



EECE321 Project

Testing Assembly Codes and Comparing to C++ Values

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May 2024

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1 Testing Classical Gram-Schmidt

1.1 Figures for Q Matrix in Assembly

Address	+0	+1	+2	+3
0x100000A8	69	41	B2	BE
0x100000A4	3A	CD	13	3F
0x100000A0	05	FB	96	BE
0x1000009C	C1	D9	4C	3F
0x10000098	0F	B1	05	3F
0x10000094	00	00	00	00
0x10000090	08	FB	16	3E
0x1000008C	BD	D9	CC	BE
0x10000088	6A	41	32	3F
0x10000084	3A	CD	13	3F

0x100000C0	C8	B9	3C	BF
0x100000BC	D4	16	36	BD
0x100000B8	69	41	B2	BE
0x100000B4	3A	CD	13	3F
0x100000B0	07	FB	16	3F
0x100000AC	9E	9C	E3	3E
0x100000A8	69	41	B2	BE

Figure 1: Q Matrix in Assembly - Part 1 and Part 2

1.2 Figures for R Matrix in Assembly

Address	+0	+1	+2	+3
0x100000F0	06	54	9F	BE
0x100000EC	74	E0	82	3F
0x100000E8	00	00	00	00
0x100000E4	00	00	00	00
0x100000E0	31	67	16	40
0x100000DC	53	1D	A7	3F
0x100000D8	F1	19	F5	3F
0x100000D4	00	00	00	00
0x100000D0	D7	B3	5D	40
0x100000CC	66	90	8A	40
0x100000C8	3A	CD	93	3F
0x100000C4	D7	B3	DD	3F

0x10000100	28	99	CF	3F
0x100000FC	00	00	00	00
0x100000F8	00	00	00	00
0x100000F4	00	00	00	00
0x100000F0	06	54	9F	BE

Figure 2: R Matrix in Assembly - Part 1 and Part 2

1.3 C++ Code Results

```
Classical Gram-Schmidt Q Matrix:

[0.57735, 0.696311, -0.400099, 0.147442]
[0, 0.522233, 0.800198, -0.294884]
[0.57735, -0.348155, 0.444554, 0.589768]
[0.57735, -0.348155, -0.0444554, -0.73721]

Classical Gram-Schmidt R Matrix:

[1.73205, 1.1547, 4.33013, 3.4641]
[0, 1.91485, 1.30558, 2.35005]
[0, 0, 1.02247, -0.311188]
[0, 0, 0, 1.62186]
```

Figure 3: C++ Code Results

1.4 Q and R Matrices from C++ Code

$$Q = \begin{bmatrix} 0.57735 & 0.696311 & -0.400099 & 0.147442 \\ 0 & 0.522233 & 0.800198 & -0.294884 \\ 0.57735 & -0.348155 & 0.444554 & 0.589768 \\ 0.57735 & -0.348155 & -0.044554 & -0.73721 \end{bmatrix}$$

$$R = \begin{bmatrix} 1.73205 & 1.1547 & 4.33013 & 3.4641 \\ 0 & 1.91485 & 1.30558 & 2.35005 \\ 0 & 0 & 1.02247 & -0.311188 \\ 0 & 0 & 0 & 1.62186 \end{bmatrix}$$

2 Testing Modified Gram-Schmidt

2.1 Figures for Q Matrix in Assembly

Address	+0	+1	+2	+3
0x10000A8	69	41	B2	BE
0x10000A4	3A	CD	13	3F
0x10000A0	06	FB	96	BE
0x100009C	C0	D9	4C	3F
0x1000098	0F	B1	05	3F
0x1000094	00	00	00	00
0x1000090	0B	FB	16	3E
0x100008C	BA	D9	CC	BE
0x1000088	6A	41	32	3F
0x1000084	3A	CD	13	3F

0x10000C0	C7	B9	3C	BF
0x10000BC	BD	16	36	BD
0x10000B8	69	41	B2	BE
0x10000B4	3A	CD	13	3F
0x10000B0	06	FB	16	3F
0x10000AC	A1	9C	E3	3E

