



# **THE VERTICAL DIMENSION OF CITIES**

**Urban Economics**

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# I COURSE COMPONENTS

the idea

- Block 1

- Introduction to Urban and Regional Economics and Course Overview
- Topic I: Regional and urban concentration forces
- Topic II: The empirics of agglomeration
- Topic III: Costs and benefits of agglomeration

- Block 2

- Topic IV: Monocentric city I (household location choice)
- Topic V: Monocentric city II (household location choice)
- Topic VI: Firm location choice
- Topic VII: The urban economy in general equilibrium

- Block 3

- **Topic VIII: The vertical dimension of cities**
- Topic IX: Suburbanization and gentrification
- Topic X: Hedonic analysis

# I INTRODUCTION

roadmap

- Last time: *The urban economy in general equilibrium*
  - 1) Recap
    - Compensating differentials, location choice, supply side
  - 2) The Ahlfeldt-Redding-Sturm-Wolf model
    - Model setup
  - 3) Separating spillovers from location fundamentals
    - Reduced-form evidence
    - Difference-in-difference analysis
  - 4) Structural estimation and simulation
    - Identifying structural parameters
    - Simulating the effect of a new metro line

# I INTRODUCTION

roadmap

- This time: *The vertical dimension of cities*

- 1) Model

- Vertical and horizontal costs and benefits
  - General equilibrium

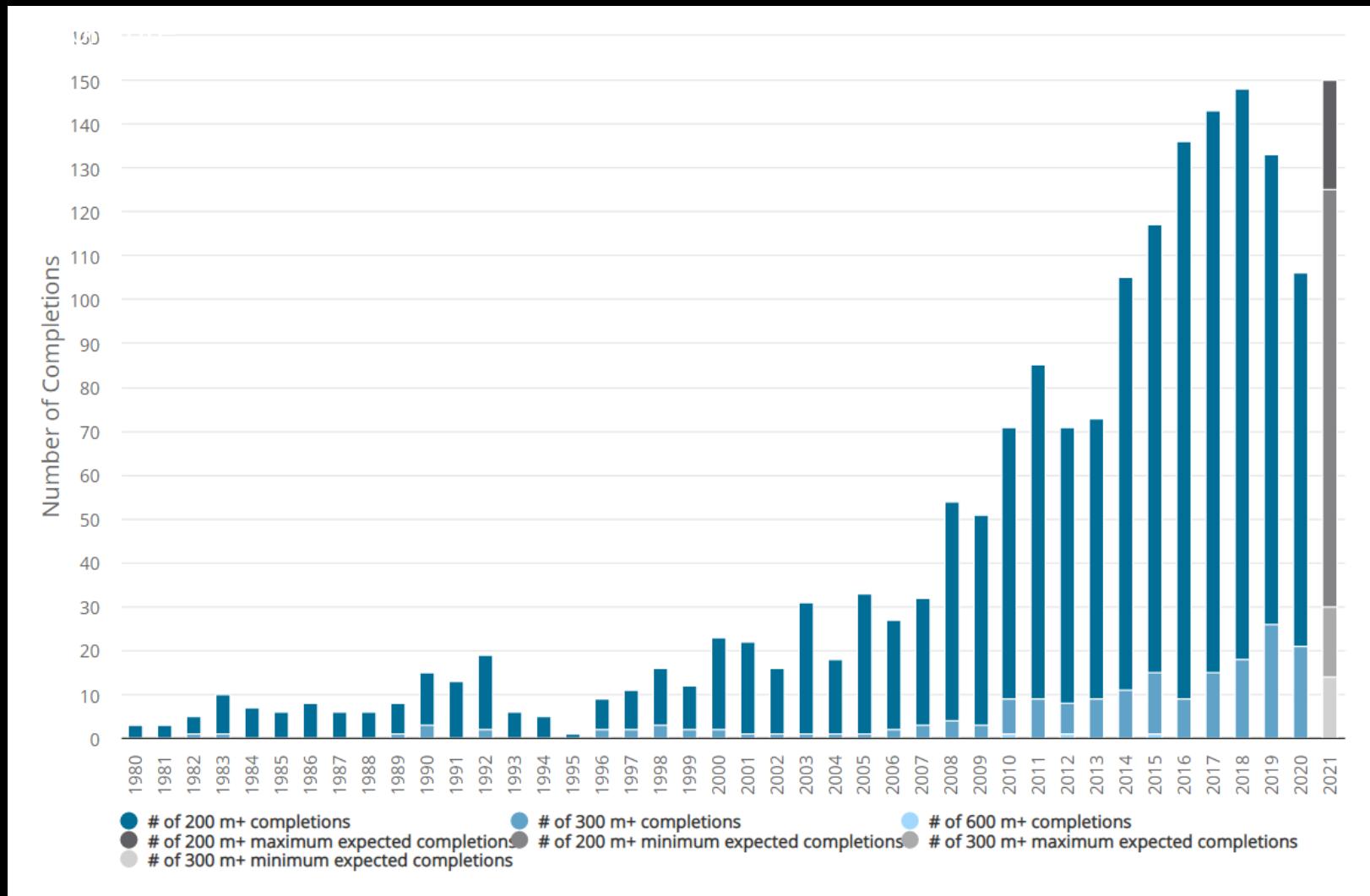
- 2) Parametrization

- Returns to height
  - Cost of height

- 3) Equilibrium and counterfactuals

- Urbanization as cause of skyscraper development
  - Skyscraper development as cause of urbanization
  - Height regulation

# I TALL BUILDINGS

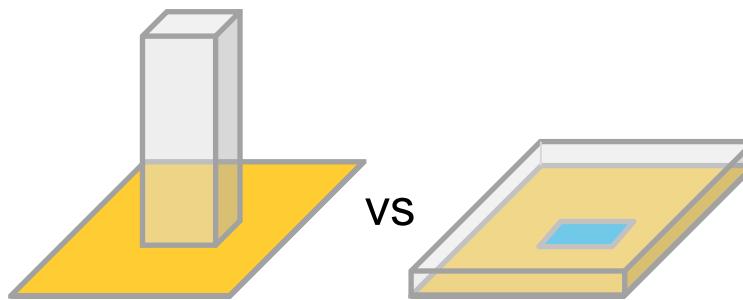


# I INTRODUCTION

roadmap

- Standard urban economics models are horizontal

- Do not explicitly model height
- Do model „structural density“
- But not exactly the same



- Ahlfeldt & Barr (2022) synthesize recent research into vertical dimension

- *Journal of Urban Economics* article: [DOI](#)
- VOXEU column: [link](#)
- Faculti podcast: [link](#)
- UCLA housing voice podcast: [link](#)

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roadmap

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# I INTRODUCTION

roadmap

- Wishlist for the model

- Endogenous land use (endogenous CBD has a mass)
  - Need residents *and* firms (like ARSW, 2015)
- Endogenous population (causes and effects of urbanization)
  - Open city model (like ARSW, 2015)
- Endogenous wage (subject to agglomeration economies)
- Account for cost and benefits in height **NEW!**
  - Can differ by land use
- General equilibrium amenable to quantitative analysis
  - But more “intuitive” than canonical quantitative spatial model a la ARSW (2015)

## II STYLIZED FACTS

**Q: Are height decisions rational?**

## II HEIGHT COMPETITION

stylized facts

- **Helsley & Strange (2008)**

- There is an intrinsic **value of being the tallest** (building)
- Game theoretical analysis reveals that skyscrapers are developed **beyond the fundamentally efficient height**
- Developers build excessively tall to “**discourage**” competitors
  - **Empire State building** tallest building for about 30 years

- **Mixed evidence**

- Barr (2010): Profit maximization not height competition!
- Barr (2012): Spatial autoregressive structure => height competition
- Barr (2013): skyscraper competition between cities

## II ARE SKYSCRAPERS TOO TALL?

stylized facts

- Tallest buildings often claimed to be irrationally height

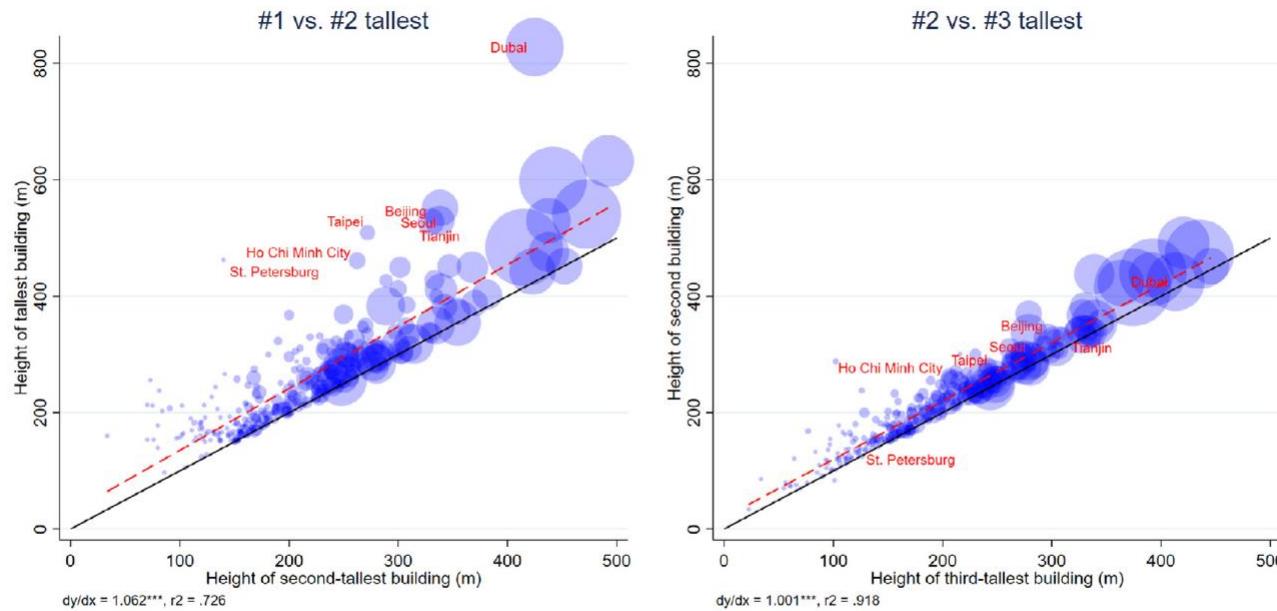


Fig. 4. Tallest building heights Note: Data covers 315 cities around the world with at least one skyscraper (heights  $\geq 150$  m. Marker size is proportionate to the number of skyscrapers in a city. Data are from Emporis (accessed in April, 2021).

