

An aerial photograph of a city, likely Chicago, showing a dense grid of streets and buildings. The city extends to the edge of a large body of water, with a prominent river or canal cutting through the urban landscape. The image is used as a background for the title slide.

THE MONOCENTRIC CITY MODEL II

Urban Economics

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I COURSE COMPONENTS

the idea

- Block I
 - Introduction to Urban and Regional Economics and Course Overview
 - Topic I: Regional and urban concentration forces
 - Topic II: The empirics of agglomeration
 - Topic III: Costs and benefits of agglomeration
- Block 2
 - Topic IV: Monocentric city I (household location choice)
 - **Topic V: Monocentric city II (household location choice)**
 - Topic VI: Firm location choice
 - Topic VII: The urban economy in general equilibrium
- Block 3
 - Topic VIII: The vertical dimension of cities
 - Topic IX: Suburbanization and gentrification
 - Topic X: Hedonic analysis

I INTRODUCTION

roadmap

- **Last time: *The monocentric city model I***
 - **1) History of thought**
 - **2) The household bid-rent model**
 - **The utility maximization problem**
 - **Compensating differentials**
 - **Rent gradients**
 - **Substitution effects in consumption**
 - **Convexity of the rent gradient**
 - **3) Density gradients**
 - **Structural density**
 - **Population density**

I INTRODUCTION

roadmap

- This time: *The monocentric city model II*
 - 1) Equilibrium conditions
 - Additional restrictions
 - 2) Comparative statics: Income
 - What happens if people get richer?
 - Where do the rich and the poor live in cities?
 - 3) Comparative statics: Transport cost
 - What happens if transport gets cheaper?
 - Did cities decentralize over time?
 - 4) Other predictions
 - Distinguishing between open-city and closed-city case

I RECALL

household bid-rent model



Price and density gradients decrease in distance from the CBD

II EQUILIBRIUM CONDITIONS

additional restrictions



Q: How to determine where the city ends?

II MCM BY BRUECKNER (1987)

additional restrictions

Variable/parameter	Description
$D(x)$	Population density at x
$h(x)$	Housing density at x
L	Total city population
$p_q(x)$	Housing rent at x
$q(x)$	Housing consumption at x
$r(x)$	Land rent at x
rA	Agricultural rent
$S(x)$	Structural density at x
t	Transport cost per unit of x
U	City-level utility
X	Distance from the CBD
Y	City-level income per capita

II EQUILIRIUM CONDITIONS

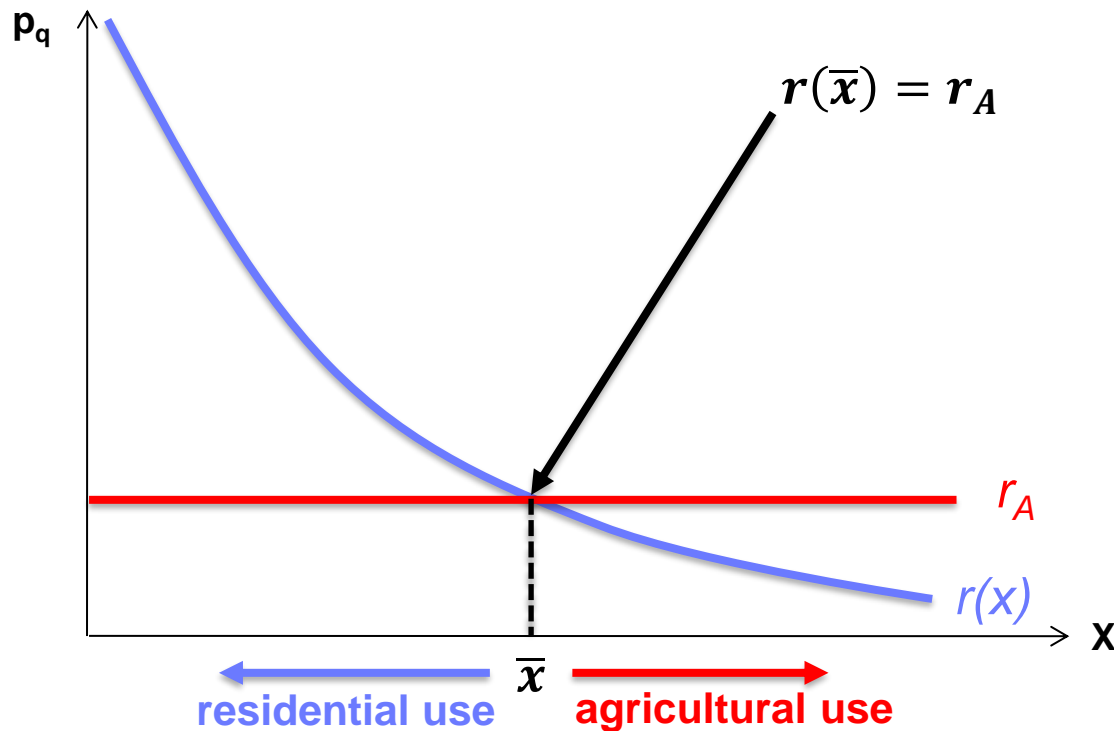
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II AGRICULTURAL RENT

additional restrictions

- For land to be „urban“, rent must exceed agricultural rent
 - Land rent paid by residents in Alonso model
 - Land rent paid by developers in Mills-Muth model



II EXOGENOUS VS. ENDOGENOUS VARIABLES

additional restrictions

- Always exogenous: ← **Determined outside the model**
 - r_A, y, t
- Always endogenous: ← **Solved within the model**
 - \bar{x}, p_q, r, S, D
- „Closed city“ case: No between-city migration
 - Population L is exogenous
 - Utility U is endogenous
- „Open city“ case: Free between-city migration
 - Population L is endogenous
 - Utility U is exogenous
 - Similar to system of cities and Rosen-Roback framework
 - See topics I-III

II COMPARATIVE STATIC ANALYSIS

additional restrictions

- Analyse impact of
 - a change in one exogenous variable
 - on endogenous variables
- Use the structure of the model to establish counterfactuals
- Derive predictions
 - What happens to rents, densities, etc. $\{ \bar{x}, p_q, r, S, D, [L, U] \}$ if incomes, transport costs, or opportunity cost of land $\{ r_A, y, t \}$ change

Central to a host of questions that concern policy makers, planners, developers, investor, etc.

**Predictions specific to open/closed-city case
Focus on close-city case first**

III COMPARATIVE STATICS: INCOME

roadmap

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III INCREASE IN INCOME

comparative statics



Q: What happens to city structure (gradients) if incomes increase?

III DECREASE IN TRANSPORT COST

comparative statics

- The slope of housing rent gradient (see topic IV):

$$\frac{\partial p_q}{\partial x} = -\frac{t}{q(y)} < 0$$

$$\left[p_q = \frac{y - tx - c}{q(y)} \right]$$

Budget constraint

- Effect of a marginal increase in income y on the slope of bid-rent curve:

$$\frac{\partial q}{\partial y} > 0$$

**Housing is a normal good \Rightarrow
Consumption increases in income**

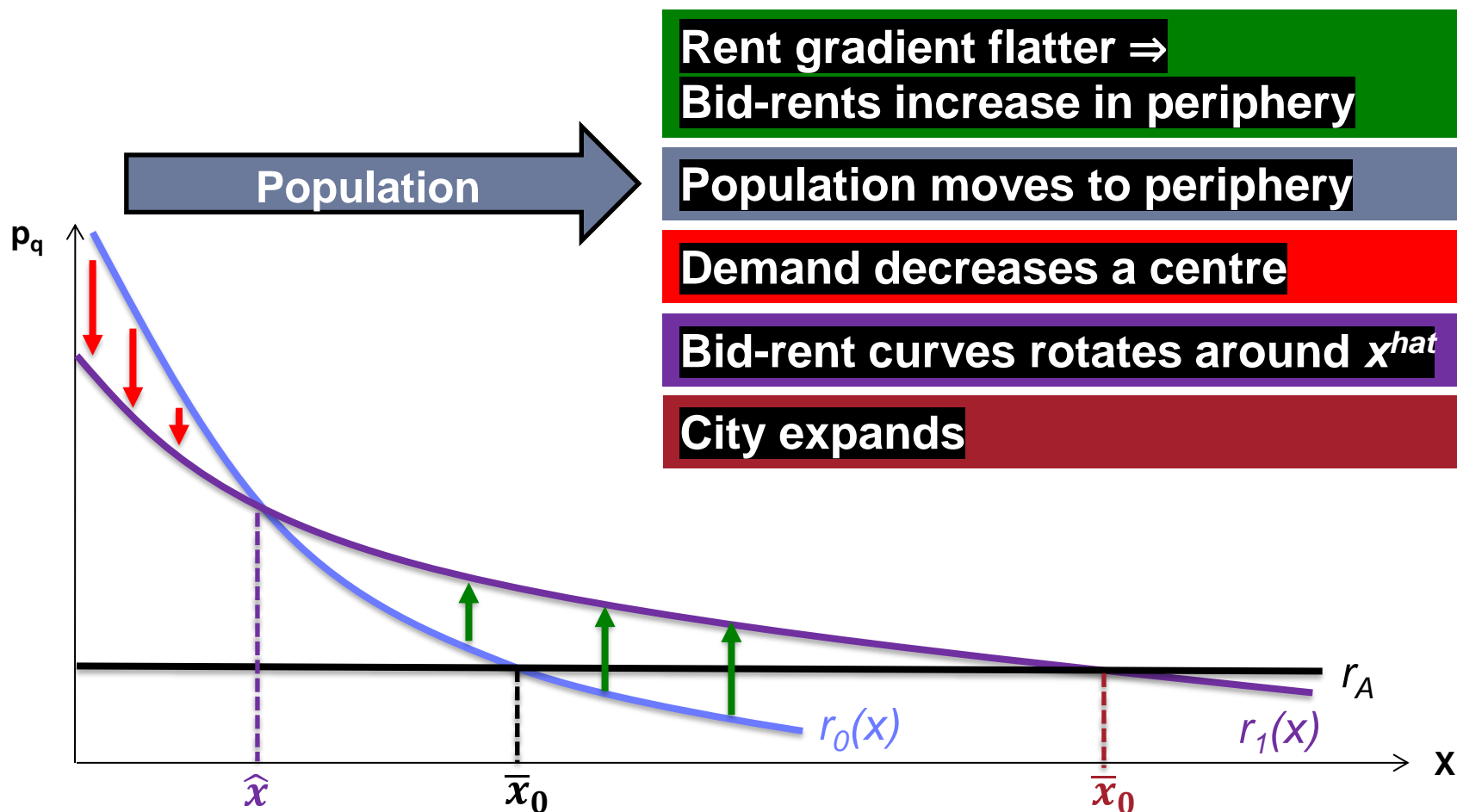
$$\frac{\partial \left(\frac{\partial p_q}{\partial x} \right)}{\partial y} = \frac{t \frac{\partial q}{\partial y}}{[q(y)]^2} > 0$$

**Income y increases \Rightarrow
rent gradient flatter!**

- Slope less negative!

