Measuring quality of life under spatial frictions

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Motivation

- **Productivity advantages** and correspondingly higher wages have been identified as potential **drivers of urbanization** (Marshall, 1890; Combes and Gobillion, 2015; many more).
- **Cities** may also be attractive places to live as they **host urban amenities** such as ethnic restaurants, music venues, or art galleries.
- Measurement of quality of life (QoL) in the tradition of Rosen (1979) and Roback (1982) has shown **little evidence for a positive urban quality-of-life premium** (Ahlfeldt and Pietrostefani, 2019; Albouy, 2011).
- An accurate measurement is important to **understand location choices** and to determine optimal **provision of public goods/transfers**.

What we do

- **Quantify measurement error in QoL when ignoring spatial frictions**
  - Compare QoL in a quantitative spatial model (QSM) with spatial frictions to quality of life in frictionless framework
  - We consider **mobility frictions (idiosyncratic preferences; local ties) and trade frictions (gravity; local non-tradable services)**
  - Use **QSM as data-generating process** in a Monte Carlo study
- **Application for Germany**
  - QoL ranking of locations & urban QoL premium
- Accessible **ABRSQOL-toolkit** with user-friendly syntax available on [GitHub]
  - Implements numerical solver in `MATLAB`, `R`, `Python`, `Stata`

What we find

- **Monte Carlos:**
  - For location with 50% higher 'true' **QoL, downward bias in** RR by about 25%
  - RR **underestimates urban QoL premium** (easily qualitatively wrong)
  - $\Rightarrow$ Priority to account for both types of **mobility frictions**
- **Application to Germany:**
  - Larger QoL differences
  - Different **QoL rankings** (average change in rank: 17)
  - Greater urban QoL premium
    Urban QoL premium: 0.29 vs. 0.12 in RR (city-cize elasticity of QoL)
    Urban wage premium: 0.06 (0.03 conditional on worker fixed effects)

Literature (selection) & contributions

- **QoL measurement and public-good valuation (no spatial frictions)**
  - **Theretical framework**: Rosen (1979), Roback (1982)
  - **QoL**: Albouy, Leibovici & Warman (2013); Albouy & Lue (2015); Albouy & Stuart (2020); Blomquist, Berger & Hoehn (1988); Gabriel & Rosenthal (2004), Glaeser (2011), Glaeser & Gottlieb (2009), Shapiro (2006)
  - **Public goods**: Chay and Greenstone (2005) for clean air; Linden and Rockoff (2008) for safety; Cellini, Ferreira & Rothstein (2010) quality of public schools; Greenstone (2017) for review

Literature (selection) & contributions

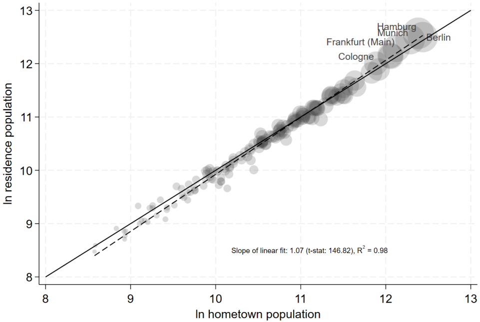
- **Spatial frictions (but no QoL measurement)**
  - **Heterogeneous tastes**: Moretti (2011), Ahlfeldt et al. (2015), Diamond (2016), Monte et al. (2018), etc.
  - **Local ties:** Zabek (2024), etc.
  - **Tradable goods prices**: Armington (1969), Allen & Arkolakis (2014), Redding (2016), Redding and Rossi-Hansberg (2017), etc.
  - **Non-tradable prices**: Moretti & Diamond (2021)

$\Rightarrow$ We quantify the role of spatial frictions for measurement of QoL

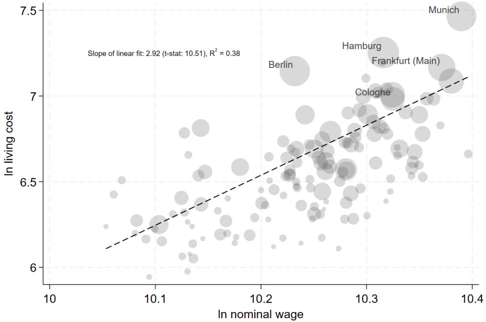
$\Rightarrow$ We use the quantitative spatial model as data-generating process

