Weighted Mean

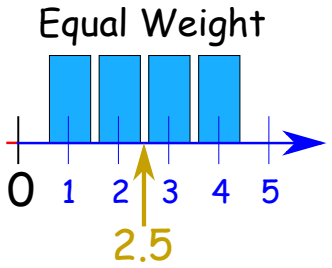
Also called Weighted Average

A mean where some values contribute more than others.

Mean

When we do a simple mean (or average), we give equal weight to each number.

Here is the mean of 1, 2, 3 and 4:



Add up the numbers, divide by how many numbers:

$$\text{Mean} = \frac{1 + 2 + 3 + 4}{4} = \frac{10}{4} = 2.5$$

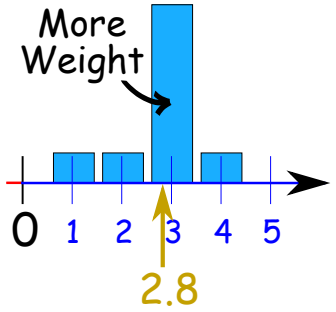
Weights

We could think that each of those numbers has a "weight" of ¼ (because there are 4 numbers):

$$\text{Mean} = \frac{1}{4} \times 1 + \frac{1}{4} \times 2 + \frac{1}{4} \times 3 + \frac{1}{4} \times 4$$
$$= 0.25 + 0.5 + 0.75 + 1 = \mathbf{2.5}$$

Same answer.

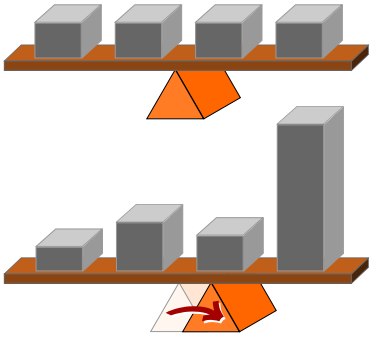
Now let's change the weight of **3** to 0.7, and the weights of the other numbers to 0.1 so **the total of the weights is still 1**:



$$\text{Mean} = 0.1 \times 1 + 0.1 \times 2 + 0.7 \times 3 + 0.1 \times 4$$
$$= 0.1 + 0.2 + 2.1 + 0.4 = \mathbf{2.8}$$

This **weighted mean** is now a little higher ("pulled" there by the weight of 3).

When some values get more weight than others, the central point (the mean) can change:



Decisions

Weighted means can help with decisions where some things are more important than others:

Example: Sam wants to buy a new camera, and decides on the following rating system:

- Image Quality **50%**
- Battery Life **30%**
- Zoom Range **20%**

The Sonu camera gets 8 (out of 10) for Image Quality, 6 for Battery Life and 7 for Zoom Range

The Conan camera gets 9 for Image Quality, 4 for Battery Life and 6 for Zoom Range

Which camera is best?

$$\text{Sonu: } 0.5 \times 8 + 0.3 \times 6 + 0.2 \times 7 = 4 + 1.8 + 1.4 = \mathbf{7.2}$$

$$\text{Conan: } 0.5 \times 9 + 0.3 \times 4 + 0.2 \times 6 = 4.5 + 1.2 + 1.2 = \mathbf{6.9}$$

Sam decides to buy the Sonu.

What if the Weights Don't Add to 1?

When the weights don't add to 1, divide by the sum of weights.

Example: Alex usually works 7 days a week, but sometimes just 1, 2, or 5 days.

Alex worked:

- on 2 weeks: 1 day each week
- on 14 weeks: 2 days each week
- on 8 weeks: 5 days each week
- on 32 weeks: 7 days each week

What is the mean number of days Alex works per week?

Use "Weeks" as the weighting:

$$\text{Weeks} \times \text{Days} = 2 \times 1 + 14 \times 2 + 8 \times 5 + 32 \times 7$$
$$= 2 + 28 + 40 + 224 = \mathbf{294}$$

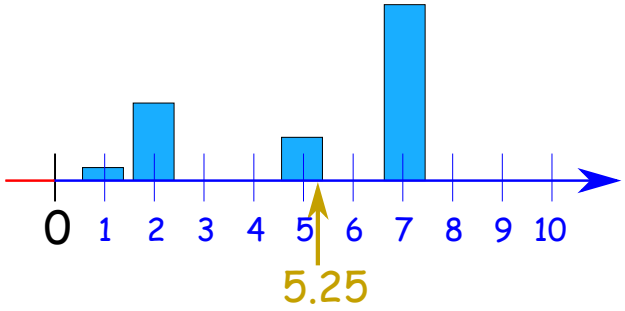
Also add up the weeks:

$$\text{Weeks} = 2 + 14 + 8 + 32 = \mathbf{56}$$

Divide:

$$\text{Mean} = \frac{294}{56} = 5.25$$

It looks like this:



But it is often better to use a table to make sure you have all the numbers correct:

Example (continued):

Let's use:

- **w** for the number of weeks (the weight)
- **x** for days (the value we want the mean of)

Multiply **w** by **x**, sum up **w** and sum up **wx**:

Weight w	Days x	wx
2	1	2
14	2	28
8	5	40
32	7	224
Σw = 56		Σwx = 294

Note: Σ (Sigma) means "Sum Up"

Divide **Σwx** by **Σx**:

$$\text{Mean} = \frac{294}{56} = 5.25$$

(Same answer.)

And that leads us to our formula:

$$\text{Weighted Mean} = \frac{\Sigma wx}{\Sigma w}$$

In other words: multiply each weight **w** by its matching value **x**, sum that all up, and divide by the sum of weights.

Summary

- **Weighted Mean**: A mean where some values contribute more than others.
- When the weights add to 1: just multiply each weight by the matching value and sum it all up
- Otherwise, multiply each weight **w** by its matching value **x**, sum that all up, and divide by the sum of weights:

$$\text{Weighted Mean} = \frac{\Sigma wx}{\Sigma w}$$

Question 1 Question 2 Question 3 Question 4 Question 5
Question 6 Question 7 Question 8 Question 9 Question 10