Assignment2-R

February 12, 2020

1 Assignment 2 - Simple and Multiple Linear Regression (I)

1.1 Overview of the steps

1. Load the data and get an overview of the data
2. Perform simple linear regressions
3. Use the simple linear regression models
4. Perform multiple linear regressions
5. Use the multiple linear regression model

1.2 Steps in detail

1.2.1 Load the data and get an overview of the data

Load the data file `Boston.rda` or `Boston.csv`.

In `R` the dataframe comes with the `MASS` library. We save the dataframe ones in `csv` and `rda` files for later use.

[47]:
```
library(MASS)
#write.csv(Boston,"../ISLR/data/Boston.csv", row.names = TRUE)
#save(Boston,file="../ISLR/data/Boston.rda")
```

Display the number of predictors (including the response `medv`) and their names:

[48]:
```
dim(Boston)[2]
names(Boston)
```

14

1. 'crim' 2. 'zn' 3. 'indus' 4. 'chas' 5. 'nox' 6. 'rm' 7. 'age' 8. 'dis' 9. 'rad' 10. 'tax' 11. 'ptratio' 12. 'black' 13. 'lstat' 14. 'medv'

Print a statistic summary of the predictors and the response `medv`:

[49]:
```
summary(Boston)
```

```
      crim                zn             indus            chas        
 Min.   : 0.00632   Min.   :  0.00   Min.   : 0.46   Min.   :0.00000  
 1st Qu.: 0.08204   1st Qu.:  0.00   1st Qu.: 5.19   1st Qu.:0.00000  
 Median : 0.25651   Median :  0.00   Median : 9.69   Median :0.00000  
 Mean   : 3.61352   Mean   : 11.36   Mean   :11.14   Mean   :0.06917  
```

```
 3rd Qu.: 3.67708   3rd Qu.: 12.50   3rd Qu.:18.10   3rd Qu.:0.00000
 Max.   :88.97620   Max.   :100.00   Max.   :27.74   Max.   :1.00000
      nox               rm              age              dis
 Min.   :0.3850   Min.   :3.561   Min.   :  2.90   Min.   : 1.130
 1st Qu.:0.4490   1st Qu.:5.886   1st Qu.: 45.02   1st Qu.: 2.100
 Median :0.5380   Median :6.208   Median : 77.50   Median : 3.207
 Mean   :0.5547   Mean   :6.285   Mean   : 68.57   Mean   : 3.795
 3rd Qu.:0.6240   3rd Qu.:6.623   3rd Qu.: 94.08   3rd Qu.: 5.188
 Max.   :0.8710   Max.   :8.780   Max.   :100.00   Max.   :12.127
      rad              tax           ptratio          black
 Min.   : 1.000   Min.   :187.0   Min.   :12.60   Min.   :  0.32
 1st Qu.: 4.000   1st Qu.:279.0   1st Qu.:17.40   1st Qu.:375.38
 Median : 5.000   Median :330.0   Median :19.05   Median :391.44
 Mean   : 9.549   Mean   :408.2   Mean   :18.46   Mean   :356.67
 3rd Qu.:24.000   3rd Qu.:666.0   3rd Qu.:20.20   3rd Qu.:396.23
 Max.   :24.000   Max.   :711.0   Max.   :22.00   Max.   :396.90
     lstat            medv
 Min.   : 1.73   Min.   : 5.00
 1st Qu.: 6.95   1st Qu.:17.02
 Median :11.36   Median :21.20
 Mean   :12.65   Mean   :22.53
 3rd Qu.:16.95   3rd Qu.:25.00
 Max.   :37.97   Max.   :50.00
```

Display the number of data points:

[50]:
```
dim(Boston)[1]
```

506

Display the data in a table (subset of rows is sufficient):

