

# Firms and Plants Across Space

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# Introduction

- **Firm location decisions** determine productivity and the set of products across regions
  - ▶ Through them, they determine prices, wages, and other equilibrium outcomes
  - ▶ These decisions are mostly missing in the **quantitative spatial economics** literature
  - ▶ Core models assume local productivities that are not a function of firm location decisions
- Key is that firms own technologies that are, at least partly, not a public good
  - ▶ The use of a firm's technology is not universal but specific to the region where they locate
  - ▶ Important because **firm sorting** affects how local productivity responds to shocks and policies
- Incorporating firm sorting into QSE models is complicated if matching is not one-to-one
- **Firms can choose multiple locations by setting multiple plants**

# A New Industrial Revolution

- The nature of firm sorting is changing because firms are setting plants in many locations (Hsieh and Rossi-Hansberg, 2023)
- Modern production relies on **scale**
  - ▶ Assembly line scaled good production in single plant
  - ▶ Useful for traded products but not for non-traded services
- ICT-based and new management technologies allowed scaling of 'service' production
  - ▶ Replicate similar establishments in multiple locations
  - ▶ **Higher fixed cost** to design model establishment, and link them with information, communication, and distribution systems
  - ▶ **Lower local marginal cost** (and local fixed costs)
- **Where do firms set their plants? What does this imply for local and aggregate productivity? For welfare?**

# Literature

- Assignment in space
  - ▶ **Firm sorting:** Baldwin and Okubo (2006); Nocke (2006); Gaubert (2018); Ziv (2019); Bilal (2023); Lindenlaub et al. (2022); Mann (2023)
  - ▶ **Worker sorting:** Behrens, et al. (2014), Eeckhout et al. (2014), Davis and Dingel (2019), Bilal and R-H (2021), Lhuillier (2024)
  - ▶ **Both:** Hong (2024); Oh (2024)
- Firm with multiple plants
  - ▶ Over time: Luttmer (2011); Cao, et al. (2019); Aghion, et al. (2019)
  - ▶ Across space: **R-H, et al. (2018), Hsieh and R-H (2023)**, Kleinman (2024)
- Multinationals and export platforms
  - ▶ Ramondo (2014), Tintelnot (2017), Arkolakis, et al. (2018), Castro-Vincenzi (2024), Alfaro-Ureña, et al. (2024)
- Solutions to plant location problem
  - ▶ Homogeneous space: **Christaller (1933), Fejes Toth (1953)**, Bollobas (1972)
  - ▶ Numerical: Jia (2008), Holmes (2011), Arkolakis, Eckert, and Shi (2023)
  - ▶ Analytical: **Oberfield et al. (2024)** and **Oberfield et al. (2025)**

# Two Examples of Service Production Scaling

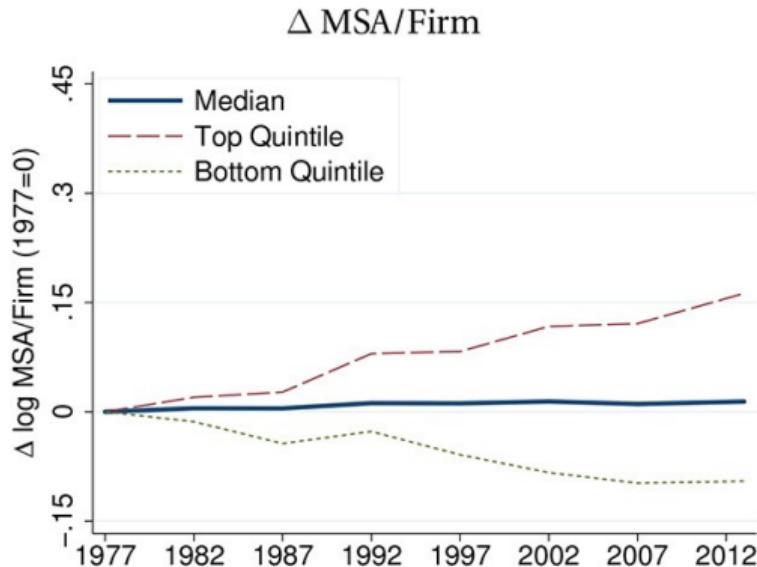
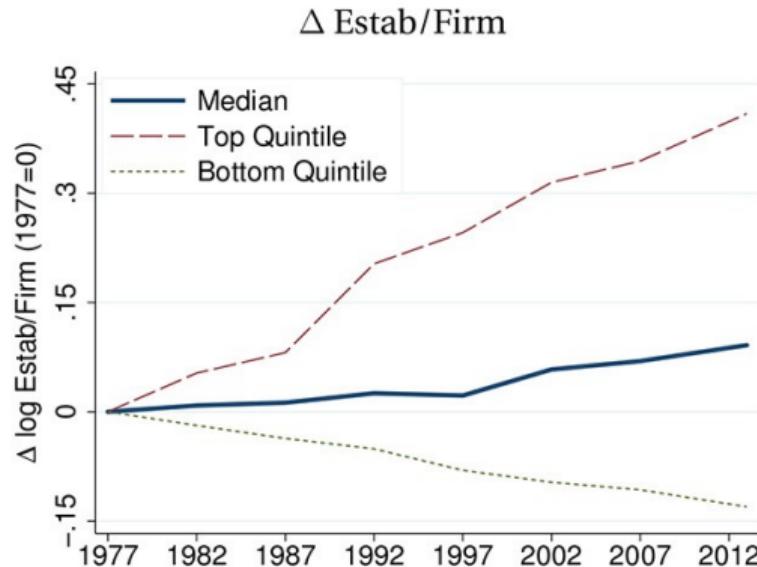
## • The Cheesecake Factory

- ▶ Industry single-establishment employment share: 85% (1977) → 52% (2013)
- ▶ "Chain Production for Sit-Down Meals"
  - ★ Kitchen structured like a manufacturing facility
  - ★ Data-driven staffing and purchasing (< 2% food waste)
  - ★ Centralized training and meal design (Calabasas, CA)
  - ★ Scaled to 210 restaurants

## • Steward Health Care

- ▶ Industry single-establishment employment share: 80% (1977) → 38% (2013)
- ▶ "Standardizing Best Practices"
  - ★ Implemented a common information and communication system
  - ★ Remote ICU allowed doctors to monitor and talk directly to local staff and patients
  - ★ Expanded to 36 hospitals in 9 states (and Malta) by 2019

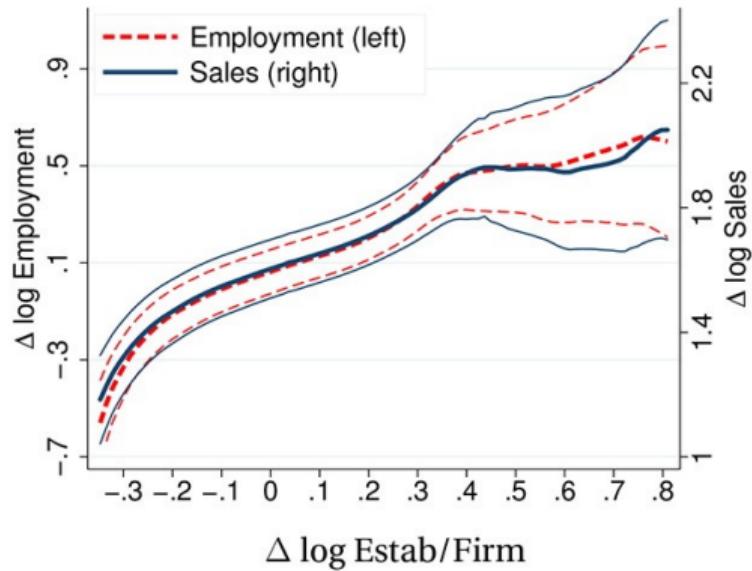
# A Few Facts: Increase in the Number of Markets per Firm



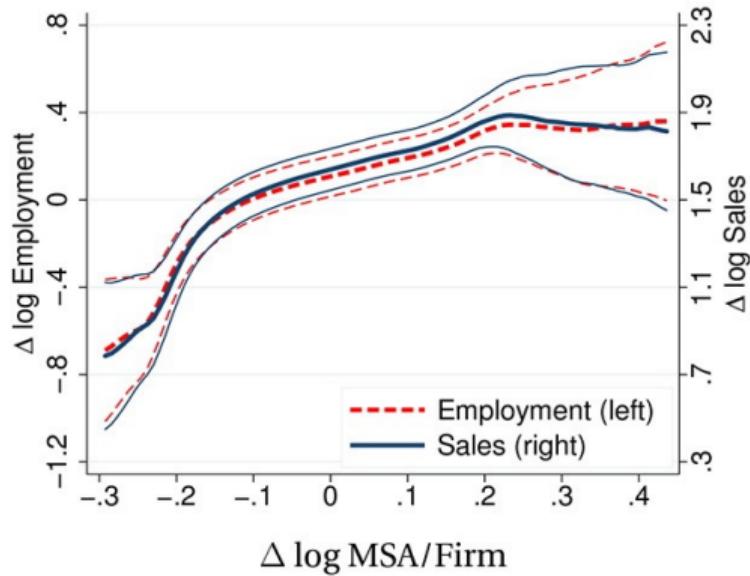
- Expansion varies across sectors [Table](#) [Figure](#)

