Assignment No 1-
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Section: ~~B~~ BEE-2C
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Course: Differential Equation

Ex 2.2

Q13) $(e^y+1)^2 e^{-y} dx + (e^x+1)^3 e^{-x} dy = 0$

$(e^y+1)^2 e^{-y} dx = -(e^x+1)^3 e^{-x} dy$

$\int \frac{-e^x}{(e^x+1)^3} dx = \int \frac{e^y}{(e^y+1)^2} dy$

$\frac{1}{2}(e^x+1)^{-2} + c = -(e^y+1)^{-1}$

$(e^x+1)^{-2} + 2(e^y+1)^{-1} = C$

Q25) $x^2 \frac{dy}{dx} = y - xy \qquad y(-1) = -1$

$\frac{1}{y} dy = \frac{1-x}{x^2} dx$

$\frac{1}{y} dy = \left(\frac{1}{x^2} - \frac{1}{x}\right) dx$

$\ln y = -\frac{1}{x} - \ln x + c$

$xy = c\, e^{-1/x}$

$y(-1) = -1 \qquad c = e^{-1}$

$xy = e^{-1-1/x} \Rightarrow y = e^{-(1+1/x)}/x$

Q27) $\sqrt{1-y^2}\, dx - \sqrt{1-x^2}\, dy = 0 \qquad y(0) = \frac{\sqrt{3}}{2}$

$\frac{dx}{\sqrt{1-x^2}} - \frac{dy}{\sqrt{1-y^2}} = 0$

$\sin^{-1} x - \sin^{-1} y = c$

$x=0, y = \sqrt{3}/2 \qquad \sin^{-1}(0) - \sin^{-1}(\sqrt{3}/2) = c$

$c = -\pi/3$

$\sin^{-1} x - \sin^{-1} y = -\pi/3$

$y = \sin\left(\sin^{-1} x + \frac{\pi}{3}\right) = x\cos\frac{\pi}{3} + \sqrt{1-x^2}\sin\frac{\pi}{3} = \frac{x}{2} + \frac{\sqrt{3}}{2}\sqrt{1-x^2}$

Ex 2.3

Q14) $xy' + (1+x)y = e^{-x}\sin 2x$

$y' + \left(1+\frac{1}{x}\right)y = \frac{1}{x}e^{-x}\sin 2x$

$P(x) = 1 + \frac{1}{x}$

$I.F = e^{\int P(x)dx}$

$= e^{\int 1+\frac{1}{x}dx} = xe^{x}$

$\frac{d}{dx}\left[xe^{x}\cdot y\right] = \sin 2x$

$y = -\frac{1}{2x}e^{-x}\cos 2x + \frac{ce^{-x}}{x}$

This is for $0 < x < \infty$. Solution is transient.

Q20) $(x+2)^2\frac{dy}{dx} = 5-8y-4xy$

$y' + \frac{4y}{x+2} = \frac{5}{(x+2)^2}$ $P(x) = \frac{4}{x+2}$

$I.F = e^{\int P(x)dx} = e^{\int (4/x+2)dx}$

$= (x+2)^4$

$\frac{d}{dx}\left[(x+2)^4 y\right] = 5(x+2)^2$

$y = \frac{5}{3}(x+2)^{-1} + c(x+2)^{-4}$

This is for $-2 < x < \infty$. Solution is transient.

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(Q27) $L\frac{di}{dt} + Ri = E \qquad i(0) = i_0$

L, R, E and i_0 constants

$$\frac{di}{dt} + \frac{Ri}{L} = \frac{E}{L} \qquad P(x) = \frac{R}{L}$$

$$I.F = e^{\int R/L\, dt} = e^{Rt/L}$$

$$\frac{d}{dt}\left[e^{Rt/L}\, i\right] = \frac{E}{L} e^{Rt/L}$$

$$i = \frac{E}{R} + c e^{-Rt/L}$$

If $i(0) = i_0$, then

$c = i_0 - \frac{E}{R}$

This is for $-\infty < t < \infty$. $\quad i = \frac{E}{R} + \left(i_0 - \frac{E}{R}\right)e^{-\frac{Rt}{L}}$