[51]:
```
Boston
```

A data.frame: 506 × 14

	crim	zn	indus	chas	nox	rm	age	dis	rad
	<dbl>	<dbl>	<dbl>	<int>	<dbl>	<dbl>	<dbl>	<dbl>	<int>
1	0.00632	18.0	2.31	0	0.538	6.575	65.2	4.0900	1
2	0.02731	0.0	7.07	0	0.469	6.421	78.9	4.9671	2
3	0.02729	0.0	7.07	0	0.469	7.185	61.1	4.9671	2
4	0.03237	0.0	2.18	0	0.458	6.998	45.8	6.0622	3
5	0.06905	0.0	2.18	0	0.458	7.147	54.2	6.0622	3
6	0.02985	0.0	2.18	0	0.458	6.430	58.7	6.0622	3
7	0.08829	12.5	7.87	0	0.524	6.012	66.6	5.5605	5
8	0.14455	12.5	7.87	0	0.524	6.172	96.1	5.9505	5
9	0.21124	12.5	7.87	0	0.524	5.631	100.0	6.0821	5
10	0.17004	12.5	7.87	0	0.524	6.004	85.9	6.5921	5
11	0.22489	12.5	7.87	0	0.524	6.377	94.3	6.3467	5
12	0.11747	12.5	7.87	0	0.524	6.009	82.9	6.2267	5
13	0.09378	12.5	7.87	0	0.524	5.889	39.0	5.4509	5
14	0.62976	0.0	8.14	0	0.538	5.949	61.8	4.7075	4
15	0.63796	0.0	8.14	0	0.538	6.096	84.5	4.4619	4
16	0.62739	0.0	8.14	0	0.538	5.834	56.5	4.4986	4
17	1.05393	0.0	8.14	0	0.538	5.935	29.3	4.4986	4
18	0.78420	0.0	8.14	0	0.538	5.990	81.7	4.2579	4
19	0.80271	0.0	8.14	0	0.538	5.456	36.6	3.7965	4
20	0.72580	0.0	8.14	0	0.538	5.727	69.5	3.7965	4
21	1.25179	0.0	8.14	0	0.538	5.570	98.1	3.7979	4
22	0.85204	0.0	8.14	0	0.538	5.965	89.2	4.0123	4
23	1.23247	0.0	8.14	0	0.538	6.142	91.7	3.9769	4
24	0.98843	0.0	8.14	0	0.538	5.813	100.0	4.0952	4
25	0.75026	0.0	8.14	0	0.538	5.924	94.1	4.3996	4
26	0.84054	0.0	8.14	0	0.538	5.599	85.7	4.4546	4
27	0.67191	0.0	8.14	0	0.538	5.813	90.3	4.6820	4
28	0.95577	0.0	8.14	0	0.538	6.047	88.8	4.4534	4
29	0.77299	0.0	8.14	0	0.538	6.495	94.4	4.4547	4
30	1.00245	0.0	8.14	0	0.538	6.674	87.3	4.2390	4
477	4.87141	0	18.10	0	0.614	6.484	93.6	2.3053	24
478	15.02340	0	18.10	0	0.614	5.304	97.3	2.1007	24
479	10.23300	0	18.10	0	0.614	6.185	96.7	2.1705	24
480	14.33370	0	18.10	0	0.614	6.229	88.0	1.9512	24
481	5.82401	0	18.10	0	0.532	6.242	64.7	3.4242	24
482	5.70818	0	18.10	0	0.532	6.750	74.9	3.3317	24
483	5.73116	0	18.10	0	0.532	7.061	77.0	3.4106	24
484	2.81838	0	18.10	0	0.532	5.762	40.3	4.0983	24
485	2.37857	0	18.10	0	0.583	5.871	41.9	3.7240	24
486	3.67367	0	18.10	0	0.583	6.312	51.9	3.9917	24
487	5.69175	0	18.10	0	0.583	6.114	79.8	3.5459	24
488	4.83567	0	18.10	0	0.583	5.905	53.2	3.1523	24
489	0.15086	0	27.74	0	0.609	5.454	92.7	1.8209	4
490	0.18337	0	27.74	0	0.609	5.414	98.3	1.7554	4
491	0.20746	0	27.74	0	0.609	5.093	98.0	1.8226	4
492	0.10574	0	27.74	0	0.609	5.983	98.8	1.8681	4
493	0.11132	0	27.74	0	0.609	5.983	83.5	2.1099	4
494	0.17331	0	9.69	0	0.585	5.707	54.0	2.3817	6
495	0.27957	0	9.69	0	0.585	5.926	42.6	2.3817	6
496	0.17899	0	9.69	0	0.585	5.670	28.8	2.7986	6

Plot some predictors (at least two) against the response values. We choose `lstat`, `rm`, and `age`.

