Edge Detection

Most of the slides in this lecture are either from or adapted from Dr. Mubarak Shah's slides (Uni. Of Central Florida)

Contents

- Gradient operators
  - Prewit
  - Sobel
- Laplacian of Gaussian (Marr-Hildreth)
- Gradient of Gaussian (Canny)

This lecture

Next lecture

Edge detection

- **Goal:** Identify sudden changes (discontinuities) in an image
  - Intuitively, most semantic and shape information from the image can be encoded in the edges
  - More compact than pixels
- **Ideal:** artist’s line drawing (but artist is also using object-level knowledge)

Example

Example

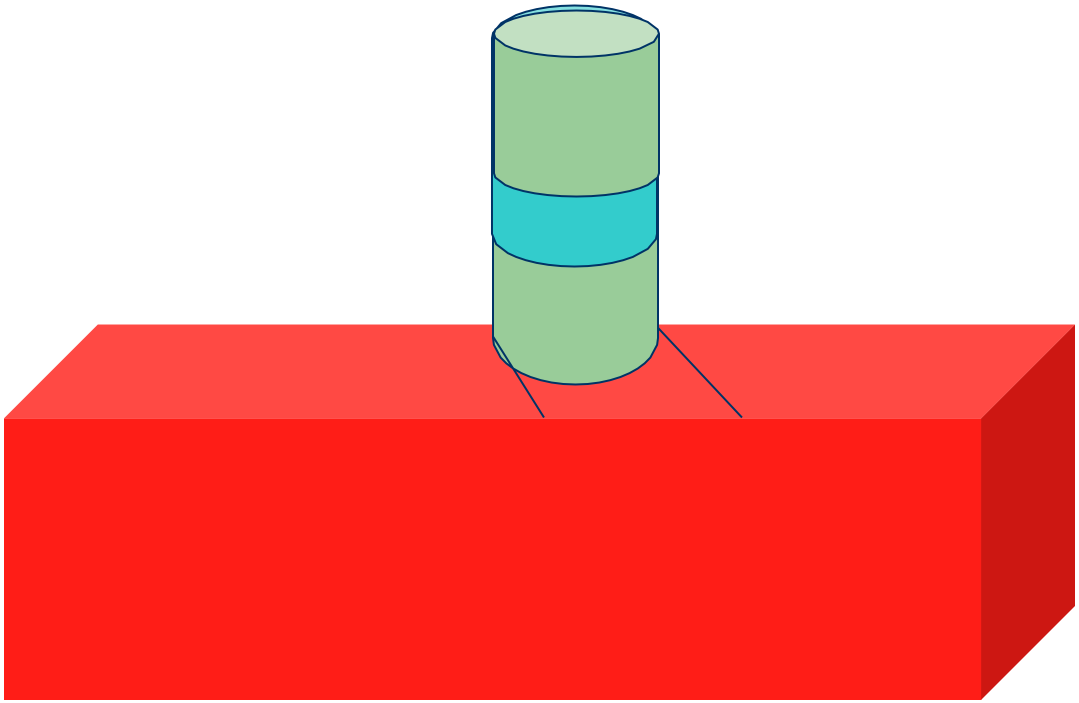
Example

Example

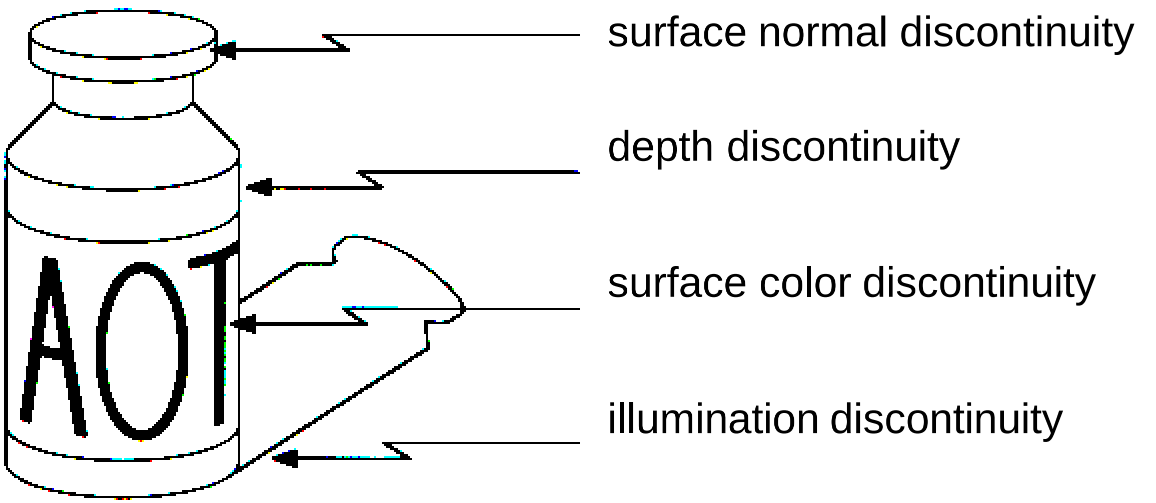
Example

Edge Detection in Images

- At edges intensity or color changes

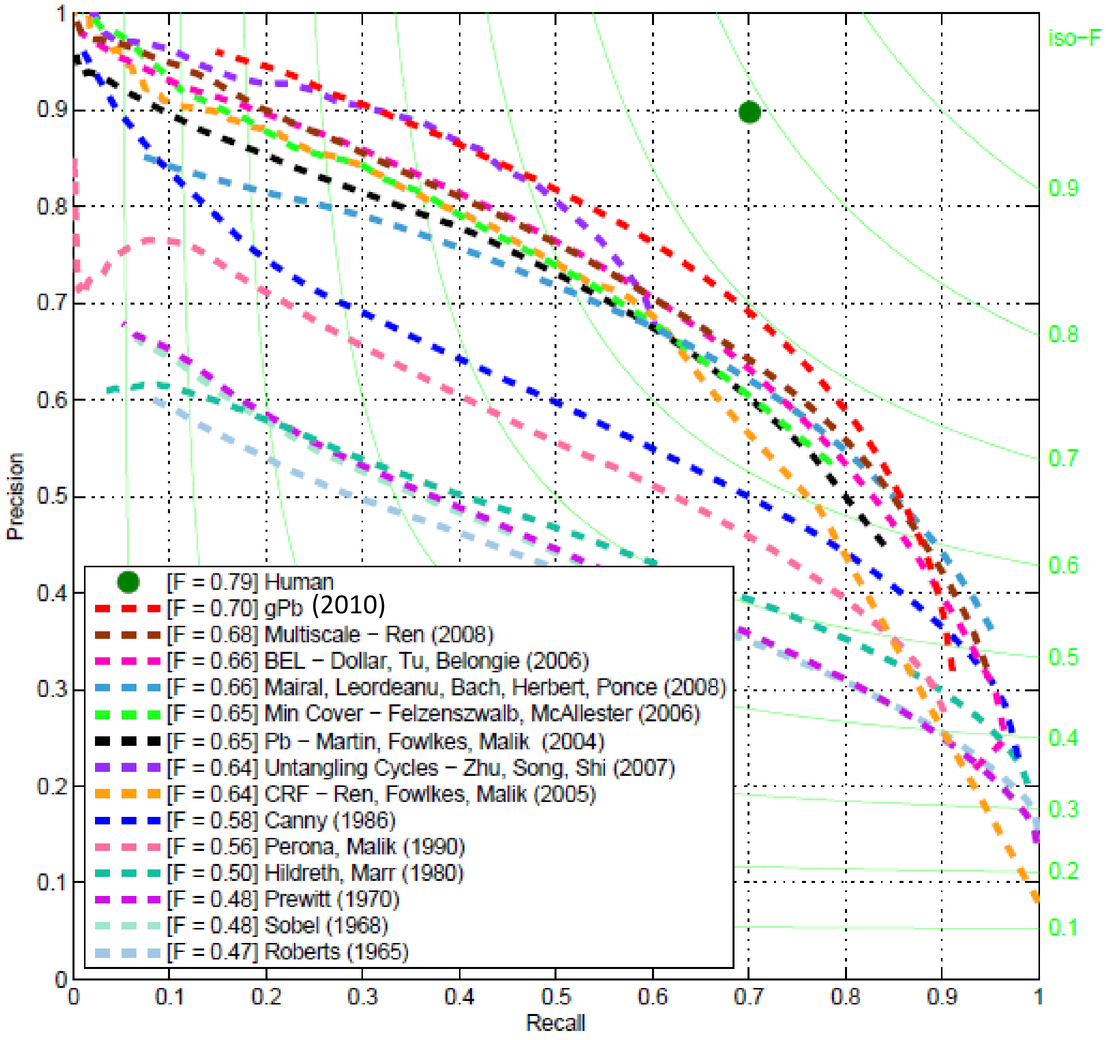


Origin of Edges



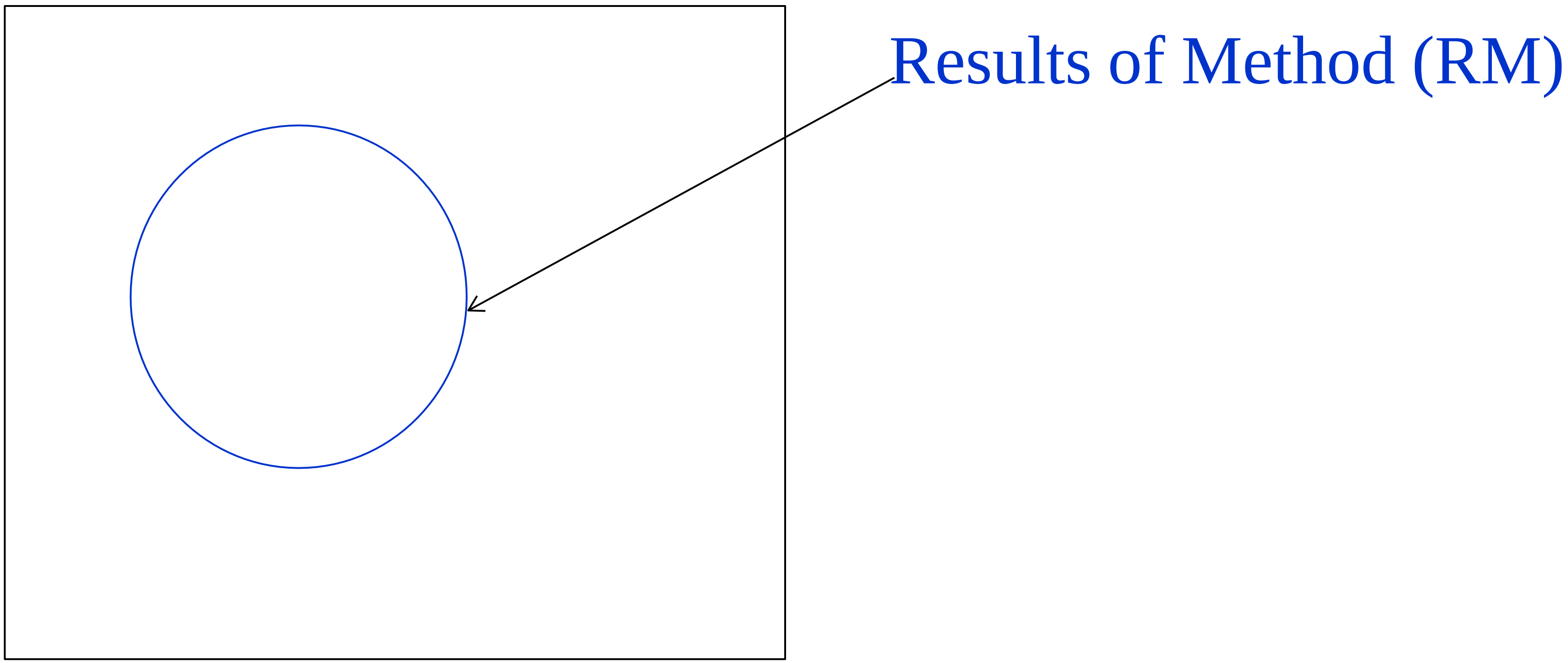
- Edges are caused by a variety of factors

45 years of boundary detection

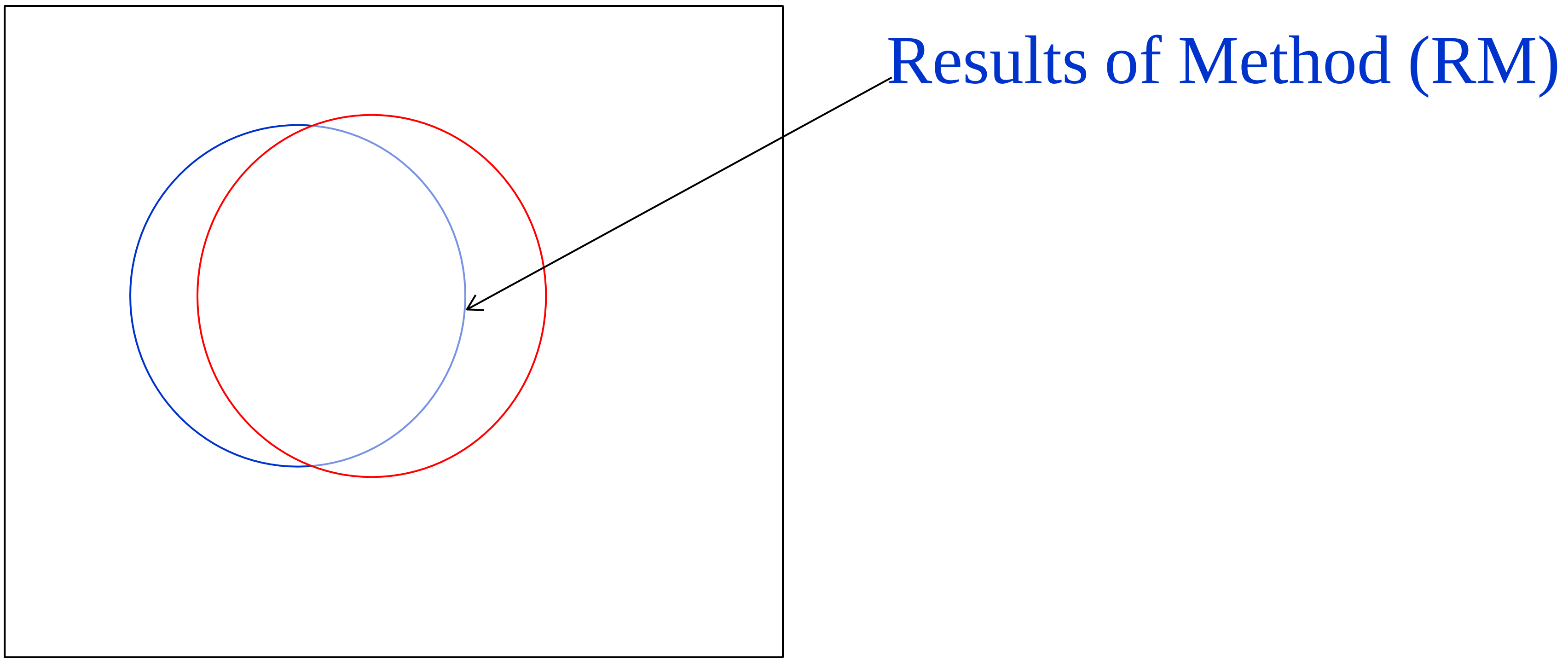


Source: Arbelaez, Maire, Fowlkes, and Malik. TPAMI 2011 (pdf)

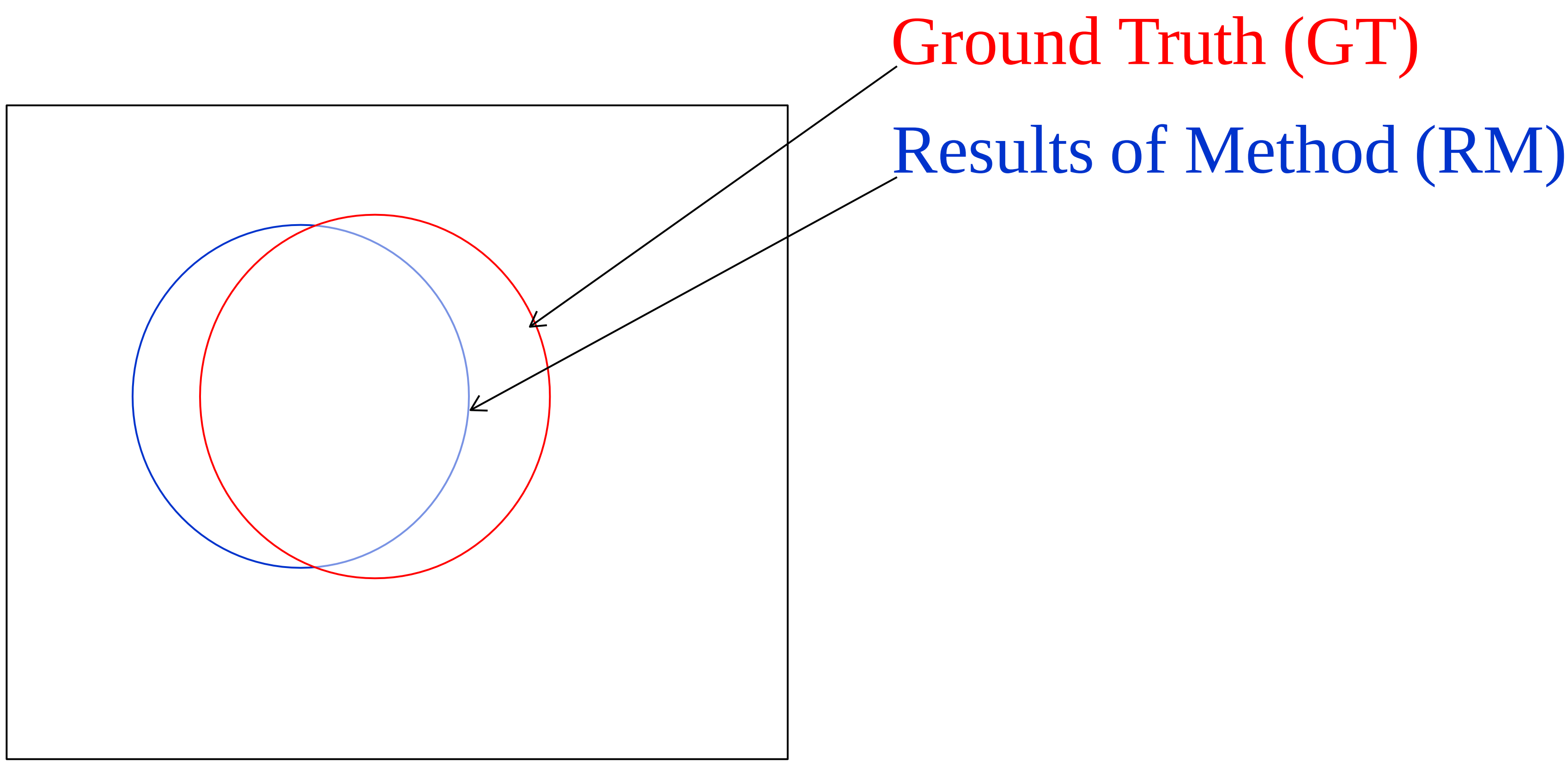
Evaluation Metrics



Evaluation Metrics



Evaluation Metrics



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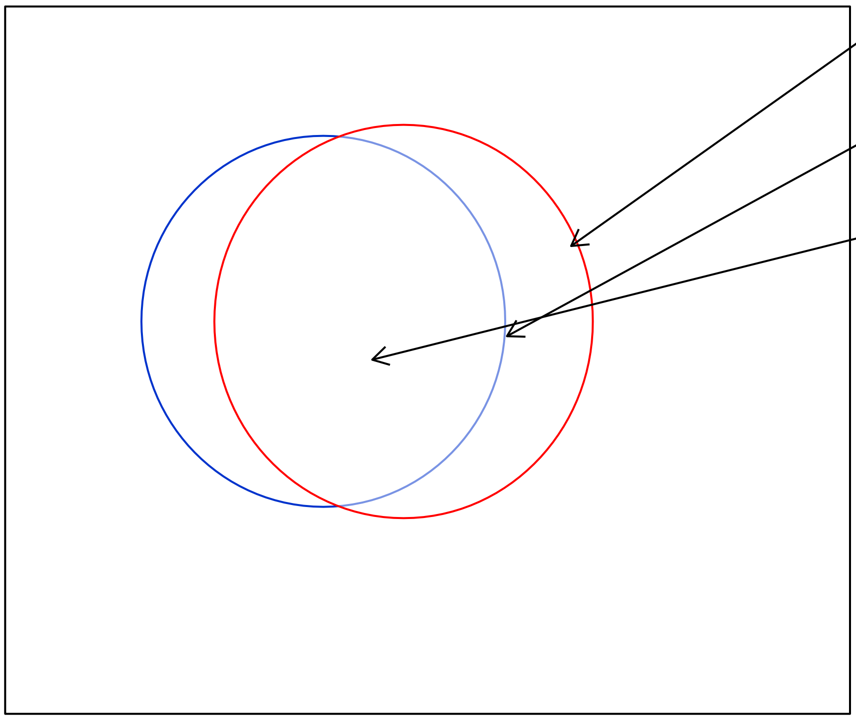
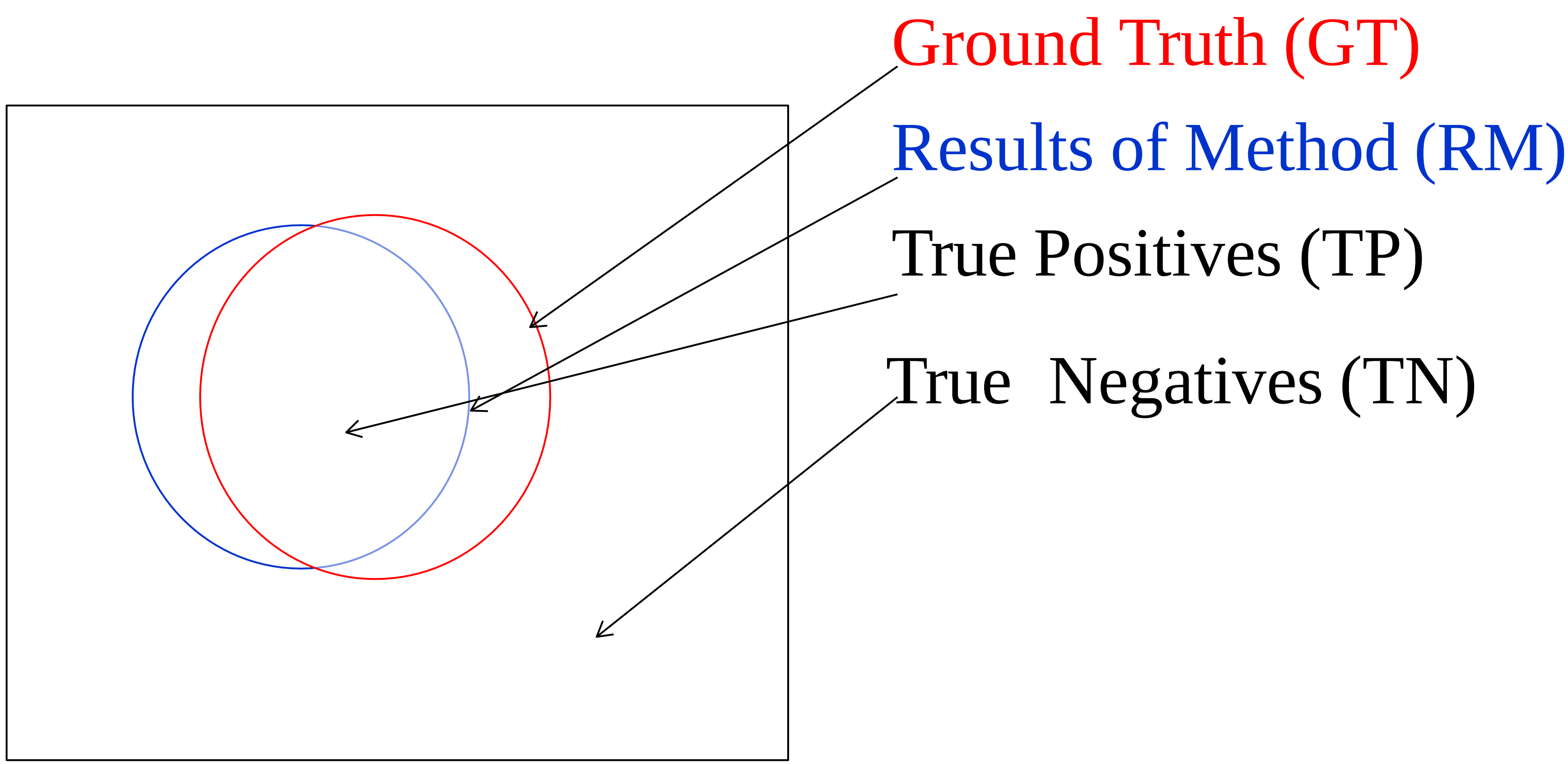


