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Supervised Learning Theory

Overview (/machine-learning/)

One Variable Linear Regression (/one-variable-linear-regression/)

Linear Algebra (/linear-algebra-machine-learning/)

Multiple Variable Linear Regression (/multi-variable-linear-regression/)

Logistic Regression (/logistic-regression/)

Neural Networks (Representation) (/neural-networks-representation/)

Neural Networks (Learning) (/neural-networks-learning/)

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Machine Learning Systems Design (/machine-learning-systems-design/)

Support Vector Machines (/machine-learning-svms-support-vector-machines/)

Unsupervised Learning Theory

Unsupervised Learning (/machine-learning-unsupervised-learning/)

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Learning Curve

> **Summary:** Evaluate bias and variance with a learning curve

Learning Curve Theory

- Graph that compares the performance of a model on training and testing data over a varying number of training instances
- We should generally see performance improve as the number of training points increases
- When we separate training and testing sets and graph them individually
  - We can get an idea of how well the model can generalize to new data
- Learning curve allows us to verify when a model has learning as much as it can about the data
- When it occurs
  1. The performances on the training and testing sets reach a plateau
  2. There is a consistent gap between the two error rates
- The key is to find the sweet spot that minimizes bias and variance by finding the right level of model complexity
- Of course with more data any model can improve, and different models may be optimal
- For a more in-depth theoretical coverage of learning curves, you can view a guide by Andrew Ng that I have compiled here (http://www.ritchieng.com/applying-machine-learning/)

Types of learning curves

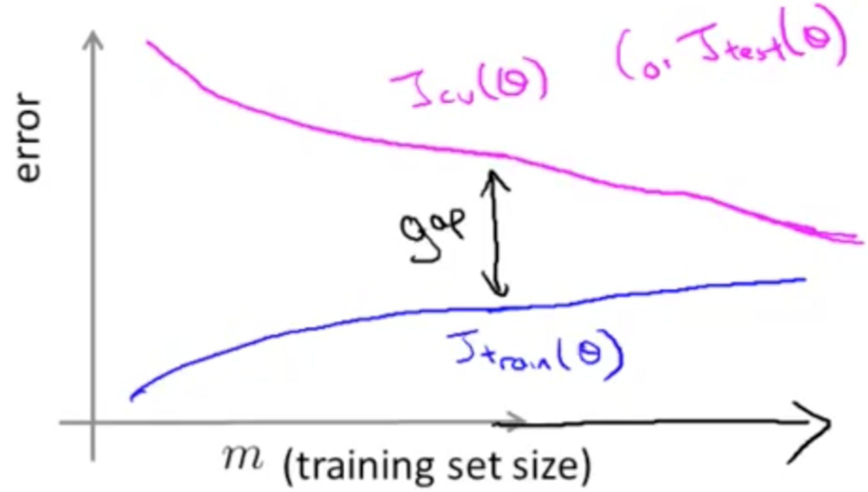
- Bad Learning Curve: High Bias
  - When training and testing errors converge and are high
    - No matter how much data we feed the model, the model cannot represent the underlying relationship and has high systematic errors
    - Poor fit
    - Poor generalization
- Bad Learning Curve: High Variance