III INCREASE IN INCOME

comparative statics



III COMPARATIVE STATIC ANALYSIS

additional restrictions

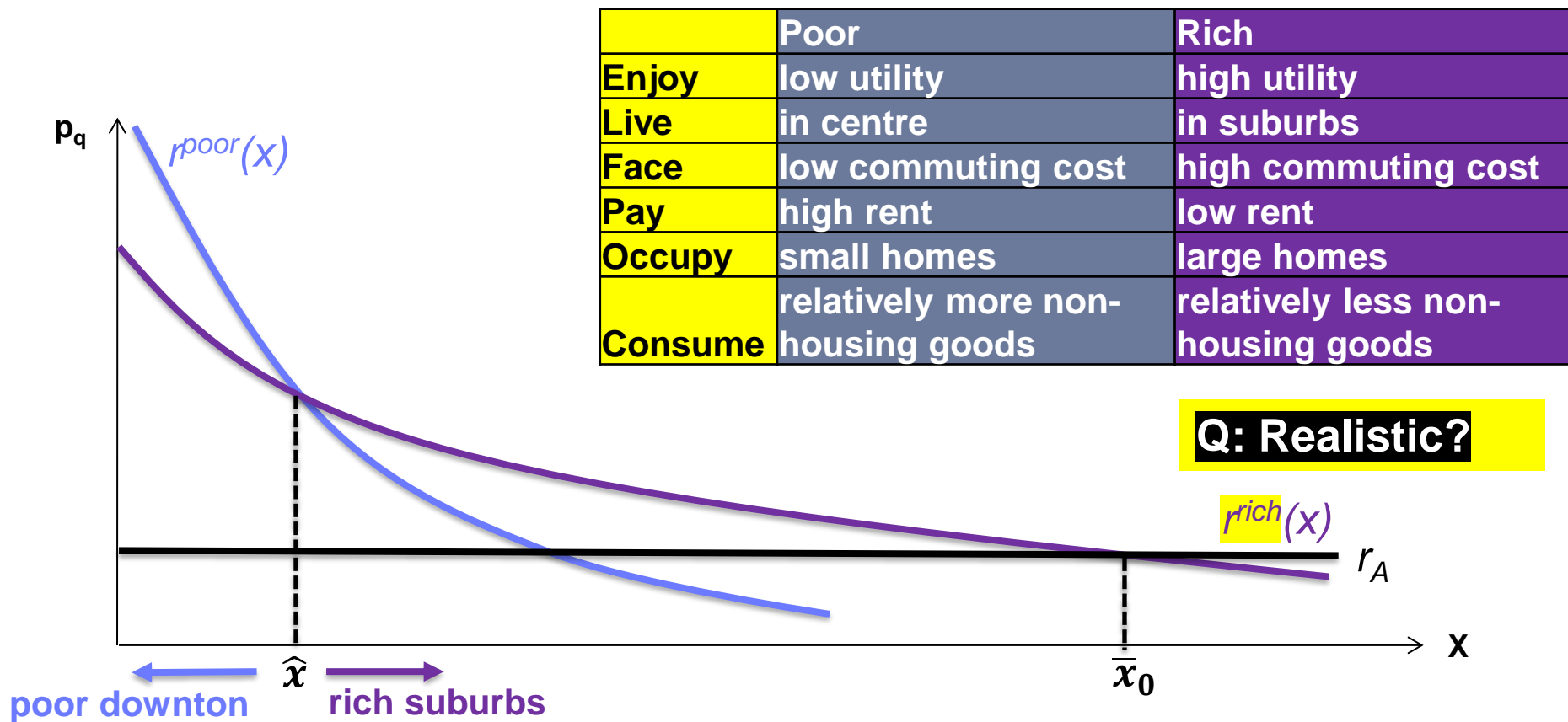
- Housing rents decrease in centre, increase in periphery
 - Structural / housing density and land rent behave similarly
 - Housing consumption q rises in centre (where p falls)
 - Population density must decrease in centre
 - q must increase at city margin (higher income, same rent)
 - Ambiguous change of q between rotation point and city new city margin
- Of course, utility increases since residents consume more

Q: Lessons for within-city income segregation?

III INCOME SEGREGATION

comparative statics

- Bid-rents by rich and poor with monetary commuting cost



III INCOME SEGREGATION

additional restrictions

- With monetary t , rich live in suburbs, poor live in centre
- If we add the value of time, t likely depends on income

$$\frac{\partial p_q}{\partial x} = -\frac{t(y)}{q(y)} < 0$$

$$\frac{d\left(\frac{\partial p_q}{\partial x}\right)}{dy} = -\frac{\overbrace{t'(y)q(y)}^{>0} - \overbrace{t(y)q'(y)}^{>0}}{[q(y)]^2}$$

Theoretically ambiguous, elasticities matter!

- Income elasticities of commuting cost and housing consumption appear to be similar (Wheaton, 1977)

$$\frac{\partial t(y)}{\partial y} \approx \frac{\partial q(y)}{\partial y}$$

**In line with observed cross-city differences
(central Paris rich, downtown Detroit poor)**

III BID-RENT CURVES VS. PRICE GRADIENTS

comparative statics

- The observed price gradient (house prices, rents, etc.) is the envelope of individual bid-rent curves

Each bid-rent curve is convex (*substitution effect*)

Price gradients are *more* convex (*sorting effect*)



IV COMPARATIVE STATICS: TRANSPORT COST

roadmap

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IV DECREASE IN TRANSPORT COST

comparative statics



Q: What happens to the spatial structure if transport cost decrease?

IV DECREASE IN TRANSPORT COST

comparative statics

- The slope of housing rent gradient (see topic IV):

$$\frac{\partial p_q}{\partial x} = -\frac{t}{q}$$

$$\left[p_q = \frac{y - tx - c}{q} \right]$$

Budget constraint

- Effect of a marginal increase in transport cost t on the slope of bid-rent curve:

**Larger transport t cost \Rightarrow
steeper bid-rent curve**

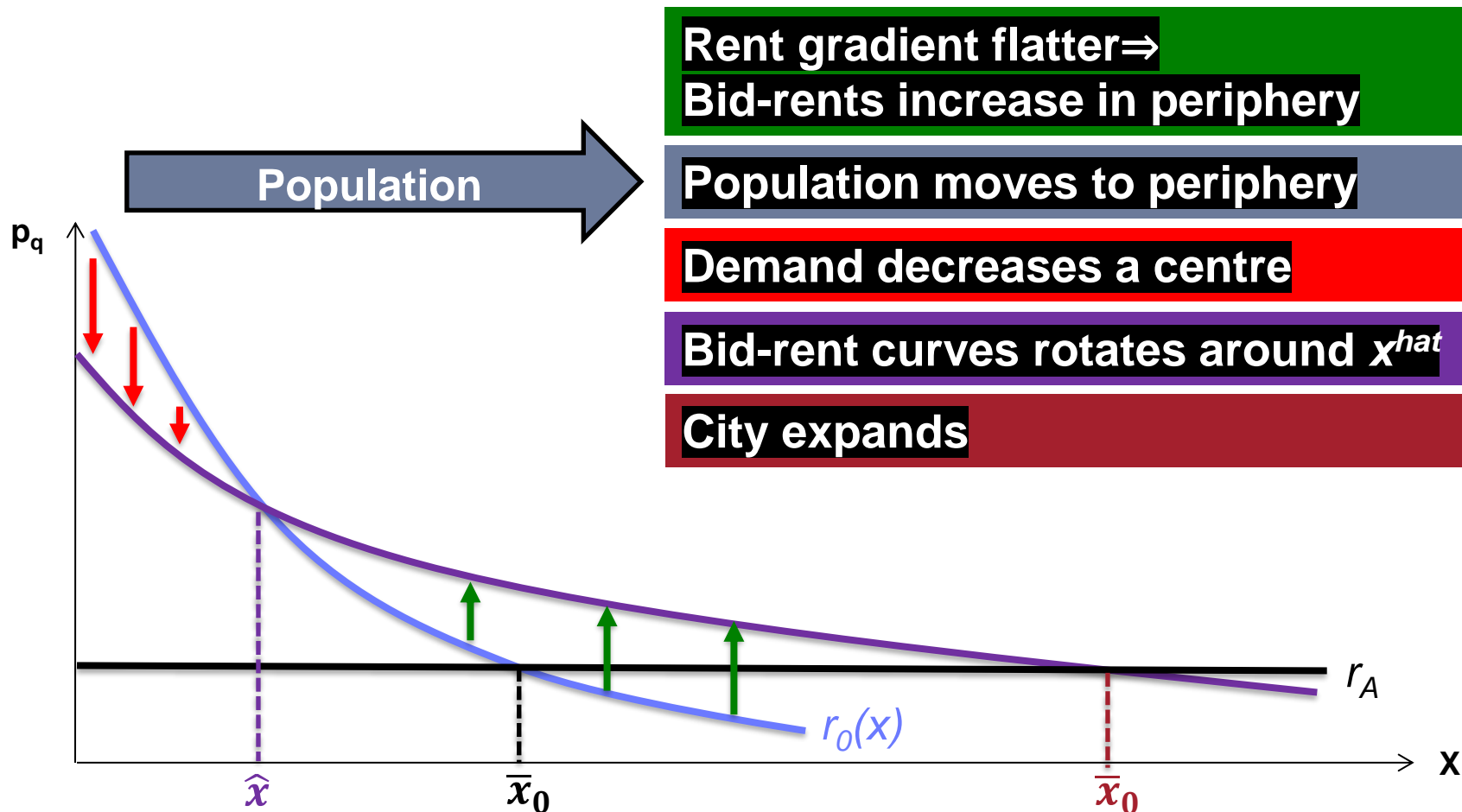
$$\frac{d\left(\frac{\partial p_q}{\partial x}\right)}{dt} = -\frac{1}{q} < 0$$

**Transport cost t decrease \Rightarrow
rent gradient flatter!**

- Slope more negative!