Ground Truth (GT)

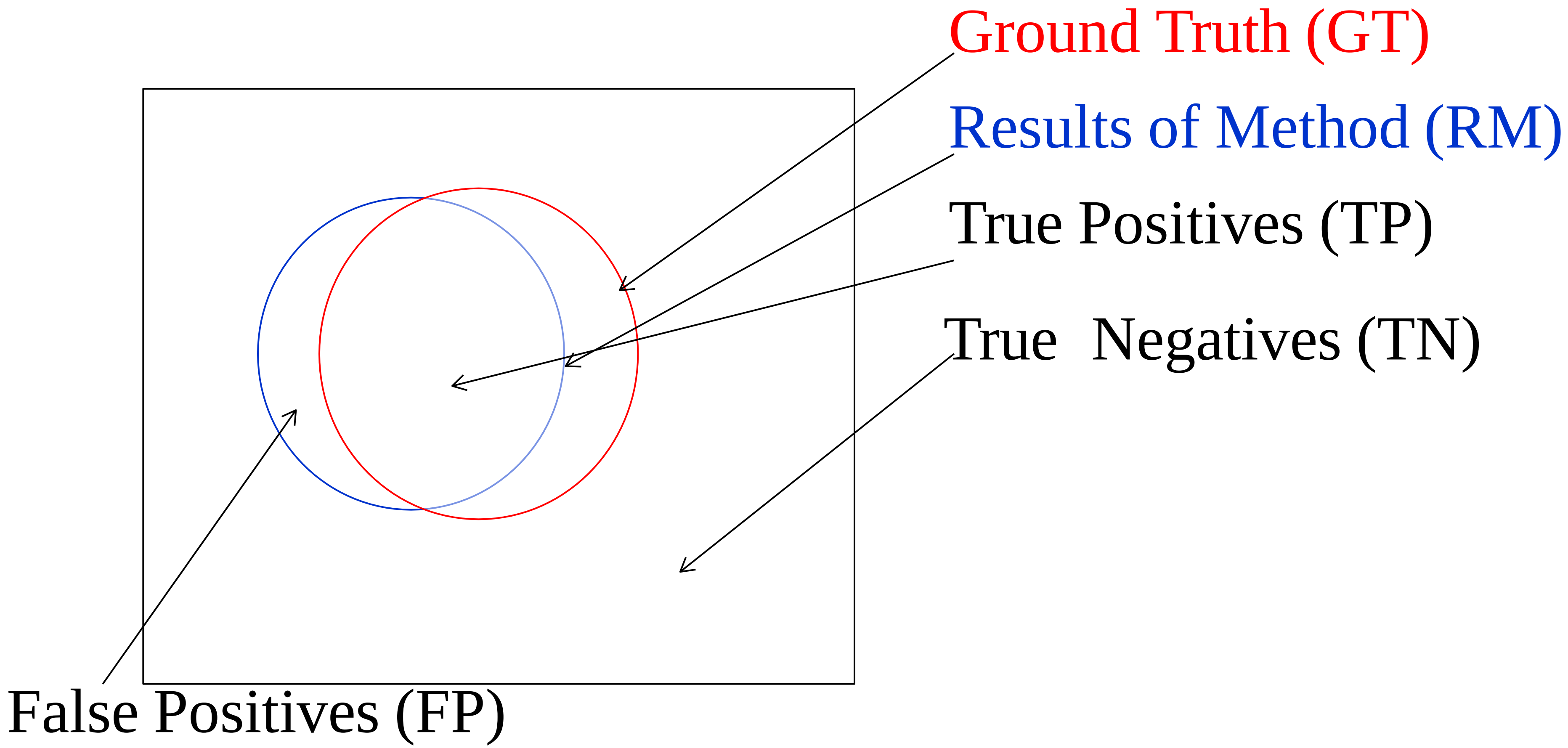
Results of Method (RM)

True Positives (TP)

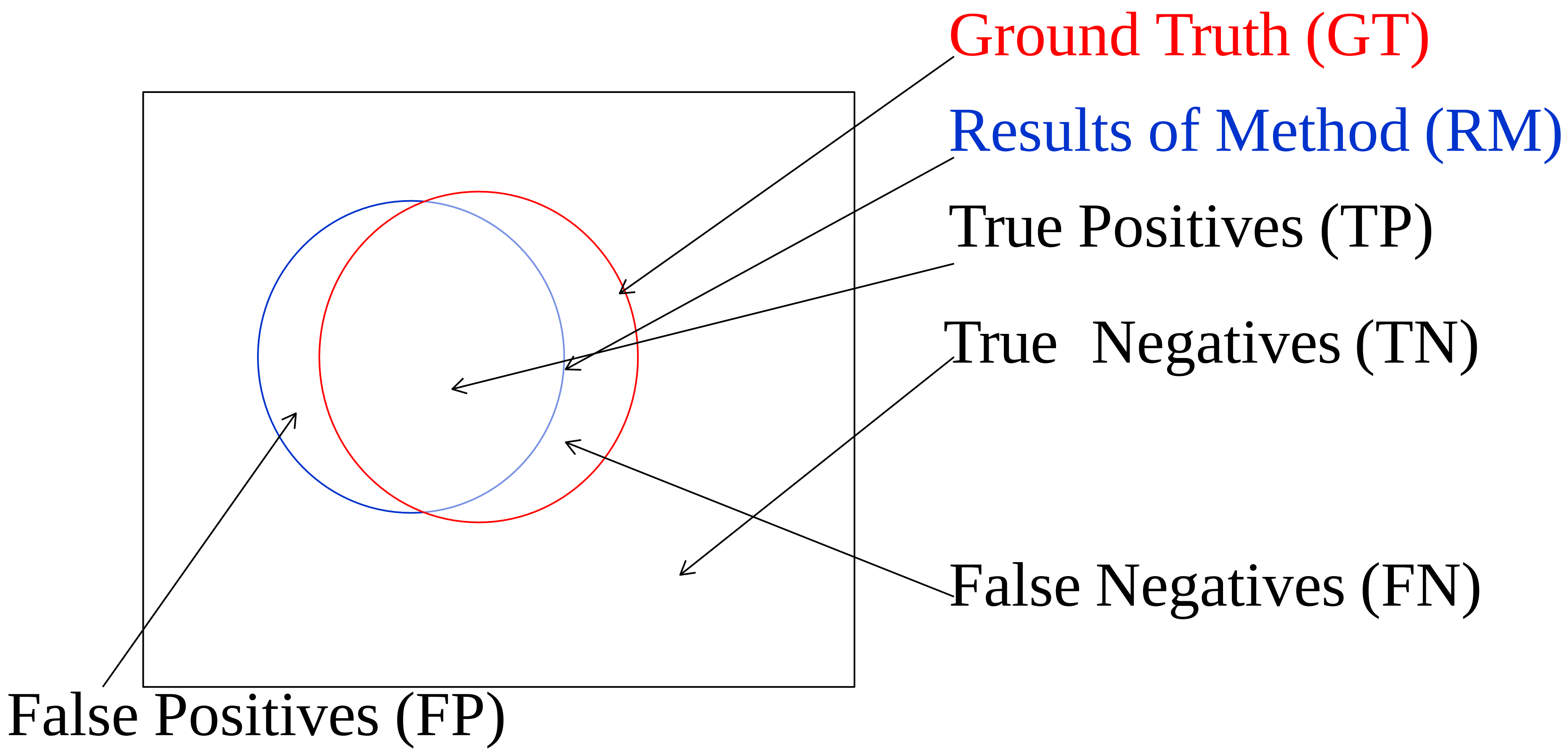
Evaluation Metrics



Evaluation Metrics



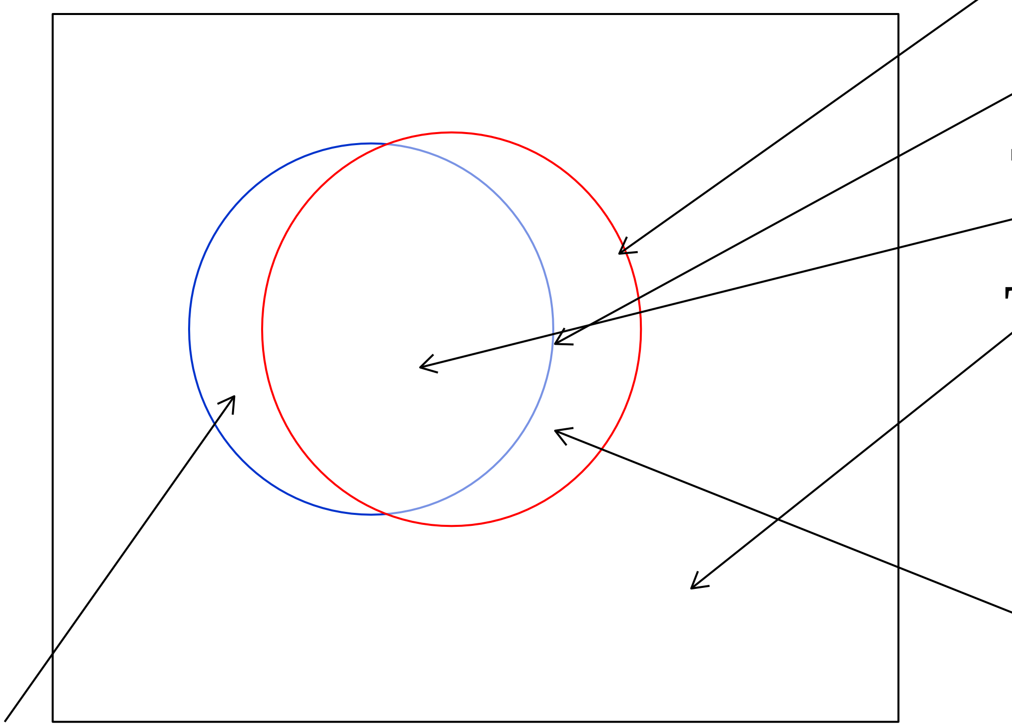
Evaluation Metrics

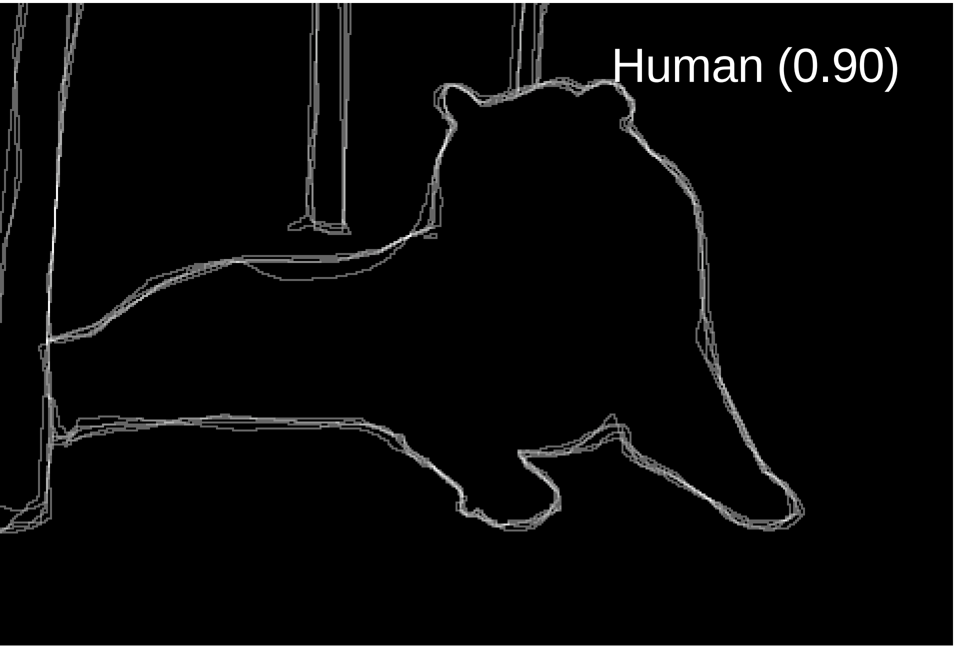


Evaluation Metrics

$$\text{precision} = \frac{GT \cap RM}{RM} = \frac{TP}{RM}$$

$$\text{recall} = \frac{GT \cap RM}{GT} = \frac{TP}{GT}$$

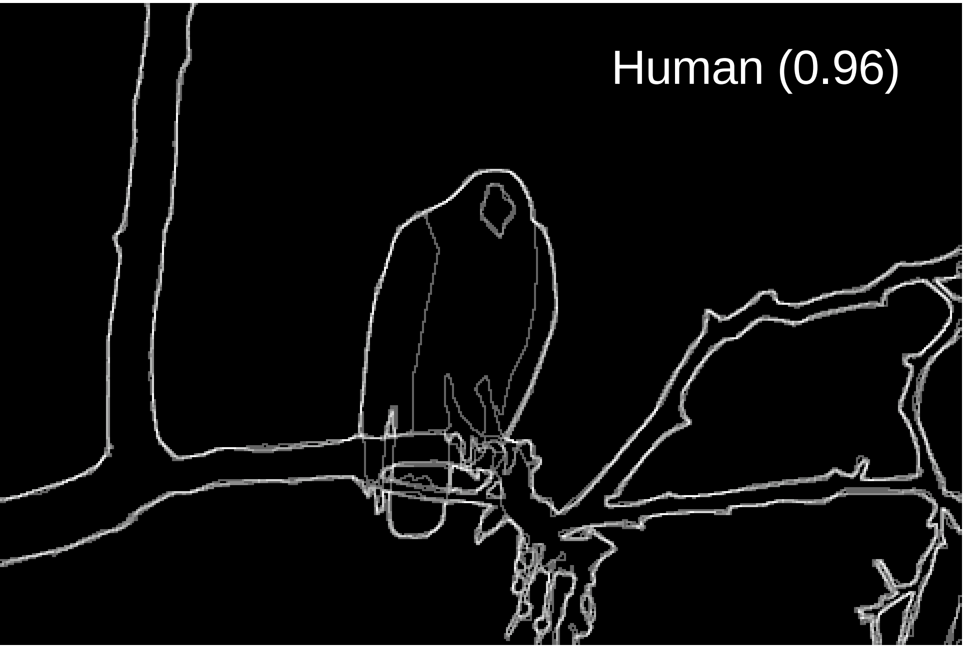




Slide Credit: James Hays

For more:
http://www.eecs.berkeley.edu/Research/Projects/CS/vision/bsds/bench/html/108082-color.html

Results



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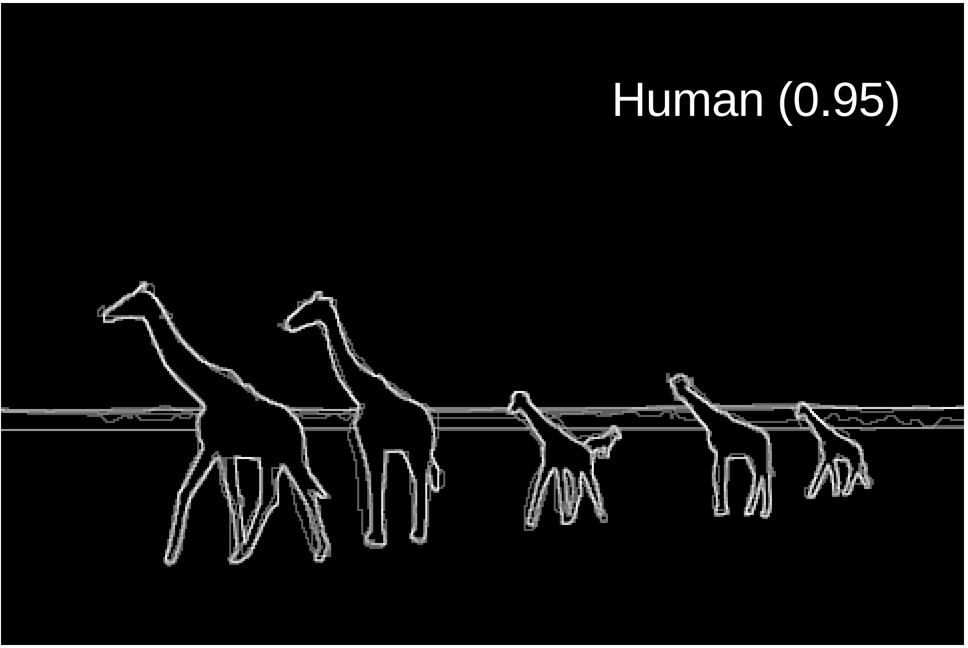
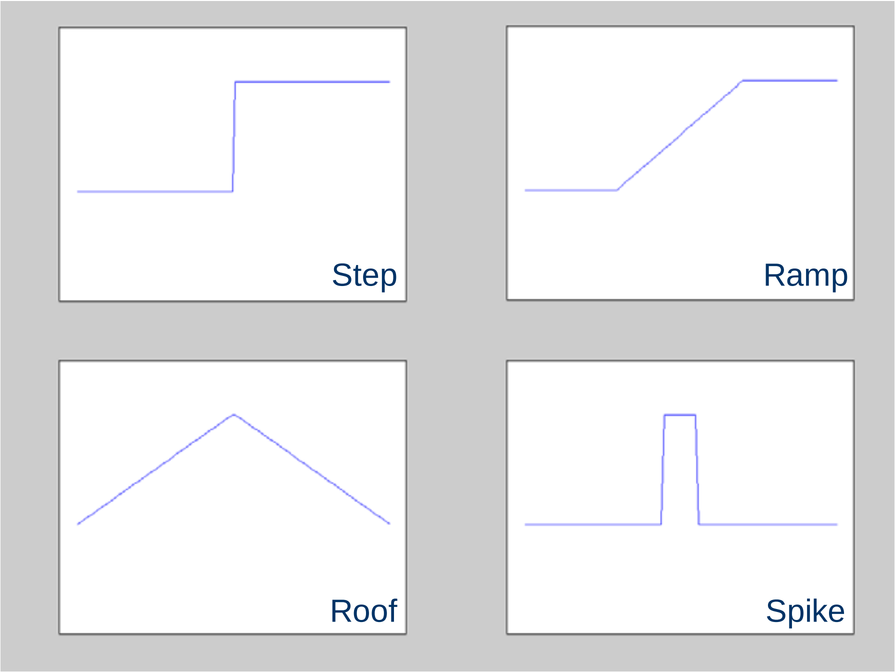


Slide Credit: James Hays

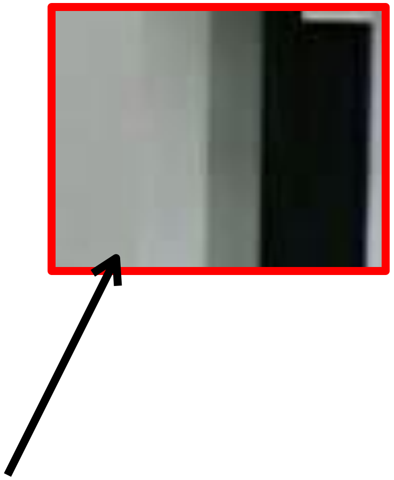
What is an Edge?