# A Few Facts: Markets per Firm and Industry Growth

$\Delta \text{Emp}$  and  $\text{Sales}$  vs.  $\Delta \text{Estab/Firm}$



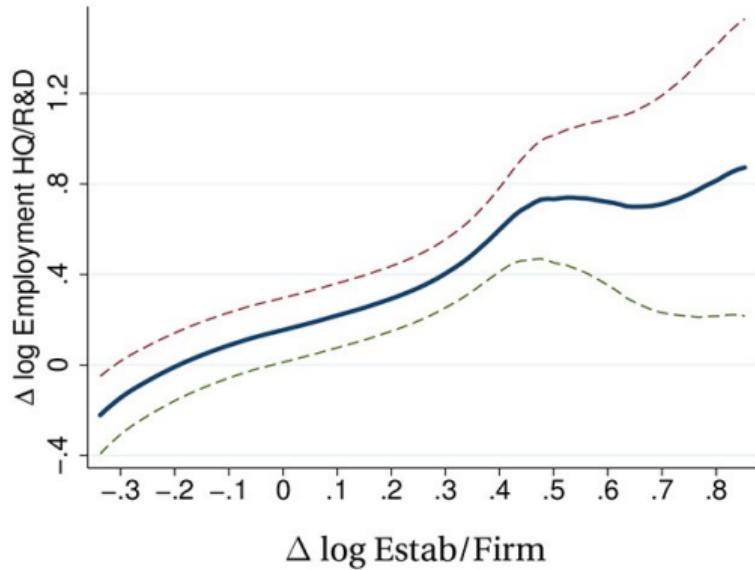
$\Delta \text{Emp}$  and  $\text{Sales}$  vs.  $\Delta \text{MSA/Firm}$



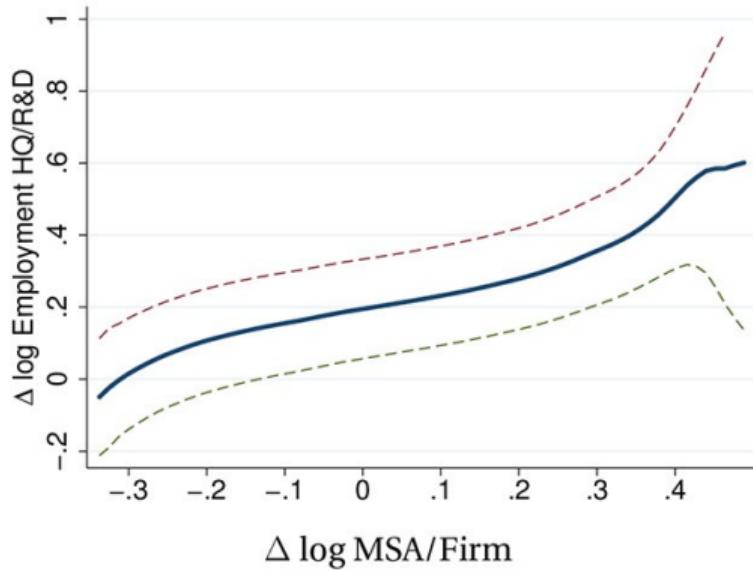
- Regression of Industry Growth on  $\Delta \text{Log Markets per Firm}$  Table

# A Few Facts: Growth in Fixed Costs

$\Delta \text{Emp in HQ/R&D}$  by  $\Delta \text{Estab/Firm}$



$\Delta \text{Emp in HQ/R&D}$  by  $\Delta \text{MSA/Firm}$

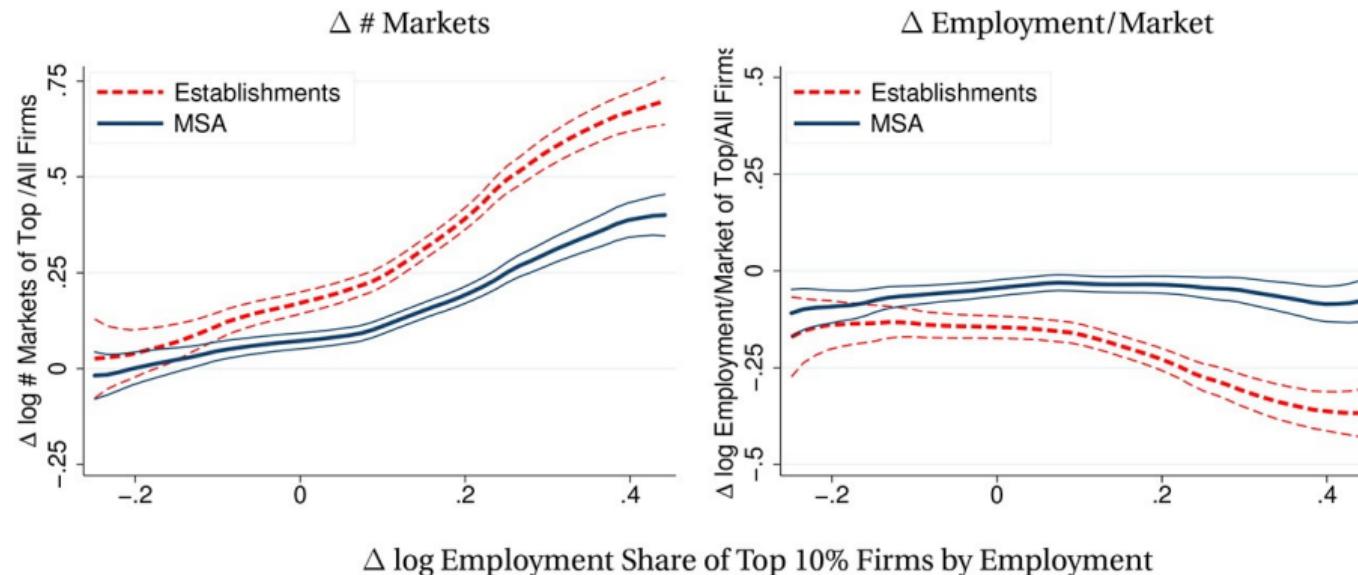


- Regression of  $\Delta \text{Log Employment in HQ/R&D}$  on  $\Delta \text{Log Markets per Firm}$ , 1977-2013 Table

# A Few Facts: Top Firms Expand by Entering New Markets

- Decomposing the rise in top firm share:

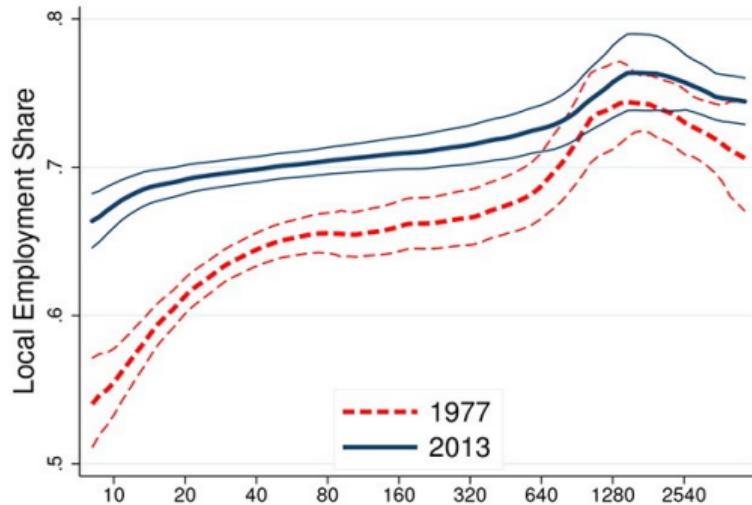
$$\Delta \log \frac{L_{\text{top}}}{L} = \underbrace{\Delta \log \frac{\#\text{MSA}_{\text{top}}}{\#\text{MSA}}}_{\text{Growth in the number of MSAs}} + \underbrace{\Delta \log \frac{L_{\text{top}}/\#\text{MSA}_{\text{top}}}{L/\#\text{MSA}}}_{\text{Changes in employment per MSA}}.$$



