Assignment No 3

Name : Ahmad Irfan

Section : BEE-2C

Roll No : 23L-6052

Course : Differential Equation

Ch No 7 Laplace
EX-7.1 TRANSFORM

Q No 13

$$f(t) = te^{4t}$$

$$L\{f(t)\} = \int_0^\infty (te^{4t}) e^{-st} dt$$

$$= \int_0^\infty te^{-t(s-4)} dt$$

By using Integration by parts

$$= \frac{t}{-(s-4)} e^{-t(s-4)} \Big|_0^\infty - \frac{e^{-t(s-4)}}{(s-4)^2} \Big|_0^\infty$$

$$\boxed{= \frac{1}{(s-4)^2}, \; s>4}$$ Ans:-

Q No 19 $L\{f(t)\} = L\{2t^4\}$

$$f(t) = 2t^4$$

$$L\{2t^4\} = 2 \times L\{2t^4\}$$

$$= 2 \times \frac{4!}{s^{4+1}} = 2 \times \frac{4!}{s^5}$$

$$\boxed{L\{2t^4\} = \frac{48}{s^5}}$$ Ans:-

Q 29 $\quad f(t) = (1+e^{2t})^2$

$L\{1+e^{4t}+2e^{2t}\} = L\{1\} - L\{e^{4t}\} + L\{e^{2t}\}$

$= \frac{1}{s} + \frac{1}{s-4} + \frac{2}{s-2}$

$L\{1+e^{2t}\}^2 = \boxed{\frac{1}{s} + \frac{1}{s-4} + \frac{2}{s-2}}$ Ans:-

Q 31 $\quad f(t) = 4t^2 - 5\sin 3t$

$L\{f(t)\} = L\{4t^2 - 5\sin 3t\}$

$= 4L\{t^2\} - 5L\{\sin 3t\}$

$\boxed{= \frac{8}{s^3} - \frac{15}{s^2+9}}$ Ans:-

Q 36 $\quad f(t) = e^{-t}\cosh t$

$L\{f(t)\} = L\{e^{-t}\cosh t\}$

$= L\{e^{-t}\left(\frac{e^{t}-e^{-t}}{2}\right)\}$

$= L\{\frac{1}{2} + \frac{e^{-2t}}{2}\}$

$= L\{\frac{1}{2}\} + L\{\frac{e^{-2t}}{2}\}$

$\boxed{= \frac{1}{2}\left(\frac{1}{s} + \frac{1}{s+2}\right)}$ Ans:-

Ex 7.2

Q5 $L^{-1}\left\{\dfrac{(s+1)^3}{s^4}\right\}$

$= L^{-1}\left\{\dfrac{s^3+1+3s^2+3s}{s^4}\right\}$

$= L^{-1}\left\{\dfrac{1}{s}\right\} + \dfrac{1}{3!}L^{-1}\left\{\dfrac{3!}{s^4}\right\} + L^{-1}(3)\left\{\dfrac{1}{s^2}\right\} + L^{-1}\left(\dfrac{3}{2!}\right)\left\{\dfrac{2!}{s^3}\right\}$

$= L^{-1}\left\{\dfrac{(s+1)^3}{s^4}\right\} = 1 + \dfrac{t^3}{6} + 3t + \dfrac{3}{2}t^2$

Q11. $L^{-1}\left(\dfrac{5}{s^2+49}\right)$

$= \dfrac{5}{7}L^{-1}\left\{\dfrac{7}{s^2+7^2}\right\}$

$= \dfrac{5}{7}\sin 7t$

Q15 $L^{-1}\left\{\frac{2s-6}{s^2+9}\right\}$

$= L^{-1}\left\{2\cdot\frac{s}{s^2+9}\right\} - L^{-1}\left\{\frac{2\cdot 3}{s^2+9}\right\}$

$= 2\cos 3t - 2\sin 3t$ Ans

using formulas!

Q19 $L^{-1}\left\{\frac{s}{s^2+2s-3}\right\} = L^{-1}\left\{\frac{s}{(s-1)(s+3)}\right\}$

$= L^{-1}\left\{\frac{1}{4}\,\frac{1}{s-1}\right\} + L^{-1}\left\{\frac{3}{4}\,\frac{1}{s+3}\right\}$