- Discontinuity of intensities in the image
- Edge models
  - Step
  - Roof
  - Ramp
  - Spike



Closeup of edges

Closeup of edges



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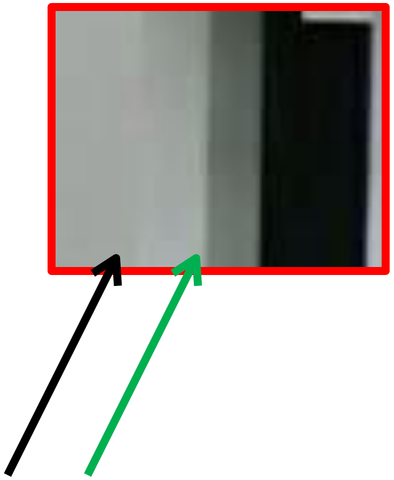

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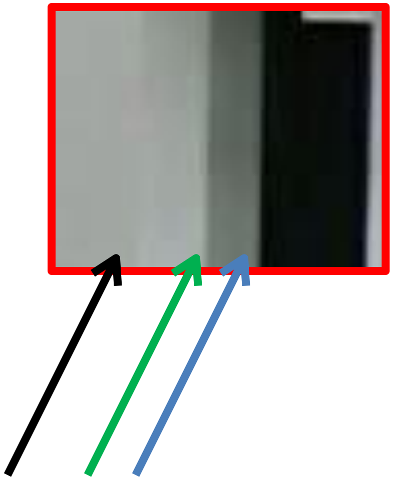
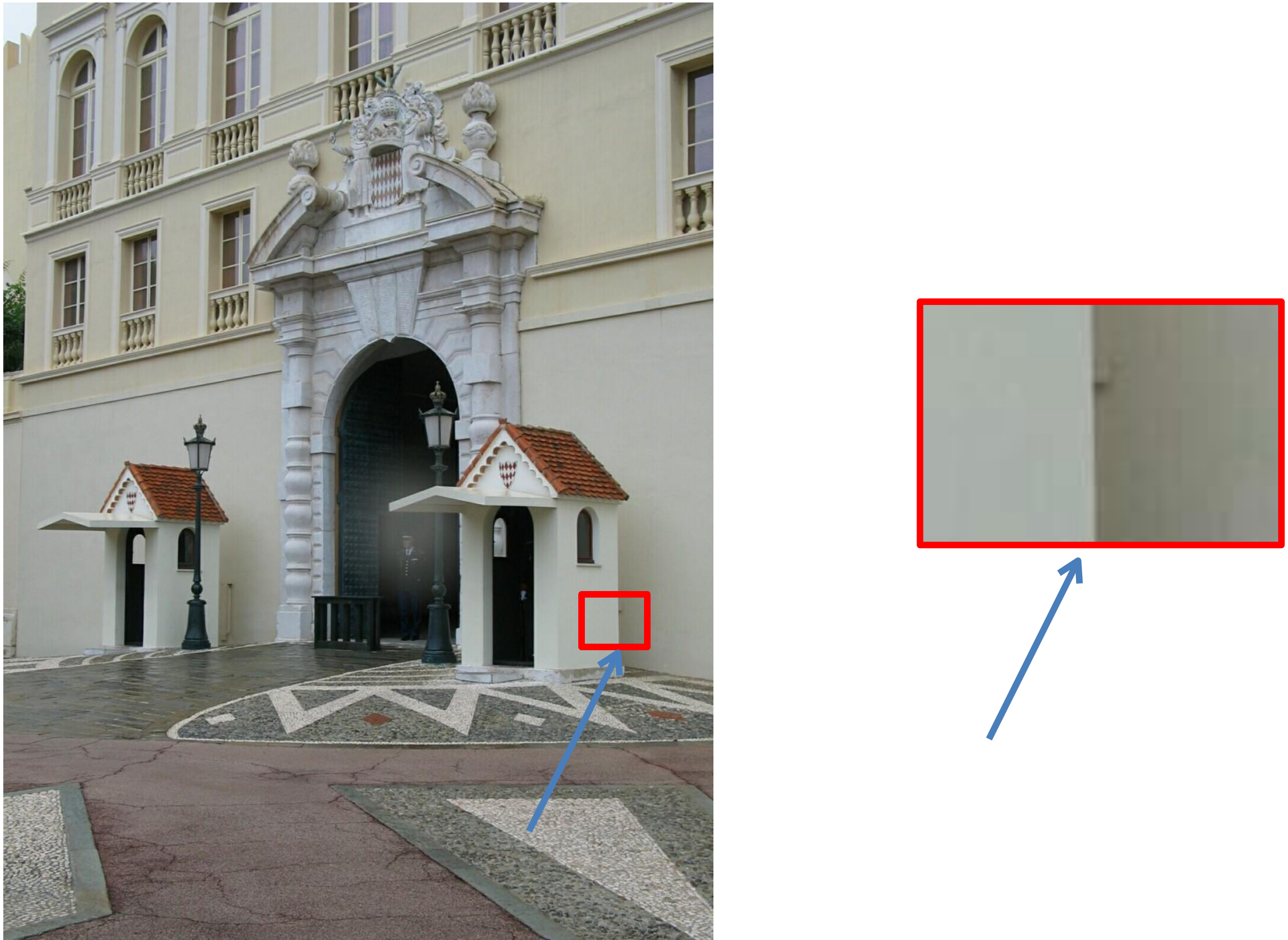


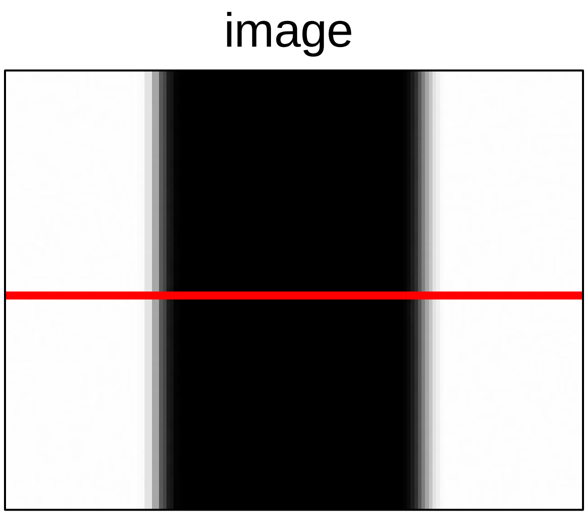
Closeup of edges



Closeup of edges

Characterizing edges

- An edge is a place of rapid change in the image intensity function



Characterizing edges

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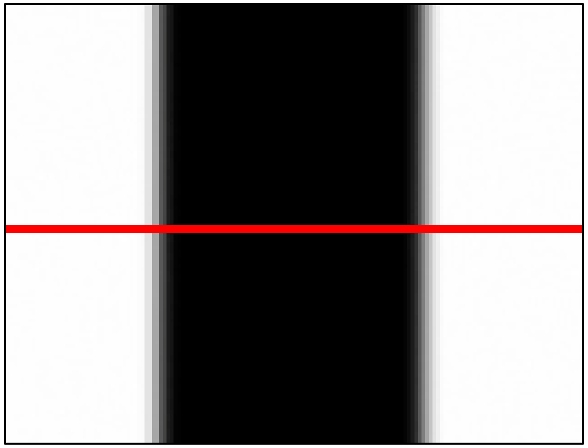
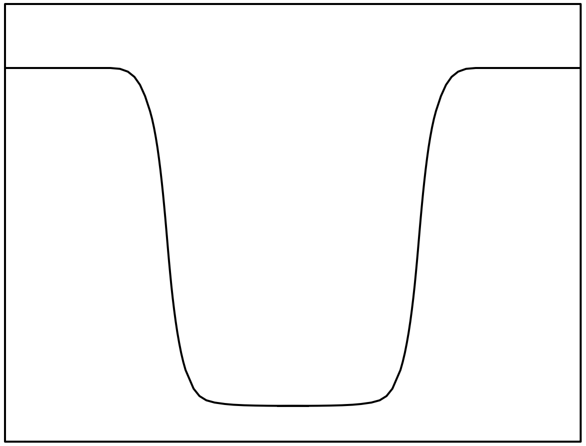
image



intensity function
(along horizontal scanline)



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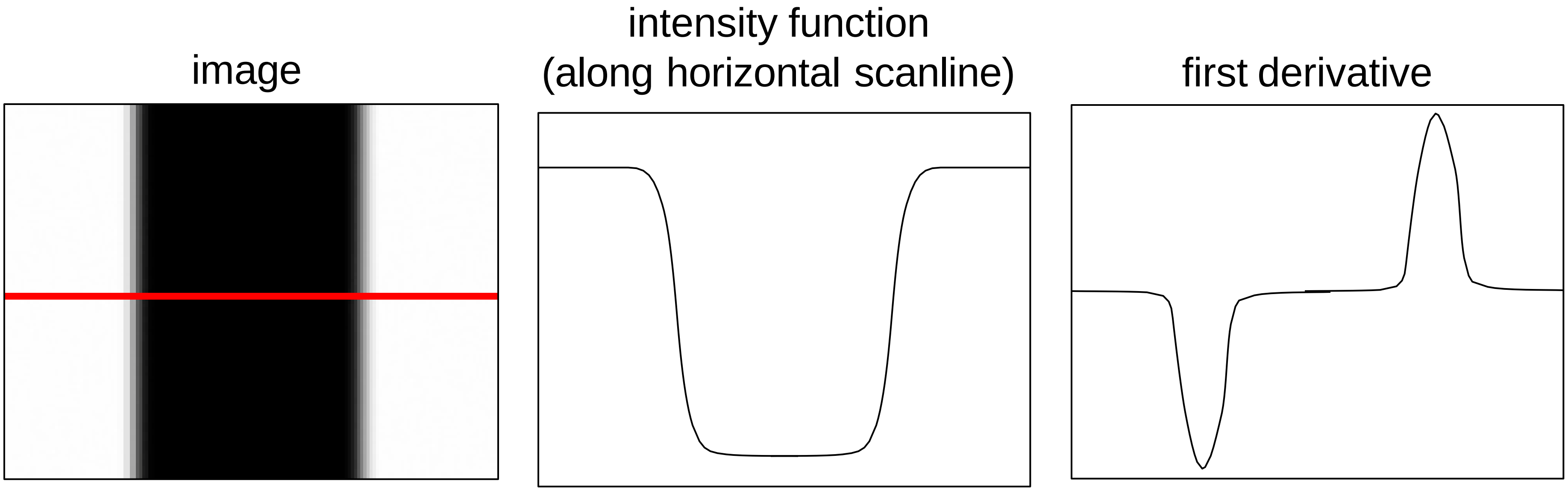

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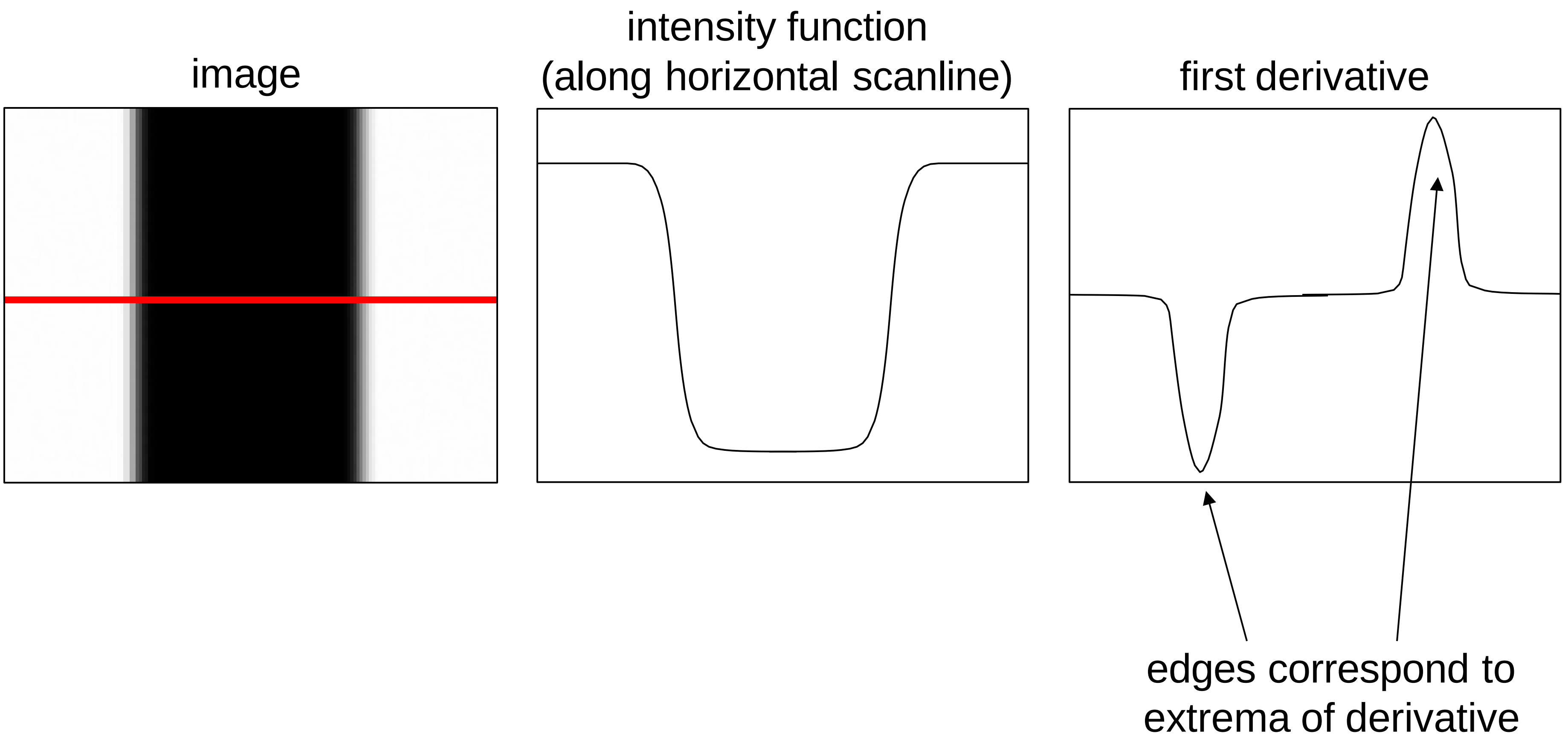
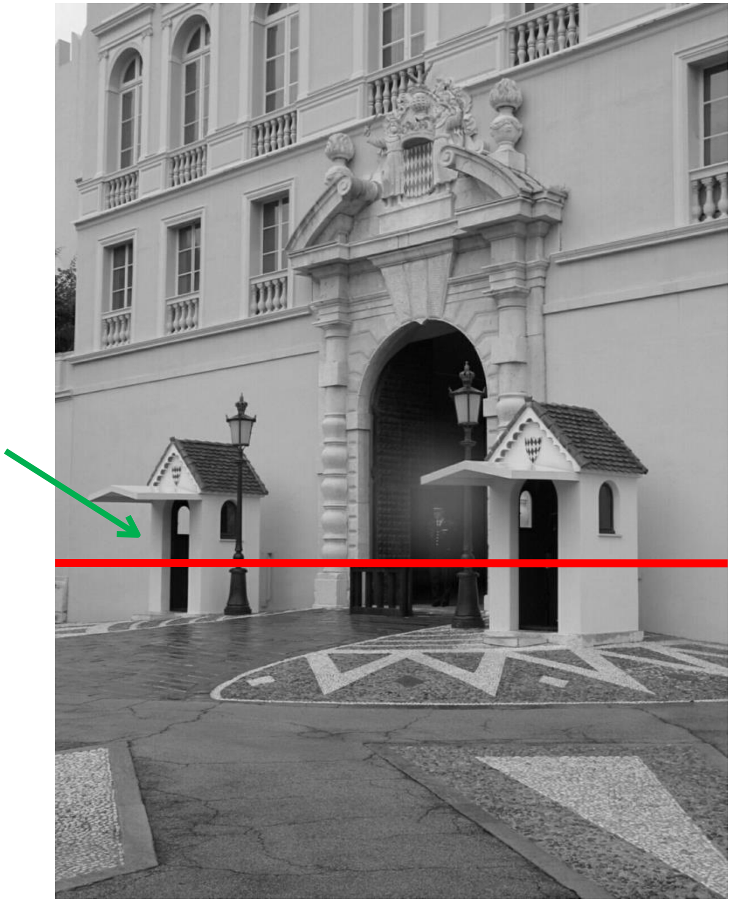
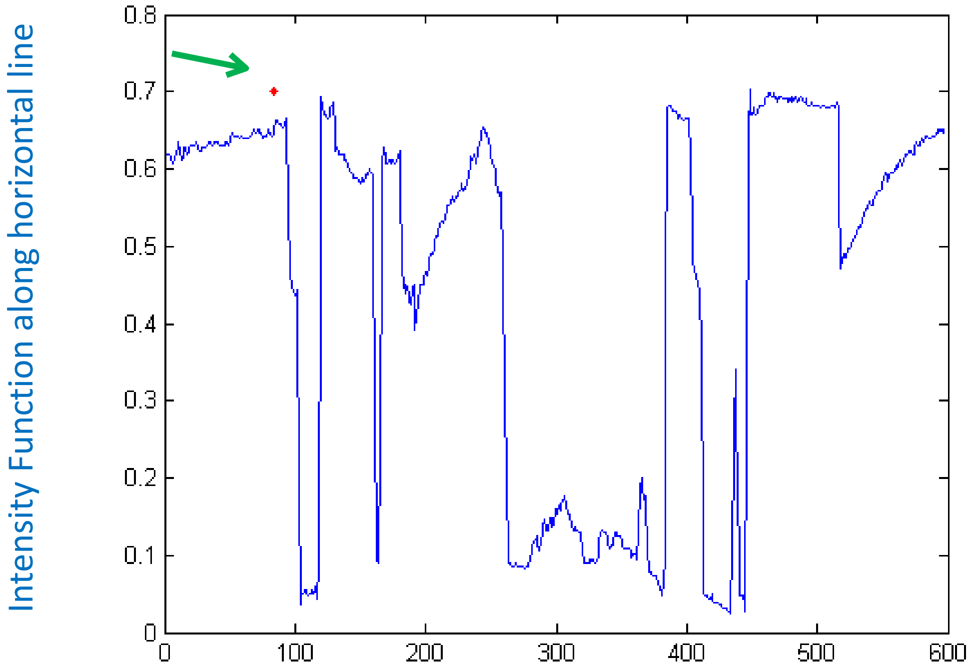


Intensity profile





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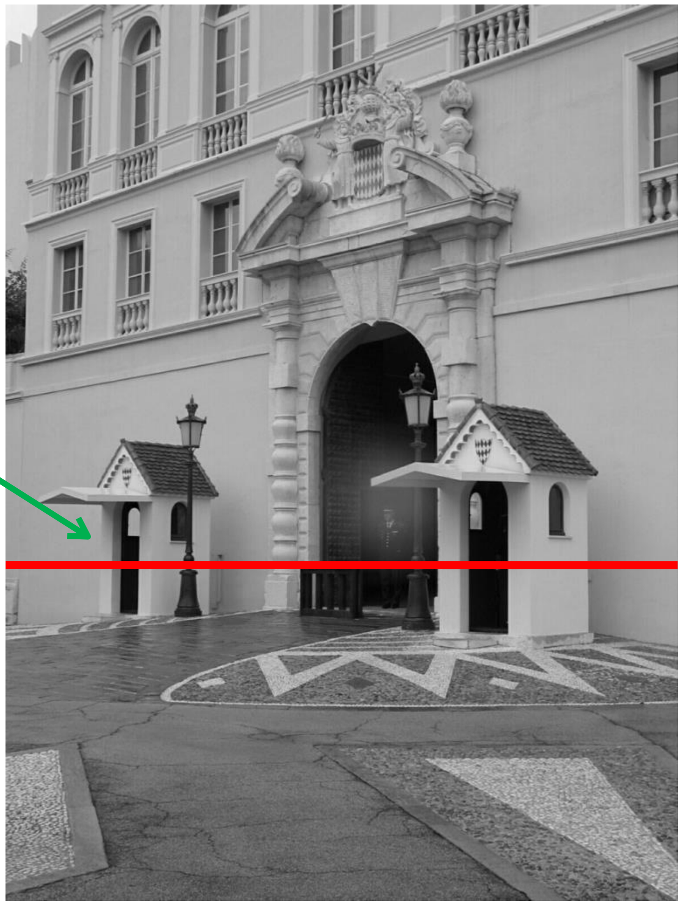
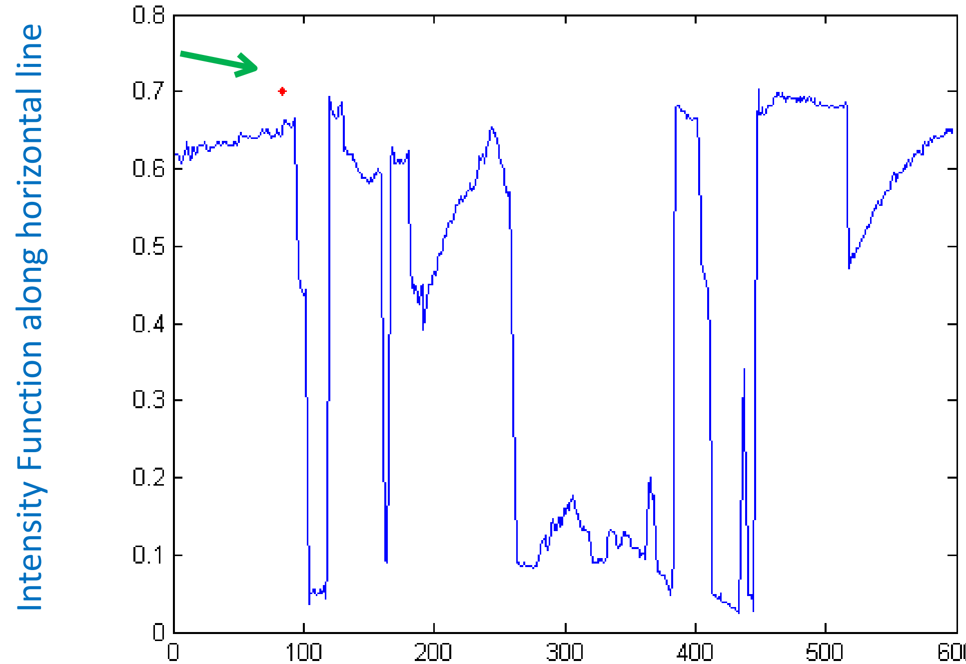
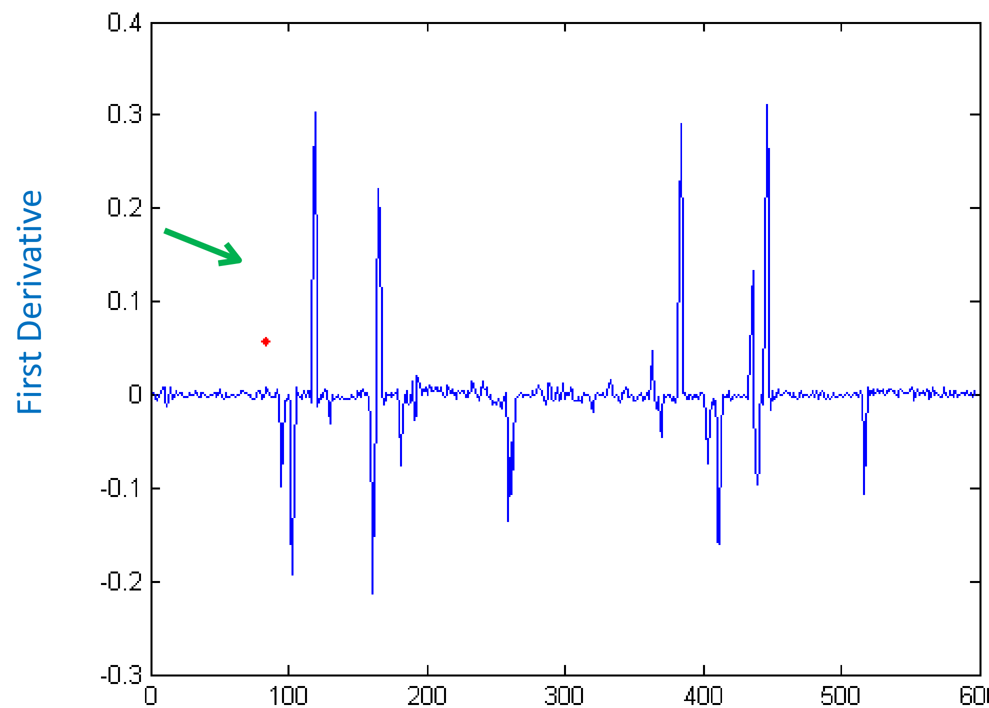
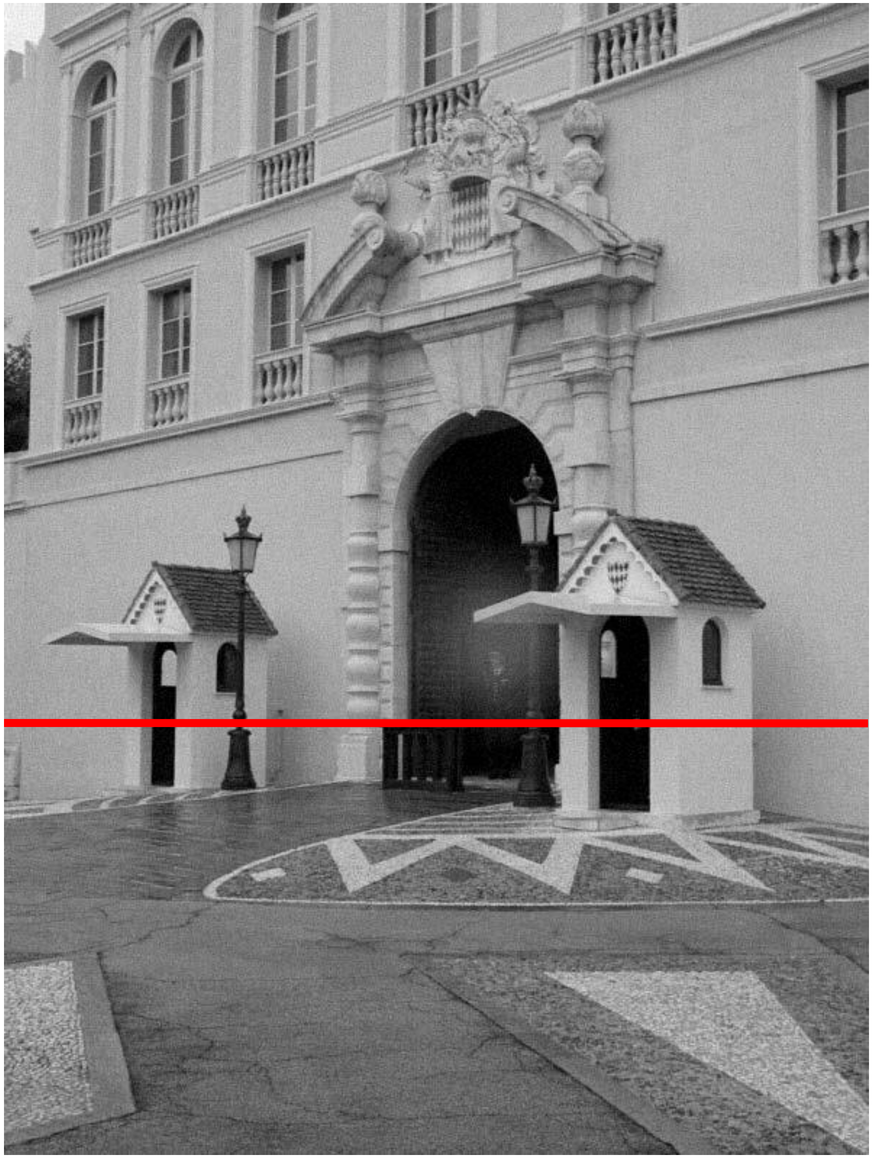
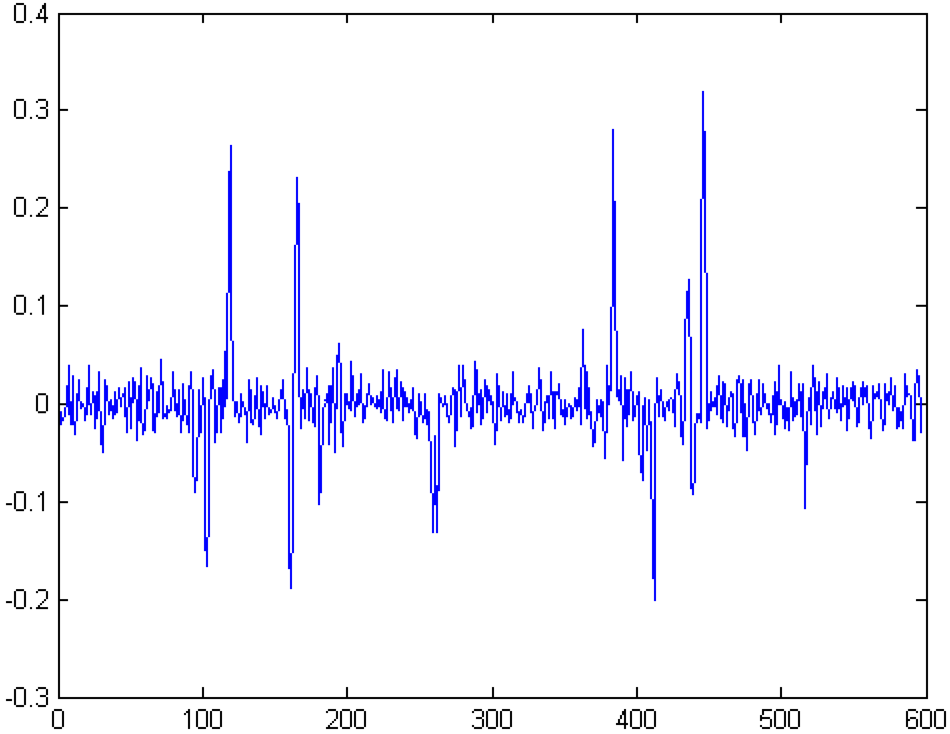