Stylized facts

Stylized facts I



(a) Hometown vs. residence population

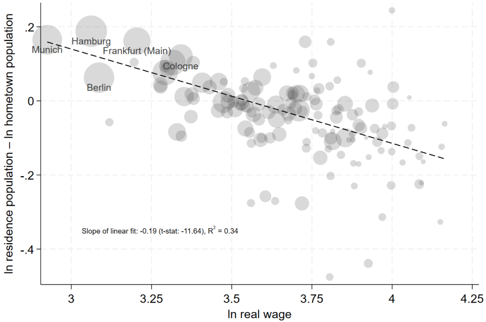


(b) Nominal wage vs. living cost

Notes: Unit of observation are the 141 local labour markets (LLM) in Germany.

Stylized facts II

(c) Housing vs. non-housing cost



(d) Real wage vs. residence pop. surplus

Notes: Unit of observation are the 141 local labour markets (LLM) in Germany.

The model

A QSM with canonical building blocks

- J **locations (local labour markets)**
  - $\bar{L}^b_m$ workers grew up in hometown $m \in J$ (exogenous)
  - $\bar{L} = \sum_m \bar{L}^b_m$ workers in the economy choose residence $i \in J$ (endogenous)
- **Utility** from tradables, non-tradables (services), floor space and QoL
  - **Tradables:** produced from labour
  - **Non-tradables (services):** produced from labour and floor space
  - **Floor space:** produced from capital and land
  - **QoL:** Locational fundamental (akin to TFP in production function)
- **Spatial frictions**
  - **Mobility**: Idiosyncratic taste shocks & local ties (dislike residence $\neq$ hometown)
  - **Trade**: Costly traded goods & non-tradable services
  - **No commuting** (workers live and work in same local labour market)

QSM

Measuring quality of life

- **Relative quality of life in full QSM:**

$$\hat{A} = \frac{(\hat{P}^t)^{\alpha\beta}(\hat{p}^n)^{\alpha(1-\beta)}(\hat{p}^H)^{1-\alpha}}{\hat{w}}\left(\hat{L}/\hat{\mathcal{L}}\right)^{\frac{1}{\gamma}},$$

where $\mathcal{L}_i \equiv \left(\sum_{m\in J}\Psi_m^b \bar{L}_m^b + (\exp[\xi]-1)\,\Psi_i^b\bar{L}_i^b\right)$.

- **QoL in Rosen-Roback:**

$$\hat{A}_{RR} = \frac{(\hat{p}^H)^{1-\alpha}}{\hat{w}}$$

- **Measurement error** when no friction is taken into account:

$$\mathcal{E} \equiv \ln\frac{\hat{A}_{RR}}{\hat{A}} = \underbrace{-(1/\gamma)\ln\hat{L}}_{\text{idiosyncratic tastes}} \underbrace{+(1/\gamma)\ln\hat{\mathcal{L}}}_{\text{local ties}} \underbrace{-\alpha\beta\ln\hat{P}^t}_{\text{trade costs}} \underbrace{-\alpha(1-\beta)\ln\hat{p}^n}_{\text{local services}}$$

Idiosyncratic tastes

$$\mathcal{E} \equiv \ln \frac{\hat{A}_{RR}}{\hat{A}} = \underbrace{-\,(1/\gamma)\ln \hat{L}}_{\textbf{idiosyncratic tastes}} \;\; \underbrace{+\,(1/\gamma)\ln \hat{\mathcal{L}}}_{\text{local ties}} \;\; \underbrace{-\,\alpha\beta \ln \hat{P}^t}_{\text{trade costs}} \;\; \underbrace{-\,\alpha(1-\beta)\ln \hat{p}^n}_{\text{local services}}$$

- **Upward-sloping labour supply** to the city
  - Small differences in wages do not trigger oceans of workers to move
  - Housing demand curve downward sloping
- Labour and housing **productivities matter** (unlike in RR)
  - *RR* **wrongly attributes productivity** (high wages and low rents) to low QoL
  - More productive cities grow larger ⇒ **downward bias of urban QoL in** ***RR***

