Assignment 3

Name : Muhammad Ahmad

Roll No:- 212-5617 BS(DS) A2

Submitted To:- Prof. Aisha Rashid

6.1

Question:- 3

$$A(x) = (\text{edges})^2$$
$$= (\sqrt{1-x^2} + \sqrt{1-x^2})^2$$
$$= (2\sqrt{1-x^2})^2$$
$$= 4(1-x^2)$$

$$V = 4\int_{-1}^{1} (1-x^2)\, dx$$

$$= 16/3$$

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Question No: 8

a)

$A(x) = \frac{1}{2}(\text{base})\,\text{hight}$

$= \frac{1}{2}(\sqrt{x} - \frac{x}{2})\,6$

$= 6\sqrt{x} - 3x$

$V = \int_0^4 6\sqrt{x} - 3x$

$V = 8$

b)

$A(x) = \frac{1}{2}\left(\frac{\text{diameter}}{2}\right)^2$

$= \frac{1}{8}(\sqrt{x} - x/2)^2$

$= \frac{\pi}{8}(\sqrt{x} - x^{3/2}) + \frac{1}{4}x^2$

$V = \frac{\pi}{8}\int_0^4 x - x^{3/2} + \frac{1}{4}x^2$

$V = \frac{\pi}{15}$

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Question 13

a)

$(\text{Side})^2 = s^2$

$V = \int_0^h s^2$

$= s^2 h$

b)

Volume of col. is same as volume of right prism, regardless of number of turns

$V = s^2 h$

Question 19

$R(y) = \tan(\frac{\pi}{4} y)$ $\quad 4\,du = \pi\,dy$

$y = 0, u = 0 \quad y = 1 \quad u = \pi/4$

$V = \pi \int_0^1 \tan(\frac{\pi}{4} y)^2\, dy$

$V = 4 - \pi$

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Question No: 42

$R(x) = \sec x$

$r(x) = \tan x$

$V = \pi \int_0 \sec^2 x - \tan^2 x$

$= \pi$

Question No: 48

$R(y) = 2$

$r(y) = 1 + \sqrt{y}$

$V = \pi \int_0^1 2^2 - (1+y)^2$

$V = \pi \int_0^1 3 - y - 2\sqrt{y}$

$V = \pi \left[3 - \frac{1}{2} - \frac{4}{3} \right]$

$V = \frac{7\pi}{6}$

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Question 51

a) line = $y=1$

$$V=\pi\int_{-1}^{1}(1-x^2)^2$$

$$=\pi\left[x-\frac{2x^3}{3}+\frac{x^5}{5}\right]_{-1}^{1}$$

$$=\frac{16\pi}{15}$$

b)

$$V=\pi\int_{-1}^{1}4+x^4-4x^2-1$$

$$=\left[3x+\frac{x^5}{5}-\frac{4x^3}{3}\right]_{-1}^{1}$$

$$V=\frac{\pi\,56}{15}$$

c) line $y=1$

$$V=\pi\int_{-1}^{1}4+x^4-4x^2-1$$

$$=\pi\left(3x+\frac{x^5}{5}-\frac{2x^3}{3}\right)$$

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$$V = \pi \left[6 - \frac{2}{5} - \frac{4}{3} \right]$$

$$= \pi \left[\frac{64}{15} \right]$$

$$V = \frac{64\pi}{15}$$

Question 59:-

$$R(y) = \sqrt{256 - y^2}$$

$$V = \pi \int_{-16}^{-7} (256 - y^2)$$

$$V = \pi \left[256y - \frac{y^3}{3} \right]_{-16}^{-7}$$

$$V = \pi \left[2304 - \frac{1}{3}(3753) \right]$$

$$V = 1053\pi \, cm^3$$

$$V \approx 3308 \, cm^3$$

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6.3

Question No: 9

$$y = \frac{x^3}{3} + \frac{1}{4x}$$

$$\frac{dy}{dx} = x^2 - \frac{1}{4x^2}$$

$$\left(\frac{dy}{dx}\right)^2 = \left(x^2 - \frac{1}{4x^2}\right)^2$$

$$= x^4 - \frac{1}{2} + \frac{1}{16x^4}$$

$$L = \int_1^3 \sqrt{1 + x^4 - \frac{1}{2} + \frac{1}{16x^4}}$$

$$L = \int_1^3 \left(x^2 + \frac{1}{4x^2}\right) dx$$

$$L = \frac{1}{3}\,26 + \frac{1}{6}$$

$$L = \frac{53}{6}$$

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Practice Ex.