Intensity Profile of a little noisy image

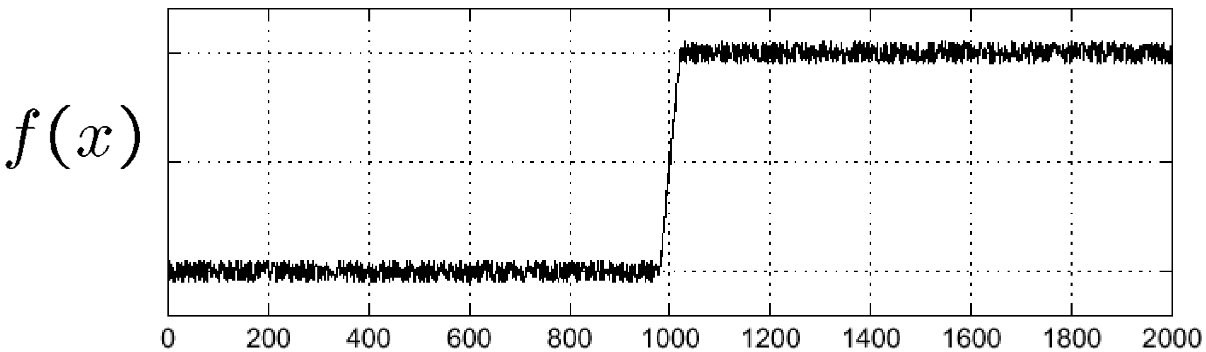




Gradient (first derivative)

Effects of noise

- Consider a single row or column of the image
  - Plotting intensity as a function of position gives a signal



Effects of noise

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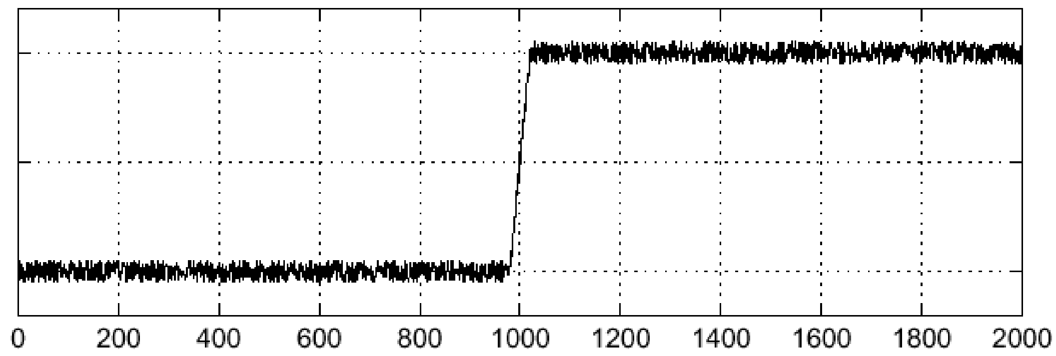
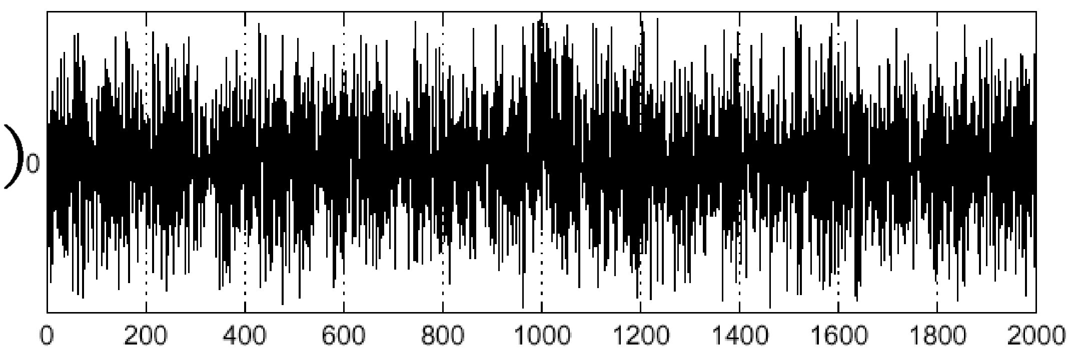
$f(x)$



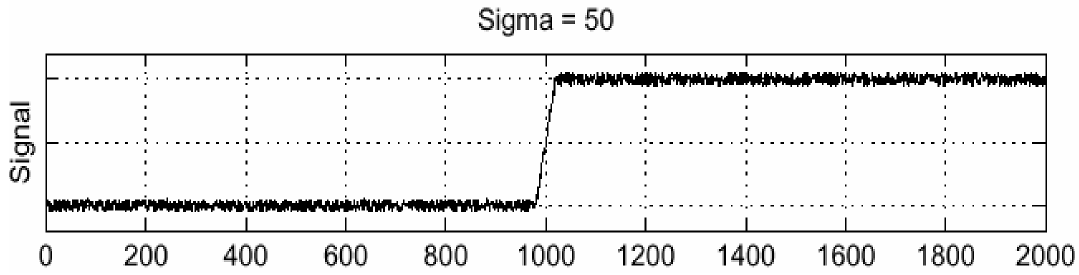
$\frac{d}{dx}f(x)$



Where is the edge?

Solution: smooth first

f



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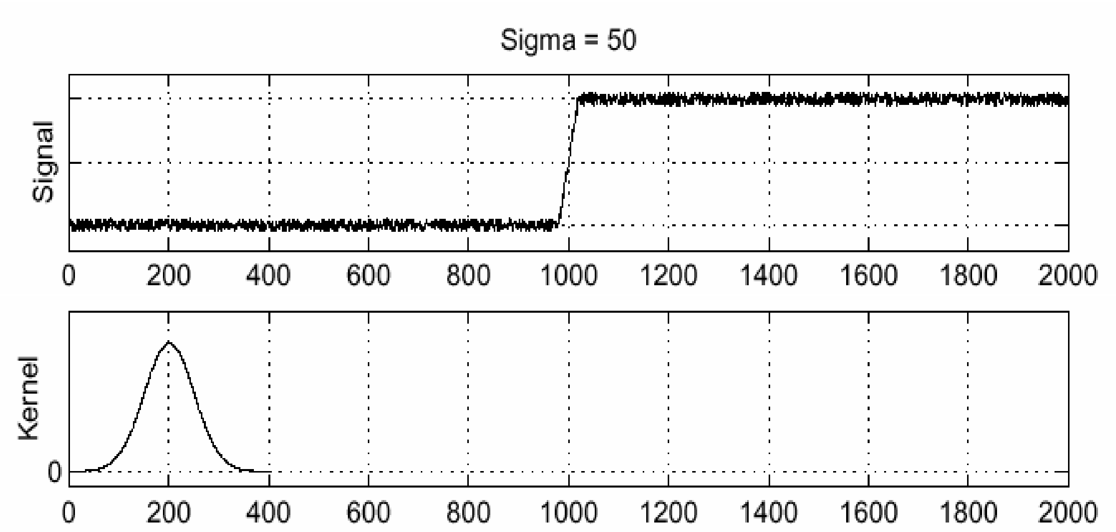
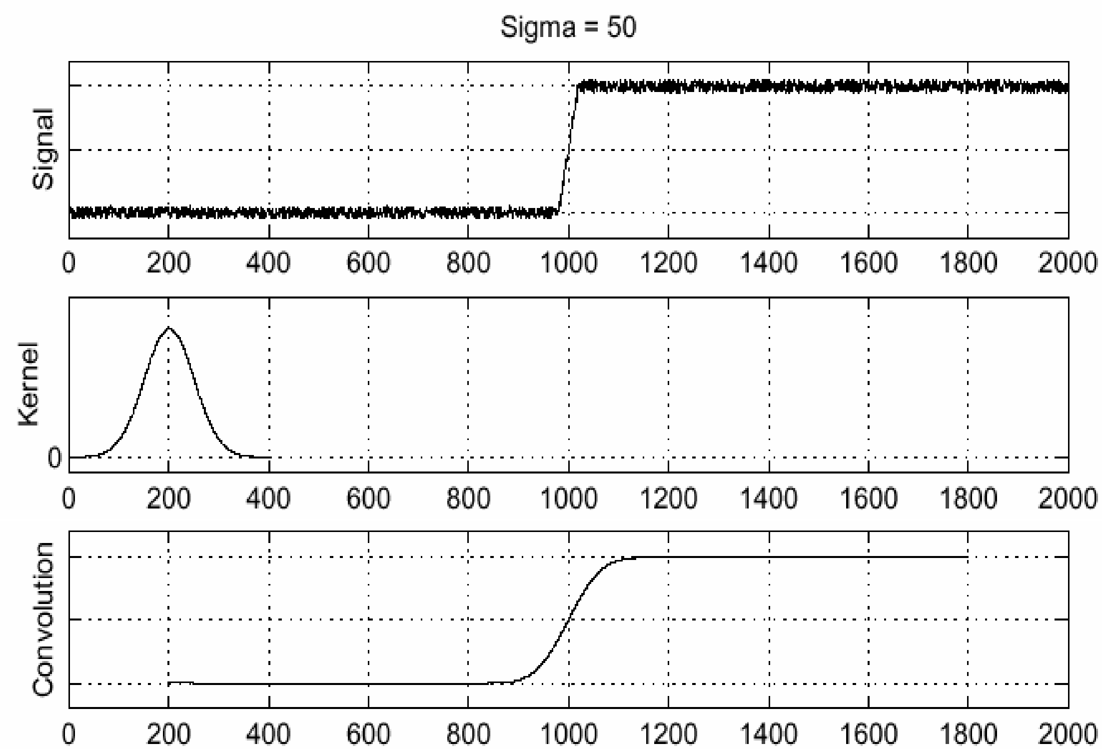


f

Smoothing function g

Solution: smooth first

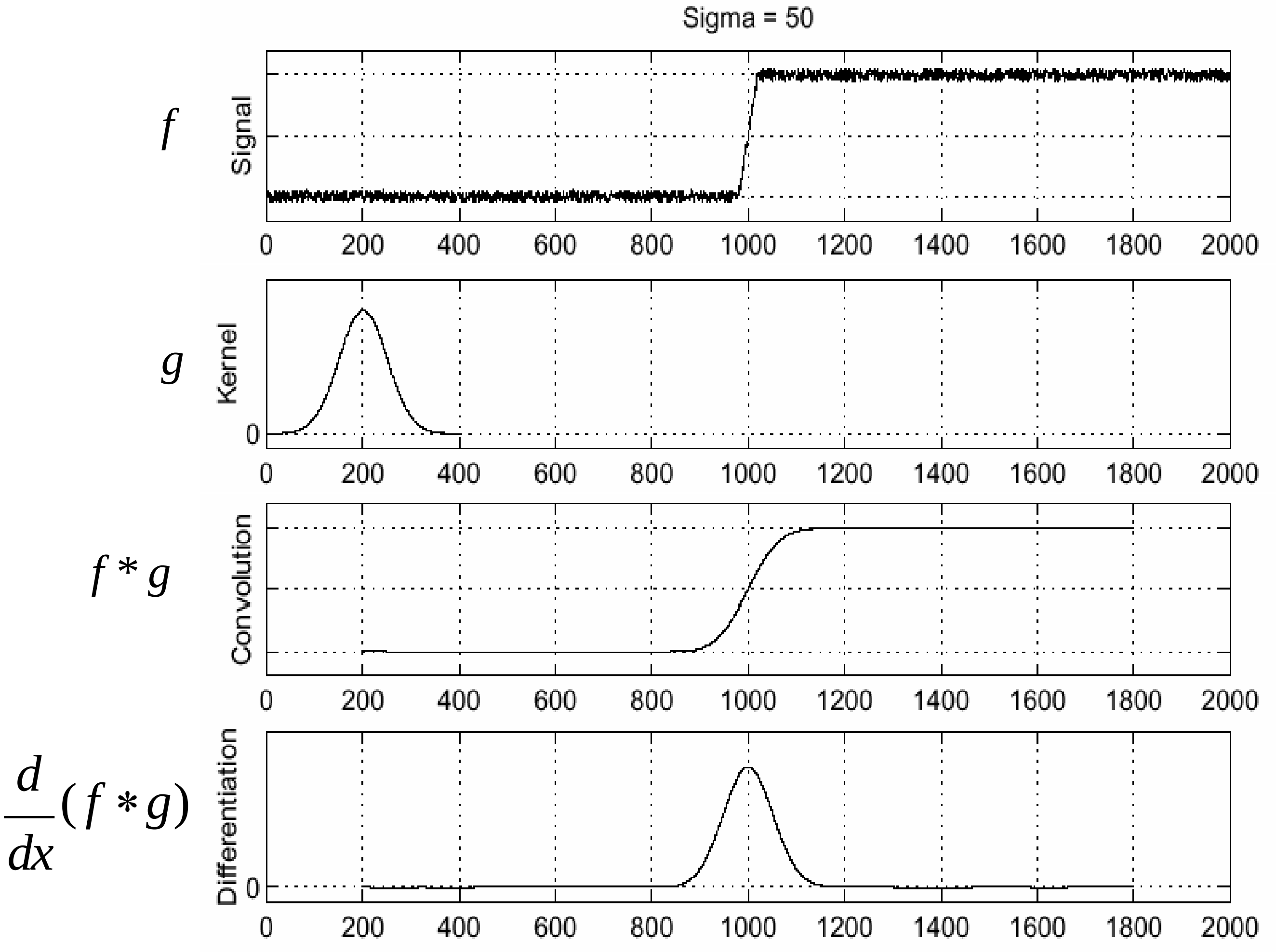


f

Smoothing function g

$f * g$

Solution: smooth first



- To find edges, look for peaks in $\frac{d}{dx}(f * g)$

Derivative theorem of convolution

- Differentiation is convolution, and convolution is associative:

$$\frac{d}{dx}(f * g) = f * \frac{d}{dx} g$$

- This saves us one operation:

Derivative theorem of convolution

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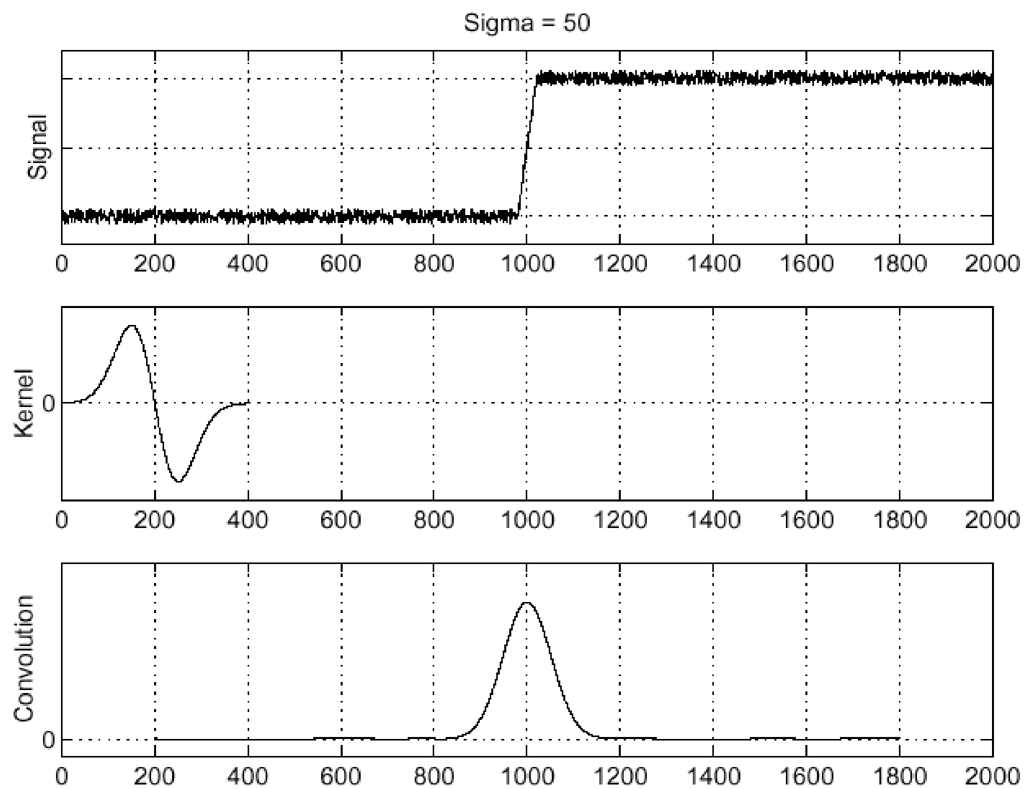
f

$\frac{d}{dx} g$

$f * \frac{d}{dx} g$



Detecting Discontinuities

- Image derivatives

$$\frac{\partial f}{\partial x} = \lim_{\varepsilon \to 0} \left(\frac{f(x+\varepsilon) - f(x)}{\varepsilon} \right) \Rightarrow \frac{\partial f}{\partial x} \approx \frac{f(x_{n+1}) - f(x)}{\partial x}$$

- Convolve image with derivative filters

Detecting Discontinuities

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- Convolve image with derivative filters

Backward difference [-1 1]

Detecting Discontinuities

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- Convolve image with derivative filters

Backward difference	[-1 1]
Forward difference	[1 -1]

Detecting Discontinuities

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- Convolve image with derivative filters

Backward difference	[-1 1]
Forward difference	[1 -1]
Central difference	[-1 0 1]

Derivative in Two-Dimensions

- Definition

- Convolution kernels

Derivative in Two-Dimensions

- Definition

$$\frac{\partial f(x,y)}{\partial x} = \lim_{\varepsilon \to 0} \left(\frac{f(x+\varepsilon, y) - f(x,y)}{\varepsilon} \right)$$

- Convolution kernels

Derivative in Two-Dimensions

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$$\frac{\partial f(x,y)}{\partial x} = \lim_{\varepsilon \to 0} \left(\frac{f(x+\varepsilon, y) - f(x,y)}{\varepsilon} \right) \qquad \frac{\partial f(x,y)}{\partial y} = \lim_{\varepsilon \to 0} \left(\frac{f(x, y+\varepsilon) - f(x,y)}{\varepsilon} \right)$$

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- Convolution kernels

$$f_x = \begin{bmatrix} 1 & -1 \end{bmatrix}$$

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- Convolution kernels

$$f_x = \begin{bmatrix} 1 & -1 \end{bmatrix} \qquad f_y = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Image Derivatives