Local ties

$$\mathcal{E} \equiv \ln \frac{\hat{A}_{RR}}{\hat{A}} = \underbrace{-\ (1/\gamma)\ln \hat{L}}_{\text{idiosyncratic tastes}} \ \underbrace{+\ (1/\gamma)\ln \hat{\mathcal{L}}}_{\textbf{local ties}} \ \underbrace{-\ \alpha\beta \ln \hat{P}^t}_{\text{trade costs}} \ \underbrace{-\ \alpha(1-\beta)\ln \hat{p}^n}_{\text{local services}}$$

- **Sticky hometown population**
  - With strong local ties, hometown population tends to stay
  - Even if wages are low and rents are high
- **Hometown population matters** (unlike in RR)
  - RR wrongly attributes low wages and high rents to high QoL
  - L increases in $\bar{L}^b \Rightarrow$ **upward bias of urban QoL in** RR

Trade costs

$$\mathcal{E} \equiv \ln \frac{\hat{A}_{RR}}{\hat{A}} = \underbrace{-\ (1/\gamma)\ln \hat{L}}_{\text{idiosyncratic tastes}} \ \underbrace{+\ (1/\gamma)\ln \hat{\mathcal{L}}}_{\text{local ties}} \ \underbrace{-\ \alpha\beta \ln \hat{P}^t}_{\textbf{trade costs}} \ \underbrace{-\ \alpha(1-\beta)\ln \hat{p}^n}_{\text{local services}}$$

- **Goods sourced from all other locations**
  - Central locations face lower trade costs and lower prices
  - Lower living costs and higher real wages
- **Market access matters** (unlike in RR)
  - *RR* underestimates real wages and overestimates QoL
  - Larger cities have greater market access ⇒ **upward bias of urban QoL in** *RR*

Local services

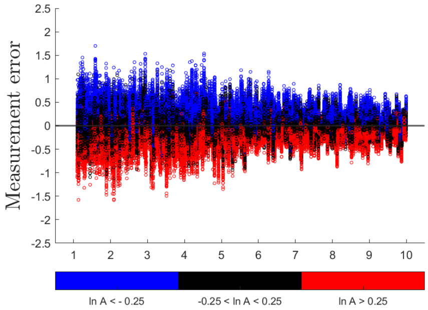
$$\mathcal{E} \equiv \ln \frac{\hat{A}_{RR}}{\hat{A}} = \underbrace{-\ (1/\gamma)\ln \hat{L}}_{\text{idiosyncratic tastes}} \ \underbrace{+\ (1/\gamma)\ln \hat{\mathcal{L}}}_{\text{local ties}} \ \underbrace{-\ \alpha\beta \ln \hat{P}^t}_{\text{trade costs}} \ \underbrace{-\ \alpha(1-\beta)\ln \hat{p}^n}_{\textbf{local services}}$$

- **QoL capitalizes into wages and rents**
  - Lower wages reduce the price of local services
  - Higher rents increase the price of local services
- **Role of local services depends on parameterization**
  - RR may underestimate or overstimate QoL
  - Positive or negative bias in urban QoL premium in RR

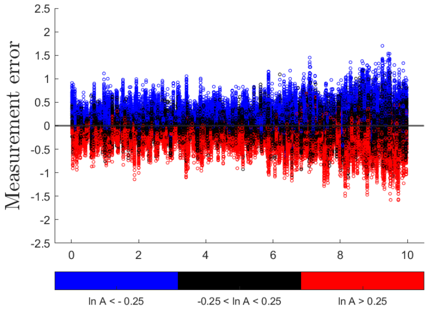
Monte Carlo Study

- $J = 144$ artificial cities (12×12) in $N = 1,000$ artificial countries (iterations) $\Rightarrow N \times J = 144,000$ equilibrium outcomes.
- Solve the full model numerically for randomly drawn fundamentals $\{\ln A_i, \ln \bar{\varphi}_i, \ln \eta_i\}$ (by city) and parameters $\{\gamma, \zeta\}$ (by country)
- Take canonical values for structural parameters from literature: $\{\alpha = 0.7; \beta = 0.5; \delta = 0.3; \mu = 0.8; \sigma = 5\}$.
- $\tau_{ij} = (\exp[-\iota * \ln dist_{ij}])^{\frac{1}{1-\sigma}}$. We take $\iota = -1$ from the literature.
- Centrality measure: $\mathcal{M}_i = \sum_{j \in J}(1/dist_{ij})$.