(Q34) $(1+x^2)\frac{dy}{dx} + 2xy = f(x), \; y(0) = 0$

where $f(x) = \begin{cases} x & 0 \le x < 1 \\ -x & x \ge 1 \end{cases}$

$$y' + \frac{2xy}{1+x^2} = \begin{cases} \frac{x}{1+x^2} & 0 \le x \le 1 \\ \frac{-x}{1+x^2} & x \ge 1 \end{cases}$$

IF is $1+x^2$ so that

$$(1+x^2)y = \begin{cases} \frac{1}{2}x^2 + c_1 & 0 \le x \le 1 \\ -\frac{1}{2}x^2 + c_2 & x \ge 1 \end{cases}$$

$y(0) = 0$ then $c_1 = 0$

For continuity we must $c_2 = 1$

$$y = \begin{cases} \frac{1}{2} - \frac{1}{2(1+x^2)} & 0 \le x \le 1 \\ \frac{3}{2(1+x^2)} - \frac{1}{2} & x > 1 \end{cases}$$

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Q 11) $(y\ln ny - e^{-ny})dx + (\frac{1}{y} + n\ln ny)dy = 0$

$M(n,y) = y\ln ny - e^{-ny}$

$N(n,y) = \frac{1}{y} + n\ln y$

$\cancel{\frac{\partial M}{\partial y} = 1 + \frac{1}{y} + ne^{-ny}}$ $\frac{\partial N}{\partial x} = \cancel{\frac{1}{y}} + \ln y$

$\frac{\partial m}{\partial y} \quad 1 + \ln y + ne^{-ny}$

The solution is not exact

Q 14) $(4t^3y - 15t^2 - y)dt + (t^4 + 3y^2 - t)dy = 0$

$M(n,y) = 4t^3y - 15t^2 - y$

$N(n,y) = t^4 + 3y^2 - t$

$\frac{\partial m}{\partial y} = 4t^3 - 1 \qquad \frac{\partial N}{\partial t} = 4t^3 - 1$

equation is exact

$\int (4t^3y - 15t^2 - y)dt + \int 3y^2 \, dy = C$

$t^4y - 5t^3 - ty + y^3 = C$

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Q36) $(y^2+xy^3)dx+(5y^2-xy+y^3\sin y)dy=0$