Image I

$$I_x = I * \begin{bmatrix} 1 & -1 \end{bmatrix}$$

$$I_y = I * \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

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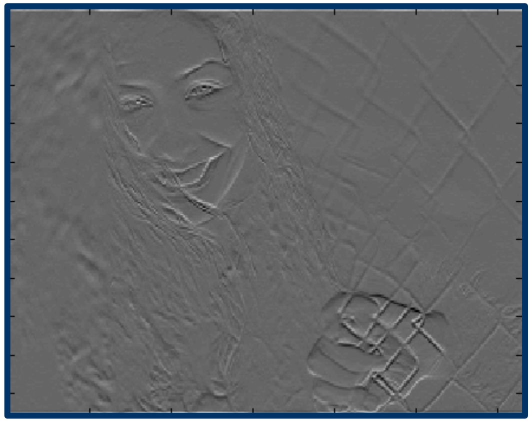


$$I_y = I * \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Edge Detectors

Edge Detectors

- Gradient operators
  - Prewit
  - Sobel
- Laplacian of Gaussian (Marr-Hildreth)
- Gradient of Gaussian (Canny)

Prewitt and Sobel Edge Detector

Prewitt and Sobel Edge Detector

- Compute derivatives
  - In x and y directions

Prewitt and Sobel Edge Detector

- Compute derivatives
  - In x and y directions
- Find gradient magnitude

Prewitt and Sobel Edge Detector

- Compute derivatives
  - In x and y directions
- Find gradient magnitude
- Threshold gradient magnitude

Prewitt Edge Detector

Prewitt Edge Detector

image

Prewitt Edge Detector

image $\longrightarrow$ average smoothing in x

Prewitt Edge Detector

Prewitt Edge Detector

image $\xrightarrow{\text{average smoothing in } x}$ blurred $\xrightarrow{\text{derivative filtering in } x}$

Prewitt Edge Detector

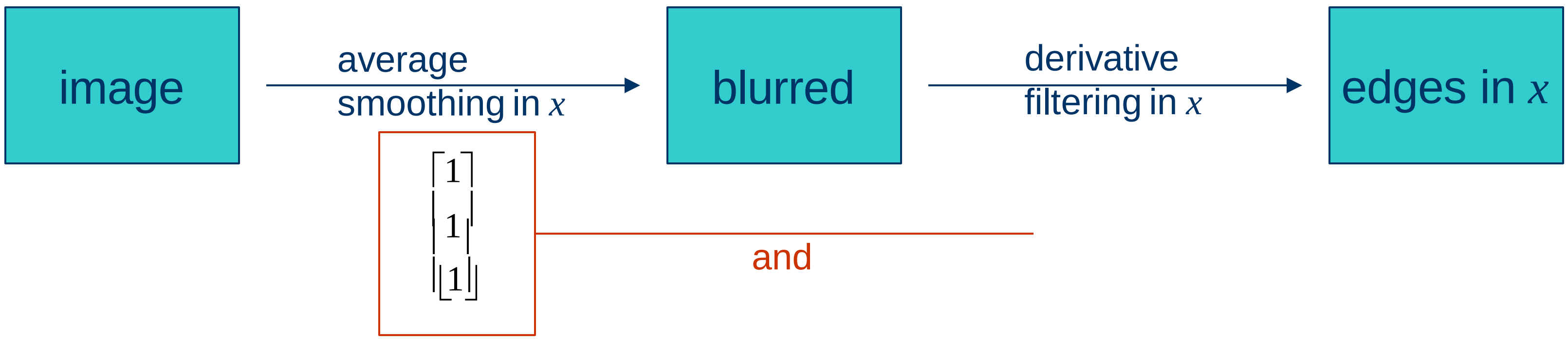
$$\boxed{\text{image}} \xrightarrow[\text{smoothing in } x]{\text{average}} \boxed{\text{blurred}} \xrightarrow[\text{filtering in } x]{\text{derivative}} \boxed{\text{edges in } x}$$

Prewitt Edge Detector

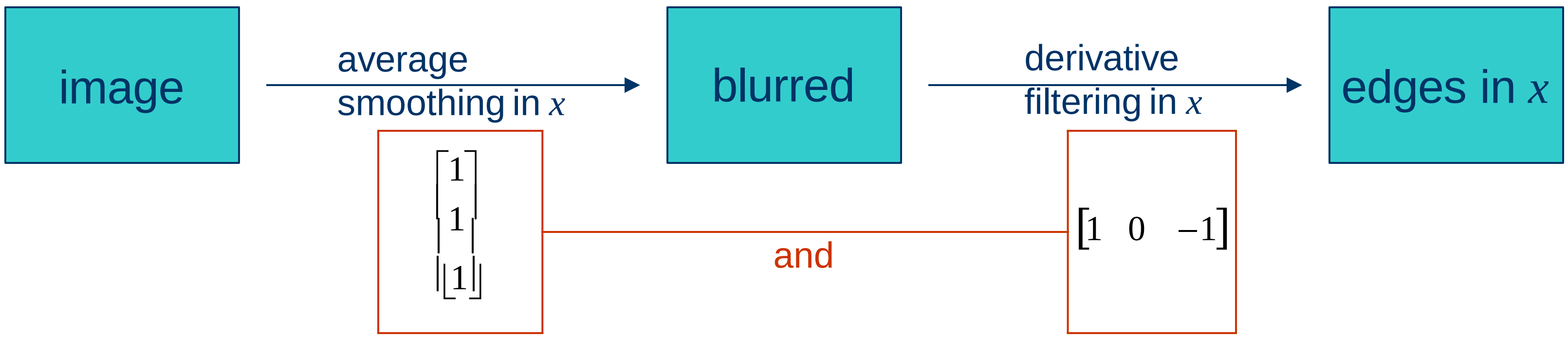
image $\xrightarrow[\text{smoothing in } x]{\text{average}}$ blurred $\xrightarrow[\text{filtering in } x]{\text{derivative}}$ edges in x

$$\boxed{\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}}$$

Prewitt Edge Detector



Prewitt Edge Detector



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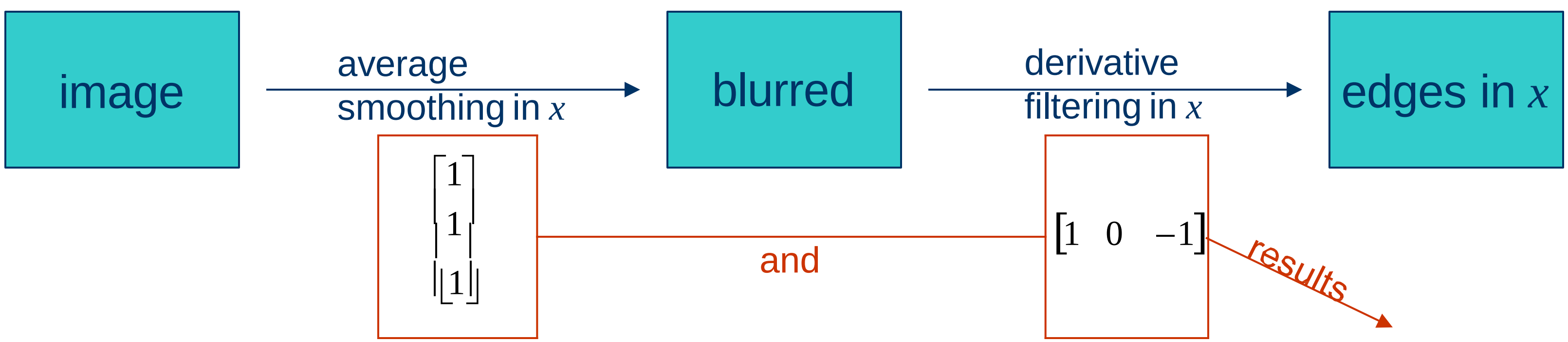
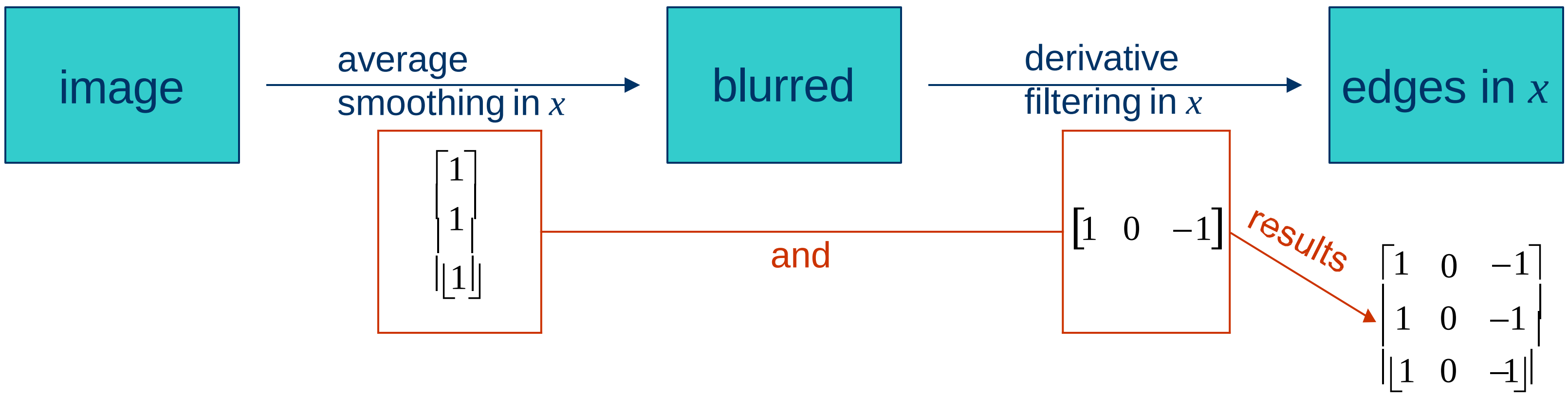


Prewitt Edge Detector



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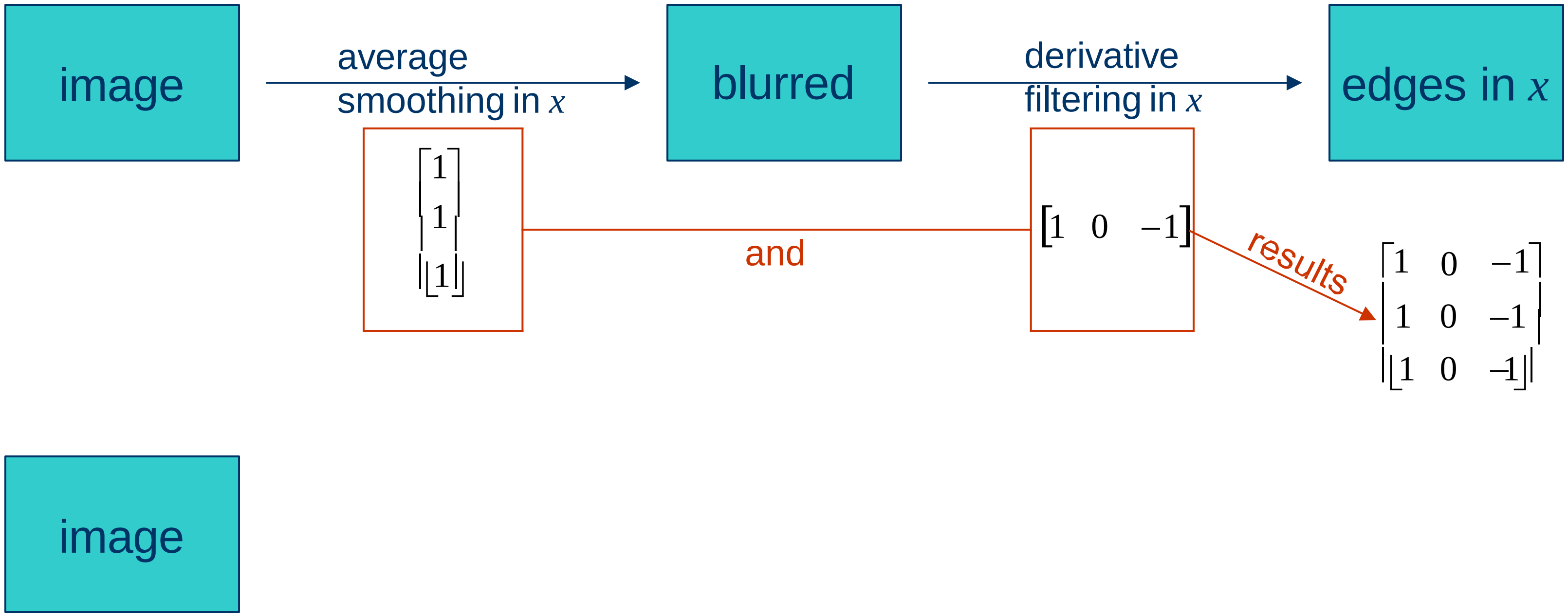

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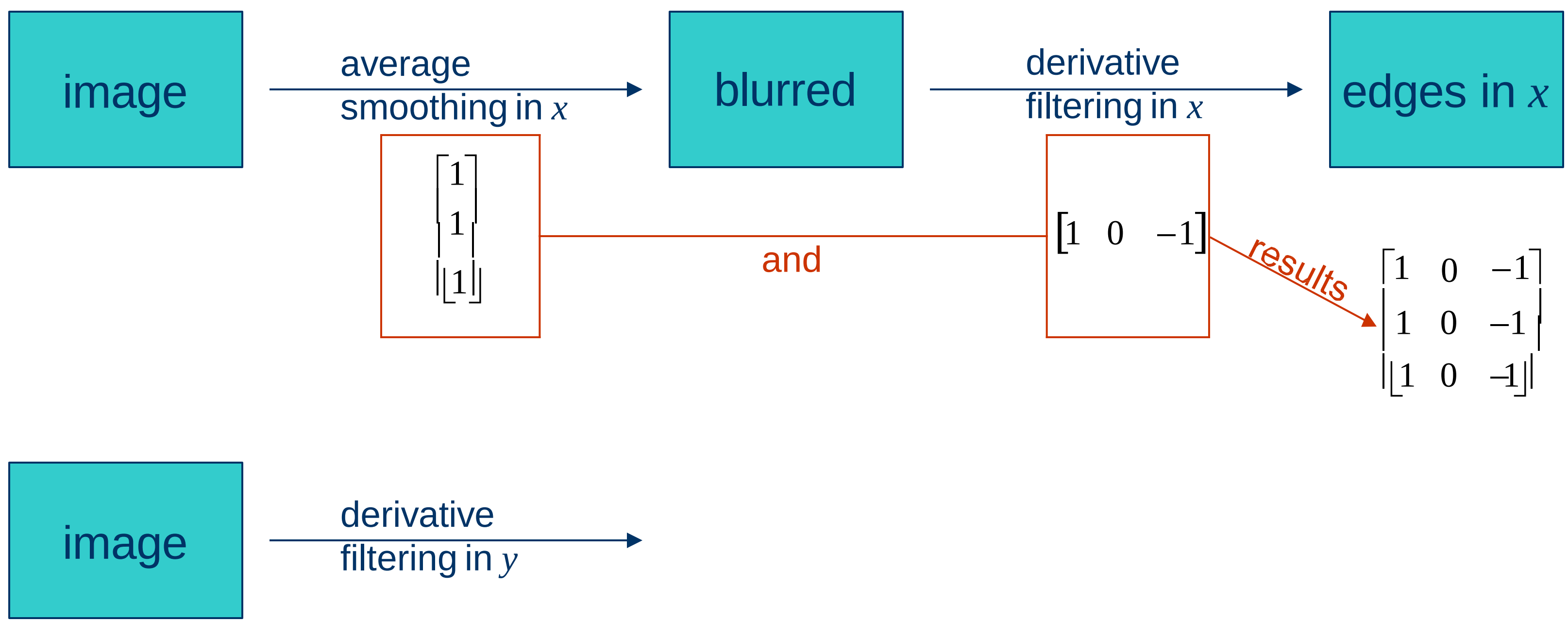
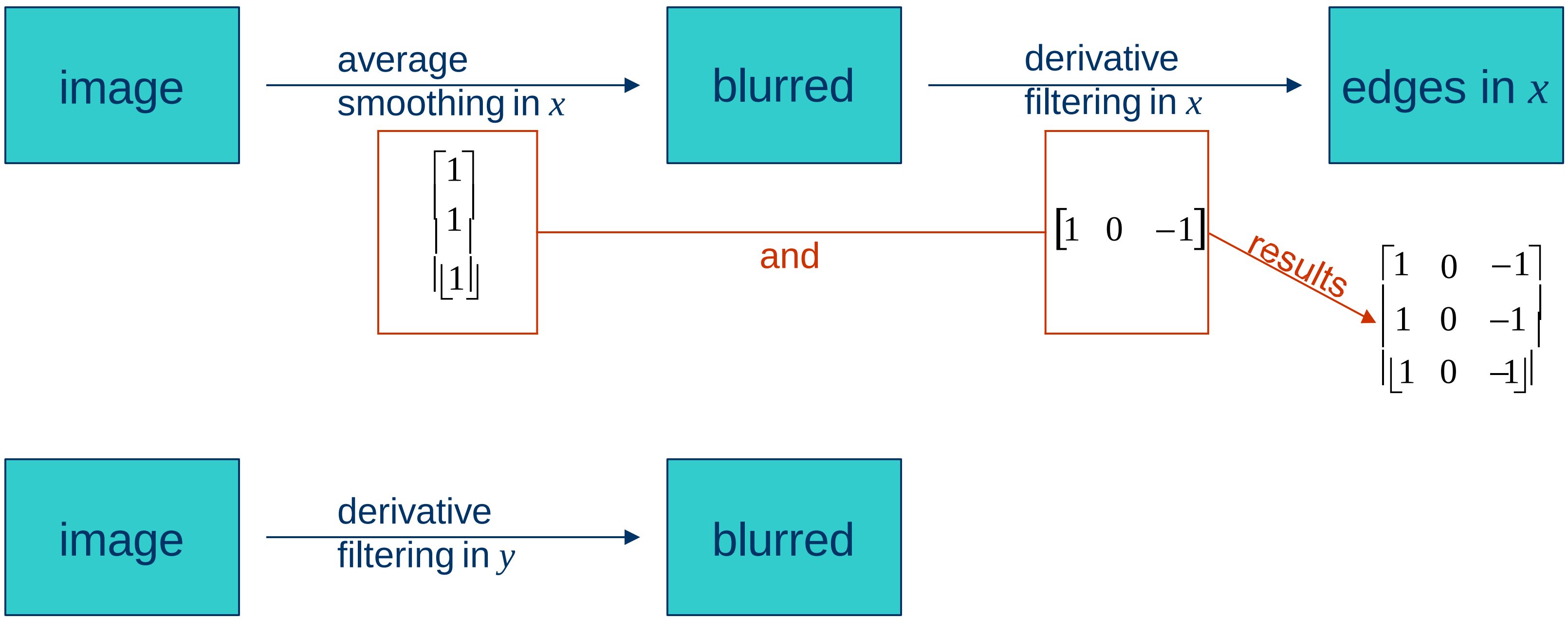


Prewitt Edge Detector



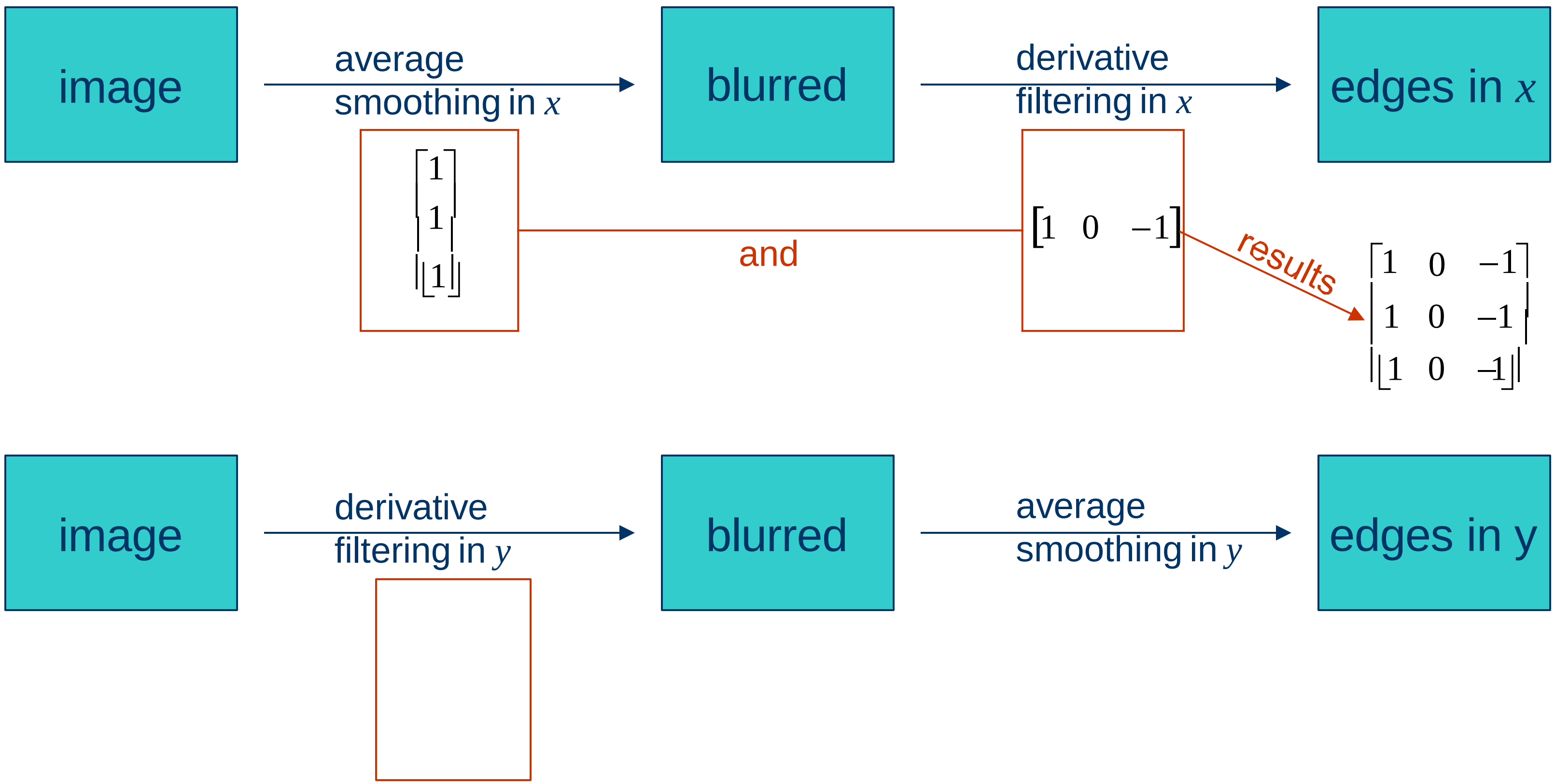
Prewitt Edge Detector

image $\xrightarrow{\text{average smoothing in } x}$ blurred $\xrightarrow{\text{derivative filtering in } x}$ edges in x

