**MATH 250 (Linear Algebra)**

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HWs: **handout every F, Due date: Every T**
HW: **20%, Midterm 35%, Final 45%**

**Tuesday & Friday: 12-13:40**
**Office Hours: T & F 14-16**
**Midterm: 05/??/2023, Final: 07/??/2023**

## Lectures:

1- **Systems of Linear Equations**
   1.1 What is a system of linear equations?
   1.2 Matrices
   1.3 Solving linear systems
   1.4 Geometric interpretation of the solution set

Objectives of This Lecture:
After this lecture Students should know the following:
• what a linear system is
• what it means for a linear system to be consistent and inconsistent
• what matrices are
• what are the matrices associated to a linear system
• what the elementary row operations are and how to apply them to simplify a linear system
• what it means for two matrices to be row equivalent
• how to use the method of back substitution to solve a linear system
• what an inconsistent row is
• how to identify using elementary row operations when a linear system is inconsistent
• the geometric interpretation of the solution set of a linear system????

2- **Row Reduction and Echelon Forms**
   2.1 Row echelon form (REF)
   2.2 Reduced row echelon form (RREF)
   2.3 Existence and uniqueness of solutions

Objectives of This Lecture:
After this lecture you should know the following:
• what the REF is and how to compute it
• what the RREF is and how to compute it
• how to solve linear systems using row reduction (Practice!!!)
• how to identify when a linear system is inconsistent

• how to identify when a linear system is consistent
• what is the rank of a matrix
• how to compute the number of free parameters in a solution set
• what are the three possible cases for the solution set of a linear system

**3- Vector Equations**

3.1 Vectors in $R^n$
3.2 The linear combination problem
3.3 The span of a set of vectors

Objectives of This Lecture:
After this lecture you should know the following:
• what a vector is • what a linear combination of vectors is
• what the linear combination problem is
• the relationship between the linear combination problem and the problem of solving linear systems of equations
• how to solve the linear combination problem
• what the span of a set of vectors is
• the relationship between what it means for a vector b to be in the span of $v_1, v_2, \ldots, v_p$ and the problem of writing b as a linear combination of $v_1, v_2, \ldots, v_p$
• the geometric interpretation of the span of a set of vectors

**4- The Matrix Equation Ax = b**

4.1 Matrix-vector multiplication
4.2 Matrix-vector multiplication and linear combinations
4.3 The matrix equation problem

Objectives of This Lecture:
After this lecture you should know the following:
• how to multiply a matrix A with a vector x
• that the product Ax is a linear combination of the columns of A
• how to solve the matrix equation Ax = b if A and b are known
• how to determine if a set of vectors $\{v_1, v_2, \ldots, v_p\}$ in $R^m$ spans all of $R^m$
• the relationship between the equation Ax = b, when b can be written as a linear combination of the columns of A, and when the augmented matrix [A b] is consistent
• when the columns of a matrix $A \in M_{m\times n}$ span all of $R^m$
• the basic properties of matrix-vector multiplication

**5- Homogeneous and Nonhomogeneous Systems**

5.1 Homogeneous linear systems
5.2 Nonhomogeneous systems

Objectives of This Lecture:
After this lecture you should know the following:
• what a homogeneous/nonhomogeneous linear system is
• when a homogeneous linear system has nontrivial solutions

• how to write the general solution set of a homogeneous system in parametric vector form
• how to write the solution set of a nonhomogeneous system in parametric vector form
• the relationship between the solution sets of the nonhomogeneous equation Ax = b and the homogeneous equation Ax = 0

**6- Linear Independence**
6.1 Linear independence
6.2 The maximum size of a linearly independent set

Objectives of This Lecture:
After this lecture you should know the following:
• the definition of linear independence and be able to explain it to a colleague
• how to test if a given set of vectors are linearly independent
• the relationship between the linear independence of $\{v_1, v_2, \ldots, v_p\}$ and the solution set of the homogeneous system Ax = 0, where $A = [v_1, v_2, \ldots, v_p]$
• that in $R^n$, any set of vectors consisting of more than n vectors is automatically linearly dependent

**7- Matrix Algebra**
7.1 Sums of Matrices
7.2 Matrix Multiplication
7.3 Matrix Transpose

Objectives of This Lecture:
After this lecture you should know the following:
• know how to add and multiply matrices
• that matrix multiplication corresponds to composition of linear mappings
• the algebraic properties of matrix multiplication
• how to compute the transpose of a matrix
• the properties of matrix transposition