IV DECREASE IN TRANSPORT COST

comparative statics



IV COMPARATIVE STATIC ANALYSIS

additional restrictions

- Housing rents decrease in centre, increase in periphery
 - Structural / housing density and land rent behave similarly
 - Housing consumption q
 - rise in centre (where p falls)
 - fall in periphery (where p increases)
 - Population density
 - Falls in centre, rises in periphery
- Utility increases since cost of commuting decreases
 - Similar to a positive income shock

IV DECENTRALIZATION

comparative statics

- Steady income growth in real terms over more than a century
 - Real GDP p.c. growth since 1900 in US, Western Europe (and world) about 2% per year (Maddison Project Data)

Could possibly lead to decentralization

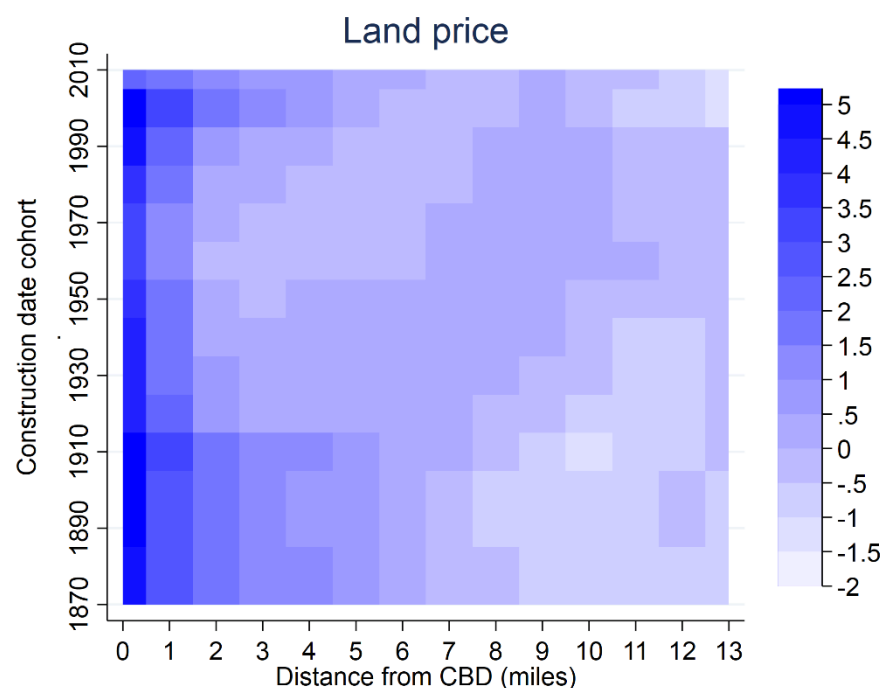
- Major transport improvements since late 19th century
 - Subway and commuter rail
 - Mass produced cars and highways
 - Passenger travel p.c. growth since 1880 in the US about 2.7% per year (Asubel et al. 1998)

Should most likely cause decentralization

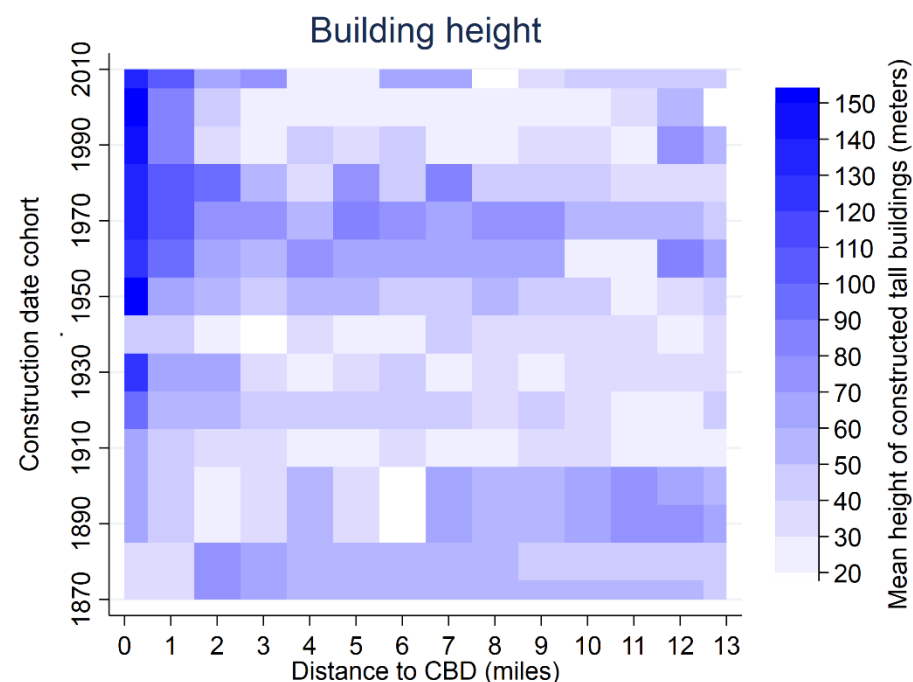
Q: Is there evidence?

IV EVIDENCE FROM CHICAGO

comparative statics



Land price

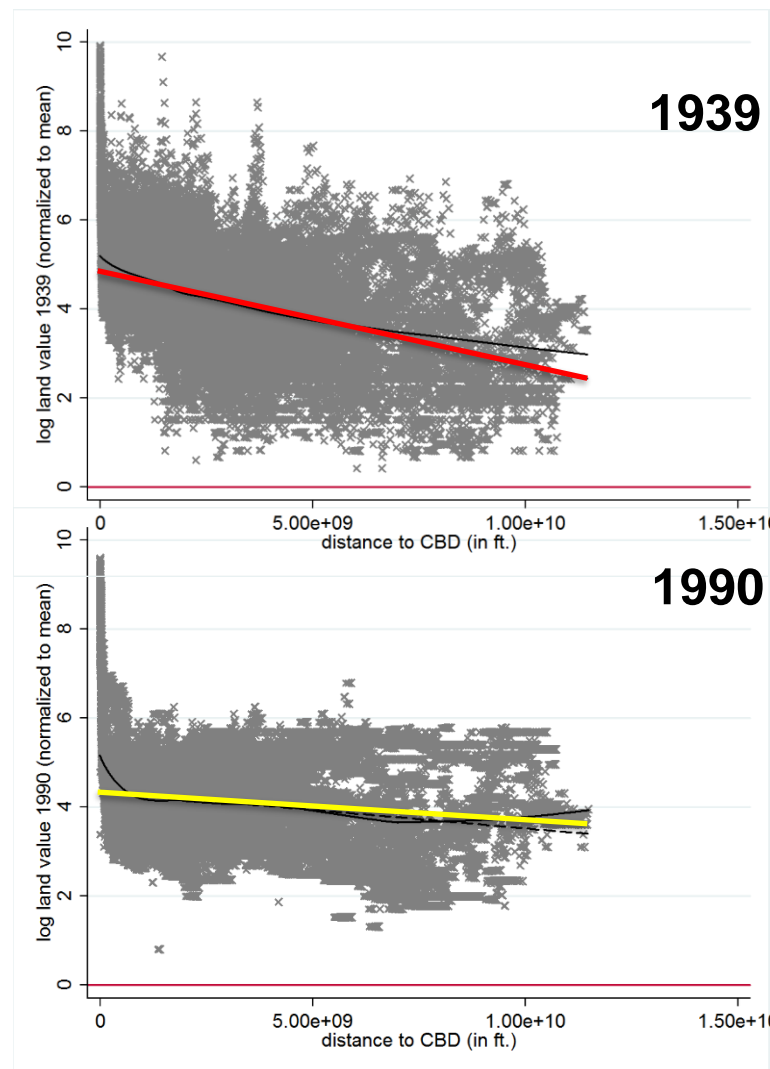
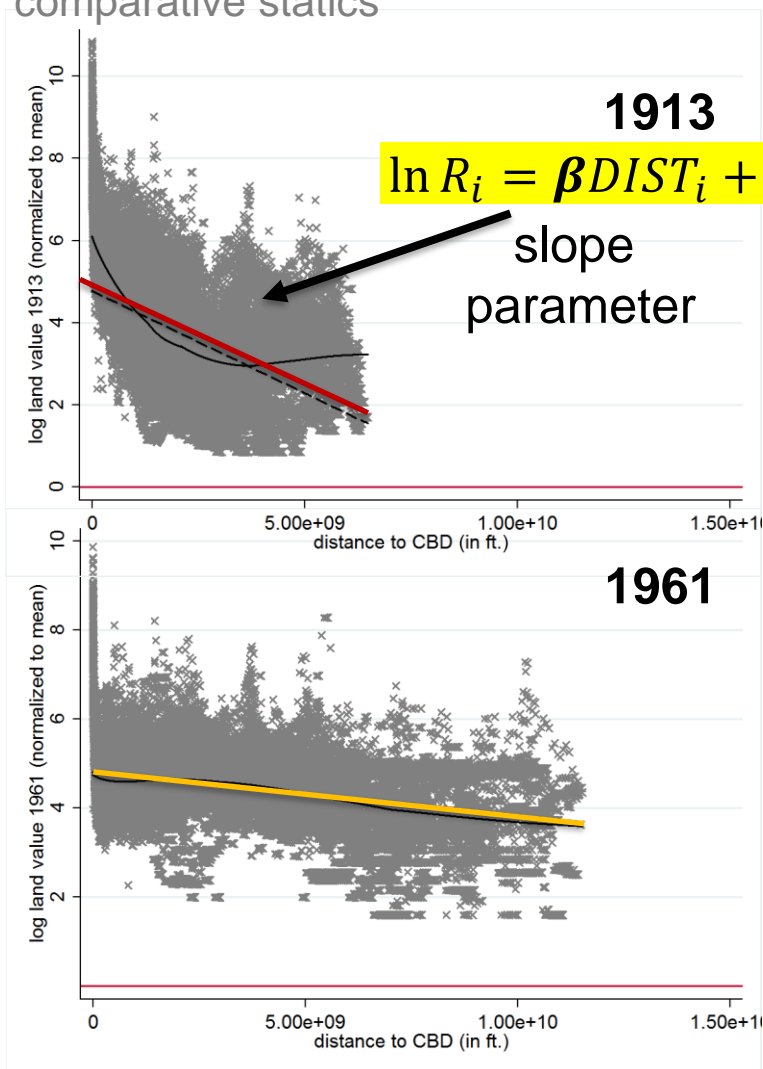


