

Unit 4

Turning Effect of Forces

STUDENT'S LEARNING OUTCOMES

After studying this unit, the students will be able to:



- define like and unlike parallel forces.
- state head to tail rule of vector addition of forces/vectors.
- describe how a force is resolved into its perpendicular components.
- determine the magnitude and direction of a force from its perpendicular components.
- define moment of force or torque as moment = force x perpendicular distance from pivot to the line of action of force.
- explain the turning effect of force by relating it to everyday life.
- state the principle of moments.
- define the centre of mass and centre of gravity of a body.
- define couple as a pair of forces tending to produce rotation.
- prove that the couple has the same moments about all points.
- define equilibrium and classify its types by quoting examples from everyday life.
- state the two conditions for equilibrium of a body.

Conceptual linkage.

This unit is built on
Lever - Science-V
Machines - Science-VI
Kinematics - Physics-IX
Trigonometry - Maths-IX

This unit leads to:
Rotational Motion, Vectors
and Equilibrium
- Physics-XI

- solve problems on simple balanced systems when bodies are supported by one pivot only.
- describe the states of equilibrium and classify them with common examples.
- explain effect of the position of the centre of mass on the stability of simple objects.

INVESTIGATION SKILLS

The students will be able to:

- determine the position of centre of mass/gravity of regularly and irregularly shaped objects.

SCIENCE, TECHNOLOGY AND SOCIETY CONNECTION

The students will be able to:

- illustrate by describing a practical application of moment of force in the working of bottle opener, spanner, door/windows handles, etc.
- describe the working principle of see-saw.
- demonstrate the role of couple in the steering wheels and bicycle pedals.
- demonstrate through a balancing toy, racing car, etc. that the stability of an object can be improved by lowering the centre of mass and increasing the base area of the objects.

Can the nut of the axle of a bike be loosened with hand? Normally we use a spanner as shown in figure 4.1. A spanner increases the turning effect of the force.

Look at the picture on the previous page. What is the joker doing? He is trying to balance himself on a wooden plank which is placed over a cylindrical pipe. Can we do the same? A baby gradually learns to stand by balancing herself. Women and children in the villages often carry pitchers with water on their heads such as shown in figure 4.2. With a little effort we can learn to balance a stick vertically up on our finger tip. Balanced objects are said to be in equilibrium. In this unit, we will learn many interesting concepts such as torque, equilibrium, etc. and their applications in daily life.

Major Concepts

- 4.1** Forces on bodies
- 4.2** Addition of Forces
- 4.3** Resolution of Forces
- 4.4** Moment of a Force
- 4.5** Principle of moments
- 4.6** Centre of mass
- 4.7** Couple
- 4.8** Equilibrium
- 4.9** Stability



Figure 4.1: We can loose a nut with a spanner.

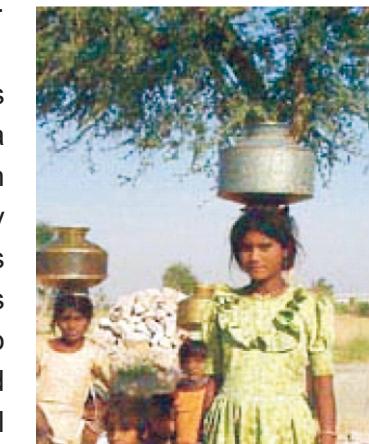
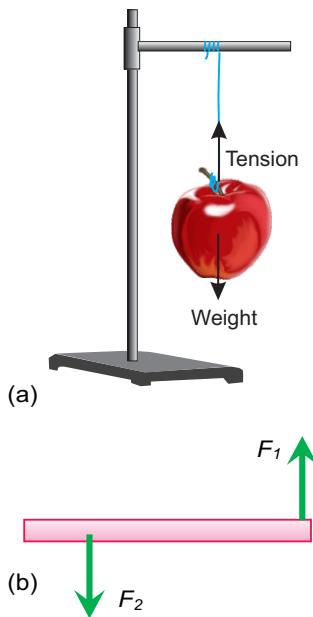


Figure 4.2: Children carrying pitchers on their heads.



Figure 4.3: Like parallel forces.

Figure 4.4: Unlike parallel forces
(a) along the same line
(b) can turn the object if not in line.

4.1 LIKE AND UNLIKE PARALLEL FORCES

We often come across objects on which many forces are acting. In many cases, we find all or some of the forces acting on a body in the same direction. For example, many people push a bus to start it. Why all of them push it in the same direction? All these forces are applied in the same direction so these are all parallel to each other. Such forces which are parallel to each other are called parallel forces.

Figure 4.3 shows a bag with apples in it. The weight of each apple weight of the bag is due to the weight of all the apples in it. Since the weight of every apple in the bag is the force of gravity acting on it vertically downwards, therefore, weights of apples are the parallel forces. All these forces are acting in the same direction. Such forces are called like parallel forces.

Like parallel forces are the forces that are parallel to each other and have the same direction.

In figure 4.4(a), an apple is suspended by a string. The string is stretched due to weight of the apple. The forces acting on it are; weight of the apple acting vertically downwards and tension in the string pulling it vertically upwards. The two forces are parallel but opposite to each other. These forces are called unlike parallel forces. In figure 4.4(b), forces F_1 and F_2 are also unlike parallel forces, because they are parallel and opposite to each other. But F_1 and F_2 are not acting along the same line and hence they are capable to rotate the body.

Unlike parallel forces are the forces that are parallel but have directions opposite to each other.

4.2 ADDITION OF FORCES

Force is a vector quantity. It has both magnitude and direction; therefore, forces are not added by ordinary arithmetical rules. When forces are added, we get a resultant force.

A resultant force is a single force that has the same effect as the combined effect of all the forces to be added.

One of the methods for the addition of forces is a graphical method. In this method forces can be added by **head to tail rule** of vector addition.

HEAD TO TAIL RULE

Figure 4.5 shows a graphical method of vector addition. First select a suitable scale. Then draw the vectors of all the forces according to the scale; such as vectors **A** and **B**.

Take any one of the vectors as first vector e.g., vector **A**. Then draw next vector **B** such that its tail coincides with the head of the first vector **A**. Similarly draw the next vector for the third force (if any) with its tail coinciding with the head of the previous vector and so on.