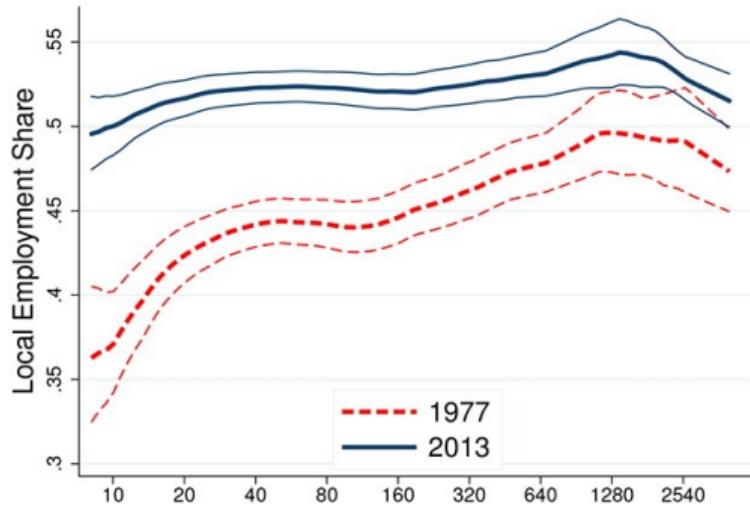
Table

# A Few Facts: Top Firms Expand to Smaller Markets

Top 10% Firms by Employment



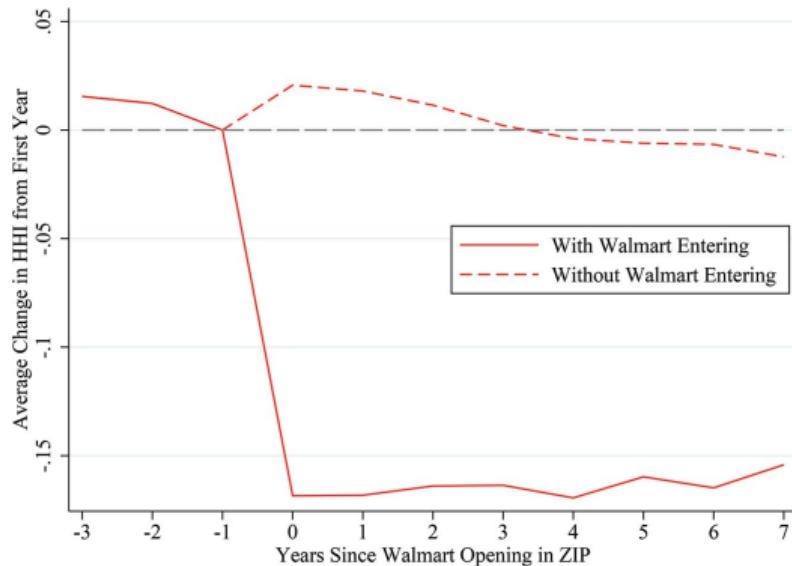
Top 10% Firms by # Establishments



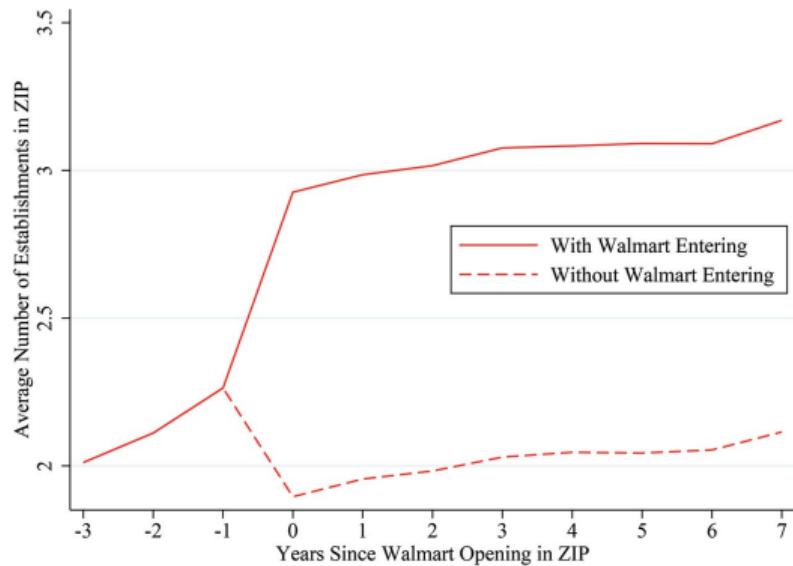
Employment (in thousands) in MSA in 1977

- Change in Market Size of Top-10% Firms Table

# A Few Facts: Concentration When Walmart Enters a Local Market



Average Change in HHI Before and After Walmart Openings; Within SIC8  
53119901, Discount Department Stores



Average Number of Estab. Before and After Walmart Openings; Within  
SIC8 53119901, Discount Department Stores

# A "Melitzian" Theory of Firms with Multiple Establishments

- Agents live in locations,  $s$ , with CES preferences over varieties produced by 'local' firms
  - Demand:  $p_{js} = E_s Y_{js}^{-1/\epsilon}$  where  $Y_{js}$  denotes the firm's output in  $j$
  - $E_s$  summarizes local industry expenditure and the local industry price index
- Firm  $j$ 's profit from producing nontraded service

$$\Pi_j = \max_{N_j, L_{js}} \int_{S_j} [p_{js} a_{js} A_j L_{js} - w_s (L_{js} + f)] ds - F$$

- $S_j$ : Set of markets firm  $j$  enters
- $A_j$ : Firm-wide productivity across locations
- $a_{js}$ : Idiosyncratic market-specific productivity
- $L_{js}$ : Labor as the only factor of production
- $f$ : Establishment local fixed cost in market  $s$ , indexed by local wage  $w_s$
- $F$  firm fixed cost in units of numeraire
- Can also add multiple industries

# Firm's Decisions

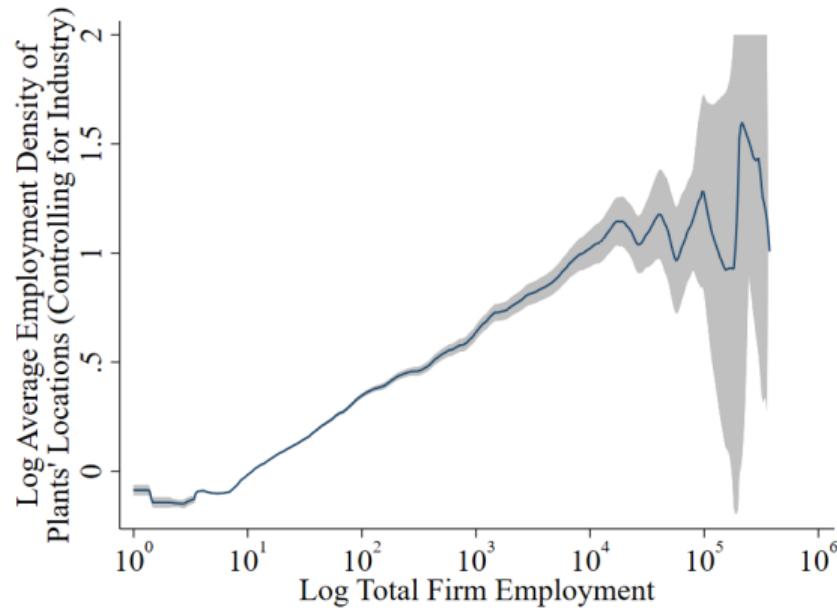
- Firm  $j$  will serve market  $s$  if local profits are positive
  - ▶ Serve market  $n$  if firm productivity,  $A_j$ , is **above a threshold**

$$A_j \geq \alpha \left( \begin{matrix} a_{js} \\ - \\ f \\ + \\ w_s \\ + \\ E_s \\ - \end{matrix} \right) \equiv \left( \frac{f}{\tilde{\epsilon} a_{js}^\epsilon w_s^{1-\epsilon} E_s^\epsilon} \right)^{1/(\epsilon-1)}$$

- ▶ **Hence, although idiosyncratic, larger markets ( $D_s$ ) are served by most and smaller ones mostly by top firms**
- Model 'New Industrial Revolution' as a menu of technologies,  $h \geq 1$ , such that
  - ▶ The fixed cost of the firm increases to  $h^\eta F \geq F$
  - ▶ The productivity of the firm increases to  $hA_j \geq A_j$
  - ▶ Implies that top firms adopt a higher  $h$
- **Model is consistent with many of the previous facts, but ...**

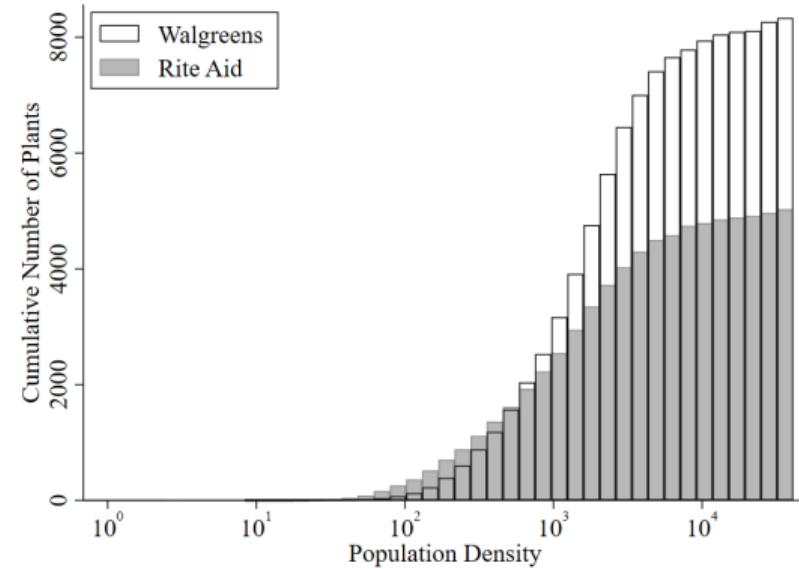
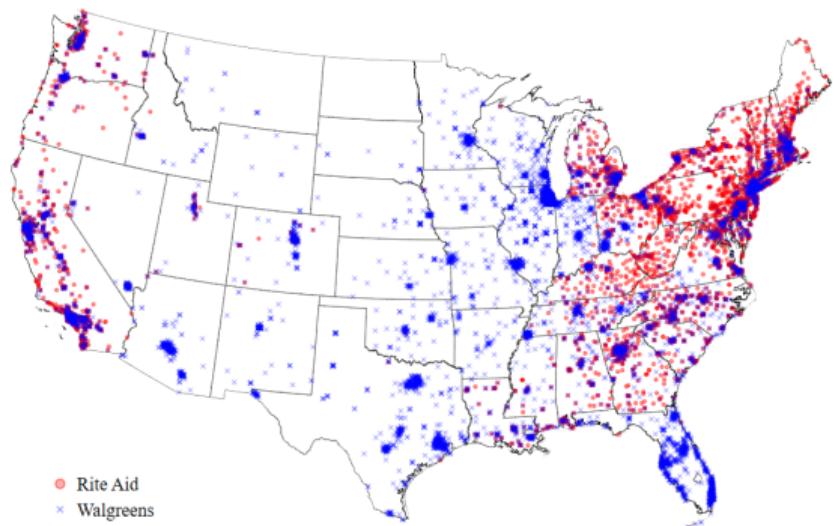
## Sorting in the Data: Average Density and Firm Size