Determinants of measurement error in Rosen-Roback



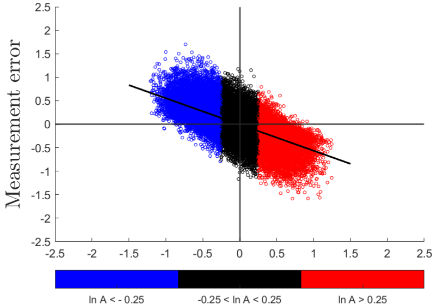
(a) Mobility frictions (γ)



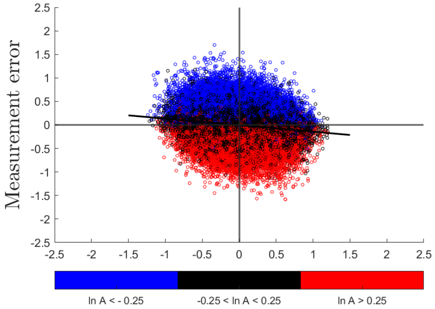
(b) Local ties (ξ)

1. **QoL is underestimated** for locations with **high QoL** $\widehat{A}$ and vice versa.
2. Measurement **error increases with greater mobility frictions**, i.e. greater idiosyncratic tastes (lower values of γ) and lower local ties (lower values of ξ).

Determinants of measurement error in Rosen-Roback



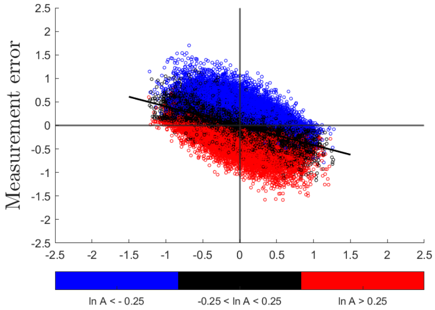
(c) Quality of life ($\ln \hat{A}$)



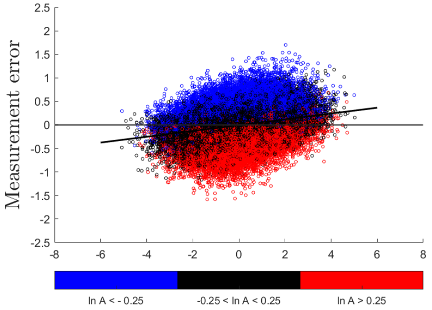
(d) Housing productivity ($\ln \hat{\eta}$)

1. In absolute terms, **measurement error increases with higher QoL** (c).
2. Similar for housing productivity, panel (d).

Determinants of measurement error in Rosen-Roback



(e) Labour productivity ($\ln \hat{\hat{\varphi}}$)



(f) Hometown population ($\ln \hat{\hat{L}}^b$)

1. ...same for labour productivity, panel (e).
2. Large **hometown pop.** works in **opposite** direction (as expected), panel (f).

Heterogeneity in measurement error

Frictions controlled:	(1)	(2)	(3)	(4)	(5)
Trade Costs		✓			
Local services			✓		
Local ties				✓	
Idiosyncratic tastes					✓
Measurement error: Intercept	-0.288	-0.305	-0.300	-0.169	-0.090
(Inverse) taste heterogeneity: γ-3	0.026	0.023	0.023	0.015	0.018
Strength of local ties: ξ-5	-0.017	-0.015	-0.015	0.015	-0.037
Market access: $\ln \hat{\mathcal{M}}$	-0.087	-0.156	-0.048	-0.053	-0.004
Quality of life: $\ln(\hat{A}/1.5)$	-0.711	-0.754	-0.741	-0.427	-0.212
Relative floor-space productivity : $\ln \hat{\tilde{\eta}}$	-0.189	-0.205	-0.270	-0.105	0.014
Relative worker productivity: $\ln \hat{\tilde{\varphi}}$	-0.547	-0.611	-0.591	-0.309	-0.130
Relative hometown population: $\ln \hat{L}^b$	0.073	0.062	0.065	-0.132	0.223
Observations	144,000	144,000	144,000	144,000	144,000
Adjusted R^2	0.977	0.985	0.984	0.783	0.807

