← Notes

Heaps and Priority Queues

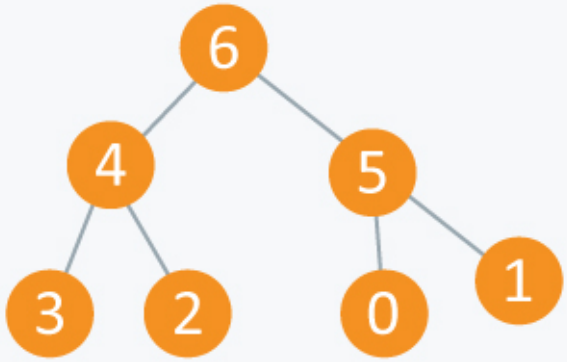
182 Code Monk Heap Heapsort Priority-queue

Heaps:

A heap is a specific tree based data structure in which all the nodes of tree are in a specific order. Let's say if X is a parent node of Y, then the value of X follows some specific order with respect to value of Y and the same order will be followed across the tree.

The maximum number of children of a node in the heap depends on the type of heap. However in the more commonly used heap type, there are at most 2 children of a note and it's known as a Binary heap.

In binary heap, if the heap is a complete binary tree with N nodes, then it has has smallest possible height which is $\log_2 N$.



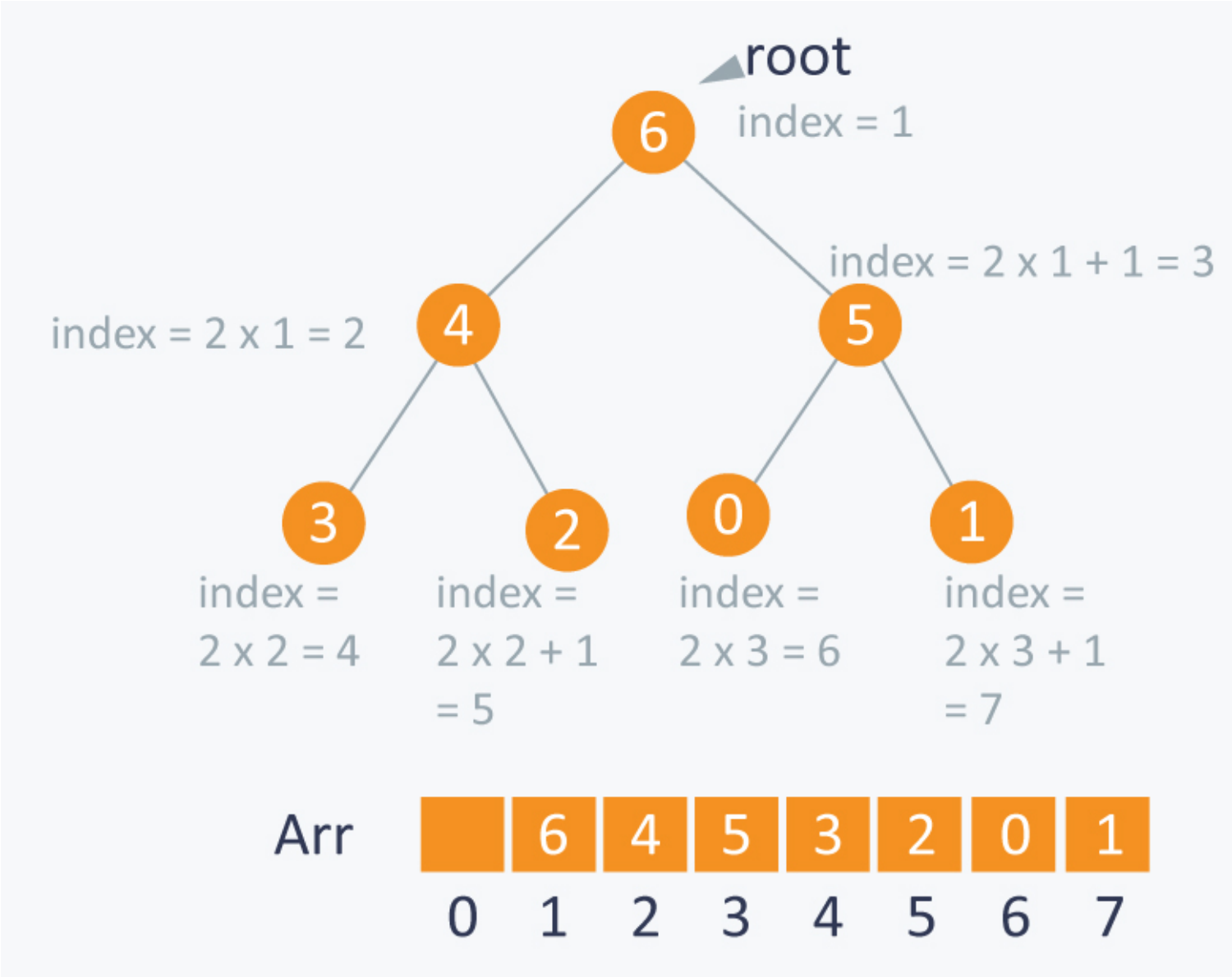
In the diagram above, you can observe a particular sequence, i.e each node has greater value than any of its children.

Suppose there are N Jobs in a queue to be done, and each job has its own priority. The job with maximum priority will get completed first than others. At each instant we are completing a job with maximum priority and at the same time we are also interested in inserting a new job in the queue with its own priority.

So at each instant we have to check for the job with maximum priority to complete it and also insert if there is a new job. This task can be very easily executed using a heap by considering N jobs as N nodes of the tree.

As you can see in the diagram below, we can use an array to store the nodes of the tree. Let's say we have 7 elements with values {6, 4, 5, 3, 2, 0, 1}.

Note: An array can be used to simulate a tree in the following way. If we are storing one element at index i in array Ar, then its parent will be stored at index i/2 (unless its a root, as root has no parent) and can be access by Ar[i/2], and its left child can be accessed by Ar[2 * i] and its right child can be accessed by Ar[2 * i +1]. Index of root will be 1 in an array.



There can be two types of heap:

Max Heap: In this type of heap, the value of parent node will always be greater than or equal to the value of child node across the tree and the node with highest value will be the root node of the tree.