Figure 4: Q Matrix in Assembly - Part 1 and Part 2

2.2 Figures for R Matrix in Assembly

Address	+0	+1	+2	+3
0x10000F0	04	54	9F	BE
0x10000EC	74	E0	82	3F
0x10000E8	00	00	00	00
0x10000E4	00	00	00	00
0x10000E0	31	67	16	40
0x10000DC	53	1D	A7	3F
0x10000D8	F1	19	F5	3F
0x10000D4	00	00	00	00
0x10000D0	D7	B3	5D	40
0x10000CC	66	90	8A	40
0x10000C8	3A	CD	93	3F
0x10000C4	D7	B3	DD	3F

0x10000100	28	99	CF	3F
0x10000FC	00	00	00	00
0x10000F8	00	00	00	00
0x10000F4	00	00	00	00
0x10000F0	04	54	9F	BE

Figure 5: R Matrix in Assembly - Part 1 and Part 2

2.3 C++ Code Results

```
Modified Gram-Schmidt Q Matrix:

[0.57735, 0.696311, -0.400099, 0.147442]
[0, 0.522233, 0.800198, -0.294884]
[0.57735, -0.348155, 0.444554, 0.589768]
[0.57735, -0.348155, -0.0444554, -0.73721]

Modified Gram-Schmidt R Matrix:

[1.73205, 1.1547, 4.33013, 3.4641]
[0, 1.91485, 1.30558, 2.35005]
[0, 0, 1.02247, -0.311188]
[0, 0, 0, 1.62186]
```

Figure 6: C++ Code Results

2.4 Q and R Matrices from C++ Code

$$Q = \begin{bmatrix} 0.57735 & 0.696311 & -0.400099 & 0.147442 \\ 0 & 0.522233 & 0.800198 & -0.294884 \\ 0.57735 & -0.348155 & 0.444554 & 0.589768 \\ 0.57735 & -0.348155 & -0.044554 & -0.73721 \end{bmatrix}$$

$$R = \begin{bmatrix} 1.73205 & 1.1547 & 4.33013 & 3.4641 \\ 0 & 1.91485 & 1.30558 & 2.35005 \\ 0 & 0 & 1.02247 & -0.311188 \\ 0 & 0 & 0 & 1.62186 \end{bmatrix}$$

3 Generalized Using Logbase2

```
C:\Users\AUB\Desktop> 321_project > - generalized_deliverable2.s

380      addi x16,x0,0
381      lop3:
382      bge x16,x13,end_lop3
383      sll x19,x16,x31 ## i*4
384      add x19,x19,x15 ## i*4+k
385      slli x19,x19,2 ## [i][k]
386      add x20,x19,x11 ## x20=8Q[i][k]
387      flw f1,0(x20) ## f1=Q[i][k]
388      fdiv.s f1,f1,f0 ## Q[i][k]/R[k][k]
389      fsw f1,0(x20) ## updated Q[i][k]
390      addi x16,x16,1
391      j lop3
392      end_lop3:
393      addi x30,x15,1 ## j=k+1
394      lop4:
395      bge x30,x13,end_lop4
396      sll x20,x15,x31 ## k*4
397      add x20,x20,x30 ## k*4+j
398      slli x20,x20,2 ## [k][j]
399      add x20,x20,x14 ## &R[k][j]
400      la x7,zero_float
401      flw f5,0(x7)
402      fsw f5,0(x20) ## R[k][j]=0
403      addi x16,x0,0 ## i=0
404      lop5: ## x20=&R[k][j]
405      bge x16,x13,end_lop5
406      sll x19,x16,x31 ## x19=i*4
407      add x22,x19,x15 ## x20=i*4+k
408      slli x22,x22,2 ## [i][k]
409      add x21,x19,x30 ## x21=i*4+j
410      slli x21,x21,2 ## [i][j]
411      add x22,x22,x11 ## &Q[i][k]
412      add x21,x21,x11 ## &Q[i][j]
413      flw f0,0(x21) ## f0=Q[i][j]
414      flw f1,0(x22) ## f1=Q[i][k]
415      flw f2,0(x20) ## f2=R[k][j]
416      fmul.s f0,f0,f1 ## Q[i][k]*Q[i][j]
```

Figure 7: Generalized Logbase2 - Figure 1


```

logbase2: ## final value in x11, input in x14
addi sp,sp,-12
sw x10,0(sp)
sw x6,4(sp)
sw x1,8(sp)
li x10,1 ## initial , maybe its 2 so logbase2(2)=1
li x6,2 ## start with 2

d1:
beq x6,x14,doned1
slli x6,x6,1
addi x10,x10,1
j d1
doned1:
addi x11,x10,0
lw x10,0(sp)
lw x6,4(sp)
lw x1,8(sp)
jalr x0,0(x1)

