

Data Structures & Algorithms

Time & Space Complexity

Time Complexity:

→ The time complexity of algorithm quantifies the time taken by the algorithm to run as a function of the length of the input.

→ Algorithm: time depends on the function of the length of the input, not on the machine in which it is running. How many operations executed depends, not how much.

Estimation of time Complexity:

need to consider the cost of each fundamental instruction and the number of times the instruction is executed.

Algorithm

```
Algorithm ADD SCALAR(A,B)
C ← A+B
return C
```

time Complexity = constant because of one operation

$$T(n) = O(1)$$

* Calculating time Complexity:

- i) a constant time C is taken to execute one operation
- ii) then the total operations for an input length on N are calculated.

* Consideration:

During analyses of the algorithm, mostly the worst-case scenario is considered.

Example for Calculating time-Complexity

Pseudo-code

```

int a[n];
for(int i=0; i < n; i++)
    cin >> a[i]
for(int i=0; i < n; i++)
    for(int j=0; j < n; j++)
        if(i != j && a[i] + a[j] == Z)
            return true
return false

```

Worst-Case Scenario: when there is no pair of elements with sum equals Z .

- i) $N * C$ operation required for input
- ii) The outer loop i loop runs N times.
- iii) For each i , the inner loop j loops runs N times.

Quadratically \Rightarrow its execution time grows as the square of the input size.

Day 01

$$\text{time execution} = N^*C + N^*N^*c + C$$

→ ignoring lower terms

→ taking higher term

$$\text{time Complexity} = O(N^2)$$

* Order of growth

is how the time of execution depends on the length of the input.

Note:

A quadratic time complexity is considered to be less efficient than a linear time complexity ($O(n)$).

$O(n^2)$

Another Example

```
int count = 0;  
for (int i=N; i>0; i/=2)  
    for (int j=0; j<i; j++)  
        count++;
```

how many time **Count++** will run.

- When $i=N$, it will run N times.

- When $i=N/2$, it will run $N/2$ times

- When $i=N/4$, it will run $N/4$ times.

Total NO of **Count++** will run
is $N + N/2 + N/4 + \dots + 1 = 2^*N$

Day 01

Time Complexity = $O(N)$

* Time Complexities with the input range for which they are accepted in competitive programming:

Input Length	Worst Accepted Time Complexity	Usually type of Solution
10-12	$O(N!)$	Recursion and Backtracking
15-18	$O(2^N * N)$	Recursion, backtracking and bit manipulation
18-22	$O(2^{N/2} * N)$	Recursion, backtracking and bit manipulation
30-40	$O(2^{N/2} * N)$	Meet in the middle, Divide and Conquer
100	$O(N^4)$	Dynamic programming and constructive
400	$O(N^3)$	Dynamic programming and constructive
2K	$O(N^2 * \log N)$	Dynamic programming, Binary Search, Sorting, Divide and Conquer
10K	$O(N^2)$	Dynamic programming, graph, trees, constructive
1M	$O(N * \log N)$	Sorting, Binary Search, Divide and Conquer
100M	$O(N), O(\log N), O(L)$	Constructive, Mathematical, Greedy Algorithm

Space Complexity

The amount of memory required by the algorithm to solve given problem is called Space Complexity.

estimation

depends on two paths

i) A fixed part:

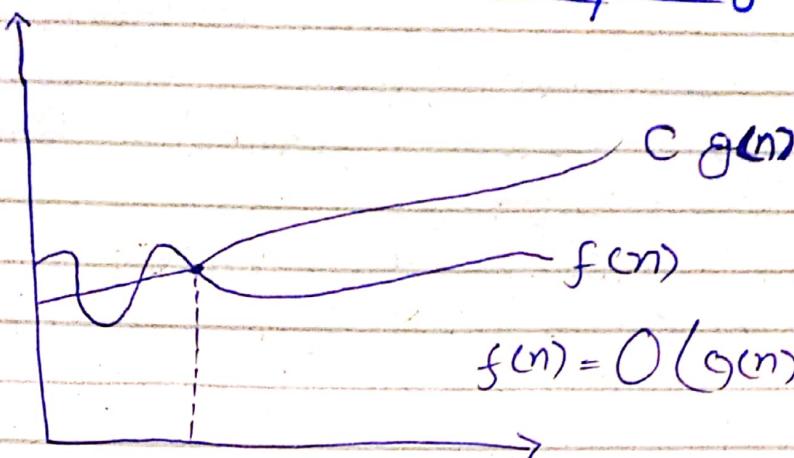
It is independent of the input size. It includes memory for instructions, constants, variable etc.

ii) A variable part:

It is dependent on the input size. It includes memory for recursion stack, referenced variable etc.

