

Measuring quality of life under spatial frictions

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Motivation

- **Productivity advantages** and correspondingly higher wages have been identified as potential **drivers of urbanization** (Marshall, 1890; Combes and Gobillon, 2015; many more).
- **Cities** may also be attractive places to live as they **host urban amenities** such as ethnic restaurants, music venues, or art galleries.
- Measurement of quality of life (QoL) in the tradition of Rosen (1979) and Roback (1982) has shown **little evidence for a positive urban quality-of-life premium** (Ahlfeldt and Pietrostefani, 2019; Albouy, 2011).
- An accurate measure of **QoL is important** to understand drivers of **urbanization** and to determine optimal **provision of public goods/transfers**.

What we do

- **Explore measurement error in QoL when ignoring spatial frictions**
 - Compare QoL in (canonical) quantitative spatial model (QSM) with spatial frictions with quality of life in frictionless framework
 - We consider mobility frictions (preference shocks; local ties) and trade frictions (gravity; local non-tradable services)
 - Use QSM as data-generating process in a Monte Carlo study
- **Application for Germany**
 - QoL ranking of locations & urban QoL premium
- Accessible **ABRSQOL-toolkit** with user-friendly syntax available on [GitHub](#)
 - Implements numerical solver in MATLAB, R, Python, Stata

How spatial frictions matter

- **Preference shocks:**

- Labor supply upward-sloping \Rightarrow QoL can no longer be inferred from real living cost (High real wage can be due to high labor or housing productivity)
- More productive cities grow larger \Rightarrow **downward bias of urban QoL in RR**

- **Local ties:**

- Locations may be large because of local ties—despite low real wages
- We expect an **upward bias of urban QoL in RR**

- **Trade costs:**

- Large locations \Rightarrow better market access \Rightarrow lower prices \Rightarrow **upward bias of urban QoL in RR**

- **Non-tradable services:**

- QoL capitalizes negatively into wages and positively into floor space prices \Rightarrow bias in RR ambiguous

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What we find

- **Monte Carlos:** For location with 50% higher ‘true’ QoL, downward bias in RR by about 25%, on average (driven more by mobility frictions than by trade frictions)

⇒ Priority to account for both types of mobility frictions

- **Application to Germany:** Accounting for spatial frictions results in larger quality-of-life differences, different quality-of-life rankings (average change in rank: 17), and an urban quality-of-life premium that exceeds the urban wage premium
- A German city that is twice as large, on average, offers a 22% higher QoL (14% under RR) which is five times the urban wage premium (4%).

Literature (selection) & contributions

- **QoL measurement and public-good valuation (no spatial frictions)**
 - **Theoretical framework:** Rosen (1979), Roback (1982)
 - **QoL:** Albouy, Leibovici & Warman (2013); Albouy & Lue (2015); Albouy & Stuart (2020); Blomquist, Berger & Hoehn (1988); Gabriel & Rosenthal (2004), Glaeser (2011), Glaeser & Gottlieb (2009), Shapiro (2006)
 - **Public goods:** Chay and Greenstone (2005) for clean air; Linden and Rockoff (2008) for safety; Cellini, Ferreira & Rothstein (2010) quality of public schools; Greenstone (2017) for review

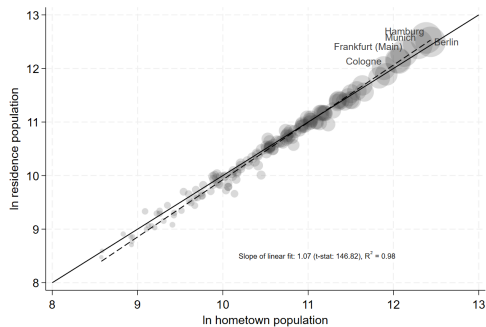
Literature (selection) & contributions

- **Spatial frictions (but no QoL measurement)**
 - **Heterogeneous tastes:** Moretti (2011), Ahlfeldt et. al. (2015), Diamond (2016), Monte et al. (2018), etc.
 - **Local ties:** Zabek (2024), etc.
 - **Tradable goods prices:** Armington (1969), Allen & Arkolakis (2014), Redding (2016), Redding and Rossi-Hansberg (2017), etc.
 - **Non-tradable prices:** Moretti & Diamond (2021)