Now draw a vector **R** such that its tail is at the tail of vector **A**, the first vector, while its head is at the head of vector **B**, the last vector as shown in figure 4.5. Vector **R** represents the resultant force completely in magnitude and direction.

EXAMPLE 4.1

Find the resultant of three forces 12 N along x-axis, 8 N making an angle of 45° with x-axis and 8 N along y-axis.

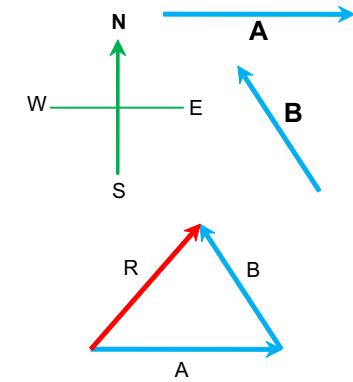


Figure 4.5: Adding vectors by head to tail rule.

It should be noted that head to tail rule can be used to add any number of forces. The vector representing resultant force gives the magnitude and direction of the resultant force.

SOLUTION

Here $F_1 = 12 \text{ N}$ along x-axis

$F_2 = 8 \text{ N}$ along 45° with x-axis

$F_3 = 8 \text{ N}$ along y-axis

Scale: 1 cm = 2 N

- Represent the forces by vectors F_1 , F_2 and F_3 according to the scale in the given direction.
- Arrange these forces F_1 , F_2 and F_3 . The tail of force F_2 coincides with the head of force at point B as shown in figure 4.6. similarly the tail of force F_3 coincides with the head of force F_2 at point C.
- Join point A the tail of the force F_1 , and point D the head of force F_3 . Let AD represents force F . According to head to tail rule, force F represents the resultant force.

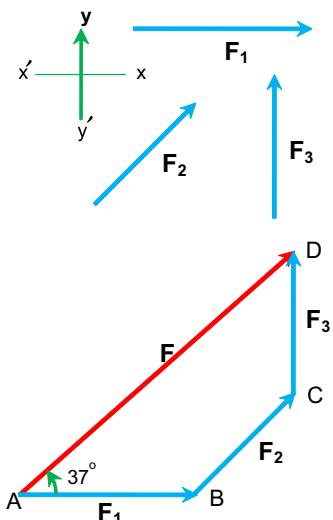
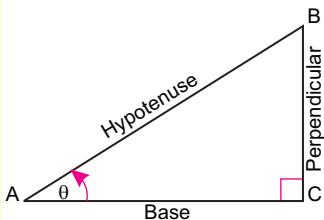


Figure 4.6: Adding forces by Head to tail rule.

Some Trigonometric Ratios

The ratios between any of its two sides of a right angled triangle are given certain names such as sine, cosine, etc. Consider a right angled triangle AABC having angle θ at A.



$$\sin \theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{BC}{AB}$$

$$\cos \theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{AC}{AB}$$

$$\tan \theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{BC}{AC}$$

- (iv) Measure AD and multiply it by 2 N cm^{-1} , the scale to find the magnitude of the resultant force F .
- (v) Measure the angle $\angle DAB$ using a protractor. Which the force F makes with x-axis. This gives the direction of the resultant force.

4.3 RESOLUTION OF FORCES

The process of splitting up vectors (forces) into their component forces is called resolution of forces. If a force is formed from two mutually perpendicular components then such components are called its **perpendicular components**.

Splitting up of a force into two mutually perpendicular components is called the resolution of that force.

Consider a force F represented by line OA making an angle θ with x-axis as shown in figure 4.7.

Draw a perpendicular AB on x-axis from A. According to head to tail rule, OA is the resultant of vectors represented by OB and BA .

$$\text{Thus } OA = OB + BA \dots \dots \dots \quad (4.1)$$

The components OB and BA are perpendicular to each other. They are called the perpendicular components of OA representing force F . Hence OB represents its x-component F_x and BA represents its y-component F_y . Therefor, equation 4.1 can be written as

$$F = F_x + F_y \dots \dots \dots \quad (4.2)$$

The magnitudes F_x and F_y of forces F_x and F_y can be found using the trigonometric ratios. In right angled triangle OBA

$$\text{Since } \frac{F_x}{F} = \frac{OB}{OA} = \cos \theta$$

$$F_x = F \cos \theta \dots \dots \dots \quad (4.3)$$

$$\text{Similarly } \frac{F_y}{F} = \frac{BA}{OA} = \sin \theta$$

$$\therefore F_y = F \sin \theta \dots \dots \dots \quad (4.4)$$

Equations 4.3 and 4.4 give the perpendicular components F_x and F_y respectively.

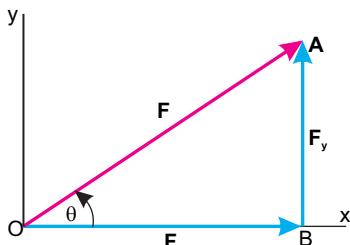


Figure 4.7: Resolution of a force

Ratio/ θ	0°	30°	45°	60°	90°
$\sin \theta$	0	0.5	0.707	0.866	-
$\cos \theta$	1	0.866	0.707	0.5	0
$\tan \theta$	0	0.577	1	1.732	8

EXAMPLE 4.2

A man is pulling a trolley on a horizontal road with a force of 200 N making 30° with the road. Find the horizontal and vertical components of its force.

SOLUTION

$$F = 200 \text{ N}$$

$\theta = 30^\circ$ with the horizontal

$$F_x = ?$$

$$F_v = ?$$

$$\text{Since } F_y = F \cos \theta$$

$$\text{or } F_x \equiv 200 \times \cos 30^\circ$$

$$= 200 \times 0.866 = 173.2 \text{ N}$$

Similarly $F_v = F \sin \theta$

$$\text{or } F_y = 200 \times \sin 30^\circ$$

$$= 200 \times 0.5 = 100 \text{ N}$$

Thus, horizontal and vertical components of the pulling force are 173.2 N and 100 N respectively.

DETERMINATION OF A FORCE FROM ITS PERPENDICULAR COMPONENTS

Since a force can be resolved into two perpendicular components. Its reverse is to determine the force knowing its perpendicular components.

Consider \mathbf{F}_x and \mathbf{F}_y as the perpendicular components of a force \mathbf{F} . These perpendicular components \mathbf{F}_x and \mathbf{F}_y , are represented by lines **OP** and **PR** respectively as shown in figure 4.8.