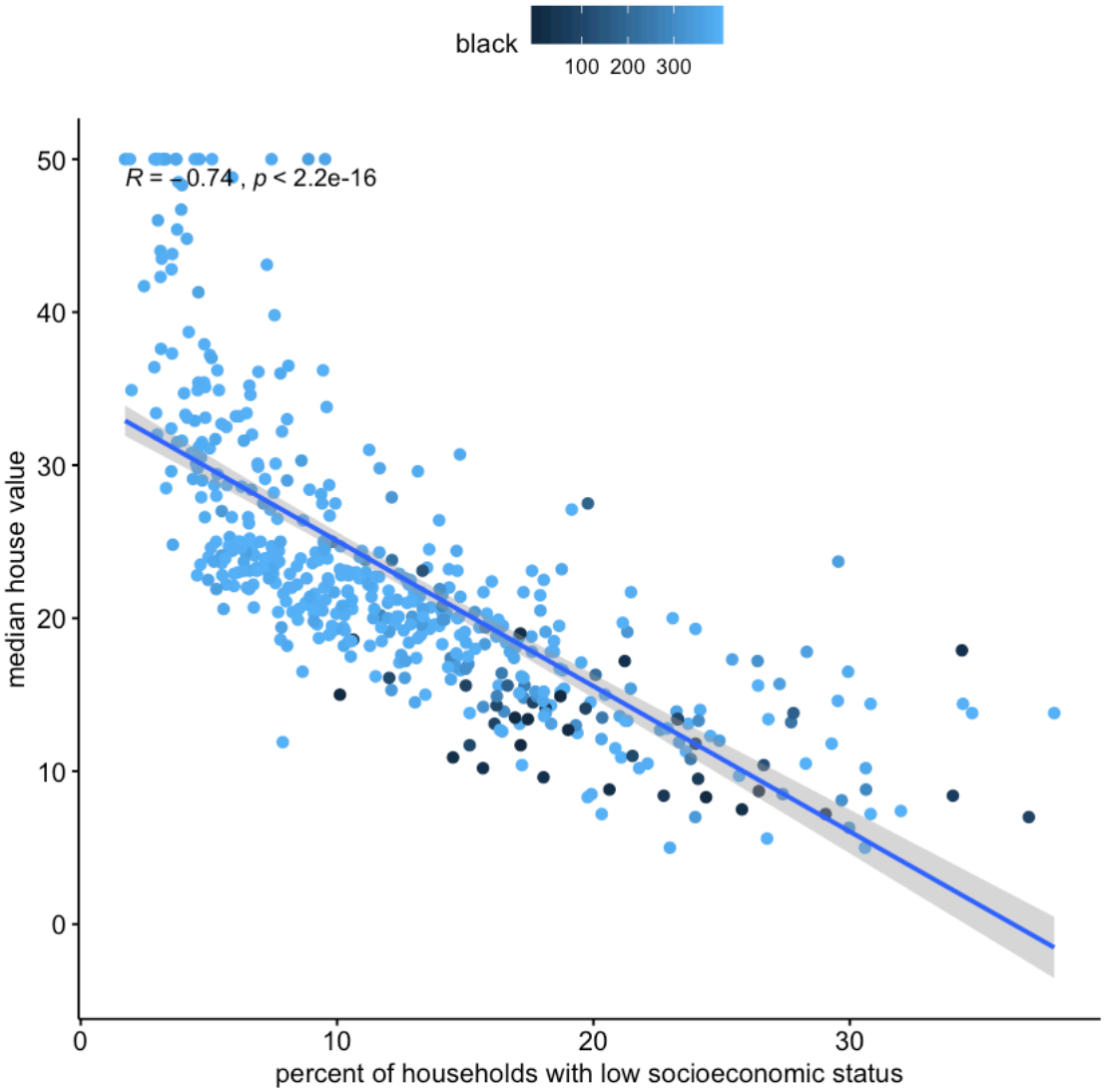
In `R`, we need to download and install a library first.