Implementation:

Let's assume that we have a heap having some elements which are stored in array Arr. The way to convert this array into a heap structure is the following. We pick a node in the array, check if the left sub-tree and the right sub-tree are max heaps, in themselves and the node itself is a max heap (it's value should be greater than all the child nodes)

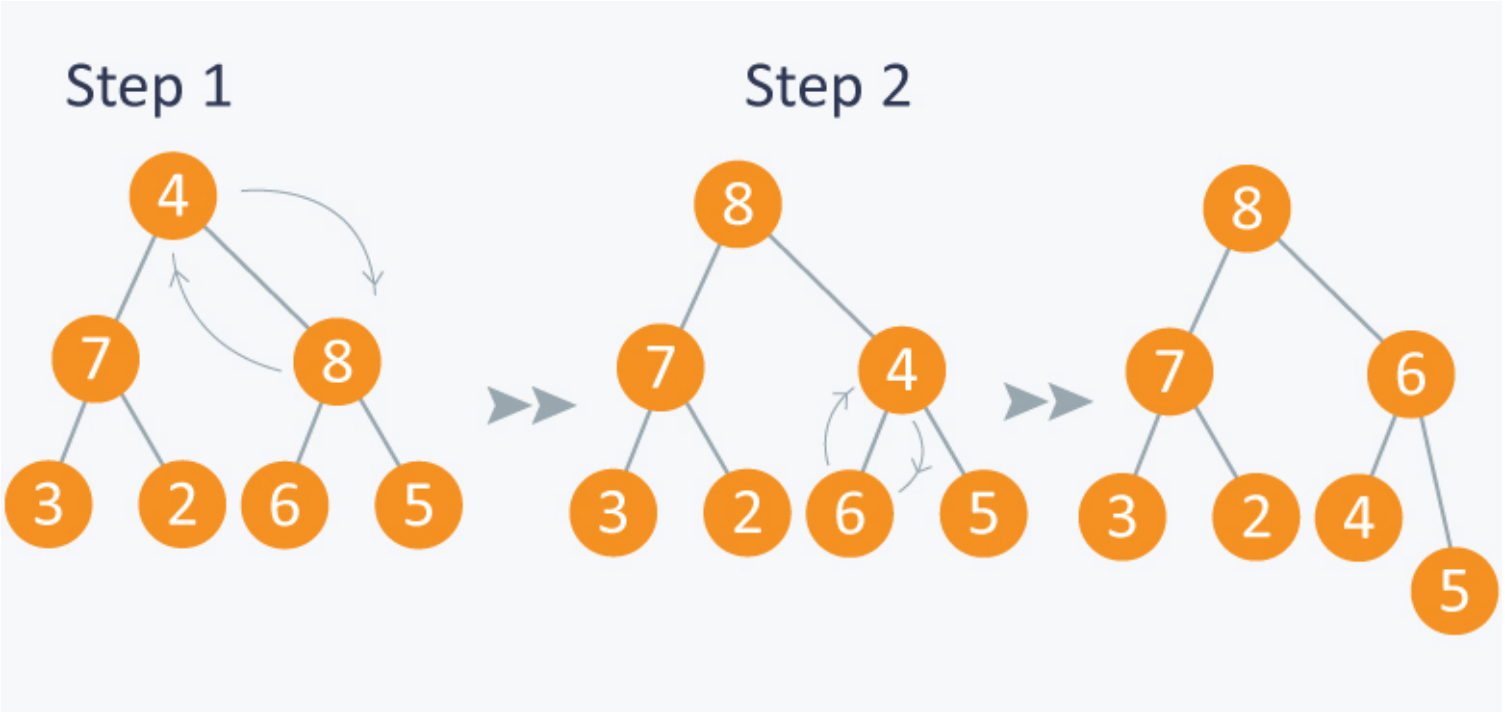
To do this we will implement a function that can maintain the property of max heap (i.e each element value should be greater than or equal to any of its child and smaller than or equal to its parent)

```
void max_heapify (int Arr[ ], int i, int N)
{
    int left = 2*i                    //left child
    int right = 2*i +1            //right child
    if(left<= N and Arr[left] > Arr[i] )
          largest = left;
    else
         largest = i;
    if(right <= N and Arr[right] > Arr[largest] )
        largest = right;
    if(largest != i )
    {
        swap (Ar[i] , Arr[largest]);
        max_heapify (Arr, largest,N);
    }
 }
```

Complexity: O(log N)

Example:

In the diagram below,initially 1st node (root node) is violating property of max-heap as it has smaller value than its children, so we are performing max_heapify function on this node having value 4.



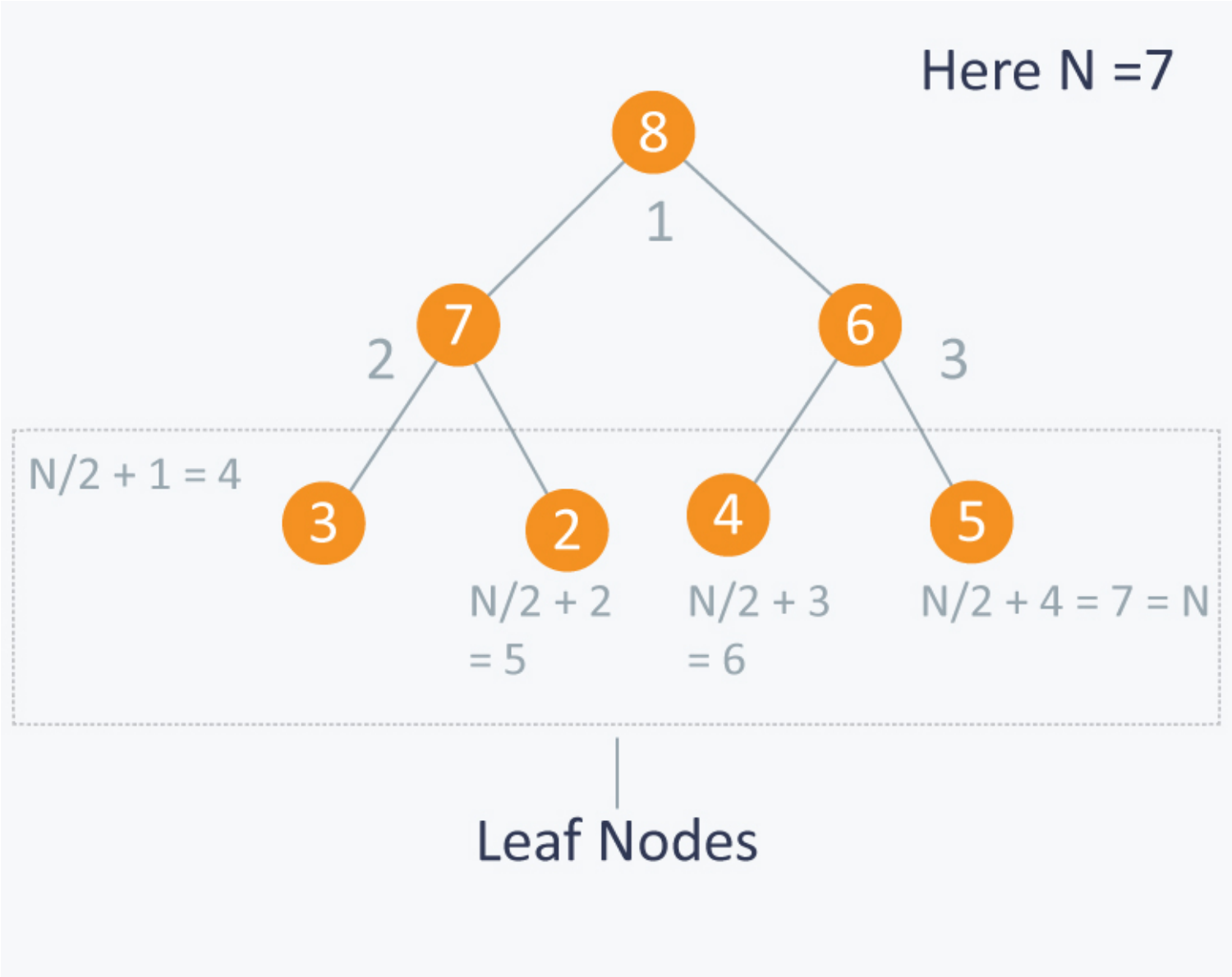
As 8 is greater than 4, then 8 is swapped with 4 and max_heapify is performed again on 4, but on different position. Now in step 2, 6 is greater than 4, so 4 is swapped with 6 and we will get a max heap, as now 4 is a leaf node, so further call to max_heapify will not create any effect on heap.