According to head to tail rule:

$$OR = OP + PR$$

Thus **OR** will completely represent the force **F**, whose x and y-components are F_x and F_y , respectively. That is

$$\mathbf{F} = \mathbf{F}_x + \mathbf{F}_y$$

The magnitude of the force \mathbf{F} can be determined using the right angled triangle OPR

$$(OR)^2 = (OP)^2 + (PR)^2$$

$$F^2 = F_x^2 + F_y^2$$

$$\text{Hence } F = \sqrt{F_x^2 + F_y^2} \dots \dots \dots \quad (4.5)$$

The direction of force F with x-axis is given by

In a right angled triangle
length of base is 4 cm and
its perpendicular is 3 cm.
Find:

- (i) length of hypotenuse
 - (ii) $\sin\theta$
 - (iii) $\cos\theta$
 - (iv) $\tan\theta$

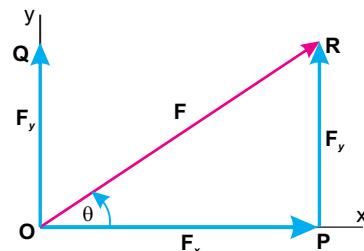


Figure 4.8: Determination of a force by its perpendicular components.

$$\tan \theta = \frac{PR}{OP} = \frac{F_y}{F_x}$$

$$\therefore \theta = \tan^{-1} \frac{F_y}{F_x} \dots \dots \dots \dots \quad (4.6)$$



Figure 4.9: It is easy to open and close the door by pulling or pushing it at its handle.



Figure 4.10: Turning effect of forces.

4.4 TORQUE OR MOMENT OF A FORCE

We open or close a door (Fig 4.9) by pushing or pulling it. Here push or pull turn the door about its hinge or axis of rotation. The door is opened or closed due to the turning effect of the force acting on it.

RIGID BODY

A body is composed of large number of small particles. If the distances between all pairs of particles of the body do not change by applying a force then it is called a rigid body. In other words, a rigid body is the one that is not deformed by force or forces acting on it.

AXIS OF ROTATION

Consider a rigid body rotating about a line. The particles of the body move in circles with their centres all lying on this line. This line is called the axis of rotation of the body.

Forces that produce turning effect are very common. Turning pencil in a sharpener, turning stopcock of a water tap, turning doorknob and so on are some of the examples where a force produces turning effect.

QUICK QUIZ

Name some more objects that work by the turning effects of forces.

The turning effect of a force is called torque or moment of the force.

Why the handle of a door is fixed near the outer edge of a door? We can open or close a door more easily by applying a force at the outer edge of a door rather than near the hinge. Thus, the location where the force is applied to turn a body is very important.

Let us study the factors on which torque or moment of a force depends. You might have seen that a mechanic uses a spanner as shown in figure 4.11 to loosen or tighten a nut or a bolt. A spanner having long arm helps him to do it with greater ease than the one

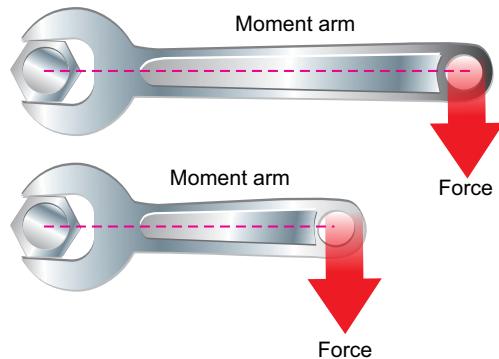


Figure 4.11: It is easy to tighten a nut using a spanner of longer arm than a spanner of shorter arm.

having short arm. It is because the turning effect of the force is different in the two cases. The moment produced by a force using a spanner of longer arm is greater than the torque produced by the same force but using a spanner of shorter arm.

LINE OF ACTION OF A FORCE

The line along which a force acts is called the line of action of the force. In figure 4.12, line BC is the line of action of force F .

MOMENT ARM

The perpendicular distance between the axis of rotation and the line of action of the force is called the moment arm of the force. It is represented by the distance L in figure 4.12.

The torque or moment of a force depends upon the force F and the moment arm L of the force. Greater is a force, greater is the moment of the force. Similarly, longer is the moment arm greater is the moment of the force. Thus the moment of the force or torque τ is determined by the product of force F and its moment arm L . Mathematically,

$$\text{Torque } \tau = F \times L \quad \dots \dots \dots \quad (4.7)$$

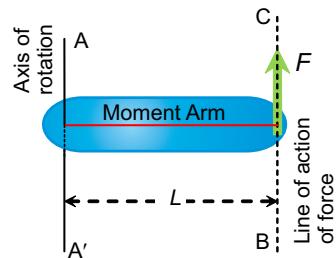


Figure 4.12: Factors affecting the moment of a force.

Mini Exercise

A force of 150 N can loosen a nut when applied at the end of a spanner 10 cm long.

- What should be the length of the spanner to loosen the same nut with a 60 N force?
- How much force would be sufficient to loosen it with a 6 cm long spanner?

SI unit of torque is newton-metre (Nm). A torque of 1 N m is caused by a force of 1 N acting perpendicular to the moment arm 1 m long.

EXAMPLE 4.3

A mechanic tightens the nut of a bicycle using a 15 cm long spanner by exerting a force of 200 N. Find the torque that has tightened it.

SOLUTION

$$\begin{aligned} F &= 200 \text{ N} \\ L &= 15 \text{ cm} = 0.15 \text{ m} \\ \text{Using} \quad \tau &= F L \\ &= 200 \text{ N} \times 0.15 \text{ m} \\ &= 30 \text{ N m} \end{aligned}$$

Thus, a torque of 30 N m is used to tighten the nut.

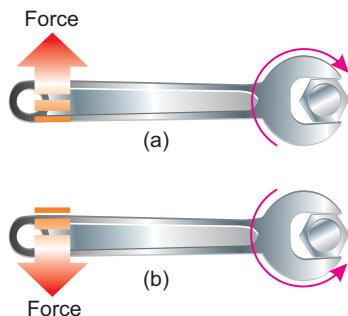


Figure 4.13: (a) to tighten, nut is turned clockwise (b) to loosen, nut is turned anticlockwise.

