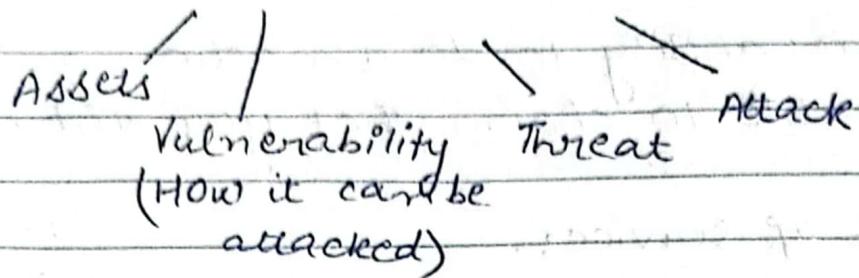




Lecture-2

Risk Estimation



→ Probabilities of a threat:

prioritizing threats | scarcity of choice → limited
with higher probabilities resource

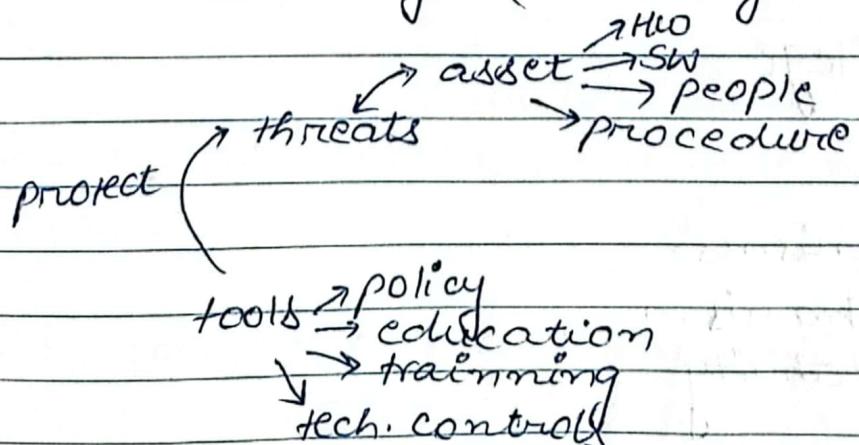
• Info-Security: (IS)

→ investment \$\$, PKR ① WASP: not for profit
people/user organization
Hardware

IS → Information System → Software
(policies) procedure → data
→ Network Infrastructure.

•) Attack surface → # of vulnerabilities in my info-system / how many ways my system can be attacked.

→ Threat Modeling: (Modeling the (possible) threats)



Microsoft STRIDE:

S: Spoofing / taking identity

T: Tampering with data

R: Repudiation

I: Info disclosure

D: Denial of service

E: Elevation of privilege

Threat Model by Microsoft



DDOS: Distributed Denial of Service.

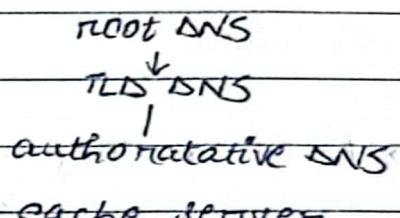


STRIDE → categories of threat

→ Repudiation: ability to do an action and later being able to claim that you did not do it.

• D: Denial of Service:-

DNS → Domain Name Service
allows access to a URL.
→ ping/DNS



→ send many requests so that the system becomes down in 1 second (DOS)

→ Priority = ease of exploitation * impact of exploitation

Lecture - 3

→ Security Design Principles:-

Principle of ...

- 1) least privilege
- 2) separation of privilege
- 3) fail-safe defaults
- 4) complete mediation
- 5) economy of mechanisms
- 6) least common mechanism
- 7) psychological acceptability



(mora)
coarse
 \nwarrow fine (bauriqi)
"granularity of privileges"

- 1) provide bare minimum privileges to a program or user to function properly. "temporary elevation" should be relinquished immediately.
- 2) access should not be granted on basis of a single condition. e.g 2 persons sign on cheque, password / login + OTC (fin. transaction).
- 3) the default config. of a system should have a conservative approach. e.g default access to an obj is 'none'.
 - examples \rightarrow AC\$ (access control list)
 - \rightarrow firewall rules
- 4) User \rightarrow OS \rightarrow object (instead of 1 time check, OS should always check whenever the obj is accessed)
- 5) simplicity in design & implementation of security measures
 - \Rightarrow a "simple security framework" provides fewer errors; development, testing & verification of security measures is easy; less assumption.
- 6) in shared systems w/ multiple users, mechanisms allowing resources to be shared by > 1 user should be minimized ("separate channel" for users, separate of



network resources".) to-do

- 7) security mechanism should not make the resources difficult to access. • UI should be well-designed & intuitive
- ordinary users should be kept in mind when planning.

→ Cryptanalysis: the process of obtaining the plain text msg from a ciphertext msg w/out knowing the keys used to perform encryption

→ Cryptography: the process of making & using codes to secure information.

→ Cryptology: the field of science that encompasses cryptography & cryptanalysis.

→ Types of cryptographic functions:

i) ⇒ secret/symmetric key cryptographic functions

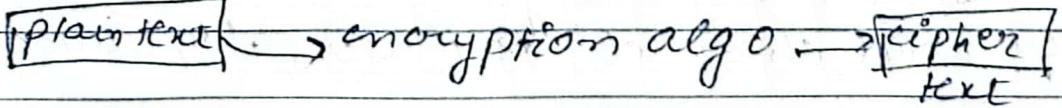
- uses 1 key
- fast computation

ii) ⇒ public/asymmetric key (PKC)

- uses 2 keys
- slow computation

iii) ⇒ Hash functions

- NO keys
- very fast computation.



ciphertext \rightarrow decryption algo \rightarrow plain text.



is also called "conventional cryptography". Sender & receiver must both know the secret key. uses techniques like 'confusion' & 'diffusion' to encrypt/decrypt data.

Sender \rightarrow Encrypt \rightarrow Decrypt \rightarrow Receiver.

\rightarrow Symmetric encryption uses:

- transmitting over a secure channel
- secure storage on insecure media

\rightarrow Authentication:

- strong authentication prove the knowledge of a secret w/out revealing it.

\rightarrow Integrity check:

- checksum vs cryptographic checksum
- Message Authorization code (MAC)

\hookrightarrow caesar's cipher.

