Assignment #3

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Solutions:-

Q1)

Let 'X' be sample I

" 'Y' " " II

X	$(X-\bar{X})$	$(X-\bar{X})^2$	Y	$(Y-\bar{Y})$	$(Y-\bar{Y})^2$
25	-1.5	2.25	44	13.83	191.27
30	3.5	12.25	34	3.83	14.67
28	1.5	2.25	22	-8.17	66.75
34	7.5	56.25	8	-22.17	491.51
24	-2.5	6.25	47	16.83	283.25
25	-1.5	2.25	31	0.83	0.69
13	-13.5	182.25	40	9.83	96.63
32	5.5	30.25	30	-0.17	0.03
24	-2.5	6.25	32	1.83	3.35
30	3.5	12.25	35	4.83	23.33
			18	-12.17	148.11
$\Sigma = 265$			21	-9.17	84.09
$\bar{X} = \frac{\Sigma X}{n}$		$\Sigma(X-\bar{X})^2 = 312.5$	$\Sigma = 362$		$\Sigma(Y-\bar{Y})^2 = 1403.7$

$$\bar{Y} = \frac{\Sigma Y}{n} = \frac{362}{12} = 30.17$$

$$\bar{X} = \frac{265}{10} = 26.5$$

$n_1 = 10$

$n_2 = 12$

$H_0 : \sigma_1^2 = \sigma_2^2$

$H_1 : \sigma_1^2 \neq \sigma_2^2$

$$F = \frac{S_1^2}{S_2^2} \quad \text{if } S_1^2 > S_2^2$$

$$S_1^2 = \frac{\Sigma(X-\bar{X})^2}{n_1-1} = \frac{312.5}{10-1} = 34.72$$

$$S_2^2 = \frac{\Sigma(Y-\bar{Y})^2}{n_2-1} = \frac{1403.7}{12-1} = 127.61$$

$$F = \frac{S_2^2}{S_1^2} = \frac{127.61}{34.72} = 3.68 \rightarrow \text{because } S_2^2 > S_1^2 \rightarrow F = \frac{S_2^2}{S_1^2}$$

Critical region reject H_0 if

$F > F_{\alpha/2}(V_1, V_2)$

$F > F_{.05/2}(11, 9)$

$F > F_{0.025}(11, 9)$

$F > 3.91$

$V_1 = n_1 - 1 = 10 - 1 = 9$

$V_2 = n_2 - 2 = 12 - 1 = 11$

we will consider V_2 first and V_1 at second because $S_2^2 > S_1^2$

$F_{cal} < F_{tab} \rightarrow 3.68 < 3.91$ So we accept H_0

$F < F_{1-\alpha/2}(V_1, V_2)$

$F < F_{1-.05/2}(V_1, V_2)$

$F < F_{.975}(V_1, V_2) \rightarrow F_{.975}(11, 9) \rightarrow \frac{1}{F_{.025}(9, 11)} \rightarrow \frac{1}{3.59} = 0.279$

$0.279 < F < 3.91$

Since our calculated value lies with in range of tabulated values so we accept null hypothesis so two populations have same variances.

Q2)

$H_0: \mu_1 = \mu_2 = \mu_3$

H_1: at least one of them is not equal

Teaching Methods

I	II	III		
40	70	45		Mean Sample I → 215/4 = 53.75
50	65	38		Mean Sample II → 251/4 = 62.75
60	66	60		Mean Sample III → 185/4 = 46.25
65	50	42		
215	251	185	651	→ Total
53.75	62.75	46.25	162.75	→ Mean Total

N = 12 no. of data in samples

K = 3 no. of groups

Correction factor : $\frac{T^2}{N} = \frac{(651)^2}{12} = 35316.75$

Total SS : $\sum\sum x_{ij}^2 - CF = (40^2+50^2+60^2+65^2+70^2+65^2+66^2+50^2+45^2+38^2 + 60^2+42^2) - 35316.75$

$\Rightarrow 36739 - 35316.75$

$\rightarrow 1422.25$

Between Treatment SS $= \frac{\sum x_{ij}^2}{n} - CF = \frac{1}{4}(215^2+251^2+185^2) - 35316.75$