```

Figure 8: Generalized Logbase2 - Figure 2

4 Unrolling

4.1 Unrolling One Loop

```
addi x16,x0,0 ## i=0
lop5: ## x20=&R[k][j]
bge x16,x13,end_lop5
slli x19,x16,2 ## x19=i*4
add x22,x19,x15 ## x20=i*4+k
slli x22,x22,2 ## [i][k]
add x21,x19,x30 ## x21=i*4+j
slli x21,x21,2 ## [i][j]
add x22,x22,x11 ## &Q[i][k]
add x21,x21,x11 ## &Q[i][j]
flw f0,0(x21) ## f0=Q[i][j]
flw f1,0(x22) ## f1=Q[i][k]
flw f2,0(x20) ## f2=R[k][j]
fmul.s f0,f0,f1 ## Q[i][k]*Q[i][j]
fadd.s f2,f0,f2 ## R[k][j]+Q[i][k]*Q[i][j]
fsw f2,0(x20) ## R[k][j] += Q[i][k] * Q[i][j]
addi x16,x16,1
j lop5
end_lop5:
```

Figure 9: Before Unrolling Loop

```

## First iteration that can be done in independent core
li x19,0 ## [0]
add x22,x19,x15 ## 0+k
slli x22,x22,2 ## [0][k] single precision
add x21,x19,x30 ## 0+j
slli x21,x21,2 ## [0][j]
add x22,x22,x11 ## &Q[0][k]
add x21,x21,x11 ## &Q[0][j]
flw f0,0(x22) ## f0=Q[0][k]
flw f1,0(x21) ## f1=Q[0][j]
flw f2,0(x20) ## f2=R[k][j]
fmul.s f0,f0,f1 ## Q[0][k]*Q[0][j]
fadd.s f2,f0,f2 ## R[k][j]+Q[0][k]*Q[0][j]
fsw f2,0(x20) ## R[k][j] += Q[0][k] * Q[0][j]
##

##Second iteration that can be done in independent core
li x19,4 ## [1]
add x22,x19,x15 ## 4+k
slli x22,x22,2 ## [1][k] single precision
add x21,x19,x30 ## 1+j
slli x21,x21,2 ## [1][j]
add x22,x22,x11 ## &Q[1][k]
add x21,x21,x11 ## &Q[1][j]
flw f0,0(x22) ## f0=Q[1][k]
flw f1,0(x21) ## f1=Q[1][j]
flw f2,0(x20) ## f2=R[k][j]
fmul.s f0,f0,f1 ## Q[1][k]*Q[1][j]
fadd.s f2,f0,f2 ## R[k][j]+Q[1][k]*Q[1][j]
fsw f2,0(x20) ## R[k][j] += Q[1][k] * Q[1][j]
##

```

Figure 10: After Unrolling Loop

4.2 Unrolling Multiple Loops

```
9  lopp6: ## x20=& R[k][j]
10 bge x16,x13,end_lopp6
11 slli x22,x16,2 ## i*4
12 add x22,x22,x30 ## i*4+j
13 slli x22,x22,2 ## [i][j]
14 slli x21,x16,2 ## i*4
15 add x21,x21,x15 ## i*4+k
16 slli x21,x21,2 ## [i][k]
17 add x22,x22,x11 ## &Q[i][j]
18 add x21,x21,x11 ## &Q[i][k]
19 flw f0,0(x21) ## Q[i][k]
20 flw f1,0(x22)## Q[i][j]
21 flw f2,0(x20) ## R[k][j]
22 fmul.s f2,f0,f2 ## R[k][j] * Q[i][k]
23 fsub.s f1,f1,f2 ## Q[i][j] - R[k][j] * Q[i][k]
24 fsw f1,0(x22) ## Q[i][j] -= R[k][j] * Q[i][k]
25 addi x16,x16,1
26 j lopp6
```

