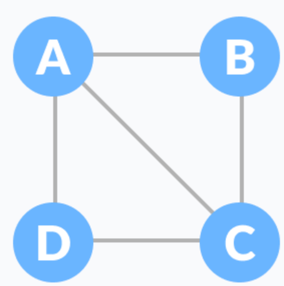
Spanning Tree and Minimum Spanning Tree

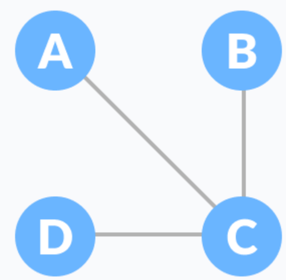
In this tutorial, you will learn about spanning tree and minimum spanning tree with help of examples and figures.

Before we learn about spanning trees, we need to understand two graphs: undirected graphs and connected graphs.

An **undirected graph** is a graph in which the edges do not point in any direction (ie. the edges are bidirectional).



Undirected Graph



Connected Graph

Spanning tree

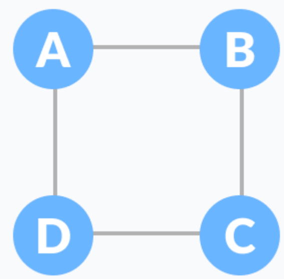
A spanning tree is a sub-graph of an undirected and a connected graph, which includes all the vertices of the graph having a minimum possible number of edges. If a vertex is missed, then it is not a spanning tree.

The edges may or may not have weights assigned to them.

The total number of spanning trees with `n` vertices that can be created from a complete graph is equal to `n(n-2)`.

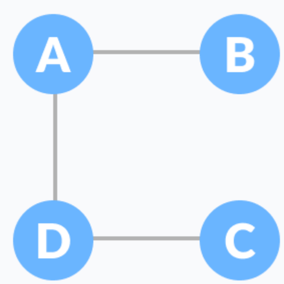
Let's understand the spanning tree with examples below:

Let the original graph be:

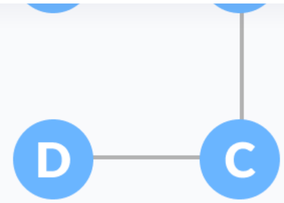


Normal graph

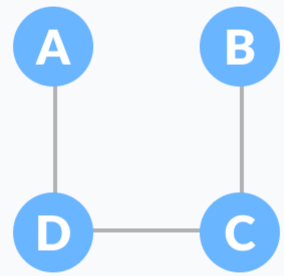
Some of the possible spanning trees that can be created from the above graph are:



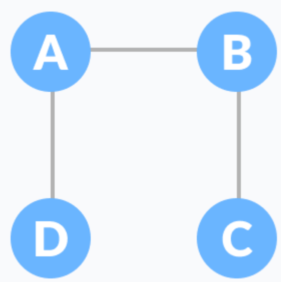
A spanning tree



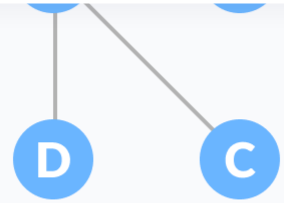
A spanning tree



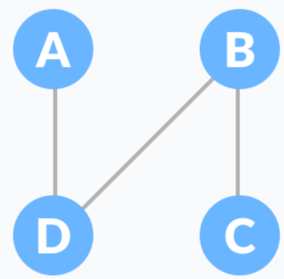
A spanning tree



A spanning tree



A spanning tree



A spanning tree

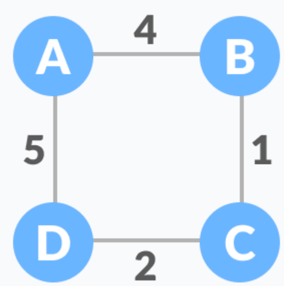
We have `n = 4`, thus the maximum number of possible spanning trees is equal to 4^{4-2} `= 16`. Thus, 16 spanning trees can be formed from the above graph.

Minimum Spanning Tree

Example of a Spanning Tree

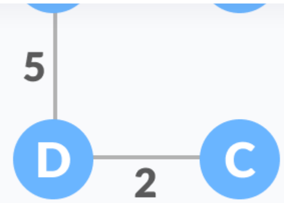
Let's understand the above definition with the help of the example below.

The initial graph is:



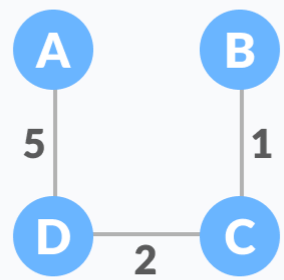
Weighted graph

The possible spanning trees from the above graph are:



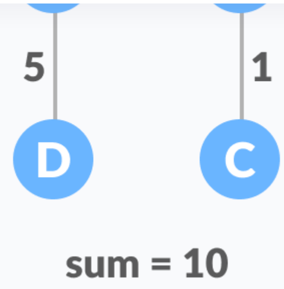
sum = 11

Minimum spanning tree - 1

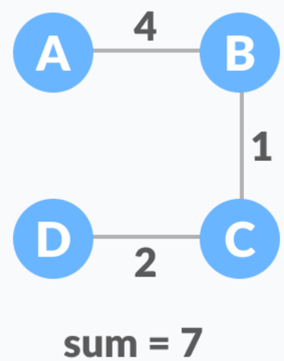


sum = 8

Minimum spanning tree - 2

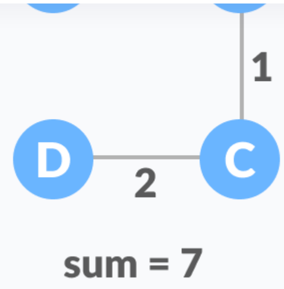


Minimum spanning tree - 3



Minimum spanning tree - 4

The minimum spanning tree from the above spanning trees is:



Minimum spanning tree

The minimum spanning tree from a graph is found using the following algorithms:

1. Prim's Algorithm (/dsa/prim-algorithm)
2. Kruskal's Algorithm (/dsa/kruskal-algorithm)

Spanning Tree Applications

- Computer Network Routing Protocol
- Cluster Analysis
- Civil Network Planning

- To find paths in the map
- To design networks like telecommunication networks, water supply networks, and electrical grids.

Next Tutorial:
Strongly Connected Components → (/dsa/strongly-connected-components)

Previous Tutorial:
Graph Data Structure (/dsa/graph)

DS & Algorithms

Linear Search

(/dsa/linear-search)

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Backtracking Algorithm

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Greedy Algorithm

(/dsa/greedy-algorithm)

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Types of Queue