3.3. Modeling the Diode Forward Characteristic

- Considering the analysis of circuits employing forward conducting diodes
- To aid in analysis, represent the diode with a model
- Define a robust set of diode models
- Discuss simplified diode models better suited for use in circuit analysis and design of diode circuits:
  - Exponential model
  - Constant voltage-drop model
  - Ideal diode model
  - Small-signal (linearization) model

3.3.1 The Exponential Model

- Exponential diode model
  - Most accurate
  - Most difficult to employ in circuit analysis
    - Due to nonlinear nature
    - V_{DD} is greater than 0.5V

$$\text{(eq 3.6)} \quad \underbrace{I_D = I_S e^{V_D/V_T}}$$

$V_D =$ voltage across diode
$I_D =$ current through diode

$$\text{(eq 3.7)} \quad I_D = \frac{V_{DD} - V_D}{R}$$

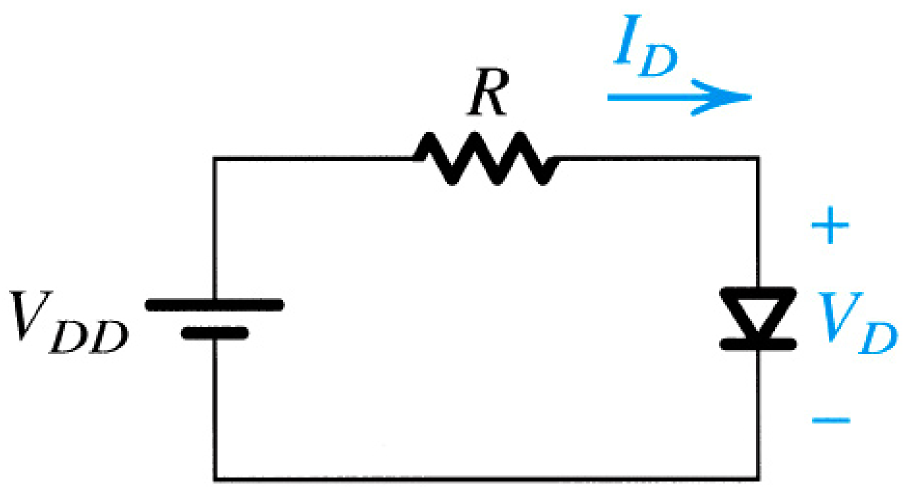
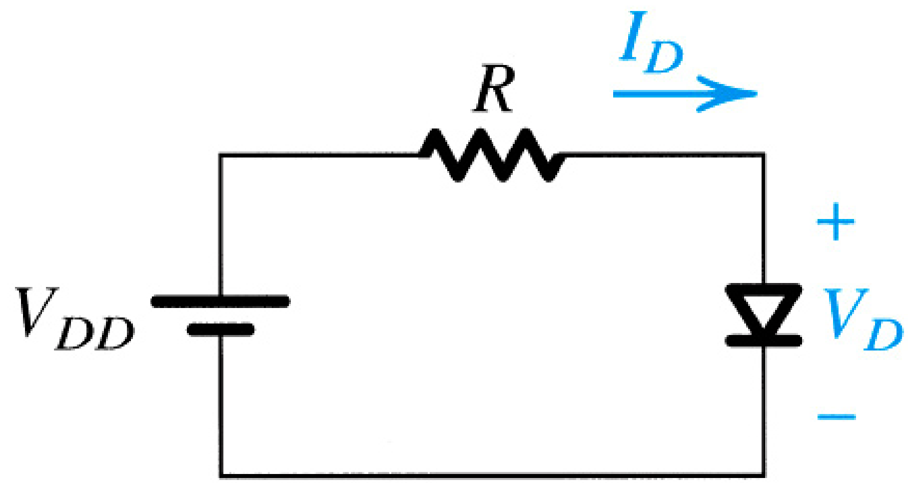


Figure 3.10: A simple circuit used to illustrate the analysis of circuits in which the diode is forward conducting

3.3.1 The Exponential Model

- **Q:** How does one solve for I_D in circuit to right?
  - $V_{DD} = 5V$
  - $R = 1kOhm$
  - $I_D = 1mA$ @ $0.7V$
- **A:** Two methods exist…
  - **graphical method**
  - **iterative method**



