What is Regression?

Regression is the process of estimating the relationship between input and output variables. One thing to note is that the output variables are continuous-valued real numbers. Hence there are an infinite number of possibilities. This is in contrast with classification, where the number of output classes is fixed. The classes belong to a finite set of possibilities.

In regression, it is assumed that the output variables depend on the input variables, so we want to see how they are related. Consequently, the input variables are called independent variables, also known as predictors, and output variables are called dependent variables, also known as criterion variables. It is not necessary that the input variables are independent of each other.

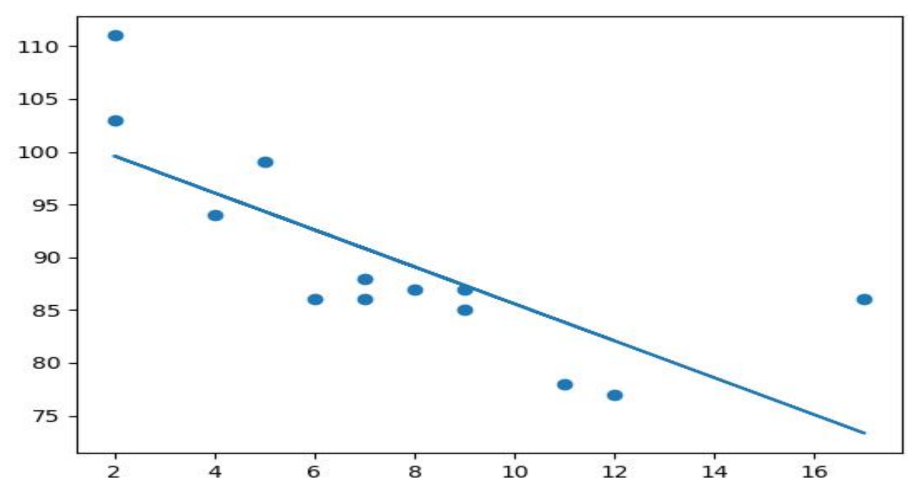
There are a lot of situations where there are correlations between input variables. Regression analysis helps us in understanding how the value of the output variable changes when we vary some input variables while keeping other input variables fixed. In linear regression, we assume that the relationship between input and output is linear. This puts a constraint on our modeling procedure, but it's fast and efficient.

Sometimes, linear regression is not sufficient to explain the relationship between input and output. Hence we use polynomial regression, where we use a polynomial to explain the relationship between input and output. This is more computationally complex, but gives higher accuracy. Depending on the problem at hand, we use different forms of regression to extract the relationship. Regression is frequently used for prediction of prices, economics, variations, and so on.

Linear Regression

Linear regression uses the relationship between the data-points to draw a straight line through all them.

This line can be used to predict future values.



In Machine Learning, predicting the future is very important.

How Does it Work?

Python has methods for finding a relationship between data-points and to draw a line of linear regression. We will show you how to use these methods instead of going through the mathematic formula.

In the example below, the x-axis represents age, and the y-axis represents speed. We have registered the age and speed of 13 cars as they were passing a tollbooth. Let us see if the data we collected could be used in a linear regression:

Example:

- Start by drawing a scatter plot:

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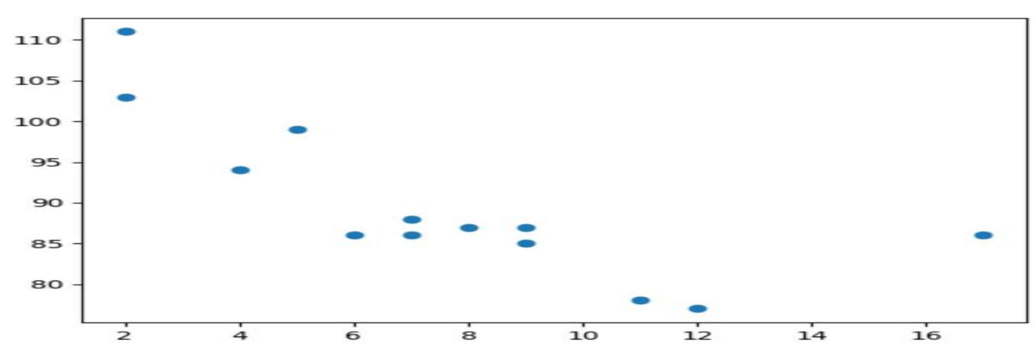
```
import matplotlib.pyplot as plt

x = [5,7,8,7,2,17,2,9,4,11,12,9,6]
y = [99,86,87,88,111,86,103,87,94,78,77,85,86]

plt.scatter(x, y)
plt.show()
```