\rightarrow Substitution cipher (monoalphabetic)

plain

text \leftarrow jeejanab

\rightarrow "+3" \leftarrow

mhhymndqyde \rightarrow ciphertext

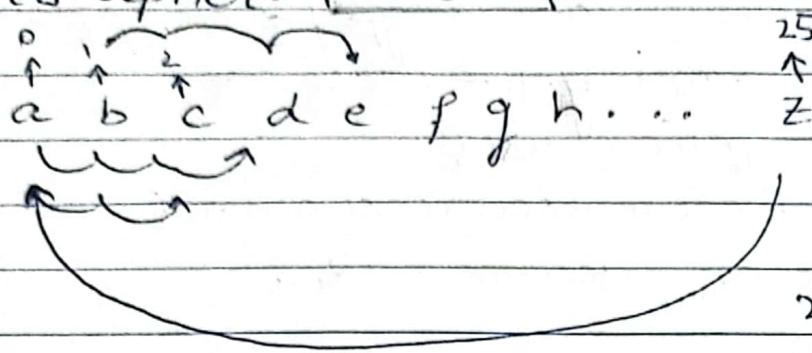
\hookrightarrow using single alphabet
to create a mapping.



type of monoalphabetic

Lecture - 4

→ caesar's cipher



$$2 + 3 = C$$

$$25 + 3 \bmod 26 = 2$$

intro to db → plain text

26 alphabets

↳ Iqwan wngc (encrypted using caesar's cipher)

$$\text{Encryption algo: } C = E(3, P) = (P+3) \bmod 26$$

ciphertext character plaintext character

Ex: abdf → plain text
degi → ciphertext

Decryption Algo → $P = D(K, C) = (C - K) \bmod 26$

→ monoalphabetic cipher: substitution.

We don't have to follow any specific pattern. Using single alphabet to create a mapping.

Ex:

$a \rightarrow e$

$g \rightarrow q$

$m \rightarrow l$

$s \rightarrow m$

$y \rightarrow z$

$b \rightarrow j$

$h \rightarrow i$

$n \rightarrow h$

$t \rightarrow u$

$z \rightarrow t$

$c \rightarrow d$

$i \rightarrow c$

$o \rightarrow f$

$u \rightarrow v$

$d \rightarrow b$

$j \rightarrow o$

$p \rightarrow g$

$v \rightarrow w$

$e \rightarrow k$

$l \rightarrow s$

$q \rightarrow p$

$w \rightarrow x$

$f \rightarrow a$

$r \rightarrow n$

$s \rightarrow n$

$x \rightarrow y$



random mapping stored to encrypt a plain text.

plain text \rightarrow zebra crossing

encrypted \rightarrow tkjne dnfmncq (cipher text)

- Generally, to decrypt this message with brute force, the time taken will be 26!.

- When not using brute force:

frequency of 2-letter \leftarrow digrams

| | | | |
|------------|------|-------------|----------|
| 12% | 8% | words | an at if |
| c, t, i, a | g 7% | of is to ok | |
| ↳ | ↳ | | |
| 9% | | | |

- We will try/use the letters with higher probabilities to decrypt the message.

\rightarrow Vigenère cipher (polyalphabetic)

Ex: Vigenère table (in grid)

keyword: Houghton column

plain text \rightarrow michigan technological university

now \leftarrow HOUGHTON HOUGHTONHOUGHTONHOUGHTON

cipher text

e consists of

\rightarrow keyword ~~is~~ multiple alphabets that is why it is known as polyalphabetic



Lecture - 5

Substitution techniques:

- One-time pad

Transposition techniques

- Rail-fence cipher
 - Row transposition cipher

→ Rail Fence (with some depth level x) no. of rows which will be used

Method: Write the plaintext in diagonal form, based on the depth level x

② Read the text in rows, to get the ciphertext.

Ex: Plain text: nespresso academy is the best

depth: 2 (rows) columns (no. of letters): 20

(white spaces are not considered in this example)

N E S O A C A D E M Y I S T H E B E

S | T

ciphertext: NSAAEYSHBSEOCDMITEET

Ex: depth = 3 (If depth = 3, total no. of letter '1' & '2' = no. of letters in middle row)

cipher text: TKVMHN YUEYUHAORC (letters = 16)

T H E A
K M N U Y V H
V Y U



| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | H | N | K | Y | U | V | E | R | Y | M | U | C | H |
| H | A | N | Y | U | E | V | R | Y | M | U | C | H | |
| A | | | | 0 | | | | | | | | | |

Plain text: THANK YOU VERY MUCH

→ Row Transposition cipher:

- ~ Writing plain text into rows & then reading of ciphertext from the columns.
- It works by rearranging the order of characters in the plain text, according to a specific pattern on key, & then writing the resulting cipher text in a grid format.

Ex:

plain text: winter is coming

key: h a c k → 2 3 1 4

2 3 1 4 (smallest in alpha numeric)

| ↓ columns 4 rows | 1 | 2 | 3 | 4 | (length of key = no. of columns) |
|---------------------|---|---|---|---|----------------------------------|
| | W | I | N | T | |
| | E | R | - | I | |
| | S | - | C | O | |

ciphertext: IR-IN-CNWESS-CO
MTIOG MING

(rowcipher.netlify.app) → to convert plain text to cipher text.

| | |
|---|-----|
| 2 | 13 |
| 2 | 6-1 |
| 2 | 3-0 |
| 2 | 1-0 |
| 2 | 0-1 |

| | |
|---|-----|
| 2 | 14 |
| 2 | 7-0 |
| 2 | 3-1 |
| 2 | 1-1 |
| 2 | 0-1 |



Ex: plain text: this life is short

key: quick

43512 (cols = 5)

| a | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| t | h | i | s | b | - |
| i | ; | f | e | - | - |
| h | s | - | g | b | h |
| o | n | t | - | - | - |

cipher text: b-e-s-i-f-t--h-+
no. of char > cols = rows
for decryption

→ One-Time Pad: key length = plain text length
key is generated randomly
based on vernam cipher

PT [] Key []
Take XOR to get CT

Ex: plain text: O A K "14" 0 1 1 0
key: S O N "18" 1 0 0 1 0
XOR ↓↓↓ 1 1 1 0 0
C O H 28 + 26 = 2 → 0 0 0 1 0
"C"

A : 0 0 0 0 0
O : 0 1 1 1 0
0 1 1 1 0 → 14 = 0

K ↗ 0 1 0 0 0
K ↗ 0 1 0 0 0

→ perfect secrecy" → CT provides no info of PT except length.