3.3.2 Graphical Analysis using Exponential Model

- Step 1: Plot the relationships of (3.6) and (3.7) on single graph
- Step 2: Find intersection of
  - **load line** and diode characteristic intersect at **operating point (Q)**

$$\underbrace{\text{(eq 3.6)}\; I_D = I_S e^{V_D/V_T}}_{}$$

$V_D =$ voltage across diode
$I_D =$ current through diode

$$\text{(eq 3.7)}\; I_D = \frac{V_{DD} - V_D}{R}$$

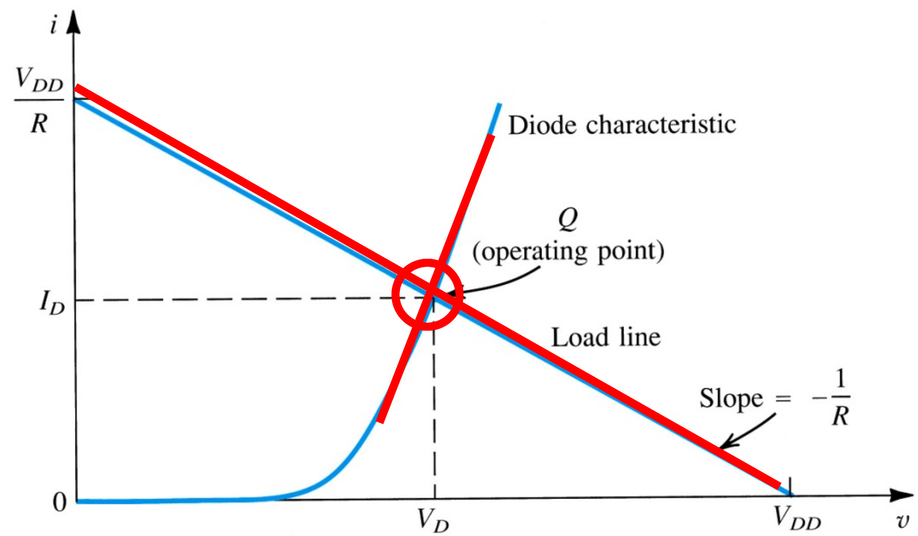


Figure 3.11: Graphical analysis of the circuit in Fig. 3.10 using the exponential diode model.

3.3.2 Graphical Analysis using Exponential Model

- **Pro's**
  - Intuitive
    - b/c of visual nature
- **Con's**
  - Poor Precision
  - Not Practical for Complex Analyses
    - Multiple lines required

3.3.3 Iterative Analysis using Exponential Model

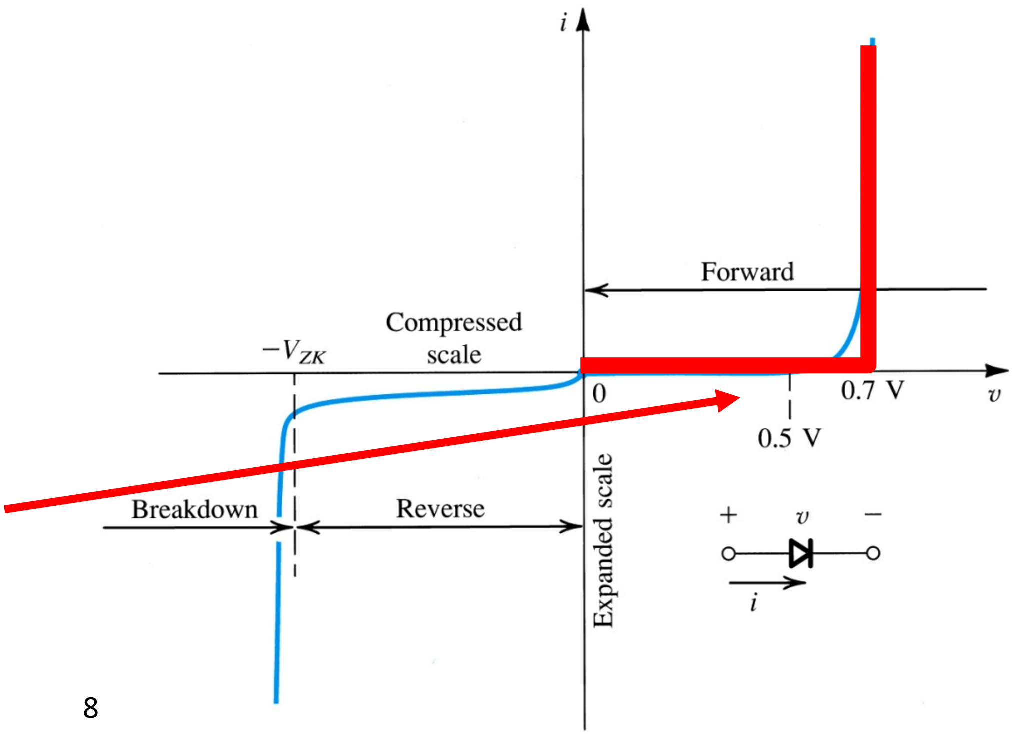
- Ex 3.4
- **Step 1:** Start with initial guess of V_D.
  - $V_D^{(0)}$
- **Step 2:** Use nodal / mesh analysis to solve I_D
- **Step 3:** Use exponential model to update V_D
  - $V_D^{(1)} = \mathbf{f}(V_D^{(0)})$
- **Step 4:** Repeat these steps until $V_D^{(k+1)} = V_D^{(k)}$
  - Upon convergence, the new and old values of V_D will match