Building height

Ahlfeldt & McMillen 2018

IV LAND PRICE GRADIENTS IN CHICAGO

comparative statics



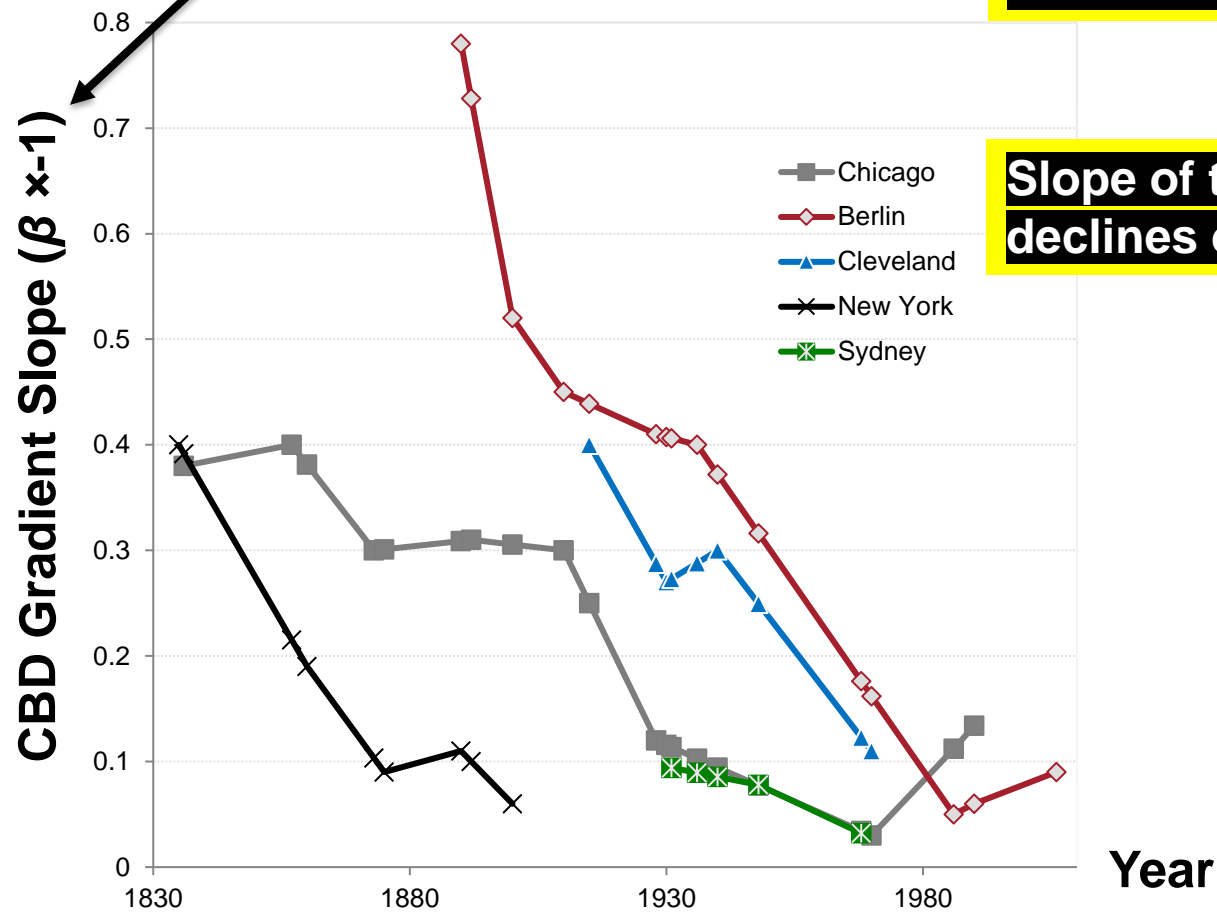
Distance gradients become flatter!

IV LAND PRICE GRADIENTS OVER TIME

comparative statics

$$\ln R_i = \beta \text{DIST}_i + \varepsilon_i$$

Based on Ahlfeldt & Wendland, 2011

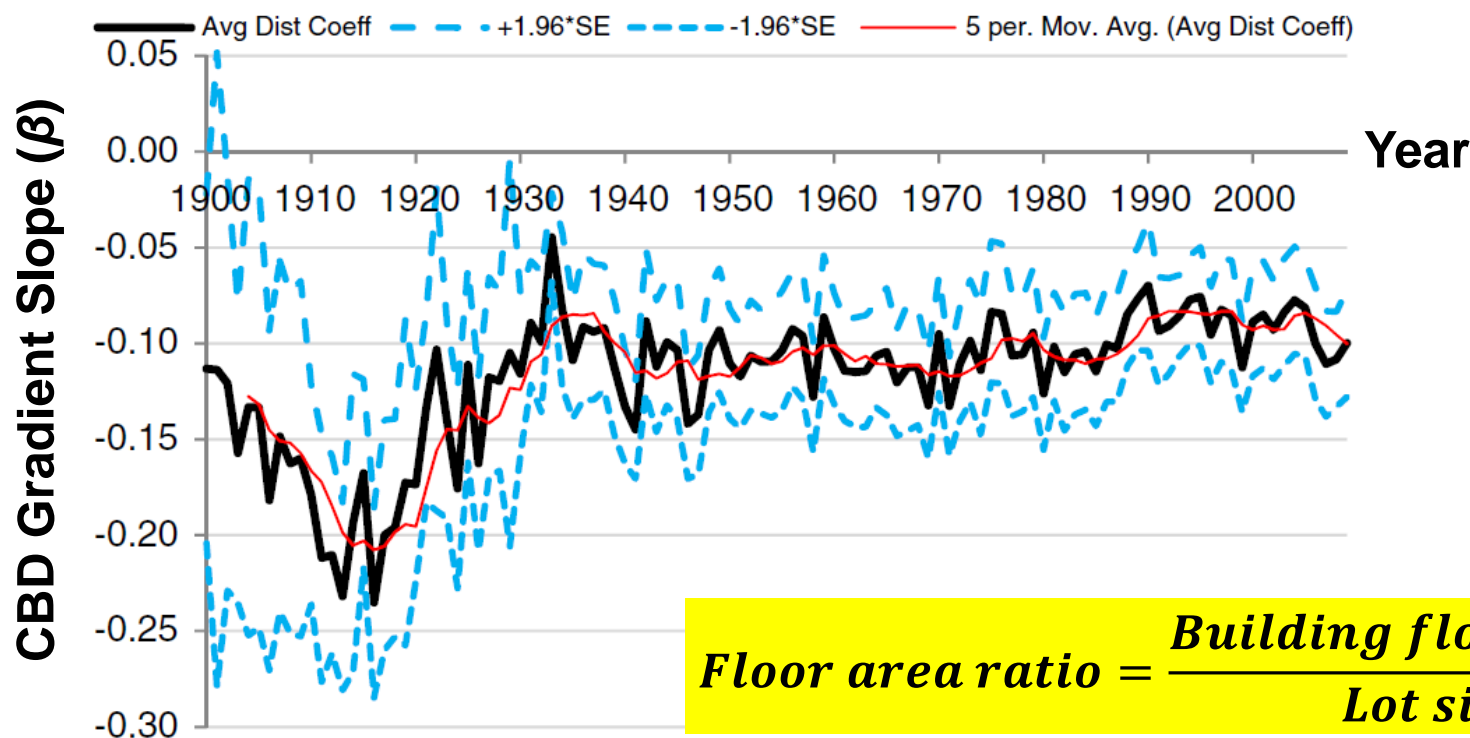


Slope of the price gradient also declines over time in other cities

IV FAR GRADIENT IN NEW YORK OVER TIME

comparative statics

$$\ln FAR_i = \beta DIST_i + \varepsilon_i$$



$$\text{Floor area ratio} = \frac{\text{Building floor space (m}^2\text{)}}{\text{Lot size (m}^2\text{)}}$$