[52]:
```
install.packages("ggpubr")
library("ggpubr")
```

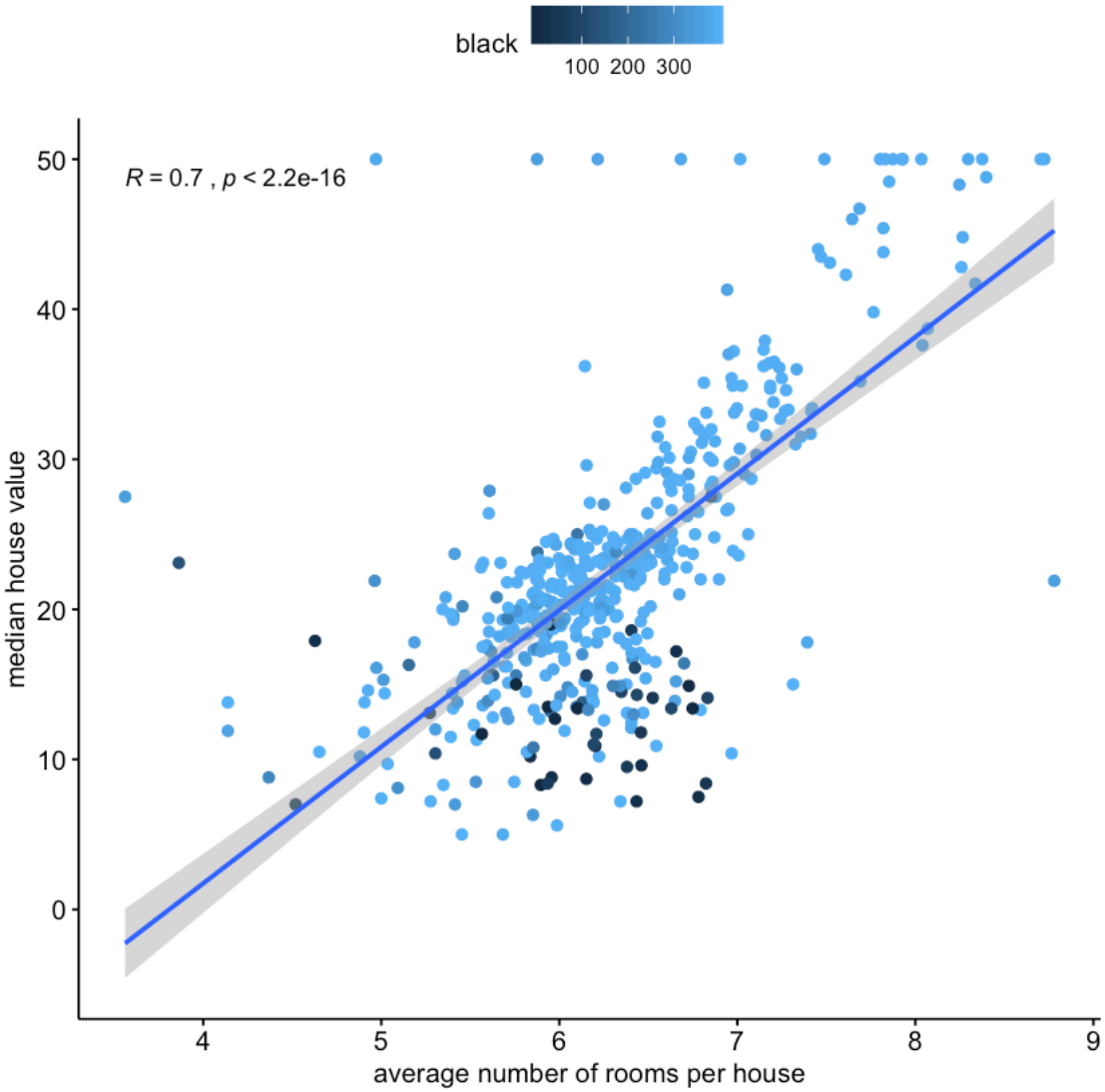
```
The downloaded binary packages are in
/var/folders/ct/4pcck8t94sdfc73rhymq4t140000gp/T//Rtmp8vvlYk/downloaded_packages
```

The `R` function `ggscatter` even displays a regression line, confidence intervals, the Pearson coefficient of correlation, and the p value. **This is not necessary at this stage.**

