Ex 7.1

Q No# 13:-

$f(t) = te^{4t}$

$L\{f(t)\} = \int_0^\infty e^{-st} \cdot te^{4t}\, dt$

$\int_0^\infty t e^{4t-st}\, dt$ ∴ apply product rule

$t \int e^{(4-s)t} - \int (t) \frac{d}{dt} \left(\int e^{(4-s)t} dt \right) dt$

$t \left\{ \frac{e^{(4-s)t}}{4-s} - \int \frac{e^{(4-s)t}}{4-s}\, dt \right.$

$\left. \frac{te^{(4-s)t}}{4-s} - \frac{1}{4-s} \cdot \left(\frac{e^{(4-s)t}}{4-s} \right) \right|_0^\infty$

$\left. \frac{te^{(4-s)t}}{4-s} - \frac{e^{(4-s)t}}{(4-s)^2} \right|_0^\infty$ applying limits

$\infty \cdot e^{\wedge\infty} \rightarrow$ undefind

$-\left(0 - \frac{1}{(4-s)^2}\right) \Rightarrow \frac{1}{(4-s)^2}$ where $s > 4$

Q19:-

$f(t) = 2t^4$

$\mathcal{L}(f(t)) = \frac{n!}{s^{n+1}}$

$2\frac{4!}{s^5}$

Q No 29:-

$f(t) = (1+e^{2t})^2$ ∴ open square

$\mathcal{L}(1+2e^{2t}+e^{4t}) = \frac{1}{s} + \frac{2}{s-2} + \frac{1}{s-4}$

$\therefore \mathcal{L}(1) = \frac{1}{s}$

$\therefore \mathcal{L}(e^{at}) = \frac{1}{s-a}$

Q No 31:-

$f(t) = 4t^2 - 5\sin 3t$

$\mathcal{L}(4t^2) - \mathcal{L}(5\sin 3t)$

$= 4\frac{2!}{s^3} - 5\cdot\frac{3}{s^2+9}$

Q No#36

$L\{e^{-t}\cosh t\}$

$= L\left\{e^{-t}\,\frac{e^{t}+e^{-t}}{2}\right\}$

$= \frac{1}{2s} + \frac{1}{2s+4}$

7.2

Q No 5:-

$L^{-1}\left\{\frac{(s+1)^3}{s^4}\right\}$

$= \left\{\frac{1}{s} + \frac{3}{s^2} + \frac{3}{2}\cdot\frac{2}{s^3} + \frac{1}{6}\cdot\frac{3!}{s^4}\right\}$

$\boxed{= 1 + 3t + \frac{3}{2}t + \frac{t^3}{6}}$

Q 11:-

$$\mathcal{L}^{-1}\left\{\frac{5}{s^2+49}\right\} = \mathcal{L}^{-1}\left\{\frac{5}{7}\cdot\frac{7}{s^2+49}\right\}$$

$$\boxed{= \frac{5}{7}\sin 7t}$$

Q 15:-

$$\mathcal{L}^{-1}\left\{\frac{2s-6}{s^2+9}\right\}$$

$$\mathcal{L}^{-1}\left\{2\cdot\frac{s}{s^2+3^2} - 2\cdot\frac{3}{s^2+3^2}\right\}$$

$$= 2\cos 3t + (-2\sin 3t)$$

$$= (2\cos 3t - 2\sin 3t)$$

Q 19:-

$$\mathcal{L}^{-1}\left\{\frac{1}{s^2+2s-3}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{3^2}\cdot\frac{1}{s-4} - \frac{1}{9}\cdot\frac{1}{s+5}\right\}$$

$$= \frac{1}{9}e^{4t} - \frac{1}{9}e^{-5t}.$$

Q 22:-

$$L^{-1}\left\{\frac{s-3}{(s-\sqrt{3})(s+\sqrt{3})}\right\}$$

$$= L^{-1}\left\{\frac{s}{s^2-3} - \sqrt{3}\cdot\frac{\sqrt{3}}{s^2-3}\right\}$$

$$\cos\sqrt{3}t - \sqrt{3}\sinh\sqrt{3}t$$

Q 32:-

$$2\frac{dy}{dt} + y = 0 \quad , y(0) = -3$$

$$L\left\{\frac{dy}{dt}\right\} = sF(s) - f(0)$$

$$= sY(s) - y(0)$$

So,

$$\cancel{-2\,sY(s) + 3 + sY(s) + 3}$$

$$2sY(s) - 2y(0) - Y(s) = 0$$

$$2sY(s) + 6 + Y(s) = 0$$

$$Y(s)\left[2s + 1\right] = 6$$

$$L\{Y(s)\} = \frac{6}{2s+1}$$

$$y(s) = L^{-1}\left\{\frac{6}{2s+1}\right\} \Rightarrow \frac{\frac{6}{2}}{s+1/2}$$