Day 02

Space and Time Complexity: Big-O Notation



$O(g(n)) = \{ f(n) : \text{There exist positive constants } C \text{ and } n_0 \text{ such that } 0 \leq f(n) \leq Cg(n) \text{ for all } n \geq n_0 \}$

Such that $0 \leq f(n) \leq Cg(n)$ for all $n \geq n_0$.

O -notation gives an upper bound for a function to within a constant factor. We write $f(n) = O(g(n))$ if there are positive constants n_0 and C such that to the right of n_0 , the value of $f(n)$ always lies on or below $Cg(n)$.

Example

Prove

$$5n^2 + 3n + 1 = O(n^2)$$

Let

$$\begin{aligned} f(n) &= 5n^2 + 3n + 1 \\ &= n^2(5 + \frac{3}{n} + \frac{1}{n^2}) \leq n^2(5+3+1) \\ &= 8n^2 \end{aligned}$$

When $n > 1$

then $5n^2 + 3n + 1 < 8n^2$

So from definition

$$0 \leq f(n) \leq Cg(n) \text{ for } n \geq 2$$

Here $C = 8$, $n_0 = 1$ and $g(n) = n^2$

Hence, $5n^2 + 3n + 1 = O(n^2)$ proved

Day 03

Recursion

Recursion

i) Terminates when base case becomes true.

ii) Used with functions

iii) Every recursive call needs extra space in the Stack memory

iv) Smaller code size

Iteration

Terminates when condition becomes false.

Used with loops

Every iteration does not require any extra space

Larger code size

Advantages

i) provides neat and simple way to write code.

DisAdvantages

ii) Smaller code is difficult to understand

ii) Takes more time to execute due to function calling and returns overhead

iii) Has greater space requirements than the iterative program as all functions will remain in the stack until base case is reached.

Day 03

Summary of Recursion

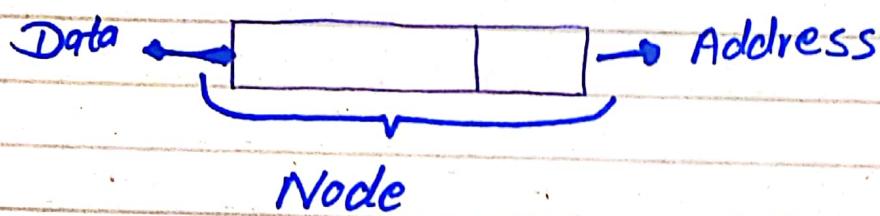
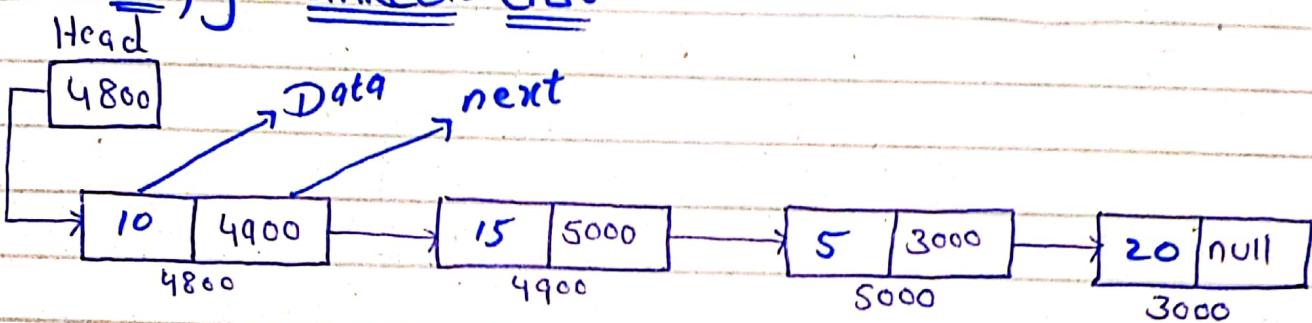
- i) Two types of cases in recursion i.e. Recursive case, base case.
- ii) The base case is ~~turns~~ out used to terminate the recursive function when the case turns out to be true.
- iii) Each recursive call makes new copy of method in the stack memory.
- iv) Infinite recursion may lead to running out of the stack.
- v) Examples of Recursive algorithms:
Merge Sort, Quick Sort, Tower of Hanoi, Fibonacci Series, Factorial Problem etc.