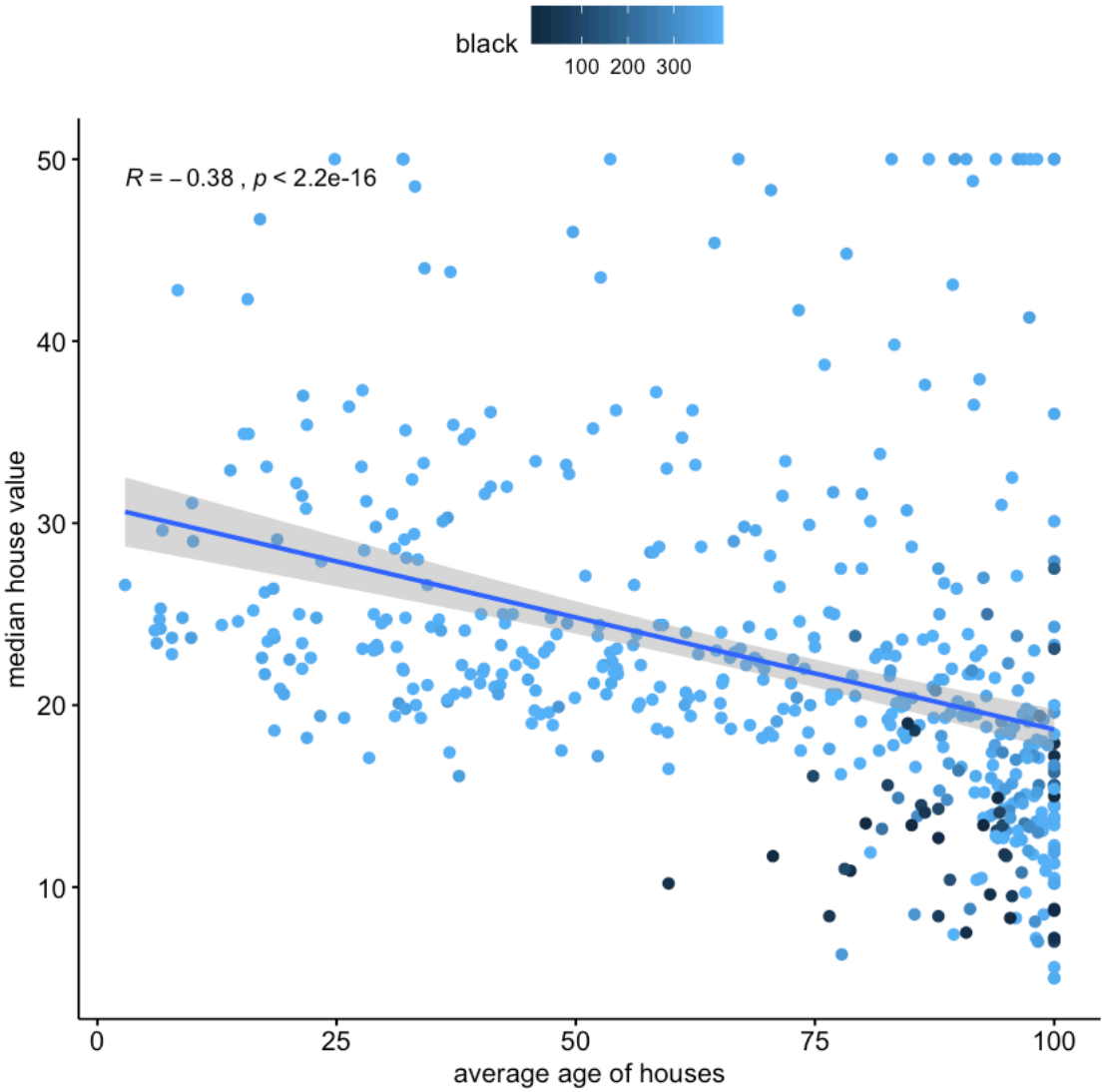
[53]:
```
ggscatter(Boston, x = "lstat", y = "medv",
          add = "reg.line", conf.int = TRUE,
          cor.coef = TRUE, cor.method = "pearson",
          xlab = "percent of households with low socioeconomic status",
          ylab = "median house value")
```



[54]:
```
ggscatter(Boston, x = "rm", y = "medv",
          add = "reg.line", conf.int = TRUE,
          cor.coef = TRUE, cor.method = "pearson",
          xlab = "average number of rooms per house",
          ylab = "median house value")
```



[55]:
```
ggscatter(Boston, x = "age", y = "medv",
          add = "reg.line", conf.int = TRUE,
          cor.coef = TRUE, cor.method = "pearson",
          xlab = "average age of houses",
          ylab = "median house value")
```



1.2.2 Perform simple linear regressions

Fit a simple linear regression model, with `medv` as the response and some (at least two) predictors individually. We choose `lstat`, `rm`, and `age`.