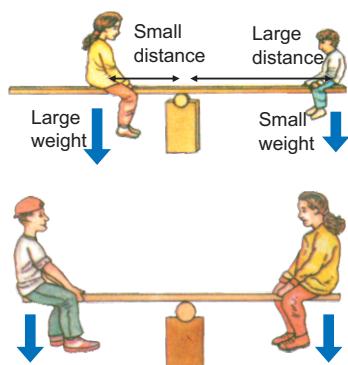


Figure 4.14: Children on see-saw.

4.5 PRINCIPLE OF MOMENTS

A force that turns a spanner in the clockwise direction is generally used to tighten a nut as shown in figure 4.13(a). The torque or moment of the force so produced is called clockwise moment. On the other hand, to loosen a nut, the force is applied such that it turns the nut in the anticlockwise direction as shown in figure 4.13(b). The torque or moment of the force so produced is called anticlockwise moment.

A body initially at rest does not rotate if sum of all the clockwise moments acting on it is balanced by the sum of all the anticlockwise moments acting on it. This is known as the principle of moments. According to the principle of moments:

A body is balanced if the sum of clockwise moments acting on the body is equal to the sum of anticlockwise moments acting on it.

QUICK QUIZ

- Can a small child play with a fat child on the seesaw? Explain how?
- Two children are sitting on the see-saw, such that they can not swing. What is the net torque in this situation?

EXAMPLE 4.4

A metre rod is supported at its middle point O as shown in figure 4.15. The block of weight 10 N is suspended at point B, 40 cm from O. Find the weight of the block that balances it at point A, 25 cm from O.

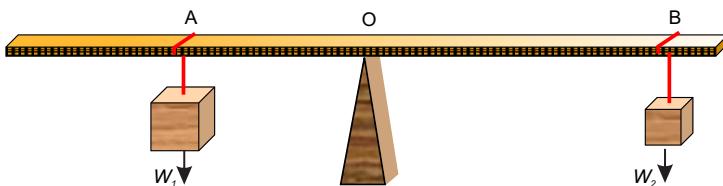


Figure 4.15: Balancing a metre rod on a wedge

SOLUTION

$$w_1 = ?$$

$$w_2 = 10 \text{ N}$$

$$\text{Moment arm of } w_1 = OA = 25 \text{ cm} = 0.25 \text{ m}$$

$$\text{Moment arm of } w_2 = OB = 40 \text{ cm} = 0.40 \text{ m}$$

Applying principle of moments;

$$\text{Clockwise moments} = \text{Anticlockwise moments}$$

$$\text{moment of } w_2 = \text{moment of } w_1$$

$$\text{or } w_1 \times \text{moment arm of } w_2 = w_1 \times \text{moment arm of } w_1$$

$$\text{Thus } w_1 \times OA = w_2 \times OB$$

$$\text{or } w_1 \times 0.25 \text{ m} = 10 \text{ N} \times 0.40 \text{ m}$$

$$w_1 = \frac{10 \text{ N} \times 0.40 \text{ m}}{0.25 \text{ m}}$$

$$= 16 \text{ N}$$

Thus, weight of the block suspended at point A is 16 N.

4.6 CENTRE OF MASS

It is observed that the centre of mass of a system moves as if its entire mass is confined at that point. A force applied at such a point in the body does not produce any torque in it i.e. the body moves in the direction of net force F without rotation.

Consider a system of two particles A and B connected by a light rigid rod as shown in figure 4.16.

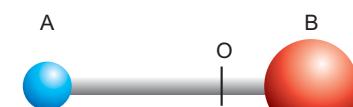


Figure 4.16: Centre of mass of two unequal masses.

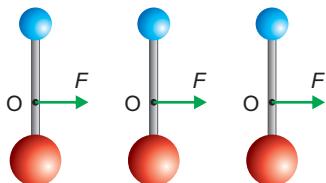


Figure 4.17: A force applied at COM moves the system without rotation.

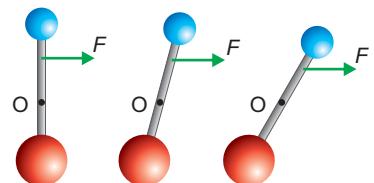


Figure 4.18: The system moves as well as rotates when a force is applied away from COM.

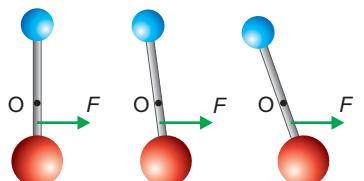


Figure 4.19: The system moves as well as rotates when a force is applied away from COM.

Let O is a point anywhere between A and B such that the force F is applied at point O as shown in figure 4.17. If the system moves in the direction of force F without rotation, then point O is the centre of mass of the system.

Does the system still move without rotation if the force acts elsewhere on it?

- (i) Let the force be applied near the lighter particle as shown in figure 4.18. The system moves as well as rotates.
- (ii) Let the force be applied near the heavier particle as shown in figure 4.19. In this case, also the system moves as well as rotates.

Centre of mass of a system is such a point where an applied force causes the system to move without rotation.

4.6 CENTRE OF GRAVITY

A body is made up of a large number of particles as illustrated in figure 4.20. Earth attracts each of these particles vertically downward towards its centre. The pull of the Earth acting on a particle is equal to its weight. These forces acting on the particles of a body are almost parallel. The resultant of all these parallel forces is a single force equal to the weight of the body. A point where this resultant force acts vertically towards the centre of the Earth is called the centre of gravity G of the body.

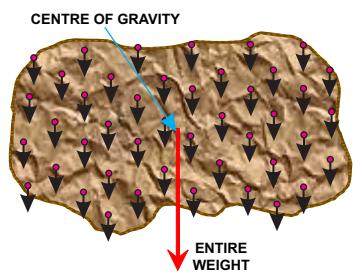


Figure 4.20: Centre of gravity of a body is a point where its entire weight is assumed to act vertically downward.

A point where the whole weight of the body appears to act vertically downward is called centre of gravity of a body.

It is useful to know the location of the centre of gravity of a body in problems dealing with equilibrium.