Linked-List-Basics

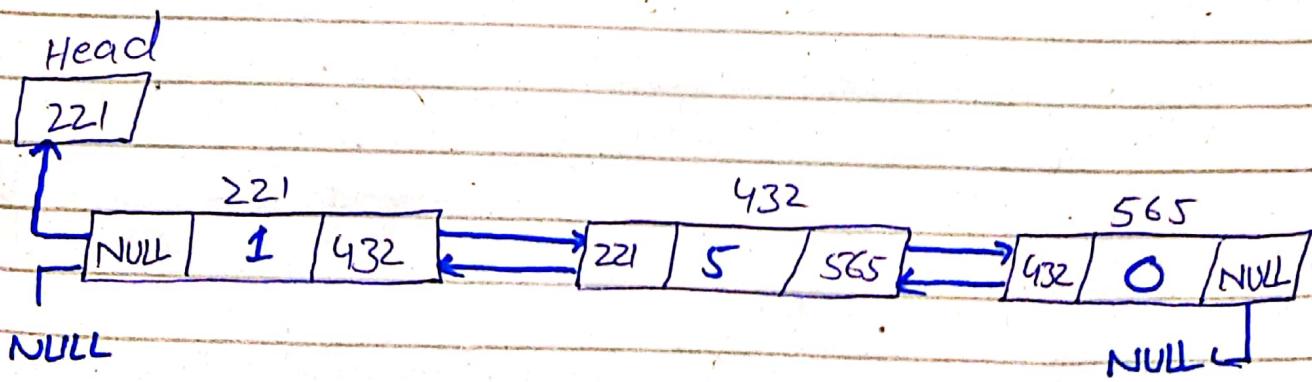
3 types:

- i) Singular Linked-List
- ii) Doubly Linked-List
- iii) Circularly Linked-List

Singly-Linked-List

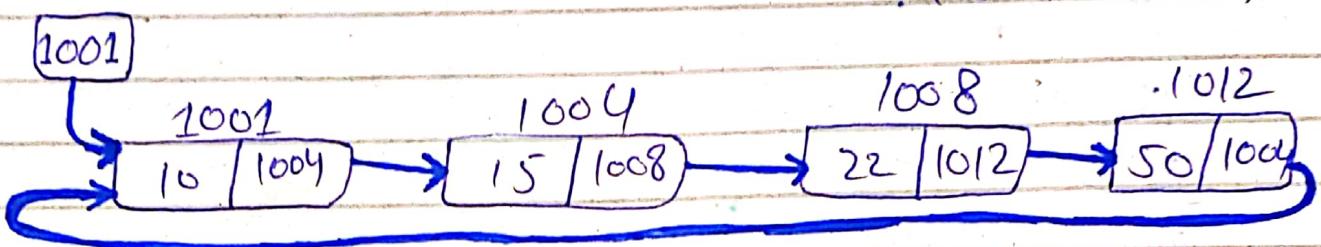


Doubly-Linked-List



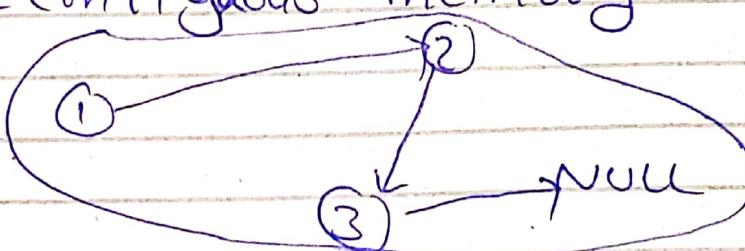
Dynamic Memory Allocation: Dynamic allocation is a concept in programming where memory is allocated at runtime, rather than at compile time. This means that memory is allocated during the execution of the program, and can be resized as needed during the program's execution.

