$X_1 = 126, 128, 128, 130, 130, 132$

$X_2 = 78, 80, 82, 82, 84, 86$

$$M_1 = \frac{126+128+128+130+130+132}{6} = 129$$

$$M_2 = \frac{78+80+82+82+84+86}{6} = 82$$

$$\sigma = \sqrt{\frac{\Sigma(x-M)^2}{n-1}} \quad \because n = 6$$

$$\sigma_1 = \sqrt{\frac{22}{5}} = 2.1$$

$$\sigma_2 = \sqrt{\frac{40}{5}} = \sqrt{8} = 2.83$$

=> Co-variance

$$= \frac{\sum_{i=1}^{6}((X_1 - M_1) \times (X_2 - M_2))}{n-1}$$

$$= \frac{28}{5} = 5.6$$

=> Eigen values

$$\sigma_1^2 = (2.1)^2 = 4.4$$

$$\sigma_2^2 = (2.83)^2 = 8.00 \approx 8$$

Covariance Matrix = $\begin{bmatrix} 4.4 & 5.6 \\ 5.6 & 8 \end{bmatrix}$ (C)

$|(C - \lambda I)| = 0$

$$\left| \begin{bmatrix} 4.4 & 5.6 \\ 5.6 & 8 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} 4.4-\lambda & 5.6 \\ 5.6 & 8-\lambda \end{vmatrix} = 0$$

$(4.4-\lambda)(8-\lambda) - (31.36) = 0$

$35.2 - 12.4\lambda + \lambda^2 - 31.36 = 0$

$\lambda^2 - 12.4\lambda + 3.84 = 0$

Using Quadratic Formula

$$\lambda = \frac{-(-12.4) \pm \sqrt{(-12.4)^2 - 4(1)(3.84)}}{2(1)}$$

$$\lambda = \frac{12.4 \pm \sqrt{138.4}}{2} = \frac{12.4 \pm 11.76}{2}$$

$$\lambda_1 = \frac{12.4 + 11.76}{2}$$

$$\lambda_1 = \frac{24.16}{2}$$

$\lambda_1 = 12.08$

$$\lambda_2 = \frac{12.4 - 11.76}{2}$$

$$= \frac{0.64}{2}$$

$\lambda_2 = 0.32$

=> Standardizing

$$Z = \frac{x_i - M_i}{S_i}$$

$(Z_1)$

FOR $X_1$ = -1.43, -0.48, -0.48, 0.48, 0.48, 1.43

FOR $X_2$ = -1.41, -0.71, 0, 0, 0.71, 1.41

$(Z_2)$

=> Eigen Vectors

~~$A = \begin{bmatrix} a \\ \end{bmatrix}$~~

For $\lambda_1 = 12.08$.

$$\begin{bmatrix} 4.4-12.08 & 5.6 \\ 5.6 & 8-12.08 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -7.68 & 5.6 \\ 5.6 & -4.08 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \Rightarrow \begin{bmatrix} 0.83 \\ 0.56 \end{bmatrix}$$

For $\lambda_2 = 0.32$

$$\begin{bmatrix} 4.4-0.32 & 5.6 \\ 5.6 & 8-0.32 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 4.08 & 5.6 \\ 5.6 & 7.68 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} -0.56 \\ 0.83 \end{bmatrix}$$

=> PC

$PC_1 = (0.83 \times Z_1) + (0.56 \times Z_2)$

$= [-1.98, -0.80, -0.4, -0.4, 0.8, 1.98]$

$PC_2 = (-0.56 \times Z_1) + (0.83 \times Z_2)$

$= [-0.37, -0.32, 0.27, -0.27, 0.32, 0.37]$

Date:

=> Entropy (Achieved Goal)

$P_{yes} = \frac{5}{9}$ $P_{NO} = \frac{4}{9}$

$$E(AG) = -\frac{5}{9}\log_2\frac{5}{9} - \frac{4}{9}\log_2\frac{4}{9}$$

$$= -\frac{5}{9}(-0.847) - \frac{4}{9}(-1.17)$$

$$= 0.47 + 0.52$$

$$= 0.99$$

=> Information Gain:

1) Exercises Daily = Yes

- AG = Yes => P = 4/5
- AG = NO => P = 1/5

$$\text{Entropy (yes)} = -\frac{4}{5}\log_2\frac{4}{5} - \frac{1}{5}\log_2\frac{1}{5}$$

$$= -\frac{4}{5}(-0.322) - \frac{1}{5}(-2.32)$$

$$= 0.257 + 0.464$$

$= 0.721$

2) Exercises Daily = NO

- AC1 = Yes $\Rightarrow P = 1/4$
- AC1 = NO $\Rightarrow P = 3/4$

$$\text{Entropy}_{(NO)} = -\frac{1}{4}\log_2\frac{1}{4} - \frac{3}{4}\log_2\frac{3}{4}$$

$$= -\frac{1}{4}(-2) - \frac{3}{4}(-0.415)$$

$$= 0.5 + 0.311$$

$$= 0.811$$

$\Rightarrow$ Weighted Average Entropy

$$= \frac{5}{9} \times 0.721 + \frac{4}{9} \times 0.811$$

$$= 0.4 + 0.36$$

$$= 0.76$$