3.3.3 Iterative Analysis using Exponential Model

- Pro's
  - High Precision
- Con's
  - Not Intuitive
  - Not Practical for Complex Analysis
    - 10+ iterations may be required

3.3.4 The Need for Rapid Analysis

- Analyze the diode-based circuit more efficiently
  - Rapid circuit analysis with a simpler model
  - Further refine and “fine-tune” the design in almost final design
  - Perform with the aid of a computer circuit analysis program (SPICE)
- One example is assume that voltage drop across the diode is constant



3.3.5 The Constant Voltage-Drop Model

- Voltage drop of a forward-conducting diode varies in a relatively narrow range (0.6-0.8V)
- The **constant voltage-drop diode model** assumes that the slope of I_D vs. V_D is vertical @ 0.7V
- Not very different
- Employed in the initial phases of analysis and design
- **Ex3.4:** solution change if CVDM is used?
    - **A:** 4.262mA to 4.3mA

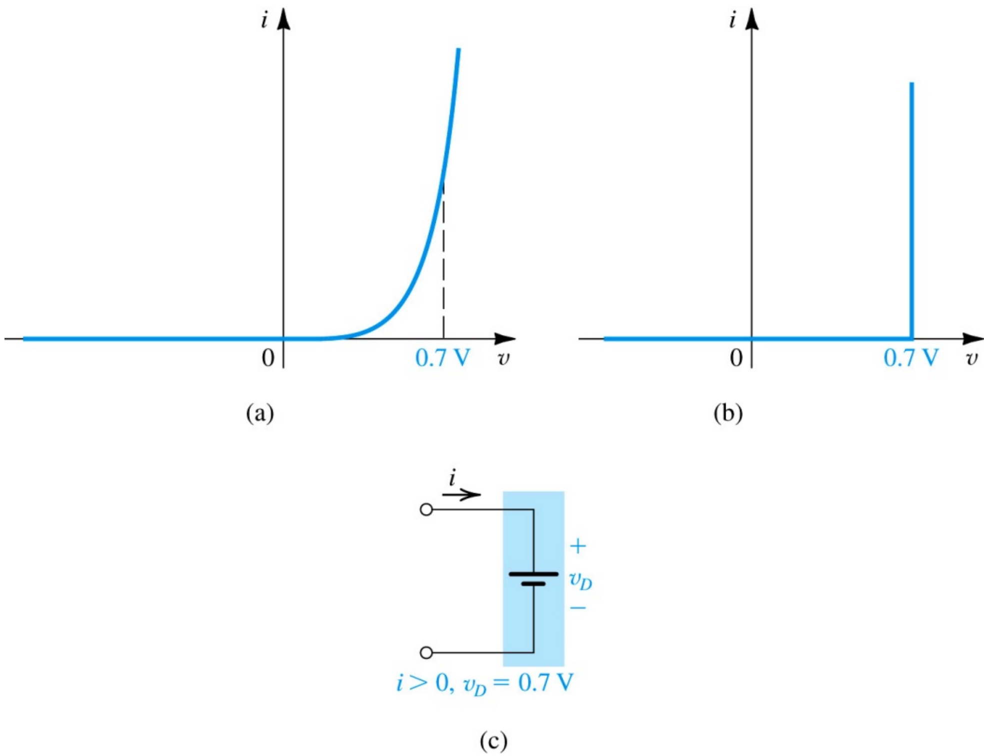
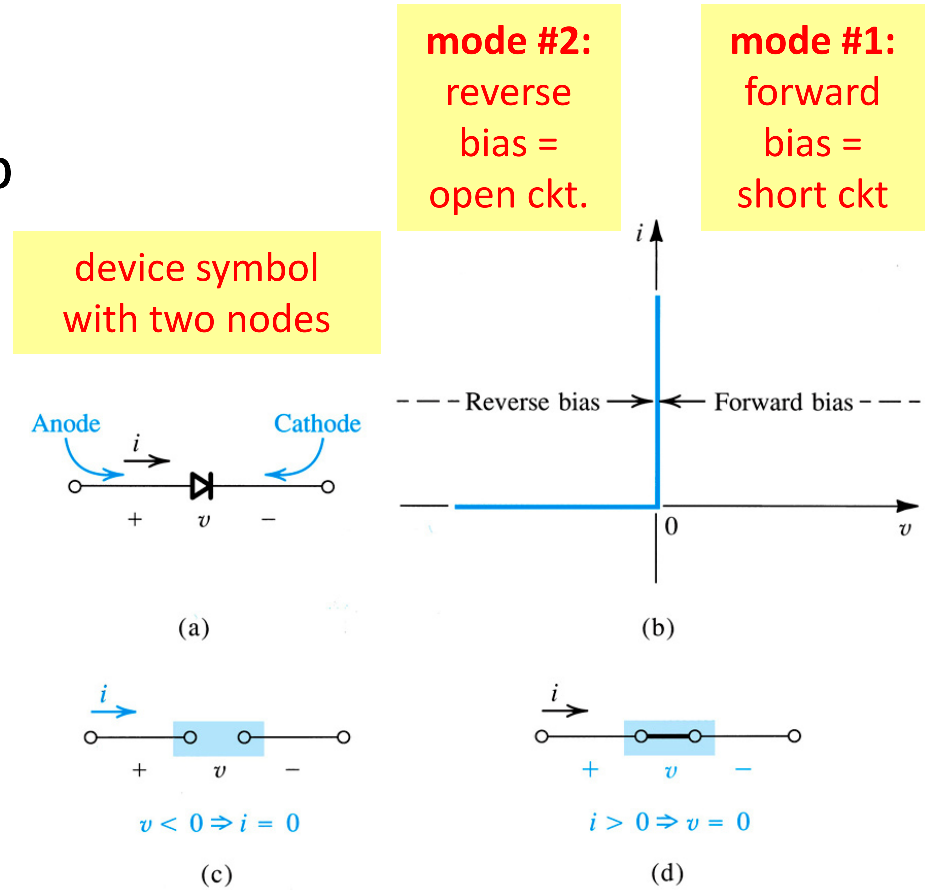


Figure 3.12: Development of the diode constant-voltage-drop model: (a) the exponential characteristic; (b) approximating the exponential characteristic by a constant voltage, usually about 0.7 Vi; (c) the resulting model of the forward-conducting diodes

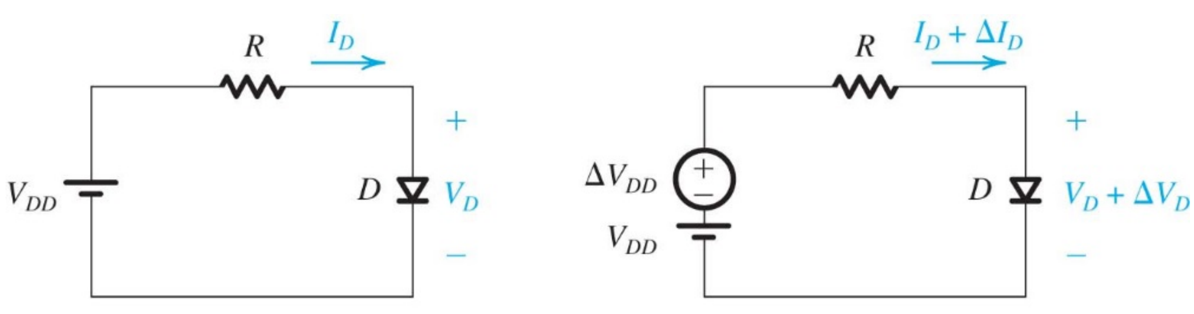
3.3.6 Ideal Diode Model

- When involving voltages much greater than the diode voltage drop
- Very quick analysis for a gross estimate
- For determine which diodes are on/off in a multidiode circuit
- The **ideal diode model** assumes that the slope of I_D vs. V_D is vertical @ $0V$
- **Ex3.4:** solution change if ideal model is used?
  - **A:** $4.262mA$ to $5mA$