[57]:
```
lm.fit=lm(medv~lstat ,data=Boston)
summary(lm.fit)
```

```
Call:
lm(formula = medv ~ lstat, data = Boston)

Residuals:
    Min      1Q  Median      3Q     Max
```

```
-15.168  -3.990  -1.318   2.034  24.500

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 34.55384    0.56263   61.41   <2e-16 ***
lstat       -0.95005    0.03873  -24.53   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 6.216 on 504 degrees of freedom
Multiple R-squared:  0.5441,Adjusted R-squared:  0.5432
F-statistic: 601.6 on 1 and 504 DF,  p-value: < 2.2e-16
```

[59]:
```
lm.fit=lm(medv~rm ,data=Boston)
summary(lm.fit)
```

```
Call:
lm(formula = medv ~ rm, data = Boston)

Residuals:
    Min      1Q  Median      3Q     Max
-23.346  -2.547   0.090   2.986  39.433

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  -34.671      2.650  -13.08   <2e-16 ***
rm             9.102      0.419   21.72   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 6.616 on 504 degrees of freedom
Multiple R-squared:  0.4835,Adjusted R-squared:  0.4825
F-statistic: 471.8 on 1 and 504 DF,  p-value: < 2.2e-16
```

[60]:
```
lm.fit=lm(medv~age ,data=Boston)
summary(lm.fit)
```

```
Call:
lm(formula = medv ~ age, data = Boston)

Residuals:
    Min      1Q  Median      3Q     Max
-15.097  -5.138  -1.958   2.397  31.338
```

```
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 30.97868    0.99911  31.006   <2e-16 ***
age         -0.12316    0.01348  -9.137   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 8.527 on 504 degrees of freedom
Multiple R-squared:  0.1421,Adjusted R-squared:  0.1404
F-statistic: 83.48 on 1 and 504 DF,  p-value: < 2.2e-16
```

Interprete the results. *Your interpretation of the results goes here!*

Obtain a confidence interval for the coefficient estimates for the indivisual models.

[62]:
```
lm.fit=lm(medv~lstat ,data=Boston)
confint(lm.fit)
```

A matrix: 2 × 2 of type dbl

	2.5 %	97.5 %
(Intercept)	33.448457	35.6592247
lstat	-1.026148	-0.8739505

[63]:
```
lm.fit=lm(medv~rm ,data=Boston)
confint(lm.fit)
```

A matrix: 2 × 2 of type dbl

	2.5 %	97.5 %
(Intercept)	-39.876641	-29.464601
rm	8.278855	9.925363

[64]:
```
lm.fit=lm(medv~age ,data=Boston)
confint(lm.fit)
```

A matrix: 2 × 2 of type dbl

	2.5 %	97.5 %
(Intercept)	29.0157516	32.94160395
age	-0.1496469	-0.09667852

Interprete the results. *Your interpretation of the results goes here!*

1.2.3 Use the simple linear regression models

Predict the `medv` response values for some selected predictor values. Calculate the prediction intervals for these values.

[66]:
```
lm.fit=lm(medv~lstat,data=Boston)
predict(lm.fit,data.frame(lstat=c(5,10,15)), interval ="prediction")
```

A matrix: 3 × 3 of type dbl

	fit	lwr	upr
1	29.80359	17.565675	42.04151
2	25.05335	12.827626	37.27907
3	20.30310	8.077742	32.52846

[69]:
```
lm.fit=lm(medv~rm,data=Boston)
predict(lm.fit,data.frame(rm=c(5,6.5,8)), interval ="prediction")
```

A matrix: 3 × 3 of type dbl

	fit	lwr	upr
1	10.83992	-2.214474	23.89432
2	24.49309	11.480391	37.50578
3	38.14625	25.058353	51.23415

[70]:
```
lm.fit=lm(medv~age,data=Boston)
predict(lm.fit,data.frame(age=c(25,50,75)), interval ="prediction")
```

A matrix: 3 × 3 of type dbl

	fit	lwr	upr
1	27.89961	11.090368	44.70885
2	24.82054	8.043748	41.59734
3	21.74147	4.971031	38.51192

Interprete the results. *Your interpretation of the results goes here!*

1.2.4 Perform multiple linear regressions

Fit `medv`as response with the predictors selected before altogether.