Structural density gradient also flattens

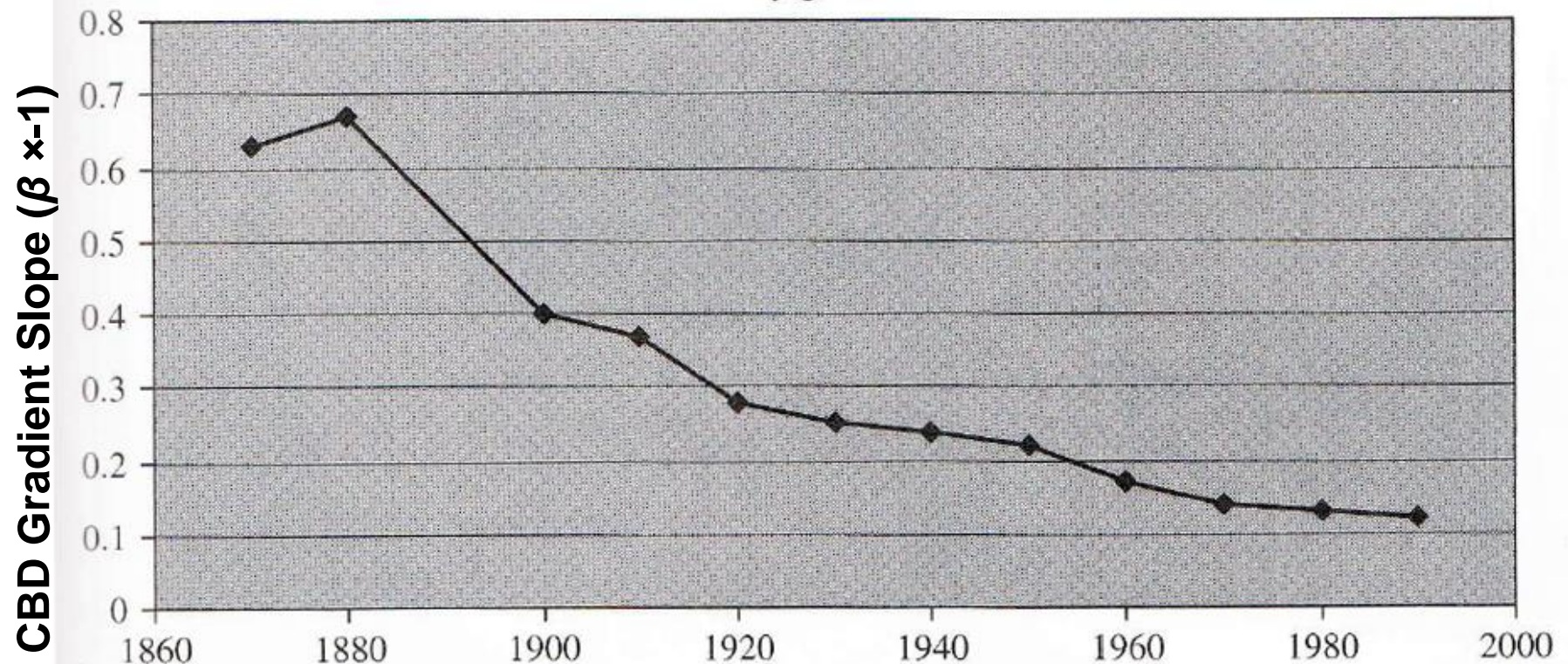
Barr & Cohen (2014)

IV POPULATION DENSITY GRADIENTS IN CHIAGO

comparative statics

$$\ln POPDENS_i = \beta DIST_i + \varepsilon_i$$

Density gradient



McMillen & McDonald, ch. 8

Population density gradient also flattens

IV CAUSES OF DECENTRALIZATION

comparative statics

- Strong evidence for urban „decentralization“
 - In line with transport improvements since mid-19th century (rail & road)
- But other explanations theoretically possible
 - Increase in income (in MCM with monetary travel cost)
 - Increase in preference for suburban amenities
 - Increase in downtown disamenities (pollution, noise)
 - Aeging of downtown housing (Brueckner & Rosenthal 2009)

Q: Is there „causal“ evidence for transport effects on decentralization?

IV HIGHWAYS AND POPULATION DECENTRALIZATION

comparative statics

TABLE I
AGGREGATE TRENDS IN SUBURBANIZATION, 1950–1990

	1950	1960	1970	1980	1990	Percent change 1950–1990
Panel A: Large MSAs						
MSA population	92.9	115.8	134.0	144.8	159.8	72
Total CC population	44.7	48.5	51.3	49.2	51.0	14
Constant geography CC population	44.7	44.2	42.6	37.9	37.1	-17
N for constant geog. CC population	139	132	139	139	139	
Panel B: Large Inland MSAs						
MSA population	39.2	48.9	57.0	65.0	73.5	88
Total CC population	16.8	19.7	22.1	22.1	23.2	38
Constant geography CC population	16.8	16.5	15.4	13.3	12.5	-26
N for constant geog. CC population	100	94	100	100	100	
Total U. S. population	150.7	178.5	202.1	225.2	247.1	64

Baum-Snow (2006)

Total US MSA population increased

Constant-geography central population decreased

IV BAUM-SNOW (2006)

comparative statics

■ Estimation equation:

$$\Delta \log N_i^c = \delta_0 + \delta_1 \Delta ray_i + \delta_2 r_{ci} + \delta_3 \Delta \tilde{w}_i + \delta_4 \Delta \log N_i^{MSA} + \delta_5 \Delta \tilde{G}_i + \varepsilon_i$$

1950-1990
central city
population

Change in
number of
highway
passing
through
central city

Controls

$\text{cov}(\text{IV}, \varepsilon) = 0$

Use number of planned highways
in 1947 plan as *instrument* to
addresses *endogeneity*

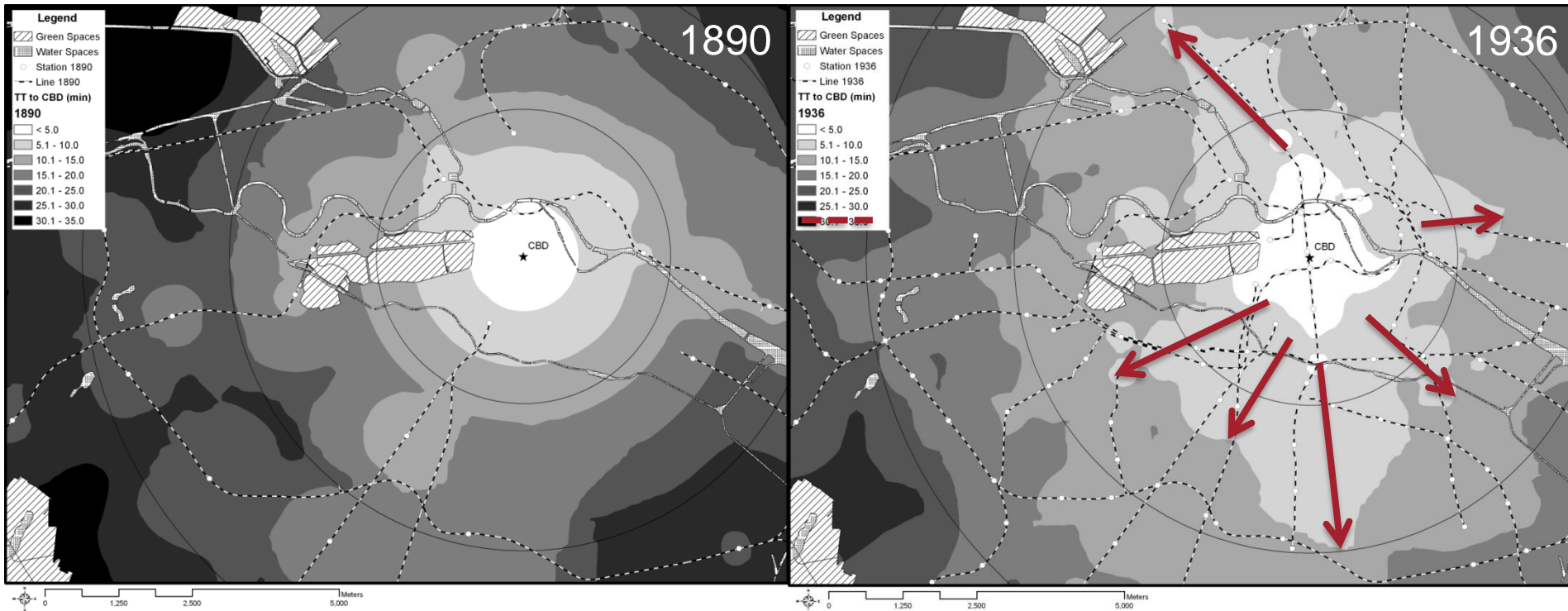
**Causal effect of an additional highway passing
through on central city population: -18%!**

Highways decentralize population

IV RAIL EXPANSION IN PRE-WWII BERLIN

comparative statics

- **Estimated travel time (min) to the CBD accounting for rapid transit (U-/S-Bahn)**

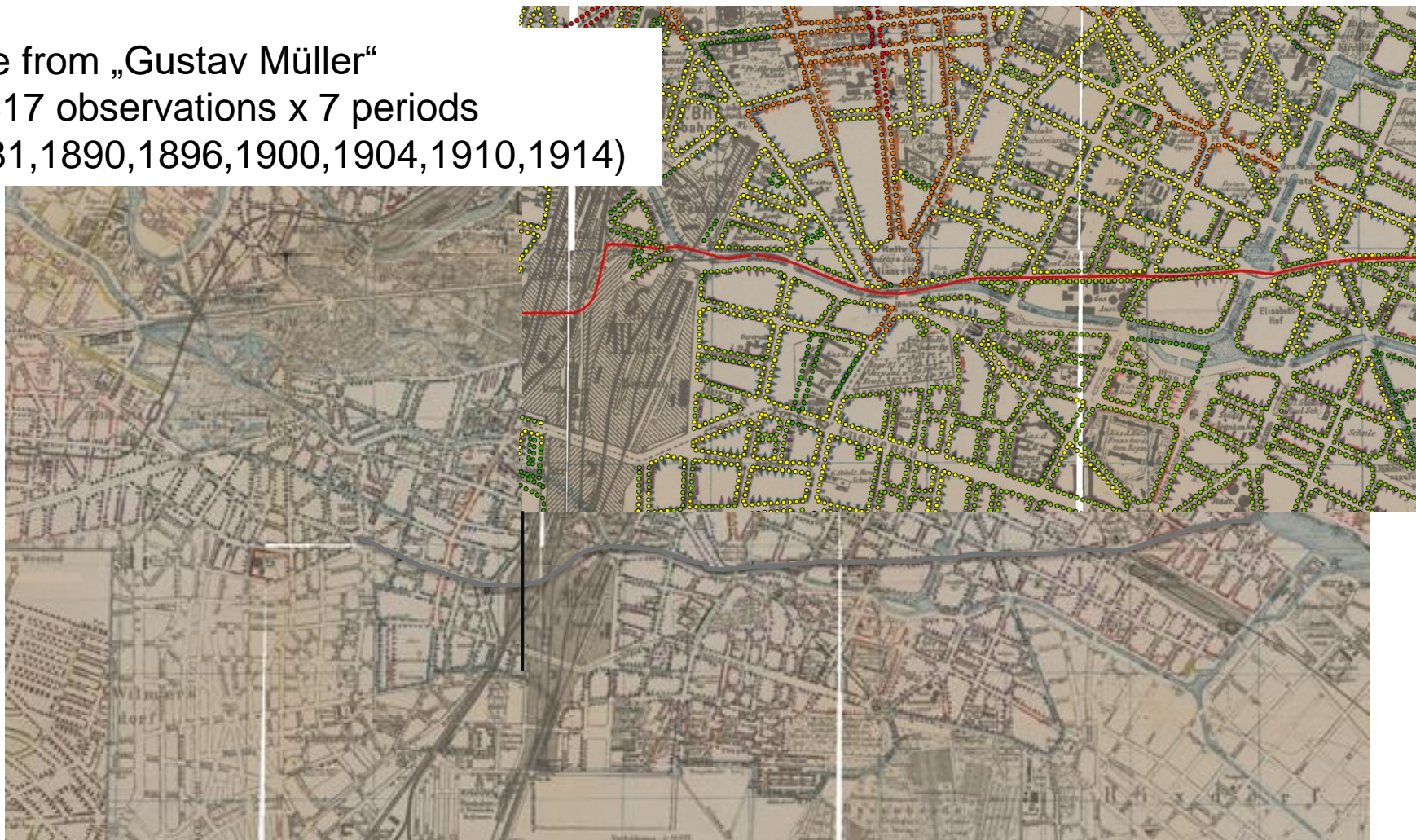


Ahlfeldt & Wendland (2011)

