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# 1.

The probability that a computer with a random input will print will print out the string x followed by any arbitrary sequence is the sum of the probabilities over all sequences starting with the string x.

This sum is lower bounded by the largest term, which corresponds to the simplest concatenated sequence.

1. The simplest program to print a sequence that starts with n 0’s is Print 0’s forever. This program has constant length c and hence the probability of strings starting with n zeroes is

1. Just as in part (a), there is a short program to print the bits of π forever. Hence
2. A program to print out n 0’s followed by a 1 must in general specify . Since most integers have a complexity, and given , the program to print out 0n1 is simple, we have
3. We know that n bits of are essentially incompressible, i.e., their complexity . Hence, the shortest program to print out n bits of Ω followed by anything must have a length at least , and hence

# 2.

1. Channel capacity

independent of the distribution of X , and hence the capacity of the channel is

which is attained when Y has a uniform distribution, which occurs (by symmetry) when X has a uniform distribution

The capacity of the channel is log bits/transmission.

1. The capacity is achieved by an uniform distribution on the inputs. for .

# 3.

We can use the same chain of inequalities as in the proof of the converse to the channel coding theorem. Hence

since by the definition of the channel, Yi depends only on Xi and is conditionally independent of everything else. Continuing the series of inequalities, we have

with equality if is chosen i.i.d. ∼ Bern(1/2). Hence

# 4.

1. This is the classical Gaussian channel capacity problem with
2. Since the transmitter and the receiver both know the means, the receiver can simply subtract the mean while decoding. Thus, we are back in case (a). Hence the capacity is

1. Let be the density of . Clearly is independent of the time index . Also

where ∗ represents convolution. From the distribution of it is obvious that the optimal input distribution is and the capacity is