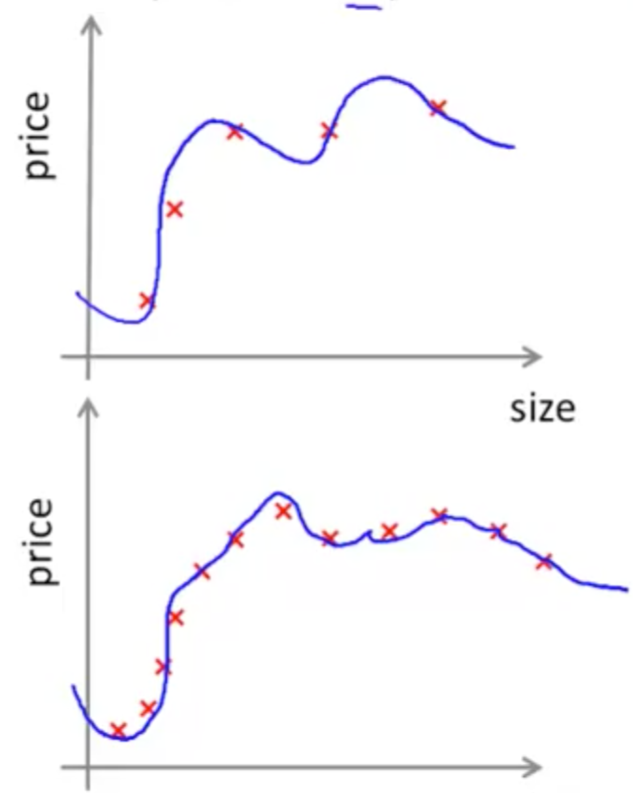
  - When there is a large gap between the errors
    - Require data to improve
    - Can simplify the model with fewer or less complex features
- Ideal Learning Curve
  - Model that generalizes to new data
  - Testing and training learning curves converge at similar values
  - Smaller the gap, the better our model generalizes

High variance

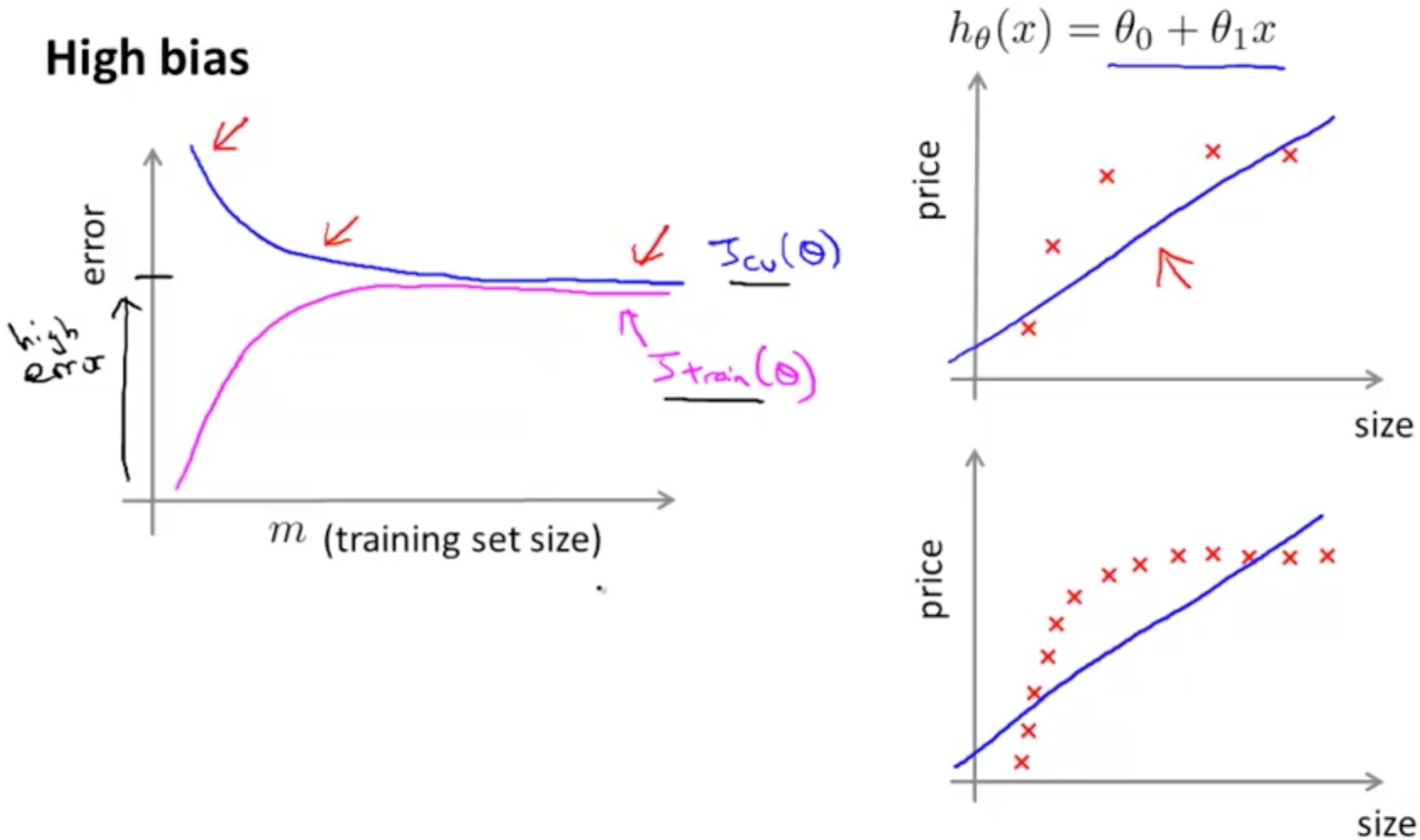
$$h_\theta(x) = \theta_0 + \theta_1 x + \cdots + \theta_{100} x^{100}$$

