

# Root finder

NUMERICAL ANALYSIS

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## PSEUDOCODE OF APPLICATION

### BISECTION METHOD

1. Substitute  $x_u$  and  $x_l$  in the given equation  $f(x)$
2. If  $f(x_u) * f(x_l) < 0$  then
  - Bisection Method is not Valid
3. Else
  - Bisection Method is Valid
4.  $x_r = \frac{(x_u + x_l)}{2}$
5. If  $f(x_r) > 0$  then
  - $x_u = x_r$
6. Else
  - $x_l = x_r$
7. Repeat step 1 again until number of iterations reached, or precision is accepted

### FALSE POSITION METHOD

1. Substitute  $x_u$  and  $x_l$  in the given equation  $f(x)$
2. If  $f(x_u) * f(x_l) < 0$  then
  - False Position Method is not Valid
3. Else
  - False Position Method is Valid
4.  $x_r = \frac{x_u * f(x_l) - x_l * f(x_u)}{f(x_u) - f(x_l)}$
5. If  $f(x_r) > 0$  then
  - $x_u = x_r$
6. Else
  - $x_l = x_r$
7. Repeat step 1 again until number of iterations reached, or precision is accepted

#### FIXED POINT METHOD

1. Construct  $g(x)$  from  $f(x)$
2. Get  $g(x_o)'$  from  $g(x)$
3. If  $\text{abs}(g(x_o)') < 1$  then
  - Fixed point position is valid
4. Else
  - Fixed point position is not valid
5.  $x_i = g(x_o)$
6.  $x_o = x_i$
7. Repeat step 5 again until number of iterations reached, or precision is accepted

#### NEWTON-RAPHSON METHOD

1.  $x_i = x_{i-1} - \frac{f(x_{i-1})}{f'(x_{i-1})}$
2. Calculate relative error from previous value
3. If error keeps diverging terminate process else
4.  $x_{i-1} = x_i$
5. Repeat step 1 again until number of iterations reached, or precision is accepted

#### SECANT METHOD

1.  $x_{i+1} = x_i - \frac{x_i - x_{i-1}}{f(x_i) - f(x_{i-1})} \times f(x_i)$
2. Calculate relative error from previous value
3. If error keeps diverging terminate process else
4.  $x_{i-1} = x_i, x_i = x_{i+1}$
5. Repeat step 1 again until number of iterations reached, or precision is accepted

## DATA STRUCTURES USED

- Arrays was used to hold equations received from the files or from the user
- UI Matlab table was used to visualize data of every method
- Arrays was used to hold parameters of each method
- Matlab App designer was used

## ANALYSIS

### EXAMPLE 1

*Equation 1 :  $x^4 - 2x^3 - 4x^2 - 4x + 4$*

### BISECTION METHOD

Output Screen

iteration	Xl	Xu	Xr	F(Xr)	es (%)
8.0000	-0.4297	-0.4219	-0.4258	-0.0122	0.9174
9.0000	-0.4258	-0.4219	-0.4238	0.0016	0.4608
10.0000	-0.4258	-0.4238	-0.4248	-0.0053	0.2299
11.0000	-0.4248	-0.4238	-0.4243	-0.0018	0.1151
12.0000	-0.4243	-0.4238	-0.4241	-0.0001	0.0576
13.0000	-0.4241	-0.4238	-0.4240	0.0008	0.0288
14.0000	-0.4241	-0.4240	-0.4240	0.0004	0.0144
15.0000	-0.4241	-0.4240	-0.4240	0.0001	0.0072
16.0000	-0.4241	-0.4240	-0.4241	0.0000	0.0036
17.0000	-0.4241	-0.4241	-0.4241	-0.0000	0.0018
18.0000	-0.4241	-0.4241	-0.4241	0.0000	0.0009

Current Equation  Precision

Approximate root  Iterations took  Execution Time (s)

*Root =  $-0.4240601543103423$  Precision = 6 iterations = 18 Time = 1.063 s*

## FALSE POSTION METHOD

iteration	xl	fxl	xu	fxu	xr	f(xr)	es (%)
2.0000	-1.0000	-6.1000	-0.2469	1.0567	-0.3581	0.4384	31.0497
3.0000	-1.0000	-6.1000	-0.3581	0.4384	-0.4011	0.1587	10.7295
4.0000	-1.0000	-6.1000	-0.4011	0.1587	-0.4163	0.0543	3.6463
5.0000	-1.0000	-6.1000	-0.4163	0.0543	-0.4215	0.0182	1.2223
6.0000	-1.0000	-6.1000	-0.4215	0.0182	-0.4232	0.0061	0.4074
7.0000	-1.0000	-6.1000	-0.4232	0.0061	-0.4238	0.0020	0.1355
8.0000	-1.0000	-6.1000	-0.4238	0.0020	-0.4240	0.0007	0.0450
9.0000	-1.0000	-6.1000	-0.4240	0.0007	-0.4240	0.0002	0.0150
10.0000	-1.0000	-6.1000	-0.4240	0.0002	-0.4241	0.0001	0.0050
11.0000	-1.0000	-6.1000	-0.4241	0.0001	-0.4241	0.0000	0.0017
12.0000	-1.0000	-6.1000	-0.4241	0.0000	-0.4241	0.0000	0.0005