Now as we can see that we can maintain violated max- heap by using **max_heapify** function.

Before moving ahead, lets observe a property which states: A N element heap stored in an array has leaves indexed by N/2+1, N/2+2 , N/2+3 .... upto N.

Let's observe this with an example:

Lets take above example of 7 elements having values {8, 7, 6, 3, 2, 4, 5}.



So you can see that elements 3, 2, 4, 5 are indexed by N/2 +1 (i.e 4), N/2+2 (i.e 5) and N/2+3 (i.e 6) and N/2+4 (i.e 7) respectively.

Building MAX HEAP:

Now let's say we have N elements stored in the array Arr indexed from 1 to N. They are currently not following the property of max heap. So we can use max-heapify function to make a max heap out of the array.

How?

From the above property we observed that elements from Arr[N/2+1] to Arr[N] are leaf nodes,and each node is a 1 element heap. We can use max_heapify function in a bottom up manner on remaining nodes, so that we can cover each node of tree.

```
void build_maxheap (int Arr[ ])
{
    for(int i = N/2 ; i >= 1 ; i-- )
    {
        max_heapify (Arr, i) ;
    }
}
```

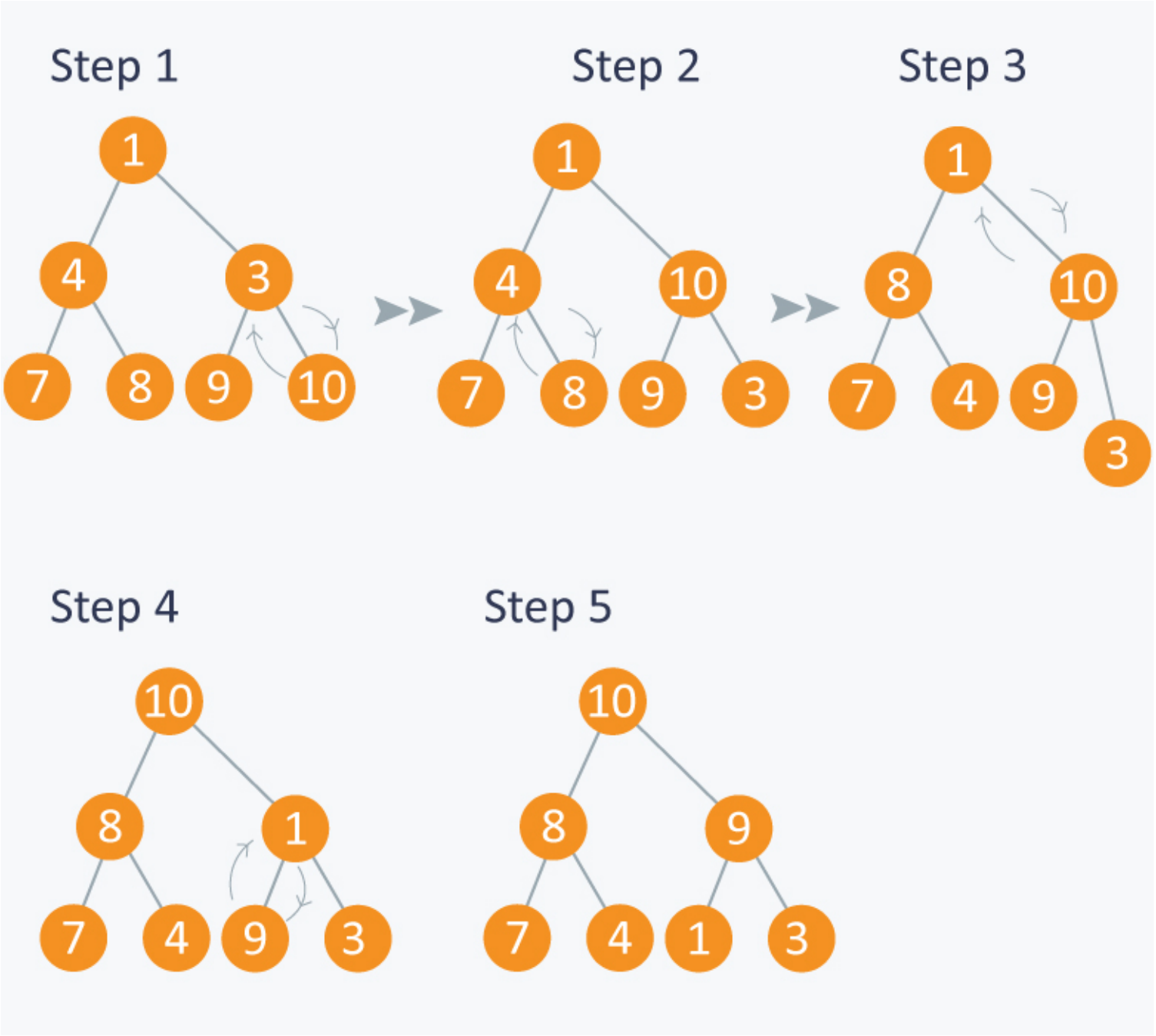
Complexity: O(N). max_heapify function has complexity log N and the build_maxheap functions runs only N/2 times, but the amortized complexity for this function is actually linear.
For more details, you can refer this.

Example:
Suppose you have 7 elements stored in array Arr.

Here N = 7, so starting from node having index N/2 = 3, (also having value 3 in the above diagram), we will call max_heapify from index N/2 to 1.

In the diagram below:

In step 1, in max_heapify(Arr, 3), as 10 is greater than 3, 3 and 10 are swapped and further call to max_heap(Arr, 7) will have no effect as 3 is a leaf node now.



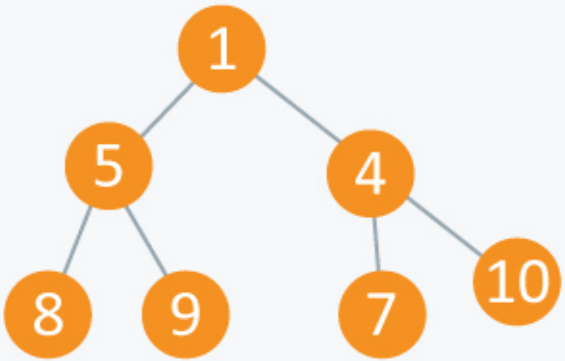
In step 2, calling max_heapify(Arr, 2) , (node indexed with 2 has value 4) , 4 is swapped with 8 and further call to max_heap(Arr, 5) will have no effect, as 4 is a leaf node now.

In step 3, calling max_heapify(Arr, 1) , (node indexed with 1 has value 1), 1 is swapped with 10 .