- Compute average weighted density of the locations of a firm  $j$  as  $\bar{L}_j \equiv \sum_s \omega_{js} \mathcal{L}_s$ 
  - ▶  $\mathcal{L}_s$  denotes population density in location  $s$
  - ▶  $\omega_{js}$  denotes the share of plants of firm  $j$  in  $s$  (similar for employment or sales)



# An Example: Pharmacies

- Consider the locations of **Walgreens** and **Rite Aid** pharmacies
- Walgreens, the larger firm, has **more** plants in denser locations, but **fewer** plants in less dense ones



# Sorting in the Data: Average Density and Firm Size by Industry

By major industry	All	Manufacturing	Services	Retail Trade	FIRE
	$\ln \bar{L}_j$	$\ln \bar{L}_j$	$\ln \bar{L}_j$	$\ln \bar{L}_j$	$\ln \bar{L}_j$
$\ln L_j$	0.165*** (0.000975)	0.0523*** (0.00272)	0.160*** (0.00153)	0.150*** (0.00247)	0.234*** (0.00320)
Observations	3,670,994	274,478	1,479,391	856,860	244,048
R-squared	0.139	0.192	0.097	0.068	0.122
SIC8 FE	Yes	Yes	Yes	Yes	Yes
M	12	12	12	12	12

Robust standard errors in parentheses

\*\*\* p<0.01, \*\* p<0.05, \* p<0.1

Robustness

- Hence, firm/plant sorting is **inconsistent with “Melitzian” theory**
  - ▶ Top firms select into largest/densest markets and less productive firms into smaller ones
- **Need a new theory of firm and plant sorting in space!**
  - ▶ One where decisions on local entry are not independent

# Plants in Space

- Customers distributed across locations  $s \in \mathcal{S} = [0, 1]^2 \subset \mathbb{R}^2$
- Each location  $s$  characterized by
  - ▶ Exogenous local productivity,  $B_s$
  - ▶ Residual demand,  $E_s(p) = E_s p^{-\epsilon}$ , with  $\epsilon > 1$ 
    - ★ Later, monopolistic competition:  $D_s$  a function of local price index, local population
  - ▶ Wage rate,  $W_s$
  - ▶ Commercial rent,  $R_s$
- Firms take local equilibrium as given

## Firms

- Each firm  $j \in J$  produces a unique variety
- Chooses set of locations  $O_j \in \mathcal{S}$  where to produce
  - ▶ Let  $N_j = |O_j|$ , denote the number of locations where  $j$  produces
- Firm productivity in location  $o \in O_j$  is  $B_o Z(A_j, N_j)$ 
  - ▶ where  $A_j$  is an exogenous component of firm productivity
  - ▶ and  $Z_N(A_j, N_j) < 0$ ,  $Z(A_j, 0) < \infty$  (**Span-of-control costs**)
- Each plant takes  $\xi$  units of commercial real estate, with rental cost  $R_s$  per unit of space
- Iceberg cost,  $T(\delta)$ , to deliver good to customer at distance  $\delta$

# The Firm's Problem

- Minimal cost of delivering one unit to  $s$  is  $\Lambda_{js} (O_j) \equiv \min_{o \in O_j} \frac{W_o T(\delta_{so})}{B_o Z(A_j, N_j)}$
- Optimal price solves  $\max_{p_{js}} E_s(p_{js}) (p_{js} - \Lambda_{js})$
- Total profit of firm  $j$  is then given by

$$\pi_j = \max_{O_j} \left\{ \int_s \max_{p_{js}} E_s p_{js}^{-\epsilon} (p_{js} - \Lambda_{js} (O_j)) ds - \sum_{o \in O_j} R_o \xi \right\}$$

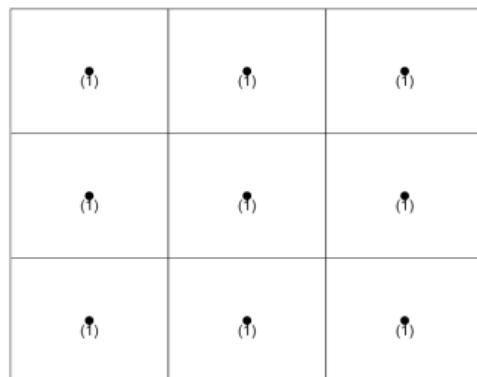
- **Catchment area of plant  $o$ :** all locations  $s$  served by plant in  $o$

# Catchment Areas

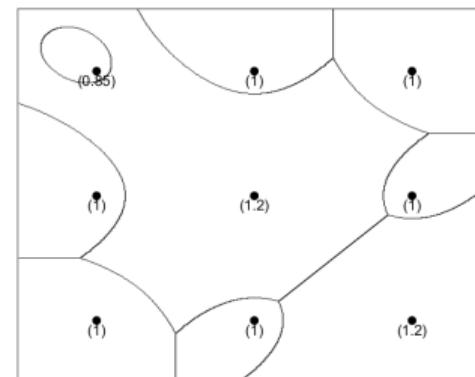
- Given plant locations, catchment areas only depend on  $T(\delta_{so})$  and  $B_o/W_o$

$$CA(o) = \left\{ s \in \mathcal{S} \text{ for which } o = \arg \max_{\tilde{o} \in O_j} \left\{ \frac{B_{\tilde{o}}/W_{\tilde{o}}}{T(\delta_{s\tilde{o}})} \right\} \right\}$$

- Example:  $T(\delta_{so}) = 1 + \delta_{so}$



$$B_o/W_o = 1 \forall o$$



$$B_o/W_o \text{ vary with } o$$

# A Limit Case

- In general, placement of plants in space is a hard problem
  - ▶ Catchment areas depend on local characteristics of plant locations
  - ▶ Plant locations depend on the whole distribution of demand across space
- Our approach is to study a limit case in which firms choose to have many plants, with small catchment areas
  - ▶ Consider an environment indexed by  $\Delta$ , in which

$$\xi^\Delta = \Delta^2$$

$$T^\Delta(\delta) = t\left(\frac{\delta}{\Delta}\right)$$

$$Z^\Delta(A, N) = z(A, \Delta^2 N)$$

- ▶ Study limit case as  $\Delta \rightarrow 0$
- **Tradeoffs** between the fixed and span-of-control costs of setting up plants and the cost of reaching consumers **remain relevant**
  - ▶ Plants continue to cannibalize each other's customers
  - ▶ But forces will apply at local level

# The Core Result

**Proposition:** Suppose that  $R_s$ ,  $D_s$ , and  $B_s/W_s$  are continuous functions of  $s$ . Then, in the limit as  $\Delta \rightarrow 0$ ,

$$\pi_j = \sup_{n: S \rightarrow \mathbb{R}^+} \int_s \left[ x_s z \left( A_j, \int n_{\tilde{s}} d\tilde{s} \right)^{\epsilon-1} n_s g(1/n_s) - R_s n_s \right] ds$$

where  $x_s \equiv \frac{(\epsilon-1)^{\epsilon-1}}{\epsilon^\epsilon} E_s (B_s/W_s)^{\epsilon-1}$  and where  $g(u)$  is the integral of  $t(\cdot)^{1-\epsilon}$  over the distances of points to the center of a **regular hexagon** with area  $u$

- $x_s$  combines local demand and effective labor costs
- $\kappa(n) \equiv ng\left(\frac{1}{n}\right)$  represents the **local efficiency of distribution**
- In the limit:
  - ▶ Maximum profits are attained by placing plants so that catchment areas are, locally, uniform infinitesimal hexagons
  - ▶ **Firm's problem is one of calculus of variations which is much simpler**

Elements of Proof

# The Local Efficiency of Distribution

- We can write the firm's problem in the limit case as

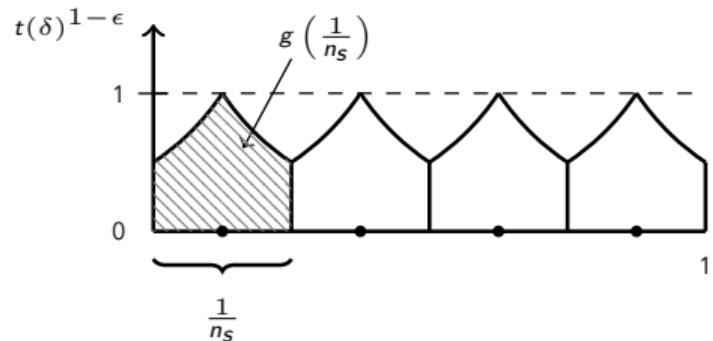
$$\pi_j = \sup_{N_j, n_j: S \rightarrow \mathbb{R}^+} \int_s [x_s z(A_j, N_j)^{\epsilon-1} \kappa(n_{js}) - R_s n_{js}] ds, \text{ s.t. } N_j = \int_s n_{js} ds$$

- The local efficiency of distribution,  $\kappa(n) \equiv ng\left(\frac{1}{n}\right)$ , is

- $\kappa(0) = 0$
- Strictly increasing and strictly concave
- $\lim_{n \rightarrow \infty} \kappa(n) = 1$  (**Saturation**)
- $1 - \kappa(n) \underset{n \rightarrow \infty}{\sim} n^{-1/2}$  (**Asymptotic power law**)

- If, additionally,  $\lim_{\delta \rightarrow \infty} T(\delta) \delta^{-4/(\epsilon-1)} = \infty$ , then