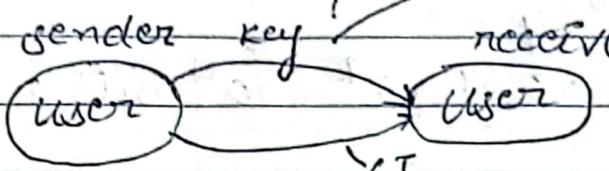
Lecture - 6

Traditional Block Cipher Structure

- Stream cipher reversible
- Block cipher irreversible

Festive cipher

→ Stream cipher:



→ To solve this, we define an algo.

which will be used to generate a key

The key should be

random.

- one that encrypts a digital data stream one bit or one byte at a time. For example Vigenere cipher, Vernam cipher
- If the cryptographic key stream is random, then this cipher is unbreakable by any means other than acquiring the key stream.

→ Block cipher:

one in which a block of plain text is treated as a whole & used to produce a cipher text block of equal length. Typically, 64-bit or 128-bit.

- Most network based systems. Cryptographic applications make use of block cipher

Reversible Mapping
 PlainText → Ciphertext

00

11

e.g. $i=1$

01

10

mapping

10

00

means $\rightarrow 2 \times 4 = 8$ bits

11

01

key length

Irreversible Mapping

PT

00

11

01

10

10

01

11

01

CS



$PT \rightarrow CT$ (n-bits)

2^n possible different PT blocks

* when $n = 4$

$\rightarrow CT$ length \times no. of bits = 4 bits \times 16

* when $n = 64$

$64 \text{ bits} \times 2^{64} = 2^{70} \approx 10^{21}$ bits

in

→ Feistel Cipher: / Structure / Network

Step 1: Split plain text into two halves L_0 & R_0

Step 2: $E = f_S(R_0)$ $f_S \rightarrow$ some algo

Step 3: $L_1 = R_0$ and $R_1 = L_0 \text{ XOR } E$ $E \rightarrow$ temp variable

Step 4: Concatenate L_1 and $R_1 \Rightarrow$ resulting in our ciphertext

$\therefore L_1 \Rightarrow$ first part of cipher text

$\therefore R_1 \Rightarrow$ second part of cipher text

$L_0 \quad R_0$

Ex: Plaintext 01110 100001

= ~~010101~~ 010100 (E)

$$L_1 = 100001$$

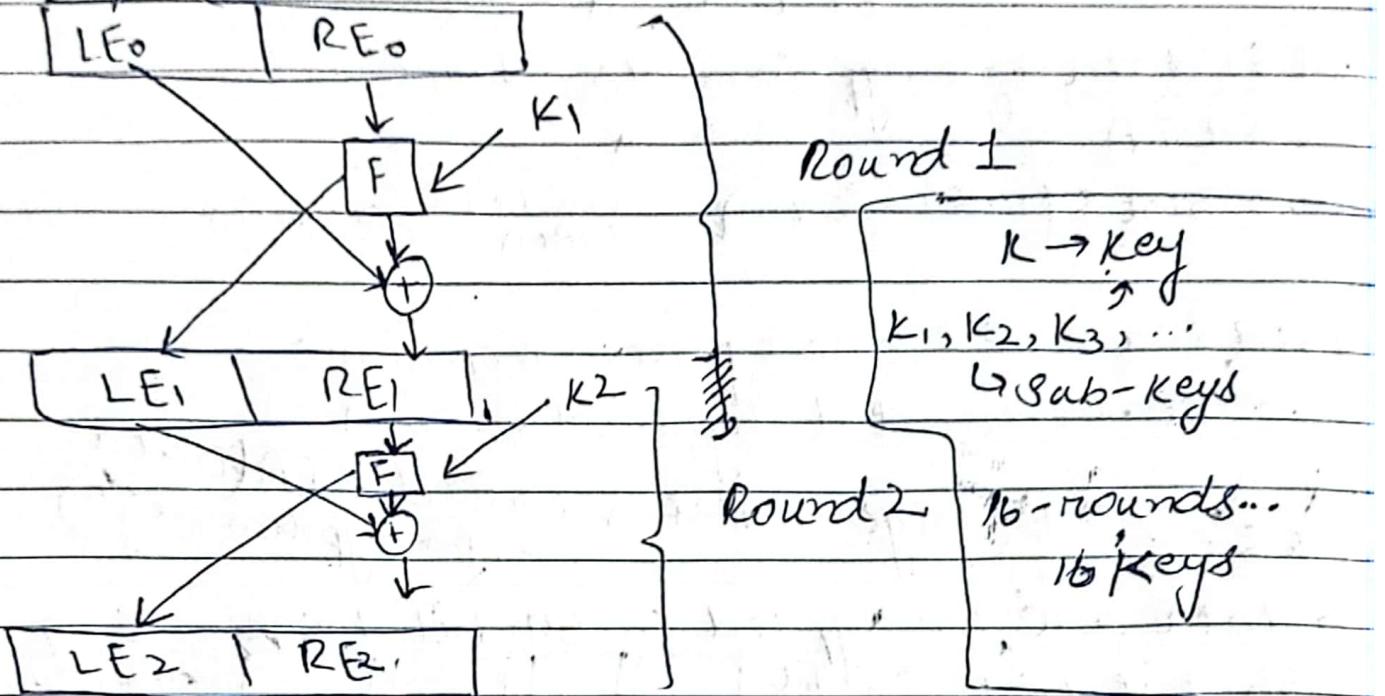
$$R_1 = 01110 \text{ XOR } 010100$$

$$\begin{array}{r} L_1 \text{ concatenate} \\ R_1 \end{array} = 001010$$

$$R_1 \rightarrow 100001001010$$



Input (Plain Text)



The exact realization of a Feistel network depends on the choice of the following parameters of design features.

- i) Block size iii) # of rounds
- ii) Key size iv) Subkey generation algo
- v) Round function F



Lecture - 7

DES: data encryption standard

- DEA
- 3DES (later form)
- LUCIFER (IBM)
- NISON

AES: advanced encryption technique

- recommended by NIST in 2001.

