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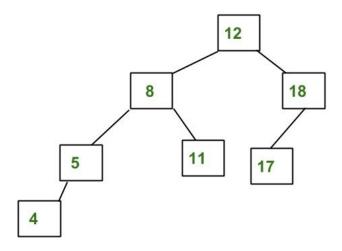
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AVL Tree | Set 1 (Insertion)

AVL tree is a self-balancing Binary Search Tree (BST) where the difference between heights of left and right subtrees cannot be more than one for all nodes.

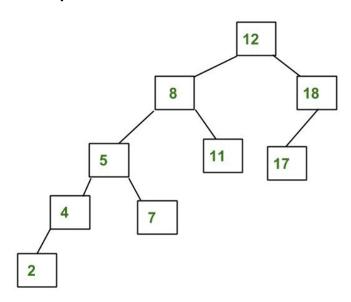
An Example Tree that is an AVL Tree



The above tree is AVL because differences between heights of left and right subtrees for every node is less than or equal to 1.



An Example Tree that is NOT an AVL Tree



The above tree is not AVL because differences between heights of left and right subtrees for 8 and 18 is greater than 1.

Why AVL Trees?

Most of the BST operations (e.g., search, max, min, insert, delete.. etc) take O(h) time where h is the height of the BST. The cost of these operations may become O(n) for a skewed Binary tree. If we make sure that height of the tree remains O(Logn) after every insertion and deletion, then we can guarantee an upper bound of O(Logn) for all these operations. The height of an AVL tree is always O(Logn) where n is the number of nodes in the tree (See this video lecture for proof).

Insertion

To make sure that the given tree remains AVL after every insertion, we must augment the standard BST insert operation to performed some re-balancing. Following are two basic operations that can be performed to re-balance a BST without violating the BST property

```
(keys(left) < key(root) < keys(right)).
```

- 1) Left Rotation
- 2) Right Rotation

```
T1, T2 and T3 are subtrees of the tree rooted with y (on the left side) or x (on the right side)

y

X

/ \ Right Rotation

x T3 ----->

T1 y

/ \ <-----

T1 T2 Left Rotation

T2 T3

Keys in both of the above trees follow the following order

keys(T1) < key(x) < keys(T2) < key(y) < keys(T3)

So BST property is not violated anywhere.
```

Steps to follow for insertion

Let the newly inserted node be w

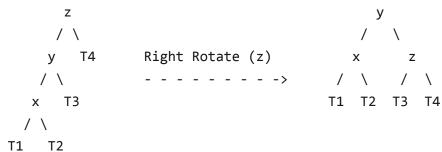
- 1) Perform standard BST insert for w.
- 2) Starting from w, travel up and find the first unbalanced node. Let z be the first unbalanced node, y be the child of z that comes on the path from w to z and x be the grandchild of z that comes on the path from w to z.
- **3)** Re-balance the tree by performing appropriate rotations on the subtree rooted with z. There can be 4 possible cases that needs to be handled as x, y and z can be arranged in 4 ways. Following are the possible 4 arrangements:
- a) y is left child of z and x is left child of y (Left Left Case)
- b) y is left child of z and x is right child of y (Left Right Case)
- c) y is right child of z and x is right child of y (Right Right Case)
- d) y is right child of z and x is left child of y (Right Left Case)

Following are the operations to be performed in above mentioned 4 cases. In all of the cases, we only need to re-balance the subtree rooted with z and the complete tree becomes balanced as the height of subtree (After appropriate rotations) rooted with z becomes

same as it was before insertion. (See this video lecture for proof)

a) Left Left Case

T1, T2, T3 and T4 are subtrees.