Figure 11: Before Unrolling second Loop - Figure 3

```

## First iteration that could be done on independent cores
##, once R[k][j] is finalized above
li x22,0 ## i=0
add x22,x22,x30 ## i*0+j
slli x22,x22,2 ## [0][j]
li x21,0
add x21,x21,x15 ## i*0+k
slli x21,x21,2 ## [0][k]
add x22,x22,x11 ## &Q[0][j]
add x21,x21,x11 ## & Q[0][k]
flw f0,0(x21) ## Q[0][k]
flw f1,0(x22)## Q[0][j]
flw f2,0(x20) ## R[k][j]
fmul.s f2,f0,f2 ## R[k][j] * Q[0][k]
fsub.s f1,f1,f2 ## Q[0][j] - R[k][j] * Q[0][k]
fsw f1,0(x22) ## Q[0][j] -= R[k][j] * Q[0][k]
##

## Second iteration that could be done on independent cores
##, once R[k][j] is finalized above
li x22,4 ## i=1
add x22,x22,x30 ## 4+j
slli x22,x22,2 ## [1][j]
li x21,4
add x21,x21,x15 ## 4+k
slli x21,x21,2 ## [1][k]
add x22,x22,x11 ## &Q[1][j]
add x21,x21,x11 ## & Q[1][k]
flw f0,0(x21) ## Q[1][k]
flw f1,0(x22)## Q[1][j]
flw f2,0(x20) ## R[k][j]
fmul.s f2,f0,f2 ## R[k][j] * Q[1][k]
fsub.s f1,f1,f2 ## Q[1][j] - R[k][j] * Q[1][k]
fsw f1,0(x22) ## Q[1][j] -= R[k][j] * Q[1][k]
##

```

Figure 12: After Unrolling second Loop - Figure 4

5 QR Extension

```

1226 solveforx: ## given y is in x8, H is in x24
1227 ## QR DECOMPOSE H
1228 ## where modified takes A in x10,
1229 ## and result it in x11
1230 ## and x14
1231 addi x10,x24,0 ## A--> H
1232 call modified_granny ## now x11--> Q, x14-> R
1233 addi x10,x23,0 ## result in Q_t
1234 addi x26,x11,0 ## input Q in x26
1235 call transposeMatrix ## our result in x10
1236 ## auto auto tghyrt ma3a x23 , same address :)
1237 ## our y is in x8, our Q_t is in x10
1238 addi x11,x25,0 ## now x11 holds new addres for y_prime
1239 addi x24,x8,0 ## now x24--> holds y
1240 addi x27,x10,0
1241 call multiplyMatrixVector
1242 ## y_prime is in x11, R--> x14
1243 addi x18,x11,0 ## y_prime --> x18
1244 addi x10,x28,0
1245 call backwardSubstitution ## our result in x10
1246 li a0,10
1247 ecall
1248

```

Figure 13: QR Extension - Assembly Code for Function

Address	+0	+1	+2	+3
0x10000180	BD	16	36	BD
0x1000017C	A1	9C	E3	3E
0x10000178	C0	D9	4C	3F
0x10000174	BA	D9	CC	BE
0x10000170	69	41	B2	BE
0x1000016C	69	41	B2	BE
0x10000168	0F	B1	05	3F
0x10000164	6A	41	32	3F
0x10000160	3A	CD	13	3F
0x1000015C	3A	CD	13	3F
0x10000158	00	00	00	00
0x10000154	3A	CD	13	3F

0x10000190	C7	B9	3C	BF
0x1000018C	06	FB	16	3F
0x10000188	06	FB	96	BE
0x10000184	0B	FB	16	3E
0x10000180	BD	16	36	BD

Figure 14: QR Extension - Transposing Q Matrix

0x10000140	A2	1B	84	BF
0x1000013C	8E	3E	33	40
0x10000138	C6	D1	5E	3F
0x10000134	30	3A	CB	40

Figure 15: QR Extension - Result of MultiplyMatrix Vector (y')

Address	+0	+1	+2	+3
0x10000150	B6	E8	22	BF
0x1000014C	C0	E8	22	40
0x10000148	13	00	00	BF
0x10000144	00	A3	8B	BF

Figure 16: QR Extension - Result of Backward Substitution (x)

```

Transposed Q Matrix:
[0.57735, 0, 0.57735, 0.57735]
[0.696311, 0.522233, -0.348155, -0.348155]
[-0.400099, 0.800198, 0.444554, -0.0444554]
[0.147442, -0.294884, 0.589768, -0.73721]

Result of Q_T * y (y_prime):
[6.35085, 0.870388, 2.80069, -1.03209]

Result of backward substitution (x):
[-1.09091, -0.5, 2.54545, -0.636364]