Notes: Each column represents a different measurement error for a location with 50% higher quality of life than the numéraire location, so $\hat{A} = 1.5$. All explanatory variables are expressed relative to a numéraire location. The mobility friction parameter γ and local ties valuation ξ are re-scaled to have a zero value at $\gamma = 3$ and $\xi = 5$, respectively, so we can interpret the intercept as the measurement error for otherwise identical locations. All coefficients are highly significant.

Insights

- **Measurement error in RR for baseline city** is -25% $(= \exp[-0.288] - 1)$
  - City with $\widehat{A} = 1.5$, country with $\gamma = 3$ (Redding 2016) and $\xi = 5$ (Zabek 2024)
  - Increasing differences in $\hat{\mathcal{M}}$, $\hat{A}$, $\hat{\eta}$ or $\hat{\bar{\varphi}}$ by 50% magnifies the bias by $2.6pp$, $29.2pp$, $5.5pp$ and $14.9pp$, respectively.
  - The same change in $\ln \hat{L}^b$ reduces the bias by $2.3pp$ (Location is more attractive, but real wage increases are falsely interpreted as a lower QoL).
- **Mobility frictions matter more than trade frictions**
  - Controlling for mobility frictions reduces the bias more
  - **Shapley:** Tradable goods, 10.1%; local services, 4.5%; idiosyncratic tastes, 18.8%; and local ties, 66.6%
- Need to **control for local ties when controlling for idiosyncratic tastes**
  - Else, measurement error may be even larger (in opposite direction)

Urban Quality of Life Premium

- **Definition:** Elasticity of QoL with respect to city size.
  - In the DGP, $\hat{L}(\hat{A}, \hat{\eta}, \hat{\hat{\varphi}}, \hat{\mathcal{M}}, \hat{\hat{L}}^b) = c\hat{A}^{\gamma} \exp\left(\epsilon(\hat{\eta}, \hat{\hat{\varphi}}, \hat{\mathcal{M}}, \hat{\hat{L}}^b)\right)$
  - Can estimate the urban quality of life premium as follows:

$$\ln \hat{A} = \tilde{c} + \rho \ln \hat{L} + \underbrace{\left[-\frac{1}{\gamma}\epsilon\right]}_{\text{error term}}$$

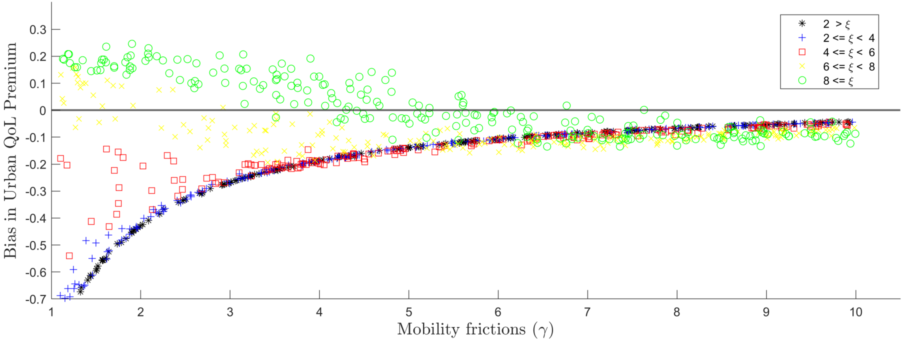
  - "Descriptive concept" since $cov(L, \epsilon) \neq 0$

- RR **does not even recover the descriptive statistic correctly**:

$$\ln \hat{A}_{RR} = \tilde{c} + \rho_{RR} \ln \hat{L} + \underbrace{\left[\mathcal{E} - \frac{1}{\gamma}\epsilon\right]}_{\text{error term}}$$

  - Additional bias since $cov(L, \mathcal{E}) \neq 0$

Urban QoL premium biases



Notes: Each dot represents an estimate of bias in the urban QoL premium $\mathcal{B} = \rho_{RR} - \rho$.