**But only few tall skyscrapers “dominate” their skylines  
#2 and #3 within cities have almost the same height**

## II ARE SKYSCRAPERS TOO TALL?

stylized facts

- Empire State Building claimed to be the irrational outcome of 1920s *skyscraper race* (nickname: Empty State Building)

Actually,  
performed  
well after  
Great  
Depression

Competitive  
model  
should be  
fine

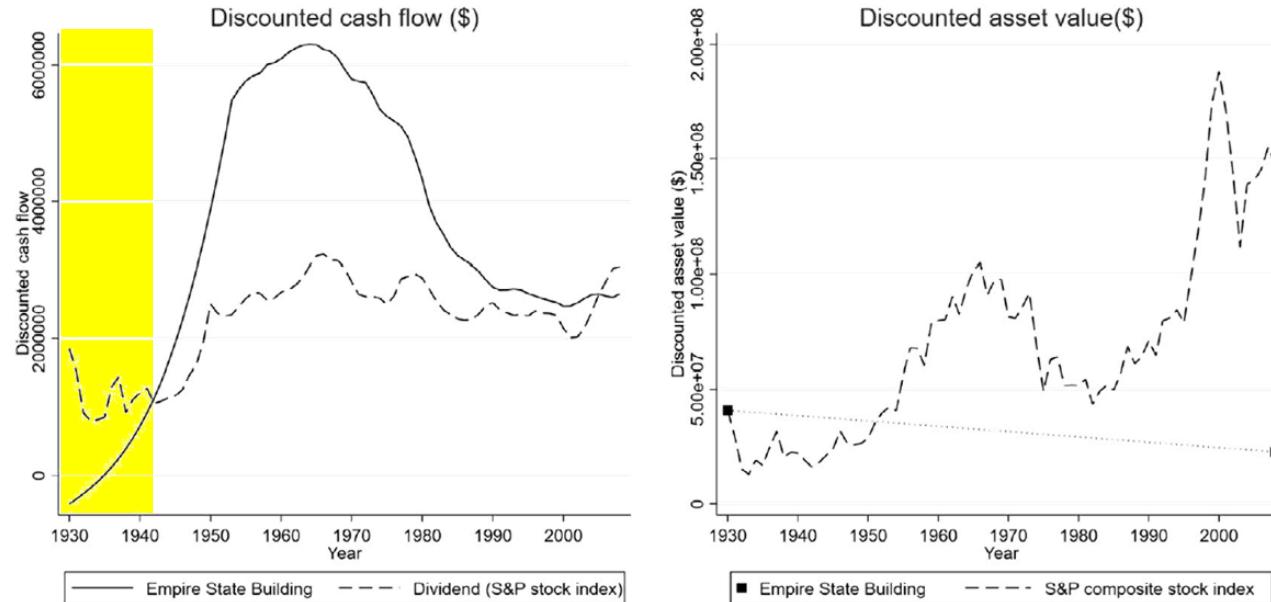


Fig. 5. Empire State Building vs. stock market Note: ESB asset value is total cost of structure and land in 1930 and land value in 2008 (as we assume that the structure has fully depreciated). For stock market, the asset value is the total investment inflated by the stock market index. Cash flow of the ESB is the net operating income (EBITDA). Cash flow of the stock market is the dividend (the product of the dividend yield and the asset value). All time series deflated by the cummulated opportunity cost of capital (risk-free rate). The net realized return for the ESB, at 5.4%, beats the stock market, at 4.3%. See Online Appendix Section B.4 for details.

# III GEOGRAPHY

model

- **Adopt stylized geography in the spirit of monocentric model**
- **Location defined by X**
  - $X = 0$  is the historic center
  - $D(x) = |X|$  is the distance from the historic center
- **Production amenity declines in distance from the CBD**
  - Captures agglomeration spillovers
- **Residential amenity declines in distance from the CBD**
  - **Captures commuting cost**
    - If dependent on income

# III WORKERS

model

- Utility depends on floor space ( $f$ ), other goods ( $g$ ) and amenities ( $A$ )

Depends on horizontal and vertical space

$$U(x, s) = A^R(x, s) \left( \frac{g}{\alpha^R} \right)^{\alpha^R} \left( \frac{f^R(x, s)}{1 - \alpha^R} \right)^{1 - \alpha^R}$$

Distance gradient

$$\tilde{A}^R(x) = \bar{a}^R e^{-\tau^R |x|}$$

$$A^R(x, s) = \tilde{A}^R(x) s^{\tilde{\omega}^R}$$

Height elasticity  
of rent

- Utility maximization wrt.  $g$  and  $f$  delivers indirect utility, marshallian demand functions.
- Setting  $U$  to reservation utility  $\bar{U}$  gives bid rent:

$$\bar{p}^R(x) = \frac{1}{S^R(x)} \int_0^{S^R} p^R(x, s) ds = \frac{a^R(x)}{1 + \omega^R} S^R(x)^{\omega^R} \quad a^R(x) = \tilde{A}^R(x)^{\frac{1}{1-\alpha^R}} (y^R)^{\frac{1}{1-\alpha^R}}$$

Horizontal bid-rent falls in distance and depends on height elasticity

### III FIRMS

model

- Output depends on floor space ( $f$ ), other labour ( $l$ ) and amenities ( $A$ )

**Depends on horizontal and vertical space**

$$g(x, s) = A^C(x, s) \left( \frac{l}{\alpha^C} \right)^{\alpha^C} \left( \frac{f^C(x, s)}{1 - \alpha^C} \right)^{1 - \alpha^C}$$

**Agglo. & distance effect**

$$\begin{aligned} \tilde{A}^C &= \bar{a}^C N^\beta e^{-\tau^C |x|} \\ A^C(x, s) &= \tilde{A}^C(x) s^{\tilde{\omega}^C} \end{aligned}$$

**Height elasticity  
of rent**

- Profit maximization wrt.  $l$  and  $f$  delivers marginal rate of substitution
- Assuming zero economic profits gives bid rent:

$$\bar{p}^R(x) = \frac{1}{S^R(x)} \int_0^{S^R} p^R(x, s) ds = \frac{a^R(x)}{1 + \omega^R} S^R(x)^{\omega^R} \quad a^C(x) = \tilde{A}^C(x)^{\frac{1}{1-\alpha^C}} (y^C)^{\frac{\alpha^C}{\alpha^C-1}}$$

**Horizontal bid-rent falls in distance, depends on height & agglomeration**

# III DEVELOPERS

model

- Profit function:

$$\pi^U(x, S^U(x)) = \bar{p}^U(x)S^U(x) - \tilde{c}^U(S^U(x))S^U(x) - r^U(x)$$

Horizontal bid rent

$$\tilde{c}^U = c^U S^U(x)^{\theta^U}$$

Unit cost increases  
in height

- Profit-maximizing building height:

$$\frac{\partial \pi}{\partial S} = 0 \text{ equivalent to } MR = MC$$

$$S^{*U}(x) = \left( \frac{a^U}{c^U(1+\theta^U)} \right)^{\frac{1}{\theta^U-\omega^U}}$$

- See also the Ahlfeldt & McMillen (2018) version

PV of rent of 1-floor building

$$S^* = \left( \frac{p^U(1 + \omega^U)}{c^U(1 + \theta^U)} \right)^{\frac{1}{\theta^U-\omega^U}}$$

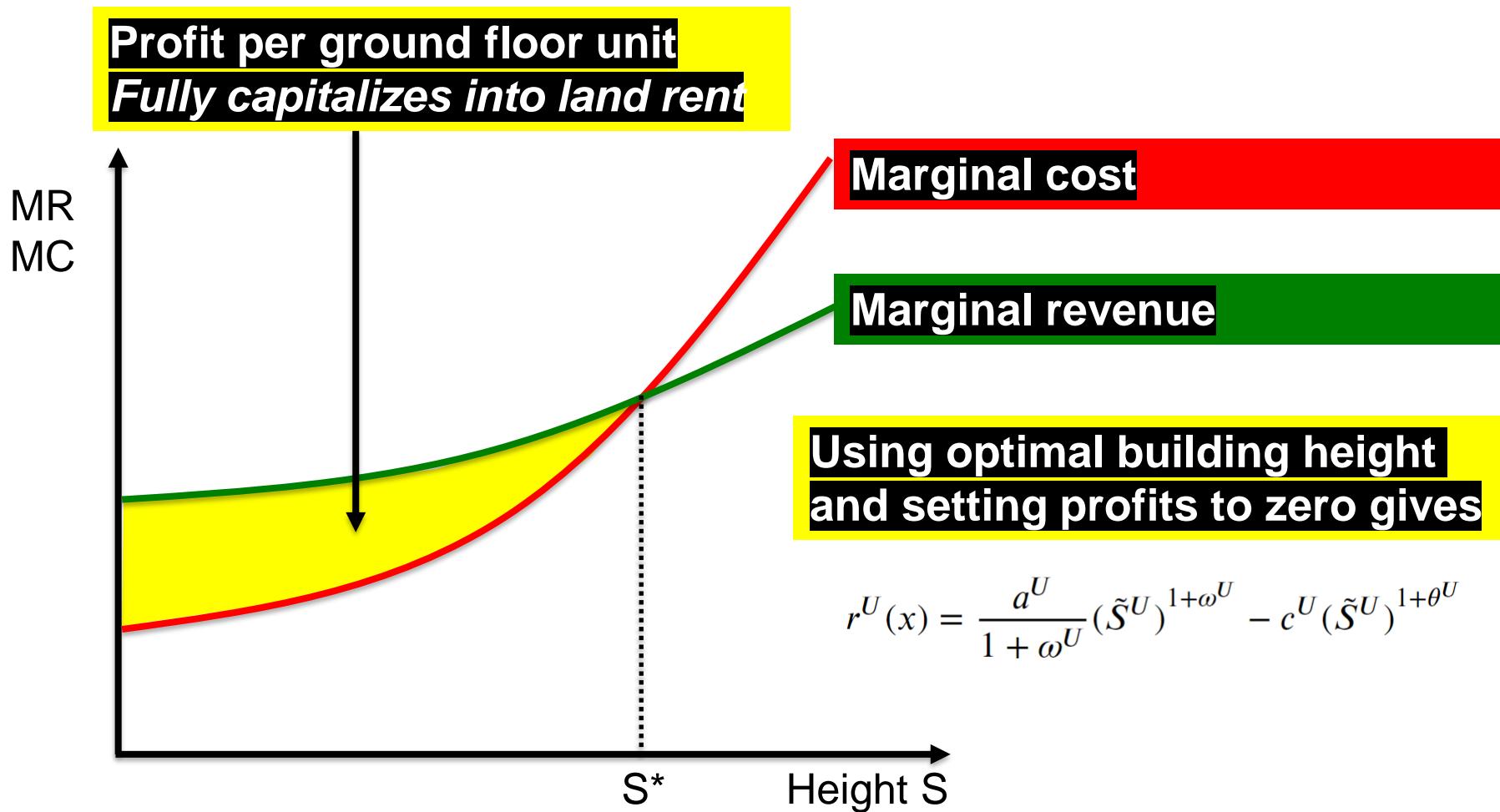


Useful for seminar

Unit cost of 1-floor building

### III LAND RENTS

model



### III IMPLICATIONS FOR DEVELOPERS

model

- 1) Compute optimal building height for given 1-floor rent and cost

$$S^* = \left( \frac{p^U(1 + \omega^U)}{c^U(1 + \theta^U)} \right)^{\frac{1}{\theta^U - \omega^U}}$$

- 2) Plug in  $S^*$  and multiply by building footprint  $T$  to get **total profit**

$$\pi^* = (pS^{*1+\omega} - cS^{*1+\theta})T - r \frac{T}{\bar{l}} \quad \text{Ground floor ratio}$$

Net operating income  
 NOI ← Internal rate of return  
 IRR ←

- Remember that  $p$  is expressed as a PV, e.g.

$$p = \frac{NOI}{IRR}$$

- “Profitable” bid as long as  $\pi^* \geq 0$

**Profitable offer for land:**

$$rT \leq \bar{l}(pS^{*1+\omega} - cS^{*1+\theta(Z)})T$$

## II EQUILIBRIUM

model

- Land is allocated to highest bidder



**Find commercial and residential zone (land use pattern)**

- Real estate and labour markets clear

- Get # workers at workplace from MRS:  $L(x) = \frac{\alpha^C}{1 - \alpha^C} \frac{\bar{p}^C(x)}{y^C} S^C(x)$