**8- Invertible Matrices**
8.1 Inverse of a Matrix
8.2 Computing the Inverse of a Matrix

Objectives of This Lecture:
After this lecture you should know the following:
• how to compute the inverse of a matrix
• properties of matrix inversion and matrix multiplication
• the characterizations of invertible matrices

**9- Determinants**
9.1 Determinants of $2 \times 2$ and $3 \times 3$ Matrices
9.2 Determinants of $n \times n$ Matrices
9.3 Triangular Matrices

Objectives of This Lecture:
After this lecture you should know the following:
• how to compute the determinant of any sized matrix
• that the determinant of A is equal to the determinant of AT
• the determinant of a triangular matrix is the product of its diagonal entries

**10- Properties of the Determinant**

10.1 Elementary Row Operations and Determinants
10.2 Determinants and Invertibility of Matrices
10.3 Properties of the Determinant

Objectives of This Lecture:
After this lecture you should know the following:
• how the determinant behaves under elementary row operations
• that A is invertible if and only if det $A \neq 0$
• that det (AB) = det(A) det(B)

**11- Applications of the Determinant**

11.1 The Cofactor Method
11.2 Cramer's Rule
11.3 Volumes

Objectives of This Lecture:
After this lecture you should know the following:
• what the Cofactor Method is
• what Cramer's Rule is
• the geometric interpretation of the determinant (volume)

**12- Vector Spaces**

12.1 Vector Spaces
12.2 Subspaces of Vector Spaces

Objectives of This Lecture:
After this lecture you should know the following:
• what a vector space/subspace is
• be able to give some examples of vector spaces/subspaces
• that the span of a set of vectors in V is a subspace of V

**13- Subspaces**

13.1 Null space, Row space and Column space

Objectives of This Lecture:
After this lecture you should know the following:
• what the null space of a matrix is and how to compute it

• what the column space of a matrix is and how to determine if a given vector is in the column space
• what the row space of a matrix is and how to determine if a given vector is in the row space

**14- Linear Independence, Bases, and Dimension**

14.1 Linear Independence
14.2 Bases
14.3 Dimension of a Vector Space
14.1 The Rank of a Matrix

Objectives of This Lecture:
After this lecture you should know the following:
• what it means for a set to be linearly independent/dependents
• what a basis is (a spanning set that is linearly independent)
• what is the meaning of the dimension of a vector space
• how to determine if a given set in $R^n$ is linearly independent
• how to find a basis for the null space and column space of a matrix A
• what the rank of a matrix is and how to compute it
• what the nullity of a matrix is and how to compute it
• the Rank Theorem

**15- Inner Products and Orthogonality**

15.1 Inner Product on $R^n$
15.2 Orthogonality
15.3 Orthonormal Basis

Objectives of This Lecture:
After this lecture you should know the following:
• how to compute inner products, norms, and distances
• how to normalize vectors to unit length
• what orthogonality is and how to check for it
• what an orthogonal and orthonormal basis is

**16- Eigenvalues and Eigenvectors**

16.1 Eigenvectors and Eigenvalues
16.2 When $\lambda = 0$ is an eigenvalue

Objectives of This Lecture:
After this lecture you should know the following:
• what eigenvalues are
• what eigenvectors are and how to find them when eigenvalues are known
• the behavior of a discrete dynamical system when the initial condition is set to an eigenvector of the system matrix

**17- The Characteristic Polynomial**

17.1 The Characteristic Polynomial of a Matrix

17.2 Eigenvalues and Similarity

Objectives of This Lecture:

After this lecture you should know the following:

• what the characteristic polynomial is and how to compute it
• how to compute the eigenvalues of a matrix
• that when a matrix A has distinct eigenvalues, we are guaranteed a basis of $R^n$ consisting of the eigenvectors of A
• that when a matrix A has repeated eigenvalues, it is still possible that there exists a basis of R n consisting of the eigenvectors of A
• what is the algebraic multiplicity and geometric multiplicity of an eigenvalue
• that eigenvalues of a matrix do not change under similarity

**18- Diagonalization**

18.1 Eigenvalues of Triangular Matrices

18.2 Diagonalization

18.3 Conditions for Diagonalization

Objectives of This Lecture:

After this lecture you should know the following:

• Determine if a matrix is diagonalizable or not
• Find the algebraic and geometric multiplicities of an eigenvalue
• Apply the Facts introduced in this lecture

**19- Diagonalization of Symmetric Matrices**

19.1 Symmetric Matrices

19.2 Eigenvectors of Symmetric Matrices

19.3 Symmetric Matrices are Diagonalizable.

Objectives of This Lecture:

After this lecture you should know the following:

• a symmetric matrix is diagonalizable with an orthonormal set of eigenvectors