$= \frac{1}{4}e^{t} + \frac{3}{4}e^{-3t}$ Ans

Q22 $L^{-1}\left\{\frac{s-3}{(s-\sqrt{3})(s+\sqrt{3})}\right\}$

$\left(\frac{s-3}{(s-\sqrt{3})(s+\sqrt{2})}\right) = \frac{3+\sqrt{3}}{2\sqrt{3}(s+\sqrt{3})} - \frac{3-\sqrt{3}}{2\sqrt{3}(s-\sqrt{3})}$

$$= L^{-1}\left\{ \frac{s}{s^2-3} - \sqrt{3}\,\frac{\sqrt{3}}{s^2-3} \right\}$$

$$= \cosh\sqrt{3}t - \sqrt{3}\sinh\sqrt{3}t$$ Ans:-

Q 32 $2\frac{dy}{dt} + y = 0$, $y(0) = -3$

$$2sL\{y\} - 2y(0) + L\{y\} = 0$$

$$L\{y\} = \frac{-6}{2s+1} = \frac{-3}{s+1/2}$$

$$y = -3e^{-t/2}$$ Ans:-

Q 38

$$s^2L\{y\} + 9L\{y\} = \frac{1}{s-1}$$

$$L\{y\} = \frac{1}{(s-1)(s^2+9)}$$

$$= \frac{1}{10}\cdot\frac{1}{s-1} - \frac{1}{10}\,\frac{1}{s^2+9} - \frac{1}{10}\,\frac{s}{s^2+9}$$

$$y = \frac{1}{10}e^{t} - \frac{1}{10}\sin 3t - \frac{1}{10}\cos 3t$$

Ans:-

Ex 7.3

Q3 $L\{t^3 e^{-2t}\}$

$\Rightarrow L\{t^3\}_{s \to s-3} = \frac{3!}{s^4}_{s \to s-3}$

$L\{t^3 e^{-2t}\} = \frac{6}{(s+2)^4}$ Ans.

Q8 $L\{e^{-2t} \cos 4t\}$

$L\{\cos 4t\}_{s \to s+2} = \frac{s}{s^2+16}_{s \to s+2}$

$= \frac{s+2}{(s+2)^2+16}$ Ans.

Q11 $L^{-1}\left\{\frac{1}{(s+2)^3}\right\} = L^{-1}\left\{\frac{\frac{1}{2!} \cdot 2!}{(s-(-2))^3}\right\}$

$= \frac{1}{2!} L^{-1}\left\{\frac{2!}{s^3}\right\}_{s \to s-(-2)} = \frac{1}{2!} t^2 e^{-2t}$

Q17 $L^{-1}\left\{\frac{s}{(s+1)^2}\right\} = L^{-1}\left\{\frac{s+1-1}{(s+1)^2}\right\}$

$$= L^{-1}\left\{ \frac{s+1}{(s+1)^2} - \frac{1}{(s+1)^2} \right\}$$

$$= L^{-1}\left\{ \left(\frac{1}{s+1}\right) - \frac{1}{(s+1)^2} \right\} = e^{-t} - te^{-t}$$

Q22 $\quad y' - y = 1 + te^{t} \qquad y(0) = 0$

$$sL\{y\} - L\{y\} = \frac{1}{s} + \frac{1}{(s-1)^2}$$

$$L\{y\} = \frac{1}{s(s-1)} + \frac{1}{(s-1)^3} = \frac{-1}{s} + \frac{1}{s-1} + \frac{1}{(s-1)^3}$$

$$y = -1 + e^{t} + \frac{1}{2} t^2 e^{t}$$

Q30 $\quad y'' - 2y' + 5y = 1 + t$

$$s^2 L\{y\} - sy(0) - y'(0) - 2\{sL\{y\} - (0)\}$$

$$+ 5L\{y\} = \frac{1}{s} + \frac{1}{s^2}$$

$$L\{y\} = \frac{4s^2 + s + 1}{s^2(s^2 - 2s + 5)} = \frac{7}{25}\frac{1}{s} + \frac{1}{5}\frac{1}{s^2} + \frac{-7s/25 - 109/2}{s^2 - 2s + 5}$$

$$= \frac{7}{25}\frac{1}{s} + \frac{1}{5}\frac{1}{s^2} - \frac{7}{25}\frac{s-1}{(s-1)^2 + 2^2} + \frac{51}{25}\frac{2}{(s-1)^2 + 2^2}$$

$$y = \frac{7}{25} + \frac{1}{5}t + \frac{51}{25}e^{t}\sin 2t - \frac{7}{25}e^{t}\cos 2t$$

Q38) $L\{e^{2-t}\, u(t-2)\}$

$= L\{e^{-(t-2)}\, u(t-2)\}$

$= \dfrac{e^{-2s}}{s+1}$ Ans

Q47) $L^{-1}\left\{\dfrac{e^{-s}}{s(s+1)}\right\}$

$\dfrac{1}{s(s+1)} = \dfrac{1}{s} + \dfrac{1}{s+1} = \dfrac{1}{s} + \dfrac{1}{s-(-1)}$

$f(t) = 1 - e^{-t}$

Hence!

$y(t) = u(t-1) - e^{-(t-1)}\, u(t-1)$ Ans