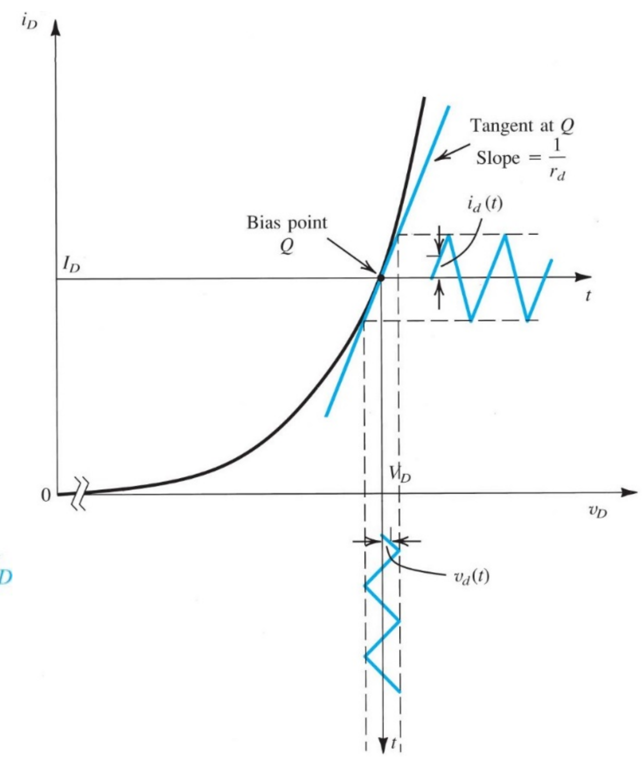


3.3.7 Small-Signal Model

- Operate at a dc biased point on the forward *i-v* characteristic and a small ac signal superimposed on dc
- dc operating point (I_D , V_D) by other model
- And then, modeled as variable resistor = inverse of the slope of the tangent to exponential *i-v* characteristic at the bias point
- Whose value is defined via linearization of exponential model
- Around bias point defined by constant voltage drop model

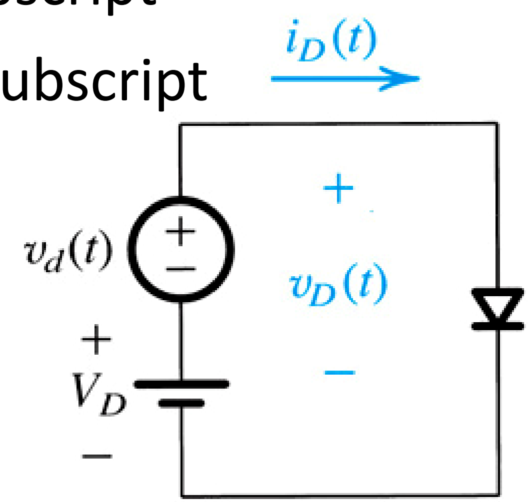


(a) (b)



3.3.7 Small-Signal Model

- Define the small-signal diode model
- Step 1: consider the conceptual circuit of figure 3.13(b)
    - DC voltage (V_D) is applied to diode
    - Upon V_D, arbitrary time-varying signal $v_d(t)$ is super-imposed
- DC only – upper-case w/ upper-case subscript
- Time-varying only – lower-case w/ lower-case subscript
- Total instantaneous – lower-case w/ upper-case subscript
    - DC + time-varying



3.3.7 Small-Signal Model

- Step 2: define DC current as in (3.8)
- Step 3: Define total instantaneous voltage (v_D) as composed of V_D and v_d
- Step 4: Define total instantaneous current (i_D) as function of v_D

(eq 3.8) $$I_D = I_S e^{V_D / V_T}$$

(eq 3.9) $$\underbrace{v_D(t) = V_D + v_d(t)}$$

$v_D(t) =$ total instantaneous voltage across diode
$V_D =$ dc component of $v_D(t)$
$v_d(t) =$ time varying component of $v_D(t)$

(eq 3.10) $$\underbrace{i_D(t) = I_S e^{v_D / V_T}}$$

note that this is different from (3.8)

