University of Glasgow

Wednesday 28 April 2021
Available from 09:15 BST
Expected Duration: 1 hour 30 minutes
Time Allowed: 3 hours
Timed exam within 24 hours

DEGREES OF MSci, MEng, BEng, BSc, MA and MA (Social Sciences)

ALGORITHMS AND DATA STRUCTURES 2
COMPSCI2007

Answer all 5 questions

This examination paper is an open book, online assessment and is worth a total of 60 marks

1. Algorithm **F** takes as input an array of integers A and two indices p, r of A. It is described by the following pseudocode:

```
1: F(A, p, r)
2:   if p < r
3:     x := A[p]
4:     A[p] := A[r]
5:     A[r] := x
6:     F(A, p + 1, r − 1)
```

(a) Briefly explain what algorithm **F** implements. [5]

(b) What is the output of $\mathbf{F}(A, 0, 6)$, where $A = [1, 5, 7, 9, 3, 4, 1]$? [2]

(c) Draw the recursion trace for $\mathbf{F}(A, 0, 6)$. [3]

(d) Is **F** tail recursive? Justify your answer. [2]

(e) Is **F** an in-place algorithm? Justify your answer. [2]

(f) Write an iterative version of algorithm **F** and compute its complexity. Suppose n is the size of the input array. Use big-Oh notation. [6]

2. For each of the following statements, prove whether it is true or false.

(a) $29 = O(\log_2 n)$ [3]

(b) $\max(n^3, 10n^2) = O(n^2)$ [3]

3. **(a)** Briefly describe using your own words the counting sort algorithm (expected word count: 100). Illustrate your description by sorting the list: 3, 7, 4, 2, 1, 5, 4. [6]

(b) Briefly describe using your own words the radix sort algorithm (expected word count: 50). Illustrate your description by sorting the list: 802, 256, 958, 938, 693, 405, 684, 854. [6]

4. Illustrate the result of each operation in the sequence PUSH(S, 5), PUSH(S, 2), PUSH(S, 4), POP(S), PUSH(S, 7), and POP(S) on an initially empty stack S. Consider the following two cases:

(a) Stack S is implemented by array $S[0..5]$. [3]

(b) Stack S is implemented by a linked list. [3]

5. **(a)** Briefly explain with your own words what a hash table is, its main components and its operations? Expected word count: 100. [3]

(b) Briefly explain with your own words what a *hash collision* is and give an example application for which hash table storage would be suitable. Expected word count: 50. [3]

(c) Briefly describe with your own words the chaining method of hash collision resolution and give one advantage of this method. Expected word count: 70. [2]

(d) Show the resulting hash table if keys 5, 28, 19, 15, 20, 33, 12, 17, 10 are inserted into an initially empty hash table assuming collisions are resolved by chaining. Suppose the table has 9 slots, and let the hash function be $h(k) = k \mod 9$. [3]

(e) Briefly describe with your own words the open addressing method of hash collision resolution. Expected word count: 70. [2]

(f) Show the resulting hash table if keys 50, 700, 76, 85, 92, 73, 101 are inserted into an initially empty hash table assuming collisions are resolved by open addressing with *linear probing*. Suppose the table has 7 slots, and let the hash function be $h(k) = k \mod 7$. [3]

END OF QUESTION PAPER

University of Glasgow

Monday, 29 April 2019
9.30 am – 11.00 am
(1 hour 30 minutes)

DEGREES OF MSci, MEng, BEng, BSc, MA and MA (Social Sciences)

ALGORITHMS AND DATA STRUCTURES 2: COMPSCI2007

Answer all questions

This examination paper is worth a total of 60 marks.

The use of calculators is not permitted in this examination.

INSTRUCTIONS TO INVIGILATORS: Please collect all exam question papers and exam answer scripts and retain for school to collect. Candidates must not remove exam question papers.

1. Consider the algorithm described by the following pseudocode:

```
1:  F(A, x, p, r)
2:    if p > r
3:      return −1
4:    q := (p + r/2)
5:    if A[q] = x
6:      return q
7:    else
8:      if A[q] > x
9:        return F(A, x, p, q − 1)
10:     else
11:       return F(A, x, q + 1, r)
```

The inputs of algorithm **F** are: a sorted array of integers A, an integer x, and two indices for A p, r.

(a) Briefly explain what algorithm **F** implements. [5]

(b) What is the output of $\mathbf{F}(A, 13, 0, 7)$, where $A = [0,4,4,5,8,7,13,14]$? [2]

(c) Draw the recursion trace for $\mathbf{F}(A, 13, 0, 7)$. [3]

(d) Is **F** tail recursive? Justify your answer. [2]

(e) Is **F** linear recursive? Justify your answer. [2]

(f) Write the recurrence equation for the running time $T(n)$ of algorithm **F** (with $n = r - p + 1$). [2]

(g) Solve the recurrence equation in **1(f)** to compute the complexity of algorithm **F** using big-Oh notation. [4]

2. Rank the following functions by order of growth; that is, find an arrangement $f_1, f_2, \ldots, f_8$ of the functions satisfying: $f_{i-1} = O(f_i)$.

$$\log\log n \qquad 1 \qquad n^3 \qquad 2^n \qquad n \qquad \sqrt{n} \qquad n\log n \qquad \log n$$

[5]

3. **(a)** Describe using pseudocode or otherwise the merge sort algorithm for sorting a list of integers contained in an array. Illustrate your description by sorting the list: 6, 3, 1, 7, 4, 5, 2. Note that you do not need to describe the merge procedure in detail. [8]

(b) What is the time complexity of merge sort? [1]

(c) Assume the quicksort algorithm is implemented using a partitioning scheme that selects the middle element (i.e. element $A[q]$ in subarray $A[p..r]$ where $q = \lfloor (p+r)/2 \rfloor$) as the pivot. Show an input of length 8 exhibiting worst case complexity $O(n^2)$. Explain your answer. [4]

4. **(a)** Briefly describe the two primary operations of the queue ADT. [2]

(b) Briefly describe the linked list implementation of the queue ADT. What is the complexity of the primary operations? [4]

5. **(a)** What is a binary search tree? Does the order in which a set of values is inserted into a binary search tree matter? Justify your answer. [6]

(b) Give a recursive definition of the preorder, inorder and postorder traversals of a binary tree. [6]

(c) Define using pseudocode or otherwise the insert operation on binary search trees. [4]

END OF QUESTION PAPER