• DES: → the algo in this is called DEA (Data Encryption algo)

→ data are encrypted in 64-bit blocks using a 56-bit key → 64-bit  every 8th bit

→ the algo transforms 64-bit input in a series of steps into a 64-bit output.

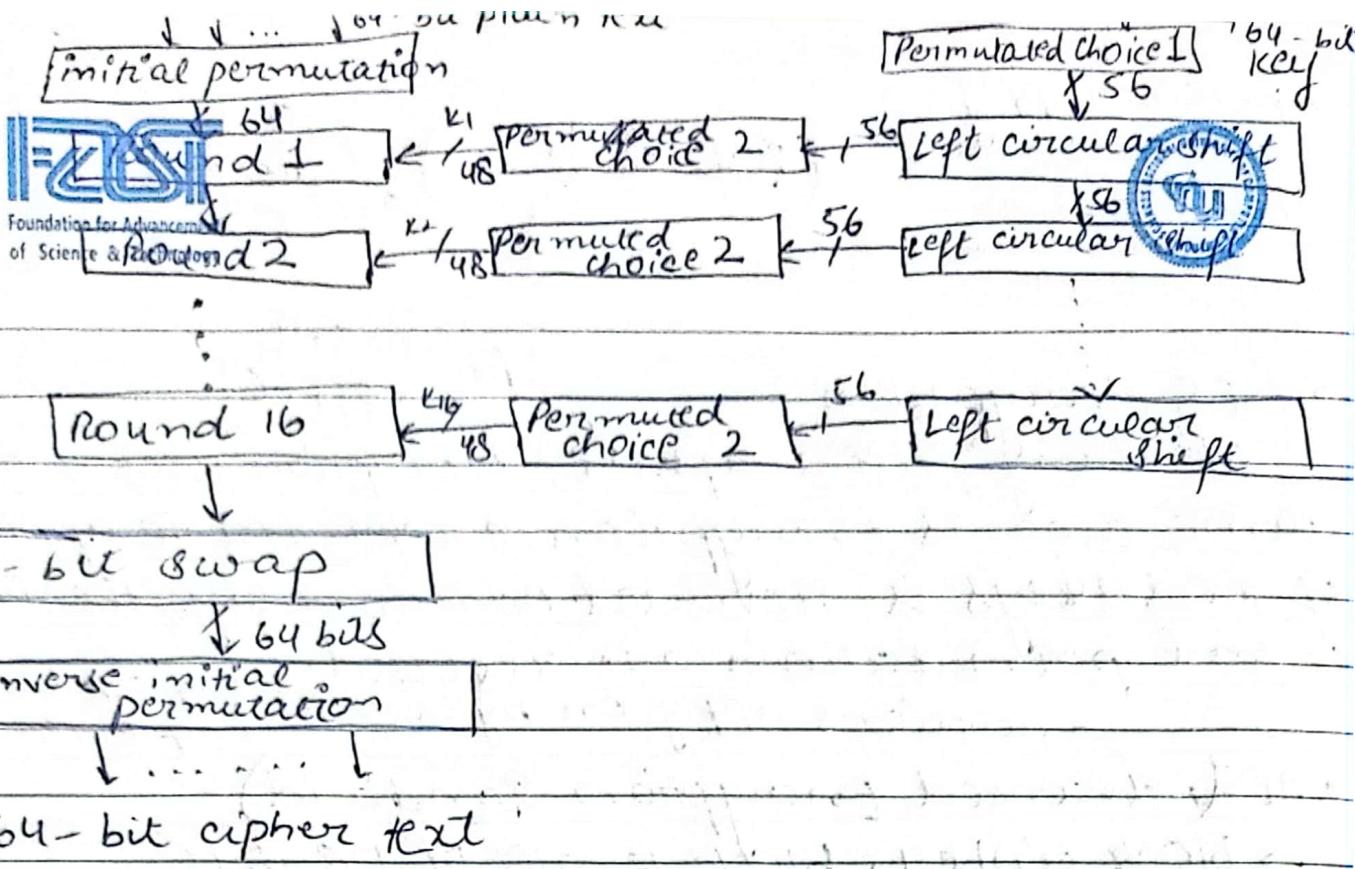
→ the same steps, with the same key, are used to reverse the encryption

~ 1999 ⇒ 3DES introduced by NIST (i.e., repeat DEA 3 times, using 2 or 3 different keys)

~ DES has the exact same situation structure of a feistal cipher, except for the initial & final permutation

→ key:

Initially, key is passed through a permutation fn. Then, for each of the 16 rounds, a subkey (k_i) is produced by the combination of a left circular shift & a permutation. In each round, the permutation f_{k_i} is same.



• DES/DEA (3 phases)

- ① 64-bit PT passes thru an initial permutation (IP) that rearranges the bits to produce the permuted bit
- ② Phase of 16 rounds of the same function, which involves both permutation & substitution functions.
~ the output of the last round (16th)... consists of a 64-bits that are a function of the input PT & the key.
- ③ Left & right halves of the output are swapped to produce the "preoutput". Then, the preoutput is passed thru a permutation [IP⁻¹] that is the inverse of the initial permutation function, to get 64-bit CT.



(Table 4.5). Avg time for key search.
Exhaustive search.



AVALANCHE EFFECT

→ DES decryption?

as w/f any feistal cipher, decryption uses the same algo as encryption, except that the application of the subkeys is reversed. Additionally, the initial & final permutation are reversed.
→ functions apply on bytes \Rightarrow quicker software implementation.

• AES(Advanced Encryption Standard)

→ block cipher, block length of 128-bits

→ allows for 3 different key lengths: 128, 192, 256 bits.

→ encryption consists of 10 rounds of processing for 128-bit keys, 12 rounds for 192-bit keys, & 14 rounds for 256-bit keys

→ except for the last round, in each case, all other rounds are identical.