- Get # workers at residence from MRS:  $n(x) = \frac{S^R(x)}{y^R} \frac{\bar{p}^R(x)}{1 - \alpha^R}$

- Total # workers at workplace and residence equate

$$\int_{-x_0}^{x_0} L(x)dx = \int_{-x_1}^{x_0} n(x)dx + \int_{x_0}^{x_1} n(x)dx = N$$

**Fixed point solved finds numerical solution to y (wage) and N (population) that clears markets**

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# IV PARAMETRIZATION

**Q: Do rents increase in height within buildings?**

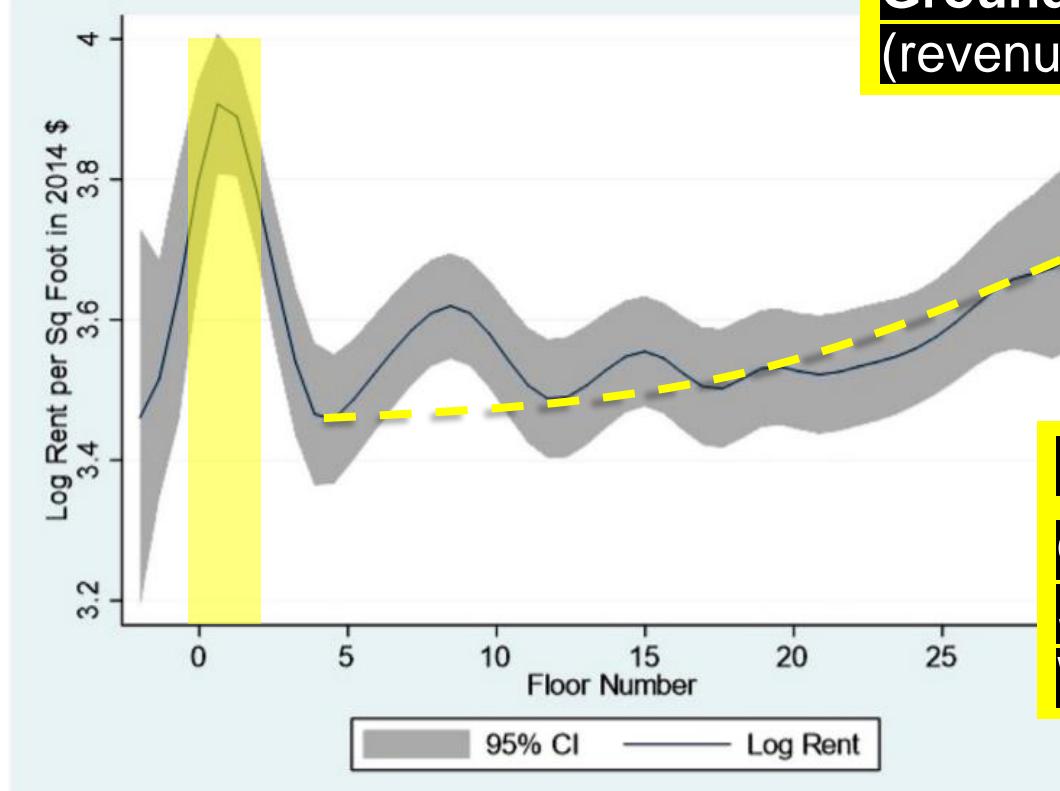
# IV COMMERCIAL VERTICAL RENT GRADIENT

Liu et al (2018)

C.H. Liu et al./Journal of Urban Economics 106 (2018) 101–122

111

**Ground floor premium driven by retail  
(revenue effect, customer proximity)**



**Positive height gradient**  
Offices higher up more attractive:  
Sigaling effect (quality)  
Wage effect (workplace amenity)

**Fig. 1.** Non-parametric estimates of offering memo rent function.

Notes: Estimated using local polynomial (lpoly) regression with a Gaussian kernel function, 3° of power and a bandwidth optimally chosen at 1.49 floors (using Stata's lpoly routine). The sample includes only suites from offering memo (OM) buildings over 30 floors in height and is restricted to suites from 2 floors below ground level up to floor 30.

# IV COMMERCIAL VERTICAL RENT GRADIENT

Liu et al (2018)

**Table 2a**

Rent gradients with building fixed effects<sup>a</sup>.

	Offering memo data		CompStak data	
	Double log	Semi-log	Double log	Semi-log
Below ground floor	-0.2621 (0.1035)	-0.4596 (0.0987)	-	-
Ground floor	0.4661 (0.0952)	0.3448 (0.1002)	0.1156 (0.0357)	0.0295 (0.0354)
Ground floor X Bldg height	0.0046 (0.0017)	0.0059 (0.0018)	0.0070 (0.0017)	0.0073 (0.0017)
Log(Floor number + k) <sup>b</sup>	0.1883 (0.0355)	- -	0.0858 (0.0049)	- -
Floor number	- -	0.0087 (0.0012)	- -	0.0058 (0.0003)
Observations	5,445	5,445	37,007	37,007
Lease quarter Fixed Effects	-	-	Yes	Yes
Building Fixed Effects	93	93	1,922	1,922
R-sq within	0.162	0.177	0.247	0.254

**Ground floor premium**  
**Up to  $\exp(0.12)-1 \approx 13\%$**

**Height elasticity of**  
**office rent = 8.6%**  
**(CS probably more**  
**representative than OM)**

**Corresponds to**  
 **$\omega = 7.1\%$  (see appendix)**

<sup>a</sup> Dependent variable for the OM regressions is gross rent per square foot in \$2014. Dependent variable for the CS regressions is in \$2014 and is net rent which adjusts gross rent for months of free rent at the start of the lease and other accommodations. Standard errors clustered at the building level are in parentheses.

<sup>b</sup>  $k$  is set to a value 1 unit larger in absolute value than the lowest basement floor in the data, -5 for the offering memo data and -1 for the CompStak data.

# IV COMMERCIAL VERTICAL RENT GRADIENT

Liu et al (2018)

**Table 2b**

Convex Rent Gradients<sup>a</sup>.

	Offering Memo Data			CompStak Data		
	(1) Floors 3 through 29	(2) Floors 30 through 59	(3) Floors 60 and above	(4) Floors 3 through 29	(5) Floors 30 through 59	(6) Floors 60 and above
<b>PANEL A: Double Log</b>						
Log(Floor number + k) <sup>b</sup>	0.1334 (0.0322)	0.1810 (0.1571)	1.036 (0.0196)	0.0760 (0.0046)	0.2873 (0.0394)	1.274 (0.3036)
Observations	3,524	774	115	29,360	4,602	116
Lease quarter Fixed Effects	No	No	No	Yes	Yes	Yes
Building Fixed Effects	93	44	4	1,862	369	18
R-sq within	0.029	0.008	0.112	0.257	0.295	0.710
<b>PANEL B: Semi-Log</b>						
Floor number	0.0075 (0.0018)	0.0045 (0.0034)	0.0124 (0.0002)	0.0058 (0.0003)	0.0068 (0.0009)	0.0161 (0.0032)
Observations	3,524	774	115	29,360	4,602	116
Lease quarter Fixed Effects	No	No	No	Yes	Yes	Yes
Building Fixed Effects	93	44	4	1,862	369	18
R-sq within	0.030	0.010	0.117	0.259	0.295	0.708

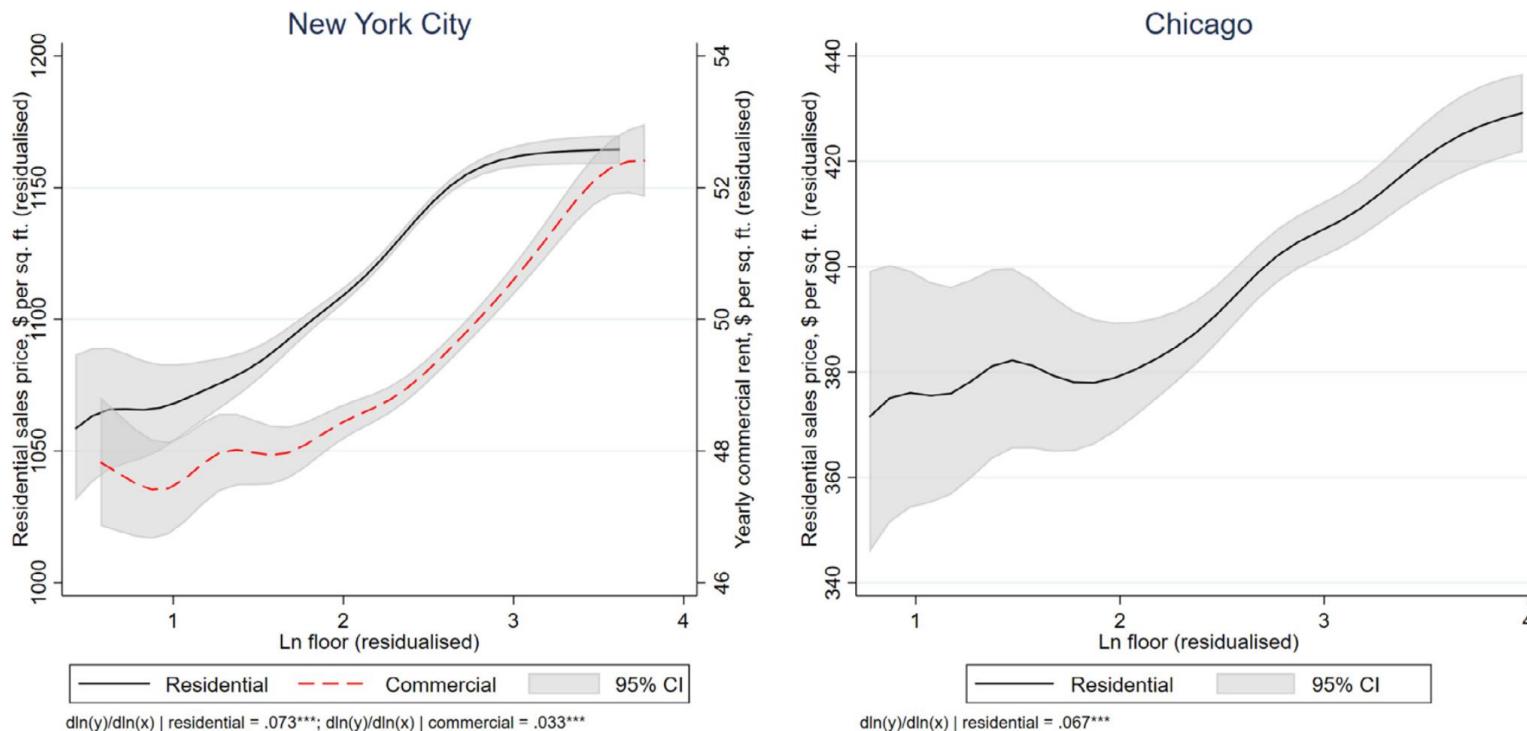
<sup>a</sup> Dependent variable for the OM regressions is gross rent per square foot in \$2014. Dependent variable for the CS regressions is in \$2014 and is net rent which adjusts gross rent for months of free rent at the start of the lease and other accommodations. Standard errors clustered at the building level are in parentheses.

