

Q no 1.

Propositional Calculus.

(a) $P = \text{you go to Mexico.}$

$q = \text{you will be far away.}$

$$P \rightarrow q.$$

(b)

$P = \text{I hear you}$

$q = \text{you are far away.}$

$$q \rightarrow \neg P$$

(c)

$P = \text{I can hear you}$

$q = \text{I forgot what you look like.}$

$$\neg P \rightarrow q.$$

(d)

$P = \text{I come to Mexico}$

$q = \text{I know what you look like}$

$r = \text{I will be able to find you.}$

$$(P \wedge \neg q) \rightarrow \neg r$$

(e)

$p = \text{you go to Mexico.}$

$q = \text{I follow you.}$

$r = \text{I will be able to find you.}$

$$(p \wedge q) \rightarrow \neg r$$

Qno. 2.

(a) All dancers love to dance.

$$\forall x (\text{dancers}(x) \rightarrow \text{love}(\text{dance}, x))$$

(b) Everyone who sings and plays an instrument loves to dance.

$$\forall x ((\text{sing}(x) \& \text{plays}(x)) \rightarrow \text{love}(\text{dance}, x))$$

(c) All dancers sing and play an instrument.

$$\forall x (\text{dancer}(x) \rightarrow (\text{sing}(x) \& \text{play}(x)))$$

conclusion:

$$\text{let } \text{dancers}(x) = p$$

$$\text{love}(\text{dance}, x) = q$$

$$\text{sing}(x) \& \text{plays}(x) = r$$

from (a)

$$p \rightarrow q$$

from (b)

$$r \rightarrow q$$

using hypothetical syllogism

$$p \rightarrow r \text{ which is (c)}$$

Qno. 3.

(a)

$$\vdash A \rightarrow A$$

A

Assumption

$$A \rightarrow A$$

→ introduction

(b)

$$\vdash ((\neg A \rightarrow \neg B) \rightarrow A) \rightarrow ((\neg B \rightarrow \neg A) \rightarrow \neg B)$$

$$\neg A \rightarrow \neg B, \neg A$$

assumption (i)

$$\neg B$$

modus ponens (ii)

$$\neg B \rightarrow \neg A$$

→ introduction (iii)

A

assumption (iv)

$$(\neg A \rightarrow \neg B) \rightarrow A$$

→ introduction (i and iv) (a)

$$(\neg B \rightarrow \neg A) \rightarrow \neg B$$

→ introduction (iii and ii) (b)

$$((\neg A \rightarrow \neg B) \rightarrow A) \rightarrow ((\neg B \rightarrow \neg A) \rightarrow \neg B)$$

→ introduction (a & b)

(c)

$$\vdash (\neg\neg\neg A \rightarrow \neg\neg\neg B) \rightarrow (\neg A \rightarrow \neg B)$$

$$\neg A, \neg B$$

assumptions

$$\neg A \rightarrow \neg B$$

→ introduction - (a)

$$\neg\neg\neg A$$

→ Elimination

$$\neg\neg\neg B$$

→ Elimination

$$\neg\neg\neg A \rightarrow \neg\neg\neg B$$

→ introduction (b)

from (a) and (b)

$$(\neg\neg\neg A \rightarrow \neg\neg\neg B) \rightarrow (\neg A \rightarrow \neg B) \rightarrow \text{introduction}$$