$M(x,y)=y^2+xy^3$

$N(x,y)=5y^2-xy+y^3\sin y$

$\frac{dM}{dy}=2y+3xy^2 \quad \frac{dN}{dx}=-y$

$\frac{N_x-M_y}{M}=\frac{-3}{y} \cdot I.F=e^{-3\int dy/y}=\frac{1}{y^3}$

Let $M=\frac{y^2+xy^3}{y^3}=\frac{1}{y}+x$

$N=\frac{5y^2-xy+y^3\sin y}{y^3}=\frac{5}{y}-\frac{x}{y^2}+\sin y$

$M_y=\frac{-1}{y^2}=N_x$

$\boxed{5\ln y+\frac{x}{y}-\cos y+\frac{1}{2}x^2=C}$

$f(x)=\frac{1}{y}+x$

$f=\frac{x}{y}+\frac{1}{2}x^2+h(y)$

$h'(y)=\frac{5}{y}+\sin y$

$h(y)=5\ln y-\cos y$

Q38) $(x^2+y^2-5)dx=(y+xy)dy \; ; \; y(0)=1$

$M_y=2y \qquad N_x=-y$

$\frac{M_y-N_x}{N}=\frac{-3}{1+x}$

$I.F=e^{-3\int dx/(1+x)}=\frac{1}{(1+x)^3}$

$M=\frac{(x^2+y^2-5)}{(1+x)^3}, \; N=\frac{-(y+xy)}{(1+x^3)}=\frac{-y}{(1+x)^2}$

$M_y=\frac{2y}{(1+x^3)}=N$

$h(x)=\frac{2}{(1+x)^2}+\frac{2}{1+x}+\ln\left(\frac{1}{x}\right)$

$$-\frac{y^2}{2(1+u)^2}+\frac{2}{(1+u)^2}+\frac{2}{(1+u)}+\ln(1+u)=C$$

Ex 2.5

(Q10) $x\frac{dy}{dx}=y+\sqrt{x^2-y^2}$, $x>0$

Let $y=vx$

$vx+\sqrt{x^2-(vx)^2}\,dx-x(vdx+xdv)/dv=0$

$\sqrt{x^2-v^2x^2}\,dx-x^2dv=0$

$x\sqrt{1-v^2}\,dx-x^2dv=0$

$\frac{dx}{x}-\frac{dv}{\sqrt{1-v^2}}=0 \Rightarrow \ln x-\sin^{-1}v=C$

$\sin^{-1}v=\ln x+C_1 \Rightarrow \sin^{-1}\frac{y}{x}=\ln x+C_2$

$\frac{y}{x}=\sin(\ln x+C_2) \Rightarrow \boxed{y=x\sin(\ln x+C_2)}$

(Q12) $(x^2+y^2)\frac{dx}{dy}=xy$, $y(-1)=1$

Let $y=vx$

$(x^2+v^2x^2)dx=vx^2(vdx-xdv)$

$x^2(1+v^2)dx-vx^3dv=0$

$$\frac{dx}{x}-\frac{v\,dv}{1+v^2}=0$$

$\ln x-\frac{1}{2}\ln(1+v^2)=C$

$$\frac{x^2}{1+v^2}=C_1$$

$x^4=C_1(x^2+y^2)$

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$y(-1) = -1$

we find $c_1 = \frac{1}{2}$

$$\boxed{2x^4 = y^2 + x^2}$$

Q20) $3(1+t^2)\frac{dy}{dt} = 2ty(y^3 - 1)$

$$y' + \frac{2y}{3(1+t^2)} = \frac{2ty^4}{3(1+t^2)}$$

$w = y^{-3}$

$$\frac{dw}{dt} - \frac{2tw}{1+t^2} = \frac{-2t}{1+t^2}$$

I.F $\frac{1}{1+t^2}$

$$\frac{w}{1+t^2} = \frac{1}{1+t^2} + c$$

$$y^{-3} = 1 + c(1+t^2)$$

Q21) $\frac{dy}{dx} = 2 + \sqrt{y - 2x + 3}$

Let $v = y - 2x + 3$

$\frac{dv}{dx} = \frac{dy}{dx} - 2$ Then $\frac{dv}{dx} + 2 = 2 + \sqrt{v}$

$$\int \frac{1}{\sqrt{u}} dx = 2\sqrt{u} = x + C$$

$$2\sqrt{y-2x+3} = x + C$$

Q1 Exact eq:

$$(3xy - y^2)dx + x(x-y)dy = 0 \quad -(1)$$

$M = 3xy - y^2 \qquad N = x(x-y)$

$M_y = 3x - 2y \neq N_x = 2x - y$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

$$\frac{M_y - N_x}{N} = \frac{3x - 2y - 2x + y}{x(x-y)} = \frac{1}{x}$$

I.F $e^{\int \frac{1}{x} dx} = e^{\ln x} = x$

Multiply x with eq (1)

$$(3x^2y - xy^2)dx + x^2(x-y)dy = 0$$

$M_y = \cancel{3}x^3y - \frac{x^2y^2}{2} \quad N_x = \cancel{x}^3 y - \frac{x^2y^2}{2}$

$M_y = N_x$

$$\int M dx + \int N dy = 0$$

$$\int \left(3x^2y - \frac{x^2y^2}{2}\right)dx + \int 0 \, dy = 0$$

$$\boxed{x^3y - \frac{x^2y^2}{2} + y = C}$$

Q No 2 Exact Eq.