Step 4 is a subpart of step 3, as after swapping 1 with 10, again a recursive call of max_heapify(Arr, 3) will be performed , and 1 will be swapped with 9. Now further call to max_heapify(Arr, 7) will have no effect, as 1 is a leaf node now.

In step 5, we finally get a max- heap and the elements in the array Arr will be :

Min Heap: In this type of heap, the value of parent node will always be less than or equal to the value of child node across the tree and the node with lowest value will be the root node of tree.



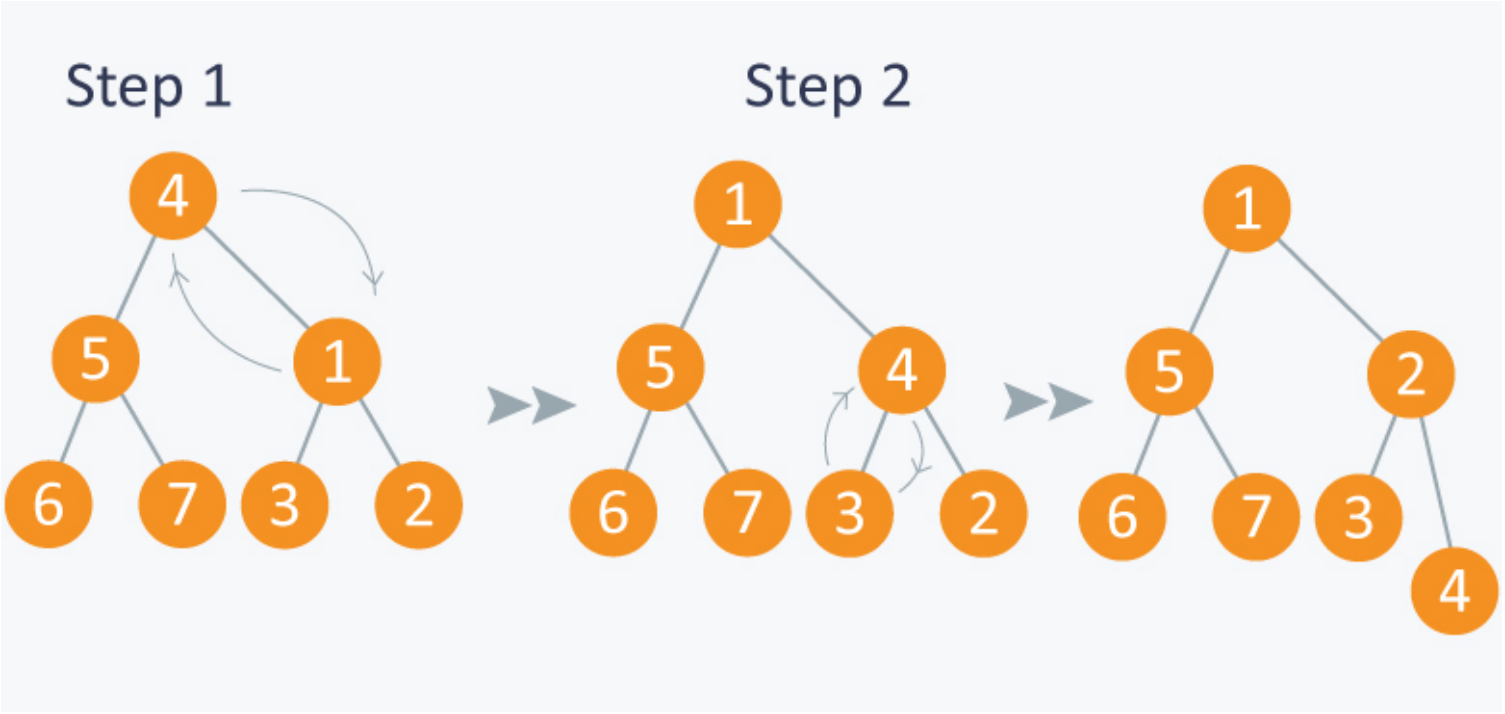
As you can see in the above diagram, each node has a value smaller than the value of their children.
We can perform same operations as performed in building max_heap.
First we will make function which can maintain the min heap property, if some element is violating it.

```
void min_heapify (int Arr[ ] , int i, int N)
{
int left  = 2*i;
int right = 2*i+1;
int smallest;
if(left <= N and Arr[left] < Arr[ i ] )
     smallest = left;
else
    smallest = i;
if(right <= N and Arr[right] < Arr[smallest] )
    smallest = right;
if(smallest != i)
{
    swap (Arr[ i ], Arr[ smallest ]);
    min_heapify (Arr, smallest,N);
}
}
```

Complexity: O (log N) .

Example:

Suppose you have elements stored in array Arr {4, 5, 1, 6, 7, 3, 2}. As you can see in the diagram below, the element at index 1 is violating the property of min -heap, so performing min_heapify(Arr, 1) will maintain the min-heap.