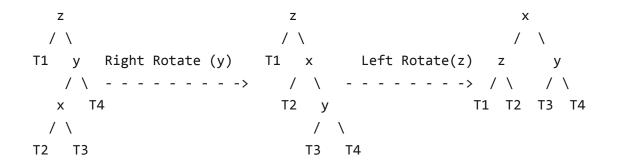


b) Left Right Case

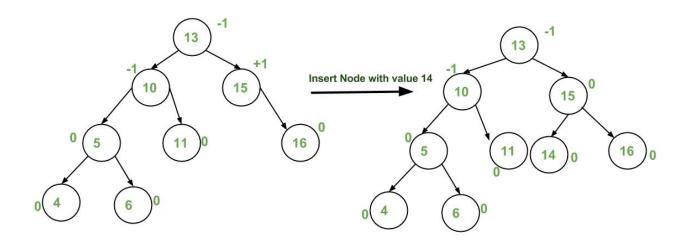
c) Right Right Case



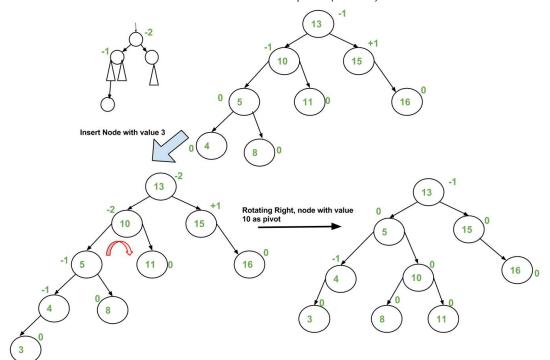
d) Right Left Case



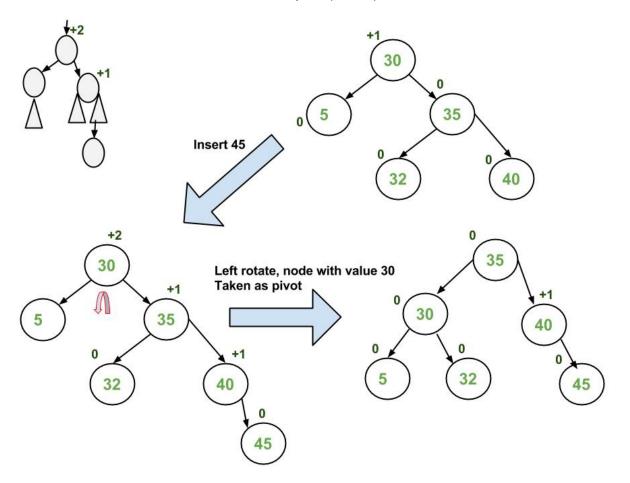
Insertion Examples:

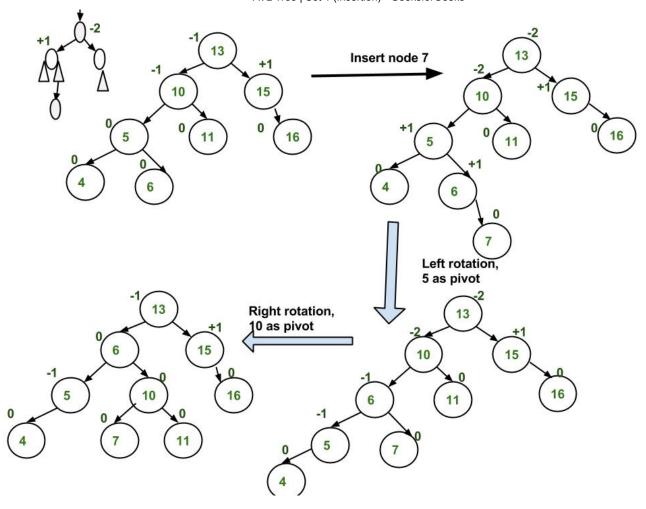




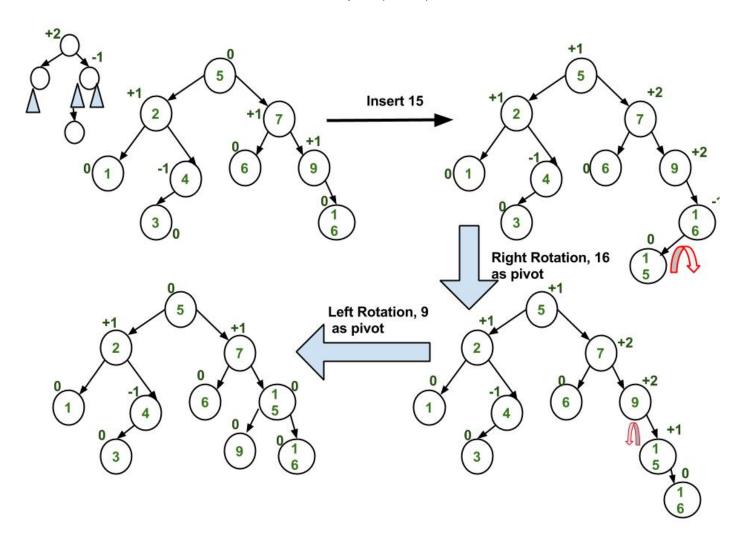












Recommended: Please solve it on "PRACTICE" first, before moving on to the solution.

