

Standard Deviation and Variance

Advanced

Deviation just means how far from the normal

Standard Deviation

The Standard Deviation is a measure of how spread out numbers are.

Its symbol is σ (the greek letter sigma)

The formula is easy: it is the **square root** of the **Variance**. So now you ask, "What is the Variance?"

Variance

The Variance is defined as:

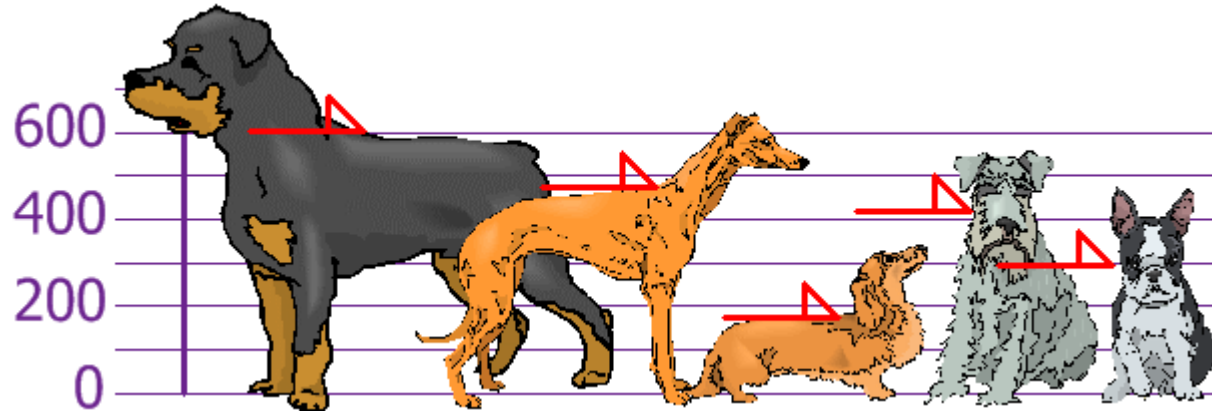
The average of the **squared** differences from the Mean.

To calculate the variance follow these steps:

- Work out the Mean (the simple average of the numbers)
- Then for each number: subtract the Mean and square the result (the *squared difference*).
- Then work out the average of those squared differences. (Why Square?)

Example

You and your friends have just measured the heights of your dogs (in millimeters):



The heights (at the shoulders) are: 600mm, 470mm, 170mm, 430mm and 300mm.

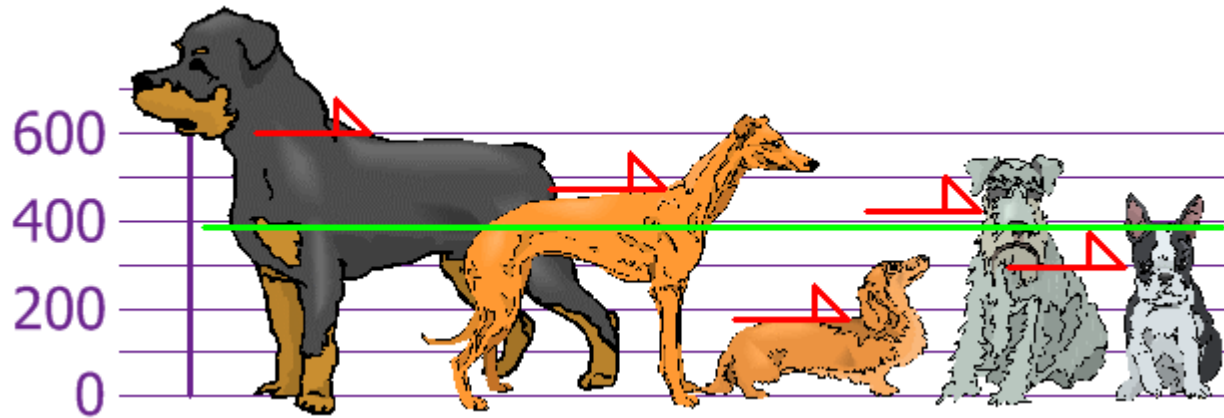
Find out the Mean, the Variance, and the Standard Deviation.