Question No:- 5

$$A(x) = \frac{\pi}{4}(\text{diameter})^2$$

$$= \frac{\pi}{4}\left(4x - x^{5/2} + x^7/16\right)$$

$$V = \frac{\pi}{4}\int_0^4 \left(4x - x^{5/2} + x^7/16\right) dx$$

$$V = \frac{32\pi}{4}\left[1 - \frac{8}{7} + \frac{2}{5}\right]$$

$$V = \frac{32\pi}{4}\left[\frac{9}{35}\right]$$

$$V = \frac{72\pi}{35}$$

Question 13:-

a)

$$V = \pi\int_0^2 (x^2 - 2x)^2$$

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$$V = \pi \left[\frac{x^5}{5} + \frac{4x^3}{3} - \frac{4x^4}{4} \right]_0^2$$

$$V = \pi \left[\frac{32}{5} - 16 + \frac{32}{3} \right]$$

$$V = \frac{16\pi}{15}$$

b)

$$V = \pi \int_0^2 1 - (x^2 - 2x + 1)^2$$

$$= \pi \int_0^2 x - \frac{(x-1)^5}{5}$$

$$V = \pi \left(\frac{8}{5} \right)$$

$$V = \frac{8\pi}{5}$$

c)

$$V = 2\pi \int_0^2 (2-x)(-(x^2-2x))$$

$$= 2\pi \int_0^2 (x^3 - 4x^2 + 4x)\,dx$$

$$= 2\pi \int_0^2 \left[\frac{x^4}{4} - \frac{4x^3}{3} + \frac{4x^2}{2} \right]_0^2$$

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$$V = 2\pi\left[4 - \frac{32}{3} + 8\right]$$

$$V = \frac{8\pi}{3}$$

d)

$$V = \pi\int_0^2 2 - (x^2 - 2x)\,dx - \pi\int 2^2\,dx$$

$$= \pi\int_0^2 4 - 4x^2 + 8x + x^4 - 4x^2 + 4x^2\,dx - 8\pi$$

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$$V = \pi\left[\frac{x^5}{5} - x^4 + 4x^2 + 4x\right]_0^2 - 8\pi$$

$$V = \pi\left[\frac{22}{5}\right] - 8\pi$$

$$V = \frac{32\pi}{5}$$

Question No: 31

$$\Delta V = \pi(\text{radius})^2(\text{thickness})$$
$$= \pi\left(\frac{5}{4}y\right)^2 \Delta y$$

$$Fy = 62.4\,\Delta y$$

$$\Delta W = \frac{(62.4)25}{16}\pi y^2(14-y)\Delta y \text{ ft-lb}$$

$$W = \int_0^4 \frac{62.4(25)}{16}\pi y^2(14-y)\,\Delta y \text{ ft-lb}$$

$$W = (62.4)\left(\frac{25\pi}{16}\right)\left[\frac{14}{3}(3^3-0) - \frac{1}{4}(3^4)\right]$$

$$W = 418208.81 \text{ ft-lb}$$

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Ex 7.1

Question 2

As the graph failed horizental line test. So it is not one-to-one function

Question No: 8

The function is one-to-one as the graph passes the horizental line test.

