

# Change of Basis and Model-view Transformation

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- CS174A, discussion 1A, Winter 2020.
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- <https://github.com/luckiday/cs174a-1a-2020w>

## Outline

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- Change of basis
- Model-view Transformation

## Change of basis

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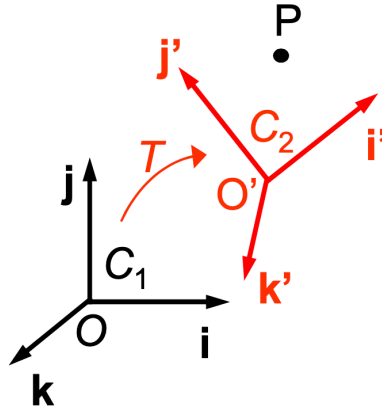
**Recall:** (Coordinate Systems) In homogeneous coordinate systems, a vector is denoted as:

$$\mathbf{v} = v_1 \mathbf{a} + v_2 \mathbf{b} + v_3 \mathbf{c} \rightarrow \mathbf{v} = [\mathbf{a} \quad \mathbf{b} \quad \mathbf{c} \quad O] \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ 0 \end{bmatrix}$$

A point is

$$P = p_1 \mathbf{a} + p_2 \mathbf{b} + p_3 \mathbf{c} + O \rightarrow P = [\mathbf{a} \quad \mathbf{b} \quad \mathbf{c} \quad O] \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ 1 \end{bmatrix}$$

## Transformations as a change of basis



**Question:** if we apply a transformation,  $M_1$ , to coordinate system  $C_1$ , and get  $C_2$ , what is the new position of  $P$  in  $C_2$ ?

We use  $[x, y, z, 1]^T$  to represent  $P$ 's position in  $C_1$ , and  $[x', y', z', 1]^T$  to represent  $P$ 's position in  $C_2$ .

**Solution:**

In coordinate system  $C_1$ :

$$P = x\mathbf{i} + y\mathbf{j} + z\mathbf{k} + O = [\mathbf{i} \quad \mathbf{j} \quad \mathbf{k} \quad O] \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

In coordinate system  $C_2$ :

$$P = x'\mathbf{i}' + y'\mathbf{j}' + z'\mathbf{k}' + O' = [\mathbf{i}' \quad \mathbf{j}' \quad \mathbf{k}' \quad O'] \begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix}$$

Since  $C_2$  is transformed from  $C_1$  with  $M_1$ ,

$$M_1 [\mathbf{i} \quad \mathbf{j} \quad \mathbf{k} \quad O] = [\mathbf{i}' \quad \mathbf{j}' \quad \mathbf{k}' \quad O'] \quad (*1)$$

Hence

$$[\mathbf{i} \quad \mathbf{j} \quad \mathbf{k} \quad O] \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = M_1 [\mathbf{i} \quad \mathbf{j} \quad \mathbf{k} \quad O] \begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix}$$

Since  $[\mathbf{i} \quad \mathbf{j} \quad \mathbf{k} \quad O]$  is the identity matrix, we have

$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = M_1 \begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} \rightarrow P_{C_1} = M_1 P_{C_2}$$

i.e.,  $P_{C_2} = M_1^{-1} P_{C_1}$

**Question:** what is  $M_1$  with respect to the basis vectors?

Referring to Eq. (\*1), and since  $\begin{bmatrix} i & j & k & O \end{bmatrix}$  is the identity matrix:

$$M_1 = \begin{bmatrix} i' & j' & k' & O' \end{bmatrix} = \begin{bmatrix} i'_x & j'_x & k'_x & O'_x \\ i'_y & j'_y & k'_y & O'_y \\ i'_z & j'_z & k'_z & O'_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

## Successive transformations

If we transform  $C_2$  to  $C_3$  with  $M_2$ . We consider the  $\begin{bmatrix} i' & j' & k' & O' \end{bmatrix}$  as the identity matrix in the section transformation, then,

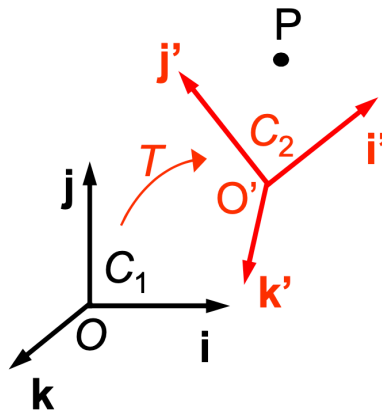
$$P_{C_2} = M_2 P_{C_3}$$

Therefore,

$$P_{C_1} = M_1 P_{C_2} = M_1 M_2 P_{C_3}$$

Notice:  $\begin{bmatrix} i'' & j'' & k'' & O'' \end{bmatrix}$  of  $C_3$  is the matrix representation in the  $C_2$ .