IV HISTORIC LAND PRICES IN BERLIN

comparative statics

Date from „Gustav Müller“
11,817 observations x 7 periods
(1881, 1890, 1896, 1900, 1904, 1910, 1914)



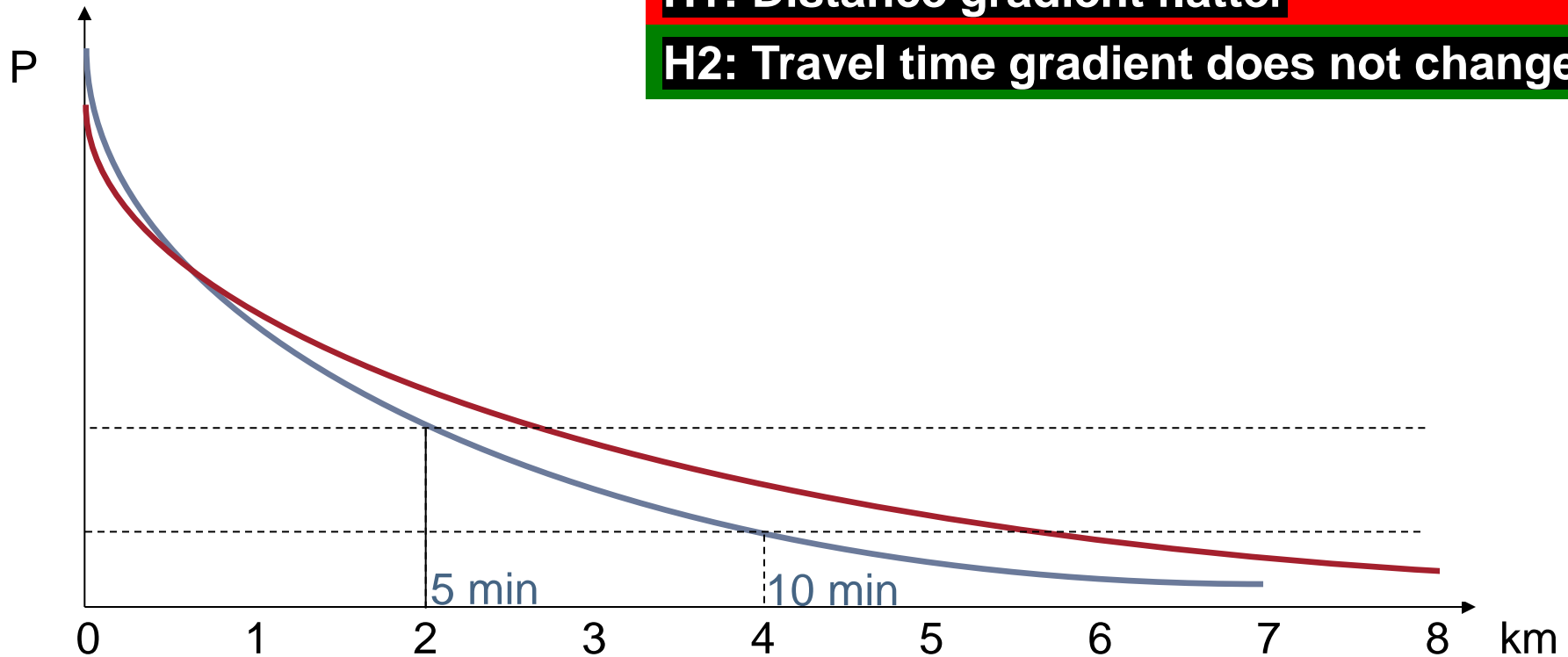
IV DECENTRALIZATION

comparative statics

City expands if transport cost decrease

H1: Distance gradient flatter

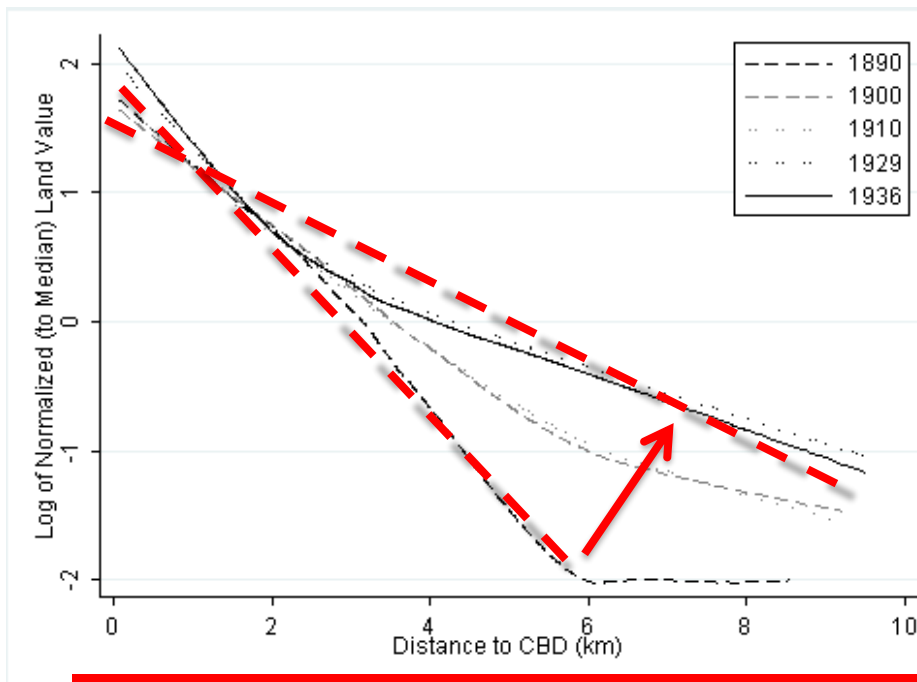
H2: Travel time gradient does not change