<sup>b</sup> k is set to a value 1 unit larger in absolute value than the lowest basement floor in the data, -5 for the offering memo data and -1 for the CompStak data.

**Height elasticity of office rent larger for higher floors  
=> highly convex rent premium**

# IV RESIDENTIAL VERTICAL RENT

Ahlfeldt & Barr (2022)

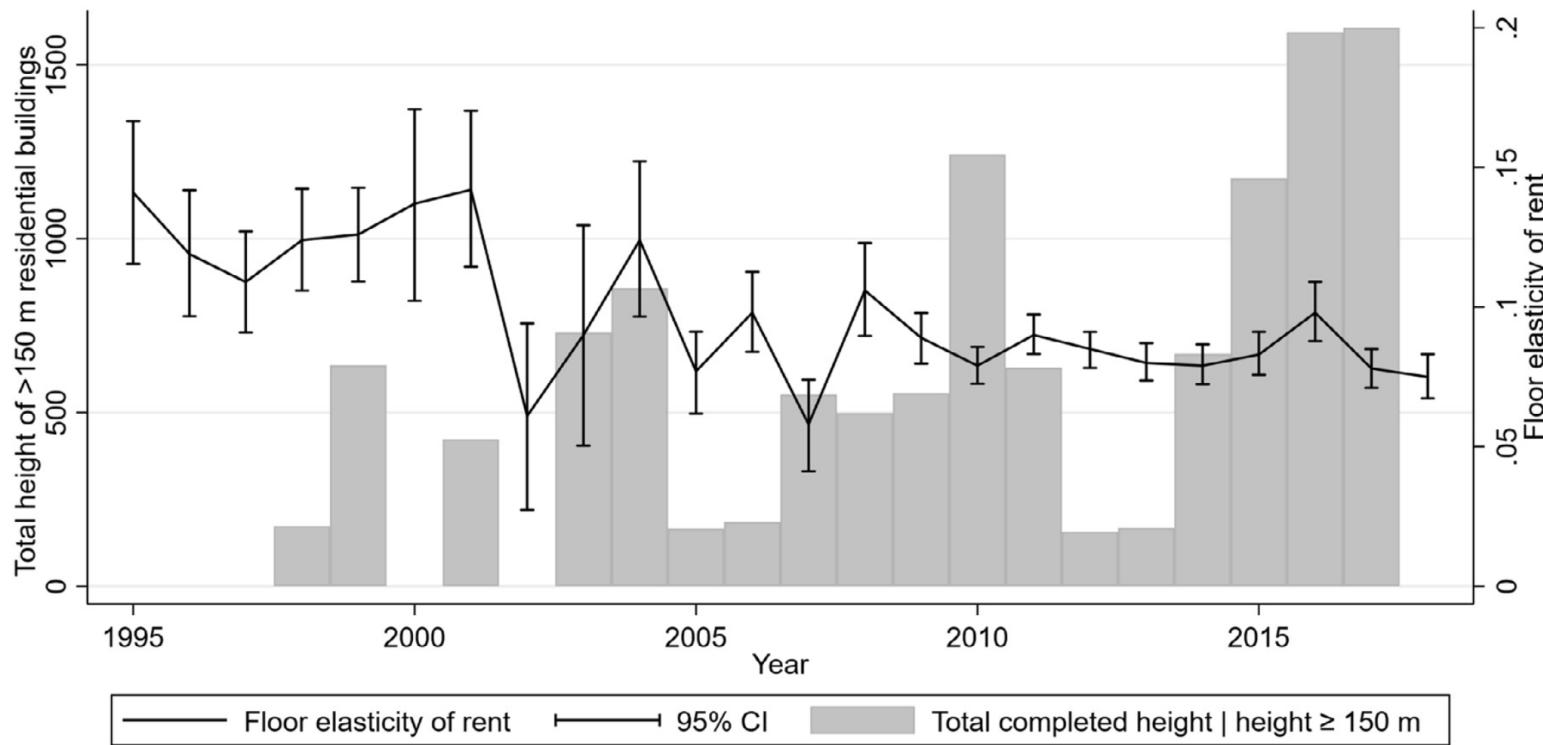


**Fig. 8.** Returns to height Note: Price, rent and floor are residualized in regressions (in logs) against unit characteristics and building fixed effects and time effects. The displayed non-linear functions are the outcome of locally weighted regressions using a Gaussian kernel and a bandwidth of 0.2. Confidence bands are at the 95% level. We trim the data set to exclude outliers that fall into the bottom or top percentile in terms of residualized ln rent/price or residualized ln floor before estimating the non-parametric gradient to improve the presentation. The parametric elasticity estimates are from the full sample. Residential prices are from StreetEasy. Commercial asking rents are from Cushman & Wakefield.

**Positive residential rent gradient due to utility effects  
(better view, less noise, etc.)**

## IV RESIDENTIAL VERTICAL RENT

Ahlfeldt & Barr (2022)



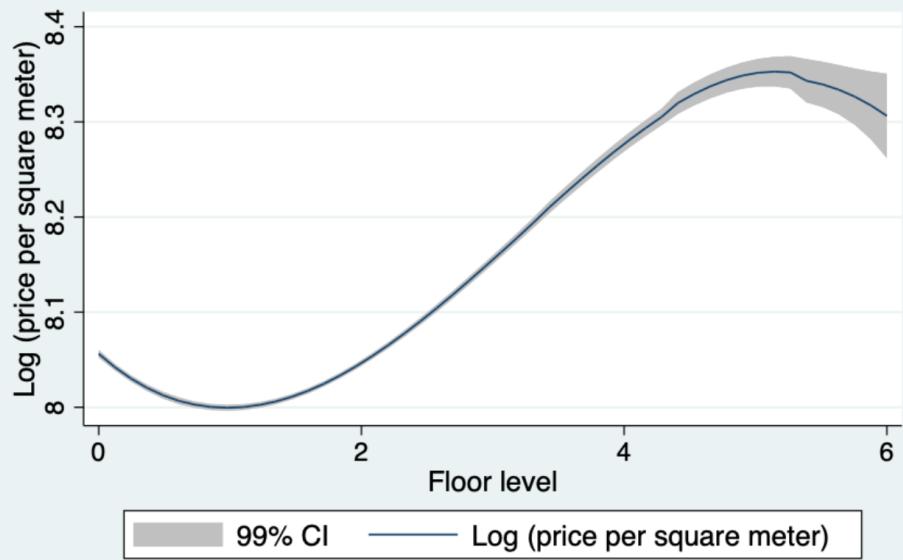
**Fig. 9.** Returns to height over time in the New York City residential market Note: The floor elasticity of rent is estimated in regressions of log rent against log floor by year, controlling for unit characteristics and building fixed effects. Confidence bands are at the 95% level. Residential rents are implied rents based on apartment prices data from StreetEasy, assuming a constant discount rate. Total completed height of residential buildings exceed 150 m are from the CTBUH Skyscraper Center.

**Q: Vertical rent gradient decreases over time, why?**

# IV RESIDENTIAL VERTICAL RENT: LONDON

2022 REEF dissertation “The vertical price gradient in London” (Candidate number: 40037)

The low-rise vertical price gradient



The medium-rise vertical price gradient

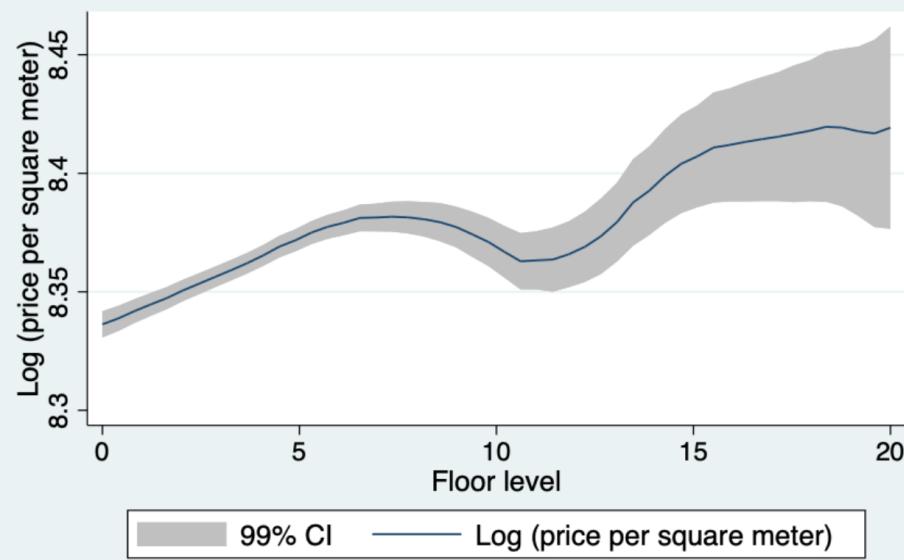


Figure 2: The non-linear vertical price gradient was yielded from estimating locally weighted regressions using Stata's `lpolyci` function. Stata's default Epanechnikov Kernel function is used in the analysis. Inspired by [Liu et al. \(2018\)](#), the bandwidth of 1.5 and function degree of three are chosen. Confidence bands are at the 99% level. Data source: [\(Dennett, et al., 2021\)](#)

$$\omega = 0.6\%$$

Figure 1: Stata's default polynomial smooth of degree 0 and the Epanechnikov Kernel function are used to yield the mid-rise VPG, with a bandwidth of 1.5. Confidence bands are at the 99% level.

Confidence bands are at the 99% level.  
Data source: [\(Dennett, et al., 2021\)](#)

$$\omega = 3\%$$

# IV PARAMETRIZATION

**Q Are taller buildings more expensive to build?**

# IV RELATED EVIDENCE

parametrization

- Liu et al (2018) provide estimates of the **vertical rent gradient** conditional on building fixed effects
  - **Control for unobserved building effects** (location quality, design, maintenance cost, sustainability, etc.)
  - Lends great credibility to their estimates (causal evidence)
- Himbert and Danton (2018) show that in a residential building the
  - **fifth floor** commands a **5% premium** over the 1st floor
  - **ground floor** commands a **1.5-3.5% premium** ← **What is going on?**
- **Additional evidence** (unconditional on building fixed effects) substantiates the notion of a height premium
  - Colwell et al (1998), Koster et al (2014), Shilton & Zaccaria (1994)
- Although Eichholtz et (2010) also find mixed evidence

# IV THE HEIGHT ELASTICITY OF CONSTRUCITON COST

Ahlfeldt & McMillen (2018)

- Rule-of-thumb

Can we confirm using data?