Current Equation  Precision

Approximate root  Iterations took  Execution Time (s)

*Root = -0.4240601543103423 Precision = 6 iterations = 12 Time = 0.8281 s*

## FIXED POINT METHOD

iteration	Xr	es (%)
1	1.0333	196.7742
2	-3.3385	130.9522
3	-41.7997	92.0131
4	-2.9031e+06	99.9986
5	-7.1030e+25	100
6	-2.5455e+103	100
7	-Inf	100
8	NaN	NaN

Current Equation  Precision

Approximate root  Iterations took  Execution Time (s)

ERROR Divergence: Root is impossible to locate

ERROR

---

## NEWTON-RAPHSON

iteration	Xr	es (%)
1.0000	-0.5414	84.7222
2.0000	-0.4346	24.5499
3.0000	-0.4242	2.4719
4.0000	-0.4241	0.0240
5.0000	-0.4241	0.0000

Current Equation

Precision

Next

Approximate root

Iterations took

Excecution Time (s)

Back

Status : Equations Inserted

*Root = -0.4240601543103423 Precision = 9 iterations = 5 Time = 0.2969 s*

---

## SECANT METHOD

iteration	X(i-1)	F(X(i-1))	X(i)	F(X(i))	X(i+1)	es (%)
1.0000	-3.0000	53.3000	-1.0000	-6.1000	-1.2054	17.0391
2.0000	-1.0000	-6.1000	-1.2054	-8.8722	-0.5481	119.9351
3.0000	-1.2054	-8.8722	-0.5481	-0.9785	-0.4666	17.4630
4.0000	-0.5481	-0.9785	-0.4666	-0.3126	-0.4283	8.9298
5.0000	-0.4666	-0.3126	-0.4283	-0.0303	-0.4242	0.9694
6.0000	-0.4283	-0.0303	-0.4242	-0.0011	-0.4241	0.0380
7.0000	-0.4242	-0.0011	-0.4241	-0.0000	-0.4241	0.0001

Current Equation

Precision

Next

Approximate root

Iterations took

Excecution Time (s)

Back

Status : Equations Inserted

*Root = -0.42406131353142507 Precision = 7 iterations = 7 Time = 0.5625 s*

## EXAMPLE 2

Equation 2:  $x - 0.75 - 0.2 \sin(x)$

### BISECTION METHOD

iteration	Xl	Xu	Xr	F(Xr)	es (%)
1.0000	0	1.5000	0.7500	-0.1363	100.0000
2.0000	0.7500	1.5000	1.1250	0.1945	33.3333
3.0000	0.7500	1.1250	0.9375	0.0263	20.0000
4.0000	0.7500	0.9375	0.8438	-0.0557	11.1111
5.0000	0.8438	0.9375	0.8906	-0.0149	5.2632
6.0000	0.8906	0.9375	0.9141	0.0057	2.5641
7.0000	0.8906	0.9141	0.9023	-0.0046	1.2987
8.0000	0.9023	0.9141	0.9082	0.0005	0.6452
9.0000	0.9023	0.9082	0.9053	-0.0020	0.3236
10.0000	0.9053	0.9082	0.9067	-0.0008	0.1616
11.0000	0.9067	0.9082	0.9075	-0.0001	0.0807

Current Equation  Precision

Approximate root  Iterations took  Execution Time (s)

Status : Equations Inserted

Root = 0.9076023101806641 Precision = 6 iterations = 18 Time = 0.9688

### FALSE POSTION METHOD

iteration	xl	fxl	xu	fxu	xr	f(xr)	es (%)
1.0000	0	-0.7500	1.5000	0.5505	0.8651	-0.0372	100.0000
2.0000	0.8651	-0.0372	1.5000	0.5505	0.9052	-0.0021	4.4370
3.0000	0.9052	-0.0021	1.5000	0.5505	0.9075	-0.0001	0.2486
4.0000	0.9075	-0.0001	1.5000	0.5505	0.9076	-0.0000	0.0140
5.0000	0.9076	-0.0000	1.5000	0.5505	0.9076	-0.0000	0.0008