$35862.75 - 35316.75 \rightarrow 546$

Within Treatment SS = Total SS − Between Treatment SS

$= 1422.25 - 546 = 876.25$

Sum of variation	Sum of Squares	d.f	Mean Square	F-ratio
Between Treatment SS	SSB=546	K-1 2	$S_1^2 = \frac{SSB}{K-1}$ $= \frac{546}{2}$ $= 273$	$F = \frac{S_1^2}{S_2^2}$ $F = \frac{273}{97.4}$
Within Treatment SS	SSE=876.25	N-K 9	$S_2^2 = \frac{SSE}{N-K}$ $= \frac{876.25}{9}$ $= 97.4$	$F = 2.80$
Total SS	SST = 1422.25	N-1 11		

Now find tabulated value: $\alpha = 0.05$

$F_\alpha (v_1, v_2)$

$F_{.05}(2, 9)$

$F_{tab} = 4.26$

Since calculated value is less than tabulated value so we accept our null hypothesis and therefore mean of three populations are equal.

Q3 (i)

X	Y	XY	X^2
5	16	80	25
6	19	114	36
8	23	184	64
10	28	280	100
12	36	432	144
13	41	533	169
15	44	660	225
16	45	720	256
17	50	850	289
$\Sigma X = 102$	$\Sigma Y = 302$	$\Sigma XY = 3853$	$\Sigma X^2 = 1308$

$\bar{X} = \frac{102}{9} = 11.33$

$\bar{Y} = \frac{302}{9} = 33.56$

$$b = \frac{n\Sigma XY - \Sigma X \Sigma Y}{n\Sigma X^2 - (\Sigma X)^2} = \frac{9(3853) - (102)(302)}{9(1308) - (102)^2}$$

$$\rightarrow \frac{34677 - 30804}{11772 - 10404} \Rightarrow \frac{3873}{1368}$$

$$\boxed{b = 2.83}$$

$$a = \bar{y} - b\bar{x}$$

$$a = 33.56 - (2.83)(11.33)$$

$$\boxed{a = 1.50}$$

$$y = a + bx$$

$$y = 1.50 + 2.83x$$ → regression line equation

(ii) $e = y - \hat{y}$ ∴ we will find $\hat{y}$ using $\hat{y} = 1.50 + 2.83x$

Show that $\Sigma e = 0$

$\hat{y}$	$y - \hat{y}$	$(y - \hat{y})^2$
15.65	0.35	0.1225
18.48	0.52	0.2704
24.14	-1.14	1.2996
29.8	-1.8	3.24
35.46	0.54	0.2916
38.29	2.71	7.3441
43.95	0.05	0.0025
46.78	-1.78	3.1684
49.61	0.39	0.1521
$\Sigma\hat{y} = 302.16$	$\Sigma(y - \hat{y}) = -0.16$	$\Sigma(y - \hat{y})^2 = 15.8912$

(ii) $\Sigma(y - \hat{y}) \cong 0$

$\Sigma(y - \hat{y}) = -0.16$

$0.16 \cong 0$ hence proved $\Sigma e = 0$

(iii) $S_{x \cdot y} = \sqrt{\dfrac{\Sigma(Y-\hat{Y})^2}{n-2}}$

$= \sqrt{\dfrac{15.8912}{9-2}}$

$\boxed{S_{x \cdot y} = 1.51}$

(iv) $\Sigma Y = 302$

$\Sigma \hat{Y} = 302.16$

$\Sigma Y \cong \Sigma \hat{Y}$

$302 \cong 302.16$ hence proved $\Sigma Y = \Sigma \hat{Y}$

(v) $r^2 = 1 - \left(\dfrac{\Sigma(Y-\hat{Y})^2}{\Sigma(Y-\bar{Y})^2}\right)$

$y-\bar{y}$	$(y-\bar{y})^2$
-17.56	308.35
-14.56	211.99
-10.56	111.51
-5.56	30.91
2.44	5.95
7.44	55.35
10.44	108.99
11.44	130.87
16.44	270.27
	$\Sigma(y-\bar{y})^2 = 1234.19$

coefficient of determination

$r^2 = 1 - \dfrac{15.8912}{1234.19}$

$\boxed{r^2 = 0.9871}$

Q4)