$$\boxed{y = 3e^{-t/2}}$$

Q No 38:-

$y'' + 9y = e^t \quad y(0) = 0, \; y'(0) = 0$

$L\{y''\} = s^2 F(s) - s f(0) - f'(0)$

applying laplace on b/s

$s^2 Y(s) - s\,y(0) - y'(0) + L\{9y\} = \frac{1}{s-1}$

$s^2 y(s) - 0 - 0 + 9L\{y\} = \frac{1}{s-1}$

$s^2 y(s) + 9Y(s) = \frac{1}{s-1}$

$y(s) = \frac{1}{(s-1)(s^2+9)}$

$1 = A(s^2+9) + \cancel{B(s+} \; Bs + C\,(s-1)$

$s = +1$

$1 = A(10) + B + C(0)$

$\boxed{A = \frac{1}{10}}$

$1 = As^2 + 9A + Bs^2 - Bs + Cs - C$

comparing co-effi

$s^2 : A + B = 0$

$s : -B + C = 0$

$1 : 9A - C = 1$

$B = -\frac{1}{10}$

$C = -\frac{1}{10}$

$\frac{1}{s-1} + \frac{1}{s+9} \cdot \frac{s}{s^2+9}$

$\frac{1}{10} \cdot \frac{1}{s-1} - \frac{1}{10} \cdot \frac{1}{s^2+9} \; \frac{-1}{10} \cdot \frac{s}{s^2+9}$

$y = \frac{1}{10} e^t - \frac{1}{10} \cos 3t - \frac{1}{30} \sin 3t$

7.3

Q NO 3:-

$$\mathcal{L}\{t^3 e^{-2t}\}$$

$$\frac{3!}{s^4}\Big|_{s \to s+2}$$

$$\frac{6}{(s+2)^4}$$

Q No 11:-

$$\mathcal{L}^{-1}\left\{\frac{1}{(s+2)^3}\right\}$$

$$t^2 = \frac{2!}{s^3}$$

and e^{-2t}

so,

$$\frac{1}{2!} \cdot \frac{2}{(s+2)^3}$$

$$\frac{1}{2} t^2 e^{-2t}$$

Q No 8:-

$$\mathcal{L}\{e^{-2t}\cos 4t\}$$

$$\frac{s}{s^2+16}\Big|_{s\to s+2}$$

$$\frac{s+2}{(s+2)^2+16}$$

Q No 17:-

$$\mathcal{L}^{-1}\left\{\frac{s}{(s+1)^2}\right\}$$

$$\cancel{e^{t}} \quad \frac{(s+1)-1}{(s+1)^2}$$

$$\frac{s+\cancel{1}}{(s+1)^{\cancel{2}}} \quad \frac{-1}{(s+1)^2}$$

$$\frac{1}{(s+1)} - \frac{1}{(s+1)^2}$$

$$\Rightarrow e^{-t}\cancel{t} - e^{-t}t$$

Q NO 22:-

$y' - y = 1 + te^t \quad y(0) = 0$

$sy_{(s)} - y(0) - y_{(s)} = \frac{1}{s} + \frac{1}{(s-1)^2}$

$y_{(s)}(s-1) = \frac{1}{s} + \frac{1}{(s-1)^2}$

$y_{(s)} = \frac{1}{s(s-1)} + \frac{1}{(s-1)^3}$

partial   formula

$\frac{1}{s(s-1)} = \frac{A}{s} + \frac{B}{(s-1)} \quad A$

$\frac{1}{(s-1)} \quad -\frac{1}{s}$

so, $e^t - 1 + \frac{1}{2} t^2 e^t$

Q No 38:-

$\mathcal{L}\{e^{2t} u(t-2)\}$

$= e^{-2s} \mathcal{L}\{e^{-t}\}$

$F(s-a)$

$$\boxed{= \frac{e^{-2s}}{s+1}}$$

Q No 47:-

$\mathcal{L}^{-1}\left\{\frac{e^{-s}}{s(s+1)}\right\}$

Here $a=1$

so,

$\frac{1}{s(s+1)}$ $A=1$, $B=-1$

$u(t-1) - e^{-(t-1)} u(t-1)$