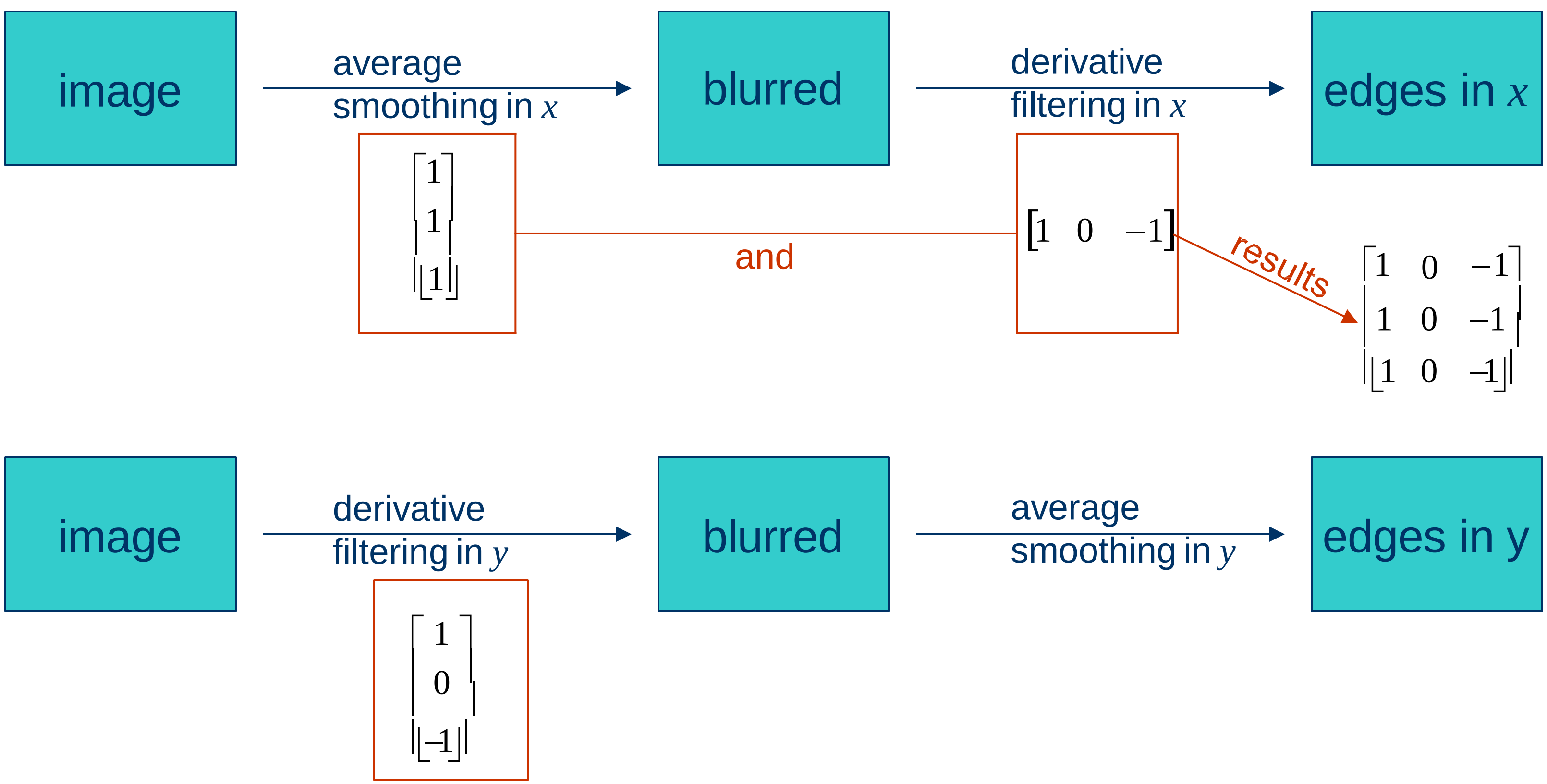
$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 1 & 0 & -1 \end{bmatrix} \quad \xrightarrow{\text{results}} \quad \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix}$$

image $\xrightarrow{\text{derivative filtering in } y}$ blurred $\xrightarrow{\text{average smoothing in } y}$

Prewitt Edge Detector



Prewitt Edge Detector



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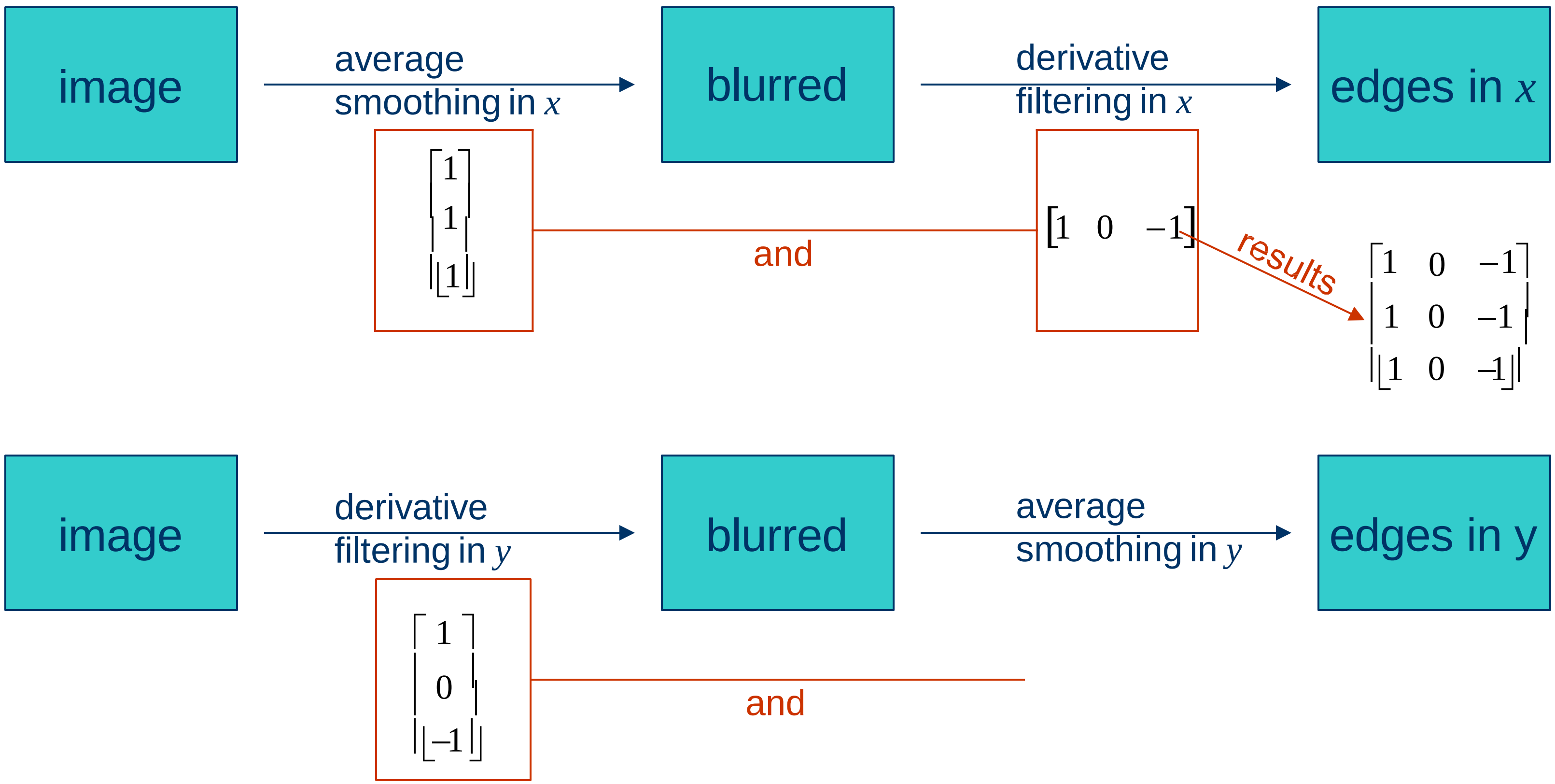

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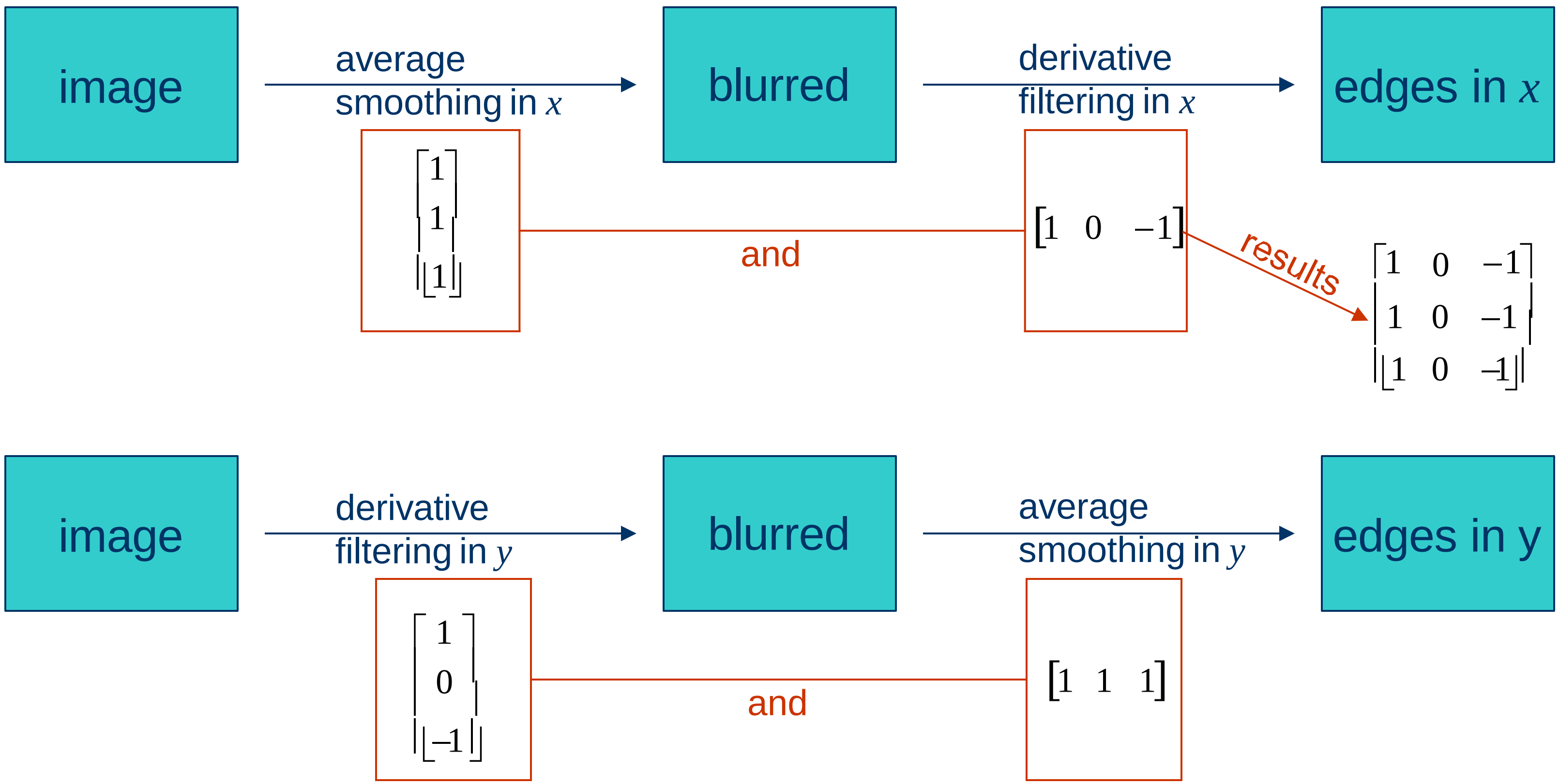
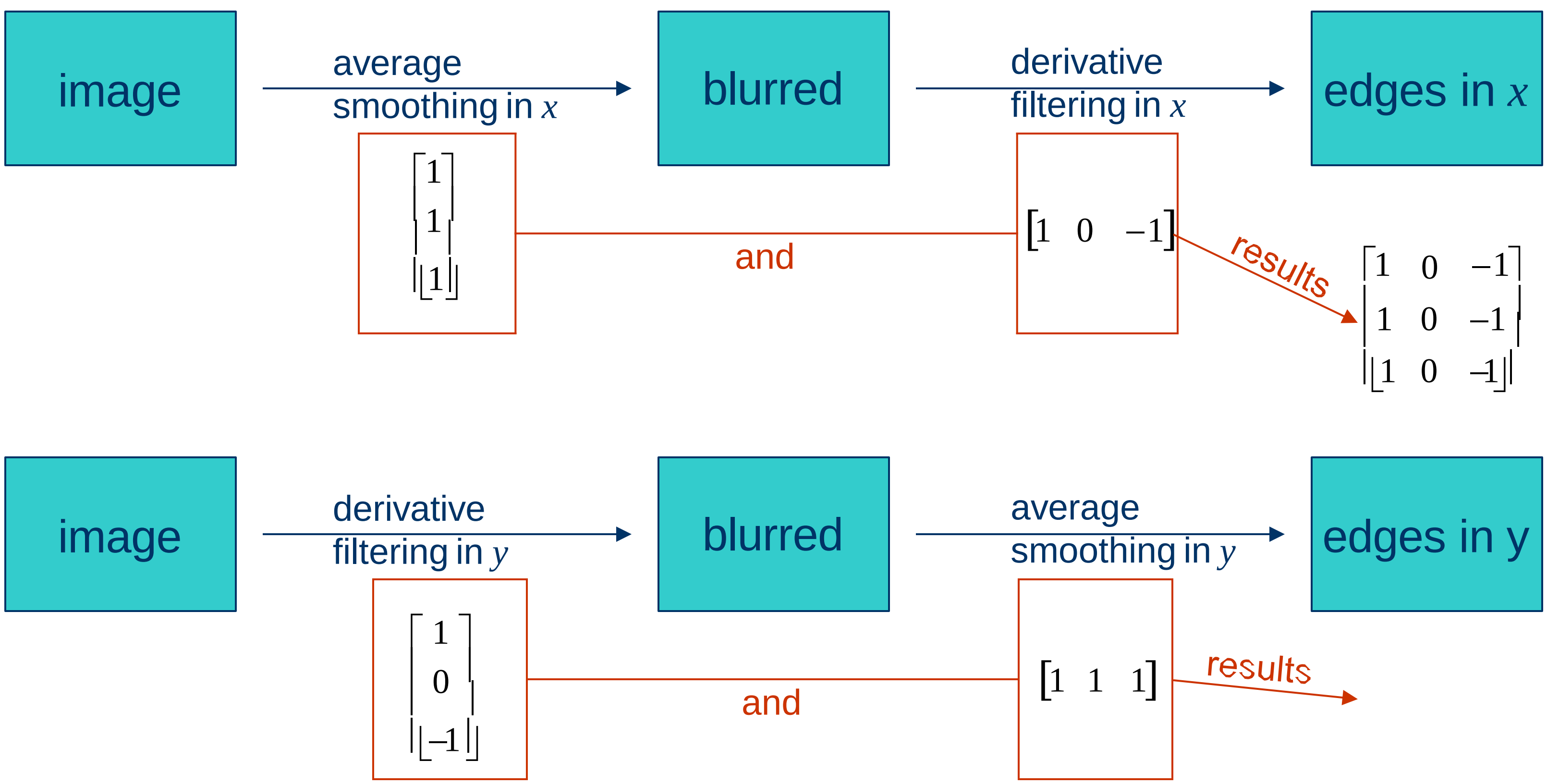
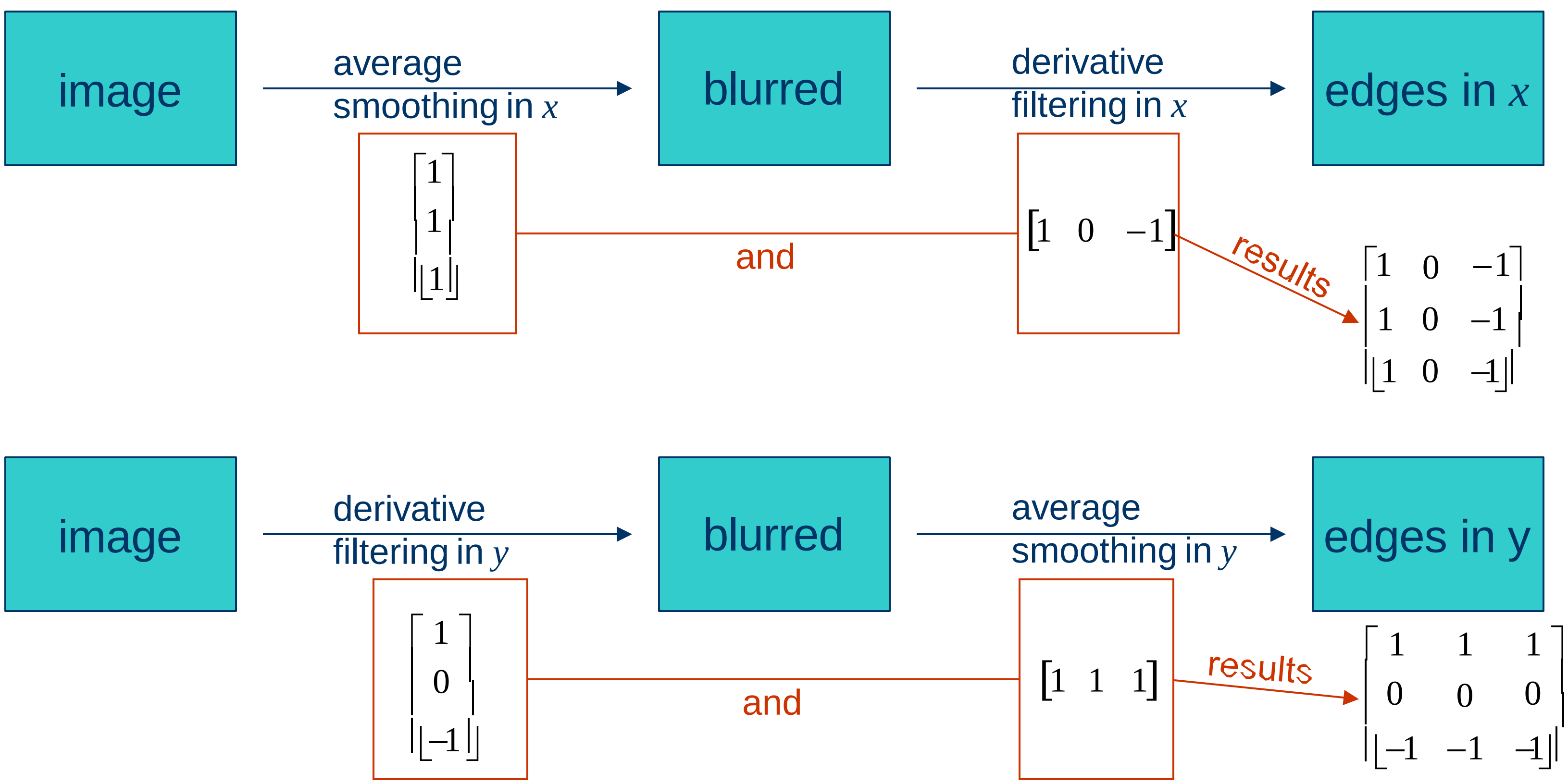


