False Position Method (Regula Falsi) Example
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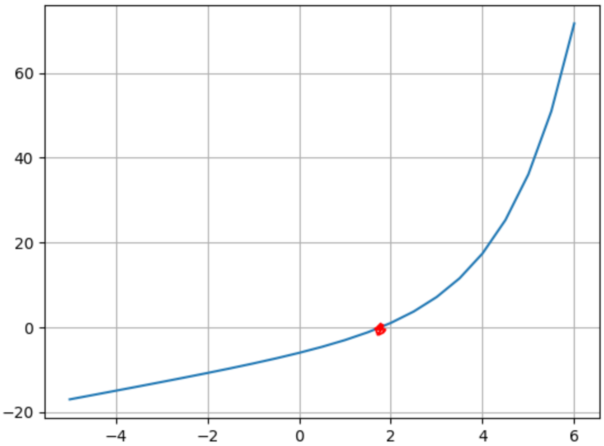
Find the root of the following equation:
$y = f(x) = e^{0.7x} + 2x - 7 = 0$
by using the False Position Method with initial $x_l = -3$, $x_u = 5$, and tolerance $\varepsilon = 10^{-3}$

Solution:



$$x_r = \frac{x_l \cdot f(x_u) - x_u \cdot f(x_l)}{f(x_u) - f(x_l)} = \frac{-3 \cdot f(5) - 5 \cdot f(-3)}{f(5) - f(-3)} = -0.89724$$

$y_r = f(x_r = -0.89724) = -8.26087$
Since $|y_r| > \varepsilon$, then we continue to the iteration

Iteration 1: ---
1. Shrinking the span
$y_l = f(x_l = -3) = e^{0.7\cdot(-3)} + 2 \cdot (-3) - 7 = -12.87754$
$y_r = f(x_r = -0.89724) = e^{0.7\cdot(-0.89724)} + 2 \cdot (-0.89724) - 7 = -8.26087$
$y_u = f(x_u = 5) = e^{0.7\cdot(5)} + 2 \cdot (5) - 7 = 36.11545$
Since the sign of $f(x_r = -0.89724)$ is the same as that of $f(x_l = -3)$, then we set $x_l \leftarrow x_r$ and $y_l \leftarrow y_r$.
Now, we have $x_l = -0.89724$, $x_u = 5$.

2. Updating x_r

$$x_r = \frac{x_l \cdot f(x_u) - x_u \cdot f(x_l)}{f(x_u) - f(x_l)} = \frac{-0.89724 \cdot f(5) - 5 \cdot f(-0.89724)}{f(5) - f(-0.89724)} = 0.65282$$

Now, we have triplet numbers: $x_l = -0.89724$, $x_r = 0.65282, x_u = 5$
$y_r = f(x_r = 0.65282) = -4.11509$
Since $|y_r| > \varepsilon$, then we continue the iteration

Iteration 2: ---
1. Shrinking the span
$y_l = f(x_l = -0.89724) = -8.26087$
$y_r = f(x_r = 0.65282) = -4.11509$
$y_u = f(x_u = 5) = 36.11545$
Since the sign of $f(x_r = 0.65282)$ is the same as that of $f(x_l = -0.89724)$, then we set $x_l \leftarrow x_r$ and $y_l \leftarrow y_r$.
Now, we have $x_l = 0.65282$, $x_u = 5$.

2. Updating x_r

$$x_r = \frac{x_l \cdot f(x_u) - x_u \cdot f(x_l)}{f(x_u) - f(x_l)} = \frac{0.65282 \cdot f(5) - 5 \cdot f(0.65282)}{f(5) - f(0.65282)} = 1.46206$$

Now, we have triplet numbers: $x_l = 0.65282$, $x_r = 1.46206, x_u = 5$

$y_r = f(x_r = 1.46206) = -1.29311$

Since $|y_r| > \varepsilon$, then we continue the iteration

Iteration 3: ---

1. Shrinking the span

$y_l = f(x_l = 0.65282) = -4.11509$

$y_r = f(x_r = 1.46206) = -1.29311$

$y_u = f(x_u = 5) = 36.11545$

Since the sign of $f(x_r = 1.46206)$ is the same as that of $f(x_l = 0.65282)$, then we set $x_l \leftarrow x_r$ and $y_l \leftarrow y_r$.

Now, we have $x_l = 1.46206$, $x_u = 5$.

2. Updating x_r

$$x_r = \frac{x_l \cdot f(x_u) - x_u \cdot f(x_l)}{f(x_u) - f(x_l)} = \frac{1.46206 \cdot f(5) - 5 \cdot f(1.46206)}{f(5) - f(1.46206)} = 1.82395$$

Now, we have triplet numbers: $x_l = 1.46206$, $x_r = 1.82395, x_u = 5$

$y_r = f(x_r = 1.82395) = 0.23293$

Since $|y_r| > \varepsilon$, then we continue the iteration

(Let's skip the detail of iteration 4 - 6 and go ahead to iteration 7)

Iteration 7: ---

1. Shrinking the span

$y_l = f(x_l = 1.77062) = -0.00509$

$y_r = f(x_r = 1.81554) = 0.19506$

$y_u = f(x_u = 1.82395) = 0.23293$

Since the sign of $f(x_r = 1.81554)$ is the same as that of $f(x_u = 1.82395)$, then we set $x_u \leftarrow x_r$ and $y_l \leftarrow y_r$.

Now, we have $x_l = 1.77062$, $x_u = 1.81554$.

2. Updating x_r

$$x_r = \frac{x_l \cdot f(x_u) - x_u \cdot f(x_l)}{f(x_u) - f(x_l)} = \frac{1.77062 \cdot f(1.81554) - 1.81554 \cdot f(1.77062)}{f(1.81554) - f(1.77062)} = 1.77158$$

Now, we have triplet numbers: $x_l = 1.77062$, $x_r = 1.77158, x_u = 1.81554$

$y_r = f(x_r = 1.77158) = -0.00085$

Since $|y_r| > \varepsilon$, then we stop the iteration, and the we have the solution $x_r = 1.77158$