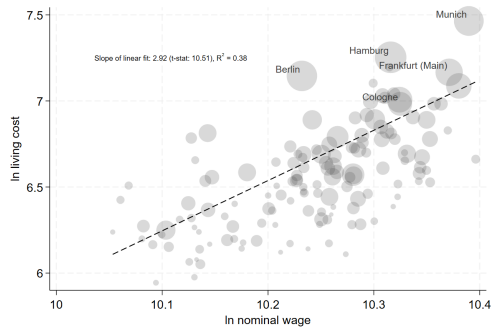
- ⇒ We quantify the role of spatial frictions for measurement of QoL
- ⇒ We use the quantitative spatial model as data-generating process

Stylized facts

Stylized facts I



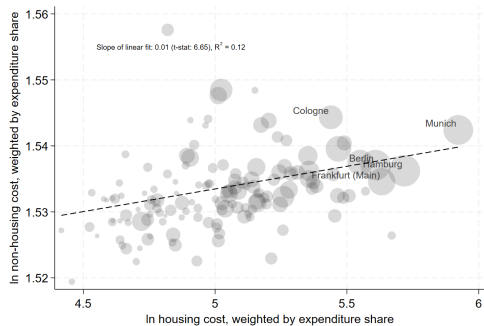
(a) Hometown vs. residence population



(b) Nominal wage vs. living cost

Notes: Unit of observation are the 141 local labour markets (LLM) in Germany.

Stylized facts II



(c) Housing vs. non-housing cost



(d) Real wage vs. residence pop. surplus

Notes: Unit of observation are the 141 local labour markets (LLM) in Germany.

The model

A canonical QSM

- J locations, \bar{L} mobile workers
- No commuting
- Utility from tradables, non-tradables (services), floor space and QoL
- **Tradables:** produced from labour
- **Non-tradables (services):** produced from labour and floor space
- **Mobility frictions** (preference shocks, local ties) and **trade costs**
- **Floor space:** produced from capital and land; demanded from residents and producers of services.

Measuring quality of life

QoL and measurement error

- **Relative quality of life in full QSM:**

$$\hat{A} = \frac{(\hat{P}^t)^{\alpha\beta} (\hat{p}^n)^{\alpha(1-\beta)} (\hat{p}^H)^{1-\alpha}}{\hat{w}} \left(\hat{L} / \hat{\mathcal{L}} \right)^{\frac{1}{\gamma}}, \quad (1)$$

where $\mathcal{L}_i \equiv \left(\sum_{m \in J} \Psi_m^b \bar{L}_m^b + (\exp[\xi] - 1) \Psi_i^b \bar{L}_i^b \right)$.

- **QoL in Rosen-Roback:**

$$\hat{A}_{RR} = \frac{(\hat{p}^H)^{1-\alpha}}{\hat{w}} \quad (2)$$

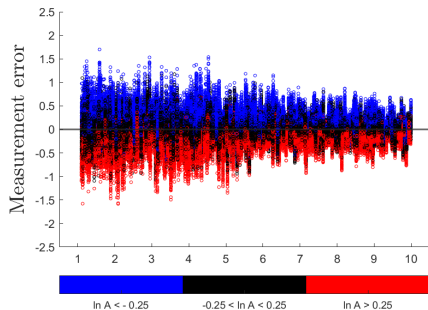
- **Measurement error** when no friction is taken into account:

$$\mathcal{E} \equiv \ln \frac{\hat{A}_{RR}}{\hat{A}} = \underbrace{-\alpha\beta \ln \hat{P}^t}_{\text{trade costs}} \underbrace{-\alpha(1-\beta) \ln \hat{p}^n}_{\text{local services}} \underbrace{-(1/\gamma) \ln \hat{L}}_{\text{idiosyncratic tastes}} \underbrace{+(1/\gamma) \ln \hat{\mathcal{L}}}_{\text{local ties}} \quad (3)$$