implementation

Following is the implementation for AVL Tree Insertion. The following implementation uses the recursive BST insert to insert a new node. In the recursive BST insert, after insertion, we get pointers to all ancestors one by one in a bottom-up manner. So we don't parent pointer to travel up. The recursive code itself travels up and visits all the ancestors of the newly inserted node.

1) Perform the normal BST insertion.

- 2) The current node must be one of the ancestors of the newly inserted node. Update the height of the current node.
- 3) Get the balance factor (left subtree height right subtree height) of the current node.
- 4) If balance factor is greater than 1, then the current node is unbalanced and we are either in Left Left case or left Right case. To check whether it is left left case or not, compare the newly inserted key with the key in left subtree root.
- 5) If balance factor is less than -1, then the current node is unbalanced and we are either in Right Right case or Right-Left case. To check whether it is Right Right case or not, compare the newly inserted key with the key in right subtree root.

C++

```
// C++ program to insert a node in AVL tree
#include<bits/stdc++.h>
using namespace std;
// An AVL tree node
class Node
    public:
    int key;
    Node *left;
    Node *right;
    int height;
};
// A utility function to get maximum
// of two integers
int max(int a, int b);
// A utility function to get the
// height of the tree
int height(Node *N)
    if (N == NULL)
        return 0;
    return N->height;
}
// A utility function to get maximum
```



```
// of two integers
int max(int a, int b)
{
    return (a > b)? a : b;
}
/* Helper function that allocates a
   new node with the given key and
   NULL left and right pointers. */
Node* newNode(int key)
{
    Node* node = new Node();
    node->key = key;
    node->left = NULL;
    node->right = NULL;
    node->height = 1; // new node is initially
                       // added at leaf
    return(node);
}
// A utility function to right
// rotate subtree rooted with y
// See the diagram given above.
Node *rightRotate(Node *y)
    Node *x = y->left;
    Node *T2 = x - right;
    // Perform rotation
    x \rightarrow right = y;
    y \rightarrow left = T2;
    // Update heights
    y->height = max(height(y->left),
                    height(y->right)) + 1;
    x->height = max(height(x->left),
                     height(x->right)) + 1;
    // Return new root
    return x;
}
```

```
// A utility function to left
// rotate subtree rooted with x
// See the diagram given above.
Node *leftRotate(Node *x)
    Node *y = x - right;
    Node *T2 = y->left;
    // Perform rotation
    y \rightarrow left = x;
    x \rightarrow right = T2;
    // Update heights
    x->height = max(height(x->left),
                     height(x->right)) + 1;
    y->height = max(height(y->left),
                     height(y->right)) + 1;
    // Return new root
    return y;
}
// Get Balance factor of node N
int getBalance(Node *N)
    if (N == NULL)
        return 0;
    return height(N->left) - height(N->right);
}
// Recursive function to insert a key
// in the subtree rooted with node and
// returns the new root of the subtree.
Node* insert(Node* node, int key)
    /* 1. Perform the normal BST insertion */
    if (node == NULL)
        return(newNode(key));
    if (key < node->key)
        node->left = insert(node->left, key);
    else if (key > node->key)
```

```
node->right = insert(node->right, key);
else // Equal keys are not allowed in BST
    return node;
/* 2. Update height of this ancestor node */
node->height = 1 + max(height(node->left),
                    height(node->right));
/* 3. Get the balance factor of this ancestor
    node to check whether this node became
    unbalanced */
int balance = getBalance(node);
// If this node becomes unbalanced, then
// there are 4 cases
// Left Left Case
if (balance > 1 && key < node->left->key)
    return rightRotate(node);
// Right Right Case
if (balance < -1 && key > node->right->key)
    return leftRotate(node);
// Left Right Case
if (balance > 1 && key > node->left->key)
{
    node->left = leftRotate(node->left);
    return rightRotate(node);
// Right Left Case
if (balance < -1 && key < node->right->key)
    node->right = rightRotate(node->right);
    return leftRotate(node);
}
/* return the (unchanged) node pointer */
return node;
```