(and small λ)





If a learning algorithm is suffering from high variance, getting more training data is likely to help.


High bias
error
high error
$J_{cv}(\theta)$
$J_{train}(\theta)$
m (training set size)
$h_\theta(x) = \theta_0 + \theta_1 x$
price
size
price
size

Example 1: High Bias

- In this example, you'll see that we'll be using a linear learner on quadratic data
- The result is that we've high bias
- We'll have a low score (high error)

In [1]:
```
# imports
from sklearn.linear_model import LinearRegression
from sklearn.learning_curve import learning_curve
import matplotlib.pyplot as plt
from sklearn.metrics import explained_variance_score, make_scorer
from sklearn.cross_validation import KFold
import numpy as np
```

In [2]:
```
size = 1000
cv = KFold(size, shuffle=True)
```

Create X array

In [3]:
```
#np.reshape(old_shape, new_shape)

# new array (-1, 1)
# -1 implies to take shape from original, hence 1000
# this creates a 1000 x 1 array
X = np.reshape(np.random.normal(scale=2,size=size),(-1,1))
X.shape
```

Out[3]:
```
(1000, 1)
```

In [4]:
```
# np.random.normal(scale=2,size=size) creates a 1000 x 1 matrix
# scale=2 is the standard deviation of the distribution
np.random.normal(scale=2,size=size).shape
```