3.3.7 Small-Signal Model

- Step 5: Redefine (3.10) as function of both V_D and v_d

(eq 3.11) $$i_D(t) = I_S e^{(V_D + v_d)/V_T}$$

- Step 6: Split this exponential in two

(eq 3.11) $$i_D(t) = \overbrace{\underbrace{I_S e^{V_D/V_T}}_{I_D} e^{v_d/V_T}}^{\textbf{action:}\text{ split this exponential using appropriate laws}}$$

- Step 7: Redefine total instant current in terms of DC component (I_D) and time-varying current (i_d)

(eq 3.12) $$i_D(t) = I_D e^{v_D/V_T}$$

3.3.7 Small-Signal Model

example: $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots$

- Step 8: Apply power series expansion to (3.12)
- Step 9: Because $v_d/V_T << 1$, certain terms may be neglected

(eq 3.12) $$i_D(t) = I_D \overbrace{\underbrace{\left(1 + \frac{v_d}{V_T} + \overbrace{\left[\left(\frac{v_d}{V_T}\right)^2 \frac{1}{2!}\right] + \left[\left(\frac{v_d}{V_T}\right)^3 \frac{1}{3!}\right] + \ldots}^{\text{because } v_d/V_T<<1\text{, these terms are assumed to be negligible}}\right)}_{\text{power series expansion of } e^{v_d/V_T}}}^{\textbf{action: }\text{apply power series expansion to (4.12)}}$$

(eq 3.14) $$i_D(t) = I_D \overbrace{\left(1 + \frac{v_d}{V_T}\right)}^{\textbf{action: }\text{eliminate negligible terms}}$$

3.3.7 Small-Signal Model

- Small signal approximation
  - Shown to right for exponential diode model
    - Total instant current (i_D)
    - Small-signal current (i_d)
    - Diode small-signal resistance / incremental resistance (r_d)
  - Valid for $v_d < 5mV$ amplitude (not peak to peak)
  - Inversely proportional to the bias current I_D

$$i_D(t) = I_D + \underbrace{\left(\frac{I_D}{V_T}\right) v_d}_{i_d}$$

$$i_D(t) = I_D + i_d$$

$$i_d = \frac{1}{r_d} v_d$$

$$r_d = \frac{V_T}{I_D}$$

3.3.7 Small-Signal Model

- Assuming that the signal amplitude is sufficiently small such that the excursion along the *i-v* curve is limited to a short almost-linear segment

$$r_d = 1 \Big/ \left[\frac{\partial i_D}{\partial v_D}\right]_{i_D = I_D}$$

- This method may be used to approximate any function y = **f**(x) around an operating point (x_0, y_0).

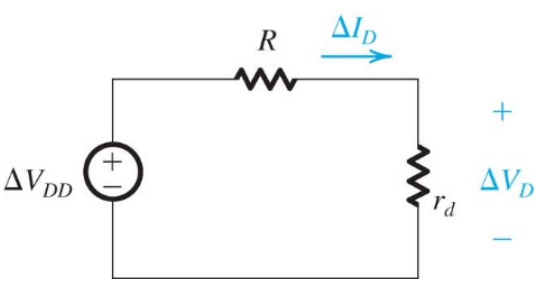
$$y(t) = y_0 + \left(\left.\frac{\partial y}{\partial x}\right|_{y=Y} \right)^{-1} \overbrace{\left(x(t) - x_0 \right)}^{\Delta x}$$

3.3.7 Small-Signal Model

- **Q:** How is **small-signal resistance** r_d defined?
  - **A:** From steady-state current (I_D) and thermal voltage (V_T) as below
    - Note this approximation is only valid for small-signal voltages $v_d < 5mV$

$$r_d = \frac{V_T}{I_D}$$

- After dc analysis (define the dc bias point = quiescent point) of the diode,
- Eliminating all dc sources (short-circuiting dc voltage sources and open-circuiting dc current sources)
- Replacing the diode by its small-signal resistance



3.3.7 Small-Signal Model

- **Q:** How is the small-signal diode model defined?
  - **A:** The total instantaneous circuit is divided into steady-state and time varying components, which may be analyzed separately and solved via algebra
    - In steady-state, diode represented as CVDM
    - In time-varying, diode represented as resistor

Neither of these circuits employ the exponential model – simplifying the “solving” process

Example 3.5:

