Institute for Geodesy and Geoinformation

University of Bonn

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Satellite Geodesy and Earth System MGE-01 WS2024-2025

Exercise Compulsory

December 8th, 2024
deadline: January 19th, 2025

Task Compulsory.1

The most common way of mathematically representing the Earth's gravitational potential is by using a (finite) series in spherical harmonics. For this assignment, please use fully normalized spherical harmonic coefficients (SHCs, or Stokes coefficients) which can be downloaded from the ecampus directory:

MGE-GES-01 Satellite Geodesy and Earth System > Exercises > Compulsory Exercise

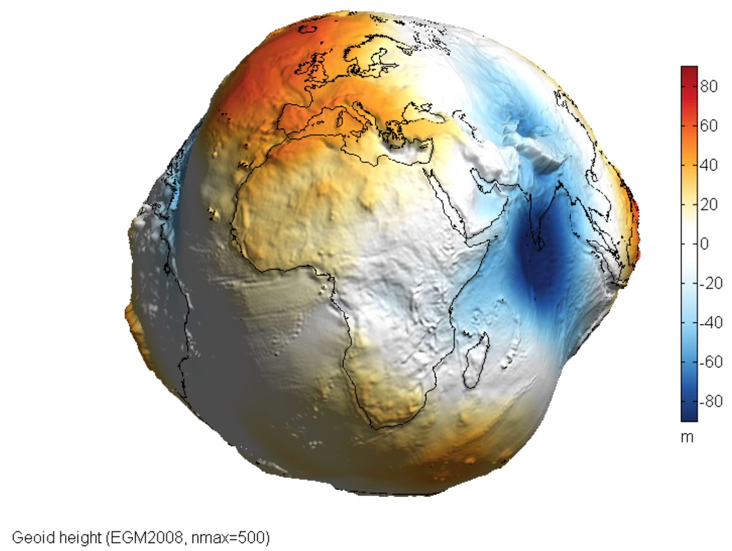

Figure 1: EGM2008 geoid height. Source: www.asu.cas.cz

Important:

1. We expect that every student submits their individual homework assignment!
2. You will have to hand in via ecampus a single zipped file that contains:
   a) In the assignment, student completes and documents the Toolbox. Directories "config, data, lib, src, and res" should be uploaded. The structure should be strictly follow, as it is expected that once copied these directories at the right place no changes in the structure is necessary.
   b) a written report as pdf-file. Please read the 'Report Guidelines' carefully before you start writing the report. Your report must contain a step-by-step description of the approach (independent of the specific implementation), the relevant equations that you have used, the results including maps and their interpretation, and the answers to the questions. The cover page must display your names and matriculation (enrollment) numbers.
3. Suggested reference is: Hofmann-Wellenhof, B., and H. Moritz. Physical geodesy. Springer Science & Business Media, 2006.

Final deadline (delivery of report): January 31st, 2024

We offer the opportunity to ask **specific** questions regarding the compulsory exercise on

- Friday, December 13nd (10:15-11:45am),
- Thursday, January 30th (9:15-10:45am),
- Friday, January 31st (10:15-11:45am)

The question hour will be held by Jiaming Chen and Hazel Hana Brodovic (jchen1@uni-bonn.de and s32hjose@uni-bonn.de).

Questions and tasks:

1 Exercise Spherical Harmonics

1. Please implement (i.e. code) a routine for reading the spherical harmonic coefficients (SHCs).

2. Please implement a function to enable evaluating the spherical harmonic series with the given SHCs, for an arbitrary position (spherical longitude and latitude). This will require evaluating the associate Legendre functions (ALFs); use the recursion relations that were recommended in the lecture (Satellite Geodesy and Earth System) and in physical geodesy textbooks.

3. Please create maps of the following functionals (for the time-mean geopotential model ITSG-Grace2018 relative to the ellipsoidal GRS80 'normal' geopotential model). Explain and interpret what you see.

   a) disturbance potential (in $\mathrm{m}^2/\mathrm{s}^2$),

   b) gravity disturbances (in mgal),

   c) geoid heights (in m).

4. Create a map of the ITSG-Grace2018 disturbance potential at a (constant) satellite altitude of 450 km (above the mean Earth radius as given with the SHCs). How do you explain the difference compared to the map of the potential at the surface?

5. Assume for the moment, the SHCs are unknown and the disturbance potential at constant satellite height would have been observed, on a given global $1° \times 1°$ grid, with known error standard deviation (which depends only on position). How would you estimate the SHCs from these observations in a least-squares sense?

6. Further along this line, now let us assume the observations of the disturbance potential are not provided on a grid but at regular epochs along a satellite orbit which is known in inertial coordinates. Which additional steps are required to estimate the SHCs?

7. Read the SHCs from the files acos.txt and asin.txt. Compute

$$\begin{Bmatrix} c_{nm}(t) \\ s_{nm}(t) \end{Bmatrix} = \begin{Bmatrix} c_{nm}^{ITSG} \\ s_{nm}^{ITSG} \end{Bmatrix} + \begin{Bmatrix} c_{nm}^{acos} \\ s_{nm}^{acos} \end{Bmatrix} * \cos\left(2\pi \frac{t - t_0}{T}\right) + \begin{Bmatrix} c_{nm}^{asin} \\ s_{nm}^{asin} \end{Bmatrix} * \sin\left(2\pi \frac{t - t_0}{T}\right) \quad (1)$$

   where c_{nm}^{ITSG} and s_{nm}^{ITSG} are the SHCs of the model ITSG-Grace2018, t_0 is set to January 1st, 2010, and T means 365.25 days, for the following epochs t (dd.mm.yyyy: 01.01.2010, 01.04.2010, 01.07.2010 and 01.10.2010). Compute the geoid heights for these epochs and compare to the geoid heights from ITSG-Grace2018. What kind of signal is added by Eq. (1) to ITSG-Grace2018? Which geophysical effects can be identified?

Maps should be created on a global $1° \times 1°$ grid; i.e. covering the entire Earth. Until mentioned otherwise, the spherical harmonic expansion should be evaluated up to harmonic degree $n = 100$ (truncation degree).

2 Exercise Altimetry

Task altimetry:

Starting from the matlab program that you find in : ECampus/GES-01: Satellite Geodesy and Earth System (20232)/Exercises/calval_MGS05_test.zip
Exercise you will:

1) Run the program and change such as figures are stored in res/alongtrack

2) Interpolate linearly the 20 Hz at 1 Hz parameters

3) Apply the necessary correction to study sea level change. Check if the correction applied is in the expected range and discuss the selection done in case of multiple possibilities.

4) Apply the necessary correction to validate altimetry against a gauge. Check if the correction applied is in the expected range and discuss the selection done in case of multiple possibilities.

5) Compare altimetric time-series of the Theia and Dahiti databases by selecting the corresponding stations. The comparison is done in terms of standard deviation different and correlation. The figures are plotted by the programs and the corresponding interpolated data are stored in the same ascii file together with the statistical parameters and the distance between the stations. The final results are all stored in an ascii file.

It is mandatory to follow the same structure given. The needed input data are found in the same directory given above.