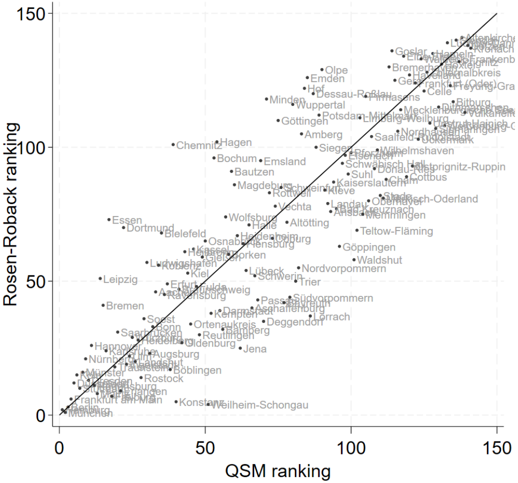
1. Urban QoL premium underestimated in RR unless $\xi > 6$ (implausible).
2. In the limit, with perfect mobility, bias converges to zero.

Quantification for Germany

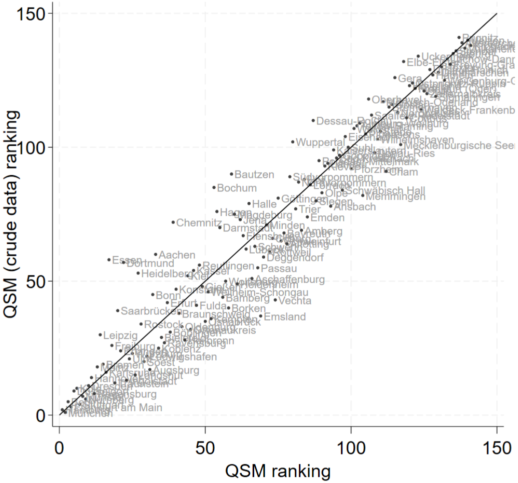
Data (2015) & parameter values

- Price indices
  - Housing prices from Ahlfeldt, Heblich & Seidel (2023)
  - Goods and services prices from Weinand & von Auer (2020)
- Employment and wages from IAB based on universe of German workers
  - Using place of vocational training (Ausbildung) as **hometown**
- $\alpha = 2/3$; from $1 - \alpha = 0.33$ (Federal Statistical Office 2020)
- $\beta = 0.34$ (own estimate)
- $\gamma = 3$ from Krebs and Pflueger (2023)
- $\xi = 5.5$ (own estimate) somewhat larger than for US
  - Consistent with a greater share of workers living in their hometowns

	QSM, best data		Rosen-Roback			QSM, crude data		
	Rank	$\hat{A}$	Rank	$\hat{A}_{RR}$	(4)/(2)	Rank	$\hat{A}_{CD}$	(7)/(2)
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Hamburg	1	2.081	2	1.737	0.834	2	1.648	0.792
München	2	2.033	1	1.963	0.965	3	1.644	0.809
Berlin	3	1.851	3	1.678	0.907	1	1.758	0.950
Frankfurt am Main	4	1.696	5	1.520	0.896	4	1.441	0.850
Düsseldorf	5	1.566	12	1.335	0.853	5	1.392	0.889
...	...	...	...	...	...	...	...	...
Freyung-Grafenau	137	0.538	126	0.688	1.278	139	0.547	1.017
Kronach	138	0.526	132	0.651	1.237	140	0.538	1.021
Stendal	139	0.522	141	0.575	1.101	130	0.617	1.183
Vulkaneifel	140	0.519	107	0.773	1.488	141	0.525	1.010
Uelzen	141	0.510	137	0.637	1.248	138	0.548	1.073
Standard deviation		0.276		0.253			0.223	



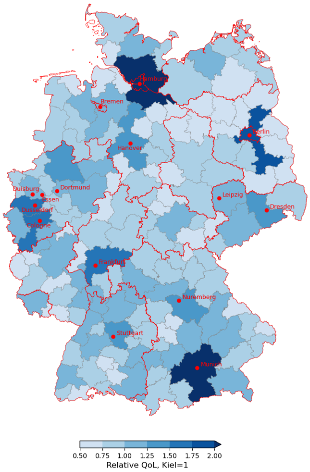
(a) Rosen-Roback (A_{RR})