}

```
// A utility function to print preorder
// traversal of the tree.
// The function also prints height
// of every node
void preOrder(Node *root)
{
    if(root != NULL)
        cout << root->key << " ";</pre>
        preOrder(root->left);
        preOrder(root->right);
    }
}
// Driver Code
int main()
    Node *root = NULL;
    /* Constructing tree given in
    the above figure */
    root = insert(root, 10);
    root = insert(root, 20);
    root = insert(root, 30);
    root = insert(root, 40);
    root = insert(root, 50);
    root = insert(root, 25);
    /* The constructed AVL Tree would be
                30
            / \
            20 40
            / \ \
        10 25 50
    cout << "Preorder traversal of the "</pre>
            "constructed AVL tree is \n";
    preOrder(root);
    return 0;
}
```

```
// This code is contributed by
// rathbhupendra
```

C

```
// C program to insert a node in AVL tree
#include<stdio.h>
#include<stdlib.h>
// An AVL tree node
struct Node
    int key;
    struct Node *left;
    struct Node *right;
    int height;
};
// A utility function to get maximum of two integers
int max(int a, int b);
// A utility function to get the height of the tree
int height(struct Node *N)
    if (N == NULL)
        return 0;
    return N->height;
}
// A utility function to get maximum of two integers
int max(int a, int b)
{
    return (a > b)? a : b;
}
/* Helper function that allocates a new node with the given key and
    NULL left and right pointers. */
struct Node* newNode(int key)
{
```

```
struct Node* node = (struct Node*)
                         malloc(sizeof(struct Node));
    node->key = key;
    node->left = NULL;
    node->right = NULL;
    node->height = 1; // new node is initially added at leaf
    return(node);
}
// A utility function to right rotate subtree rooted with y
// See the diagram given above.
struct Node *rightRotate(struct Node *y)
{
    struct Node *x = y->left;
    struct Node *T2 = x->right;
    // Perform rotation
    x \rightarrow right = y;
    y \rightarrow left = T2;
    // Update heights
    y->height = max(height(y->left), height(y->right))+1;
    x->height = max(height(x->left), height(x->right))+1;
    // Return new root
    return x;
}
\ensuremath{//} A utility function to left rotate subtree rooted with x
// See the diagram given above.
struct Node *leftRotate(struct Node *x)
    struct Node *y = x->right;
    struct Node *T2 = y->left;
    // Perform rotation
    y \rightarrow left = x;
    x \rightarrow right = T2;
    // Update heights
    x->height = max(height(x->left), height(x->right))+1;
    y->height = max(height(y->left), height(y->right))+1;
```

```
// Return new root
    return y;
}
// Get Balance factor of node N
int getBalance(struct Node *N)
    if (N == NULL)
        return 0;
    return height(N->left) - height(N->right);
}
// Recursive function to insert a key in the subtree rooted
// with node and returns the new root of the subtree.
struct Node* insert(struct Node* node, int key)
{
    /* 1. Perform the normal BST insertion */
    if (node == NULL)
        return(newNode(key));
    if (key < node->key)
        node->left = insert(node->left, key);
    else if (key > node->key)
        node->right = insert(node->right, key);
    else // Equal keys are not allowed in BST
        return node;
    /* 2. Update height of this ancestor node */
    node->height = 1 + max(height(node->left),
                           height(node->right));
    /* 3. Get the balance factor of this ancestor
          node to check whether this node became
          unbalanced */
    int balance = getBalance(node);
    // If this node becomes unbalanced, then
    // there are 4 cases
    // Left Left Case
    if (balance > 1 && key < node->left->key)
```

```
return rightRotate(node);
    // Right Right Case
    if (balance < -1 && key > node->right->key)
        return leftRotate(node);
    // Left Right Case
   if (balance > 1 && key > node->left->key)
        node->left = leftRotate(node->left);
        return rightRotate(node);
   // Right Left Case
    if (balance < -1 && key < node->right->key)
        node->right = rightRotate(node->right);
        return leftRotate(node);
    /* return the (unchanged) node pointer */
    return node;
}
// A utility function to print preorder traversal
// of the tree.
// The function also prints height of every node
void preOrder(struct Node *root)
{
    if(root != NULL)
        printf("%d ", root->key);
        preOrder(root->left);
        preOrder(root->right);
    }
}
/* Drier program to test above function*/
int main()
  struct Node *root = NULL;
```

```
/* Constructing tree given in the above figure */
root = insert(root, 10);
root = insert(root, 20);
root = insert(root, 30);
root = insert(root, 40);
root = insert(root, 50);
root = insert(root, 25);
/* The constructed AVL Tree would be
          30
       20
     10 25
               50
*/
printf("Preorder traversal of the constructed AVL"
       " tree is \n");
preOrder(root);
return 0;
```