CENTRE OF GRAVITY OF SOME SYMMETRICAL OBJECTS

The centre of gravity of objects which have symmetrical shapes can be found from their geometry. For example, the centre of gravity of a uniform rod lies at a point where it is balanced. This balance point is its middle point G as shown in figure 4.21.

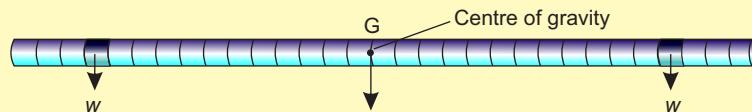


Figure 4.21: Centre of gravity is at the middle of a uniform rod.

The centre of gravity of a uniform square or a rectangular sheet is the point of intersection of its diagonals as shown in figure 4.22 (a) and (c). The centre of gravity of a uniform circular disc is its centre as shown in figure 4.22(b). Similarly, the centre of gravity of a solid sphere or hollow sphere is the centre of the spheres as shown in figure 4.22(b).

The centre of gravity of a uniform triangular sheet is the point of intersection of its medians as shown in figure 4.22 (d). The centre of gravity of a uniform circular ring is the centre of the ring as shown in figure 4.22(e). The centre of gravity of a uniform solid or hollow cylinder is the middle point on its axis as shown in figure 4.22(f).

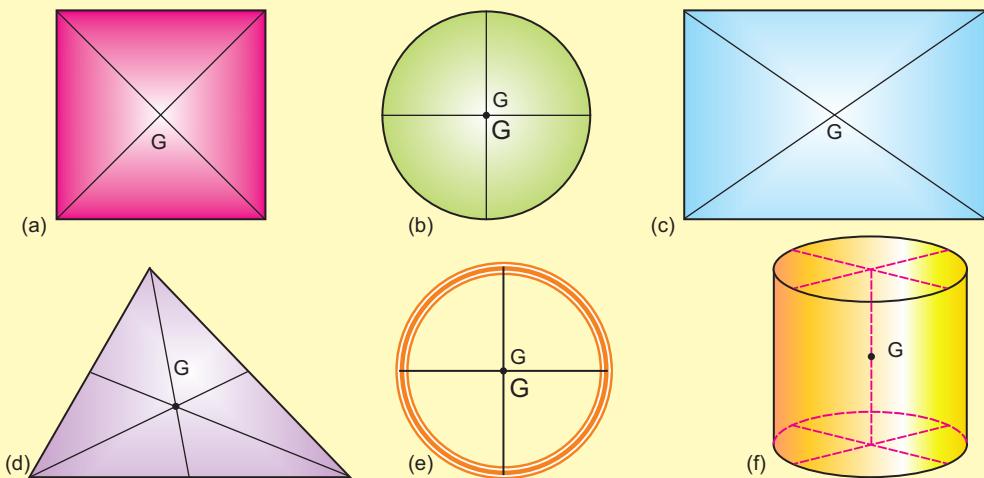


Figure 4.22: Centre of gravity of some symmetrical objects.

CENTRE OF GRAVITY OF AN IRREGULAR SHAPED THIN LAMINA

A simple method to find the centre of gravity of a body is by the use of a plumbline. A plumbline consists of a small metal bob (lead or brass) supported by a string. When the bob is suspended freely by the string, it rests along the vertical direction due to its weight acting vertically downward as shown in figure 4.23 (a). In this state, centre of gravity of the bob is exactly below its point of suspension.

EXPERIMENT

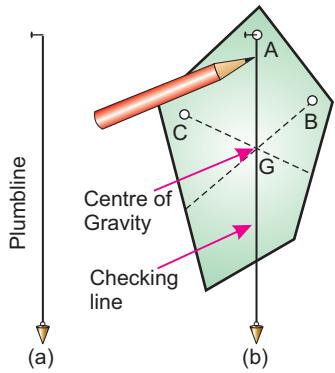


Figure 4.23: (a) Plumblne (b) Locating the centre of gravity of a piece of cardboard by using plumblne.

Take an irregular piece of cardboard. Make holes A, B and C as shown in figure 4.23(b) near its edge. Fix a nail on a wall. Support the cardboard on the nail through one of the holes (let it be A), so that the cardboard can swing freely about A. The cardboard will come to rest with its centre of gravity just vertically below the nail. Vertical line from A can be located using a plumblne hung from the nail. Mark the line on the cardboard behind the plumblne. Repeat it by supporting the cardboard from hole B. The line from B will intersect at a point G. Similarly, draw another line from the hole C. Note that this line also passes through G. It will be found that all the vertical lines from holes A, B and C have a common point G. This Common point G is the **centre of gravity** of the cardboard.

4.7 COUPLE

When a driver turns a vehicle, he applies forces that produce a torque. This torque turns the steering wheel. These forces act on opposite sides of the steering wheel as shown in figure 4.24 and are equal in magnitude but opposite in direction. These two forces form a couple.



Figure 4.24: It is easy to turn a steering wheel by applying a couple.

A couple is formed by two unlike parallel forces of the same magnitude but not along the same line.



Figure 4.25: A double arm spanner.

A double arm spanner is used to open a nut. Equal forces each of magnitude F are applied on ends A and B of a spanner in opposite direction as shown in figure 4.25. These forces form a couple that turns the spanner about point O. The torques produced by both the forces of a couple have the same direction. Thus, the total torque produced by the couple will be

$$\text{Total torque of the couple} = F \times OA + F \times OB$$

$$= F(OA + OB)$$

$$\text{Torque of the couple} = F \times AB \dots \dots (4.8)$$

\therefore Equation 4.8 gives the torque produced by a couple of forces F and F separated by distance AB . The torque of a couple is given by the product of one of

the two forces and the perpendicular distance between them.

4.8 EQUILIBRIUM

Newton's first law of motion tells us that a body continues its state of rest or of uniform motion in a straight line if no resultant or net force acts on it. For example, a book lying on a table or a picture hanging on a wall, are at rest. The weight of the book acting downward is balanced by the upward reaction of the table. Consider a log of wood of weight w supported by ropes as shown in figure 4.26. Here the weight w is balanced by the forces F_1 and F_2 pulling the log upward. In case of objects moving with uniform velocity, the resultant force acting on them is zero. A car moving with uniform velocity on a levelled road and an aeroplane flying in the air with uniform velocity are the examples of bodies in equilibrium.

A body is said to be in equilibrium if no net force acts on it.