- $\kappa''(0) = 0$
- $\kappa'(0) < \infty$  (**No INADA condition**)



# FOCs and Span-of-Control

- The problem in the limit case is

$$\pi_j = \sup_{N_j, n_j: \mathcal{S} \rightarrow \mathbb{R}^+} \int_s [x_s z(A_j, N_j)^{\epsilon-1} \kappa(n_{js}) - R_s n_{js}] ds, \text{ s.t. } N_j = \int_s n_{js} ds$$

- Differentiating with respect to the number of plants in  $s$ ,  $n_{js}$ , we obtain

$$x_s z(A_j, N_j)^{\epsilon-1} \kappa'(n_{js}) \leq R_s + \lambda_j, \quad \text{with } = \text{ if } n_{js} > 0 \quad (n_{js} \text{ FOC})$$

- Lagrange multiplier,  $\lambda_j$ : **marginal span-of-control cost** of firm  $j$

$$\lambda_j = -\frac{dz(A_j, N_j)^{\epsilon-1}}{dN_j} \int_s x_s \kappa(n_{js}) ds \quad (N_j \text{ FOC})$$

# Sorting and Span-of-Control

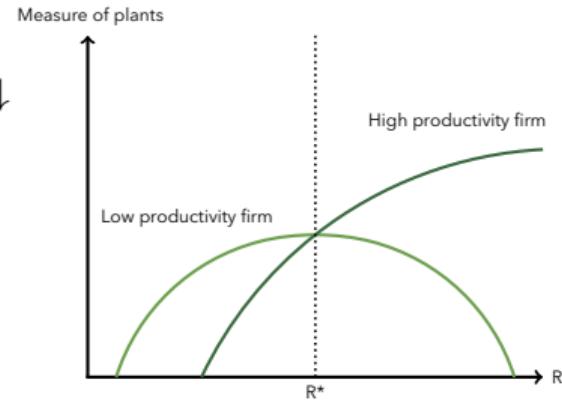
- In “**Melitzian**” models, more productive firms enter more marginal markets
- Here, less productive firms have more plants in worse locations. **Why?**
  - ▶ Productive firms have more profits per plant, but also larger effective fixed costs

$$x_s z(A_j, N_j)^{\epsilon-1} \kappa'(n_{js}) = R_s + \underbrace{\lambda_j}_{\text{effective fixed cost}}$$

- ▶ High productivity firms are less sensitive to rents since

$$\lambda_j \uparrow \Rightarrow \frac{d \ln(R_s + \lambda_j)}{d \ln R_s} \downarrow$$

so they sort into high-rent locations



Proposition

# Industry Equilibrium

- Consider an infinitesimal industry with many firms
  - ▶ Dixit-Stiglitz preferences, monopolistic competition
  - ▶ Firms take as given distribution of population, rent, wages, activity of other industries
- **Local price index** for the industry is

$$P_s \equiv \left( \int_j p_{js}^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}} = \frac{\epsilon}{\epsilon - 1} \frac{W_s}{B_s Z_s}$$

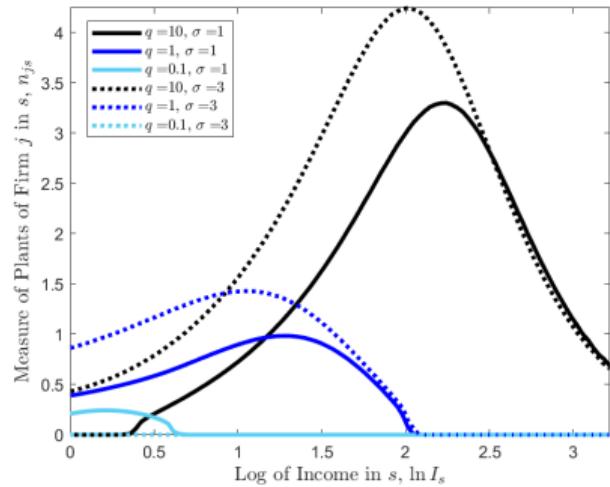
where **local productivity** is

$$Z_s \equiv \left( \int_j z(A_j, N_j)^{\epsilon-1} \kappa(n_{js}) dj \right)^{\frac{1}{\epsilon-1}}$$

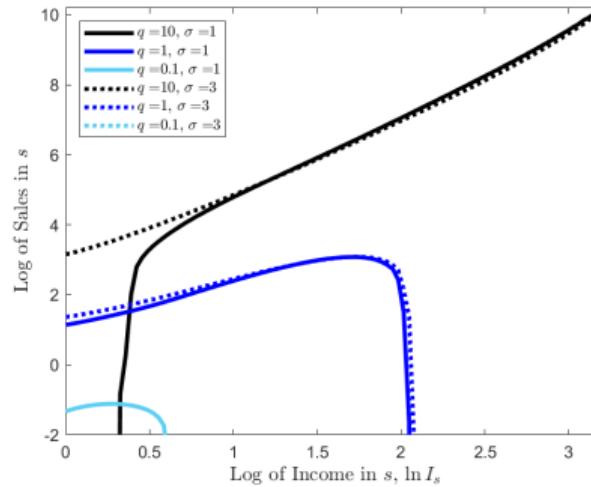
- In the background, GE: People freely mobile, land used for housing or commercial real estate, location characterized by exogenous amenities and productivity

# Improvements in Span-of-Control: $z(q, N) = qe^{-N/\sigma}$

Plants



Sales



- **Top firms** expand to low income locations
  - ▶ Also, reduce presence in top locations due to competition
- **Intermediate firms** have larger presence in worse locations and **bottom firms** exit

# The Case of Banks: The Bank's Problem

- The bank's choice set is  $\mathcal{C}_j \equiv \{n_j : \mathcal{O} \rightarrow \mathbb{R}_+, \bar{Q}_j^D, \bar{Q}_j^L, r_j^D, r_j^L\}$

$$\pi_j = \sup_{\mathcal{C}_j} \underbrace{(r_j^L - \theta_j^L)L_j - (r_j^D + \theta_j^D)D_j}_{\text{net revenue from retail branching}} - \underbrace{F(W_j/D_j) \times W_j}_{\text{wholesale funding costs}}$$
$$- \underbrace{\int_{\mathcal{O}} \psi_s n_{js} ds}_{\text{branching costs}} - \underbrace{w_{s_j^{HQ}} h \left( \int_{\mathcal{O}} n_{js} ds \right)}_{\text{span of control costs}} - \underbrace{w_{s_j^{HQ}} C(\bar{Q}_j^D, \bar{Q}_j^L)}_{\text{appeal investment costs}}$$

$$\text{with } D_j \equiv \int_{\mathcal{O}} E_s^D Q_{js}^D \kappa^D(n_{js}) \mathcal{G}^D(r_j^D) ds \text{ and } L_j \equiv \int_{\mathcal{O}} E_s^L Q_{js}^L \kappa^L(n_{js}) \mathcal{L}^D(r_j^L) ds$$

- As before,  $\kappa^x(n_{js})$  are known functions that depend on travel costs
- Location-specific bank appeal is  $Q_{js}^x \equiv \bar{Q}_j^x J_{jl}^x \phi_{jl}$  for  $x = \{D, L\}$
- Local competition indices are determined by  $E_s^x \equiv \bar{E}_s^x / P_s^x = \bar{E}_s^x \left( \int_{j \in \mathcal{J}} Q_{js}^x \kappa^x(n_{js}) \mathcal{G}^x(r_j^x) dj \right)^{-1}$

# The Case of Banks: What Determines Location Choices?

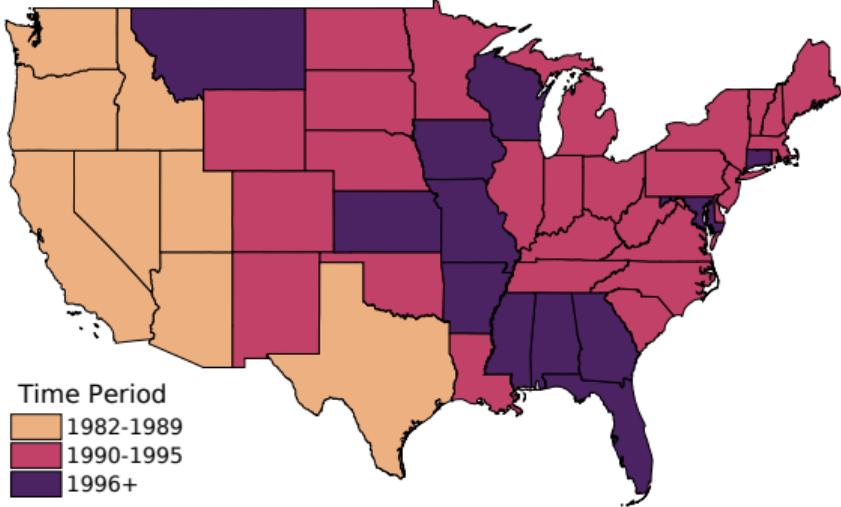
Previous formulation already imposes the following result:

- **Lemma:** Bank  $j$  chooses  $r_j^D$  and  $r_j^L$  and sets  $r_{jo}^D = r_j^D$  and  $r_{jo}^L = r_j^L$  for all  $o \in O_j$

Location choices are determined by:

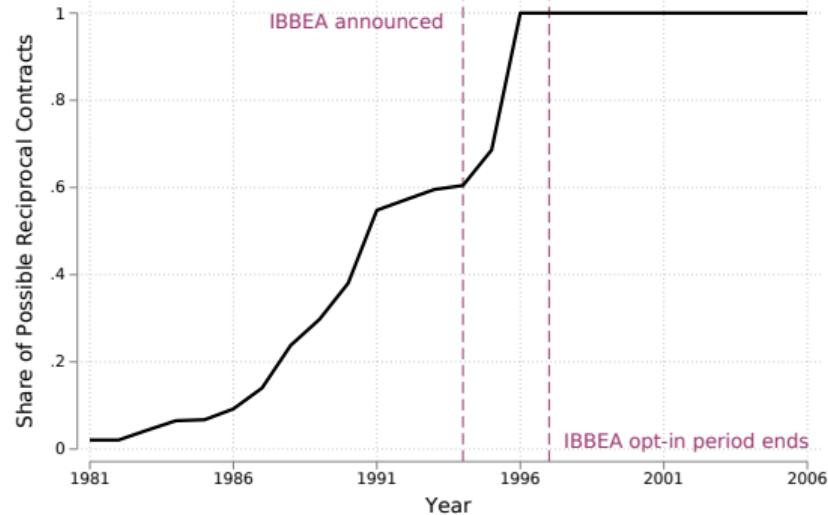
1. **Span-of-control sorting:** "Productive" banks sort toward larger but more costly locations
2. **Mismatch sorting:** High WFE banks sort toward (relatively) deposit-abundant locations
3. **Appeal investments and branch spillovers:** returns to scale, specialization vs. mismatch
4. **Distance to headquarters:** bank appeal is higher when closer to HQ

# The Case of Banks: Geographic Deregulation



## Example: California Agreements

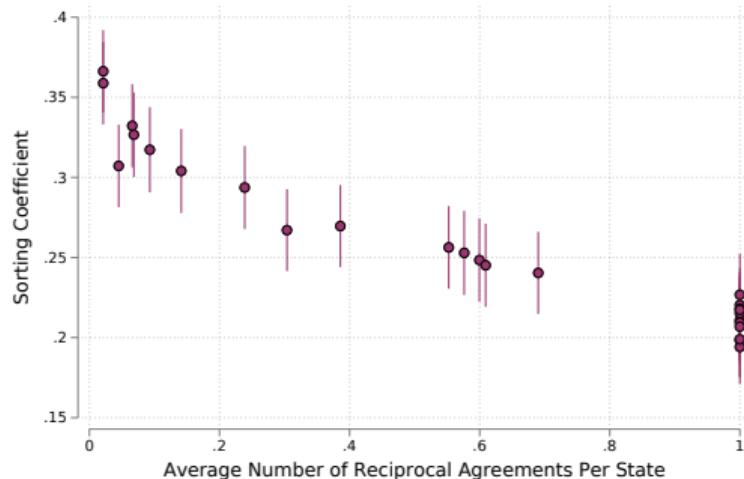
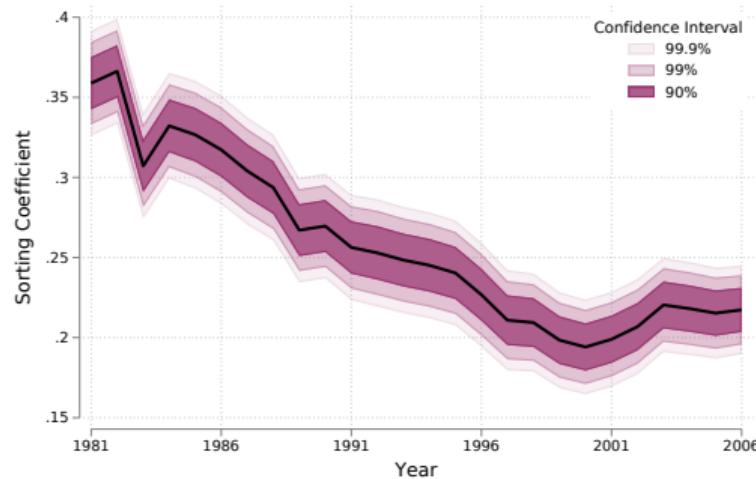
- Bank Holding Company Act of 1956: Set up branches only within HQ state
- Reciprocal agreements among some states: Starts in 1982 with Maine and NY
- Riegle-Neal Act of 1994/1997: Full opening



## Reciprocal Inter-State Agreements

# Deregulation and Sorting Over Time

Test of Span-of-Control Sorting:  $\log(\text{Density}_{jst}) = \beta_t \text{Size}_{jt} + \gamma_{st} + \varepsilon_{jst}$



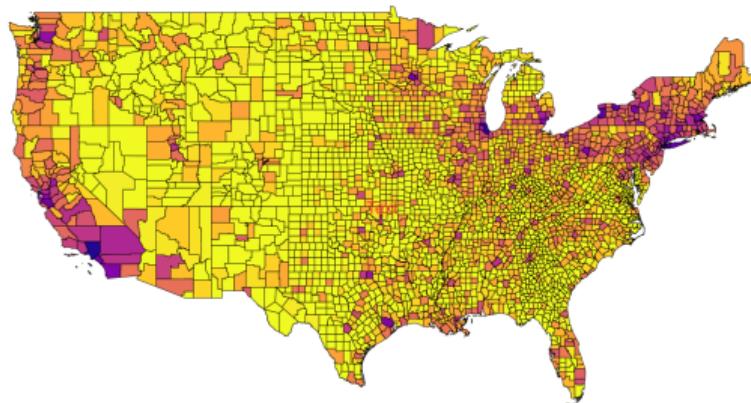
- Sorting present throughout, but **weakens** with deregulation in the time series
  - ▶ Rationalization: **deregulation induces mismatch sorting**

# A Quantitative Model of Bank Location Choices

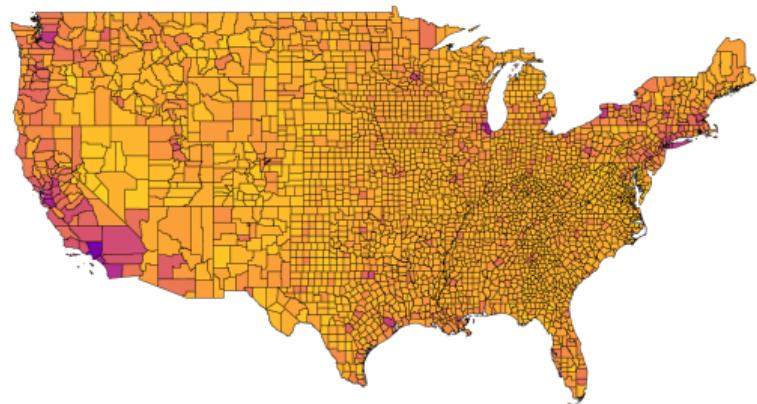
- Location-specific fundamentals  $\{w_s, \psi_s, \bar{E}_s^D, \bar{E}_s^L\}$  for 1981
  - ▶ Use wages  $w_s$  from QCEW and estimate rents  $\psi_s$  with wages and population density from Census
  - ▶ Estimate local demands  $\bar{E}_s^D$  and  $\bar{E}_s^L$  using QCEW, FDIC, and Call Reports
- Set of banks
  - ▶ Originally, there are  $\approx 10,000$  banks in FDIC data
  - ▶ Keep 10% of the largest banks by deposits
  - ▶ Collapse the remaining banks to "headquarters-location representative banks"
- Bank-specific fundamentals  $\{s_{jHQ}, \theta_j^D, \theta_j^L\}$ 
  - ▶  $s_{jHQ}$  known from FDIC data
  - ▶ Estimate  $\theta_j^D$  and  $\theta_j^L$  using FDIC data on total deposits and wholesale funding
- Compute industry equilibrium with  $\approx 4000$  banks and  $\approx 3000$  locations (15-20 min at  $10^{-4}$ )
  - ① Pre-liberalization episode with no branching outside the home state
  - ② Post-liberalization episode with branching allowed across states