[72]:
```
lm.fit=lm(medv~lstat+rm+age ,data=Boston)
summary(lm.fit)
```

```
Call:
lm(formula = medv ~ lstat + rm + age, data = Boston)

Residuals:
    Min      1Q  Median      3Q     Max
-18.210  -3.467  -1.053   1.957  27.500

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) -1.175311   3.181924  -0.369    0.712
lstat       -0.668513   0.054357 -12.298   <2e-16 ***
rm           5.019133   0.454306  11.048   <2e-16 ***
age          0.009091   0.011215   0.811    0.418
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 5.542 on 502 degrees of freedom
Multiple R-squared:  0.639,Adjusted R-squared:  0.6369
F-statistic: 296.2 on 3 and 502 DF,  p-value: < 2.2e-16
```

Interprete the results. *Your interpretation of the results goes here!*

Fit `medv`as response with all available predictors altogether.

[73]:
```
lm.fit=lm(medv~. ,data=Boston)
summary(lm.fit)
```

```
Call:
lm(formula = medv ~ ., data = Boston)

Residuals:
    Min      1Q  Median      3Q     Max
-15.595  -2.730  -0.518   1.777  26.199

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  3.646e+01  5.103e+00   7.144 3.28e-12 ***
crim        -1.080e-01  3.286e-02  -3.287 0.001087 **
zn           4.642e-02  1.373e-02   3.382 0.000778 ***
indus        2.056e-02  6.150e-02   0.334 0.738288
chas         2.687e+00  8.616e-01   3.118 0.001925 **
nox         -1.777e+01  3.820e+00  -4.651 4.25e-06 ***
rm           3.810e+00  4.179e-01   9.116  < 2e-16 ***
age          6.922e-04  1.321e-02   0.052 0.958229
dis         -1.476e+00  1.995e-01  -7.398 6.01e-13 ***
rad          3.060e-01  6.635e-02   4.613 5.07e-06 ***
tax         -1.233e-02  3.760e-03  -3.280 0.001112 **
ptratio     -9.527e-01  1.308e-01  -7.283 1.31e-12 ***
black        9.312e-03  2.686e-03   3.467 0.000573 ***
lstat       -5.248e-01  5.072e-02 -10.347  < 2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 4.745 on 492 degrees of freedom
Multiple R-squared:  0.7406,Adjusted R-squared:  0.7338
F-statistic: 108.1 on 13 and 492 DF,  p-value: < 2.2e-16
```

Interprete the results. *Your interpretation of the results goes here!*

[90]:
```
install.packages("corrplot")
source("http://www.sthda.com/upload/rquery_cormat.r")
```

```
The downloaded binary packages are in
/var/folders/ct/4pcck8t94sdfc73rhymq4t140000gp/T//Rtmp8vvlYk/downloaded_packages
```

Check the correlation between the predictors.

In R, we need to download and install a library and an external function first.