Figure (b.1) AES \rightarrow chp 4
 AES \rightarrow chp 6

- Each round of processing includes one single byte based substitution step, a row wise permutation step, a column wise mixing step & the addition of the round key.

in phase 1



"IP → initial permutation"

Lecture-8

→ DES: (Data Encryption Standard)

DES-key size = 56 bits (64 bits, but 8 bits are used as
→ ^{Plaintext} parity check bits).IP(x) = $\{R_0 R_1 \dots R_{15}\}$ - Phase 1: Initial Permutation $L_i = R_{i-1}$ } - Phase 2) repeats 16x $R_i = L_{i-1} \oplus f(R_{i-1}, k_i)$ $y = IP^{-1}(R_{16} L_{16})$ } → Phase-3
→ cipher text

| | | | | | | | | |
|----|----|----|----|----|----|----|---|-------------------|
| 58 | 50 | 42 | 34 | 26 | 18 | 10 | 2 | → This table |
| 60 | 52 | 44 | 36 | 28 | 20 | 12 | 4 | specifies the |
| 62 | 54 | 46 | 38 | 30 | 22 | 14 | 6 | input permutation |
| 64 | 56 | 48 | 40 | 32 | 24 | 16 | 8 | on a 64-bit |
| 57 | 49 | 41 | 33 | 25 | 17 | 9 | 1 | block |
| 59 | 51 | 43 | 35 | 27 | 19 | 11 | 3 | |
| 61 | 53 | 45 | 37 | 29 | 21 | 13 | 5 | |
| 63 | 55 | 47 | 39 | 31 | 23 | 15 | 7 | |

• The meaning is as follows

The 1st bit of the output is taken from the 5th bit of the input, the 2nd bit from the 58th bit, etc. so on, with the last bit of the output taken from 7th bit of the input.

• This info is presented as a table for ease of presentation, it is a vector, not a matrix



NOTE that, as usual

$$R_{16} = L_{15} \oplus f(R_{15}, K_{14})$$

$$L_{16} = R_{15}$$

~ but they are switched in the pre output $\rightarrow (R_{16}, L_{16})$
 $\rightarrow (R_{16}, L_{16})$

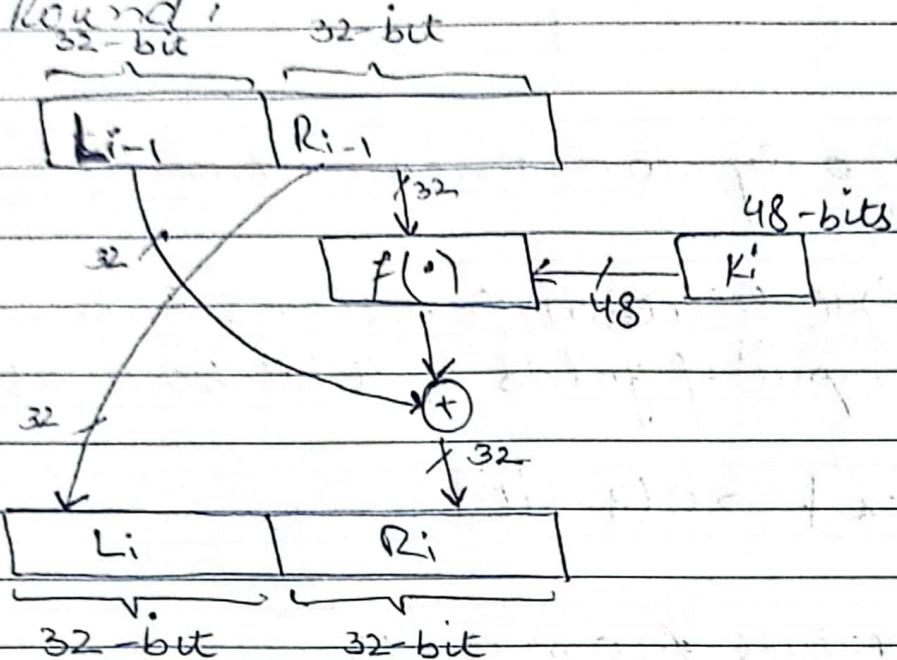
Final Permutation

| | | | | | | | |
|----|---|----|----|----|----|----|----|
| 40 | 8 | 48 | 16 | 56 | 24 | 64 | 32 |
| 39 | 7 | 47 | 15 | 55 | 23 | 63 | 31 |
| 38 | 6 | 46 | 14 | 54 | 22 | 62 | 30 |
| 37 | 5 | 45 | 13 | 53 | 21 | 61 | 29 |
| 36 | 4 | 44 | 12 | 52 | 20 | 60 | 28 |
| 35 | 3 | 43 | 11 | 51 | 19 | 59 | 27 |
| 34 | 2 | 42 | 10 | 50 | 18 | 58 | 26 |
| 33 | 1 | 41 | 9 | 49 | 17 | 57 | 25 |

Interpretation (similar to IP)