Circular Linked list :: { malloc, calloc, realloc, free }
useage { linkedlist, trees, graphs }



Basic Properties

- i) Variable Size
- ii) Non-Contiguous memory



- iii) Dynamic Memory Allocation
- iv) Insertion and Deletion
- v) Random Access

time Complexity = $O(n)$

- vi) Efficiency

a) Efficient for insertion and deletion.

b) unEfficient for random access operations.

Correct Cases: That are different from normal cases.

Day #04

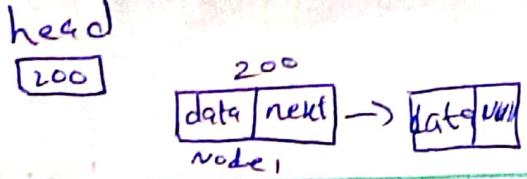
ArrayList VS LinkedList

Insert: $O(n) > O(1)$

Search: $O(1) < O(n)$

Java linkedList Implementation

```
import java.util.*;
class BuiltInLL {
    public static void main(String args[]) {
        LinkedList<String> list = new LinkedList<String>();
        list.addFirst("Es");
        list.addFirst("Ahmad");
        System.out.println(list);
        list.addLast("a");
        list.addLast("Software");
        list.addLast("Engineer");
        System.out.println(list);
        System.out.println("Size!" + list.size());
        for (int i = 0; i < list.size(); i++) {
            if (i == list.size() - 1) {
                System.out.println(list.get(i));
            }
            System.out.println(list.get(i) + " → ");
        }
        list.removeFirst();
        list.removeLast();
        System.out.println(list);
        list.remove(2);
    }
}
```



Day #05

Linked List Scratch Implementation

Class LL {

```

    Node head;
    private int size;
    LL() {
        this.size = 0;
    }
}
  
```

class Node {

```

    String data;
    Node next;
}
  
```

Node(String data) {

```

    this.data = data;
    this.next = null;
    size++;
}
  
```

// add - last

public void addlast(String data) {

```

    Node newNode = new Node(data);
    if (head == null) { // corner case
        head = newNode;
        return;
    }
}
  
```

↑
These are
different
cases.
from normal.

Node currNode = head;

↳ gt means that gt is of type
node

Day #05

```
while (currNode.next != null) {  
    currNode = currNode.next;
```

}

```
currNode.next = newNode;
```

}

// add - first

```
public void addFirst (String data) {  
    Node newNode = new Node(data);  
    if (head == null) {  
        head = newNode;  
        return;  
    }
```

```
newNode.next = head
```

```
head = newNode;
```

↳ This line will store address of newNode in
head

// delete - first

```
public void deleteFirst () {  
    if (head == null) {
```

```
        System.out.println("The list is empty!");  
        return;
```

size --;

```
head = head.next;
```

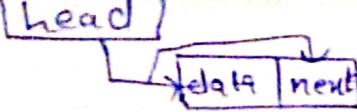
 // delete - last

```
public void deleteLast () {
```

```
    if (head == null) {
```

```
        System.out.println("The list is empty!");  
        return;
```

}



Day #05

Size --;

```
if (head.next == null) {
    head.next
    head = null;
    return;
}
```

Node secondLast = head;

Node lastNode = head.next;

```
while (lastNode.next != null) {
    lastNode = lastNode.next;
    secondLast = secondLast.next;
}
```

secondLast.next = null;

}

// Size

```
public int getSize() {
    return size;
}
```

```
public static void main(String args[]) {
```

LL list = new LL();

list.addFirst("a");

list.addFirst("is");

list.addFirst("Ahmad");

list.printList();

Day #05

```
list.addlast("Software Engineer");
```

```
list.printlist();
```

```
list.deleteFirst();
```

```
list.printlist();
```

```
list.deletelast();
```

```
list.printlist();
```

```
list.addlast("jaaja");
```

```
list.printlist();
```

```
System.out.println("size:" + list.getsize());
```

{