Out[4]:
```
(1000,)
```

Create y array

In [5]:
```
y = np.array([[1 - 2*x[0] +x[0]**2] for x in X])
y.shape
```

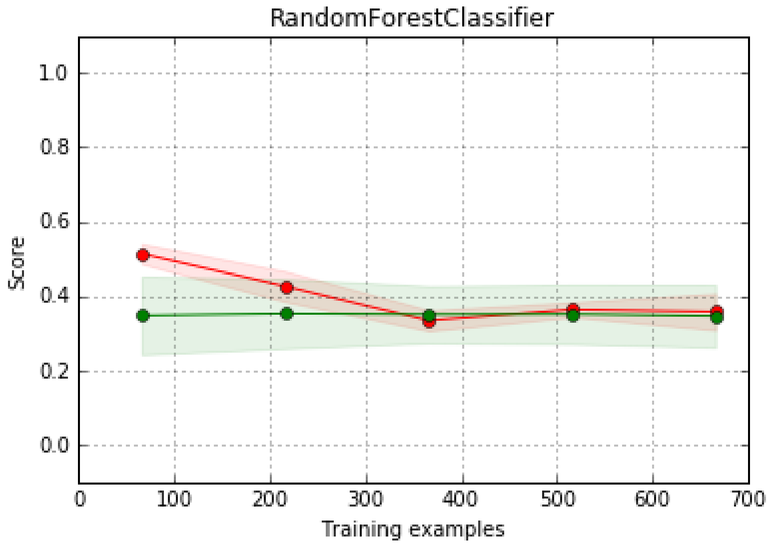
Out[5]:
```
(1000, 1)
```

Plot learning curve

In [6]:
```
def plot_curve():
    # instantiate
    lg = LinearRegression()

    # fit
    lg.fit(X, y)


    """
    Generate a simple plot of the test and traning learning curve.

    Parameters
    ----------
    estimator : object type that implements the "fit" and "predict" methods
        An object of that type which is cloned for each validation.

    title : string
        Title for the chart.

    X : array-like, shape (n_samples, n_features)
        Training vector, where n_samples is the number of samples and
        n_features is the number of features.

    y : array-like, shape (n_samples) or (n_samples, n_features), optional
        Target relative to X for classification or regression;
        None for unsupervised learning.

    ylim : tuple, shape (ymin, ymax), optional
        Defines minimum and maximum yvalues plotted.

    cv : integer, cross-validation generator, optional
        If an integer is passed, it is the number of folds (defaults to 3).
        Specific cross-validation objects can be passed, see
        sklearn.cross_validation module for the list of possible objects

    n_jobs : integer, optional
        Number of jobs to run in parallel (default 1).
```

```
    x1 = np.linspace(0, 10, 8, endpoint=True) produces
        8 evenly spaced points in the range 0 to 10
    """

    train_sizes, train_scores, test_scores = learning_curve(lg, X, y, n_jobs=-1, cv=cv, train_sizes=np.linspace(.1
, 1.0, 5), verbose=0)

    train_scores_mean = np.mean(train_scores, axis=1)
    train_scores_std = np.std(train_scores, axis=1)
    test_scores_mean = np.mean(test_scores, axis=1)
    test_scores_std = np.std(test_scores, axis=1)

    plt.figure()
    plt.title("RandomForestClassifier")
    plt.legend(loc="best")
    plt.xlabel("Training examples")
    plt.ylabel("Score")
    plt.gca().invert_yaxis()

    # box-like grid
    plt.grid()

    # plot the std deviation as a transparent range at each training set size
    plt.fill_between(train_sizes, train_scores_mean - train_scores_std, train_scores_mean + train_scores_std, alph
a=0.1, color="r")
    plt.fill_between(train_sizes, test_scores_mean - test_scores_std, test_scores_mean + test_scores_std, alpha=0.
1, color="g")

    # plot the average training and test score lines at each training set size
    plt.plot(train_sizes, train_scores_mean, 'o-', color="r", label="Training score")
    plt.plot(train_sizes, test_scores_mean, 'o-', color="g", label="Cross-validation score")

    # sizes the window for readability and displays the plot
    # shows error from 0 to 1.1
    plt.ylim(-.1,1.1)
    plt.show()
```

In [8]:
```
%matplotlib inline
plot_curve()
```



Compared to the theory we covered, here our y-axis is 'score', not 'error', so the higher the score, the better the performance of the model.

- Training score (red line) decreases and plateau
  - Indicates underfitting
  - High bias
- Cross-validation score (green line) stagnating throughout
  - Unable to learn from data
- Low scores (high errors)
  - Should tweak model (perhaps increase model complexity)

Example 2: High Variance

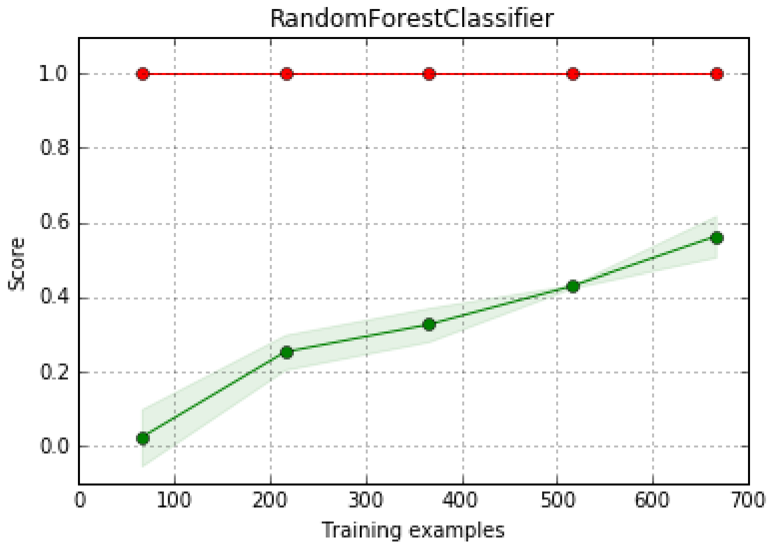
- Noisy data and complex model

There're no inline notes here as the code is exactly the same as above and are already well explained.