A body in equilibrium thus remains at rest or moves with uniform velocity.

CONDITIONS FOR EQUILIBRIUM

In the above examples, we see that a body at rest or in uniform motion is in equilibrium if the resultant force acting on it is zero. For a body in equilibrium, it must satisfy certain conditions. There are two conditions for a body to be in equilibrium.

FIRST CONDITION FOR EQUILIBRIUM

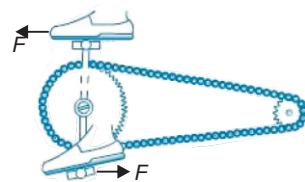
A body is said to satisfy first condition for equilibrium if the resultant of all the forces acting on it is zero. Let n number of forces $\mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_3, \dots, \mathbf{F}_n$ are acting on a body such that

$$\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \dots + \mathbf{F}_n = 0$$

$$\text{or } \sum \mathbf{F} = 0 \quad \dots \quad \dots \quad \dots \quad (4.9)$$

The symbol Σ is a Greek letter called sigma used for summation. Equation 4.9 is called the first condition for equilibrium.

DO YOU KNOW?



A cyclist pushes the pedals of a bicycle. This forms a couple that acts on the pedals. The pedals cause the toothed wheel to turn making the rear wheel of the bicycle to rotate.

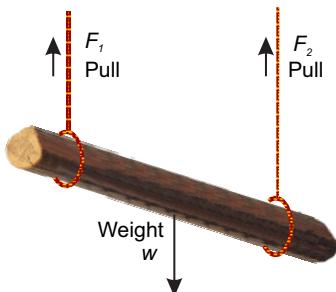


Figure 4.26: The forces acting on the log are; upward forces F_1, F_2 and its weight w in the downward direction.



Figure 4.27: A wall hanging is in equilibrium



Figure 4.28: A paratrooper coming down with terminal velocity is in equilibrium.

The first condition for equilibrium can also be stated in terms of x and y -components of the forces acting on the body as:

$$F_{1x} + F_{2x} + F_{3x} + \dots + F_{nx} = 0$$

$$\text{and } F_{1y} + F_{2y} + F_{3y} + \dots + F_{ny} = 0$$

$$\text{or } \sum F_x = 0 \dots \dots \dots \quad (4.10)$$

$$\text{and } \sum F_y = 0 \dots \dots \dots \quad (4.11)$$

A book lying on a table or a picture hanging on a wall, are at rest and thus satisfy first condition for equilibrium. A paratrooper coming down with terminal velocity (constant velocity) also satisfies first condition for equilibrium and is thus in equilibrium.

EXAMPLE 4.5

A block of weight 10 N is hanging through a cord as shown in figure 4.29. Find the tension in the cord.

SOLUTION

Weight of the block $w = 10 \text{ N}$

Tension in the cord $T = ?$

Applying first condition for equilibrium

$$\sum F_x = 0$$

There is no force acting along x -axis.

$$\sum F_y = 0$$

$$T - w = 0$$

$$T = w$$

$$T = 10 \text{ N}$$

Thus, the tension in the cord is 10 N.

SECOND CONDITION FOR EQUILIBRIUM

First condition for equilibrium does not ensure that a body is in equilibrium. This is clear from the following example. Consider a body pulled by the forces F_1 and F_2 as shown in figure 4.30(a). The two forces are equal but opposite to each other. Both are acting along the same line, hence their resultant will be zero. According to the first condition, the body will be in

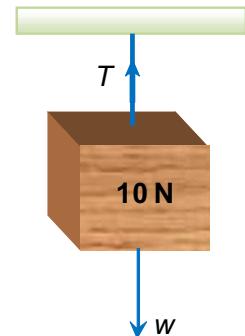


Figure 4.29

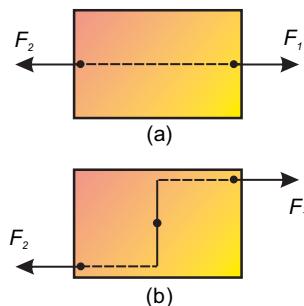


Figure 4.30: (a) Two equal and opposite forces acting along the same lines (b) Two equal and opposite forces acting along different lines

equilibrium. Now shift the location of the forces as shown in figure 4.30(b). In this situation, the body is not in equilibrium although the first condition for equilibrium is still satisfied. It is because the body has the tendency to rotate. This situation demands another condition for equilibrium in addition to the first condition for equilibrium. This is called **second condition for equilibrium**. According to this, a body satisfies second condition for equilibrium when the resultant torque acting on it is zero. Mathematically,

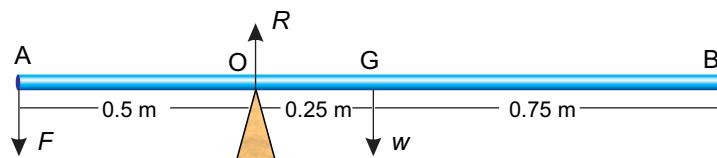
$$\sum \tau = 0 \dots \dots \dots \dots \quad (4.12)$$

QUICK QUIZ

1. A ladder leaning at a wall as shown in figure 4.31 is in equilibrium. How?
2. The weight of the ladder in figure 4.31 produces an anticlockwise torque. The wall pushes the ladder at its top end thus produces a clockwise torque. Does the ladder satisfy second condition for equilibrium?
3. Does the speed of a ceiling fan go on increasing all the time?
4. Does the fan satisfy second condition for equilibrium when rotating with uniform speed?

EXAMPLE 4.6

A uniform rod of length 1.5 m is placed over a wedge at 0.5 m from its one end. A force of 100 N is applied at one of its ends near the wedge to keep it horizontal. Find the weight of the rod and the reaction of the wedge.



SOLUTION

A rod balanced over a wedge

$$\begin{aligned}
 F &= 100 \text{ N} \\
 OA &= 0.5 \text{ m} \\
 AG = BG &= 0.75 \text{ m} \\
 OG = AG - AO &= 0.75 \text{ m} - 0.5 \text{ m} \\
 &= 0.25 \text{ m}
 \end{aligned}$$



Figure 4.31: A ladder leaning at a wall.