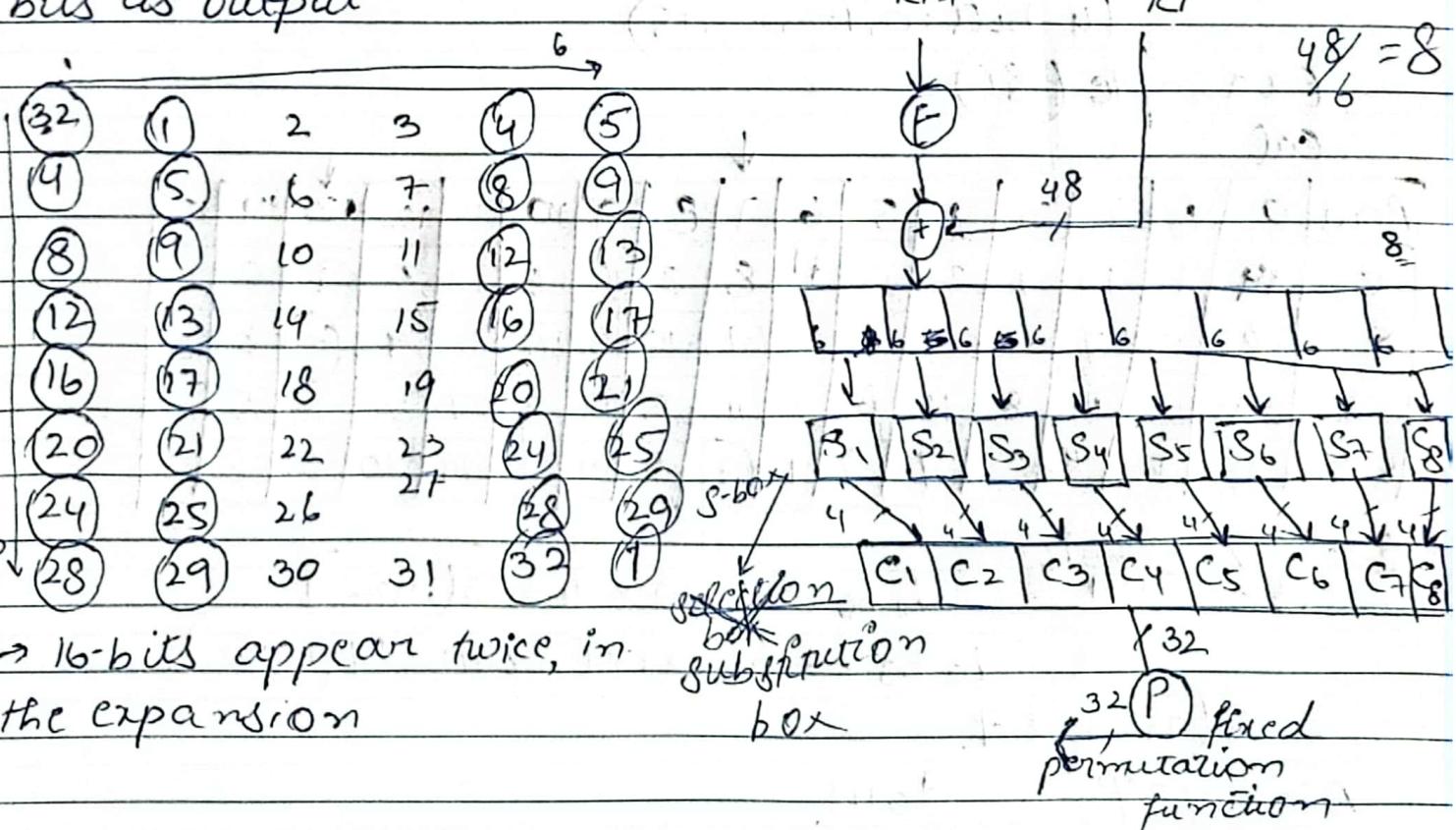
\hookrightarrow output of final permutation has bit 40 of the preoutput block as it's 1st bit, but 8 as 2nd bit & so on, until bit 25 of the preoutput block is the last bit of the output.

DES Round i



→ DES ' $f(\cdot)$ ' function.

E is an expansion function which takes a block of 32 bits as input & produces a block of 48-bit bits as output



→ 16-bits appear twice, in the expansion

~~block substitution~~

box

³²P fixed permutation function



- S-boxes:

S-boxes are the only non-linear elements in DES design.

~ each of the unique selection functions S_1, S_2, \dots, S_8 takes a 6-bit input ^{block as} yields a 4-bit block as output

$$B(6\text{-bit}) \rightarrow [S\text{-box}] \rightarrow C(4\text{-bits})$$

S -matrix 4×16 , values from 0 ~ 15

$$B(6\text{-bit long}) = b_1 b_2 b_3 b_4 b_5 b_6$$

$b_1 b_6 \Rightarrow r = \text{row of the matrix}$,
(2-bits : 0, 1, 2, 3)

$b_2 b_3 b_4 b_5 \Rightarrow c = \text{column of the matrix}$
(4-bits : 0, 1, 2, ..., 15)

Example (S_1)

| Row | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|-----|----|----|----|---|----|----|----|----|----|----|----|----|----|----|----|----|
| Col | | | | | | | | | | | | | | | | |
| 0 | 14 | 4 | 13 | 1 | 2 | 15 | 11 | 8 | 3 | 10 | 6 | 12 | 5 | 9 | 0 | 7 |
| 1 | 0 | 15 | 7 | 4 | 14 | 2 | 13 | 1 | 10 | 6 | 12 | 11 | 9 | 5 | 3 | 8 |
| 2 | 4 | 1 | 14 | 8 | 13 | 6 | 2 | 11 | 15 | 12 | 9 | 7 | 3 | 10 | 5 | 0 |
| 3 | 15 | 12 | 18 | 2 | 4 | 9 | 1 | 7 | 5 | 11 | 3 | 14 | 10 | 0 | 6 | 13 |

\therefore $b_1 b_2 b_3 b_4 b_5 b_6$

$$B = 101111 \quad b_1 b_6 \Rightarrow \text{row} = 11 = 3(\text{row})$$

$$b_2 b_3 b_4 b_5 \Rightarrow \text{col} = 011 = 7(\text{row})$$

$$S(r, c) = 7$$

Binary rep $\rightarrow 0111$



$$\text{Ex: } B = \begin{matrix} b_1 & b_2 & b_3 & b_4 & b_5 & b_6 \\ 0 & 1 & 1 & 0 & 1 & 1 \end{matrix}$$

row = $b_1 b_6 = 1^{\text{st}}$ row (01)

col = $b_2 b_3 b_4 b_5 = 1101$ $(13^{\text{th}} \text{ col})$

$$S(n=1, c=13) = 5 (0101)$$

• Finite Field (ⁱⁿAES):

$\text{GF}(P)$

$\text{mod } p$

$\hookrightarrow p$ is a prime

$+, -, \times, /$

\hookrightarrow all results should remain in the field

| | 0 | 1 | 2 | \doteq |
|---|---|---|---|-----------------|
| 0 | 0 | 1 | 2 | $\text{mod } 3$ |
| 1 | 1 | 2 | 0 | |
| 2 | 2 | 0 | 1 | |

inverse

Public Key

Lecture - 9

 (Public key
cryptography)

RPKC

 (private key,
public key)


→ RSA:

↳ invented by Ron Rivest, Adi Shamir & Len Adleman at MIT, 1978

- Block size can be variable
- Key length can be variable
- Plain text must be smaller than the key length
- CT block will be the length of key.
- Product of prime numbers, factoring of result

Applications: secrecy (secret exchange of keys, digital signatures)

→ Euler Totient function (ϕ):

if n is a tive integer, $\phi(\text{phi})$ function counts all the tive integers less than n that are relatively prime to n .

$$\text{Ex: } \phi(10) = ?$$

(1)

(7)

$$8 = 2 \times 2 \times 2$$

(3)

(9)

$$4 = 2 \times 2$$

8

$$\phi(10) = 4$$

$$6 = 2 \times 3$$

co-prime? / relatively prime?