Functional Forms

# Quantitative Model: Distribution of $\bar{E}_s^L$ and $E_s^L$

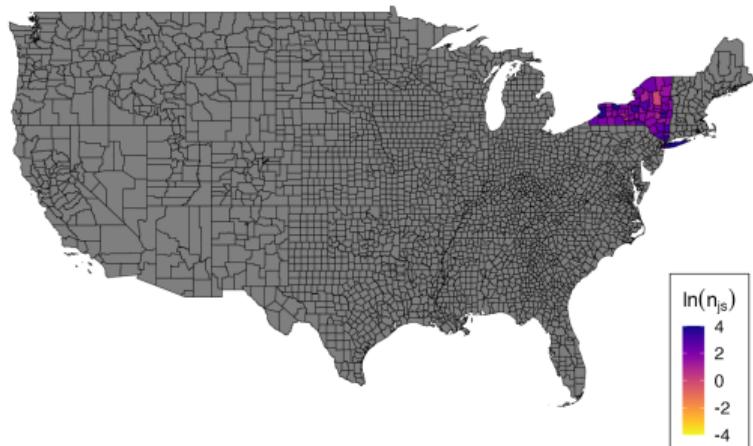


Values of  $\bar{E}_s^L$  in the post-liberalization episode

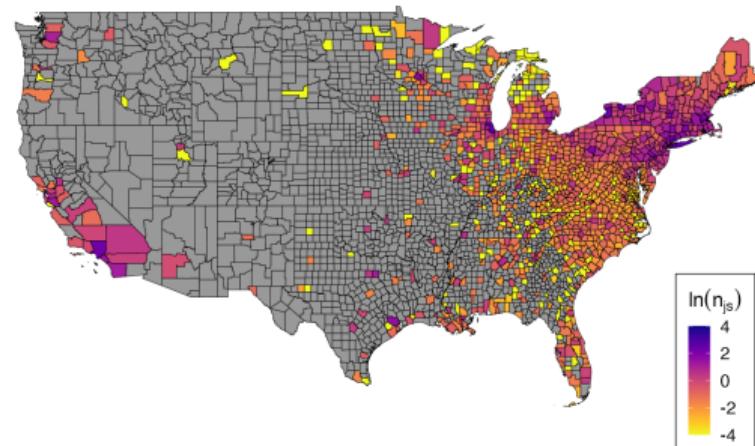


Values of  $E_s^L$  in the post-liberalization episode

# Quantitative Model: Spatial Expansion of a NYC Bank



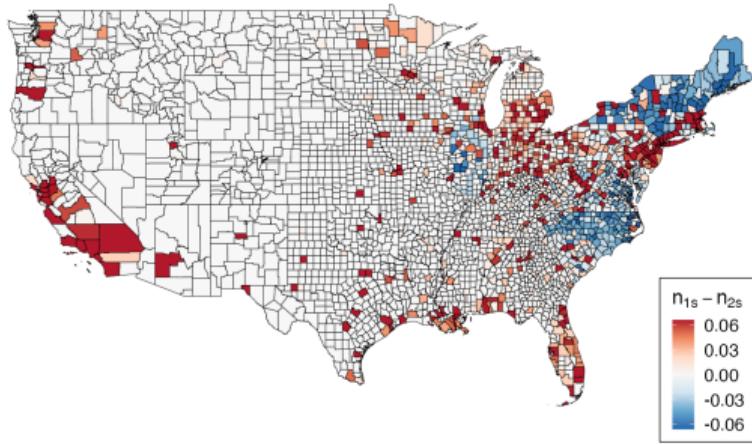
Density of the banks' plants in the **pre-liberalization episode**



Density of the banks' plants in the **post-liberalization episode**

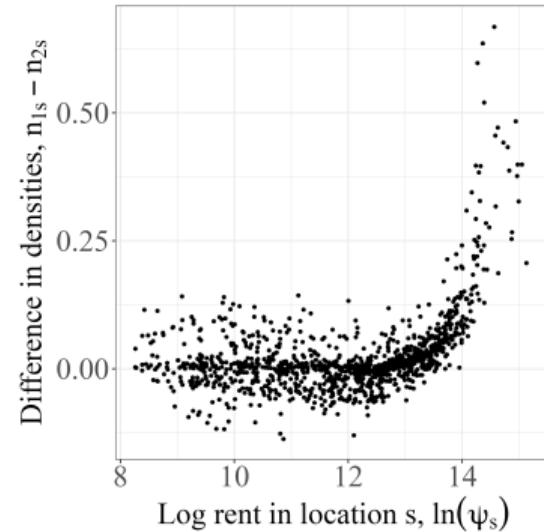
# Quantitative Model: Spatial Sorting of Two NYC Banks

- Compare productive bank (1) and an unproductive (2) bank, both headquartered in NYC
  - ▶ Initial wholesale sale funding of productive bank is larger:  $W_1/D_1 > W_2/D_2$



Difference in densities between NYC banks across counties

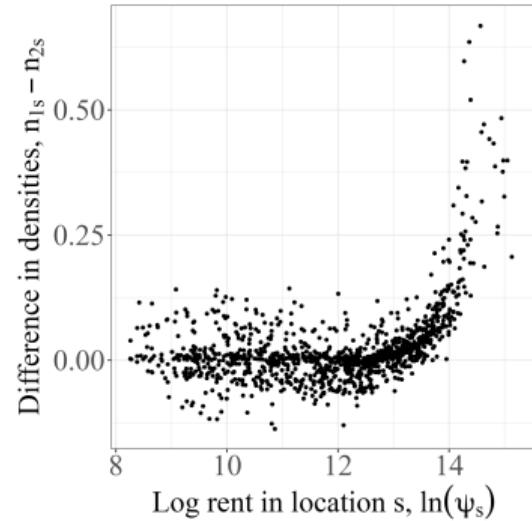
(Winsorized at 1st and 99th quantiles)



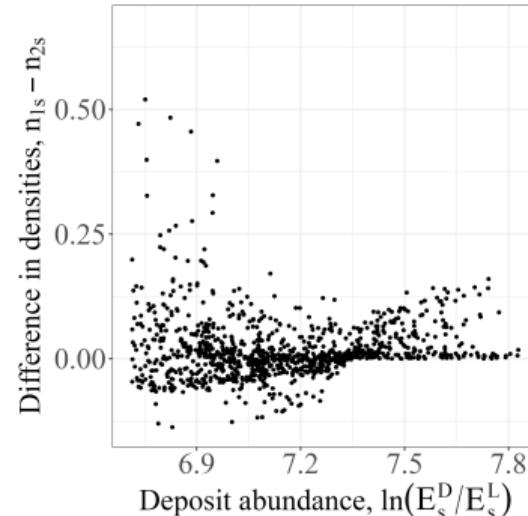
Difference in densities between NYC banks across counties

# Quantitative Model: Span of Control and Mismatch Sorting

- **Span of control sorting:**  $n_{1s} - n_{2s}$  increasing in rents ( $\psi_s$ )
- **Mismatch sorting:**  $n_{1s} - n_{2s}$  increasing in deposit abundance ( $E_s^D / E_s^L$ )



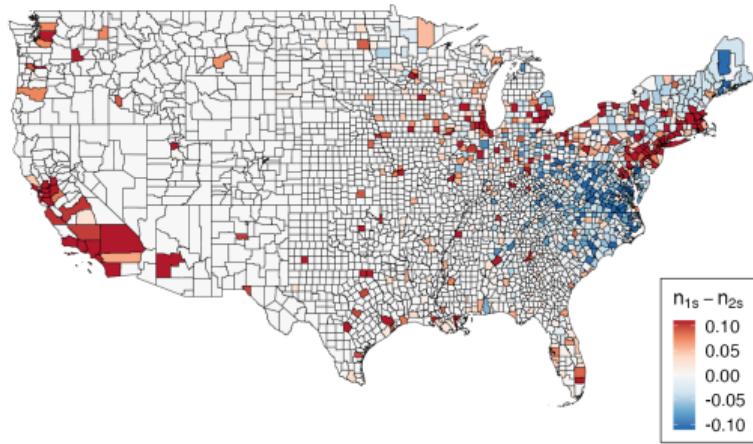
Difference in densities between NYC banks across counties



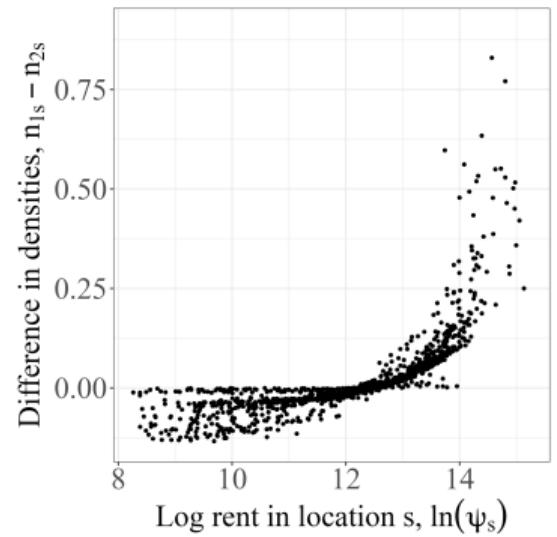
Difference in densities between NYC banks across counties.

# Quantitative Model: Counterfactual without Mismatch Sorting

- Remove motives for mismatch sorting by setting  $E_s^D = E_s^L$



Difference in densities between NYC banks across counties  
(Winsorized at 1st and 99th quantiles)



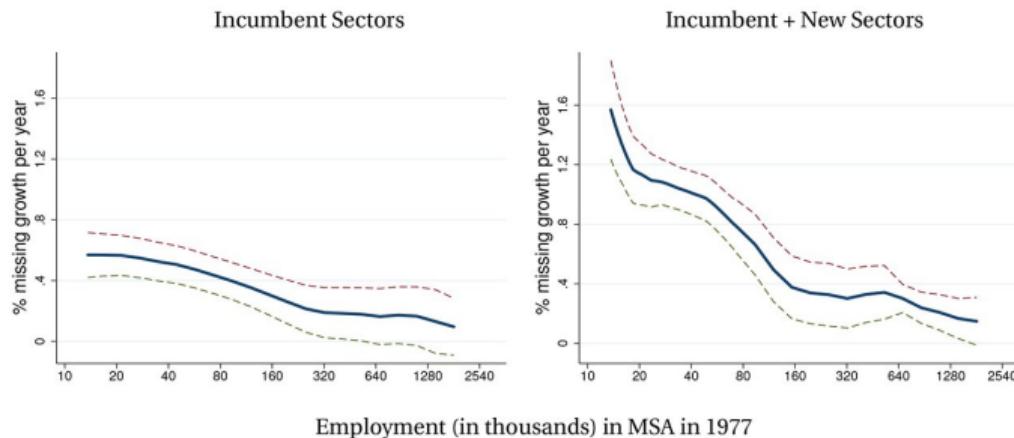
Difference in densities between NYC banks across counties