Figure 4.32: A ceiling fan rotating at constant speed is in equilibrium as net torque acting on it is zero.

$$W = ?$$

$$R = ?$$

Applying second condition for equilibrium,
taking torques about O.

$$\sum \tau = 0$$

$$F \times AO + R \times 0 - w \times OG = 0$$

$$100 \text{ N} \times 0.5 \text{ m} - w \times 0.25 \text{ m} = 0$$

$$\text{or } w \times 0.25 \text{ m} = 100 \text{ N} \times 0.5 \text{ m}$$

$$w = \frac{100 \text{ N} \times 0.5 \text{ m}}{0.25 \text{ m}}$$

$$w = 200 \text{ N}$$

Applying first condition for equilibrium

$$\sum F_y = 0$$

$$R - F - w = 0$$

$$R - 100 \text{ N} - 200 \text{ N} = 0$$

$$\text{or } R = 300 \text{ N}$$

Thus, weight of the rod is 200 N and reaction of the wedge is 300 N.

STATES OF EQUILIBRIUM

There are three states of equilibrium; stable equilibrium, unstable equilibrium and neutral equilibrium. A body may be in one of these three states of equilibrium.

STABLE EQUILIBRIUM

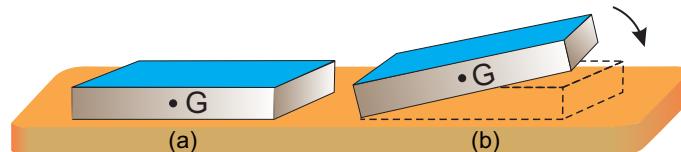


Figure 4.33: Stable equilibrium (a) A book is lying on a table (b) The book returns to its previous position when let free after a slight tilt.

Consider a book lying on the table. Tilt the book slightly about its one edge by lifting it from the opposite side as shown in figure 4.33. It returns to its previous

position when sets free. Such a state of the body is called **stable equilibrium**. Thus

A body is said to be in stable equilibrium if after a slight tilt it returns to its previous position.

When a body is in stable equilibrium, its centre of gravity is at the lowest position. When it is tilted, its centre of gravity rises. It returns to its stable state by lowering its centre of gravity. A body remains in stable equilibrium as long as the centre of gravity acts through the base of the body.

Consider a block as shown in figure 4.34. When the block is tilted, its centre of gravity G rises. If the vertical line through G passes through its base in the tilted position as shown in figure 4.34 (b), the block returns to its previous position. If the vertical line through G gets out of its base as shown in figure 4.34(c), the block does not return to its previous position. It topples over its base and moves to new

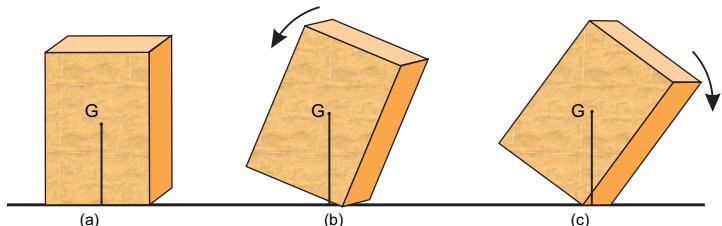


Figure 4.34 (a) Block in stable equilibrium (b) Slightly tilted block is returning to its previous position, (c) A more tilted block topples over its base and does not return to its previous position.

stable equilibrium position. That is why a vehicle is made heavy at its bottom to keep its centre of gravity as low as possible. A lower centre of gravity keeps it stable. Moreover, the base of a vehicle is made wide so that the vertical line passing through its centre of gravity should not get out of its base during a turn.

UNSTABLE EQUILIBRIUM

Take a pencil and try to keep it in the vertical position on its tip as shown in figure 4.36. Whenever you leave it, the pencil topples over about its tip and falls down. This is called the **unstable equilibrium**. In unstable equilibrium, a body may be made to stay only



Can you do this without falling?



Figure 4.35: A double decker bus being under test for stability.

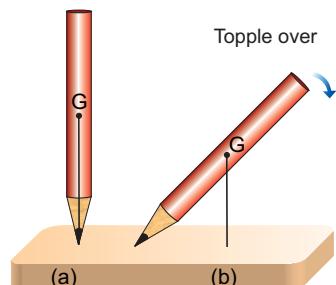


Figure 4.36: Unstable equilibrium
(a) pencil just balanced at its tip with centre of gravity G at the highest position, (b) Pencil topples over caused by the torque of its weight acting at G.

DO YOU KNOW?

Vehicles are made heavy at the bottom. This lowers their centre of gravity and helps to increase their stability.

If a body does not return to its previous position when set free after a slightest tilt is said to be in unstable equilibrium.

The centre of gravity of the body is at its highest position in the state of unstable equilibrium. As the body topples over about its base (tip), its centre of gravity moves towards its lower position and does not return to its previous position.

NEUTRAL EQUILIBRIUM

Figure 4.37: Neutral equilibrium
(a) a ball is placed on a horizontal surface (b) the ball remains in its new displaced position.

If a body remains in its new position when disturbed from its previous position, it is said to be in a state of neutral equilibrium.

In neutral equilibrium, all the new states in which a body is moved, are the stable states and the body, remains in its new state. In neutral equilibrium, the centre of gravity of the body remains at the same height, irrespective to its new position. There are various objects which have neutral equilibrium such as a ball, a sphere, a roller, a pencil lying horizontally, an egg lying horizontally on a flat surface etc.

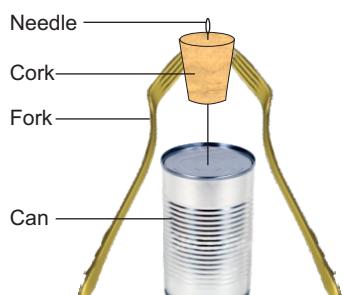
4.9 STABILITY AND POSITION OF CENTRE OF MASS

Figure 4.38: A needle is made to balance at its tip.

As we have learnt that position of centre of mass of an object plays an important role in their stability. To make them stable, their centre of mass must be kept as low as possible. It is due to this reason, racing cars are made heavy at the bottom and their height is kept to be minimum. Circus artists such as tight rope walkers use long poles to lower their centre of mass. In this way they are prevented from toppling over. Here are few examples in which lowering of centre of mass make the objects stable. These

objects return to their stable states when disturbed. In each case centre of mass is vertically below their point of support. This makes their equilibrium stable.