IV DECENTRALIZATION

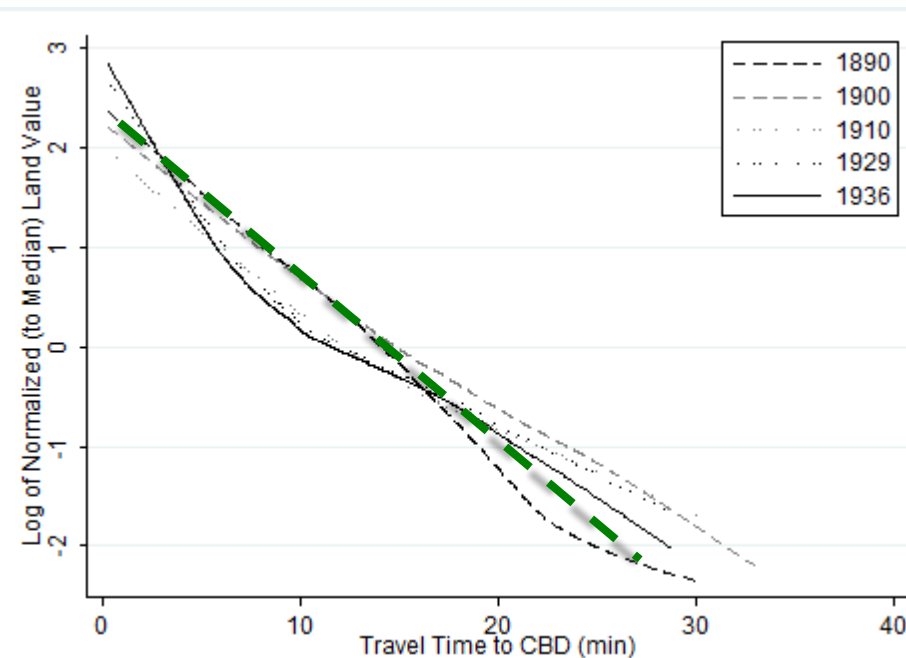
comparative statics

H1: Distance Gradients



β for distance *does* change

H2: Travel Time Gradients



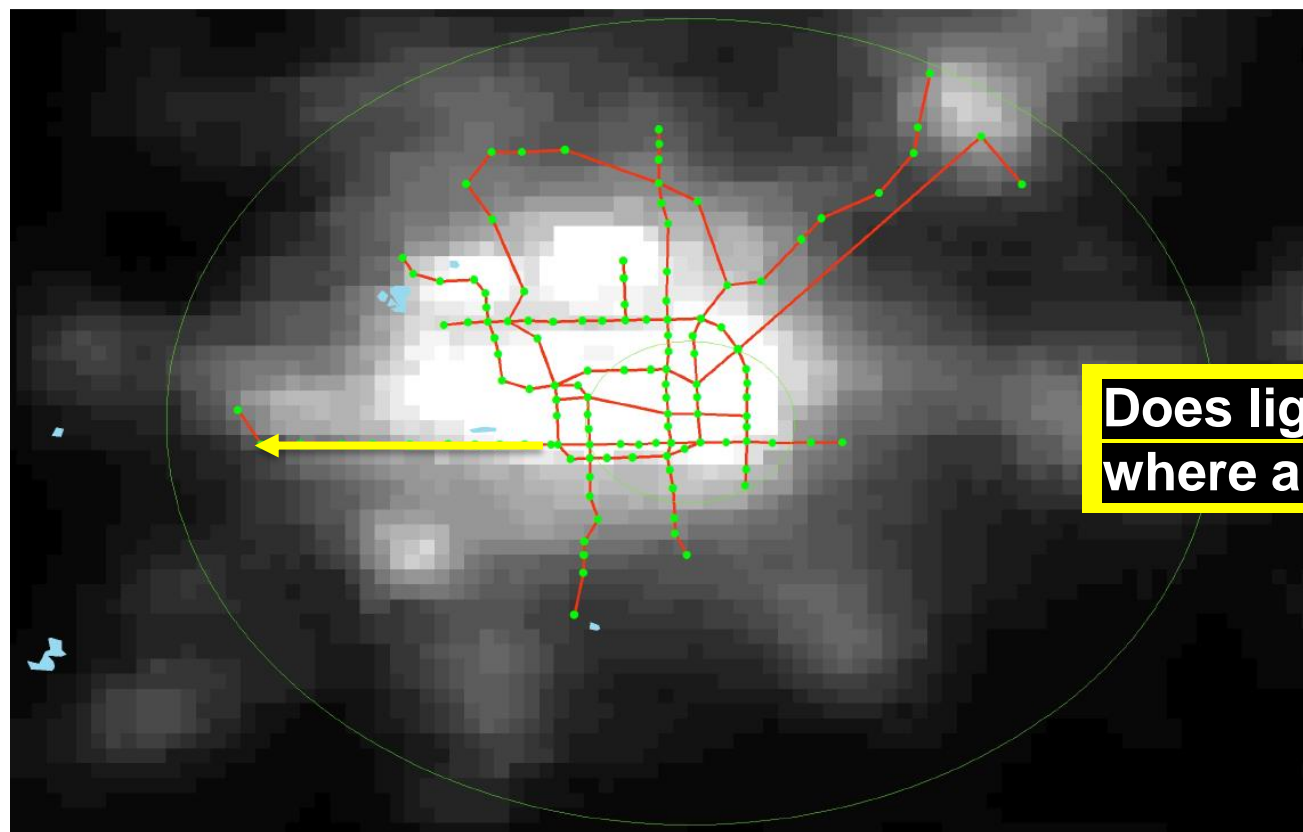
β for travel time *does not* change

Almost three quarters of “decentralization” (change in distance gradient β over time) can be explained by changes in travel time!

IV SUBWAYS AND “LIGHT” DECENTRALIZATION

comparative statics

- **Gonzalez-Navarro and Turner (2016)** look at decentralization of „lights at night“ in response to subway expansions in 138 cities



**Does light intensity go up
where a new line is built?**

IV GONZALEZ-NAVARRO AND TURNER (2016)

comparative statics

▪ 1st step

- Estimate „light“ gradient at five-year intervals t for city c

$$\ln LIGHTINTENSITY_{ict} = \beta_{ct} \ln DIST_{it} + \varepsilon_{ict}$$

▪ 2nd-step

- Estimate elasticity of gradient with respect to network size s

$$\Delta \ln \hat{\beta}_{ct} = \gamma \Delta \ln s_{ct} + X_{ct} b + \epsilon_{ct}$$

- *X is a vector of controls*

- Find that $\frac{\partial \beta}{\partial s} = 0.06$

Elasticity of light gradient with respect to network size is 6%
Subways decentralize light emissions

IV URBAN DECENTRALIZATION

comparative statics

- Cities around the world have been decentralizing in terms of prices (housing rent, land price) and quantities (densities)
- Causal evidence that part of this trend is due to better transport
 - If we build radial connections, urban structure will decentralize
 - Direct implications
 - People will move to periphery and travel longer distances
 - Demand for housing will increase in periphery
 - Prices/values will increase in periphery

Causal evidence important for predictions. Relevant for planners, developers, investors, etc.

IV COMPARATIVE STATICS: TRANSPORT COST

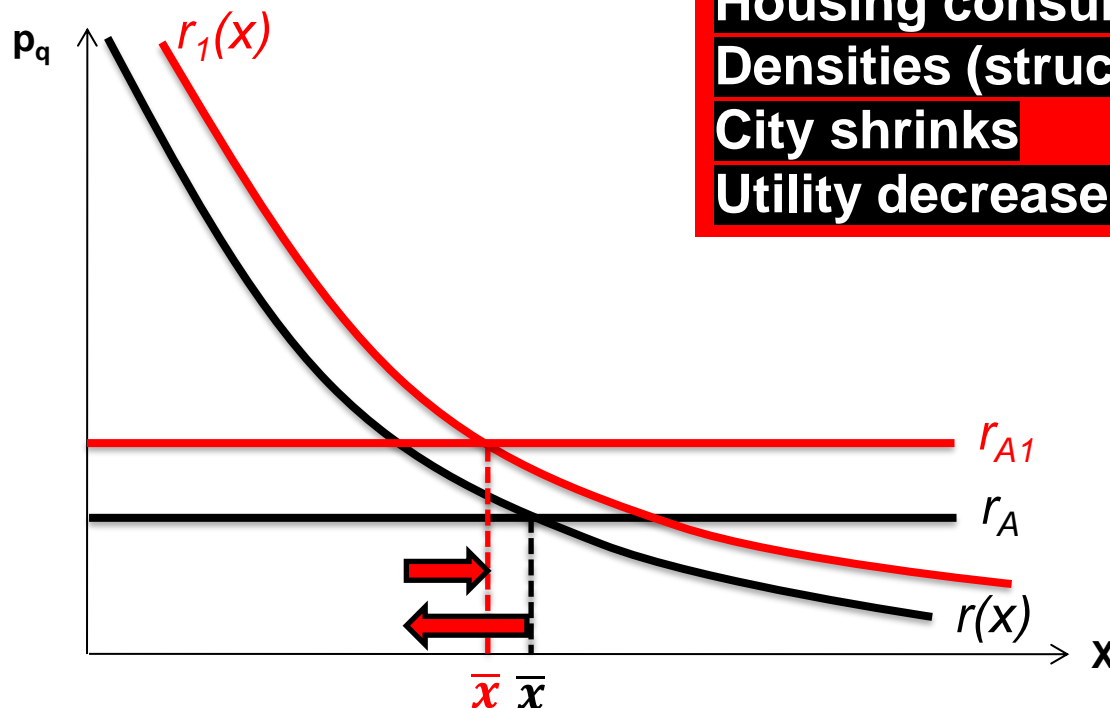
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V INCREASE IN AGRICULTURAL RENT

other predictions

- Increase in r_A creates scarcity at all locations in the city

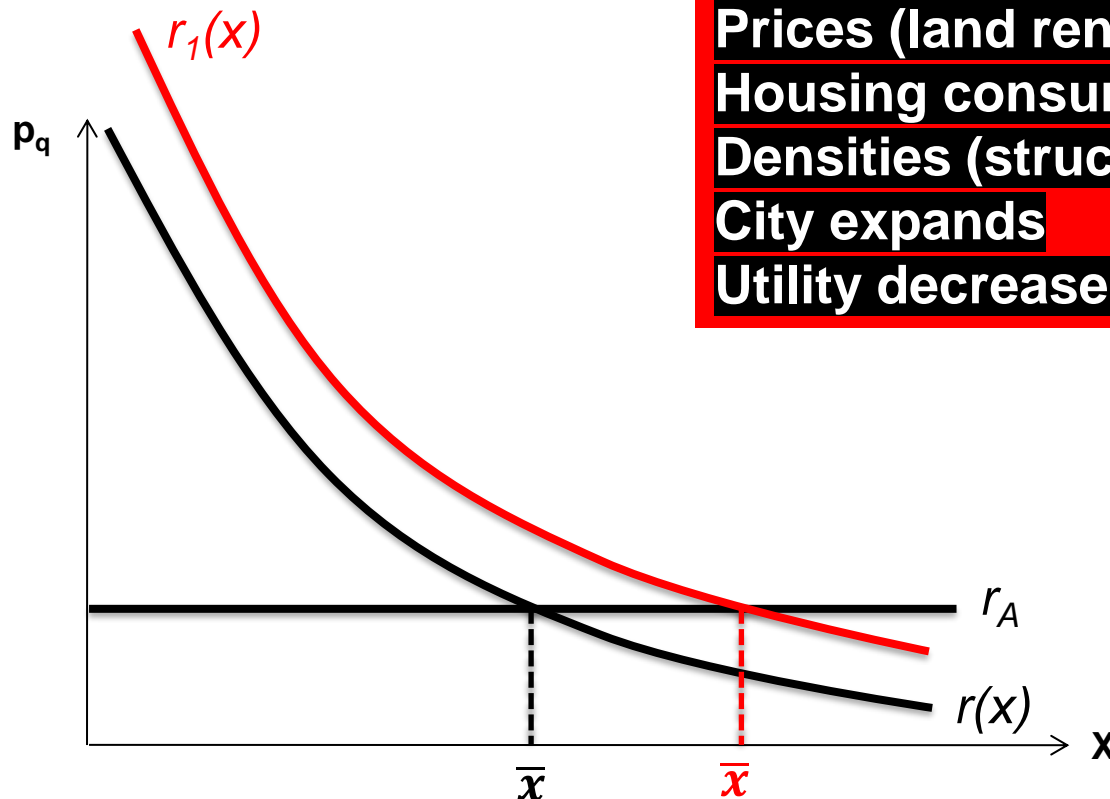


Prices (land rent, housing rent) increase
Housing consumption decreases
Densities (structural, population) increase
City shrinks
Utility decreases (ignoring density benefits)

V INCREASE IN POPULATION

other predictions

- Increase in L increases demand at all locations in the city



Prices (land rent, housing rent) increase
Housing consumption decreases
Densities (structural, population) increase
City expands
Utility decreases (ignoring density benefits)

IV OPEN-CITY CASE

other predictions

- „Open city“ case: Free between-city migration
 - Population L is endogenous
 - Utility U is exogenous
- Predictions in open-city case are similar, but require an additional step
 - 1) Hold L constant and evaluate the change using closed-city model
 - Same as before in closed-city case
 - 2) Adjust population to cancel the utility change
 - Larger $L \Rightarrow$ higher prices and lower housing consumption

Q: What does an increase in income mean for the centre?