Question No: 21

$f(x) = x^3 - 1$

$y = x^3 - 1$

$y + 1 = x^3$

$(y+1)^{1/3} = x$

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Replacing x with y

*=

$y = (x+1)^{1/3}$

$f^{-1}(x) = \sqrt[3]{x+1}$

Question No: 34

$f(x) = \sqrt[5]{x+1} - 3$

$y = (x+1)^{1/5} - 3$

$y+3 = (x+1)^{1/5}$

$(y+3)^5 = x+1$

$(y+3)^5 - 1 = x$

$y^{-1} = (x+3)^5 - 1$

$f^{-1}(x) = (x+3)^5 - 1$

Question No:- 53

$f(x) = (1-x)^3$

$y^{1/3} = 1 - x$

$x = 1 - y^{1/3}$

$y = 1 - y^{1/3}$

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$f(x) = 1 - x^{1/3}$

$\frac{df}{dx} = 3(1-x^2)_{(-1)}$

$\frac{df^{-1}}{dx} = \frac{1}{-3(1-x)^2}\Big|_{1-x^{1/3}}$

$= \frac{-1}{3(x^{2/3}}$

$\frac{df^{-1}}{dx} = -\frac{1}{3} x^{-2/3}$

Ex 7.2

Question 19:-

$y = \frac{x^4}{4} \ln x - \frac{x^4}{16}$

$\frac{dy}{dx} = \frac{4x^3}{4} \ln x + \frac{x^4}{4} \frac{1}{x} - \frac{4x^3}{16}$

$= x^3 \ln x$

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Question 30:-

$$y = \frac{1}{2} \ln \frac{1+x}{1-x}$$

$$= \frac{1}{2}\left[\ln(1+x) - \ln(1-x)\right]$$

$$\frac{dy}{dx} = \frac{1}{2}\left[\frac{1}{1+x} + \frac{1}{1-x}\right]$$

$$= \frac{1}{2}\left[\frac{2}{1-x^2}\right]$$

$$\frac{dy}{dx} = \frac{1}{1-x^2}$$

Question No: 49

$$y = \int \frac{3\sec^2 t}{6+3\tan t}$$

$u = 6 + 3\tan t$

$du = 3\sec^2 t \, dt$

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$$\int \frac{1}{u}\, du$$

$$= \ln u + C$$

$$= \ln(6+3\tan t) + C$$

Question 60:-

$$y = \sqrt{\frac{t^2+1}{t^2-1}}$$

$$y = \sqrt{\frac{1}{t(t+1)}}$$

$$y = \sqrt{\frac{t}{t+1}}$$

$$\ln y = -\frac{1}{2}\ln t - \ln(t+1)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{-1}{2}\left[\frac{1}{t} + \frac{1}{t+1}\right]$$

$$\frac{dy}{dx} = \frac{-2(t+1)}{2(t^2+t)^{3/2}}$$

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Question No: 70:-

$$y = \frac{x^5}{(1-10x)\sqrt{x^2+2}}$$

$$y' = \frac{5x^4 \times (1-10x)\sqrt{x^2+2} - x^5 \times \left(-10\sqrt{x^2+2} + (1-10x) \times \frac{2x}{2\sqrt{x^2+2}}\right)}{\left[(1-10x)(x^2+2)\right]^2}$$

$$y' = \frac{-30x^7 + 4x^6 - 80x^5 + 10x^4}{\sqrt{x^2+2}\,(100x^4 - 20x^3 + 201x^2 - 40x + 2)}$$

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Ex. 7.5

Question No: 20

$$\lim_{x \to 1} \frac{x-1}{\ln x - \pi \sin(\pi x)}$$

$$\lim_{x \to 1} = \frac{1}{1/x - \pi \cos(\pi x)}$$

$$= \frac{1}{1 - \pi \cos \pi}$$

$$= \frac{1}{1+\pi}$$

Question 30:-

$$\lim_{x \to 0} \frac{3^x - 1}{2^x - 1}$$

$$\lim_{x \to 0} \frac{3^x \ln 3}{2^x \ln 2}$$

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$$= \frac{3^c \ln 3}{2^c \ln 2}$$