Current Equation  Precision

Approximate root  Iterations took  Execution Time (s)

Status : Equations Inserted

Root = 0.9076060665401231 Precision = 6 iterations = 5 Time = 0.6875



## FIXED POINT METHOD

iteration	Xr	es (%)
1.0000	0.7500	100.0000
2.0000	0.8863	15.3812
3.0000	0.9050	2.0579
4.0000	0.9073	0.2566
5.0000	0.9076	0.0316
6.0000	0.9076	0.0039
7.0000	0.9076	0.0005

Current Equation  Precision

Approximate root  Iterations took  Excecution Time (s)

Status : Equations Inserted

*Root = 0.9076058837577499 Precsion = 7 iterations = 7 Time = 0.2969*

## NEWTON-RAPHSON

iteration	Xr	es (%)
1.0000	0.9375	100.0000
2.0000	0.9077	3.2844
3.0000	0.9076	0.0089
4.0000	0.9076	0.0000

Current Equation  Precision

Approximate root  Iterations took  Excecution Time (s)

Status : Equations Inserted

*Root = 0.9076064952492678 Precsion = 10 iterations = 4 Time = 0.3909*

## SECANT METHOD

iteration	X(i-1)	F(X(i-1))	X(i)	F(X(i))	X(i+1)	es (%)
1.0000	0	-0.7500	0.5000	-0.3459	0.9280	46.1180
2.0000	0.5000	-0.3459	0.9280	0.0179	0.9069	2.3188
3.0000	0.9280	0.0179	0.9069	-0.0006	0.9076	0.0750
4.0000	0.9069	-0.0006	0.9076	-0.0000	0.9076	0.0001

Current Equation  Precision    
Approximate root  Iterations took  Excecution Time (s)

Status : Equations Inserted

*Root = 0.9076064953259306 Precsion = 7 iterations = 4 Time = 0.375*

### EXAMPLE 3

Equation:  $e^x + 2^{-x} + 2 \cos(x) - 6$

#### BISECTION METHOD

iteration	Xl	Xu	Xr	F(Xr)	es (%)
7.0000	1.8125	1.8438	1.8281	-0.0052	0.8547
8.0000	1.8281	1.8438	1.8359	0.0270	0.4255
9.0000	1.8281	1.8359	1.8320	0.0109	0.2132
10.0000	1.8281	1.8320	1.8301	0.0029	0.1067
11.0000	1.8281	1.8301	1.8291	-0.0012	0.0534
12.0000	1.8291	1.8301	1.8296	0.0008	0.0267
13.0000	1.8291	1.8296	1.8293	-0.0002	0.0133
14.0000	1.8293	1.8296	1.8295	0.0003	0.0067
15.0000	1.8293	1.8295	1.8294	0.0001	0.0033
16.0000	1.8293	1.8294	1.8294	-0.0000	0.0017
17.0000	1.8294	1.8294	1.8294	0.0000	0.0008

Current Equation  Precision

Approximate root  Iterations took  Execution Time (s)

Status : Equations Inserted

Root = 1.8293914794921875 Precision = 6 iterations = 17 Time = 0.9688

#### FALSE POSTION METHOD

iteration	xl	fxl	xu	fxu	xr	f(xr)	es (%)
1.0000	0	-2.0000	2.0000	0.8068	1.4251	-1.1789	100.0000
2.0000	1.4251	-1.1789	2.0000	0.8068	1.7664	-0.2449	19.3217
3.0000	1.7664	-0.2449	2.0000	0.8068	1.8208	-0.0349	2.9870
4.0000	1.8208	-0.0349	2.0000	0.8068	1.8282	-0.0047	0.4059
5.0000	1.8282	-0.0047	2.0000	0.8068	1.8292	-0.0006	0.0540
6.0000	1.8292	-0.0006	2.0000	0.8068	1.8294	-0.0001	0.0072
7.0000	1.8294	-0.0001	2.0000	0.8068	1.8294	-0.0000	0.0010

Current Equation  Precision

Approximate root  Iterations took  Execution Time (s)

Status : Equations Inserted

Root = 1.8293809448663378 Precision = 6 iterations = 7 Time = 0.5938

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## FIXED POINT POSTION

iteration	Xr	es (%)
1.0000	-1.1382	187.8554
2.0000	-2.2754	49.9759
3.0000	-2.8465	20.0650
4.0000	-2.9737	4.2769
5.0000	-2.9856	0.3997
6.0000	-2.9865	0.0272
7.0000	-2.9865	0.0018
8.0000	-2.9865	0.0001

Current Equation

-log2(6-exp(x)-2\*cos(x))

Precsion

7

Next

Approximate root

-2.987

Iterations took

8

Excecution Time (s)

