**Final Examination: Mathematical Thinking – Section 3**

**(Dec 01, 2021): 1730-17:30 (Dec 04, 2023)**

**72 hours (Marks 360/2 = 180 Marks)**

1. The Exam is open textbook only if you have bought one☺
2. Internet and any help using Internet (like ChatGPT, Google, etc) is not allowed and would be considered as Plagiarism.
3. You will assemble in Capstone project groups and will discuss and solve Sections 1 and 2. This is Question 1. Upload your solution on DevOps.
4. Question 2 is: Reading the chapter 2 “Encryption” and understand the mathematical language of encryption. You are tasked with understanding the chapter and solve 2.18 exercise problem and help Election Commission of Pakistan by developing in Python a Secure Voting Machine based on the idea of 2.18. Submit your solution and Python code on DevOps.
5. Any person at Random will be called for an oral examination on October 26, 2021, to explain solutions and whatever marks he gets will be the marks of the whole group (please make sure that all group members understand the solutions).
6. In all proofs, show your complete rough work to illustrate your thought process and then at the end neatly convert them into a literary masterpiece of mathematical poems and prose.
7. Remember, this course rewards the entire process of thinking and not a particular endpoint and result. The thinking journey and discovery of different paths is more important than reaching the destination itself (that is also important in real life though).
8. BEST OF LUCK on your thinking endeavor

Table

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**Question 1: Prove that** if x, y ∈ R. Then |x + y| ≤ |x| + |y|.

*Thinking Process Evidence:*

*Final and Neat Proof (follow Rubrics):*

**Question 2: Prove are infinitely many Fibonacci numbers and they grow exponentially [***Hint: Remember Fibonacci numbers fn = fn−1 + fn−2* and induction helps in recursive reasoning]

*Thinking Process Evidence:*

*Final and Neat Proof (follow Rubrics):*

**Question 3: Prove If n ∈ N and θ ∈ R, then [cos(θ) + i sin(θ)]n = cos(nθ) + i sin(nθ).** *[Hint: “i” shows the complex number and try PMI for doing the proof]*

*Thinking Process Evidence:*

*Final and Neat Proof (follow Rubrics):*

**Question 4: Prove that only prime triplet (i.e three primes, each at a difference of 2 from the next) is 3,5,7.**

*Thinking Process Evidence:*

*Final and Neat Proof (follow Rubrics):*

**Question 5: If a data structure of type tree has *n* vertices, then it has *n*−1 edges.**

*Thinking Process Evidence:*

*Final and Neat Proof (follow Rubrics):*

**Question 6: Prove that any odd prime number p can be represented as sum of squares of two integers if and only if *p ≡ 1 mod 4* where mod operator is modulo operator.**

*Thinking Process Evidence:*

*Final and Neat Proof (follow Rubrics):*