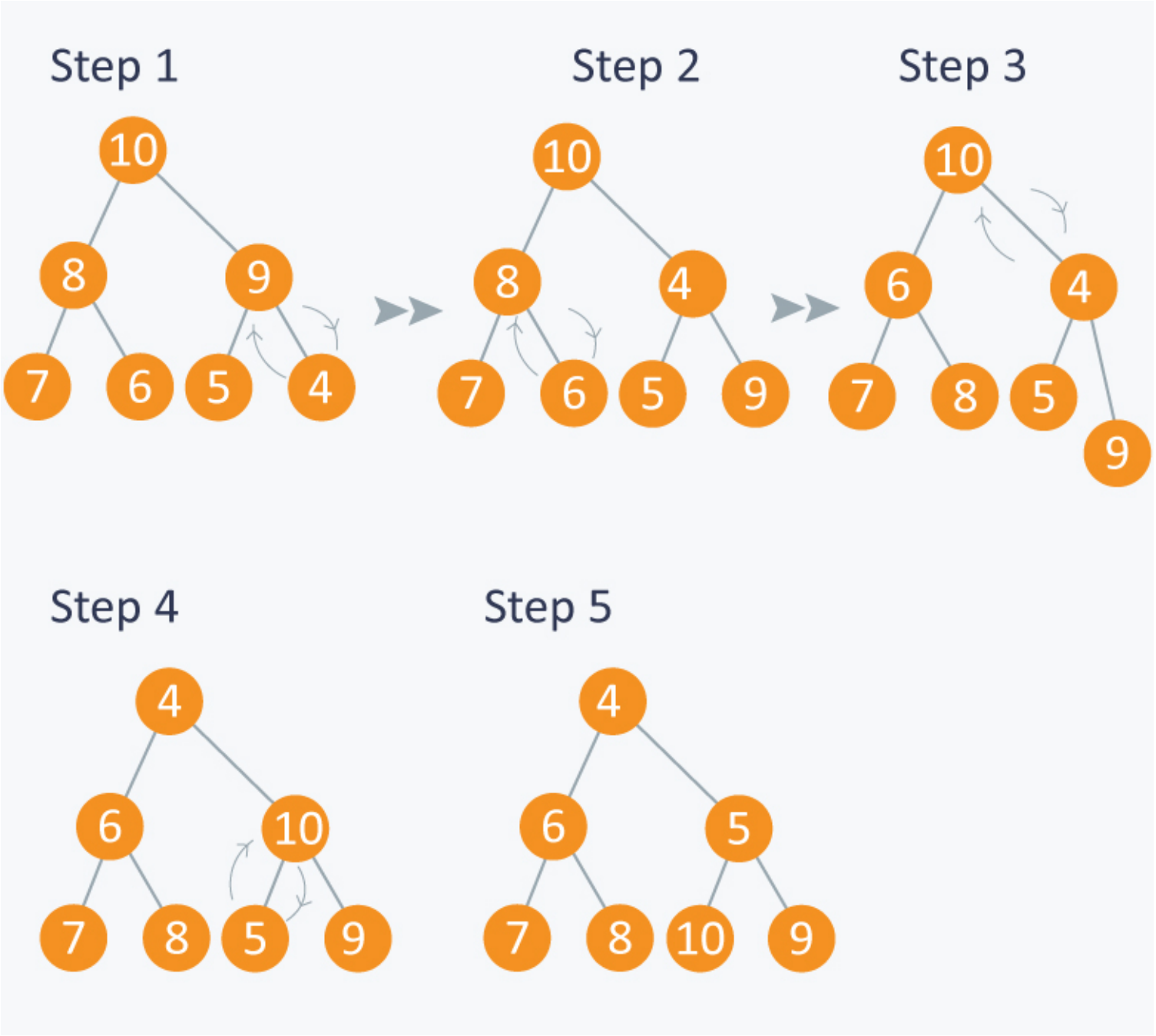


Now let's use above function in building min - heap. We will run the above function on remaining nodes other than leaves as leaf nodes are 1 element heap.

```
void build_minheap (int Arr[ ])
{
    for( int i = N/2 ; i >= 1 ; i--)
    min_heapify (Arr, i);
}
```

Complexity: O(N). The complexity calculation is similar to that of building max heap.

Example:
Consider elements in array {10, 8, 9, 7, 6, 5, 4} . We will run min_heapify on nodes indexed from N/2 to 1. Here node indexed at N/2 has value 9. And at last, we will get a min_heap.



Heaps can be considered as partially ordered tree, as you can see in the above examples that the nodes of tree do not follow any order with their siblings(nodes on the same level). They can be mainly used when we give

more priority to smallest or the largest node in the tree as we can extract these node very efficiently using heaps.

APPLICATIONS:

1) **Heap Sort:**

We can use heaps in sorting the elements in a specific order in efficient time.
Let's say we want to sort elements of array Arr in ascending order. We can use max heap to perform this operation.

Idea: We build the max heap of elements stored in Arr, and the maximum element of Arr will always be at the root of the heap.

Leveraging this idea we can sort an array in the following manner.

Processing:

1) Initially we will build a max heap of elements in Arr.

2) Now the root element that is Arr[1] contains maximum element of Arr. After that, we will exchange this element with the last element of Arr and will again build a max heap excluding the last element which is already in its correct position and will decrease the length of heap by one.

3) We will repeat the step 2, until we get all the elements in their correct position.

4) We will get a sorted array.

Implementation:

Suppose there are N elements stored in array Arr.

```
void heap_sort(int Ar[ ])
{
    int heap_size = N;
    build_maxheap(Arr);
```

```
    for(int i = N; i>=2 ; i-- )
    {
        swap|(Arr[ 1 ], Arr[ i ]);
        heap_size = heap_size-1;
        max_heapify(Arr, 1, heap_size);
    }
}
```

Complexity: As we know max_heapify has complexity O(logN), build_maxheap has complexity O(N) and we run max_heapify N-1 times in heap_sort function, therefore complexity of heap_sort function is O(N logN).

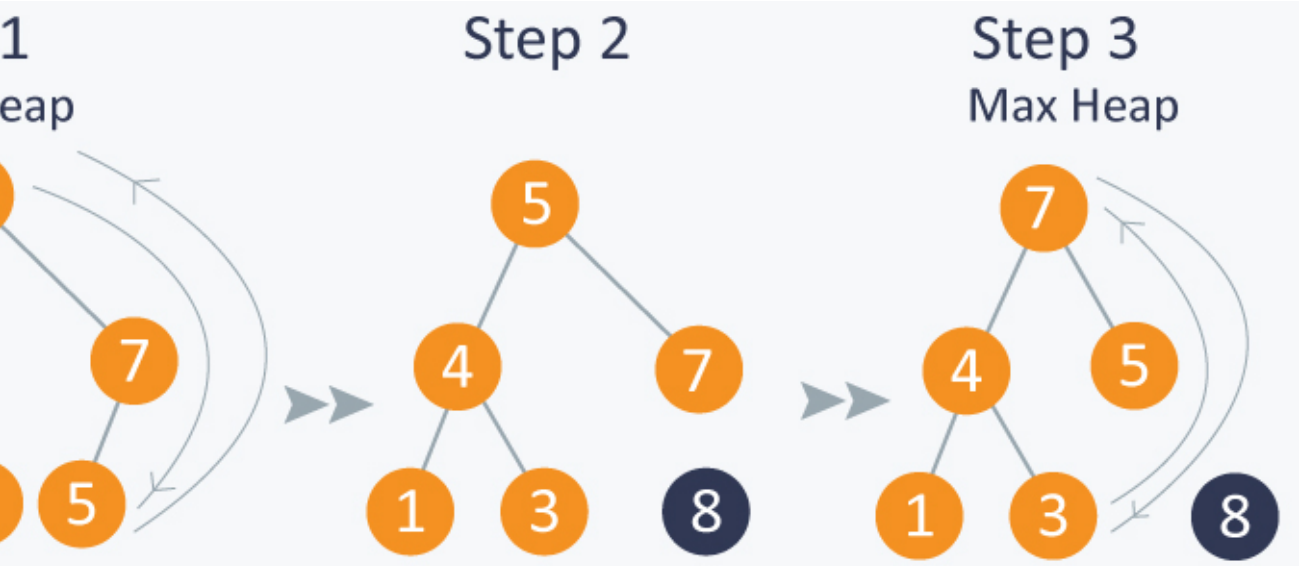
Example:

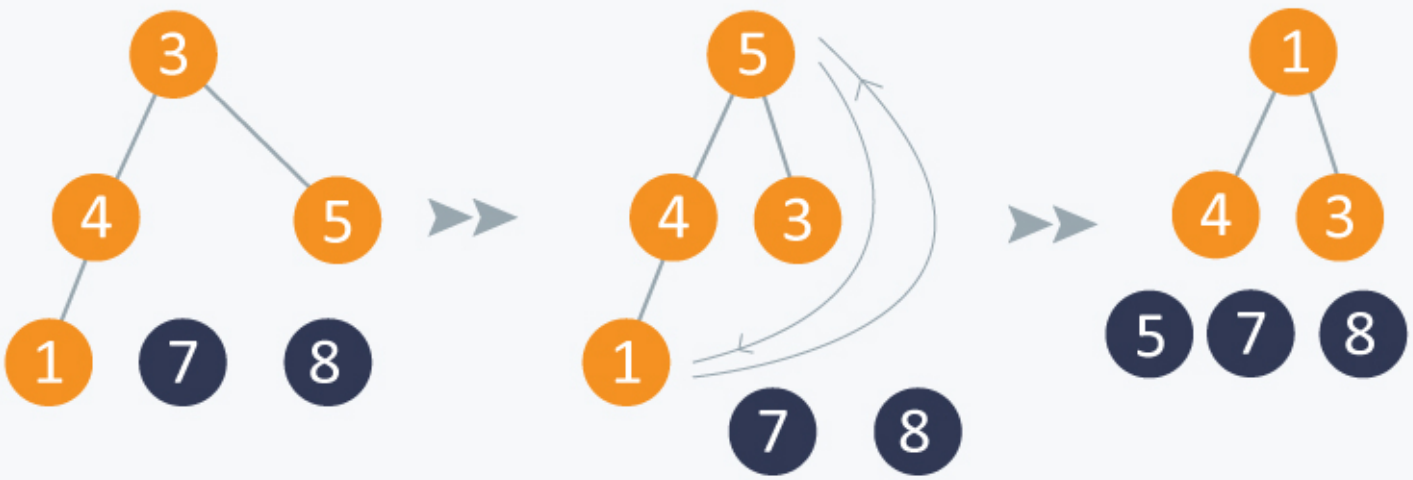
In the diagram below,initially there is an unsorted array Arr having 6 elements and then max-heap will be built.

After building max-heap, the elements in the array Arr will be:

Processing:

Step 1: 8 is swapped with 5.
Step 2: 8 is disconnected from heap as 8 is in correct position now and.
Step 3: Max-heap is created and 7 is swapped with 3.
Step 4: 7 is disconnected from heap.
Step 5: Max heap is created and 5 is swapped with 1.
Step 6: 5 is disconnected from heap.
Step 7: Max heap is created and 4 is swapped with 3.
Step 8: 4 is disconnected from heap.
Step 9: Max heap is created and 3 is swapped with 1.
Step 10: 3 is disconnected.





After all the steps, we will get a sorted array.

2) **Priority Queue:**

Priority Queue is similar to queue where we insert an element from the back and remove an element from front, but with a one difference that the logical order of elements in the priority queue depends on the priority of the elements. The element with highest priority will be moved to the front of the queue and one with lowest

priority will move to the back of the queue. Thus it is possible that when you enqueue an element at the back in the queue, it can move to front because of its highest priority.

Example:

Let's say we have an array of 5 elements : {4, 8, 1, 7, 3} and we have to insert all the elements in the max-priority queue.

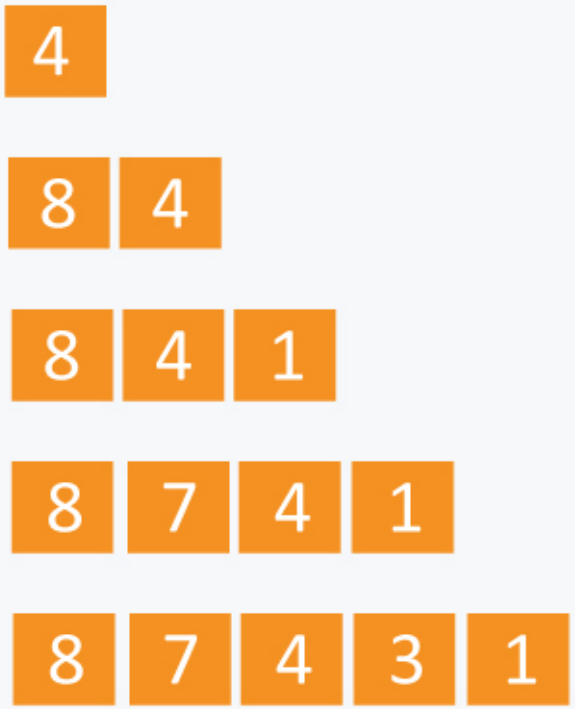
First as the priority queue is empty, so 4 will be inserted initially.

Now when 8 will be inserted it will moved to front as 8 is greater than 4.

While inserting 1, as it is the current minimum element in the priority queue, it will remain in the back of priority queue.

Now 7 will be inserted between 8 and 4 as 7 is smaller than 8.

Now 3 will be inserted before 1 as it is the 2^{nd} minimum element in the priority queue. All the steps are represented in the diagram below:



We can think of many ways to implement the priority queue.

Naive Approach:
Suppose we have N elements and we have to insert these elements in the priority queue. We can use list and can insert elements in O(N) time and can sort them to maintain a priority queue in O(N logN) time.

Efficient Approach:
We can use heaps to implement the priority queue. It will take O(log N) time to insert and delete each element in the priority queue.

Based on heap structure, priority queue also has two types max- priority queue and min - priority queue.

Let's focus on Max Priority Queue.

Max Priority Queue is based on the structure of max heap and can perform following operations:

maximum(Arr) : It returns maximum element from the Arr.
extract_maximum (Arr) - It removes and return the maximum element from the Arr.
increase_val (Arr, i , val) - It increases the key of element stored at index i in Arr to new value **val**.
insert_val (Arr, val) - It inserts the element with value **val** in Arr.

Implementation:

length = number of elements in Arr.

Maximum :

```
int maximum(int Arr[ ])
 {
      return Arr[ 1 ];  //as the maximum element is the root element in the max heap.
 }
```

Complexity: O(1)

Extract Maximum: In this operation, the maximum element will be returned and the last element of heap will be placed at index 1 and max_heapify will be performed on node 1 as placing last element on index 1 will violate the property of max-heap.

```
int extract_maximum (int Arr[ ])
{
     if(length == 0)
     {
cout<< "Can't remove element as queue is empty";
          return -1;
     }
     int max = Arr[1];
     Arr[1] = Arr[length];
```

```
    length = length -1;
    max_heapify(Arr, 1);
    return max;
}
```

Complexity: O(logN).

Increase Value: In case increasing value of any node, may violate the property of max-heap, so we will swap the parent's value with the node's value until we get a larger value on parent node.

```
void increase_value (int Arr[ ], int i, int val)
{
    if(val < Arr[ i ])
    {
        cout<<"New value is less than current value, can't be inserted" <<endl;
        return;
    }
    Arr[ i ] = val;
    while( i > 1 and Arr[ i/2 ] < Arr[ i ])
    {
        swap|(Arr[ i/2 ], Arr[ i ]);
        i = i/2;
    }
}
```

Complexity : O(log N).