# Why All This Matters: “Missing Growth”

- From Feenstra 1994:

$$\hat{P}_{n,t} = \underbrace{\left( \int_{i \in I_{n,t}} \varrho_{in,t} \hat{P}_{in,t} di \right)}_{\text{measured by BLS}} - \frac{1}{\sigma - 1} \underbrace{\left( \int_{i \in I_{n,t}} \varrho_{in,t} \hat{\chi}_{in,t} di \right)}_{\text{missing growth new varieties}} - \frac{1}{\varrho - 1} \underbrace{\hat{\chi}_{n|i \in I_{n,t}}}_{\text{missing growth new industries}}$$

- $\varrho_{in,t}$  is the Sato-Vartia weight
- $I_{n,t}$  is the set of incumbent industries
- $\hat{\chi}_{in,t}$  is the change in the sales share of the incumbent firms



- Aggregate missing growth of 0.5% per year from 1977 to 2012

# Conclusions

- Growth of top firms mostly due to entry into new markets
- Firms sort into new markets in ways that are not "Melitzian"
  - ▶ Span-of-control sorting: top firms enter more into large and expensive markets and lesser firms into smaller and cheaper ones
- Need a theory of plant sorting in space to explain observed patterns
  - ▶ Hard to circumvent high-dimensional combinatorial problem in realistic settings
  - ▶ **Important to develop the next generation of QSE models where firms make realistic location decisions**
  - ▶ Determines how technology diffuses in space in response to incentives
- Continuous density approximation useful to analyze these problems
  - ▶ Yields analytical results and base for new QSE models with thousands of firms and locations
- **Relevant since local entry of varieties and industries affects local welfare and its effect are mostly miss-measured**

# **Appendix**

# Number of Markets per Firm Across Sectors

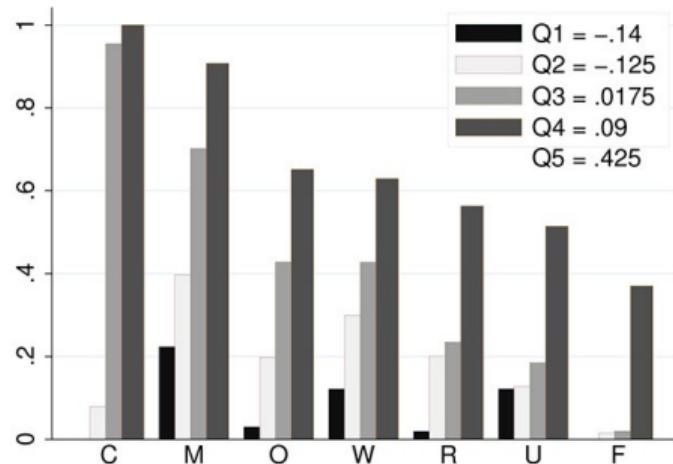
Weighted Average of  $\Delta \text{Log}$  Markets per Firm by Sector, 1977-2013

	Establishments	Zip Codes	Counties	MSAs
Construction	.016 (.034)	.017 (.031)	.015 (.038)	.012 (.020)
Manufacturing	.019 (.141)	.017 (.132)	.012 (.115)	.006 (.089)
Other	.180 (.239)	.128 (.290)	.031 (.150)	.050 (.094)
Wholesale	.156 (.248)	.139 (.239)	.076 (.156)	.030 (.084)
Retail	.216 (.237)	.186 (.185)	.096 (.136)	.040 (.078)
Utilities and transportation	.172 (.234)	.126 (.202)	.101 (.180)	.070 (.148)
Finance	.299 (.215)	.211 (.170)	.099 (.137)	.044 (.109)

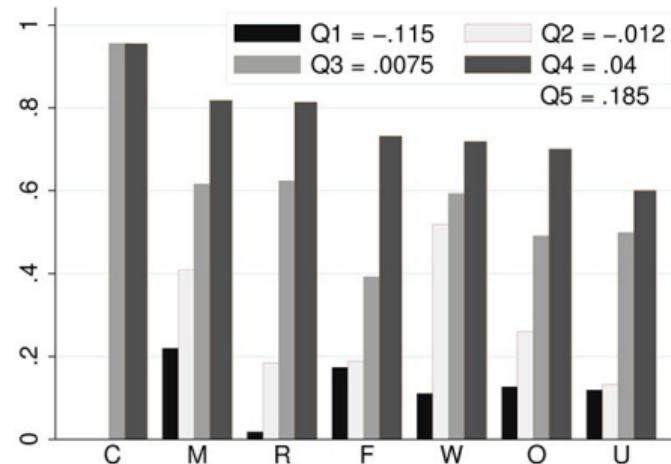
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# CDF of Number of Markets per Firm Across Sectors

CDF of  $\Delta$  Estab/Firm



CDF of  $\Delta$  MSA/Firm



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# Industry Growth and Expansion in the Number of Markets

Regression of Industry Growth on  $\Delta \text{Log Markets per Firm}$ , 1977-2013

	$\Delta \text{Log Employment}$	$\Delta \text{Log Sales}$
$\Delta \text{Log establishments/firm}$	0.845 (0.169)	1.192 (0.206)
$\Delta \text{Log MSAs/firm}$	1.444 (0.415)	2.926 (0.468)

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# Number of Markets versus Size per Markets

## Change in Share of Top 10% Firms by Employment or Sales

	Employment Share		Sales Share	
	Markets	Size	Markets	Size
Establishments	1.522 (0.092)	-0.522 (0.092)	1.289 (0.094)	-0.289 (0.094)
MSA	0.941 (0.072)	-0.059 (0.072)	0.789 (0.073)	0.211 (0.073)

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# Top Firms Entry by Market Size

## Change in Market Size of Top-10% Firms

	Top-10% Firms	
	By Employment	By Establishments/Firm
$\Delta$ MSA size of top firms/all firms	-.262 (.065)	-.059 (.014)

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# Sorting in the Data: Average Density and Firm Size

	Baseline $M = 12$	Baseline $M = 48$	Firms with $\geq 100$ plants	Industries in which largest firm has $\geq 100$ plants	HQ fixed effects, Firms with $\geq 100$ plants
	$\ln \bar{L}_j$	$\ln \bar{L}_j$	$\ln \bar{L}_j$	$\ln \bar{L}_j$	$\ln \bar{L}_j$
$\ln L_j$	0.165*** (0.000975)	0.0952*** (0.000848)	0.146*** (0.0249)	0.172*** (0.00164)	0.0791** (0.0350)
Observations	3,670,994	3,673,053	876	1,387,742	652
R-squared	0.139	0.099	0.384	0.080	0.664
SIC8 FE	Yes	Yes	Yes	Yes	Yes
HQ Location FE	No	No	No	No	Yes
M	12	48	12	12	12

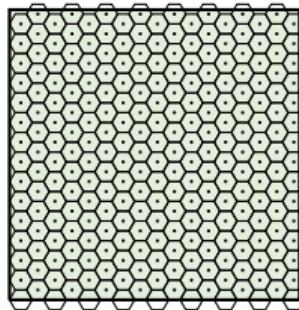
Robust standard errors in parentheses

\*\*\* p<0.01, \*\* p<0.05, \* p<0.1

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# Elements of the Proof

- When economic characteristics are **uniform across space** solution is known
  - Fejes Toth (1953) shows that if number of plants grows large, catchment areas are uniform regular hexagons



- We show that logic can be generalized to **heterogeneous space**
  - Construct upper and lower bounds for  $\pi_j$  in original problem for all  $\Delta$
  - Use hexagons for the bounds, as  $\Delta \downarrow 0$
  - Upper and lower bound approach the same limit value. Thus, so does  $\pi_j$

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# Firms Sort According to Rents

- **Assumption:**  $z(q, N) \equiv q\zeta(N)$ ,  $\zeta$  log-concave
- **Lemma:** More productive firms have larger span-of control costs

► That is, if  $z_1 < z_2$  then  $\lambda_1 < \lambda_2$ , in fact  $\frac{\lambda_1}{z_1^{\epsilon-1}} < \frac{\lambda_2}{z_2^{\epsilon-1}}$

**Proposition:** If  $z_1 < z_2$ , there is a unique cutoff  $R^*(z_1, z_2)$  for which  $\frac{R^*(z_1, z_2) + \lambda_2}{R^*(z_1, z_2) + \lambda_1} = \frac{z_2^{\epsilon-1}}{z_1^{\epsilon-1}}$

- If  $R_s = R^*(z_1, z_2)$  then  $n_{1s} = n_{2s}$
- If  $R_s > R^*(z_1, z_2)$  then  $n_{2s} \geq n_{1s}$ , with strict inequality if  $n_{2s} > 0$
- If  $R_s < R^*(z_1, z_2)$  then  $n_{1s} \geq n_{2s}$ , with strict inequality if  $n_{1s} > 0$

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# Functional Forms

$\mathcal{D}(r) = r^{b_D}$	(Demand for deposits)
$\mathcal{L}(r) = r^{-b_L}$	(Demand for loans)
$R(\omega) = a_R \exp(b_R \omega)$	(Wholesale funds interest rate)
$C(\bar{Q}^D, \bar{Q}^L) = a_C [(\bar{Q}^D)^{b_C} + (\bar{Q}^L)^{b_C}]$	(Common appeal cost function)
$h(N) = a_h \exp(b_h N)$	(Span of control costs)
$\mathcal{G}^D(r) = r^{b_D}$	(Mean utility, deposits)
$\mathcal{G}^L(r) = r^{-b_L}$	(Mean utility, loans)
$t^D(\delta) = t^L(\delta) = \exp(\delta / \sqrt{\phi})$	(Transportation costs)
$J^D(\delta) = J^L(\delta) = \exp(-\nu \delta)$	(Distance-related appeal)

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