., Barr, J. (2022), The economics of skyscrapers: A synthesis. Journal of Urban Economics
  - Alonso, W. (1964): Location and Land Use: Toward a General Theory of Land Rent. Harvard University Press.
  - Fujita, M; Ogawa, H. (1982) Regional Science and Urban Economics, Vol.12(2), pp.161-196
  - Liotta, C. Viguie, V., Lepetit, Q. (2022), Testing the monocentric standard urban model in a global sample of cities. Regional Science and Urban Economics.
  - Lucas, R. E. ; Rossi–Hansberg, E. (2002) On the Internal Structure of Cities. Econometrica, July 2002, Vol.70(4), pp.1445-1476
  - Riccardo, D. (1817): On the Principles of Political Economy and Taxation. John Murray
  - Von Thünen, J. (1826). Der isolierte Staat.
  - Mills, E. (1967): An aggregative model of resource allocation in a metropolitan area. American Economic Review, 57: 197-210
  - Mills, E. (1972): Studies in the structure of the urban economy. John Hopkins University Press.
  - Muth, R. (1969): Cities and housing: University of Chicago Press.

II COMPENSATING DIFFERENTIALS

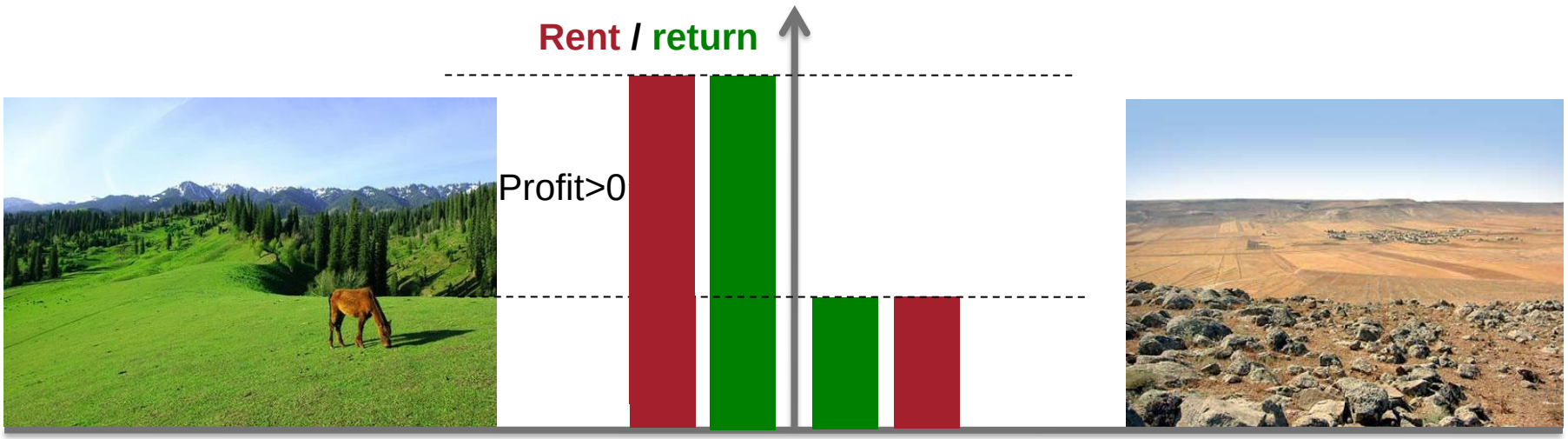
prices and location advantages

Why paying higher prices for better locations?

- Higher prices must offset for location advantages
  - **Ricardo (1817)** – Founder of "compensating differentials"
    - Bid **higher rents** for more **fertile land** to **equalize profits**

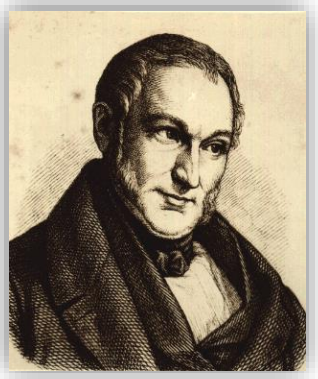
Spatial competition ensures zero econ. profit

Land rent compensates for loc. advantages



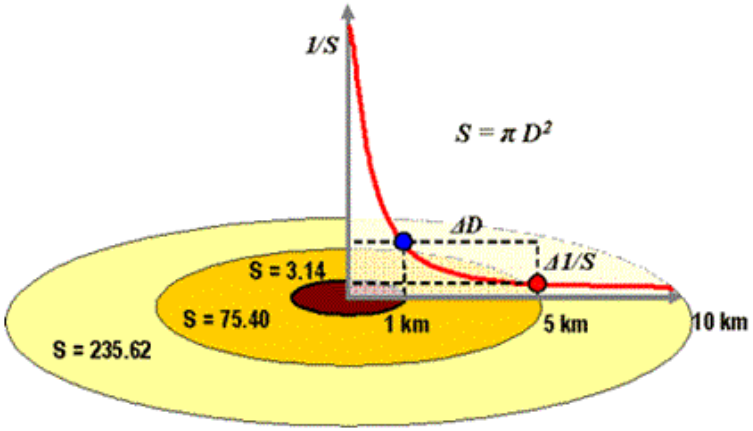
II COMPENSATING DIFFERENTIALS

prices and location advantages



Why paying higher prices for better (more central locations)?

