Bisection Method Example

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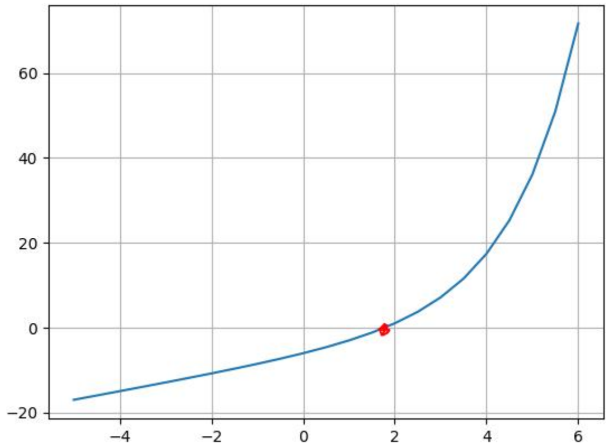
Find the root of the following equation:

$y = f(x) = e^{0.7x} + 2x - 7 = 0$

by using the Bisection Method with initial $x_l = -3$, $x_u = 5$, and tolerance $\varepsilon = 10^{-3}$

Solution:



$$x_r = \frac{x_l + x_u}{2} = \frac{-3 + 5}{2} = 1$$

$y_r = f(x_r = 1) = e^{0.7 \cdot 1} + 2 \cdot 1 - 7 = -2.98625$

Since $|y_r| > \varepsilon$, (ε=stopping criterion / tolerance) then we continue to the iteration

Iteration 1: ---

1. Shrinking the span

$y_l = f(x_l = -3) = e^{0.7 \cdot (-3)} + 2 \cdot (-3) - 7 = -12.87754$

$y_r = f(x_r = 1) = e^{0.7 \cdot (1)} + 2 \cdot (1) - 7 = -2.98625$

$y_u = f(x_u = 5) = e^{0.7 \cdot (5)} + 2 \cdot (5) - 7 = 36.11545$

Since the sign of $f(x_r = 1)$ is the same as that of $f(x_l = -3)$, then we set $x_l \leftarrow x_r$.

Now, we have $x_l = 1$, $x_u = 5$.

2. Updating x_r

$$x_r = \frac{x_l + x_u}{2} = \frac{1 + 5}{2} = 3$$

Now, we have triplet numbers: $x_l = 1$, $x_r = 3$, $x_u = 5$

$y_r = f(x_r = 3) = e^{0.7 \cdot (3)} + 2 \cdot (3) - 7 = 7.16617$

Since $|y_r| > \varepsilon$, then we continue the iteration

Iteration 2: ---

1. Shrinking the span

$f(x_l = 1) = -2.98625$ (has been calculated)

$f(x_r = 3) = 7.16617$ (has been calculated)

$f(x_u = 5) = 36.11545$ (has been calculated)

Since the sign of $f(x_r = 3)$ is the same as that of $f(x_u = 5)$, the we set $x_u \leftarrow x_r$.

Now, we have $x_l = 1$, $x_u = 3$.

2. Updating x_r

$$x_r = \frac{x_l + x_u}{2} = \frac{1 + 3}{2} = 2$$

Now, we have triplet numbers: $x_l = 1, x_r = 2, x_u = 3$
$y_r = f(x_r = 2) = e^{0.7\cdot(2)} + 2\cdot(2) - 7 = 1.05520$
Since $|y_r| > \varepsilon$, then we continue the iteration

Iteration 3: ---
1. Shrinking the span
$f(x_l = 1) = -2.98625$
$f(x_r = 2) = 1.05520$
$f(x_u = 3) = 7.16617$
Since the sign of $f(x_r = 2)$ is the same as that of $f(x_u = 3)$, the we set $x_u \leftarrow x_r$.
Now, we have $x_l = 1, x_u = 2$.

2. Updating x_r
$$x_r = \frac{x_l + x_u}{2} = \frac{1+2}{2} = 1.5$$
Now, we have triplet numbers: $x_l = 1, x_r = 1.5, x_u = 2$
$y_r = f(x_r = 1.5) = e^{0.7\cdot(1.5)} + 2\cdot(1.5) - 7 = -1.14235$
Since $|y_r| > \varepsilon$, then we continue the iteration

(Let's skip the detail of iteration 4 - 12 and go ahead to iteration 13)

Iteration 13: ---
1. Shrinking the span
$f(x_l = 1.77148) = -0.00128$
$f(x_r = 1.77246) = 0.00303$
$f(x_u = 1.77344) = 0.00735$
Since the sign of $f(x_r = 1.77246)$ is the same as that of $f(x_u = 1.77344)$, the we set $x_u \leftarrow x_r$.
Now, we have $x_l = 1.77148, x_u = 1.77246$.

2. Updating x_r
$$x_r = \frac{x_l + x_u}{2} = \frac{1.77148 + 1.77246}{2} = 1.77197$$
Now, we have triplet numbers: $x_l = 1.77148, x_r = 1.77197, x_u = 1.77246$
$y_r = f(x_r = 1.77197) = 0.00088$
Since $|y_r| < \varepsilon$, then we stop the iteration, and the we have the solution $x_r = 1.77197$