X	Y	XY	X^2	Y^2
18	62	1116	324	3844
29	42	1218	841	1764
39	59	2301	1521	3481
9	19	171	81	361
19	38	722	361	1444
31	42	1302	961	1764
42	65	2730	1764	4225
51	75	3825	2601	5625
$\Sigma X = 238$	$\Sigma Y = 402$	$\Sigma XY = 13385$	$\Sigma X^2 = 8454$	$\Sigma Y^2 = 22508$

$$r = \frac{n\Sigma XY - \Sigma X \Sigma Y}{\sqrt{(n\Sigma X^2 - (\Sigma X)^2)(n\Sigma Y^2 - (\Sigma Y)^2)}}$$

$$-1 \le r \le 1$$

$$r = \frac{8(13385) - (238)(402)}{\sqrt{[8(8454) - (238)^2][8(22508) - (402)^2]}} = \frac{11404}{\sqrt{202838480}}$$

$$\boxed{r = 0.801}$$ strong positive correlation

Q5)

Y	X_1	X_2	X_1X_2	X_1^2	X_2^2	X_1y	X_2y
12	2	1	2	4	1	24	12
10	5	4	20	25	16	50	40
9	7	3	21	49	9	63	27
13	4	5	20	16	25	52	65
20	6	3	18	36	9	120	60
$\Sigma y = 64$	$\Sigma X_1 = 24$	$\Sigma X_2 = 16$	$\Sigma X_1X_2 = 81$	$\Sigma X_1^2 = 130$	$\Sigma X_2^2 = 60$	$\Sigma X_1y = 309$	$\Sigma X_2y = 204$

$y = a + bx$

$y = a + b_1x_1 + b_2x_2 \rightarrow$ (A)

$\Sigma y = na + b_1\Sigma x_1 + b_2\Sigma x_2$ (i)

$\Sigma X_1y = a\Sigma X_1 + b_1\Sigma X_1^2 + b_2\Sigma X_1X_2 \rightarrow$ (ii)

$\Sigma X_2y = a\Sigma X_2 + b_1\Sigma X_1\Sigma X_2 + b_2\Sigma X_2^2 \rightarrow$ (iii)

$64 = 5a + 24b_1 + 16b_2$ ___ (i)

$309 = 24a + 130b_1 + 81b_2$ ___ (ii)

$204 = 16a + 81b_1 + 60b_2$ ___ (iii)

eq (i) and eq (ii)

$64 = 5a + 24b_1 + 16b_2 \quad \times 24$

$309 = 24a + 130b_1 + 81b_2 \quad \times 5$

eq (i) becomes $\rightarrow$ $1536 = \cancel{120a} + 576b_1 + 384b_2$

eq (ii) becomes $\rightarrow$ $1545 = \cancel{120a} + 650b_1 + 405b_2$

$-9 = -74b_1 - 21b_2 \rightarrow$ multiply by -1

$9 = 74b_1 + 21b_2 \rightarrow$ (B)

eq (ii) and eq (iii)

$309 = 24a + 130b_1 + 81b_2 \quad \times 16$

$204 = 16a + 81b_1 + 60b_2 \quad \times 24$

eq (ii) becomes $\rightarrow$ $4944 = \cancel{384a} + 2080b_1 + 1296b_2$

eq (iii) becomes $\rightarrow$ $4896 = \cancel{384a} + 1944b_1 + 1440b_2$

$48 = 136b_1 - 144b_2 \rightarrow$ (C)

From eq B: $\boxed{\dfrac{9 - 21b_2}{74} = b_1}$ put in eq (C)

$$48 = 136\left(\frac{9 - 21b_2}{74}\right) - 144b_2$$

$$48 = \frac{1224 - 2856b_2}{74} - \frac{144b_2}{1}$$

$$48 = \frac{1224 - 2856b_2 - 10656b_2}{74}$$

$$3552 = 1224 - 2856b_2 - 10656b_2$$

$$3552 - 1224 = -2856b_2 - 10656b_2$$

$$2328 = -13512b_2$$

$$\frac{-2328}{13512} = b_2$$

$$\boxed{b_2 = -0.17}$$ put in eq (B)

$$b_1 = \frac{9 - 21(-0.17)}{74}$$

$$\boxed{b_1 = 0.17}$$

put b_1 and b_2 in eq (i)

$$64 = 5a + 24(0.17) + 16(-0.17)$$

$$64 = 5a + 1.36$$

$$\frac{62.64}{5} = a \qquad \boxed{a = 12.53}$$

$\rightarrow \therefore y = 12.53 + 0.17x_1 - 0.17x_2$