Java

```
// Java program for insertion in AVL Tree
class Node {
   int key, height;
   Node left, right;

   Node(int d) {
      key = d;
      height = 1;
   }
}
class AVLTree {
   Node root;
```

```
// A utility function to get the height of the tree
int height(Node N) {
    if (N == null)
        return 0;
    return N.height;
}
// A utility function to get maximum of two integers
int max(int a, int b) {
    return (a > b) ? a : b;
}
// A utility function to right rotate subtree rooted with y
// See the diagram given above.
Node rightRotate(Node y) {
    Node x = y.left;
    Node T2 = x.right;
    // Perform rotation
   x.right = y;
   y.left = T2;
   // Update heights
   y.height = max(height(y.left), height(y.right)) + 1;
    x.height = max(height(x.left), height(x.right)) + 1;
    // Return new root
    return x;
}
// A utility function to left rotate subtree rooted with x
// See the diagram given above.
Node leftRotate(Node x) {
    Node y = x.right;
    Node T2 = y.left;
    // Perform rotation
   y.left = x;
   x.right = T2;
```

```
// Update heights
   x.height = max(height(x.left), height(x.right)) + 1;
   y.height = max(height(y.left), height(y.right)) + 1;
    // Return new root
    return y;
// Get Balance factor of node N
int getBalance(Node N) {
    if (N == null)
        return 0;
    return height(N.left) - height(N.right);
}
Node insert(Node node, int key) {
    /* 1. Perform the normal BST insertion */
    if (node == null)
        return (new Node(key));
    if (key < node.key)</pre>
        node.left = insert(node.left, key);
    else if (key > node.key)
        node.right = insert(node.right, key);
    else // Duplicate keys not allowed
        return node;
    /* 2. Update height of this ancestor node */
    node.height = 1 + max(height(node.left),
                          height(node.right));
    /* 3. Get the balance factor of this ancestor
          node to check whether this node became
          unbalanced */
    int balance = getBalance(node);
   // If this node becomes unbalanced, then there
   // are 4 cases Left Left Case
    if (balance > 1 && key < node.left.key)</pre>
        return rightRotate(node);
```

```
// Right Right Case
    if (balance < -1 && key > node.right.key)
        return leftRotate(node);
    // Left Right Case
    if (balance > 1 && key > node.left.key) {
        node.left = leftRotate(node.left);
        return rightRotate(node);
    }
    // Right Left Case
    if (balance < -1 && key < node.right.key) {</pre>
        node.right = rightRotate(node.right);
        return leftRotate(node);
    }
    /* return the (unchanged) node pointer */
    return node;
// A utility function to print preorder traversal
// of the tree.
// The function also prints height of every node
void preOrder(Node node) {
    if (node != null) {
        System.out.print(node.key + " ");
        preOrder(node.left);
        preOrder(node.right);
}
public static void main(String[] args) {
    AVLTree tree = new AVLTree();
    /* Constructing tree given in the above figure */
    tree.root = tree.insert(tree.root, 10);
    tree.root = tree.insert(tree.root, 20);
    tree.root = tree.insert(tree.root, 30);
    tree.root = tree.insert(tree.root, 40);
    tree.root = tree.insert(tree.root, 50);
    tree.root = tree.insert(tree.root, 25);
```