- Result:

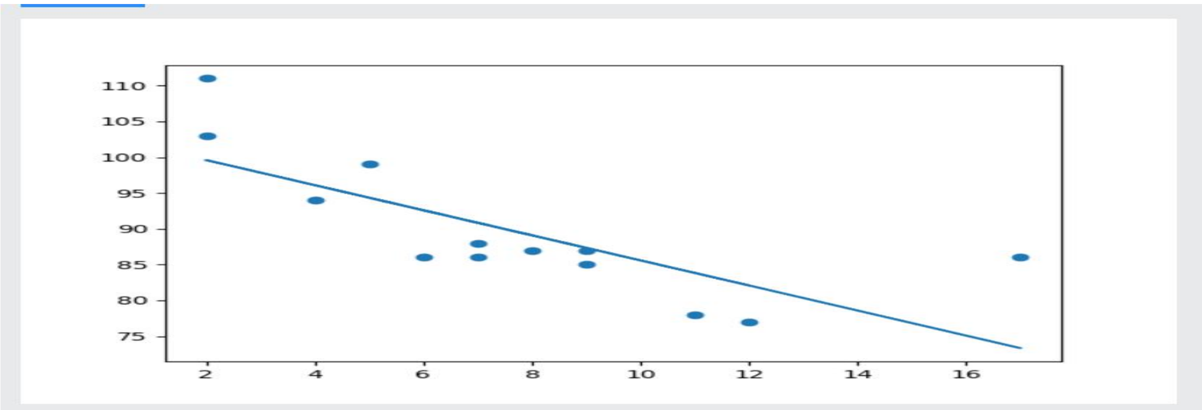


Example:

- Import scipy and draw the line of Linear Regression:

```
import matplotlib.pyplot as plt
from scipy import stats

x = [5,7,8,7,2,17,2,9,4,11,12,9,6]
y = [99,86,87,88,111,86,103,87,94,78,77,85,86]

slope, intercept, r, p, std_err = stats.linregress(x, y)

def myfunc(x):
  return slope * x + intercept

mymodel = list(map(myfunc, x))

plt.scatter(x, y)
plt.plot(x, mymodel)
plt.show()
```

- Result:



R for Relationship

It is important to know how the relationship between the values of the x-axis and the values of the y-axis is, if there are no relationship the linear regression cannot be used to predict anything.

This relationship - the coefficient of correlation - is called r. The r value ranges from -1 to 1, where 0 means no relationship, and 1 (and -1) means 100% related.

Python and the Scipy module will compute this value for you, all you have to do is feed it with the x and y values.

Note: The result -0.76 shows that there is a relationship, not perfect, but it indicates that we could use linear regression in future predictions.

Predict Future Values

Now we can use the information we have gathered to predict future values.

Example: Let us try to predict the speed of a 10 years old car. To do so, we need the same myfunc() function from the example above:

```
from scipy import stats

x = [5,7,8,7,2,17,2,9,4,11,12,9,6]
y = [99,86,87,88,111,86,103,87,94,78,77,85,86]

slope, intercept, r, p, std_err = stats.linregress(x, y)

def myfunc(x):
  return slope * x + intercept

speed = myfunc(10)

print(speed)
```

Bad Fit?

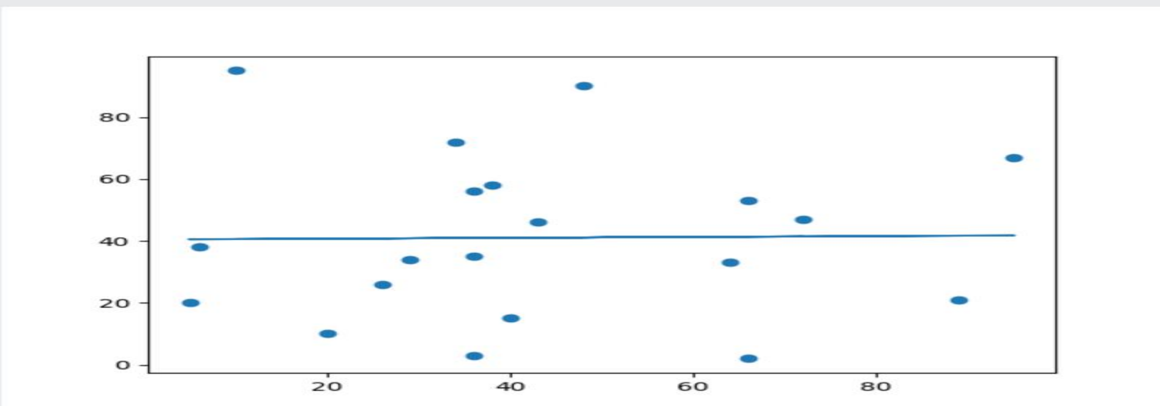
Let us create an example where linear regression would not be the best method to predict future values.