## Coordinate system transformation with rotation and transformation



Since the  $i'$ ,  $j'$ , and  $k'$  are orthogonal, the rotation from  $C_1$  to  $C_2$  is

$$R = \begin{bmatrix} i'_x & j'_x & k'_x & 0 \\ i'_y & j'_y & k'_y & 0 \\ i'_z & j'_z & k'_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The translation from  $O$  to  $O'$  is

$$T = \begin{bmatrix} 1 & 0 & 0 & O'_x \\ 0 & 1 & 0 & O'_y \\ 0 & 0 & 1 & O'_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

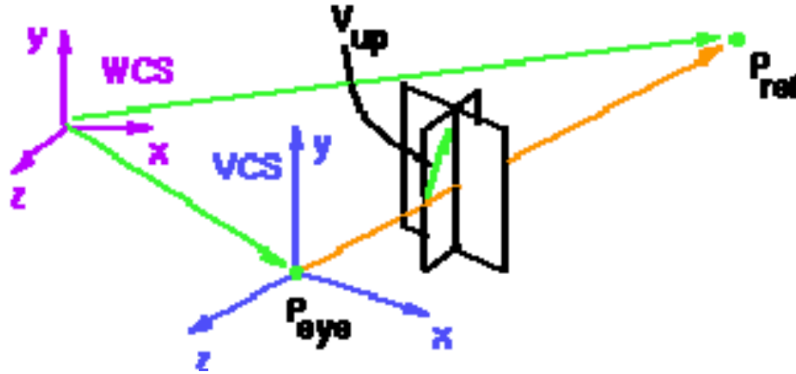
Thus,

$$M_1 = TR = \begin{bmatrix} i'_x & j'_x & k'_x & O'_x \\ i'_y & j'_y & k'_y & O'_y \\ i'_z & j'_z & k'_z & O'_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$P$ 's position in  $C_2$ ,  $[x', y', z', 1]^T$ , can be calculated with the inverse of the transformations

$$P_{C_2} = M_1^{-1} P_{C_1} = R^{-1} T^{-1} P_{C_1} = \begin{bmatrix} i'_x & j'_x & k'_x & 0 \\ i'_y & j'_y & k'_y & 0 \\ i'_z & j'_z & k'_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^T \begin{bmatrix} 1 & 0 & 0 & -O'_x \\ 0 & 1 & 0 & -O'_y \\ 0 & 0 & 1 & -O'_z \\ 0 & 0 & 0 & 1 \end{bmatrix} P_{C_1}$$

## Model-view transformation



Given eye point  $P_{eye}$ , reference point  $P_{ref}$ , and up vector  $v_{up}$ , build  $M_{cam}$  for model-view transformation.

$$\begin{aligned} k' &= \frac{P_{eye} - P_{ref}}{|P_{eye} - P_{ref}|} \\ i' &= \frac{v_{up} \times k'}{|v_{up} \times k'|} \\ j' &= k' \times i' \end{aligned}$$

The new origin  $O' = P_{eye}$ . Thus,

$$M_{cam} = \begin{bmatrix} i'_x & j'_x & k'_x & O'_x \\ i'_y & j'_y & k'_y & O'_y \\ i'_z & j'_z & k'_z & O'_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Transform from WCS (world coordinate system) to VCS (view coordinate system)

$$P_{vcs} = M_{cam}^{-1} P_{wcs} = \begin{bmatrix} i'_x & j'_x & k'_x & 0 \\ i'_y & j'_y & k'_y & 0 \\ i'_z & j'_z & k'_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^T \begin{bmatrix} 1 & 0 & 0 & -O'_x \\ 0 & 1 & 0 & -O'_y \\ 0 & 0 & 1 & -O'_z \\ 0 & 0 & 0 & 1 \end{bmatrix} P_{wcs}$$

## Quiz

Using the following WCS information about the camera location and orientation, derive the 4x4 homogeneous matrix,  $M_{cam}$ , needed to transform points from WCS to VCS.

- Camera location is located at:  $(-5, 0, 5)$
- $P_{ref}$  is the origin:  $(0, 0, 0)$
- Up vector is the y-axis:  $(0, 1, 0)$