IV OPEN-CITY CASE: INCREASE IN INCOME

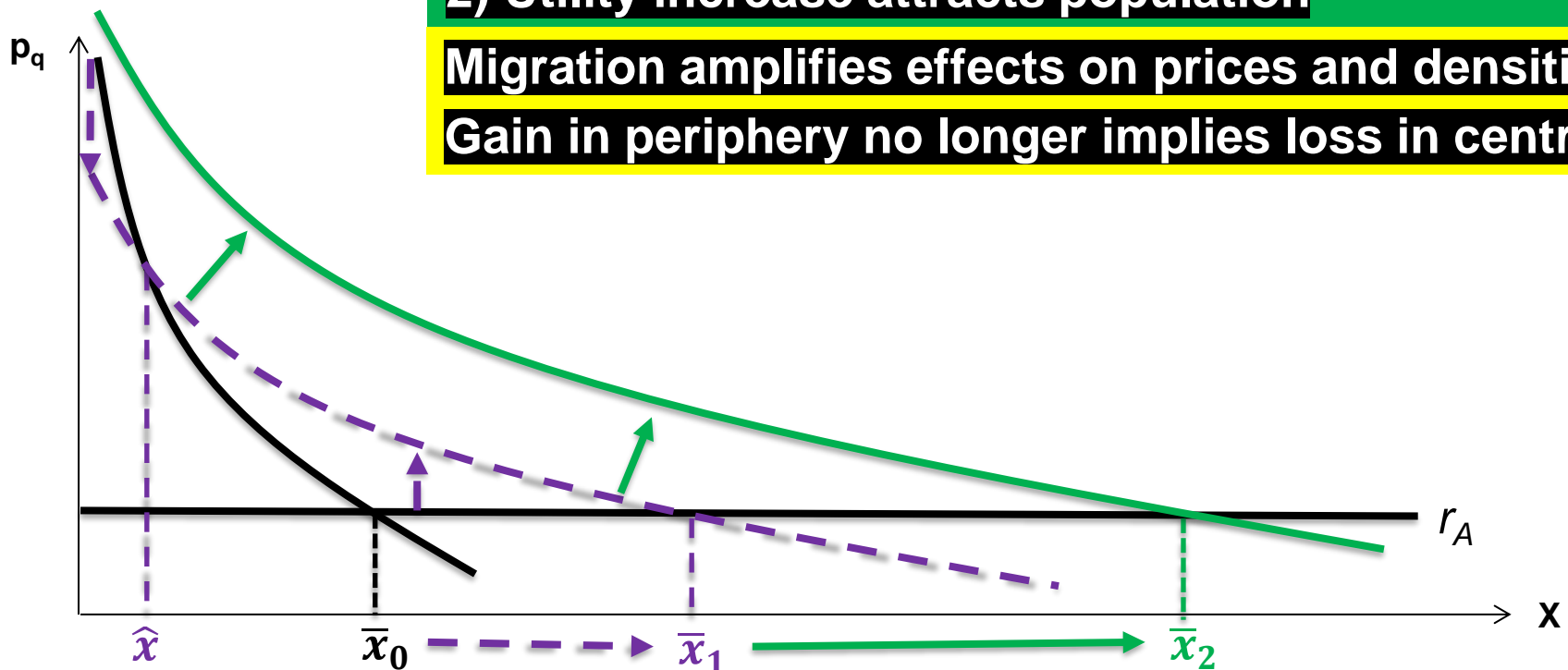
other predictions

1) Bid-rent curves rotates around x^{hat}

2) Utility increase attracts population

Migration amplifies effects on prices and densities

Gain in periphery no longer implies loss in centre



IV SUMMARY OF PREDICTIONS

comparative statics

Closed-city case		Effect on	Housing rent		Housing cons.		Str. density		Pop. density		Land rent		
Increase in	City area	C	P	C	P	C	P	C	P	C	P	Utility	
Income					?								
Commuting cost													
Agricultural land rent													
Population													
Open-city case		Effect on	Housing rent		Housing cons.		Str. density		Pop. density		Land rent		
Increase in	City area	C	P	C	P	C	P	C	P	C	P	Population	
Income													
Commuting cost													
Agricultural land rent													
	Increase	Own summary of Brueckner (1987)											
	Decrease												
	No effect												
	Ambiguous												
?													

Homework: Can you derive these predictions?

IV WHEN TO APPLY OPEN- OR CLOSD-CITY MODEL

other predictions

- Recall
 - „Closed city“ Case: No between-city migration
 - „Open city“ case: Free between-city migration
- When doing predictions, ask the quesiton:
 - Do we expect cross-city migration?
 - If all cities change in the same way => no migration
 - If just one city changes => migration incentive

Use closed-city model to predict changes occuring in all cities in country
Use open-city model to predict changes occuring in one city in a country

SUMMARY

Conclusion

- **Monocentric city model provides useful predictions**
 - Higher income leads to decentralization (monetary commuting cost)
 - Better transport leads to decentralization
 - Flatter gradients
 - Larger city areas
 - Higher agricultural rent shrinks cities
- **Some predictions substantiated by causal evidence**
 - City decentralizes with better transport
- **Predictions are useful for planners, developers, investors**
 - Insights into expected effects on demand, prices, densities
- **Next: Firm location**
 - Determinants of firm locations within cities

An aerial photograph of Chicago, Illinois, showing the city's grid pattern and the Chicago River. The word "THANKS" is overlaid in a white box on the left side of the image. The image is split horizontally, with the top half showing a wider view of the city and the bottom half showing a more detailed view of the downtown area.

THANKS

READING

- Core readings:

- Ahlfeldt, Gabriel M., Wendland, Nicolai (2011), Fifty years of urban accessibility: The impact of the urban railway network on the land gradient in Berlin 1890–1936. *Regional Science and Urban Economics*. Vol.41(2), pp.77-88
- Brueckner J., 1987, “The Structure of Urban Equilibria”, *Handbook of Regional and Urban Economics*, Vol II, 821-846.
- DiPasquale-Wheaton, chapter 3: The urban land market: Rents and prices
- McDonald & McMillen, chapter 6 & 7

- Complementary readings and references:

- Ahlfeldt, McMillen (2018): Tall buildings and land values. Height and construction cost elasticities in Chicago 1870-2010. *Review of Economics and Statistics*, forthcoming
- Asubel, J., Marchetti, C., Meyer, P. (1998): Toward green mobility: the evolution of transport. *European review* 6(2), 137-156.
- Barr, J., & Cohen, J. P. (2014). The Floor Area Ratio Gradient: New York City, 1890-2009. *Regional Science and Urban Economics*, 48(0), 110-119
- Baum-Snow, N. (2006): Did highways cause suburbanization? *The Quarterly Journal of Economics* 122 (2), 775-805.
- Gonzalez-Navarro, Marco, Turner, Matthew A. (2016): Subways and Urban Growth: Evidence from Earth. Working paper.
- Wheaton, W. (1977): *American Economic Review*, 67(4), pp. 620-631

APPENDIX: URBAN POPULATION

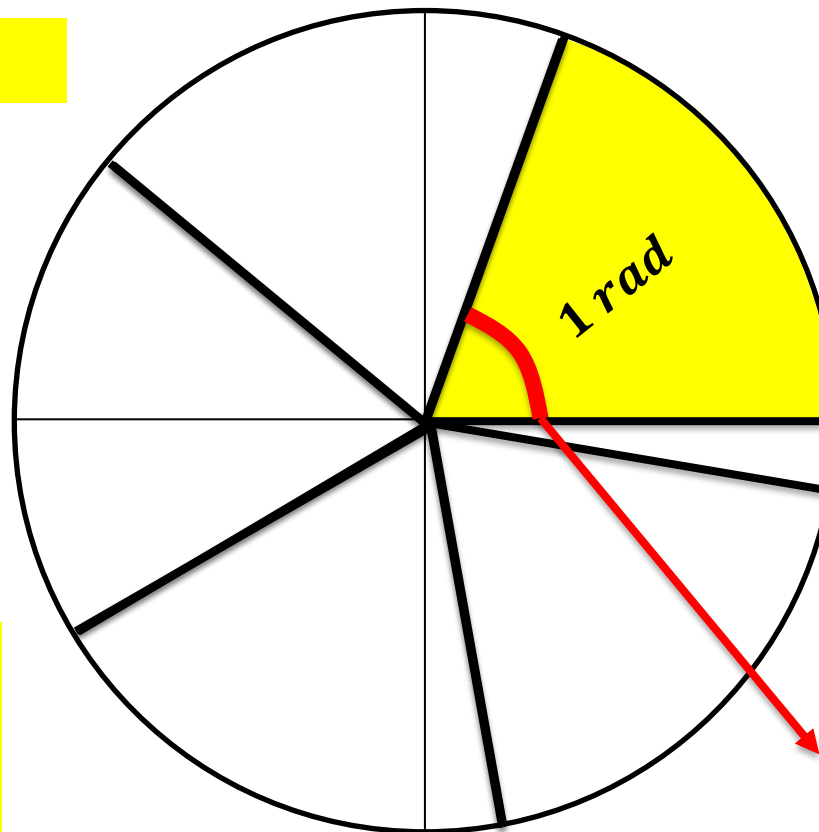
additional restrictions

- Urban population must fit into the urban area marked by \bar{x}

1 radian $\approx 57.3^\circ$

$0 < \theta < 2\pi$ is the number of radians available for housing (rest for e.g. infrastructure)

With $\theta = 2\pi$ all area for a given radius x will be used for housing



$$\int_0^{\bar{x}} \theta x D(x, t, y, u) dx = L$$

**At every x ,
population is the
product of area and
population density**

**Integrate over all x to
get population**

Note: With $\theta = 1$, a ring with a width of $dx = 1\text{km}$ at $x = 5\text{km}$ from the CDB has an area of $A = 1 \times 1\text{km} \times 5\text{km} = 5\text{km}^2$