A & B are co-prime iff $\gcd(A, B) = 1$ (common factor)
 ⇒ not necessary that the 2 numbers are prime. They have to be relatively prime to each other, e.g. 6 & 11 are co-prime.

$(11, 1)$
 $(2, 3)$

$(6, 11) \rightarrow \text{co-prime}$



Generally, $\phi(p) = p-1$ if p is a prime number.

Ex: $\phi(7) = 7-1 = 6$ ($7-p$ is prime naturally)

①, ②, ③, ④, ⑤, ⑥

→ Multiplicative Inverse of a number x is y iff y multiplied by x yields 1.

So, $x * y = 1 \rightarrow$ therefore, both x and y are multiplicative inverses of each other.

• PKC → how does it work?

≈ Plain text M and cipher text C are integers b/w 0 and $(n-1) \Rightarrow n$ is the product of two prime numbers (p, q)

Ex:

"key 1 {e, n} , key 2 {d, n}"

$$\begin{array}{|c|} \hline p, q \\ \hline n = p * q \\ \hline \end{array}$$

→ where e and d are 2 secret numbers.

$$\left. \begin{array}{l} C = M^e \text{ mod } n \\ M = C^d \text{ mod } n \end{array} \right\} \rightarrow C \text{ & } M \text{ are integers. } C \text{ is cipher text. } M \text{ is plain text.}$$

→ Sender does this
"encryption"

Receiver does this
"decryption"

→ RSA Key construction:

- Select 2 large primes, p, q when $p \neq q$

- $n = p * q$

- Euler's Totient Function $\phi(n) = (p-1)(q-1)$



Ex: $p=2, q=5 \quad n = p \times q = 10$

$$\phi(10) = (p-1)(q-1) = (2-1)(5-1) = 1(4) = 4$$

Select "e" relatively prime to ϕ

$$d \times e \pmod{\phi} = 1 \quad \text{or}$$

$$d \times e \pmod{(p-1)(q-1)} = 1$$

~~PUBLIC KEY $\Rightarrow (e, n)$~~ e & d are interchangeable

~~PRIVATE KEY $\Rightarrow (d, n)$~~ $(x^d)^e \pmod{n} = (x^e)^d \pmod{n}$

Ex 2: $p=5, q=11$

$$n = p \times q = 5 \times 11 = 55 \quad (\text{1 advertise } n=55)$$

$$e = ?$$

$$\begin{aligned} \phi(n) &= \phi(55) = (p-1)(q-1) \\ &= (5-1)(11-1) = 4 \times 10 \\ &= 40 \end{aligned}$$

$e: 3$ (assumption \Rightarrow starting with smallest co-prime)

$(e, n) \rightarrow$ we let people know this

$$\begin{array}{l} e = 3 \\ n = 55 \end{array} \rightarrow \text{public key}$$

$M = 7 \Rightarrow$ original text, plain text

$$\begin{array}{r} 6 \\ 55 \sqrt{343} \\ \underline{330} \\ 13 \end{array}$$

$$C = M^e \pmod{n}$$

$$= 7^3 \pmod{55} = 343 \pmod{55} = 13$$

$$C = 13$$

receiver receives $C = 13$

He needs to solve $d \times e = 1 \pmod{\phi} = (4)(10)$



$$3d \equiv 1 \pmod{40}$$

$$\therefore e = 3$$

11
 81
 121

woh numbers jin ka mod lena
 use karo. koi bhi value select
 to jo 3 se divide ho jaye

$$3d = 81$$

$$d = \frac{81}{3} = 21$$

$$M = C^d \pmod{n}$$

$$M = 13^{21} \pmod{4055} = 77$$

$$\hookrightarrow 13^2 = 169 \cdot 55 = 4 \text{ (example)}$$

Ex 2:

$$p=23, q=41$$

$$\text{Suppose } M=35$$

$$n = p \times q = 23 \times 41 = 943$$

$$\phi(943) = (23-1)(41-1) = 22 \times 40 = 880$$

$$\text{Supposing } e=3, n=943$$

$$C = M^e \pmod{n}$$

$$C = 35^3 \pmod{943} = 42875 \pmod{943}$$

$$C = 40$$

$$3d = 1 \pmod{880} \quad (d \times e = 1)$$

$$3d = 880 + 1761$$

$$d = \frac{880 + 1761}{3} = 887$$

$$M = C^d \pmod{n}$$

$$= 440^{887} \pmod{943} = 26535$$

$$2 | 880$$

$$2 | 440$$

$$2 | 220$$

$$2 | 110$$

$$5 | 55$$

$$11 | 11$$

$$1 | 1$$

$$880 | 1761$$

$$1760$$



Assuming $c = 7$

$$c \equiv 1 \pmod{n}$$

$$c = (35)^7 \pmod{943}$$

$$c = 545$$

$$d \times c = 1 \pmod{\phi}$$

$$7d = 1 \pmod{880}$$

$$d = 503$$

$$(880 \times 4) + 1 / 7$$

$$M = c^d \pmod{n} = 545^{503} \pmod{943}$$

$$M = 35$$



Lecture - 10

Diffie-Hellman (key exchange) ^{not used for encryption} used for transfer of keys

- 1st public key algorithm invented
- published in 1976
- specific method for securely exchanging cryptographic keys over a public channel.
- concept given by Ralph Merkle
- named after Whitfield Diffie & Martin Hellman
- public key exchange algorithm, neither encryption nor signature.

$$g \bmod p$$

$g \sim$ generator

$p \sim$ prime, used for modular arithmetic

Ex:

$$p = 11$$

when is g is primitive root of p , where $1 < g < p$, and p is a prime.

primitive:

$$g^0 \pmod{p}, g^1 \pmod{p}, \dots, g^{p-1} \pmod{p}$$

$\{1, 2, \dots, p-1\}$

should

possible map to all values from

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|----|----|---|----|----|----|----|---|----|----|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 2 | 4 | 8 | 5 | 10 | 9 | 7 | 3 | 6 | 1 | |
| 3 | 3 | 9 | 27 | 1 | 10 | 4 | 13 | 8 | 5 | 11 | |
| 4 | 4 | 16 | 54 | 2 | 7 | 11 | 14 | 10 | 3 | 6 | |