Example:

- These values for the x- and y-axis should result in a very bad fit for linear regression:

```
import matplotlib.pyplot as plt
from scipy import stats

x = [89,43,36,36,95,10,66,34,38,20,26,29,48,64,6,5,36,66,72,40]
y = [21,46,3,35,67,95,53,72,58,10,26,34,90,33,38,20,56,2,47,15]

slope, intercept, r, p, std_err = stats.linregress(x, y)

def myfunc(x):
  return slope * x + intercept

mymodel = list(map(myfunc, x))

plt.scatter(x, y)
plt.plot(x, mymodel)
plt.show()
```

Result:



And the r for relationship?

Example:

- You should get a very low r value.

The result: 0.013 indicates a very bad relationship, and tells us that this data set is not suitable for linear regression.

Finding the best fit line:

When working with linear regression, our main goal is to find the best fit line that means the error between predicted values and actual values should be minimized. The best fit line will have the least error.

The different values for weights or the coefficient of lines (a0, a1) gives a different line of regression, so we need to calculate the best values for a0 and a1 to find the best fit line, so to calculate this we use cost function.

Cost function-

The different values for weights or coefficient of lines (a0, a1) gives the different line of regression, and the cost function is used to estimate the values of the coefficient for the best fit line.

Cost function optimizes the regression coefficients or weights. It measures how a linear regression model is performing.

We can use the cost function to find the accuracy of the mapping function, which maps the input variable to the output variable. This mapping function is also known as Hypothesis function.

For Linear Regression, we use the Mean Squared Error (MSE) cost function, which is the average of squared error occurred between the predicted values and actual values. It can be written as:

For the above linear equation, MSE can be calculated as:

$$\text{MSE}= 1\frac{1}{N}\sum_{i=1}^{n}(y_i \ - (a_1x_i + a_0))^2$$

Where,

N=Total number of observation

Yi = Actual value

$(a_1x_i+a_0)$= Predicted value.

Residuals: The distance between the actual value and predicted values is called residual. If the observed points are far from the regression line, then the residual will be high, and so cost function will high. If the scatter points are close to the regression line, then the residual will be small and hence the cost function.

Gradient Descent:

- Gradient descent is used to minimize the MSE by calculating the gradient of the cost function.
- A regression model uses gradient descent to update the coefficients of the line by reducing the cost function.
- It is done by a random selection of values of coefficient and then iteratively update the values to reach the minimum cost function.

Model Performance:

The Goodness of fit determines how the line of regression fits the set of observations. The process of finding the best model out of various models is called optimization. It can be achieved by below method:

1. R-squared method:

- R-squared is a statistical method that determines the goodness of fit.
- It measures the strength of the relationship between the dependent and independent variables on a scale of 0-100%.
- The high value of R-square determines the less difference between the predicted values and actual values and hence represents a good model.
- It is also called a **coefficient of determination,** or **coefficient of multiple determination** for multiple regression.
- It can be calculated from the below formula:

$$\text{R-squared}=\frac{\text{Explained variation}}{\text{Total Variation}}$$

Assumptions of Linear Regression

Below are some important assumptions of Linear Regression. These are some formal checks while building a Linear Regression model, which ensures to get the best possible result from the given dataset.

Linear relationship between the features and target:

Linear regression assumes the linear relationship between the dependent and independent variables.

Small or no multicollinearity between the features:

Multicollinearity means high-correlation between the independent variables. Due to multicollinearity, it may difficult to find the true relationship between the predictors and target variables. Or we can say, it is difficult to determine which predictor variable is affecting the target variable and which is not. So, the model assumes either little or no multicollinearity between the features or independent variables.

Homoscedasticity Assumption:

Homoscedasticity is a situation when the error term is the same for all the values of independent variables. With homoscedasticity, there should be no clear pattern distribution of data in the scatter plot.

Normal distribution of error terms:

Linear regression assumes that the error term should follow the normal distribution pattern. If error terms are not normally distributed, then confidence intervals will become either too wide or too narrow, which may cause difficulties in finding coefficients.

It can be checked using the q-q plot. If the plot shows a straight line without any deviation, which means the error is normally distributed.

No autocorrelations:

The linear regression model assumes no autocorrelation in error terms. If there will be any correlation in the error term, then it will drastically reduce the accuracy of the model. Autocorrelation usually occurs if there is a dependency between residual errors.