- **Von Thuenen (1826)**
  - Replace **fertility** with **accessibility** of land
  - **Lower transport** cost of shipping agricultural products to the urban market increase profits => **higher rents**



Already features key features of modern "bid-rent" models

Assumption: Higher land rent offsets for lower transport cost

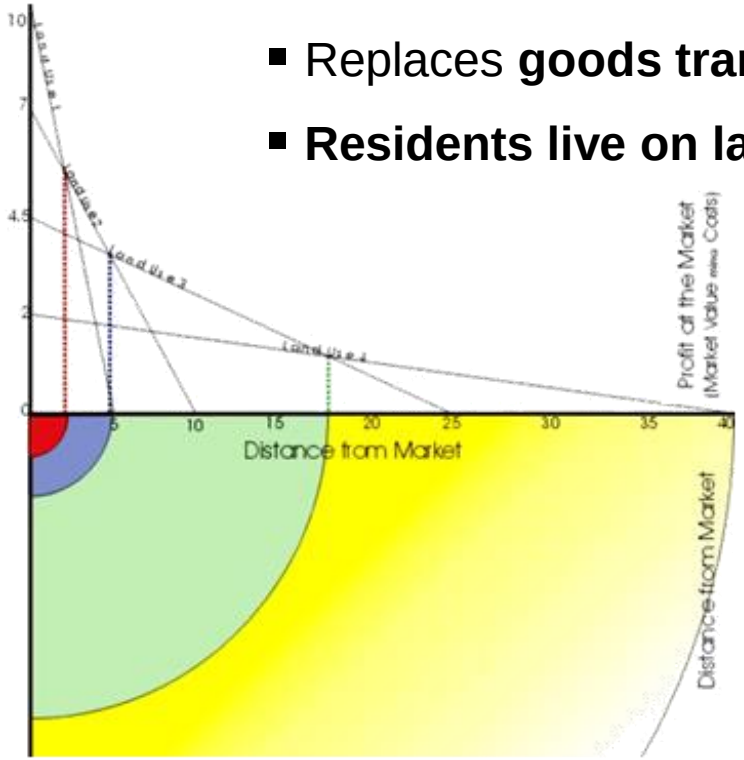
Prediction: Prices decline in distance from the city centre

II COMPENSATING DIFFERENTIALS

prices and location advantages

Why paying higher prices for better (more central locations)?

- **William Alonso (1964)**
  - Generalizes von Thünen concept to urban context
  - Replaces **goods transport costs** with **worker commuting costs**
  - **Residents live on land (realistic?)**



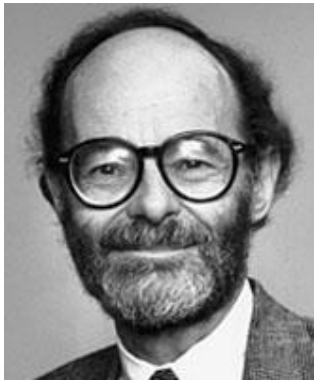
Utility must be equalized in spatial equilibrium

Commuting cost differences within urban area mirrored by price differences

Prediction: Prices decline & dwelling sizes increase in distance from the city centre

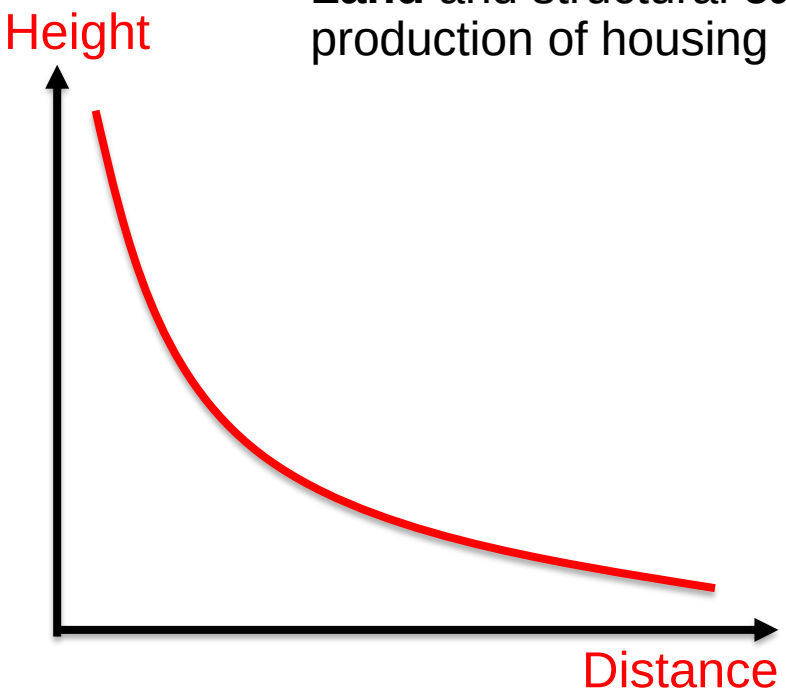
II COMPENSATING DIFFERENTIALS

prices and location advantages



What is the role of developers?