```

Figure 17: QR Extension - C++ Code Results

5.1 Transposed Q Matrix from C++ Code

$$Q^T = \begin{bmatrix} 0.57735 & 0 & -0.400099 & 0.147442 \\ 0.57735 & 0.522233 & 0.800198 & -0.294884 \\ 0.57735 & -0.348155 & 0.444554 & 0.589768 \\ 0.57735 & -0.348155 & -0.044554 & -0.73721 \end{bmatrix}$$

5.2 y' Vector from C++ Code

$$y' = \begin{bmatrix} 6.35085 \\ 0.870388 \\ 2.80069 \\ -1.03209 \end{bmatrix}$$

5.3 x Vector from C++ Code

$$x = \begin{bmatrix} -1.09091 \\ -0.5 \\ 2.54545 \\ -0.636364 \end{bmatrix}$$

6 Caching

```
17  loop2:
18      bge x16, x13, end_loop2
19      slli x19, x16, 2
20      add x19, x17, x19 # &q[i]
21      slli x20, x16, 2 #
22      add x20, x20, x15
23      slli x20, x20, 2
24      add x20, x20, x10 # &A[i][k]
25      flw f1, 0(x20)
26      fsw f1, 0(x19) # q[i] = A[i][k]
27      addi x16, x16, 1
28      j loop2
29  end_loop2:
```

Figure 18: Code Before Caching

```
390
391  ✓ loopcache:
392
393      bge x16, x13, end_loopcache
394      slli x19, x16, 2
395      add x19, x4, x19 # &a_cache[i]
396      slli x20, x16, 2 #
397      add x20, x20, x15
398      slli x20, x20, 2
399      add x20, x20, x11 # &q[i][k]
400      flw f1, 0(x20)
401      fsw f1, 0(x19) # a_cache[i] = A[i][k]
402      addi x16, x16, 1
403      j loopcache
404  end_loopcache:
405      li x16, 0
406  ✓ lop2:
407      bge x16, x13, end_lop2
408      slli x19, x16, 2
409      add x19, x17, x19 # &q[i]
410      slli x20, x16, 2 #
411      add x20, x20, x4
412      flw f1, 0(x20)
413      fsw f1, 0(x19) # q[i] = a_cache[i]
414      addi x16, x16, 1
415      j lop2
416  end_lop2:
417
418
```

Figure 19: Code After Caching

7 Verifications

7.1 First Identity: $Q \times R = A$

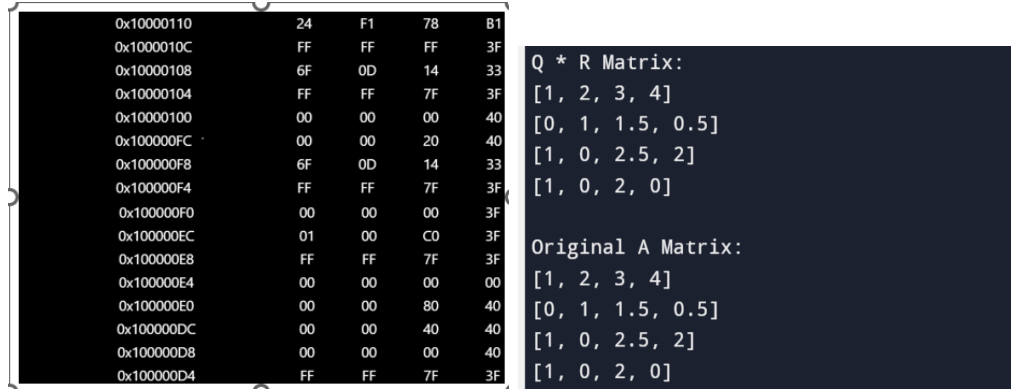


Figure 20: First Identity: $Q \times R = A$ - Assembly (left) and C++ (right)

