Discrete Assignment

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Question
1

$A = \{3,4,5\}$;; $B = \{4,5,6\}$

$x S y \iff x | y$

$\{(3,6),(4,4),(5,5)\} \in S$

$S^{-1} = \{(y,x) \in B \times A \mid (x,y) \in S\}$

Then

$\{(6,3),(4,4),(5,5)\} \in S^{-1}$

Result:

$\{(3,6),(4,4),(5,5)\} \in S$
$\{(6,3),(4,4),(5,5)\} \in S^{-1}$

Question
11

A real number r is rational if and only if there exist two integer a & b such that $r = \frac{a}{b}$ $(b \neq 0)$. A real number r is irrational if r is not rational.

$A = R$

$I = \{x, y \in R \mid x-y$ is irrational $\}$

Reflexive?

relation is ~~reflexive~~ reflective if $(a,a) \in I$ for every element $a \in A$.

since $A = Z$, I is reflective if it contains (x,x) for all $x \in R$

since I donot have (x,x) because $x - x = 0$ and 0 is irrational (since $0 = \frac{0}{1}$) thus I is not reflexive

Symmetric?

relation is symmetric if $(b,a) \in I$ whenever $(a,b) \in I$

let's assume that $(a,b) \in I$. By def. of I

$a-b$ is irrational

Let's assume that $b-a$ is rational. By def. of rational, there exist integer c & d such that

$$b-a = \frac{c}{d}$$

$$-(b-a) = \frac{-c}{d}$$

Distributive property:

$$a-b = \frac{-c}{d}$$

last inequality then implies that $a-b$ is irrational since $\frac{-c}{d}$ is rational

so our assumption that $b-a$ is rational is incorrect and thus $b-a$ is irrational

so

I is symmetric

Transitive ?

The relation of I is on set A is transitive if $(a,b) \in I$ and $(b,c) \in I$ implies $(a,c) \in I$

let us consider $a = \sqrt{2}$, $b = 1$ & $c = \sqrt{2}$

$a-b = \sqrt{2} - 1$ is irrational

$b-c = 1 - \sqrt{2}$ is irrational

$a-c = \sqrt{2} - \sqrt{2} = 0$ is rational

so I cannot be transitive too

Result:

NOT reflexive

IS symmetric

NOT transitive

Question NO.
3

set $A = \{-4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$

descriptive property:

$$\forall x, y \in A \quad x R y \Leftrightarrow 3 \mid (x-y)$$

equivalance classes for 0, 1, 2, 3 obtained

equivalanc class for 0:

$$[0] = \{x \in A \mid x R 0\}$$

start checking wheather the integer from A belong to [0]

→ $3 \nmid (-4-0)$, $3 \mid (-3-0)$, $3 \nmid (-2-0)$,
$3 \nmid (-1-0)$, $3 \mid (0-0) = 0$

→ $3 \nmid (1-0)$, $3 \nmid (2-0)$, $3 \mid (3-0)$,
$3 \nmid (4-0)$, $3 \nmid (5-0)$

therefore

$$[0] = \{-3, 0, 3\}$$

"Basically we all elements of those equivalent classes are obtained when we add 3 consequently to ~~last~~ least element that belong to the class hence,

$[1] = \{-2, 1, 4\}$

observe $[2]$, we have this $3|(-4-2)$ other elements

$-4+3, -4+3+3, -4+3+3+3$

hence

$[2] = \{-4, -1, +2, 5\}$

when we obtain $[3]$, we have this

$3|(-4-3), 3|(-3-3)$

so lowest integer from A that is in $[3]$ is -3 so

$-3+3, -3+3+3$, hence

$[3] = \{-3, 0, 3\}$

so two equal classes $[0]$ & $[3]$ exist. Hence we have 3 distinct equivalence classes

$\{-3, 0, 3\}, \{-2, 1, 4\}, \{-4, -1, +2, 5\}$