$$= \frac{\ln 3}{\ln 2}$$

Question 48:-

$$\lim_{x \to 0} \frac{(e^x - 1)^2}{x \sin x}$$

$$\lim_{x \to 0} \frac{2(e^x - 1)e^x}{x \cos x + \sin x}$$

$$\lim_{x \to 0} \frac{2e^{2x} - 2e^x}{x \cos x + \sin x}$$

$$\lim_{x \to 0} \frac{4e^{2x} - 2e^x}{-x \sin x + 2 \cos x}$$

$$= \frac{4 - 2}{-0 + 2}$$

$$= 1$$

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Question 64

$$\lim_{x \to 0^+} x(\ln x)^2$$

$$= \lim_{x \to 0^+} \frac{(\ln x)^2}{1/x}$$

$$= \lim_{x \to 0} \frac{2(\ln x)1/x}{-1/x^2}$$

$$= \lim_{x \to 0} \frac{2\ln x}{-1/x}$$

$$= \lim_{x \to 0^+} \frac{2/x}{1/x^2}$$

$$= \lim_{x \to 0^+} \frac{2x^2}{x}$$

$$= \lim_{x \to 0} (2x)$$

$$= 0$$

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Ex 8.1

Question No:- 10

$$\int_1^2 \frac{6\,dx}{x^2-4x+2}$$

$u = x-1 \quad du = dx$

$$6\int_0^1 \frac{du}{u^2-1}$$

$= 2x$

Question 10:-

$$\int \csc t \sin 3t \, dx$$

$3t = \sin(2t+t)$

$$\int 2\cos 2t \, dx = \int 1$$

$\sin 2t + t + C$

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Question 17:-

$$\int \frac{\ln y}{y + 4y\ln^2 y}$$

$$\int \frac{\ln y}{y} \cdot \frac{1}{1 + 4\ln^2 y}$$

$$= \frac{\ln}{8}\left(1 + 4\ln^2 y\right) + C$$

Question 22:-

$$\int \frac{x + 2\sqrt{x-1}}{2x\sqrt{x-1}}$$

$$\int \frac{1}{2\sqrt{x-1}}\,dx + \int \frac{1}{x}\,dx$$

$$\sqrt{x-1} + \ln|x| + C$$

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Question 28

$$\int \frac{du}{(x-2)\sqrt{x^2-4x+3}}$$

$$\int \frac{1}{u\sqrt{u^2-1}}$$

$$= \sec^{-1}(u) + C$$

$$= \sec^{-1}(x-2) + C$$

Question 34:-

$$\int e^x + e^2 \, dz$$

$$\int e^2 e^2 \, dz$$

$$= \int e^u \, du$$

$$= e^u + C$$

$$= e^{e^2} + C$$

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Question 51

$$\int (1+3x^3)e^{x^3}$$

$$= \int (1+3x^2)e^{x^3}\,dx$$

$$= xe^{x^3} + C$$

Question 9:-

$$\int x^2 e^{-x}\,dx$$

$$= -x^2e^{-x} - 2xe^{-x} - 2e^{-x} + C$$

$$e^{-x}$$

$$= x^2 + (-e^{-x})$$

$$= 2x - (e-x)$$

$$= 0$$

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Question 16

$$\int p^4 e^{-p}$$

p^4 (+) $-e^{-p}$

$4p^3$ (−) e^{-p}

$12p^2$ (+) $-e^{-p}$

$24p$ (−) e^{-p}

24 (+) $-e^{-p}$

$$(-p^4-4p^3-12p^2-24p-24)e^{-p}+C$$

Question 24

$$\int e^{-2x}\sin 2x\,dx$$

$$=-\frac{1}{2}\int e^{-y}\sin y\,dy = I$$

$$I = -\frac{1}{2}e^{-y}(\sin y + \cos y) - 1 + C'$$

$$= -\frac{e^{-2x}}{4}(\sin 2x + \cos 2x) + C$$

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Question 29

$$\int \sin(\ln x)$$

$$\int (\sin u)\, e^u \, du$$

$$= \frac{1}{2}\left[-x\cos(\ln x) + u\sin(\ln x)\right] + C$$

Question 32

$$= \int \frac{\cos\sqrt{u}}{\sqrt{u}}$$

$$= 2\int \cos u$$

$$= 2\sin u + C$$

$$= 2\sin\sqrt{x} + C$$

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Question 56

$$\int \frac{(\sin^{-1} x)^2}{\sqrt{1-x^2}}\, dx$$

$u = \sin^{-1} x$

$du = \frac{1}{\sqrt{1-u^2}}\, du$

$= \int u^2\, du$

$= \frac{u^3}{3} + C$

$= \frac{1}{3}(\sin^{-1} x)^3 + C$

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