In [12]:
```
from sklearn.tree import DecisionTreeRegressor

X = np.round(np.reshape(np.random.normal(scale=5,size=2*size),(-1,2)),2)
y = np.array([[np.sin(x[0]+np.sin(x[1]))] for x in X])

def plot_curve():
    # instantiate
    dt = DecisionTreeRegressor()

    # fit
    dt.fit(X, y)


    """
    Generate a simple plot of the test and traning learning curve.

    Parameters
    ----------
    estimator : object type that implements the "fit" and "predict" methods
        An object of that type which is cloned for each validation.

    title : string
        Title for the chart.

    X : array-like, shape (n_samples, n_features)
        Training vector, where n_samples is the number of samples and
        n_features is the number of features.

    y : array-like, shape (n_samples) or (n_samples, n_features), optional
        Target relative to X for classification or regression;
        None for unsupervised learning.

    ylim : tuple, shape (ymin, ymax), optional
        Defines minimum and maximum yvalues plotted.

    cv : integer, cross-validation generator, optional
        If an integer is passed, it is the number of folds (defaults to 3).
        Specific cross-validation objects can be passed, see
```

```
        sklearn.cross_validation module for the list of possible objects

    n_jobs : integer, optional
        Number of jobs to run in parallel (default 1).

    x1 = np.linspace(0, 10, 8, endpoint=True) produces
        8 evenly spaced points in the range 0 to 10
    """

    train_sizes, train_scores, test_scores = learning_curve(dt, X, y, n_jobs=-1, cv=cv, train_sizes=np.linspace(.1
, 1.0, 5), verbose=0)

    train_scores_mean = np.mean(train_scores, axis=1)
    train_scores_std = np.std(train_scores, axis=1)
    test_scores_mean = np.mean(test_scores, axis=1)
    test_scores_std = np.std(test_scores, axis=1)

    plt.figure()
    plt.title("RandomForestClassifier")
    plt.legend(loc="best")
    plt.xlabel("Training examples")
    plt.ylabel("Score")
    plt.gca().invert_yaxis()

    # box-like grid
    plt.grid()

    # plot the std deviation as a transparent range at each training set size
    plt.fill_between(train_sizes, train_scores_mean - train_scores_std, train_scores_mean + train_scores_std, alph
a=0.1, color="r")
    plt.fill_between(train_sizes, test_scores_mean - test_scores_std, test_scores_mean + test_scores_std, alpha=0.
1, color="g")

    # plot the average training and test score lines at each training set size
    plt.plot(train_sizes, train_scores_mean, 'o-', color="r", label="Training score")
    plt.plot(train_sizes, test_scores_mean, 'o-', color="g", label="Cross-validation score")

    # sizes the window for readability and displays the plot
    # shows error from 0 to 1.1
```

```
    plt.ylim(-.1,1.1)
    plt.show()

plot_curve()
```



Compared to the theory we covered, here our y-axis is 'score', not 'error', so the higher the score, the better the performance of the model.

- Training score (red line) is at its maximum regardless of training examples
    - This shows severe overfitting
- Cross-validation score (green line) increases over time
- Huge gap between cross-validation score and training score indicates high variance scenario
    - Reduce complexity of the model or gather more data

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Datasec • a year ago

I think this is a very good article showing the practical considerations of choosing ML models.

Question: in your example where the training score is near perfect throughout (and thus indicates over-fitting). What would be the obvious next step? Choose a simpler model and bring down the training score? What is a good range for the training score?

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AYUSH JAIN • 2 years ago

Awesome tutorial Ritchie!

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Ritchie Ng Moderator AYUSH JAIN • 2 years ago

Thanks!

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ayush kaul — I have tried problem 1. I got Kernal Restart error in jupyter notebook. I am running on CPU i3 4 GB memory

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Aitor Lopez Beltran — First of all, congratulations! Excuse me, but I'm having a hard time finding out how to get certified as a DLI Instructor. Could you help me, please? Thanks …

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