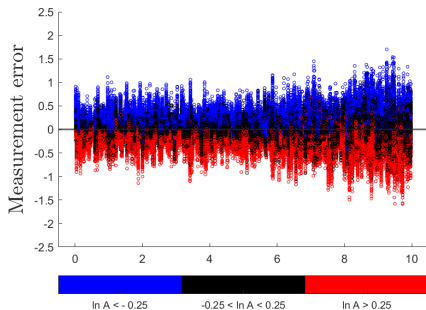
Monte Carlo Study

- Solve the full model numerically for randomly drawn fundamentals $\{\ln A_i, \ln \bar{\varphi}_i, \ln \eta_i\}$ and parameters $\{\gamma, \zeta\}$
- $J = 144$ artificial cities (12×12) in $N = 1,000$ artificial countries (iterations)
 $\Rightarrow N \times J = 144,000$ equilibrium outcomes.
- Same values for structural parameters as in our quantification exercise for Germany below: $\{\alpha = 0.66; \beta = 0.5; \delta = 0.3; \mu = 0.8; \sigma = 5\}$.
- $\tau_{ij} = (\exp[-\iota * \ln dist_{ij}])^{\frac{1}{1-\sigma}}$. We take $\iota = -1$ and $\sigma = 5$ from the literature.
- Centrality measure: $\mathcal{M}_i = \sum_{j \in J} (1/dist_{ij})$.

Determinants of measurement error in Rosen-Roback



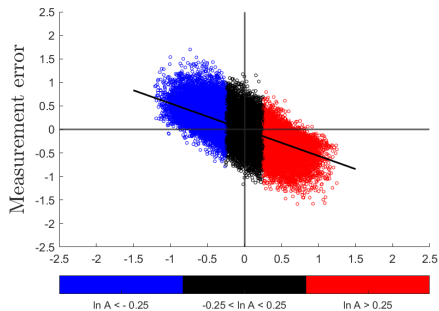
(a) Mobility frictions (γ)



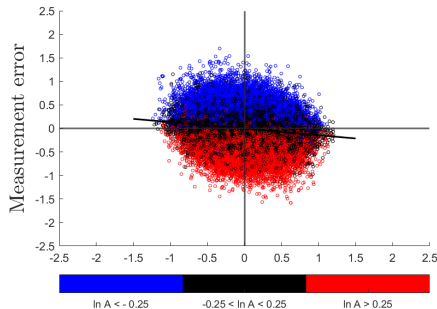
(b) Local ties (ξ)

- 1 QoL is underestimated for locations with high \hat{A} and vice versa.
- 2 Measurement error declines with lower mobility frictions (higher values of γ) and lower local ties (lower values of ξ).

Determinants of measurement error in Rosen-Roback



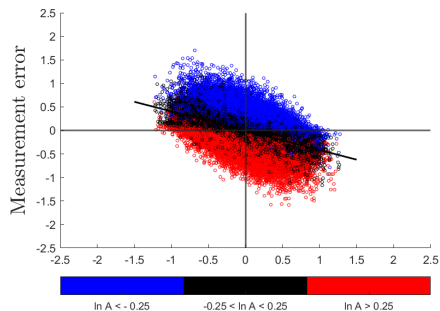
(c) Quality of life ($\ln \hat{A}$)



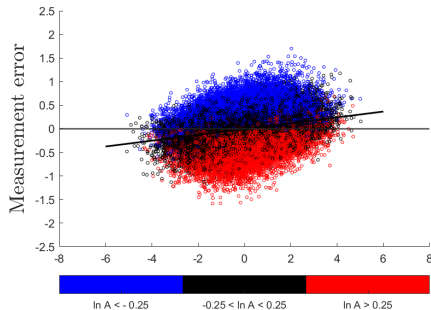
(d) Housing productivity ($\ln \hat{\eta}$)

- ① Measurement error declines with higher QoL, panel (c).
- ② Slightly negative relationship between measurement error and housing productivity, panel (d).

Determinants of measurement error in Rosen-Roback



(e) Labour productivity ($\ln \hat{\varphi}$)



(f) Hometown population ($\ln \hat{L}^b$)

- ① Impact of labour productivity is more pronounced negative, panel (e).
- ② Measurement error increases in population share with local ties, panel (f).