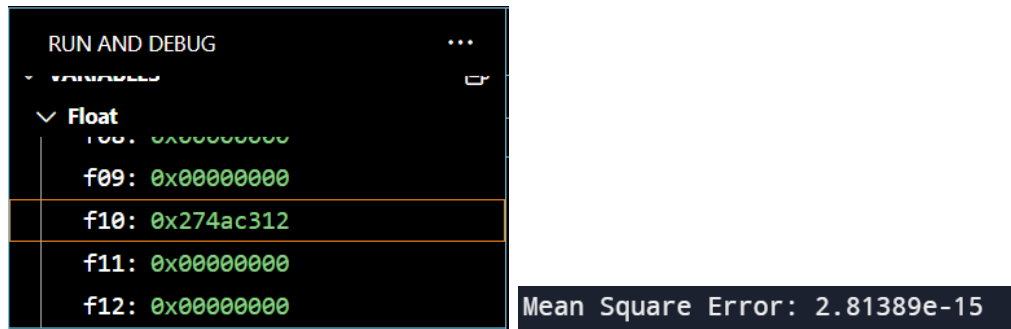


Figure 21: Mean Squared Error for $Q \times R = A$ - Assembly (left) and C++ (right)

7.2 Second Identity: $Q^T \times Q = I$

0x10000150	01	00	80	3F
0x1000014C	87	90	AB	32
0x10000148	43	48	02	33
0x10000144	3A	CD	13	33
0x10000140	87	90	AB	32
0x1000013C	00	00	80	3F
0x10000138	82	E3	41	33
0x10000134	7E	2D	F0	33
0x10000130	43	48	02	33
0x1000012C	82	E3	41	33
0x10000128	00	00	80	3F
0x10000124	3A	CD	13	33
0x10000120	3A	CD	13	33
0x1000011C	7E	2D	F0	33
0x10000118	3A	CD	13	33
0x10000114	FF	FF	7F	3F

Q_T * Q Matrix:

```
[1, -2.22045e-16, -4.96131e-16, -2.77556e-16]
[-2.22045e-16, 1, -1.78677e-16, -5.55112e-16]
[-4.96131e-16, -1.78677e-16, 1, 6.93889e-18]
[-2.77556e-16, -5.55112e-16, 6.93889e-18, 1]
```

Q_T * A Matrix:

```
[1.73205, 1.1547, 4.33013, 3.4641]
[-3.33067e-16, 1.91485, 1.30558, 2.35005]
[-8.88178e-16, -9.99201e-16, 1.02247, -0.311188]
[-5.55112e-16, -1.44329e-15, -1.9984e-15, 1.62186]
```

Figure 22: Second Identity: $Q^T \times Q = I$ - Assembly (left) and C++ (right)

RUN AND DEBUG	
▼ Float	
f09:	0x00000000
f10:	0x27743c9c
f11:	0x00000000
f12:	0x00000000

Mean Squared Error: 2.50129e-15

Figure 23: Mean Squared Error for $Q^T \times Q = I$ - Assembly (left) and C++ (right)

7.3 Third Identity: $Q^T \times A = R$

0x1000018C	00	00	70	34
0x10000188	00	00	C0	33
0x10000184	00	00	80	33
0x10000180	F7	53	9F	BE
0x1000017C	78	E0	82	3F
0x10000178	00	00	80	34
0x10000174	00	00	50	34
0x10000170	32	67	16	40
0x1000016C	54	1D	A7	3F
0x10000168	F2	19	F5	3F
0x10000164	00	00	80	33
0x10000160	D7	B3	5D	40
0x1000015C	66	90	8A	40
0x10000158	3A	CD	93	3F
0x10000154	D7	B3	DD	3F
0x10000150	01	00	80	3F

Q_T * A Matrix:

[1.73205, 1.1547, 4.33013, 3.4641]

[-3.33067e-16, 1.91485, 1.30558, 2.35005]

[-8.88178e-16, -9.99201e-16, 1.02247, -0.311188]

[-5.55112e-16, -1.44329e-15, -1.9984e-15, 1.62186]

[1.73205, 1.1547, 4.33013, 3.4641]

[0, 1.91485, 1.30558, 2.35005]

[0, 0, 1.02247, -0.311188]

[0, 0, 0, 1.62186]

Figure 24: Third Identity: $Q^T \times A = R$ - Assembly (left) and C++ (right)

RUN AND DEBUG	
▼ VARIABLES	
▼ Float	
f08:	0x00000000
f09:	0x00000000
f10:	0x294d2000
f11:	0x00000000
f12:	0x00000000

Mean Squared Error: 4.55469e-14

Figure 25: Mean Squared Error for $Q^T \times A = R$ - Assembly (left) and C++ (right)