Converting a CFG into a PDA

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Let G = (Σ, N, *S*, P) be a CFG. Then, define a nondeterministic PDA, M = (Q, Σ, Γ, δ, q_0 , *S*, $\varnothing$), as follows, where acceptance is by empty stack:

- $Q = \{q_0, q_1\}$, where q_0 is the start state,
- Σ is the same for G and M,
- S = the stack start symbol (bottom of stack symbol),
- $\Gamma = \Sigma \cup N$,
- δ: is the transition function over $Q \times \Sigma \times \Gamma \rightarrow Q \times \Gamma^*$, defined as follows, where $x \in \Sigma$, $A \in N$:
  - $\delta(q_0, \varepsilon, \varepsilon) = \{(q_1, S)\}$
  - $\delta(q_1, x, x) = \{(q_1, \varepsilon)\}$ for each $x \in \Sigma$
  - $\delta(q_1, \varepsilon, A) = \{(q_1, \alpha)\}$ for each production $A \rightarrow \alpha$

Example

Design a push-down automaton, M, to recognize the language generated by following context-free grammar:

$$E \rightarrow T \mid E * E \mid E + E \mid (E)$$

$$T \rightarrow a \mid b \mid Ta \mid Tb \mid T0 \mid T1$$

Note

$$E \rightarrow T \mid E * E \mid E + E \mid (E)$$

is the same as:

$$E \rightarrow T$$

$$E \rightarrow E * E$$

$$E \rightarrow E + E$$

$$E \rightarrow (E)$$

Example Solution

E $\rightarrow$ T | E * E | E + E | (E)

T $\rightarrow$ *a* | *b* | T*a* | T*b* | T*0* | T*1*

Define a nondeterministic PDA,

M = ($\{q_0, q_1\}$, Σ, Γ, δ, q_0, E, $\varnothing$), as follows, where acceptance is by *empty stack*:

N = {E, T}

Σ = {*a*, *b*, *0*, *1*, (,), +, *}

Γ = { *a*, *b*, *0*, *1*, (,), +, *, E, T}

Example Solution Continued

E $\rightarrow$ T | E * E | E + E | (E)

T $\rightarrow$ *a* | *b* | T*a* | T*b* | T*0* | T*1*

δ is defined as follows:

N = {E, T}

$\delta(q_0, \varepsilon, \varepsilon) = \{(q_1, E)\}$

$\delta(q_1, \varepsilon, E) = \{(q_1, T), (q_1, E + E), (q_1, E * E), (q_1, (E))\}$

$\delta(q_1, \varepsilon, T) = \{(q_1, a), (q_1, b), (q_1, Ta), (q_1, Tb), (q_1, T0), (q_1, T1)\}$

$\delta(q_1, a, a) = \{(q_1, \varepsilon)\}$

Σ = {*a*, *b*, *0*, *1*, (,), +, *}

$\delta(q_1, b, b) = \{(q_1, \varepsilon)\}$

$\delta(q_1, 0, 0) = \{(q_1, \varepsilon)\}$

$\delta(q_1, 1, 1) = \{(q_1, \varepsilon)\}$

$\delta(q_1, +, +) = \{(q_1, \varepsilon)\}$

$\delta(q_1, *, *) = \{(q_1, \varepsilon)\}$

$\delta(q_1, `(', `(') = \{(q_1, \varepsilon)\}$

$\delta(q_1, `)', `)') = \{(q_1, \varepsilon)\}$