0.375

Back

Status : Equations Inserted

*Root = -2.9865078257279154 Precsion = 7 iterations = 8 Time = 0.375*

## ERROR IN READING OF FIXED POINT

---

## NEWTON-RAPHSON

iteration	Xr	es (%)
1.0000	6.5178	100.0000
2.0000	5.5231	18.0103
3.0000	4.5465	21.4804
4.0000	3.6320	25.1786
5.0000	2.8536	27.2785
6.0000	2.2800	25.1566
7.0000	1.9498	16.9362
8.0000	1.8403	5.9466
9.0000	1.8295	0.5937
10.0000	1.8294	0.0055
11.0000	1.8294	0.0000

Current Equation

exp(x)+2^(-x)+2\*cos(x)-6

Precsion

10

Next

Approximate root

333849

Iterations took

11

Excecution Time (s)

0.7344

Back

Status : Equations Inserted

*Root = 1.829383601933849 Precsion = 10 iterations = 11 Time = 0.7344*

## SECANT METHOD

iteration	X(i-1)	F(X(i-1))	X(i)	F(X(i))	X(i+1)	es (%)
3.0000	6.6915	801.3786	1.0121	-1.6927	1.0240	1.1690
4.0000	1.0121	-1.6927	1.0240	-1.6840	3.3521	69.4508
5.0000	1.0240	-1.6840	3.3521	20.7034	1.1991	179.5371
6.0000	3.3521	20.7034	1.1991	-1.5209	1.3465	10.9419
7.0000	1.1991	-1.5209	1.3465	-1.3180	2.3036	41.5502
8.0000	1.3465	-1.3180	2.3036	2.8752	1.6473	39.8406
9.0000	2.3036	2.8752	1.6473	-0.6406	1.7669	6.7678
10.0000	1.6473	-0.6406	1.7669	-0.2431	1.8401	3.9752
11.0000	1.7669	-0.2431	1.8401	0.0442	1.8288	0.6152
12.0000	1.8401	0.0442	1.8288	-0.0023	1.8294	0.0310
13.0000	1.8288	-0.0023	1.8294	-0.0000	1.8294	0.0003

Current Equation  Precision

Approximate root  Iterations took  Excecution Time (s)

Status : Equations Inserted

*Root = 1.829383604380253 Precsion = 7 iterations = 11 Time = 1.391s*

## USER INTERFACE MANUAL

**Solving Equation using Numerical Methods**

Enter the required equation separated by comma of browse OR select file contating the equation !!

Equation

File directory

Max iterations  Epsilon

Method Selection

Output Screen

Column 1	Column 2	Column 3	Column 4
----------	----------	----------	----------

Method Selected

Current Equation  Precision

Approximate root  Iterations took  Excecution Time (s)

Status : Not Runing

1. Equations can be written in the 'Equation Text Field' or it can be read from text file browsed using browse button. Functions must be separated using ',' with no spaces when writing
2. Max iterations : has max iterations before program will stop
3. Epsilon : Has precision target value
4. Method Selection Dropdown box tochose from (Bisection , False position , Fixed Point, Newton-Raphson , Secant)
5. Go Button to run program
6. Clear button to clear table

7. Next and back buttons are used to navigate through multiple inserted equations that was separated by comma

MATLAB App

### Solving Equation using Numerical Methods

Enter the required equation separated by comma or browse OR select file containing the equation !!

Equation

File directory

Max iterations  Epsilon

Method Selection

Output Screen

**Bisection Method**

XL

XU

iteration	Xl	Xu	Xr	F(Xr)	es (%)
1.0000	-1.0000	0	-0.5000	-0.5750	100.0000
2.0000	-0.5000	0	-0.2500	1.0414	100.0000
3.0000	-0.5000	-0.2500	-0.3750	0.3314	33.3333
4.0000	-0.5000	-0.3750	-0.4375	-0.0962	14.2857
5.0000	-0.4375	-0.3750	-0.4063	0.1239	7.6923
6.0000	-0.4375	-0.4063	-0.4219	0.0154	3.7037
7.0000	-0.4375	-0.4219	-0.4297	-0.0400	1.8182
8.0000	-0.4297	-0.4219	-0.4258	-0.0122	0.9174
9.0000	-0.4258	-0.4219	-0.4238	0.0016	0.4608
10.0000	-0.4258	-0.4238	-0.4248	-0.0053	0.2299
11.0000	-0.4248	-0.4238	-0.4243	-0.0018	0.1151

Current Equation  Precision

Approximate root  Iterations took  Execution Time (s)

Status : Equations Inserted

An example shown here where selected method is bisection, and selected equation is shown in current equation text field, precision, root, iterations, time are shown. Everything was shown