Python3

```
# Python code to insert a node in AVL tree
# Generic tree node class
class TreeNode(object):
    def __init__(self, val):
        self.val = val
        self.left = None
        self.right = None
        self.height = 1
# AVL tree class which supports the
# Insert operation
class AVL_Tree(object):
    # Recursive function to insert key in
    # subtree rooted with node and returns
    # new root of subtree.
    def insert(self, root, key):
        # Step 1 - Perform normal BST
        if not root:
            return TreeNode(key)
```

```
elif key < root.val:</pre>
        root.left = self.insert(root.left, key)
    else:
        root.right = self.insert(root.right, key)
    # Step 2 - Update the height of the
    # ancestor node
    root.height = 1 + max(self.getHeight(root.left),
                       self.getHeight(root.right))
    # Step 3 - Get the balance factor
    balance = self.getBalance(root)
    # Step 4 - If the node is unbalanced,
    # then try out the 4 cases
    # Case 1 - Left Left
    if balance > 1 and key < root.left.val:</pre>
        return self.rightRotate(root)
    # Case 2 - Right Right
    if balance < -1 and key > root.right.val:
        return self.leftRotate(root)
    # Case 3 - Left Right
    if balance > 1 and key > root.left.val:
        root.left = self.leftRotate(root.left)
        return self.rightRotate(root)
    # Case 4 - Right Left
    if balance < -1 and key < root.right.val:</pre>
        root.right = self.rightRotate(root.right)
        return self.leftRotate(root)
    return root
def leftRotate(self, z):
    y = z.right
    T2 = y.left
    # Perform rotation
    y.left = z
```

```
z.right = T2
   # Update heights
    z.height = 1 + max(self.getHeight(z.left),
                     self.getHeight(z.right))
   y.height = 1 + max(self.getHeight(y.left),
                     self.getHeight(y.right))
    # Return the new root
    return y
def rightRotate(self, z):
   y = z.left
   T3 = y.right
    # Perform rotation
   y.right = z
    z.left = T3
   # Update heights
    z.height = 1 + max(self.getHeight(z.left),
                    self.getHeight(z.right))
   y.height = 1 + max(self.getHeight(y.left),
                    self.getHeight(y.right))
    # Return the new root
    return y
def getHeight(self, root):
    if not root:
        return 0
    return root.height
def getBalance(self, root):
    if not root:
        return 0
    return self.getHeight(root.left) - self.getHeight(root.right)
def preOrder(self, root):
```

```
if not root:
            return
        print("{0} ".format(root.val), end="")
        self.preOrder(root.left)
        self.preOrder(root.right)
# Driver program to test above function
myTree = AVL Tree()
root = None
root = myTree.insert(root, 10)
root = myTree.insert(root, 20)
root = myTree.insert(root, 30)
root = myTree.insert(root, 40)
root = myTree.insert(root, 50)
root = myTree.insert(root, 25)
"""The constructed AVL Tree would be
            30
         20
       10 25
# Preorder Traversal
print("Preorder traversal of the",
      "constructed AVL tree is")
myTree.preOrder(root)
print()
# This code is contributed by Ajitesh Pathak
C#
// C# program for insertion in AVL Tree
using System;
```

```
class Node
   public int key, height;
   public Node left, right;
   public Node(int d)
        key = d;
        height = 1;
}
public class AVLTree
   Node root;
   // A utility function to get
   // the height of the tree
   int height(Node N)
    {
        if (N == null)
            return 0;
        return N.height;
    }
   // A utility function to get
   // maximum of two integers
   int max(int a, int b)
        return (a > b) ? a : b;
   // A utility function to right
   // rotate subtree rooted with y
   // See the diagram given above.
   Node rightRotate(Node y)
        Node x = y.left;
        Node T2 = x.right;
```

```
// Perform rotation
   x.right = y;
   y.left = T2;
    // Update heights
   y.height = max(height(y.left),
                height(y.right)) + 1;
    x.height = max(height(x.left),
                height(x.right)) + 1;
    // Return new root
    return x;
// A utility function to left
// rotate subtree rooted with x
// See the diagram given above.
Node leftRotate(Node x)
    Node y = x.right;
   Node T2 = y.left;
    // Perform rotation
   y.left = x;
   x.right = T2;
    // Update heights
    x.height = max(height(x.left),
                height(x.right)) + 1;
    y.height = max(height(y.left),
                height(y.right)) + 1;
    // Return new root
    return y;
}
// Get Balance factor of node N
int getBalance(Node N)
    if (N == null)
        return 0;
```

```
return height(N.left) - height(N.right);
}
Node insert(Node node, int key)
    /* 1. Perform the normal BST insertion */
    if (node == null)
        return (new Node(key));
    if (key < node.key)</pre>
        node.left = insert(node.left, key);
    else if (key > node.key)
        node.right = insert(node.right, key);
    else // Duplicate keys not allowed
        return node;
    /* 2. Update height of this ancestor node */
    node.height = 1 + max(height(node.left),
                        height(node.right));
    /* 3. Get the balance factor of this ancestor
        node to check whether this node became
        unbalanced */
    int balance = getBalance(node);
    // If this node becomes unbalanced, then there
    // are 4 cases Left Left Case
    if (balance > 1 && key < node.left.key)</pre>
        return rightRotate(node);
    // Right Right Case
    if (balance < -1 && key > node.right.key)
        return leftRotate(node);
    // Left Right Case
    if (balance > 1 && key > node.left.key)
        node.left = leftRotate(node.left);
        return rightRotate(node);
```

```
// Right Left Case
    if (balance < -1 && key < node.right.key)</pre>
        node.right = rightRotate(node.right);
        return leftRotate(node);
    /* return the (unchanged) node pointer */
    return node;
// A utility function to print preorder traversal
// of the tree.
// The function also prints height of every node
void preOrder(Node node)
    if (node != null)
        Console.Write(node.key + " ");
        preOrder(node.left);
        preOrder(node.right);
}
// Driver code
public static void Main(String[] args)
    AVLTree tree = new AVLTree();
    /* Constructing tree given in the above figure */
    tree.root = tree.insert(tree.root, 10);
    tree.root = tree.insert(tree.root, 20);
    tree.root = tree.insert(tree.root, 30);
    tree.root = tree.insert(tree.root, 40);
    tree.root = tree.insert(tree.root, 50);
    tree.root = tree.insert(tree.root, 25);
    /* The constructed AVL Tree would be
        30
        / \
    20 40
```