Insert Value :

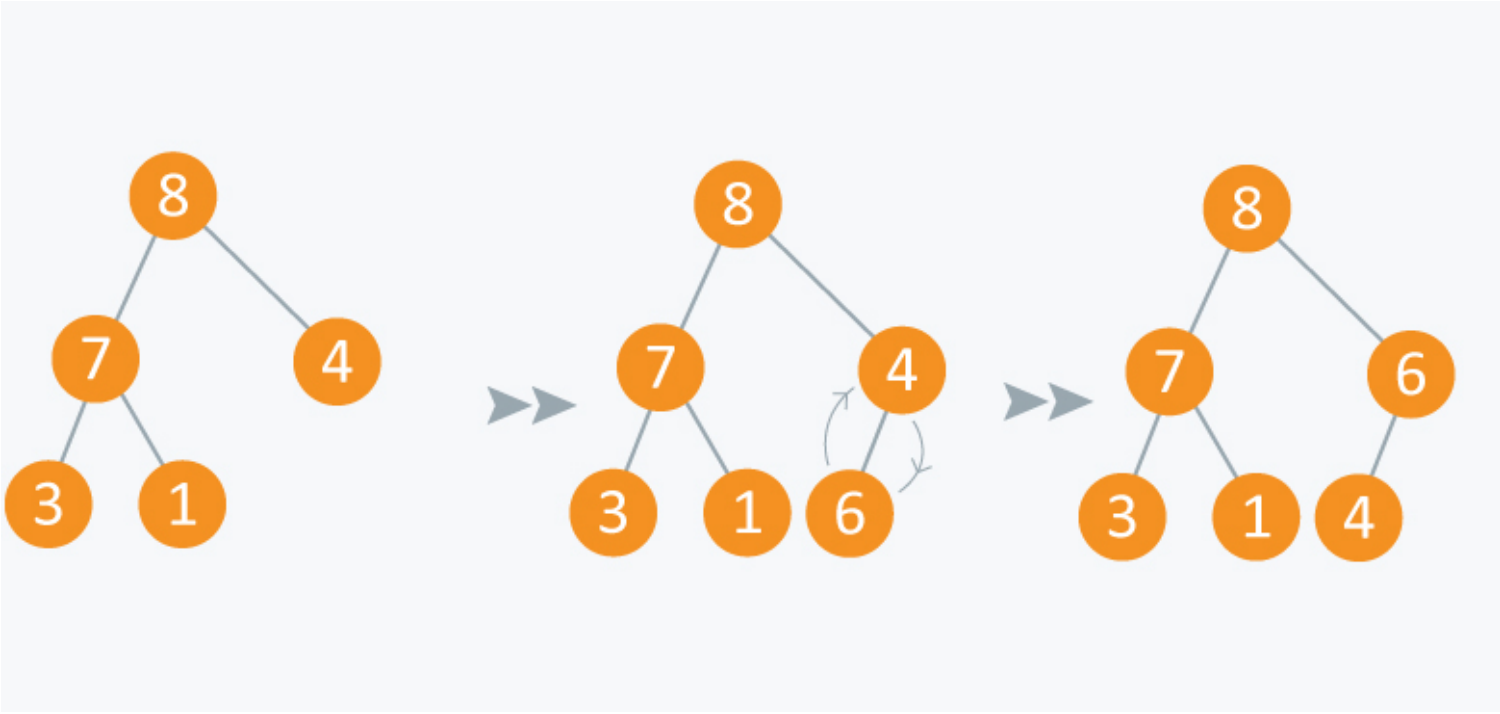
```
void insert_value (int Arr[ ], int val)
{
    length = length + 1;
```

```
    Arr[ length ] = -1;  //assuming all the numbers greater than 0 are to be inserted in queue.
    increase_val (Arr, length, val);
}
```

Complexity: O(log N).

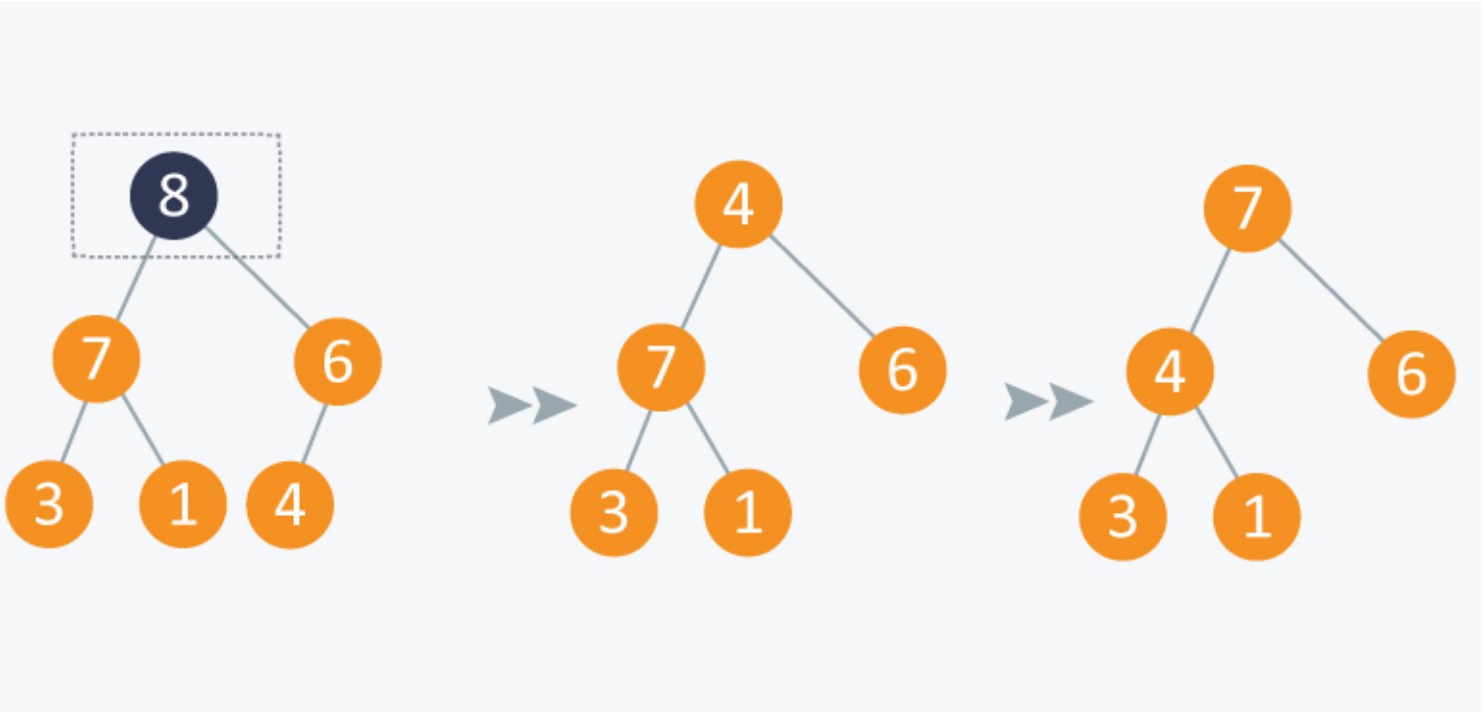
Example:

Initially there are 5 elements in priority queue.
Operation: Insert Value(Arr, 6)
In the diagram below, inserting another element having value 6 is violating the property of max-priority queue, so it is swapped with its parent having value 4, thus maintaining the max priority queue.



Operation: Extract Maximum:
In the diagram below, after removing 8 and placing 4 at node 1, violates the property of max-priority queue. So

max_heapify(Arr, 1) will be performed which will maintain the property of max - priority queue.



As discussed above, like heaps we can use priority queues in scheduling of jobs. When there are N jobs in queue, each having its own priority. If the job with maximum priority will be completed first and will be removed from the queue, we can use priority queue's operation extract_maximum here. If at every instant we have to add a new job in the queue, we can use **insert_value** operation as it will insert the element in O(log N) and will also maintain the property of max heap.