Heterogeneity in measurement error

<i>Frictions controlled:</i>	(1)	(2)	(3)	(4)	(5)
Trade Costs		✓			
Local services			✓		
Local ties				✓	
Idiosyncratic tastes					✓
Measurement error: Intercept	-0.288	-0.305	-0.300	-0.169	-0.090
(Inverse) taste heterogeneity: γ -3	0.026	0.023	0.023	0.015	0.018
Strength of local ties: ξ -5	-0.017	-0.015	-0.015	0.015	-0.037
Market access: $\ln \hat{\mathcal{M}}$	-0.087	-0.156	-0.048	-0.053	-0.004
Quality of life: $\ln(\hat{A}/1.5)$	-0.711	-0.754	-0.741	-0.427	-0.212
Relative floor-space productivity : $\ln \hat{\eta}$	-0.189	-0.205	-0.270	-0.105	0.014
Relative worker productivity: $\ln \hat{\varphi}$	-0.547	-0.611	-0.591	-0.309	-0.130
Relative hometown population: $\ln \hat{L}^b$	0.073	0.062	0.065	-0.132	0.223
Observations	144,000	144,000	144,000	144,000	144,000
Adjusted R^2	0.977	0.985	0.984	0.783	0.807

Notes: Each column represents a different measurement error for a location with 50% higher quality of life than the numéraire location, so $\hat{A} = 1.5$. All explanatory variables are expressed relative to a numéraire location. The mobility friction parameter γ and local ties valuation ξ are re-scaled to have a zero value at $\gamma = 3$ and $\xi = 5$, respectively, so we can interpret the intercept as the measurement error for otherwise identical locations. All coefficients are highly significant.

- Intercept can be interpreted as the measurement bias for otherwise identical locations at a value of $\gamma = 3$ (Redding 2016) and $\xi = 5$ (Zabek 2024).
- Ignoring all spatial frictions: Error is -25% ($= \exp[-0.288] - 1$).
- **Shapley:** Tradable goods, 10.1%; local services, 4.5%; idiosyncratic tastes, 18.8%; and local ties, 66.6%
- Increasing differences in $\hat{\mathcal{M}}$, \hat{A} , $\hat{\eta}$ or $\hat{\varphi}$ by 50% magnifies the bias by 2.6pp, 18.7pp, 5.5pp and 14.9pp, respectively. The same change in $\ln \hat{L}^b$ reduces the bias by 2.3pp (Location is more attractive, but real wage increases are falsely interpreted as a lower QoL).

Urban Quality of Life Premium

- **Definition:** Elasticity of QoL with respect to city size.
- Housing supply and labour demand shifters increase city population, such that measurement bias will be more pronounced in big cities.
- In the DGP, $\hat{L}(\hat{A}, \hat{\eta}, \hat{\varphi}, \hat{\mathcal{M}}, \hat{\hat{L}}^b) = c\hat{A}^\gamma \exp\left(\epsilon(\hat{\eta}, \hat{\varphi}, \hat{\mathcal{M}}, \hat{\hat{L}}^b)\right)$, so we can estimate the urban quality of life premium as follows:

$$\ln \hat{A} = \tilde{c} + \rho \ln \hat{L} + \left[-\frac{1}{\gamma} \epsilon \right], \quad (4)$$

where $\tilde{c} \equiv -\frac{1}{\gamma} \ln c$, the term in brackets constitutes the regression residual.

Note: \hat{L} and ϵ are correlated \Rightarrow premium only descriptive!

Urban Quality of Life Premium

- The Rosen-Roback framework, however, does not even recover this descriptive statistic correctly. We obtain:

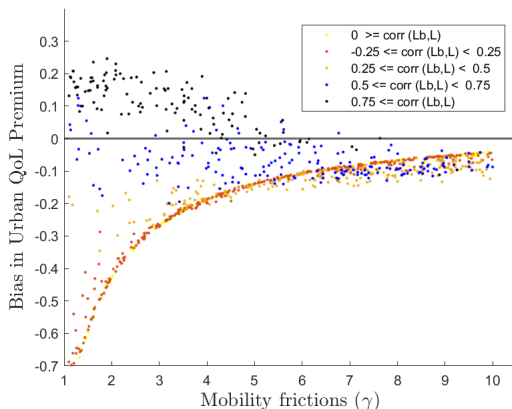
$$\ln \hat{A}_{RR} = \tilde{c} + \rho_{RR} \ln \hat{L} + \left[\mathcal{E} - \frac{1}{\gamma} \epsilon \right]. \quad (5)$$

⇒ Since \hat{L} and \mathcal{E} are correlated, ρ_{RR} will be estimated with bias, so $\mathcal{B} = \hat{\rho}_{RR} - \hat{\rho}$.