- Per-floor space construction cost increases by 2% per floor  
(Department of the Environment, 1971)

- Ahlfeldt & McMillen (2018)

- First to estimate the elasticity using actual **micro data**

$$\ln\left(\frac{K}{F}\right)_{it} = \alpha_t + \theta \ln S_{it} + \varepsilon_{it}$$

- where  $i$  refers to a tall building construction at time  $t$

- **$K$  is total construction cost** (excluding land acquisition cost)  $F$  is total building floor space, and  $S$  is a measure of height  
(data set from Emporis)

# IV THE HEIGHT ELASTICITY OF CONSTRUCITON COST

Ahlfeldt & McMillen (2018)

TABLE 5.—CONSTRUCTION COST ELASTICITY ESTIMATES

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Log Floor Space Cost							
Log building height	0.251*** (0.006)	0.475*** (0.032)	0.634*** (0.040)	0.541*** (0.034)	0.612*** (0.034)	0.523*** (0.036)	0.611*** (0.041)	1.729*** (0.050)
Country effects	Yes	Yes	Yes	-	-	-	-	-
Decade effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-
Mean number of floors	2.5	15.9	15.6	12.3	9.2	20.600	20.100	110
Semielasticity	0.099	0.030	0.041	0.044	0.067	0.025	0.030	0.016
Data	Obs.	Obs.	Obs.	Obs.	Obs.	Obs.	Obs.	Engin.
Building use	All	Com.	Res.	Com.	Res.	Com.	Res.	Com.
Building type	Small	Tall	Tall	Tall	Tall	Tall	Tall	ST
Region	World	World	World	US	US	PC	PC	-
R <sup>2</sup>	0.109	0.492	0.614	0.620	0.451	0.730	0.522	0.997
N	32,016	1,219	1,570	818	1,402	818	1,401	9

Observed data (obs.) are from Emporis. Engineering estimates (Engin.) are from Lee et al. (2011). Ln Floor space cost is computed using predicted usable floor space values based on the regressions reported in appendix section 4.8. Small buildings have fewer than five floors. Tall buildings have five or more floors. Super-tall buildings (ST) have ninety or more floors. The semielasticity is computed by dividing the cost elasticity by the mean number of floors. Pseudo-Chicago (PC) sample is the U.S. sample, reweighted to resemble the distribution of ln building heights in Chicago (using a propensity score matching; see appendix section 4.9). For a small percentage of buildings, height is imputed based on floors using an auxiliary regression of height against floors (on average height increases by 3.6 meters per floor). Standard errors are in parentheses. \* $p < 0.1$ , \*\* $p < 0.05$ , and \*\*\* $p < 0.01$ .

**Height elasticity of construction cost increases in building height**

**2%-per-floor rule of thumb roughly applies to very tall buildings**

**For „reasonably tall buildings“:**  
 $\theta \approx 0.52$  (commercial) to 0.61 (residential) >

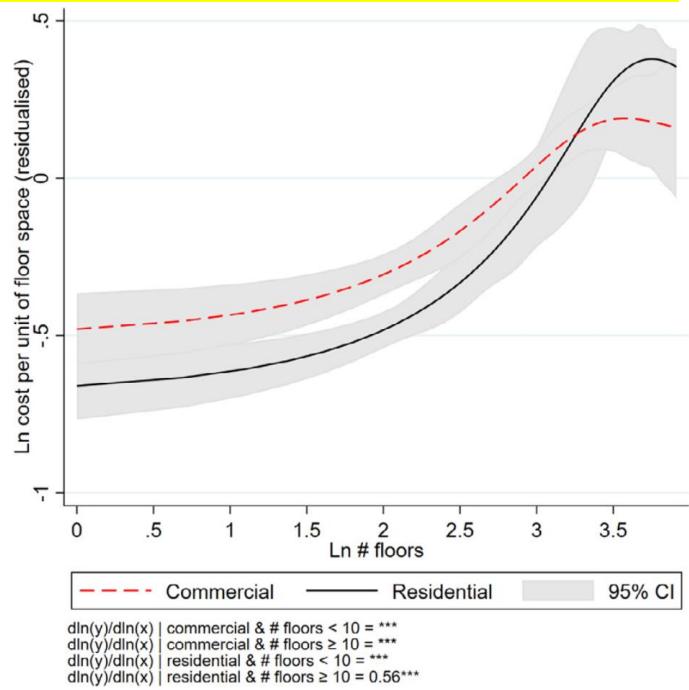
$\omega \Rightarrow$  single solution for optimal height

**Cost of height greater for residential than commerical buildings – Q: why?**

# IV THE HEIGHT ELASTICITY OF CONSTRUCITON COST

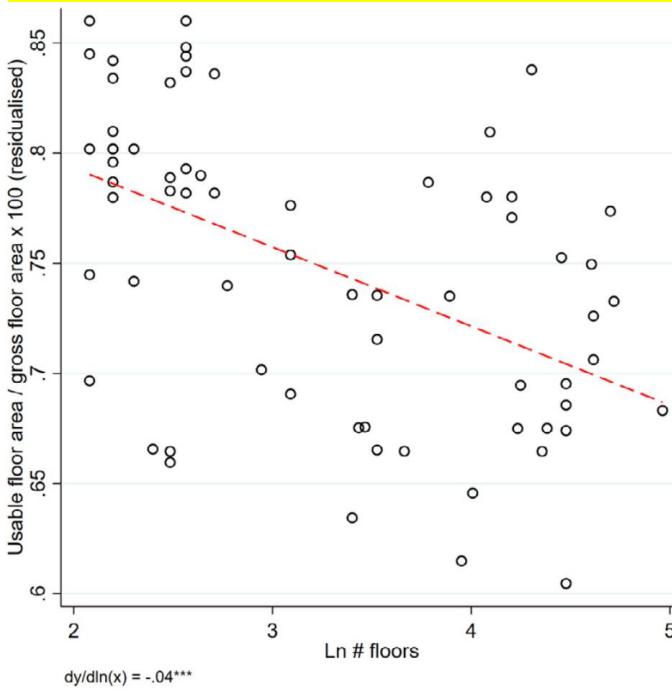
Ahlfeldt & Barr (2022)

## Evidence from a global cross-section



(a) Unit cost vs. height

## Less net floor space in taller buildings



(b) Usable floor area vs. height

**Fig. 6.** Cost of height Note: In panel (a), we first regress the log of the ratio of building construction cost over building floor space against decade fixed effects, country fixed effects, and number-of-floors fixed effects. The displayed non-linear functions are the outcome of locally weighted regressions of the estimated number-of-floor fixed effects against the number of floors, using a Gaussian kernel and a bandwidth of 10. Confidence bands are at the 95% level. The vertical line marks the natural log of 10. Data are from <https://www.emporis.com/> (see Ahlfeldt and McMillen (2018) for details). In panel (b), each marker is an individual building. The dependent variable is residualized in regressions against country fixed effects and a time trend. Note that the natural log of 10 (floors) is about 2.3. Observations were culled from Sev and Özgen (2009), Watts et al. (2007), Kim (2004), and Berger (1967).

# IV CANONICAL PARAMETER VALUES

Ahlfeldt & Barr (2022)

**Table 1**

Parameter values.

	Parameter	Value	Further reading
$1 - \alpha^C$	Share of floor space at inputs	0.15	<a href="#">Lucas and Rossi-Hansberg (2002)</a>
$1 - \alpha^R$	Share of floor space at consumption	0.33	<a href="#">Combes et al. (2019)</a>
$\beta$	Agglomeration elasticity of production amenity	0.03	<a href="#">Combes and Gobillon, 2015</a>
$\theta^C$	Commercial height elasticity of construction cost	0.5	<a href="#">Ahlfeldt and McMillen (2018)</a>
$\theta^R$	Residential height elasticity of construction cost	0.55	<a href="#">Ahlfeldt and McMillen (2018)</a>
$\omega^C$	Commercial height elasticity of rent	0.03	<a href="#">Liu et al. (2018)</a>
$\omega^R$	Residential height elasticity of rent	0.07	<a href="#">Danton and Himbert (2018)</a>
$\tau^C$	Production amenity decay	0.01	<a href="#">Ahlfeldt et al. (2015)<sup>a</sup></a>
$\tau^R$	Residential amenity decay	0.005	<a href="#">Ahlfeldt et al. (2015)</a>

Notes: These parameter values are not taken from individual papers and do not necessarily correspond to our own estimates. Instead, they represent what we view as canonical values that are suitable for stylized presentations and simple counterfactual analysis. The last column provides a references for the interested reader for further reading, but not necessarily the source of a point estimate. <sup>a</sup>The parameter value is consistent with the commercial rent gradient estimated for a large set of global cities, assuming  $\alpha^C = 0.15$  (see Online Appendix Section F.1.1). We set the following scale parameters arbitrarily to generate a plausible land use pattern:  $\bar{a}^C = 2$ ,  $\bar{a}^R = 1$ ,  $\bar{c}^C = 1.4$ ,  $\bar{c}^R = 1.4$ ,  $r^a = 30$ ,  $\bar{U} = 1$ . There are no binding height limits in the baseline parametrization ( $\bar{S}^C = \bar{S}^R = \infty$ ).

# I INTRODUCTION

roadmap

- This time: *The vertical dimension of cities*

- 1) Model

- Vertical and horizontal costs and benefits
    - General equilibrium

- 2) Parametrization

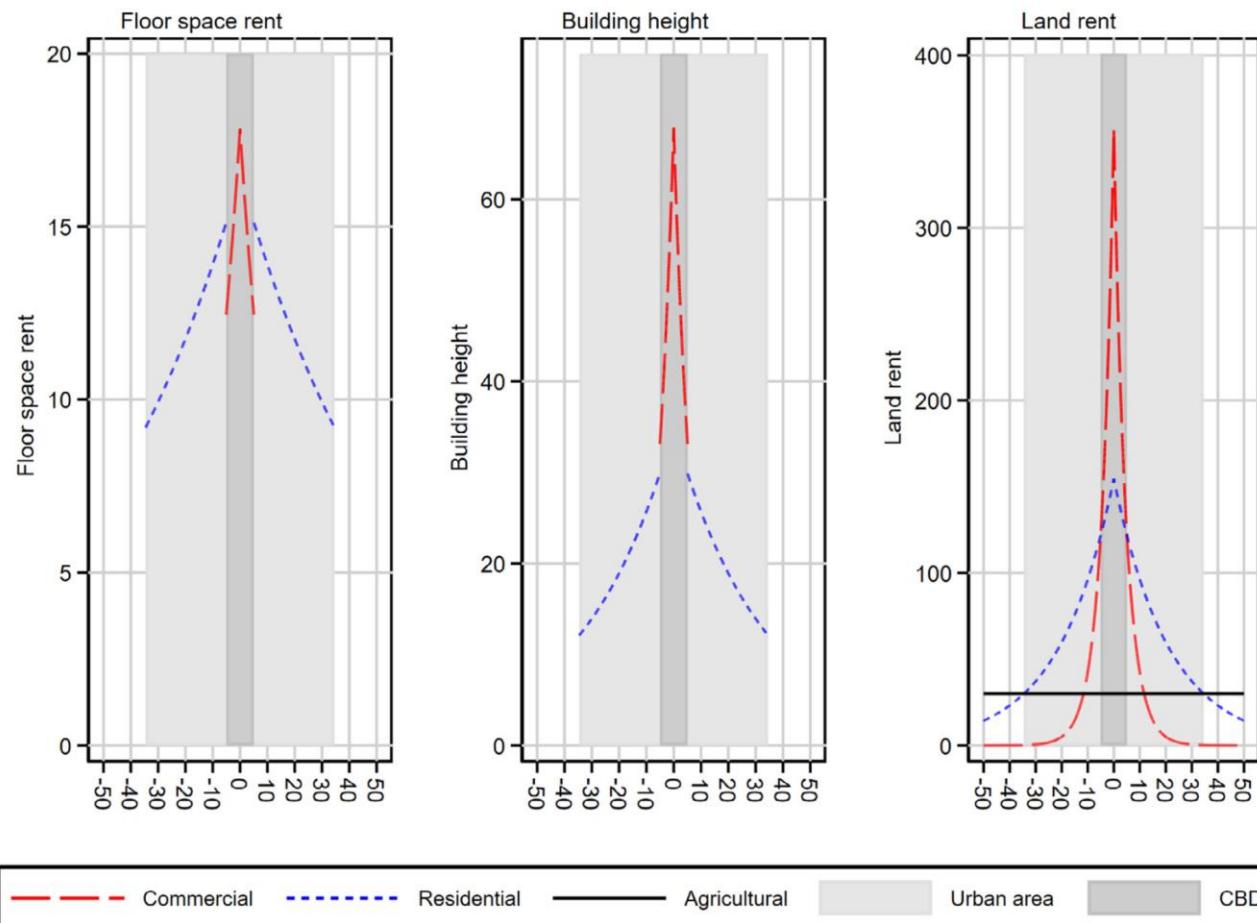
- Returns to height
    - Cost of height

- 3) Equilibrium and counterfactuals

- Urbanization as cause of skyscraper development
    - Skyscraper development as cause of urbanization
    - Height regulation

# V GRADIENTS

Ahlfeldt & Barr (2022)



**Fig. 10.** Urban gradients Note: The figure illustrates the solution to the model laid out in [Section 3](#) using the parameter values reported in [Table 1](#). Floor space rent is the average per-unit rent within a building  $\bar{p}^U(x)$ . The x-axis gives the distance from the historic center in units that roughly correspond to kilometers in real-world cities.