Output:

```
Preorder traversal of the constructed AVL tree is 30 20 10 25 40 50
```

Time Complexity: The rotation operations (left and right rotate) take constant time as only a few pointers are being changed there. Updating the height and getting the balance factor also takes constant time. So the time complexity of AVL insert remains same as BST insert which is O(h) where h is the height of the tree. Since AVL tree is balanced, the height is O(Logn). So time complexity of AVL insert is O(Logn).

Comparison with Red Black Tree

The AVL tree and other self-balancing search trees like Red Black are useful to get all basic operations done in O(log n) time. The AVL trees are more balanced compared to Red-Black Trees, but they may cause more rotations during insertion and deletion. So if your application involves many frequent insertions and deletions, then Red Black trees should be preferred. And if the insertions and deletions are less frequent and search is the more frequent operation, then AVL tree should be preferred over Red Black Tree.

Following is the post for delete.

```
AVL Tree | Set 2 (Deletion)
```

Following are some posts that have used self-balancing search trees.



Median in a stream of integers (running integers)

Maximum of all subarrays of size k

Count smaller elements on right side

AVL Tree - Insertion | GeeksforGeeks

References:

IITD Video Lecture on AVL Tree Introduction

IITD Video Lecture on AVL Tree Insertion and Deletion

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

Recommended Posts:

Threaded Binary Tree | Insertion

ScapeGoat Tree | Set 1 (Introduction and Insertion)

C Program for Red Black Tree Insertion

Left Leaning Red Black Tree (Insertion)

Optimal sequence for AVL tree insertion (without any rotations)

Insertion in a Binary Tree in level order

Binary Search Tree | Set 1 (Search and Insertion)

Skip List | Set 2 (Insertion)

Fibonacci Heap - Insertion and Union

Insertion in Unrolled Linked List

Insertion at Specific Position in a Circular Doubly Linked List

Maximum sub-tree sum in a Binary Tree such that the sub-tree is also a BST

Complexity of different operations in Binary tree, Binary Search Tree and AVL tree

Overview of Data Structures | Set 3 (Graph, Trie, Segment Tree and Suffix Tree)

Given level order traversal of a Binary Tree, check if the Tree is a Min-Heap

Improved By: princiraj1992, rathbhupendra

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