- **Intuitively:**

- For given $\hat{\tilde{L}}^b$, a city must compensate the marginal worker for a lower idiosyncratic utility through a higher QoL compared to $RR \Rightarrow$ downward bias in RR
 - If $\hat{\tilde{L}}^b$ is large, local ties explain city size despite potentially low QoL \Rightarrow upward bias in RR
- ⇒ $corr(\hat{\tilde{L}}^b, \hat{L})$ should quantitatively and qualitatively determine the bias in the urban QoL premium.

Urban QoL premium biases



- ① Unless $\text{corr}(\hat{L}^b, \hat{L})$ is very large (>0.75) **bias will be negative**
- ② In the limit, with perfect mobility, bias converges to zero

Quantification for Germany

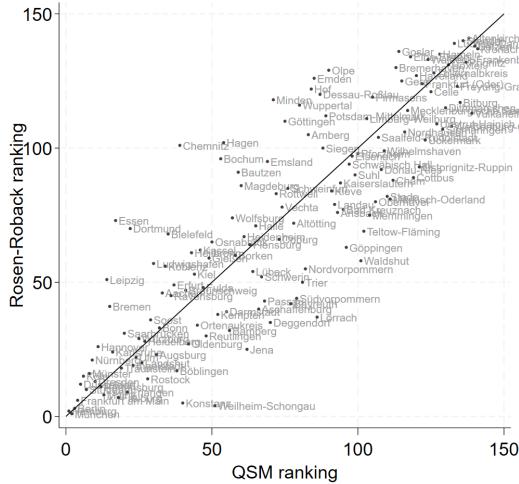
Data

- We use data on house prices (Ahlfeldt, Heblich & Seidel, 2023) and tradables/non-tradables (Weinand & von Auer 2020) for 2015
- Employment and wages from IAB based on universe of German workers
- $\alpha = 2/3$: From expenditure share on housing $1 - \alpha$ (Federal Statistical Office 2020)
- $\beta = 0.5$ from Lombardo and Ravenna (2014)
- $\gamma = 3$ from Krebs and Pflueger (2023)
- $\xi = 5.5$ (own estimate), somewhat larger than for US, consistent with a greater share of workers living in their hometowns

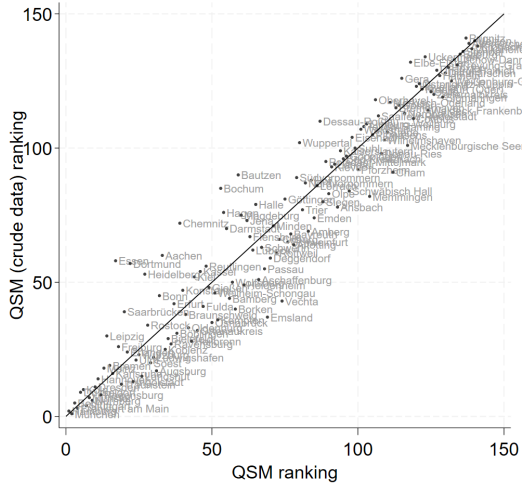
Table: Quality of life rankings

	QSM, best data		Rosen-Roback			QSM, crude data		
	Rank	\hat{A}	Rank	\hat{A}_{RR}	(4)/(2)	Rank	\hat{A}_{CD}	(7)/(2)
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Hamburg	1	2.078	2	1.726	.831	3	1.828	.944
München	2	2.04	1	1.968	.964	2	1.910	1.03
Berlin	3	1.887	3	1.683	.892	1	1.926	.874
Frankfurt am Main	4	1.717	6	1.500	.874	4	1.587	.945
Düsseldorf	5	1.562	12	1.334	.853	7	1.382	.965
...
Prignitz	137	0.535	132	0.647	1.209	140	0.509	1.271
Altenkirchen	138	0.525	141	0.576	1.096	136	0.543	1.062
Vulkaneifel	139	0.510	113	0.737	1.446	139	0.511	1.443
Uelzen	140	0.505	138	0.618	1.224	138	0.522	1.184
Kronach	141	0.504	137	0.620	1.230	141	0.487	1.272
Standard deviation		0.276		0.253			0.259	