# V EQUILIBRIUM CITY STRUCTURE

Ahlfeldt & Barr (2022)

- **1) Relative floor space rent offsets location advantages**
  - Downward sloping in distance due to commuting costs and decay of agglomeration spillovers
- **2) Heights adjust to floor space rents**
  - Different optimal heights for different uses
- **3) Land rents determined by floor space rent and height**
  - Different land rents for different uses
- **4) Land use is allocated to the highest bidder**
  - Pins down the size of the CBD and the residential zone
- **5) Level of floor space rent adjusts to clear markets**
  - Level adjusts so that aggregate demand = supply, by use

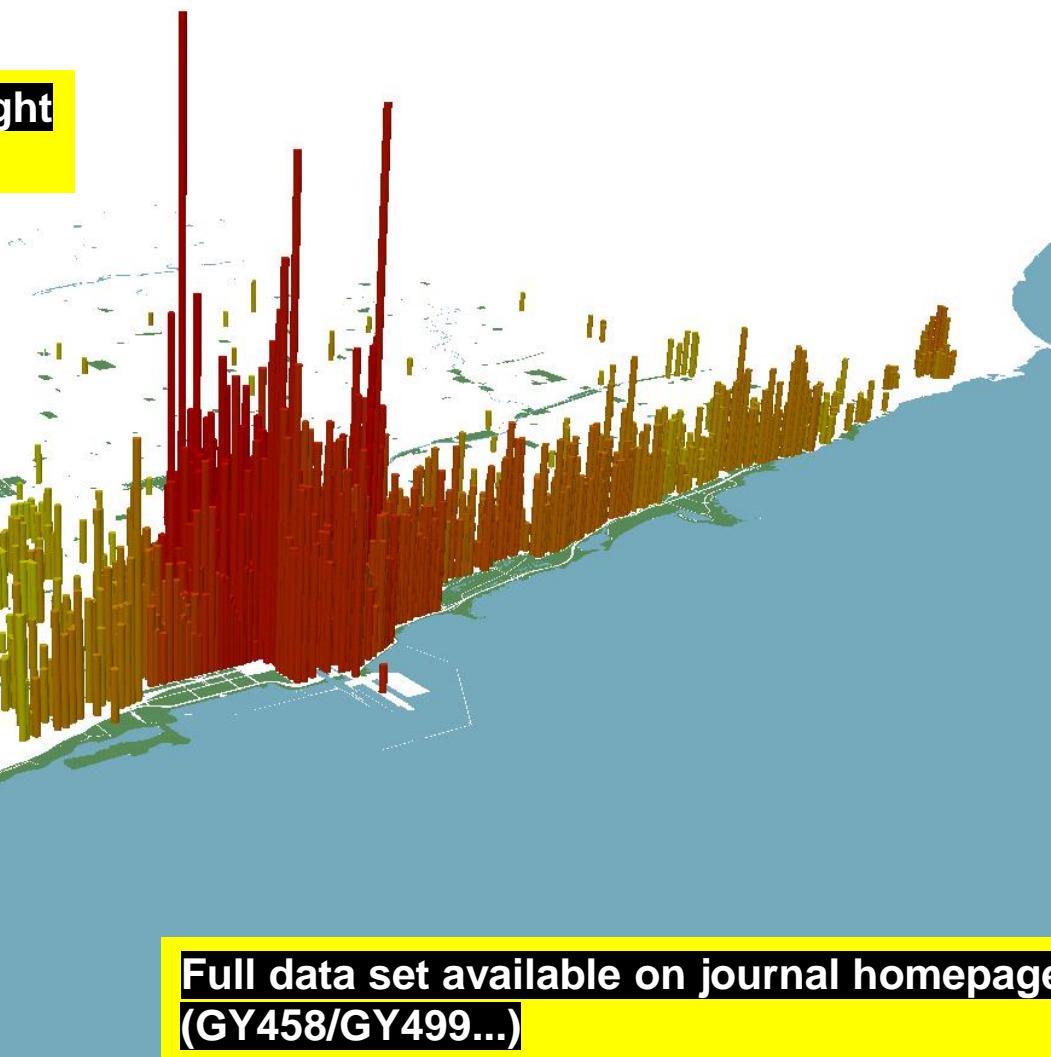
# V LAND PRICE AND BUILDING HEIGHT IN CHICAGO

Ahlfeldt & McMillen (2018)

Bars are proportionate to building height

Color coding based on land price

Heights and land prices follow same  
trend as predicted by the model



# V COUNTERFACTUALS

**Q: Are skyscrapers a cause or an effect of urbanization?**

# V COMPARATIVE STATICS: URBANIZATION

Ahlfeldt & Barr (2022)

- To understand the causal effect of urbanization
  - Solve model under varying values of agglomeration elasticity
  - $\beta = (0.025, 0.035)$
  - Return to living in city increases

**Industrialization, transition to  
knowledge-based tradable  
services economy**

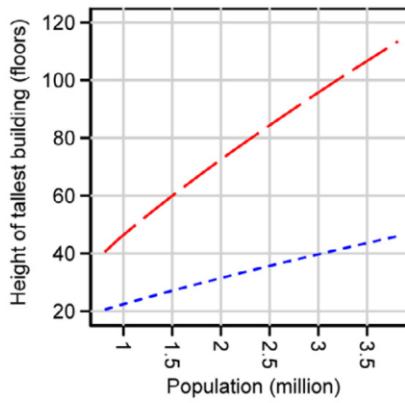
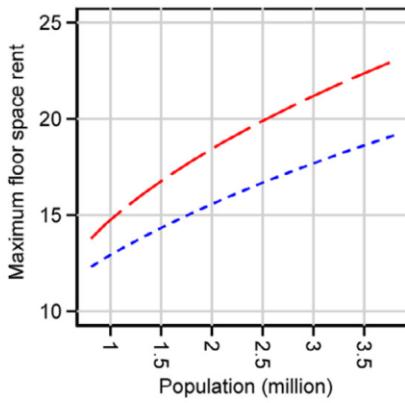
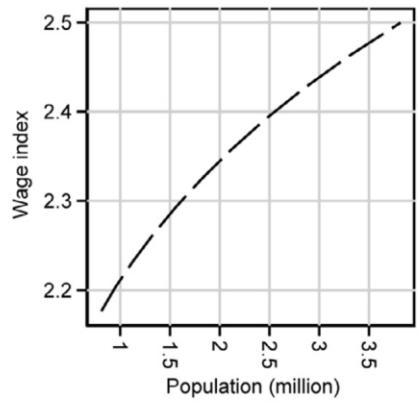
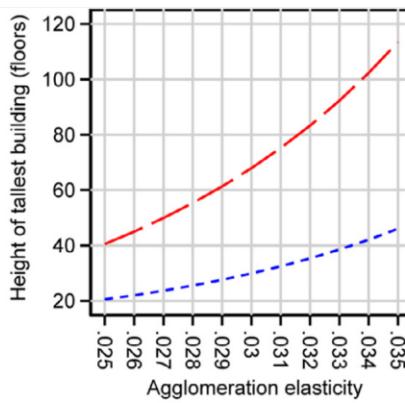
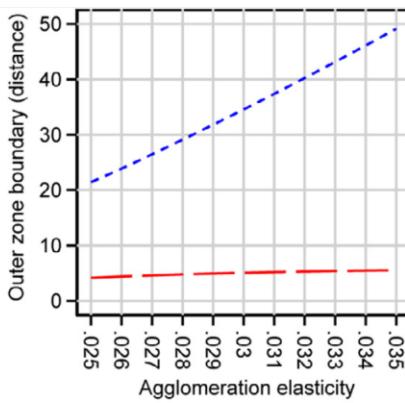
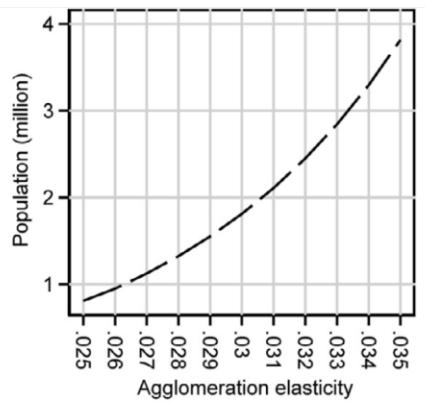
Table 1

Parameter values.

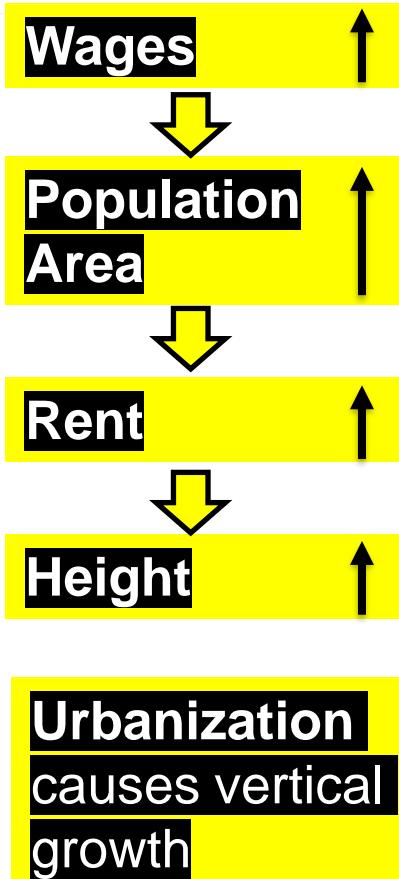
Parameter	Value	Further reading
$1 - \alpha^C$	Share of floor space at inputs	0.15
$1 - \alpha^R$	Share of floor space at consumption	0.33
$\beta$	Agglomeration elasticity of production amenity	0.03
$\theta^C$	Commercial height elasticity of construction cost	0.5
$\theta^R$	Residential height elasticity of construction cost	0.55
$\omega^C$	Commercial height elasticity of rent	0.03
$\omega^R$	Residential height elasticity of rent	0.07
$\tau^C$	Production amenity decay	0.01
$\tau^R$	Residential amenity decay	0.005

# V COMPARATIVE STATICS: URBANIZATION

Ahlfeldt & Barr (2022)



— Commercial      - - - Residential



**Fig. 13.** Variation in external returns to agglomeration Note: We report solutions to the model developed in [Section 3](#) under varying values of the agglomeration elasticity,  $\beta$ . All other parameter values are kept constant at the levels reported in [Table 1](#).