Your first step is to find the Mean:

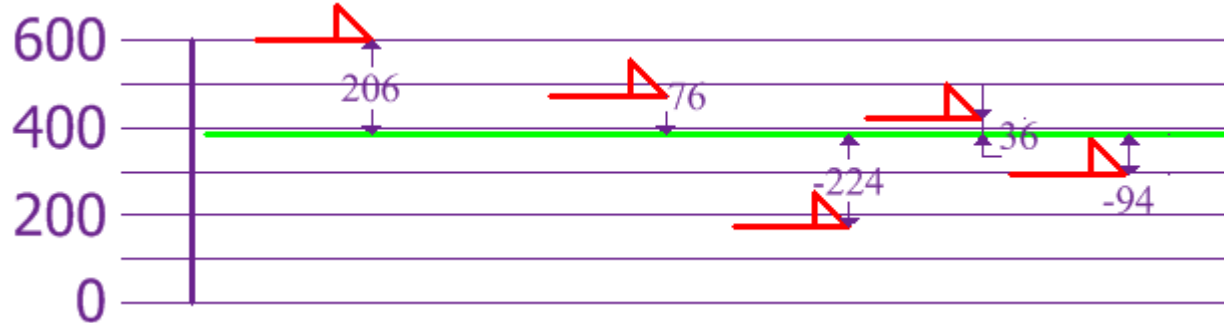
Answer:

$$\begin{aligned}
 \text{Mean} &= \frac{600 + 470 + 170 + 430 + 300}{5} \\
 &= \frac{1970}{5} \\
 &= 394
 \end{aligned}$$

so the mean (average) height is 394 mm. Let's plot this on the chart:



Now we calculate each dog's difference from the Mean:



To calculate the Variance, take each difference, square it, and then average the result:

Variance

$$\begin{aligned}
 \sigma^2 &= \frac{206^2 + 76^2 + (-224)^2 + 36^2 + (-94)^2}{5} \\
 &= \frac{42436 + 5776 + 50176 + 1296 + 8836}{5} \\
 &= \frac{108520}{5} \\
 &= 21704
 \end{aligned}$$

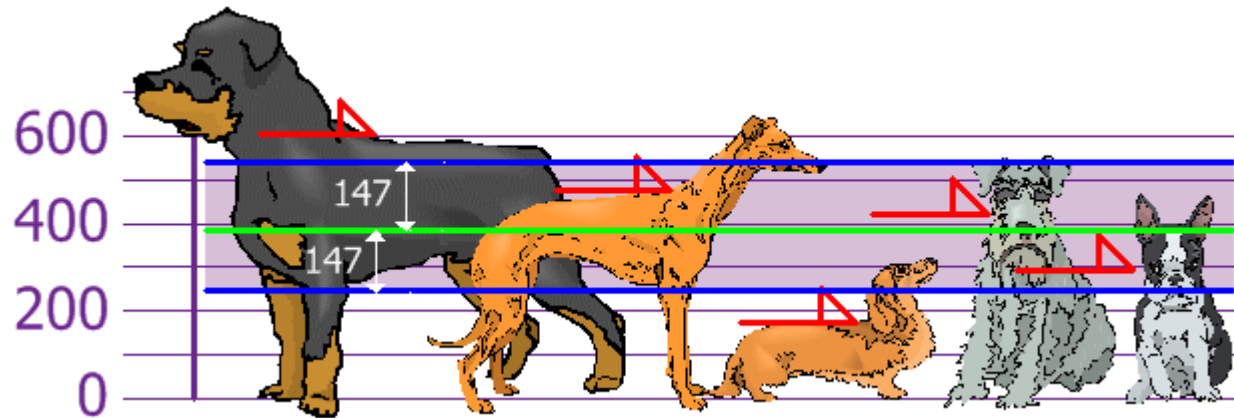
So the Variance is **21,704**

And the Standard Deviation is just the square root of Variance, so:

Standard Deviation

$$\begin{aligned}\sigma &= \sqrt{21704} \\ &= 147.32... \\ &= \mathbf{147} \text{ (to the nearest mm)}\end{aligned}$$

And the good thing about the Standard Deviation is that it is useful. Now we can show which heights are within one Standard Deviation (147mm) of the Mean:



So, using the Standard Deviation we have a "standard" way of knowing what is normal, and what is extra large or extra small.