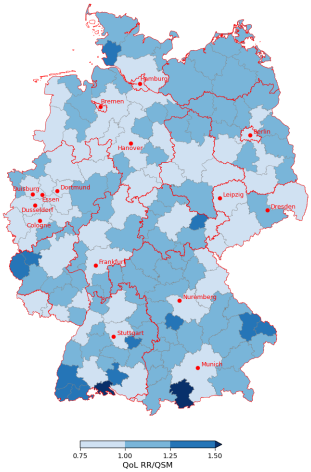
(b) QSM, crude data (A_{CD})

Variation over time

Relative QoL

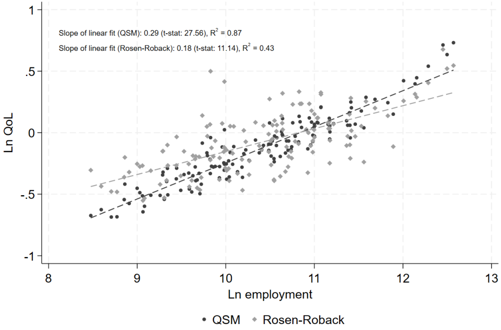


(a) QSM

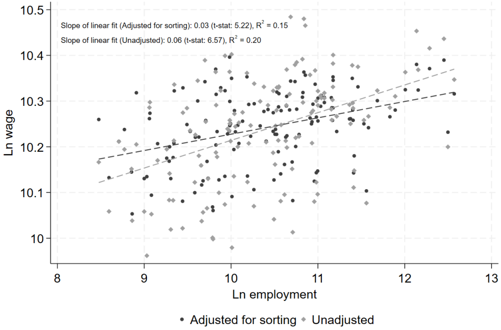


(b) RR/QSM

Urban quality-of-life premium vs. urban wage premium



(a) Urban quality of life premium



(b) Urban wage premium

1. Larger urban QoL premium when we account for spatial frictions
2. **Urban QoL premium exceeds the urban wage premium!**

Conclusions

Take-aways

- By **abstracting from spatial frictions**, estimates of QoL derived from the Rosen-Roback framework suffer from a **downward measurement error that increases in city size**.
- We document a **positive urban QoL premium for Germany.**
- **Quality of life** may be **an important agglomeration force** driving urbanisation—**even more so than productivity**.
- **Our results matter for 'correct' valuation of amenities and the optimal design of spatial transfers.**

Additional material

Theoretical Framework

- Worker ω from hometown m living in city i derives utility from the consumption of goods ($C_{i\omega}$) and floor space ($h_{i\omega}$) according to

$$U_{im\omega} = \left(\frac{C_{i\omega}}{\alpha}\right)^{\alpha} \left(\frac{h_{i\omega}}{1-\alpha}\right)^{1-\alpha} \exp[a_{im\omega}], \tag{1}$$

where $C_{i\omega} = (Q^t_{i\omega}/\beta)^{\beta}(q^n_{i\omega}/(1-\beta))^{1-\beta}$ with

$$Q^t_{i\omega} = \left[\sum_{j\in J} \left(q^t_{ji\omega}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{2}$$

and $q^n_{i\omega}$ (local non-traded services).

Mobility frictions

- The idiosyncratic amenity component $\exp[a_{im\omega}] = \exp[a_{i\omega} + \mathbb{1}\{m = i\} \cdot (\xi/\gamma)]$ is modelled as a stochastic preference shock for each location i, that is shifted upwards if the residence corresponds to the hometown. In particular, $a_{i\omega}$ is drawn from a type-I-extreme value (Gumbel) distribution:

$$F_i(a) = \exp\left(-\tilde{A}_i \exp\left\{-\left[\gamma a + \Gamma\right]\right\}\right) \quad \text{with} \quad \gamma > 0, \tag{3}$$

  where $\tilde{A}_i \equiv (A_i)^{\gamma}$ represents the mean of the amenity shock and Γ is the Euler-Mascheroni constant.