Prewitt Edge Detector



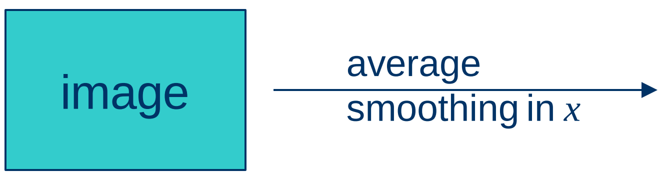
Prewitt Edge Detector



Sobel Edge Detector

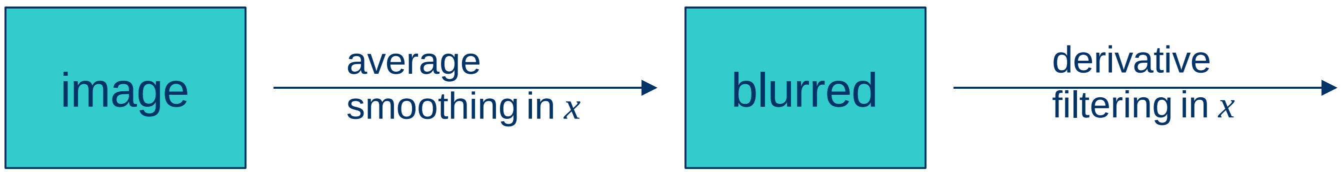
Sobel Edge Detector

Sobel Edge Detector



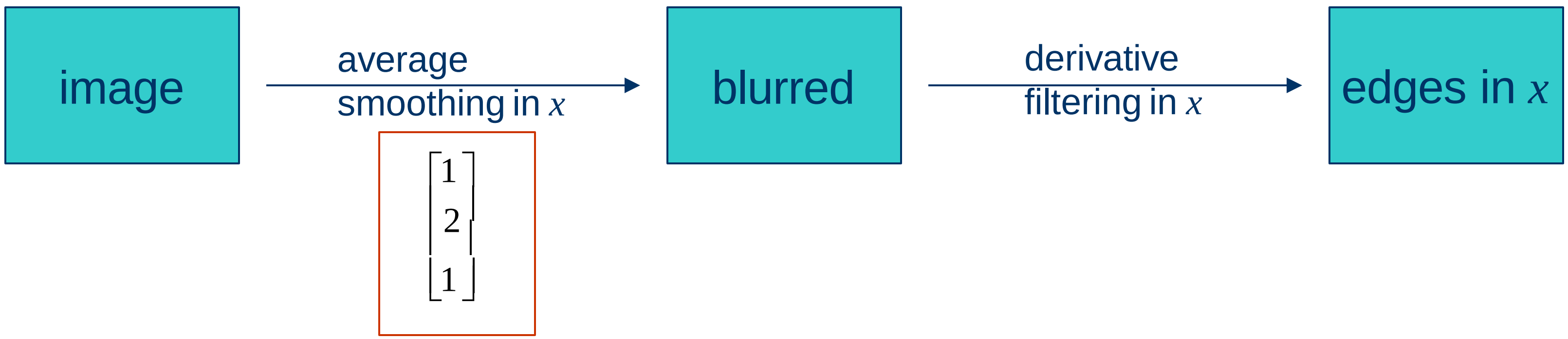
Sobel Edge Detector

Sobel Edge Detector

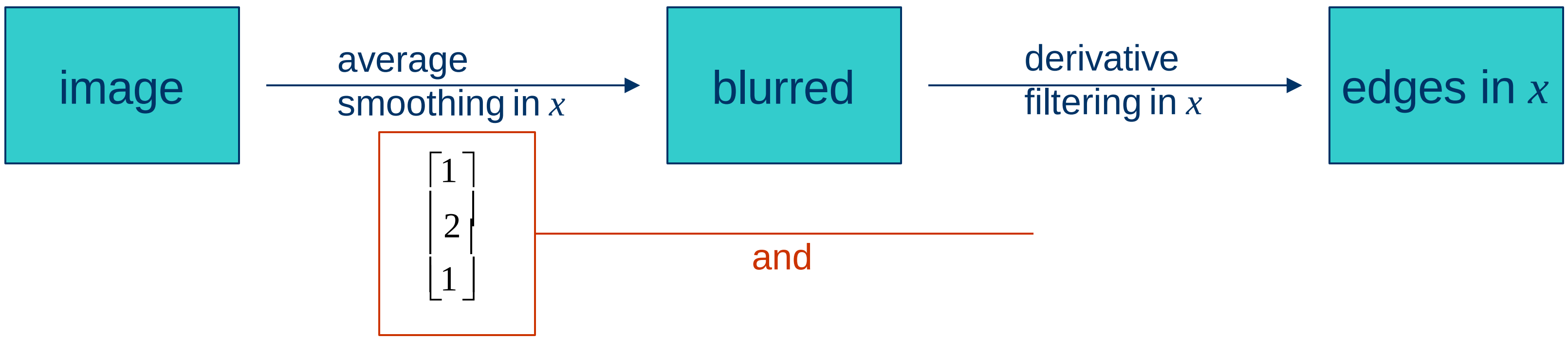


Sobel Edge Detector

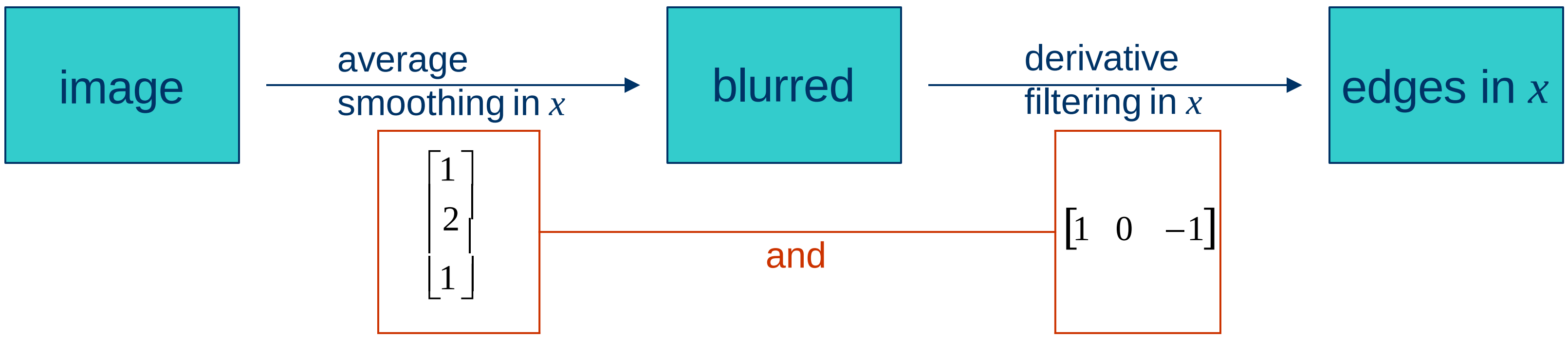
Sobel Edge Detector



Sobel Edge Detector



Sobel Edge Detector



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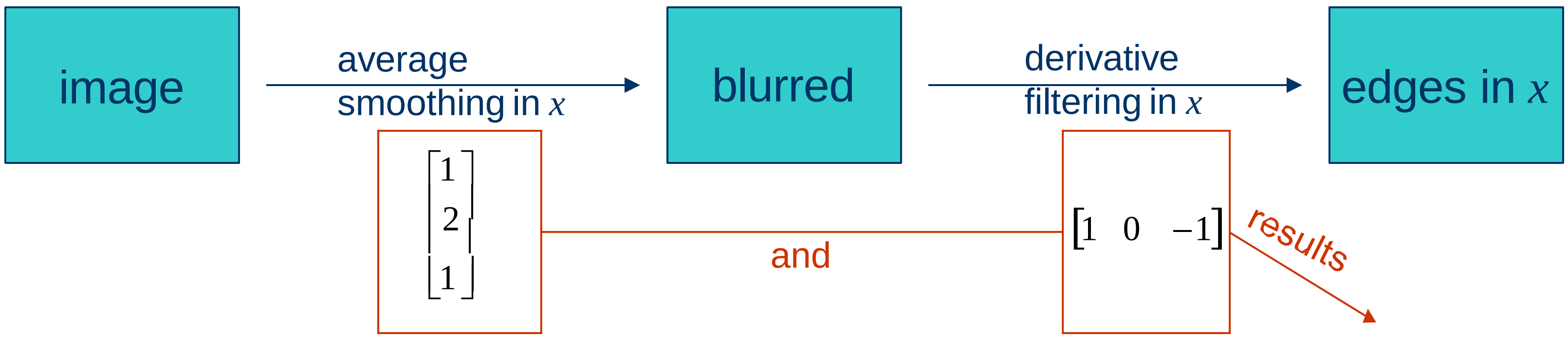
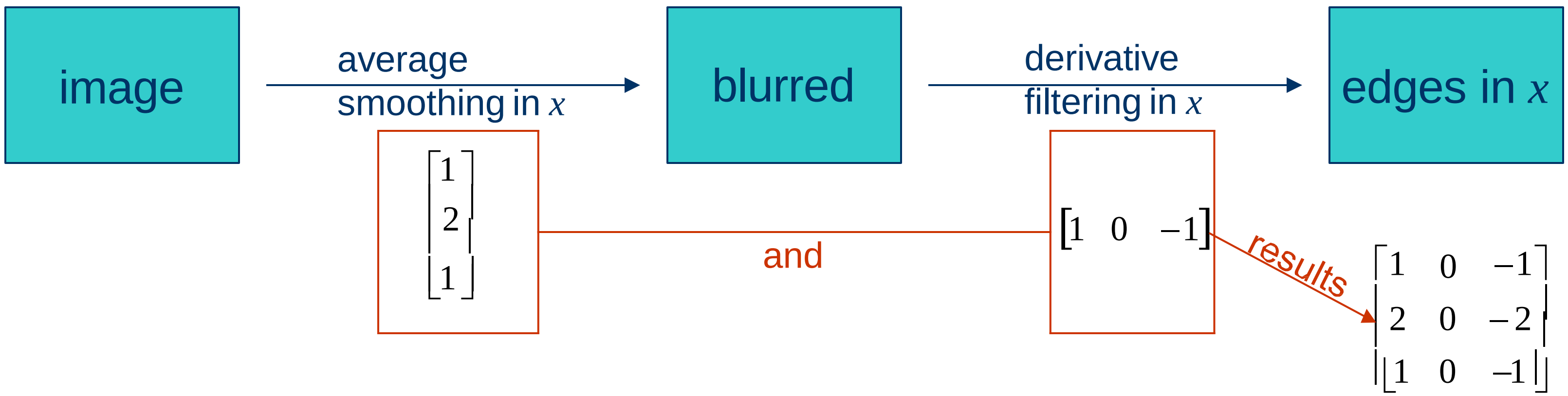


Sobel Edge Detector



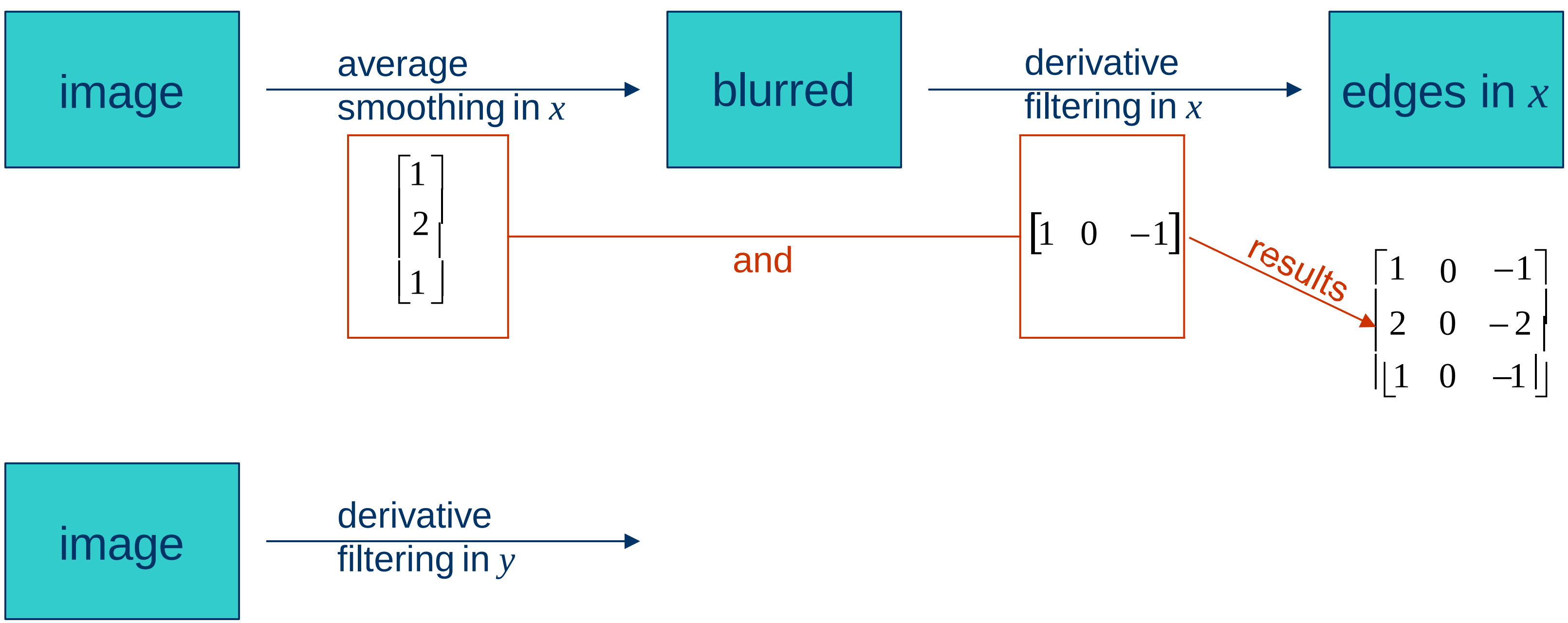
Sobel Edge Detector

image $\xrightarrow{\text{average smoothing in } x}$ blurred $\xrightarrow{\text{derivative filtering in } x}$ edges in x

$$\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 1 & 0 & -1 \end{bmatrix} \quad \text{results} \quad \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$$

image

Sobel Edge Detector



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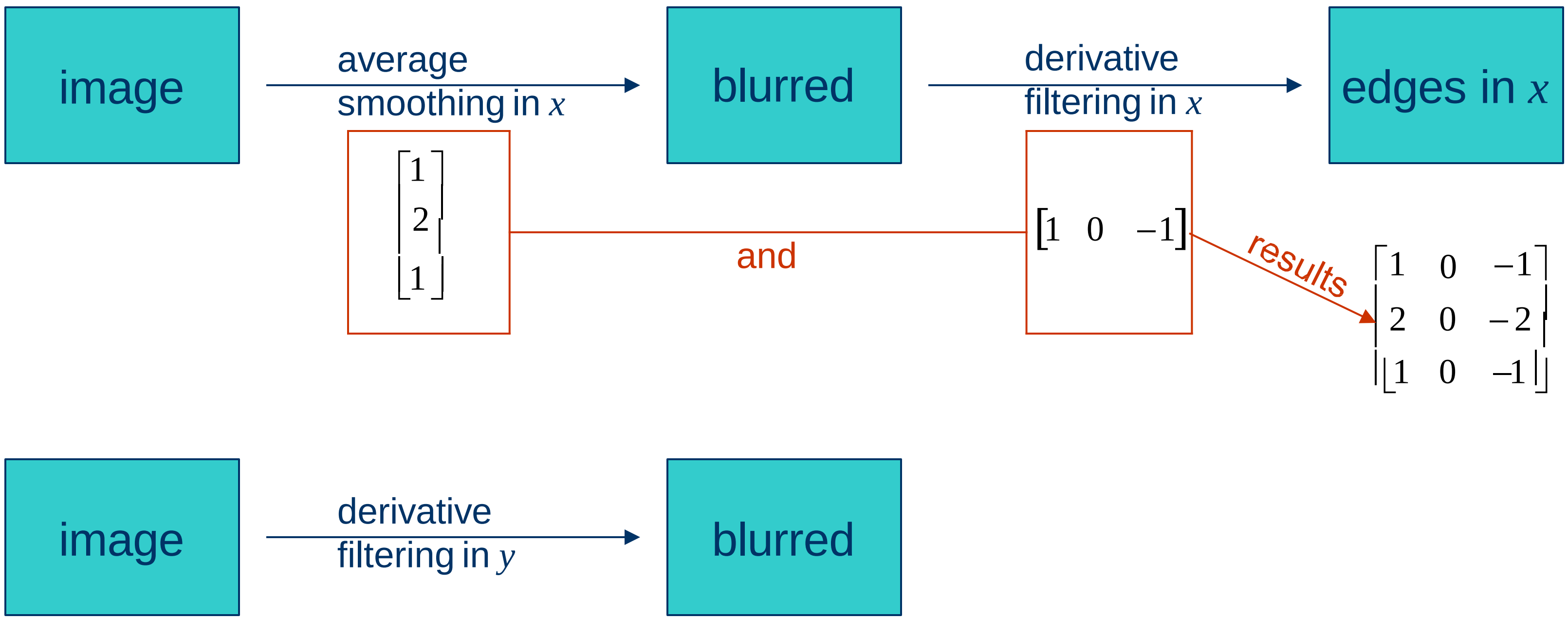

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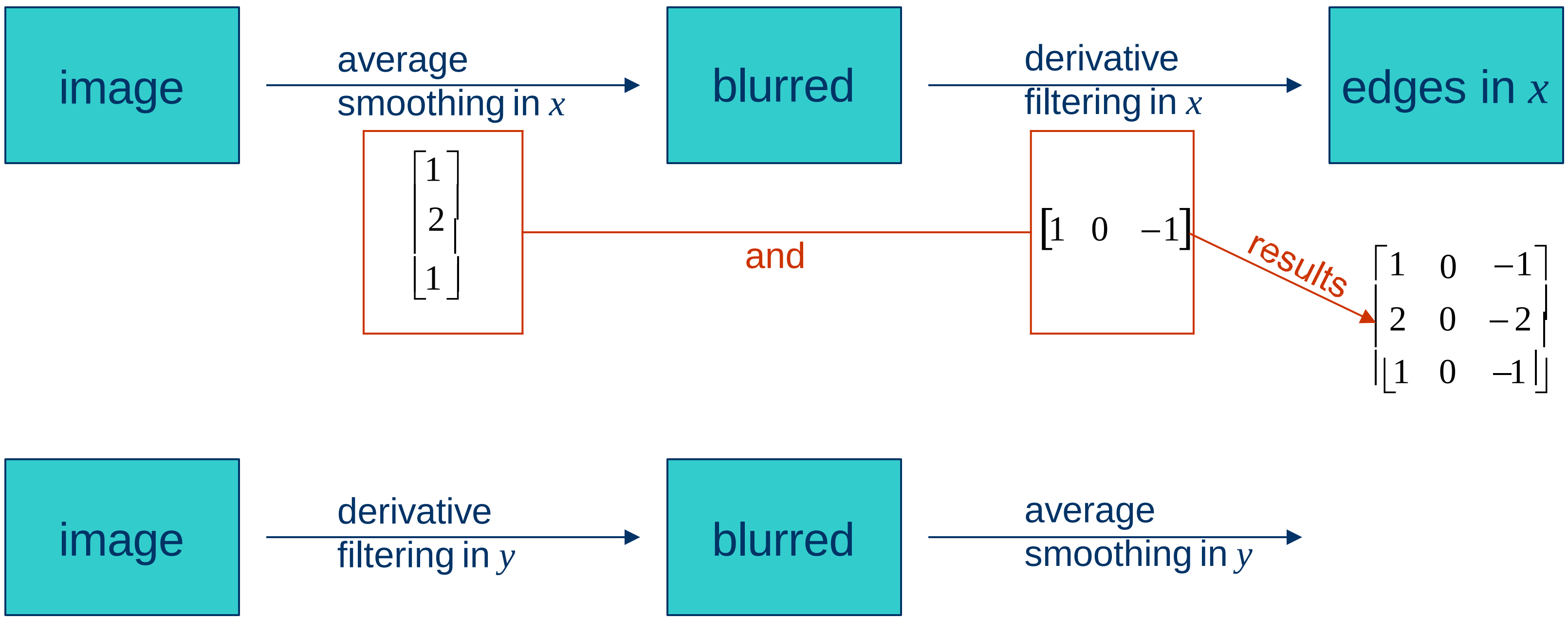

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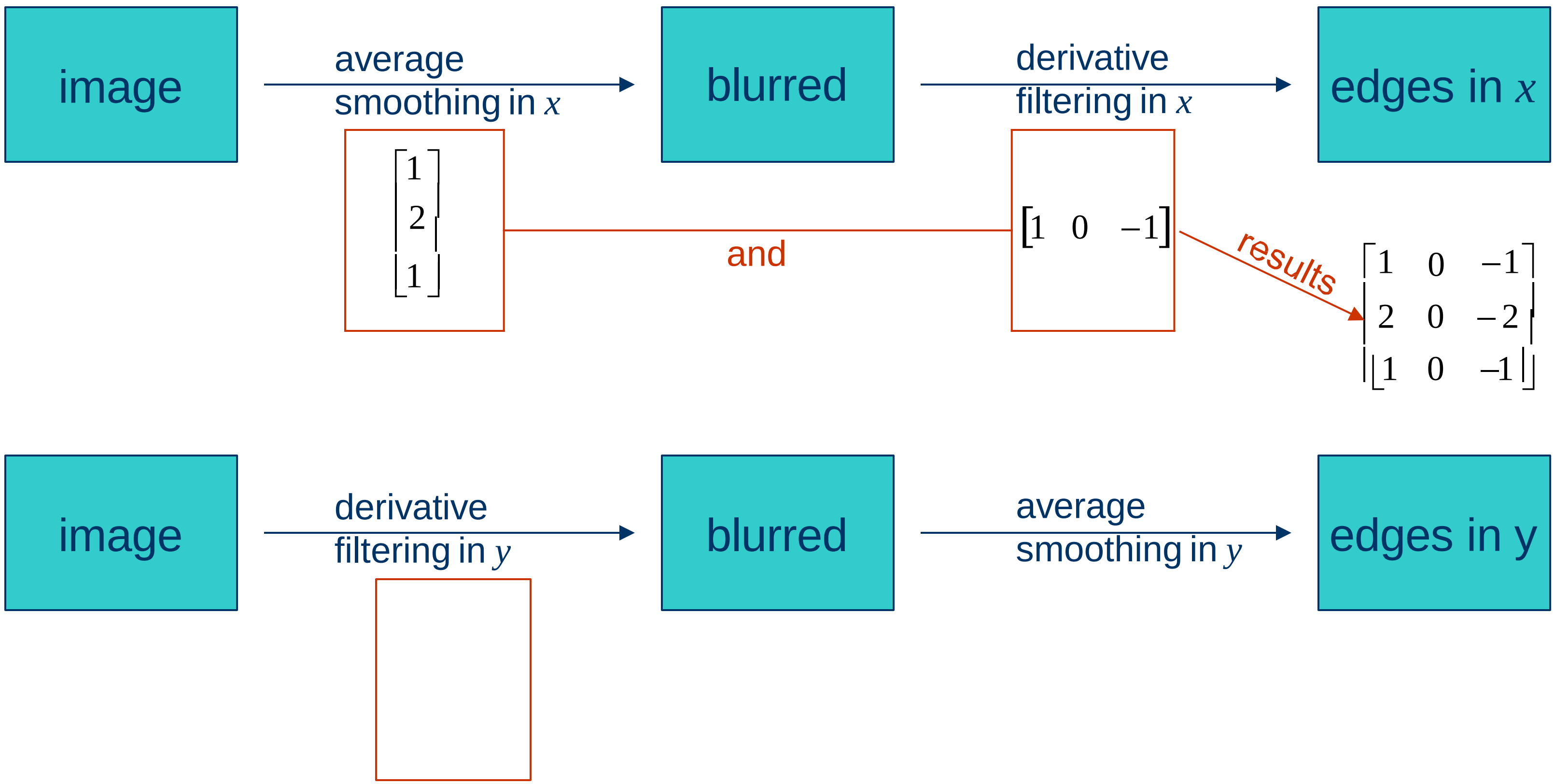

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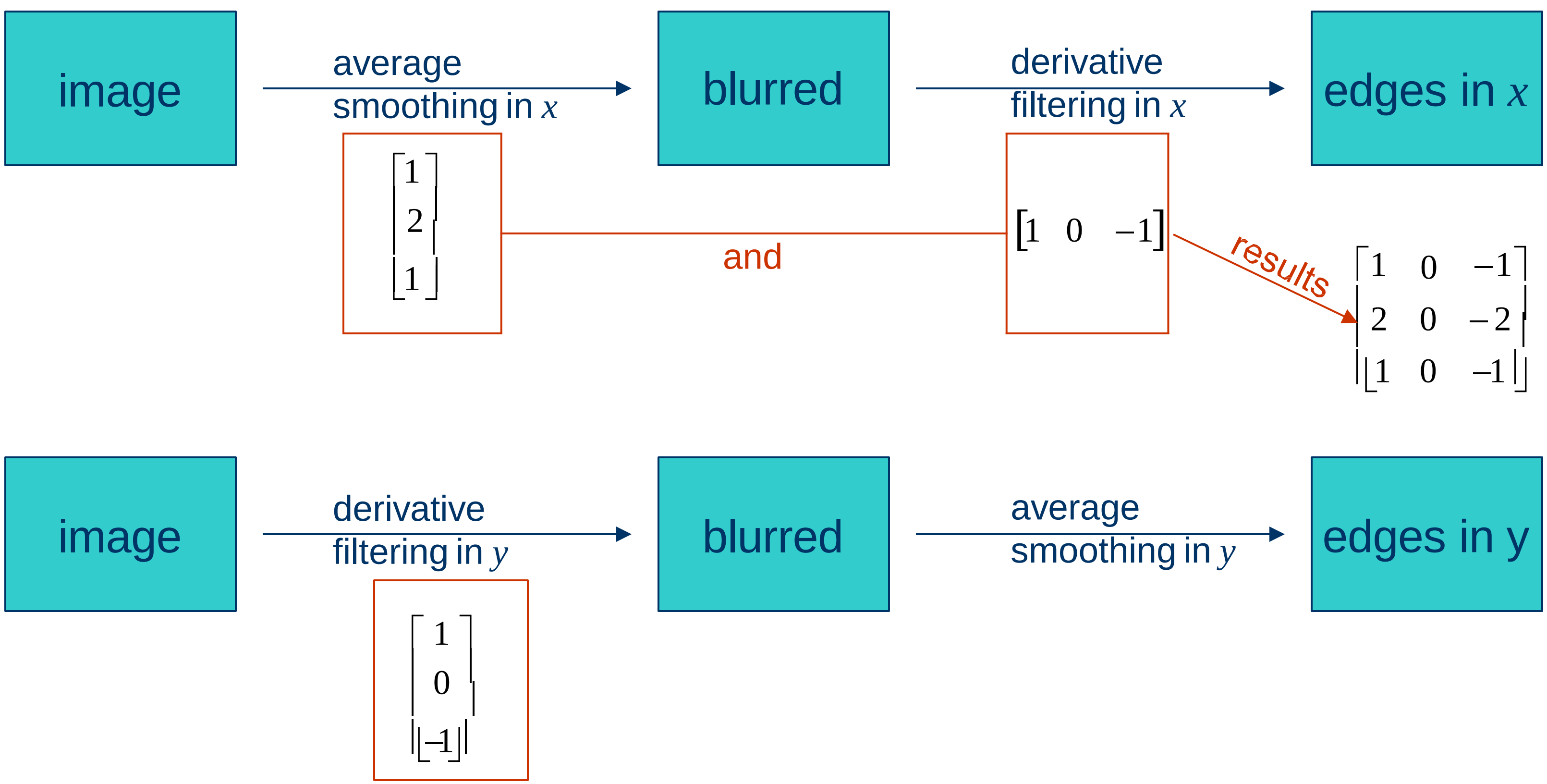