Now, let's give you a problem to ponder on:

1) How will you perform below operations for min priority queue?

a. minimum (Arr) - return minimum element from Arr.
b. extract_minimum (Arr) - removes and returns minimum element from Arr.
c. decrease_value (Arr, i, val) - decrease value of element index at i in Arr to val. It is assumed that val is at least as small as current value of Arr[i].
d. insert_value (Arr, val) - insert value in Arr.

Solve Problems

COMMENTS (35) SORT BY: Relevance

Login/Signup to Comment

Ankur Singh 5 years ago

or we can use std::priority_queue in c++ instead of all this :)

3 votes

Ayush Chaturvedi 5 years ago

how will you implement min_heap ??

0 votes

Ankur Singh 5 years ago

priority_queue<int,std::vector<int>,std::greater<int> >

13 votes

Amarjeet Sinha 2 years ago

what does above decleration means

0 votes

Joy Chatterjee 2 years ago

go to this link
https://www.geeksforgeeks.org/implement-min-heap-using-stl/

0 votes

manish626kapil 2 years ago

how will you implement a priority queue of pair such that first is in ascending order but second is in descending order

0 votes

Satyam Raj 4 months ago

Use a comparator function instead of greater<int> with boolean return type

1 vote

SOUMYA GHOSH 5 years ago

can someone explain to me increase_value method?
void increase_value (int Arr[ ], int i, int val)
{
if(val < Arr[i])
{
cout<<"New value is less than current value, can't be inserted" <<endl;
return;
}
Arr[i] = val;// here we are putting the value(val) in ith location--> now we need to adjust the heap because of this new entry
while(i > 1 and Arr[i/2] < Arr[i])// for a max heap A[i/2] shud be less than A[i]
{
swap|(Arr[i/2], Arr[i]);// why swap?
i = i/2;// why i=i/2 ?
}
}
I have wrote within comments as far as I have understood and my questions too..... someone plz tell me whether my thought process is correct and explain to me the queries.

0 votes

Prateek Garg Author 5 years ago

1) Why i/2 ?
As for node indexed at i, its parent node will be i/2, left child will be 2*i and right child will be 2*i+1.
2) Why swap ?
As we are placing the new value of Arr[i], it might violates the property of max_heap (which states that parent node must have greater value than any of its children). So we are swapping it with the parent node until we again get a max_heap.
I hope this will clear your doubt.

4 votes

Liangze Yu Edited 2 years ago

No. The parent should be (i-1)/2 and the left child should be 2*i+1 and right child should be 2*i+2.
If you use 2*i as left child, consider index 0. It will only have right child. Then it is not a binary heap.
---
nvm, I didn't see your heap starts at 1.

0 votes

Narendra koli 4 years ago

Very much helpful.

1 vote

Sarvagya Agarwal 5 years ago

In recursive call of min_heapify parameter N is missing.

0 votes

Rohit Kumar 5 years ago
http://www.spoj.com/problems/PRO/
0 votes

Sarvagya Agarwal 5 years ago
can someone suggest problems ?
0 votes

Rohit Kumar 5 years ago
http://www.spoj.com/problems/PRO/
0 votes

Sarvagya Agarwal 5 years ago
thanks ! i did this already though:)
0 votes

Rohit Kumar 5 years ago
in the increse_value() ....arr[i] is not incresed to val...
0 votes

Rohit Kumar 5 years ago
it is too simple to learn from this...good discription...thnks...
0 votes

Vivek Mangal 5 years ago
nice description & very helpful
0 votes

mohd nazar mubeen 4 years ago
Please find all the implementation here ..
http://learncoce.blogspot.in/2016/01/heap-implementation-in-java.html
0 votes

Aniruddha Paul 4 years ago
waoo a great explanation.The link to describe the order of build_heap was the best part
0 votes

Shivasurya S 4 years ago
really useful tutorial !
0 votes

Sarthak Gupta 4 years ago

If in the priority queue case we increase the value of 1 to 5 at the very first heap image, then although the max_heapify property is satisfied but the queue will not remain in descending order.
If I am going somewhere wrong , help me .

0 votes

Suvrajyoti Chatterjee 4 years ago

Initial condition is that it has to already be a max heap, in the first image the tree we are talking about is not a max heap and will not satisfy the condition.

0 votes

Sivanandan Sankaraswami 3 years ago

I like this detailed explanation.

0 votes

abhinav sharma 3 years ago

nice article

0 votes

RAHUL DEV MISHRA Edited 3 years ago

Hello All, I am new to Heaps, Binary Heap and I am trying to understand why do we need to implement Priority Queue using a Binary Heap. I also understand that the underlying data structure of Binary Heap is again an array. So my question is why can we not use an array, sorted in either descending (for max heap) or ascending (for min heap) order to represent a priority queue ? I might be wrong here, but I think the time complexity of the operations like, findMax, findMin, insert and delete will remain almost the same if implemented this way. So can we not not a use a sorted array to represent the priority queue ?

0 votes

NISHANT KUMAR Edited 2 years ago

No, If you are going with the sorted array you can find max or min element in 0(1) time. But the cost of insertion will be high i.e O(n) and each time you insert an element you will do a binary search for upper_bound and then insertion which will take O(n) time.

0 votes

Rajeevkumar Shridhar Yadav 3 years ago

Nice Explanation, Thanks.

0 votes

Deepesh Singh 3 years ago

Hi,

This code will not work in case if we have this array : [4,7,8,3,2,6,10] and we call the function like max_heapify (ary, 7, 7). i.e. at index 7 we have values as 10. This code will never satisfy this condition (largest != i) and hence swap will not happen and 10 will remain as leafnode.

```
void max_heapify (int Arr[ ], int i, int N)
{
int left = 2*i //left child
int right = 2*i +1 //right child
if(left<= N and Arr[left] > Arr[i] )
largest = left;
else
largest = i;
if(right <= N and Arr[right] > Arr[largest] )
largest = right;
if(largest != i )
{
swap (Ar[i] , Arr[largest]);
max_heapify (Arr, largest,N);
}
}
```

0 votes

Prince kumar Bhagat 2 years ago

We will run the above function on remaining nodes other than leaves as leaf nodes are 1 element heap.

0 votes

manish626kapil 2 years ago

how will you implement a priority queue of pair such that first is in ascending order but second is in descending order

0 votes

Nikhil Joshi a year ago

Here it is:
https://www.dotnetlovers.com/article/231/priority-queue

0 votes

Lobabah Saad a year ago

khanki khanki

0 votes

saitejavelagandula a year ago

why only the leaf node is moved to root when the root node is extracted?

0 votes

AUTHOR

Prateek Garg
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7 notes

TRENDING NOTES

Python Diaries Chapter 3 Map | Filter | For-else | List Comprehension
written by Divyanshu Bansal

Bokeh | Interactive Visualization Library | Use Graph with Django Template
written by Prateek Kumar

Bokeh | Interactive Visualization Library | Graph Plotting
written by Prateek Kumar

Python Diaries chapter 2
written by Divyanshu Bansal

Python Diaries chapter 1
written by Divyanshu Bansal

more ...