# V COMPARATIVE STATICS: “SKYSCRAPERIZATION”

Ahlfeldt & Barr (2022)

- To understand the causal effect of tall buildings
  - Solve model under varying value of height elasticity of cost
  - $\theta^C = (0.4, 0.6)$  ( $\theta^R$  adjusts to maintain relative difference)
  - Cost of height decreases

Innovations in construction  
technology, elevator, steel frame,  
mainframe computing...

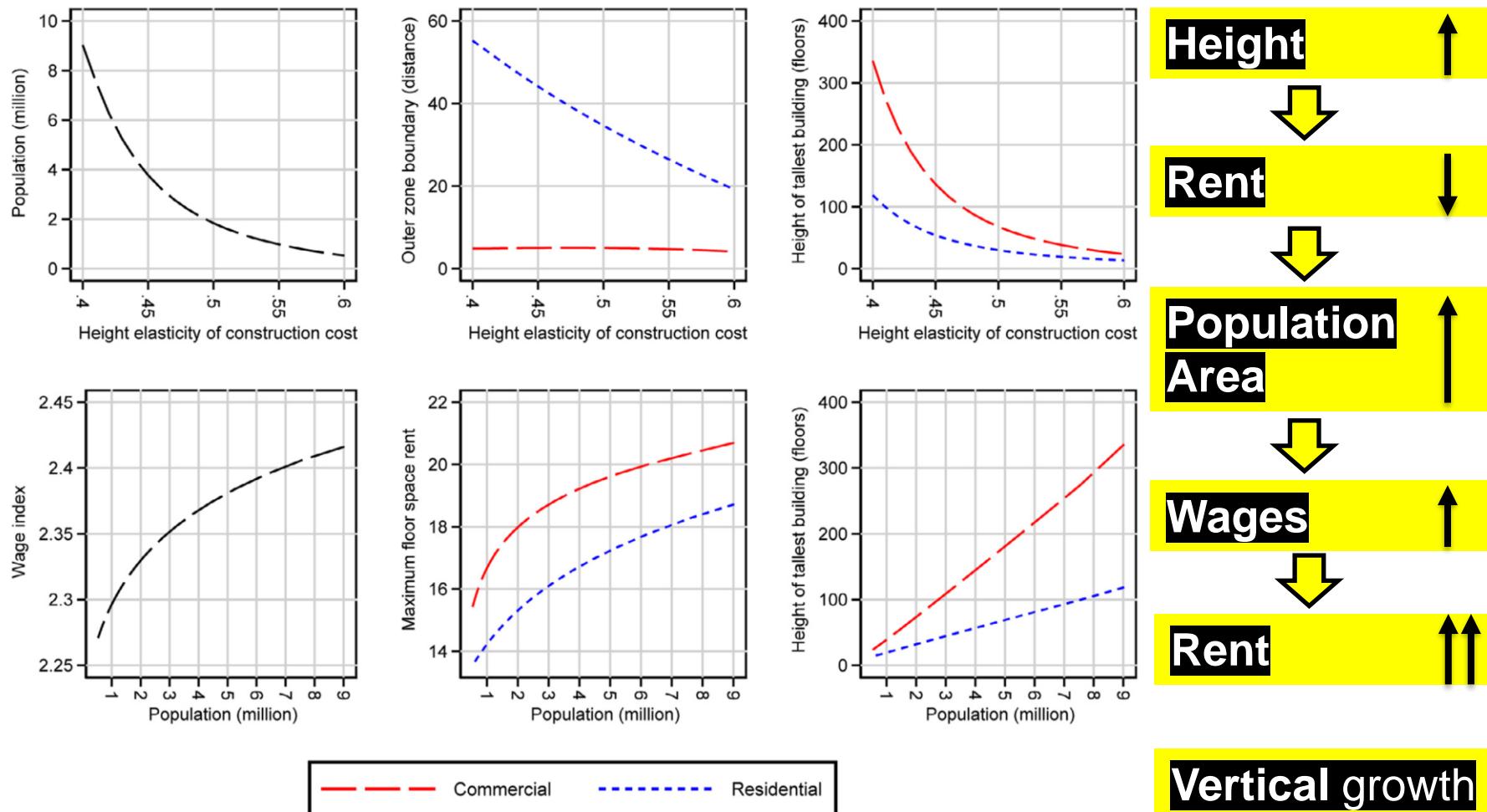
**Table 1**

Parameter values.

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$\tau^C$	Production amenity decay	0.01
$\tau^R$	Residential amenity decay	0.005

# V COMPARATIVE STATICS: “SKYSCRAPERIZATION”

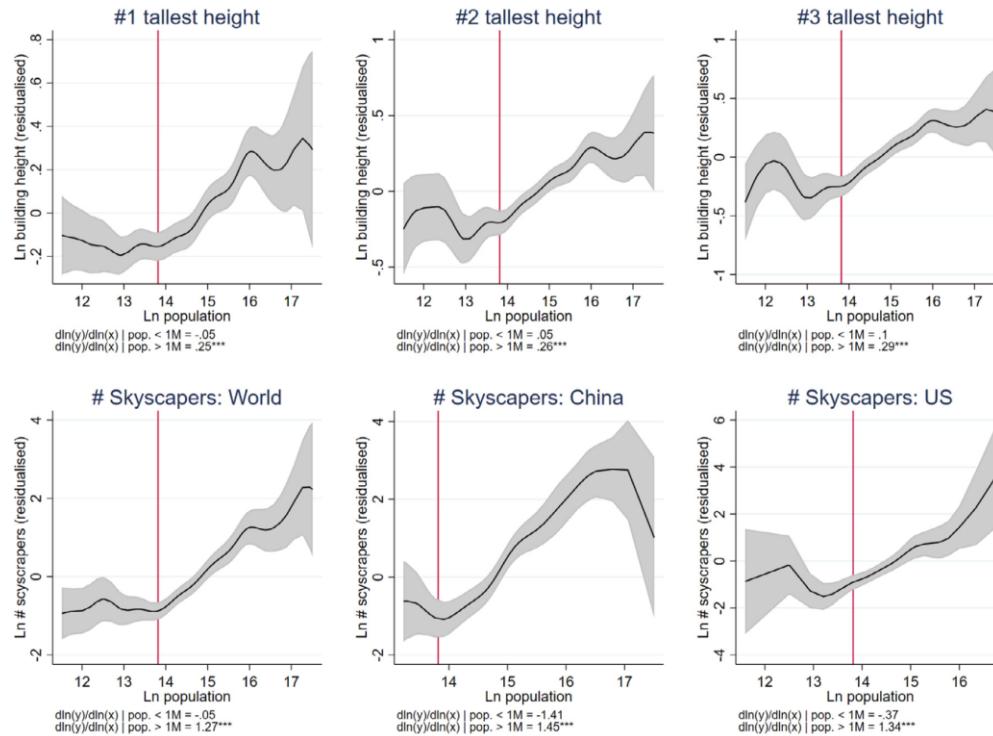
Ahlfeldt & Barr (2022)



**Fig. 15.** Variation in cost of height Note: We report solutions to the model developed in Section 3 under varying values of the amenity decay parameter. We keep the relative size constant at  $\theta^C - \theta^R = 0.05$ . All other parameter values are kept constant at the levels reported in Table 1.

# V URBANIZATION VS. “SKYSCRAPERIZATION”

Ahlfeldt & Barr (2022)



**Fig. 16.** Skyscrapers and city size Note: Skyscrapers and population for cities around with world, with at least one 150 m + skyscraper.  $\ln(\text{building height})$  and  $\ln(\#\text{skyscrapers})$  are residualized in regressions against country fixed effects. The black solid lines are from locally weighted regressions using a Gaussian kernel and a bandwidth of 0.25. Confidence bands are at the 95% level. Vertical line marks the log of one million. Skyscraper data is from <https://www.skyscrapercenter.com/>; accessed Feb. 2020. Population data are from several sources (available upon request) and are the most currently available counts for the metropolitan regions, which includes the central city and surrounding population agglomerations.

**Positive correlation between population originates from demand side (urbanization effect) and supply side (skyscraperization effect)**

# V COUNTERFACTUALS

**Q: What happens if we introduce a binding height limit?**

# V COMPARATIVE STATICS: HEIGHT REGULATION

Ahlfeldt & Barr (2022)

- Developer actually chooses height according to rule:

$$\tilde{S}^U(x) = \min(S^{*U}(x), \bar{S}^U)$$



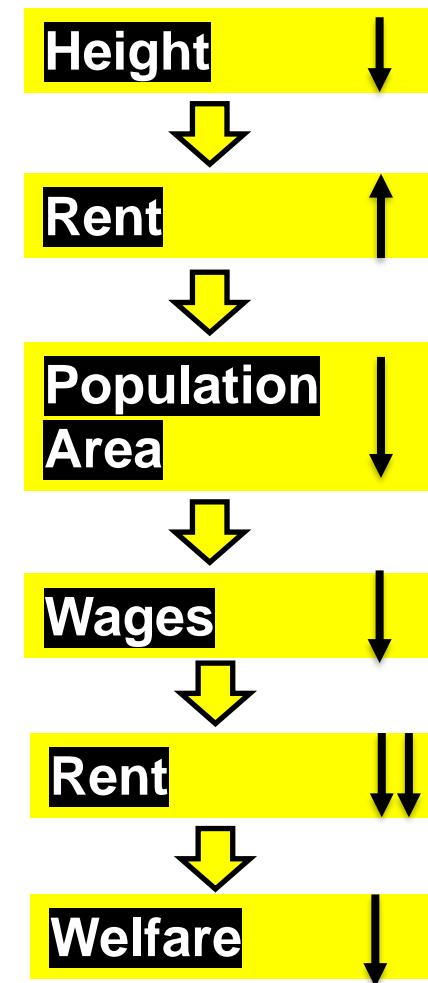
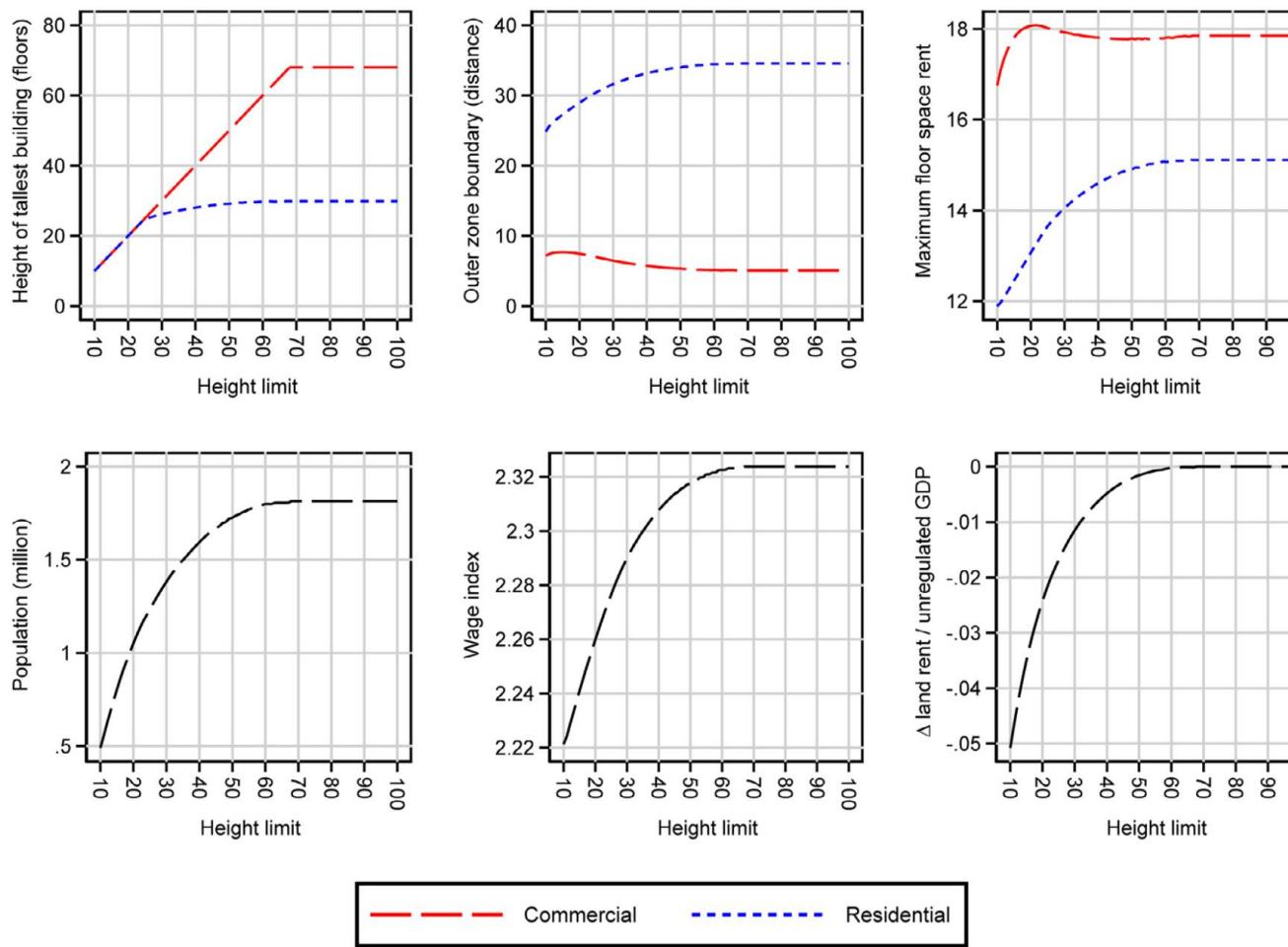
Profit-maximizing height