- **Mills (1967, 1972) and Muth (1969)**
  - Further add a competitive **construction sector**
  - **Land** and structural **capital** are a **intermediate inputs** in the production of housing



Utility must be equalized in spatial equilibrium

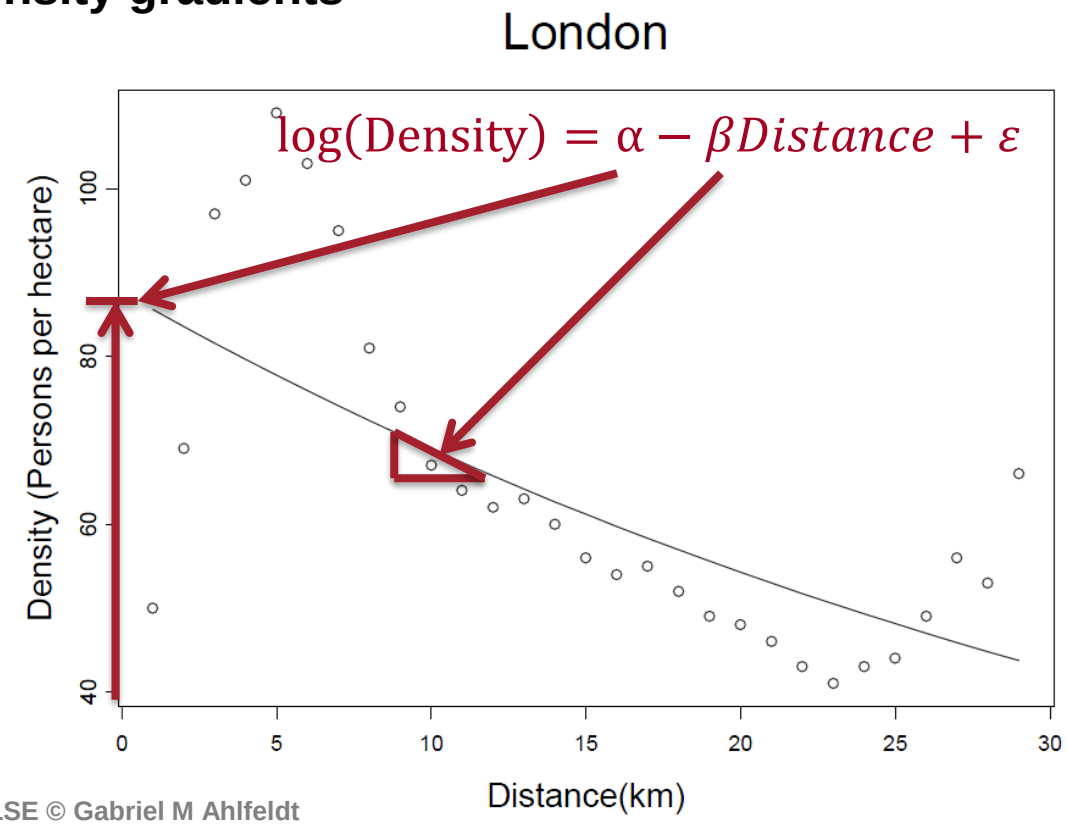
Zero profits in construction sector

Prediction: Prices, dwelling size, and structural density decline in distance from the city centre

III VARIATION IN GRADIENTS ACROSS CITIES

Density gradients

- **Bertaud & Maelpezzi (2003)**
  - **Collect population density by district for 50 "world cities"**
  - **Estimate population density gradients**



IV VARIATION IN GRADIENTS

Density gradients

- **39/50 cities relatively clearly centralized**

Ahmedabad Atlanta Bangalore Bangkok Barcelona Beijing Berlin Budapest Buenos Aires Chicago Cracow Guangzhou Hong Kong Hyderabad Jakarta Ljubljana London Los Angeles Marseille Mexico City New York City New York CMSA Paris Portland Prague Riga Rio de Janeiro San Francisco San Francisco Bay Shanghai Singapore Sofia Stockholm Tianjin Toulouse Tunis Warsaw Washington, DC Yerivan

- **6/50 cities relatively centralized**

Abidjan Bombay Curitiba Houston Seoul + New Towns St Petersburg

- **5/50 cities clearly decentralized**

Brasilia Capetown Johannesburg Moscow Seoul