$2xy - 9x^2 + (2y + x^2 + 1)\frac{dy}{dx} = 0 \; ; \; y(0) = -3$

$M(x,y) = 2xy - 9x^2 \; ; \; N(x,y) = 2y + x^2 + 1$

$M_y = 2x = N_x = 2x$

$\int M(x,y)\,dx + \int (\text{Term of } y \text{ free from } x)\, N(x,y)\,dy = 0$

$\int (2xy - 9x^2)\,dx + \int (2y + 1)\,dy = 0$

$\frac{\cancel{2}x^2y}{\cancel{2}} - \frac{\cancel{9}^{3}x^3}{\cancel{3}} + \frac{2y^2}{2} + y = c$

$x^2y - 3x^3 + \frac{\cancel{2}y^2}{\cancel{2}} + y = c$

$y(0) = -3, \; x = 0$

$0 - 0 + (-3)^2 - 3 = c$

$c = 9 - 3$

$\boxed{c = 6}$

$x^2y - 3x^3 + y^2 + y = 6$

Q No 3 $\frac{dy}{dx} = (1 + e^{-x})(y^2 - 1)$ separation

$\int \left(\frac{1}{y^2 - 1}\right) dy = \int (1 + e^{-x})\,dx$

$\ln(y^2 - 1) = x - e^{-x} + c$

$\frac{\ln(y-1) - \ln(y+1)}{2} = x - e^{-x} + c$

$$\frac{\ln(y-1)-\ln(y+1)}{2} = x - e^{-x} + c$$

Q No 4 : Solve D.Eq:

$$\frac{dy}{dx} - \frac{3}{x+1} y = (x+1)^4 \quad \text{--- (1)}$$

$$P(x) = \frac{-3}{x+1}$$

$$\text{I.F} = e^{\int -3/x+1 \, dx}$$

$$= e^{-3\int \frac{1}{x+1} dx}$$

$$= e^{-3\ln(x+1)}$$

$$= e^{\ln(x+1)^{-3}} = (x+1)^{-3} = \frac{1}{(x+1)^3}$$

Multiply I.F with eq (1):

$$(x+1)^{-3} \frac{dy}{dx} - \frac{3}{x+1}(x+1)^{-3} = (x+1)^1$$

$$\cancel{\int \frac{d}{dx} \frac{y}{(x+1)}} \quad \cancel{= \int (x+1)^4}$$

$$\int \frac{d}{dx}\left[\frac{1}{(x+1)^3} \cdot y\right] = \int (x+1)^{-2}$$

$$\frac{y}{(x+1)^3} = \frac{(x+1)^{-2+1}}{-2+1} + C$$

$$\frac{y}{(n+1)^3} = -\frac{1}{n+1} + c$$

$$y = -(n+1)^3 (n+1)^{-1} + c(n+1)^3$$

$$y = \frac{-1}{(n+1)^2} + c(n+1)^3$$

$$\boxed{y = -(n+1)^2 + c(n+1)^3}$$ Ans:

Q NO 5:- solve

$$y' = 5y + e^{-2x} y^{-2} \quad ; \ y(0) = 2$$

$$\frac{dy}{dx} - 5y = + e^{-2x} y^{-2} \qquad P(n) = -5$$

$$I.F = e^{\int -5 dx} = e^{-5x}$$

Multiply I.F with D.Eq:

$$e^{-5x} y' - 5e^{-5x} y = e^{-7x} y^{-2}$$

$$\cancel{e^{-5x}} \frac{dy}{dx} - 5e^{-5x} y = e^{-7x} y^{-2} \cancel{e^{-5x}}$$

$$\int \frac{d}{dx}\left[e^{-5x} y\right] = \int e^{-7x}$$

$$ye^{-5x} = \frac{-1}{7} e^{-7x} + c$$

$$y = \frac{-1}{7} e^{-2x} + ce^{5x}$$

$y(0) = 2$

$$2 = -\frac{1}{7} e^{-2(0)} + ce^{5(0)}$$

$$2 = -\frac{1}{7} + c \Rightarrow c = 15/7$$

$$y = -\frac{1}{7} e^{-2x} + \frac{15}{7} e^{5x}$$