- A_i serves as an exogenous measure of quality of life (QoL).

- γ governs the dispersion of individual amenity shocks for each group $\Rightarrow$ introduces imperfect spatial arbitrage $\Rightarrow$ inverse measure of mobility frictions.

Technology: Floor space

- Supplied under perfect competition combining capital (supplied at an exogenous rate r_i^K) with location-specific land, $\bar{T}_i$:

$$H_i^S = \eta_i \left(\frac{\bar{T}_i}{\delta} \right)^{\delta} \left(\frac{K_i}{1-\delta} \right)^{1-\delta} . \tag{4}$$

- Total factor productivity η_i captures the role of regulatory (e.g. height regulations) and physical (e.g. a rugged surface) constraints (Saiz 2010).

- Floor space used for housing and as an input for local services.

Technology: Tradables

- Each location produces a unique variety of a tradable intermediate good (Armington 1969) using a CES-aggregate of labor, L_i^t, as the sole production input under perfect competition according to $q_i^t = \varphi_i L_i^t$

- Labour productivity $\varphi_i = \bar{\varphi}_i L_i^{\zeta}$ is increasing in local employment according to agglomeration elasticity ζ.

- $\tau_{ji} > 1$ units have to be shipped from j for one unit to arrive in i.

- Perfect competition implies $p_{ji}^t = \tau_{ji} w_j / \varphi_j$

Technology: Local services

- Requires both labour and floor space:

$$q_i^n = \nu_i^n \left(\frac{L_i^n}{\mu} \right)^{\mu} \left(\frac{H_i^n}{1 - \mu} \right)^{1-\mu}, \tag{5}$$

where H_i^n denotes floor space input.

- Perfect competition implies $p_i^n = \nu_i^n (w_i / \varphi_i)^{\mu} (p_i^H)^{1-\mu}$ and floor-space prices p_i^H.

Location choice

- Under the distributional assumptions on the idiosyncratic utility component, we obtain the probability that a worker from hometown m lives in location i:

$$\lambda_{im} = \frac{(A_i w_i / \mathcal{P}_i)^{\gamma} \cdot \exp[\mathbb{1}\{m = i\} \cdot \xi]}{\sum_{j \in J} (A_j w_j / \mathcal{P}_j)^{\gamma} \cdot \exp[\mathbb{1}\{m = j\} \cdot \xi]}, \tag{6}$$

where $\mathcal{P}_i \equiv \left(P_i^t\right)^{\alpha\beta} (p_i^n)^{\alpha(1-\beta)} (p_i^H)^{1-\alpha}$ is the aggregate consumer price index.

- Summing over all hometown probabilities, we obtain the residential choice probability:

$$\lambda_i = \sum_m \lambda_{im} = \frac{(A_i w_i/\mathcal{P}_i)^\gamma}{\sum_{j\in J}(A_j w_j/\mathcal{P}_j)^\gamma} \left(\sum_{m\neq i} \Psi_m^b \bar{L}_m^b + \Psi_i^b \cdot \exp[\xi]\bar{L}_i^b \right) / \bar{L}, \tag{7}$$

with $\Psi_m^b = \left(1 + \frac{(\exp[\xi]-1)(A_m w_m/\mathcal{P}_m)^\gamma}{\sum_{j\in J}(A_j w_j/\mathcal{P}_j)^\gamma}\right)^{-1} < 1$ being the utility discount associated with having left the hometown.

- Mobility of workers equalises expected utility in equilibrium.

General equilibrium

- Given model primitives, a general equilibrium of the economy is referenced by a vector of the endogenous objects $\mathbf{V} = \{L_i^n, L_i^t, p_i^H, p_i^n, P_i^t, r_i, w_i\}$.
- They are jointly determined by
  1. Market clearing for tradables and services
  2. Floor-space and land market clearing
  3. National labour-market clearing: $L_i = \lambda_i \bar{L} \quad \text{with} \quad \sum_{i \in J} \lambda_i = 1$
  4. Local labour resource constraint: $L_i = L_i^n + L_i^t$
  5. Aggregate consumer price index: $\mathcal{P}_i$

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