Comparison of QoL rankings

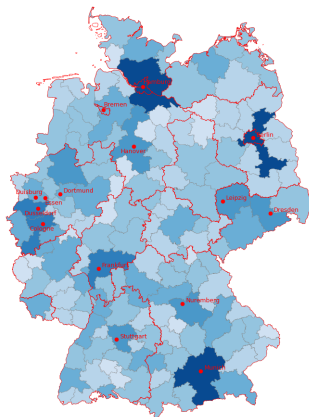


(a) Rosen-Roback (A_{RR})



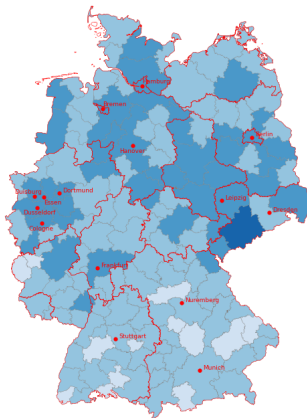
(b) QSM, crude data (A_{CD})

Relative QoL



0.4 0.8 1.2 1.6 2.0
Relative QoL, Kiel=1

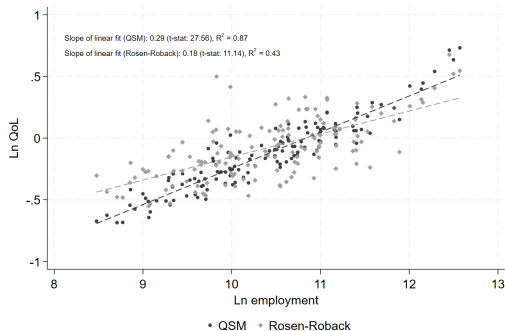
(a) QSM



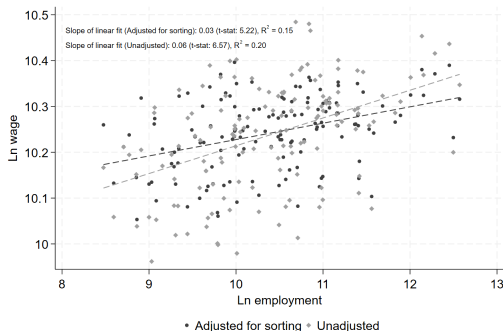
0.50 0.75 1.00 1.25 1.50
QoL QSM/RR

(b) QSM/RR

Urban quality-of-life premium vs. urban wage premium



(a) Urban quality of life premium



(b) Urban wage premium

- ① Larger urban QoL premium when we account for spatial frictions
- ② Urban QoL premium exceeds the urban wage premium!

Conclusions

Take-aways

- By **abstracting from spatial frictions**, estimates of QoL derived from the Rosen-Roback framework suffer from a **downward measurement error that increases in city size**.
- We document a **positive urban QoL premium for Germany**.
- **Quality of life** may be as important an agglomeration force driving urbanisation as **productivity**.
- Our results matter for ‘correct’ valuation of amenities and the optimal design of spatial transfers.

Additional material

Theoretical Framework

Preferences

- Worker ω from hometown m living in city i derives utility from the consumption of goods ($C_{i\omega}$) and floor space ($h_{i\omega}$) according to

$$U_{im\omega} = \left(\frac{C_{i\omega}}{\alpha} \right)^\alpha \left(\frac{h_{i\omega}}{1-\alpha} \right)^{1-\alpha} \exp[a_{im\omega}], \quad (6)$$

where $C_{i\omega} = (Q_{i\omega}^t/\beta)^\beta (q_{i\omega}^n/(1-\beta))^{1-\beta}$ with

$$Q_{i\omega}^t = \left[\sum_{j \in J} (q_{ji\omega}^t)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \quad (7)$$

and $q_{i\omega}^n$ (local non-traded services).

Mobility frictions

- The idiosyncratic amenity component $\exp[a_{im\omega}] = \exp[a_{i\omega} + \mathbb{1}\{m = i\} \cdot (\xi/\gamma)]$ is modelled as a stochastic preference shock for each location i , that is shifted upwards if the residence corresponds to the hometown. In particular, $a_{i\omega}$ is drawn from a type-I-extreme value (Gumbel) distribution:

$$F_i(a) = \exp\left(-\tilde{A}_i \exp\{-[\gamma a + \Gamma]\}\right) \quad \text{with } \gamma > 0, \quad (8)$$

where $\tilde{A}_i \equiv (A_i)^\gamma$ represents the mean of the amenity shock and Γ is the Euler-Mascheroni constant.