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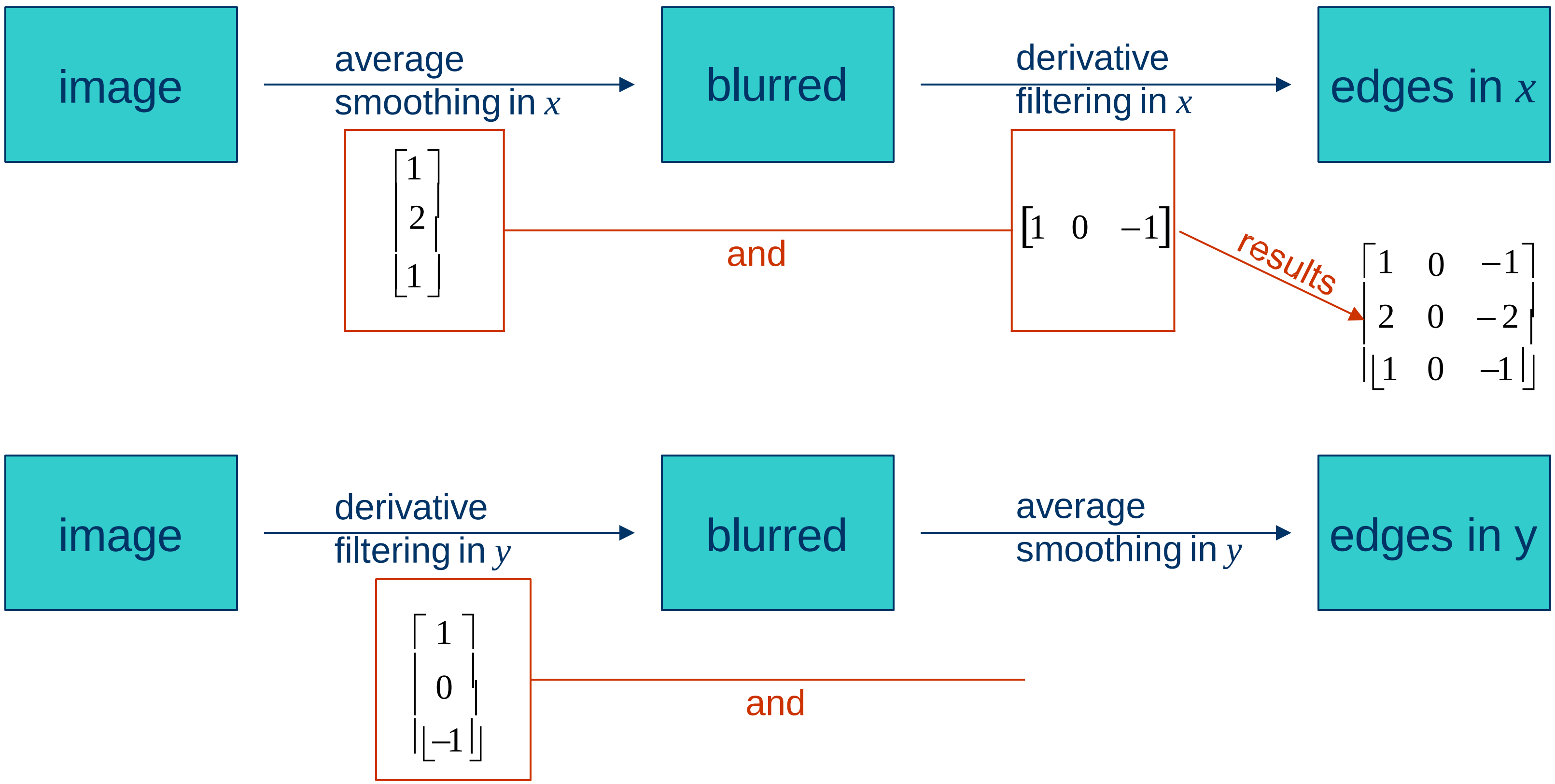

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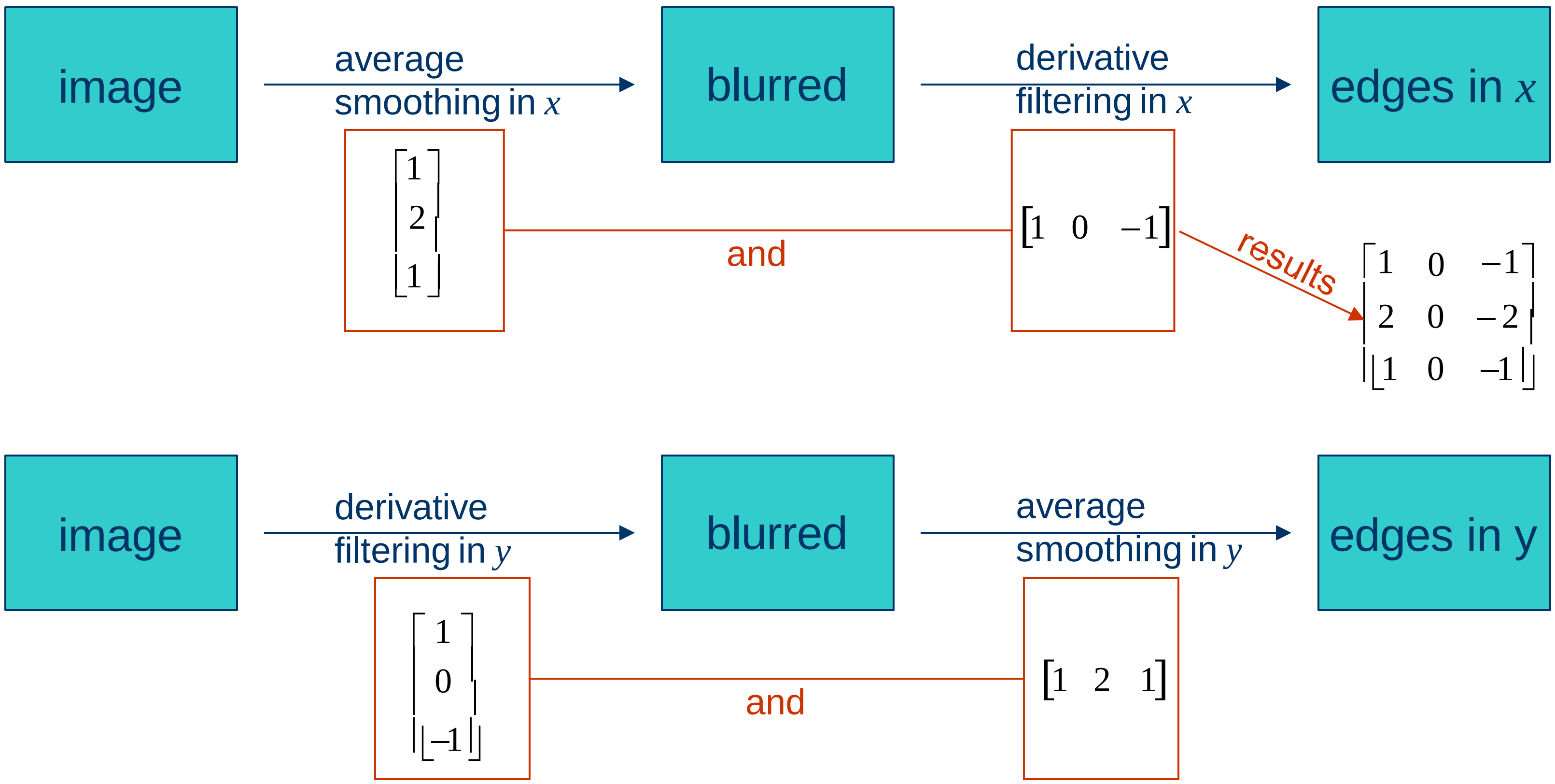

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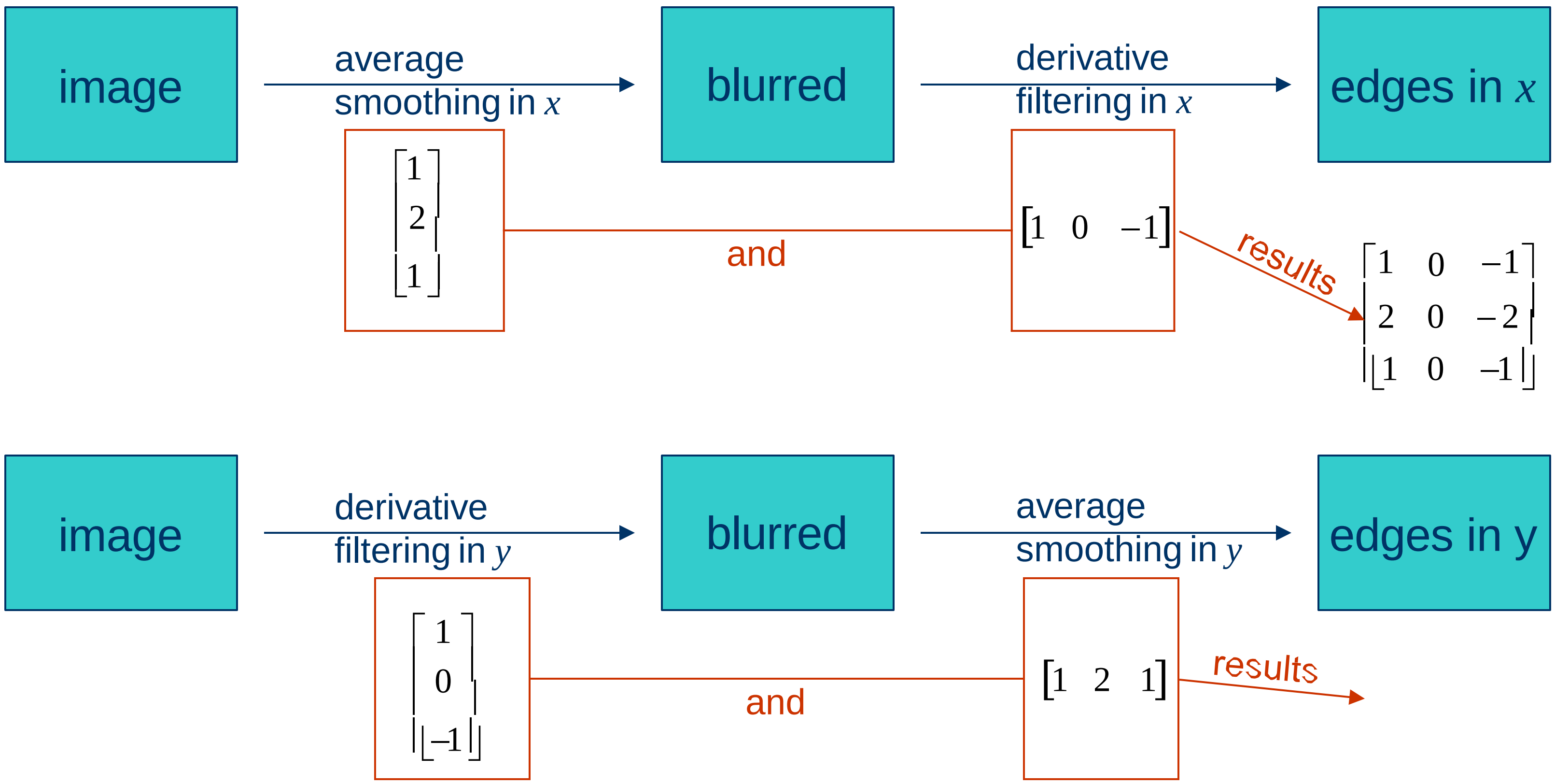

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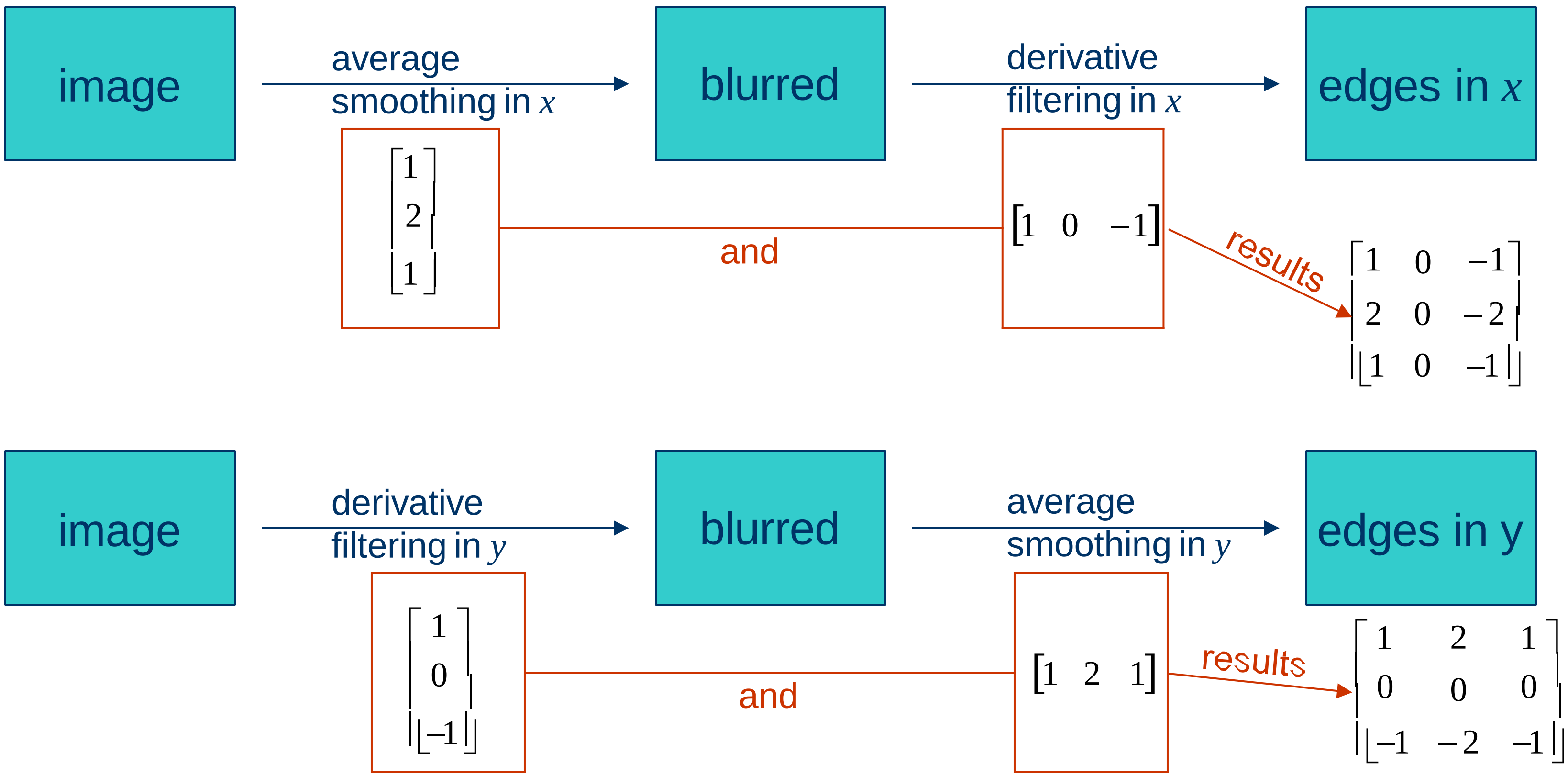
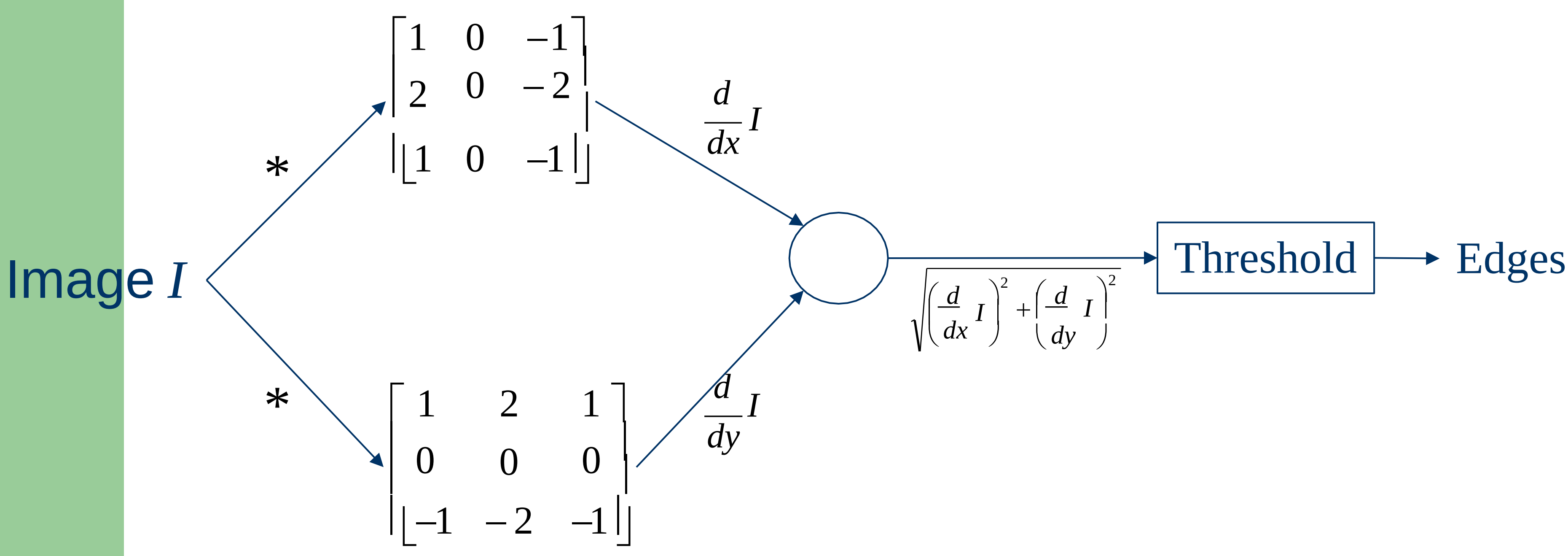
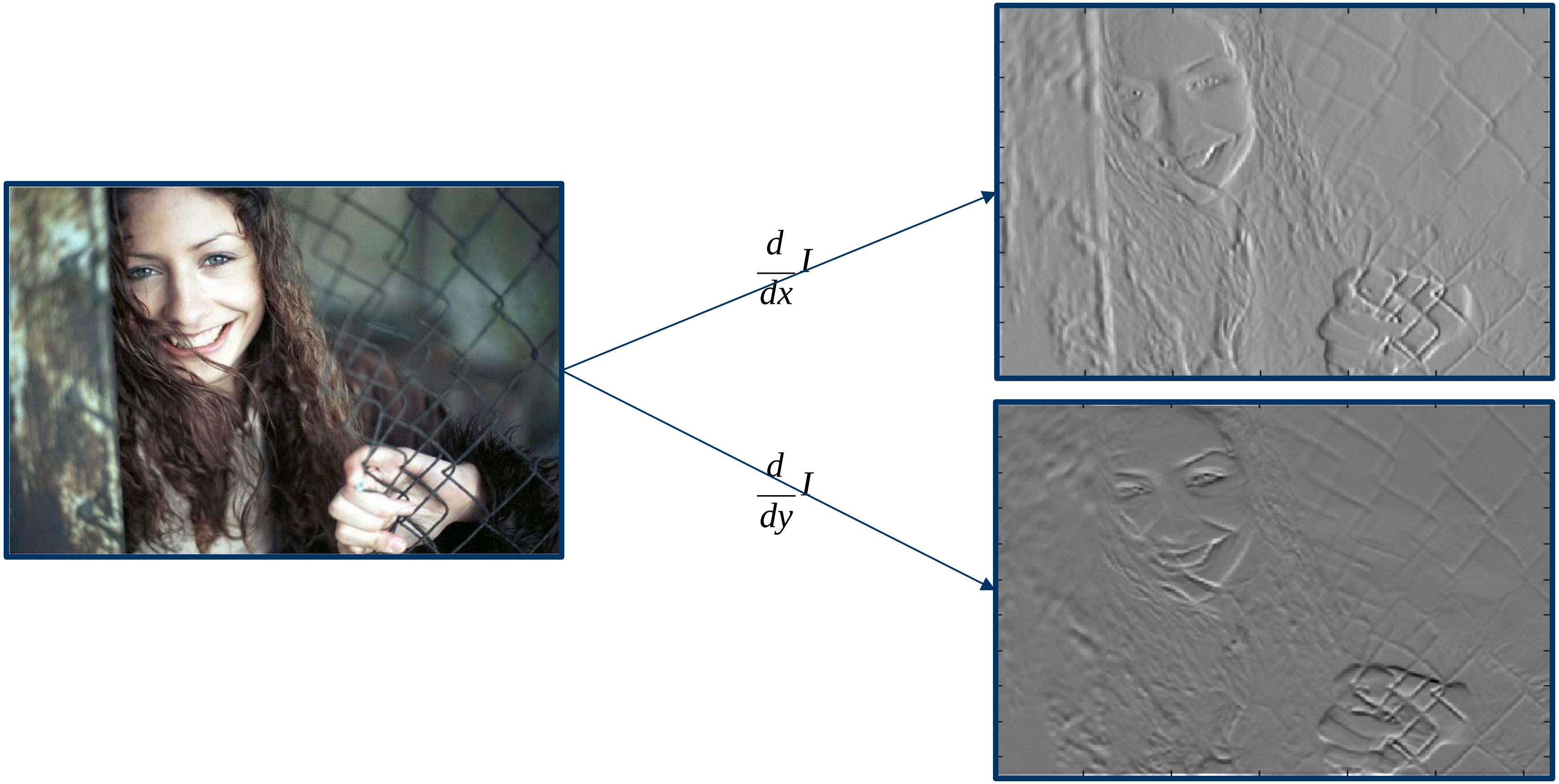


Sobel Edge Detector



Sobel Edge Detector



Sobel Edge Detector

$$\Delta = \sqrt{\left(\frac{d}{dx} I\right)^2 + \left(\frac{d}{dy} I\right)^2}$$

$$\Delta \geq Threshold = 100$$