Rottweilers **are** tall dogs. And Dachshunds **are** a bit short ... but don't tell them!

Now try the [Standard Deviation Calculator](https://www.mathsisfun.com/data/standard-deviation.html).

But ... there is a small change with Sample Data

Our example has been for a **Population** (the 5 dogs are the only dogs we are interested in).

But if the data is a **Sample** (a selection taken from a bigger Population), then the calculation changes!

When you have "N" data values that are:

- **The Population:** divide by **N** when calculating Variance (like we did)
- **A Sample:** divide by **N-1** when calculating Variance

All other calculations stay the same, including how we calculated the mean.

Example: if our 5 dogs are just a **sample** of a bigger population of dogs, we divide by **4** instead of **5** like this:

$$\text{Sample Variance} = 108,520 / 4 = 27,130$$

$$\text{Sample Standard Deviation} = \sqrt{27,130} = 164 \text{ (to the nearest mm)}$$

Think of it as a "correction" when your data is only a sample.

Formulas

Here are the two formulas, explained at [Standard Deviation Formulas](https://www.mathsisfun.com/data/standard-deviation.html) if you want to know more:

The "**Population** Standard Deviation":

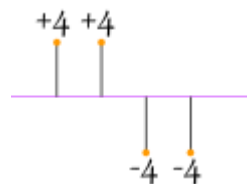
$$\sigma = \sqrt{\frac{1}{N} \sum_{i=1}^N (x_i - \mu)^2}$$

The "**Sample** Standard Deviation": $s = \sqrt{\frac{1}{N-1} \sum_{i=1}^N (x_i - \bar{x})^2}$

Looks complicated, but the important change is to divide by **N-1** (instead of **N**) when calculating a Sample Variance.

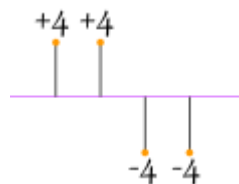
*Footnote: Why *square* the differences?

If we just add up the differences from the mean ... the negatives cancel the positives:



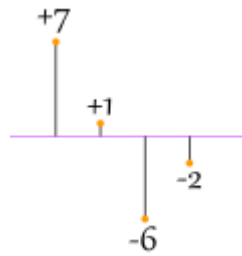
$$\frac{4 + 4 - 4 - 4}{4} = 0$$

So that won't work. How about we use absolute values?



$$\frac{|4| + |4| + |-4| + |-4|}{4} = \frac{4 + 4 + 4 + 4}{4} = 4$$

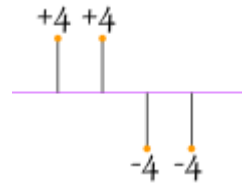
That looks good (and is the Mean Deviation), but what about this case:



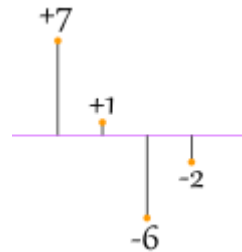
$$\frac{|7| + |1| + |-6| + |-2|}{4} = \frac{7 + 1 + 6 + 2}{4} = 4$$

Oh No! It also gives a value of 4, Even though the differences are more spread out.

So let us try squaring each difference (and taking the square root at the end):



$$\sqrt{\left(\frac{4^2 + 4^2 + 4^2 + 4^2}{4}\right)} = \sqrt{\left(\frac{64}{4}\right)} = 4$$



$$\sqrt{\left(\frac{7^2 + 1^2 + 6^2 + 2^2}{4}\right)} = \sqrt{\left(\frac{90}{4}\right)} = 4.74...$$

That is nice! The Standard Deviation is bigger when the differences are more spread out ... just what we want.

In fact this method is a similar idea to [distance between points](#), just applied in a different way.

And it is easier to use algebra on squares and square roots than absolute values, which makes the standard deviation easy to use in other areas of mathematics.

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