Figure 4.38 shows a sewing needle fixed in a cork. The cork is balanced on the tip of the needle by hanging forks. The forks lower the centre of mass of the system. Figure 4.39(a) shows a perched parrot which is made heavy at its tail. Figure 4.39(b) shows a toy that keeps itself upright when tilted. It has a heavy semi-spherical base. When it is tilted, its centre of mass rises. It returns to its upright position at which its centre of mass is at the lowest.

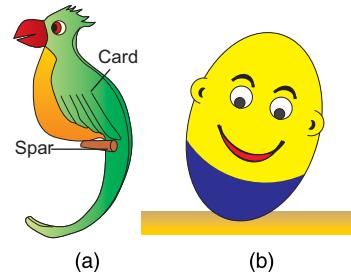


Figure 4.39: (a) A perched parrot
(b) A self-righting toy

SUMMARY

- Parallel forces have their lines of action parallel to each other.
- If the direction of parallel forces is the same, they are called like parallel forces. If two parallel forces are in opposite direction to each other, then they are called unlike parallel forces.
- The sum of two or more forces is called the resultant force.
- A graphical method used to find the resultant of two or more forces is called head to tail rule.
- Splitting up a force into two components perpendicular to each other is called resolution of that force. These components are

$$F_x = F \cos \theta, \quad F_y = F \sin \theta$$
- A force can be determined from its perpendicular components as

$$F = \sqrt{F_x^2 + F_y^2}, \quad \theta = \tan^{-1} \frac{F_y}{F_x}$$
- Torque or moment of a force is the turning effect of the force. Torque of a force is equal to the product of force and moment arm of the force.
- According to the principle of moments, the sum of clockwise moments acting on a body in equilibrium is equal to the sum of anticlockwise moments acting on it.
- Centre of mass of a body is such a point where a net force causes it to move without rotation.
- The centre of gravity of a body is a point where the whole weight of a body acts vertically downward.
- A couple is formed by two parallel forces of the same magnitude but acting in opposite direction along different lines of action.
- A body is in equilibrium if net force acting on it is zero. A body in equilibrium either remains at rest or moves with a uniform velocity.
- A body is said to satisfy second condition for equilibrium if the resultant torque acting on it is zero.
- A body is said to be in stable equilibrium if after a slight tilt it returns to its previous position.
- If a body does not return to its previous position when set free after slightly tilt is said to be in unstable equilibrium.

- A body that remains in its new position when disturbed from its previous position is said to be in a state of neutral equilibrium.

QUESTIONS

- 4.1** Encircle the correct answers from the given choices:
- Two equal but unlike parallel forces having different line of action produce
 - a torque
 - a couple
 - equilibrium
 - neutral equilibrium
 - The number of forces that can be added by head to tail rule are:
 - 2
 - 3
 - 4
 - any number
 - The number of perpendicular components of a force are:
 - 1
 - 2
 - 3
 - 4
 - A force of 10 N is making an angle of 30° with the horizontal. Its horizontal component will be :
 - 4 N
 - 5 N
 - 7 N
 - 8.7 N
 - A couple is formed by
 - two forces perpendicular to each other
 - two like parallel forces
 - two equal and opposite forces in the same line
 - two equal and opposite forces not in the same line
 - A body is in equilibrium when its:
 - acceleration is uniform
 - speed is uniform
 - speed and acceleration are uniform
 - vii. A body is in neutral equilibrium when its centre of gravity:
 - is at its highest position
 - is at the lowest position
 - keeps its height if displaced
 - is situated at its bottom
 - viii. Racing cars are made stable by:
 - increasing their speed
 - decreasing their mass
 - lowering their centre of gravity
 - decreasing their width
- 4.2** Define the following:
- resultant vector
 - torque
 - centre of mass
 - centre of gravity
- 4.3** Differentiate the following:
- like and unlike forces
 - torque and couple
 - stable and neutral equilibrium
- 4.4** How head to tail rule helps to find the resultant of forces?
- 4.5** How can a force be resolved into its perpendicular components?
- 4.6** When a body is said to be in equilibrium?
- 4.7** Explain the first condition for equilibrium.
- 4.8** Why there is a need of second condition for equilibrium if a body satisfies first condition for equilibrium?

- 4.9 What is second condition for equilibrium?
- 4.10 Give an example of a moving body which is in equilibrium.
- 4.11 Think of a body which is at rest but not in equilibrium.
- 4.12 Why a body cannot be in equilibrium due to single force acting on it?
- 4.13 Why the height of vehicles is kept as low as possible?
- 4.14 Explain what is meant by stable, unstable and neutral equilibrium. Give one example in each case.

PROBLEMS

- 4.1 Find the resultant of the following forces:
 (i) 10 N along x-axis
 (ii) 6 N along y-axis and
 (iii) 4 N along negative x-axis.
 (8.5 N making 45° with x-axis)
- 4.2 Find the perpendicular components of a force of 50 N making an angle of 30° with x axis. (43.3 N, 25 N)
- 4.3 Find the magnitude and direction of a force, if its x-component is 12 N and y-component is 5 N.
 (13 N making 22.6° with x-axis)
- 4.4 A force of 100 N is applied perpendicularly on a spanner at a distance of 10 cm from a nut. Find the torque produced by the force. (10 Nm)
- 4.5 A force is acting on a body making an angle of 30° with the horizontal. The horizontal component of the force is 20 N. Find the force. (23.1 N)
- 4.6 The steering of a car has a radius 16 cm. Find the torque produced by a couple of 50 N.
 (16 Nm)
- 4.7 A picture frame is hanging by two vertical strings. The tensions in the strings are 3.8 N and 4.4 N. Find the weight of the picture frame. (8.2 N)
- 4.8 Two blocks of masses 5 kg and 3 kg are suspended by the two strings as shown. Find the tension in each string. (80 N, 30 N)
- 4.9 A nut has been tightened by a force of 200 N using 10 cm long spanner. What length of a spanner is required to loosen the same nut with 150 N force?
 (13.3 cm)
- 4.10 A block of mass 10 kg is suspended at a distance of 20 cm from the centre of a uniform bar 1 m long. What force is required to balance it at its centre of gravity by applying the force at the other end of the bar? (40 N)