Example

$$2^0 \pmod{11}$$

$$2^1 \pmod{11}$$

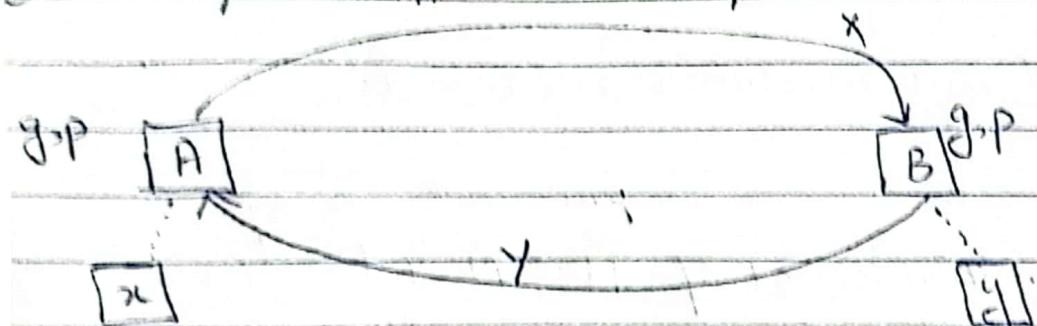
2 is primitive root of 11 as it contains all the possible values from 1 to 10



- A prime can have more than 1 primitive roots
 - will use $g \bmod p$
 - $1 \leq g \leq p$
 - g is primitive root of p

Q:

- g, p publicly shared b/w Alice (A) and Bob (B)
- A computes $x = g^x \bmod p$ $\{x$ is a secret key of Alice $\}$
- B computes $y = g^y \bmod p$ $\{y$ is a secret key of Bob $\}$
- A and B exchange x and y
- A computes $K_{AB} = y^x \bmod p$
- B computes $K_{BA} = x^y \bmod p$



$$K_{AB} = K_{BA} = g^{xy} \bmod p$$

$\begin{cases} \text{secret} \\ \{x, y\} \end{cases} \rightarrow \begin{cases} \text{public} \\ \{g, p\} \end{cases}$

Ex:

$$\begin{aligned} p &= 11 & g &= 2 & \{g, p\} \\ x &\rightarrow \text{secret key of Alice} \rightarrow x = 8 & & & \{y, y^x\} \\ y &\rightarrow \text{secret key of Bob} \rightarrow y = 4 & & & \{x, x^y\} \end{aligned}$$

secret public



private/public

$$① X = g^x \bmod p = 2^8 \bmod 11 = 3$$

Alice 38, 3

$$② Y = g^y \bmod p = 2^4 \bmod 11 = 5$$

Bob = 3, 5

↓ private ↓ public

$$③ K_{AB} = Y^x \bmod p = 5^8 \bmod 11 = 4$$

$$④ K_{BA} = X^y \bmod p = 3^4 \bmod 11 = 9$$

$$K_{AB} = K_{BA} = 4$$

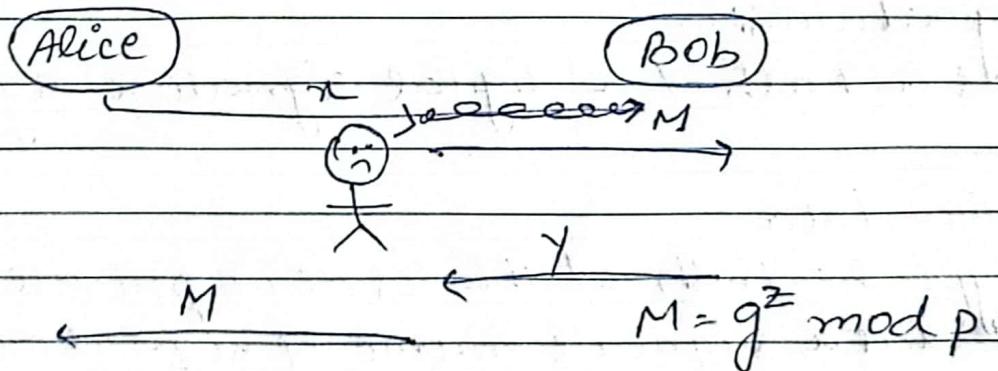
$$K_{AB} = K_{BA} = 2^{8 \times 4} \bmod 11 = 2^{32} \bmod 11 = 4$$

→ Problem:

Man in the middle act on DH

$$X = g^x \bmod p$$

$$Y = g^y \bmod p$$



$$K_{AM} = M^x \bmod p$$

$$K_{BM} = M^y \bmod p$$

$$g^{xz} \bmod p \neq g^{xy} g^{yz} \bmod p$$

$$g = 3 \quad p = 353$$

Alice \rightarrow x = 97

Bob \rightarrow y = 233

$$X = g^x \bmod p = 3^{97} \bmod 353 = 40 \quad \{97, 40\}$$

$$Y = g^y \bmod p = 3^{233} \bmod 353 = 248 \quad \{233, 248\}$$

$$K_{AB} = Y^x \bmod p = 248^{97} \bmod 353 = 160$$

$$K_{BA} = X^y \bmod p = 40^{233} \bmod 353 = 160$$

$$K_{AB} = K_{BA} = 160$$

→ Digital Signatures:

- Combines a hash w/ a digital signature algo
- To sign
 - ~ hash the data
 - ~ encrypt the hash w/ the sender's private key.
 - ~ send data signer's name & signature.
- To verify
 - ~ hash the data
 - ~ find the sender's public key
 - ~ decrypt the signature w/ the sender's public key
 - ~ the result of which should match the hash.



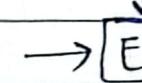
→ Digital Certificate:

unsigned certificate
containing user's ID, user's
public key

generate hashcode of
unsigned certificate



key



Encrypt hash
code with CA's
private key to
form signature

Signed
certificate
receipt →
can verify
signature using
CA's public key



∴ CA = certificate authority

↳ many vendors

⇒ Open SSL, Netscape, Entrust, VeriSign,
RSA Security.

∴ RA = Registration Authority

• authenticates user for authentication user

→ banking app

→ online registration