Height limit

- To simulate the effect of height limits in open-city model
  - Solve the model under varying values of  $\bar{S} = (10, 100)$
  - Compute the welfare effect
    - Incidence on the immobile factor (land)

# V COMPARATIVE STATICS: HEIGHT LIMITS

Ahlfeldt & Barr (2022)



**Fig. 17.** Variation in height limit Note: We report solutions to the model developed in Section 3 under varying values of the height limit,  $\bar{S}^R = \bar{S}^C$ . All other parameter values are kept constant at the levels reported in Table 1. In the bottom-left corner, we measure aggregate land rent over the area occupied by the unregulated city. We do not vary this interval across simulation runs and aggregate over the maximum land bid rent,  $\max(r^C(x), r^R(x), r^a(x))$ , at each location. GDP is the wage bill,  $(yN)$ , weighted by the inverse of the labor factor share,  $\alpha^C$ .

# V REGULATION AND GAMING THE SYSTEM

regulation

- **Many cities impose binding height regulations**
  - Historically to protect views, control density, fire protection!
- **Tension between developers and planners**
  - Developers wish to build taller to make profits
  - Planners are concerned with negative externalities
- **How can developers “convince” planners?**
  - Developers can make some form of **“social” contribution**
    - Social housing
    - Cost of infrastructure
    - Open spaces
  - **Corruption**

**Really depends on the context**

# V THE ROLE OF TROPHY ARCHITECTS

Cheshire & Derricks (2020)

- Profit-maximizing building height in the City of London: 156 floors
  - “Normal” new office building: 8 floors
- To “convince” planners, one can employ “Trophy architects” (TA)
  - Architects who won the most prestigious awards
  - To address planners’ concern about negative visual effects
- Key empirical finding
  - With a TA, planners approve 14 additional floors
  - Huge source of rental revenue!
  - But Trophy architect also adds 13% to building cost  
(C&D refer to Gardiner & Theobald)

From 2020 journal version

# V LSE GLOBAL CENTRE FOR THE SOCIAL SCIENCES

regulation

**C&D argue: TA extracts “scarcity rent”**

**Planner argues: TA generates a positive externality**

**In any case, height restrictions can create significant social cost in form of higher rents (see counterfactuals) or increased commuting cost (Bertaud & Brueckner, 2005)**

**By Roger Stirk Harbour + Partners...**

# SUMMARY

conclusion

- **Vertical rent gradients**

- There is a ground floor premium driven by retail
- Positive price gradient at higher floors driven by offices

- **Can use height elasticities to find profit-maximizing building height**

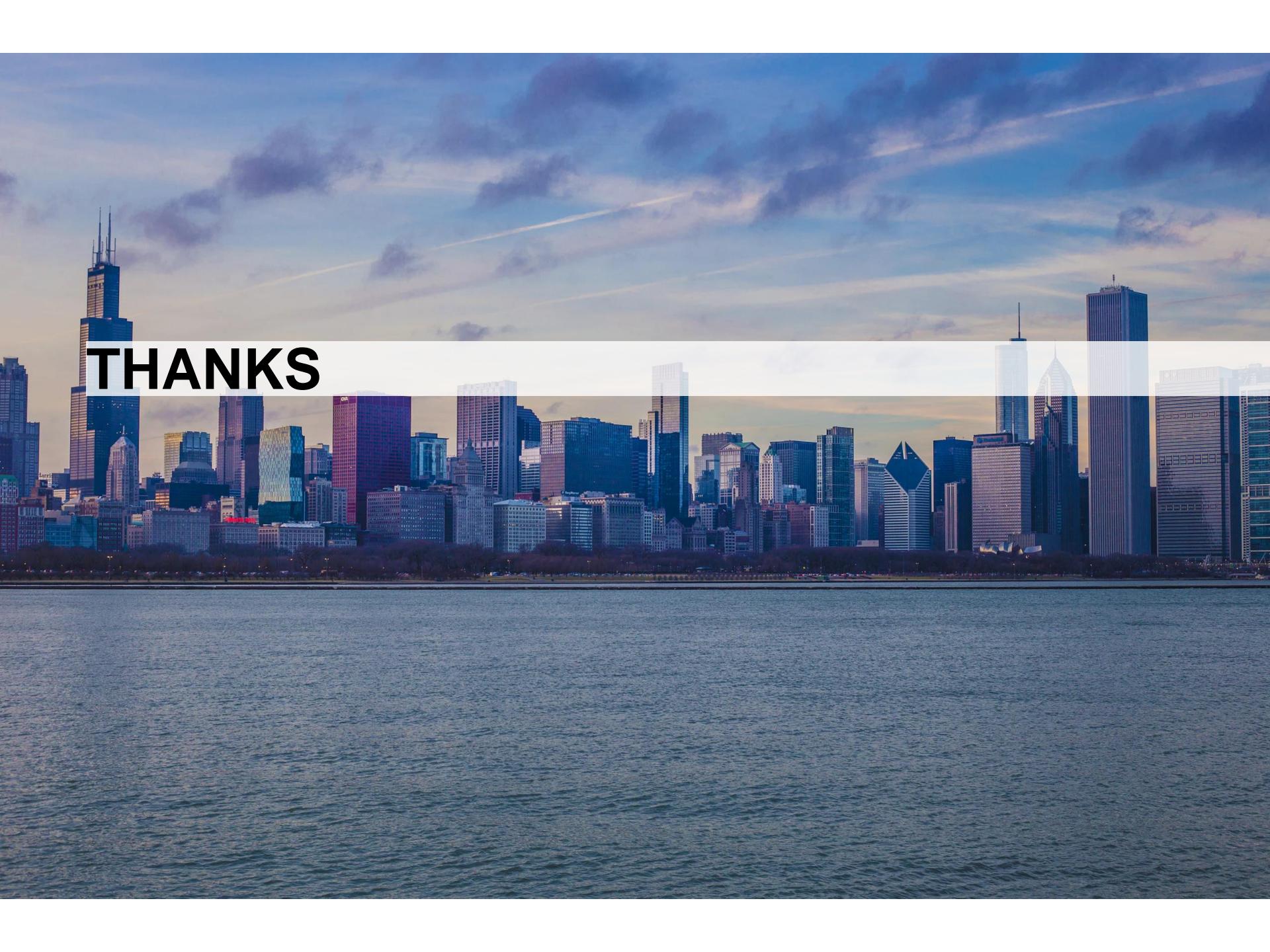
- Marginal average unit cost equals marginal average unit rent
- Determine the maximum bid for a given internal rate of return

- **Spatial equilibrium**

- Skyscrapers are cause and effect of urbanization
- Height regulation leads to welfare loss, but not necessarily higher rents

- **Next: Suburbanization and gentrification**

- Why did cities suburbanize?
- Why are some areas gentrifying?

A panoramic photograph of the Chicago skyline at sunset, viewed across Lake Michigan. The sky is filled with dramatic, colorful clouds ranging from deep purple to bright yellow and orange. The city's iconic skyscrapers, including the Willis Tower (formerly Sears Tower) and the Chicago Spire, stand tall against the horizon. In the foreground, the calm surface of the lake reflects the warm light of the setting sun.

**THANKS**

# READING

- Core readings:
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  - Ahlfeldt, G. M., McMillen, D. P. (2018) Tall Buildings and Land Values: Height and Construction Cost Elasticities in Chicago, 1870 – 2010. *Review of Economics and Statistics*.
  - Liu, Crocker H.; Rosenthal, Stuart S.; Strange, William C. (2018) The vertical city: Rent gradients, spatial structure, and agglomeration economies. *Journal of Urban Economics* 106 101–122
- Complementary readings and references:
  - Barr, Jason. (2010). Skyscrapers and the Skyline: Manhattan, 1895–2004. *Real Estate Economics*, 38(3), 567-597.
  - Barr, Jason. (2012). Skyscraper Height. *The Journal of Real Estate Finance and Economics*, 45(3), 723-753.
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  - Chesire, P.C.; Dericks, G. (2018). ‘Trophy architects’ as deadweight loss: rent acquisition by design in the constrained London office market. Working paper.
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  - Shilton, L., and Zaccaria, A. (1994). The avenue effect, landmark externalities, and cubic transformation: Manhattan office valuation. *The Journal of Real Estate Finance and Economics*, 8(2), 151-165.

### III APPENDIX: HEIGHT ELASTICITY OF AV. RENT

profit-maximizing building heights

- Liu et al (2018) estimate **elasticity of rent with respect to floor**
  - Not the same as elasticity of average rent wrt building height
- Average rent for a building with  $s=1,2,\dots,S$  floors is defined as:

$$p_S = \frac{\bar{p}}{S} \sum_{s=1}^S s^\alpha \quad \text{8.5% rent elasticity (Liu et al 2018)}$$

- Compute  $p_S$  for a series of buildings with  $S=1$  to  $S=50$  floors
- Get  $\omega$  from a simple bivariate regression:

$$\ln p_S = c + \omega \ln S + \varepsilon_S \quad \Rightarrow \quad \begin{array}{l} \text{Height elasticity of average rent} \\ \omega = 7.1\% \end{array}$$