- A_i serves as an exogenous measure of quality of life (QoL).
- γ governs the dispersion of individual amenity shocks for each group \Rightarrow introduces imperfect spatial arbitrage \Rightarrow inverse measure of mobility frictions.

Technology: Floor space

- Supplied under perfect competition combining a share of the globally available capital stock, K_i (available at unit prices), with location-specific land, \bar{T}_i :

$$H_i^S = \eta_i \left(\frac{\bar{T}_i}{\delta} \right)^\delta \left(\frac{K_i}{1-\delta} \right)^{1-\delta}. \quad (9)$$

- Total factor productivity η_i captures the role of regulatory (e.g. height regulations) and physical (e.g. a rugged surface) constraints (Saiz 2010).
- Floor space used for housing and as an input for local services.

Technology: Tradables

- Each location produces a unique variety of a tradable intermediate good (Armington 1969) using a CES-aggregate of labor, L_i^t , as the sole production input under perfect competition according to $q_i^t = \varphi_i L_i^t$
- Labour productivity $\varphi_i = \bar{\varphi}_i L_i^\zeta$ is increasing in local employment according to agglomeration elasticity ζ .
- $\tau_{ji} > 1$ units have to be shipped from j for one unit to arrive in i .
- Perfect competition implies $p_{ji}^t = \tau_{ji} w_j / \varphi_j$

Technology: Local services

- Requires both labour and floor space:

$$q_i^n = \nu_i^n \left(\frac{L_i^n}{\mu} \right)^\mu \left(\frac{H_i^n}{1-\mu} \right)^{1-\mu}, \quad (10)$$

where H_i^n denotes floor space input.

- Perfect competition implies $p_i^n = (w_i/\varphi_i)^\mu (p_i^H)^{1-\mu}$ and floor-space prices p_i^H .

Location choice

- Under the distributional assumptions on the idiosyncratic utility component, we obtain the probability that a worker from hometown m lives in location i :

$$\lambda_{im} = \frac{(A_i w_i / \mathcal{P}_i)^\gamma \cdot \exp[\mathbb{1}\{m = i\} \cdot \xi]}{\sum_{j \in J} (A_j w_j / \mathcal{P}_j)^\gamma \cdot \exp[\mathbb{1}\{m = j\} \cdot \xi]}, \quad (11)$$

where $\mathcal{P}_i \equiv (P_i^t)^{\alpha\beta} (p_i^n)^{\alpha(1-\beta)} (p_i^H)^{1-\alpha}$ is the aggregate consumer price index.

Location choice

- Summing over all hometown probabilities, we obtain the residential choice probability:

$$\lambda_i = \sum_m \lambda_{im} = \frac{(A_i w_i / \mathcal{P}_i)^\gamma}{\sum_{j \in J} (A_j w_j / \mathcal{P}_j)^\gamma} \left(\sum_{m \neq i} \Psi_m^b \bar{L}_m^b + \Psi_i^b \cdot \exp[\xi] \bar{L}_i^b \right) / \bar{L}, \quad (12)$$

with $\Psi_m^b = \left(1 + \frac{(\exp[\xi] - 1)(A_m w_m / \mathcal{P}_m)^\gamma}{\sum_{j \in J} (A_j w_j / \mathcal{P}_j)^\gamma} \right)^{-1} < 1$ being the utility discount associated with having left the hometown.

- Mobility of workers equalises expected utility in equilibrium.

General equilibrium

- Given model primitives, a general equilibrium of the economy is referenced by a vector of the endogenous objects $\mathbf{V} = \{L_i^n, L_i^t, w_i, r_i, p_i^H, \mathcal{P}_i\}$ and a scalar \bar{W} .
- They are jointly determined by
 - ① Market clearing for tradables and services
 - ② Floor-space and land market clearing
 - ③ National labour-market clearing: $L_i = \lambda_i \bar{L}$ with $\sum_{i \in J} \lambda_i = 1$
 - ④ Local labour resource constraint: $L_i = L_i^n + L_i^t$
 - ⑤ Aggregate consumer price index: \mathcal{P}_i