- R = 10$kOhm$
- Power supply $V+$: dc value of 10V + 60Hz sinusoid of 1V peak amplitude (known as the power supply ripple)
- Assume diode to have 0.7V drop at 1mA current
- **Q:** Calculate both amplitude of the dc and sine-wave signal observed across the diode
  - **A:** v_d(peak) = 2.68mV

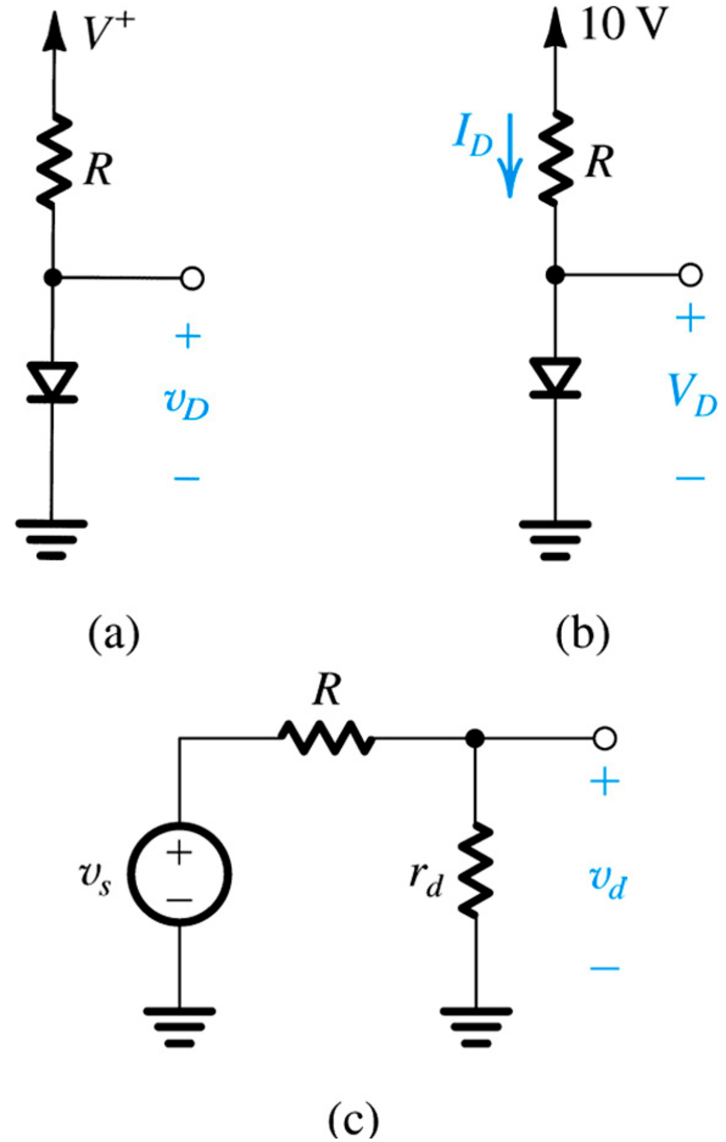


Figure 3.16: (a) circuit for Example 3.5. **(b)** circuit for calculating the dc operating point. **(c)** small-signal equivalent circuit.

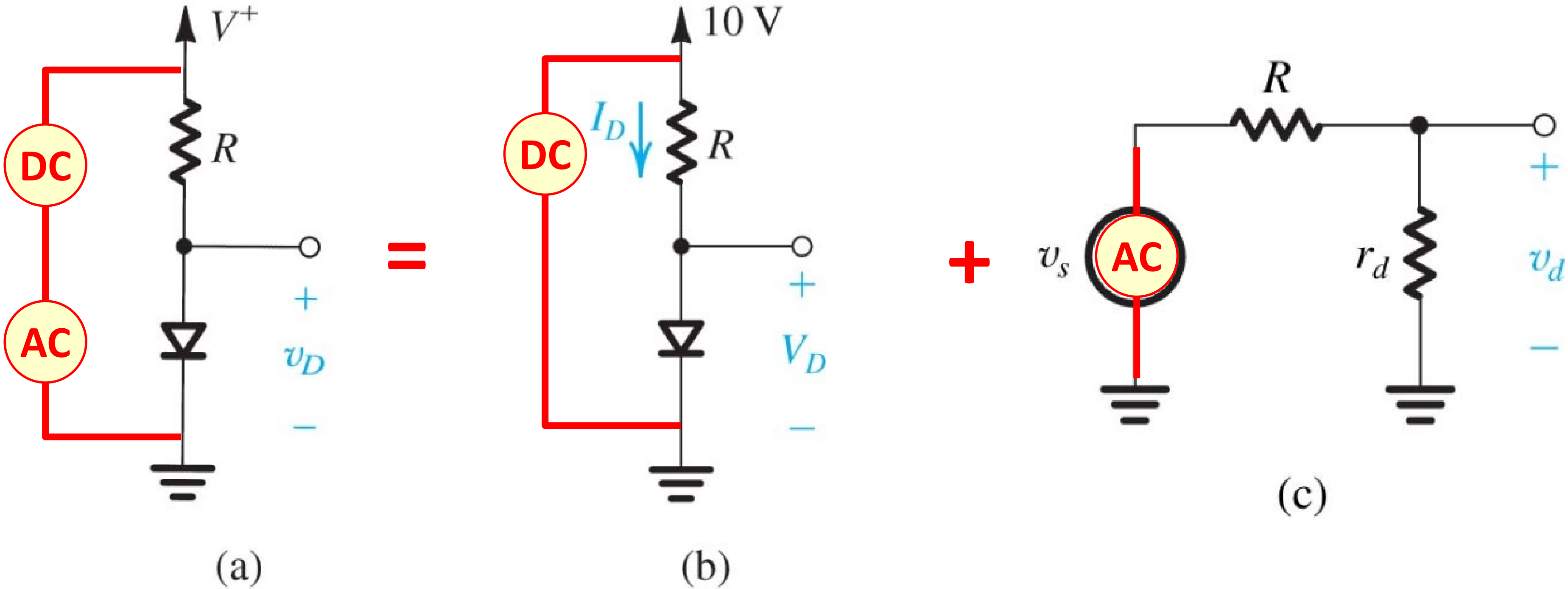


Figure 3.14: (a) Circuit for Example 3.5. **(b)** Circuit for calculating the dc operating point. **(c)** Small-signal equivalent circuit.

When to use these models?

- Exponential model
  - Low voltages
  - Less complex circuits
  - Emphasis on accuracy over practicality
- constant voltage-drop mode:
  - Medium voltages = 0.7V
  - More complex circuits
  - Emphasis on practicality over accuracy
- Ideal diode model
  - High voltages >> 0.7V
  - Very complex circuits
  - Cases where a difference in voltage by 0.7V is negligible
- Small-signal model

3.3.8 Diode Forward Drop in Voltage Regulation

- **Voltage regulator**
  - Provide a constant dc voltage between its output terminals
  - To remain output as constant as possible in spite of changes in dc power supply voltage and load current
- **Q:** What characteristic of the diode facilitates voltage regulation?
  - **A:** The approximately constant voltage drop across it (0.7V)

Example 3.6: Diode-Based Voltage Regulator

- Consider circuit shown in Figure 3.17. A string of three diodes is used to provide a constant voltage of 2.1*V*
  - **Q:** What is the change in this regulated voltage caused by **(a)** a +/- 10*%* change in supply voltage and **(b)** connection of 1*kOhm* load resistor

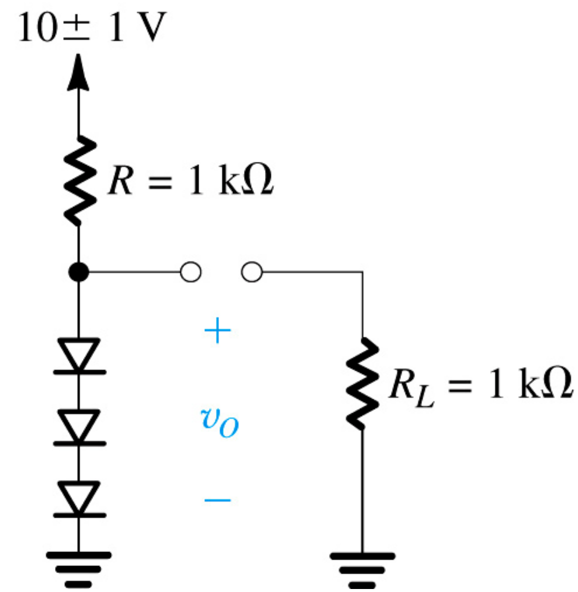


Figure 3.17: Circuit for Example 3.6