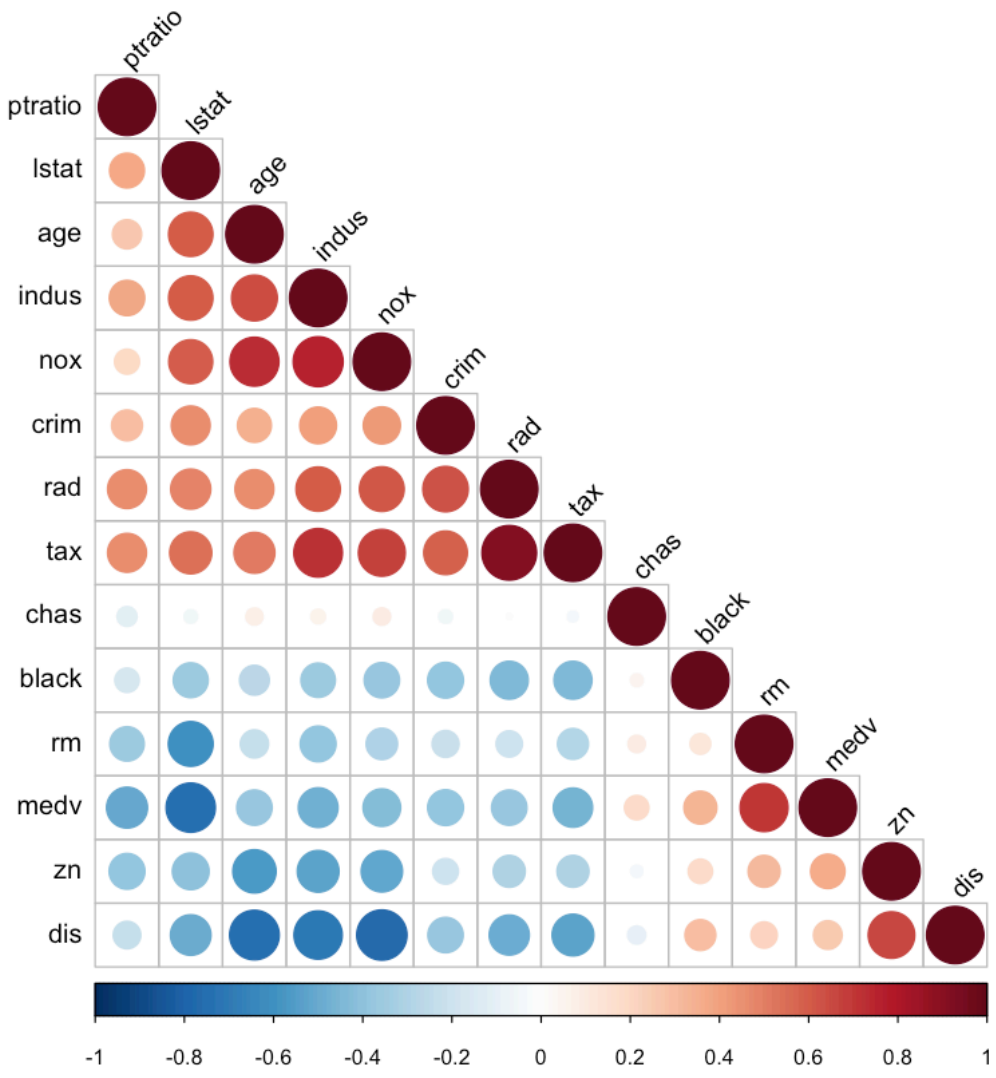
[92]:
```
rquery.cormat(Boston)
```

```
$r
        ptratio  lstat   age indus   nox   crim     rad    tax   chas black
ptratio       1
lstat      0.37      1
age        0.26    0.6     1
indus      0.38    0.6  0.64     1
nox        0.19   0.59  0.73  0.76     1
crim       0.29   0.46  0.35  0.41  0.42      1
rad        0.46   0.49  0.46   0.6  0.61   0.63       1
tax        0.46   0.54  0.51  0.72  0.67   0.58    0.91      1
chas      -0.12 -0.054 0.087 0.063 0.091 -0.056 -0.0074 -0.036      1
black     -0.18  -0.37 -0.27 -0.36 -0.38  -0.39   -0.44  -0.44  0.049     1
rm        -0.36  -0.61 -0.24 -0.39  -0.3  -0.22   -0.21  -0.29  0.091  0.13
medv      -0.51  -0.74 -0.38 -0.48 -0.43  -0.39   -0.38  -0.47   0.18  0.33
zn        -0.39  -0.41 -0.57 -0.53 -0.52   -0.2   -0.31  -0.31 -0.043  0.18
dis       -0.23   -0.5 -0.75 -0.71 -0.77  -0.38   -0.49  -0.53 -0.099  0.29
           rm medv   zn dis
ptratio
lstat
age
indus
nox
crim
rad
tax
chas
black
rm         1
medv     0.7    1
zn      0.31 0.36    1
dis     0.21 0.25 0.66   1

$p
        ptratio   lstat     age   indus      nox    crim      rad     tax
ptratio       0
lstat     3e-18       0
age     2.3e-09 2.8e-51       0
indus   3.8e-19 1.4e-51 8.4e-61       0
nox     1.9e-05   6e-49 7.5e-86 7.9e-98        0
crim    2.9e-11 2.7e-27 2.9e-16 1.5e-21  3.8e-23       0
rad     1.8e-28 9.9e-32 2.4e-27 8.4e-50  3.3e-53 2.7e-56        0
tax     5.7e-28 2.6e-40 2.6e-34   3e-82  1.1e-66 2.4e-47 4.1e-195       0
chas     0.0062    0.23   0.052    0.16     0.04    0.21     0.87    0.42
black     6e-05 1.7e-17 3.9e-10 1.2e-16  7.8e-19 2.5e-19  6.6e-26 1.4e-25
rm      1.6e-16   1e-53 4.5e-08 5.3e-20  3.8e-12 6.3e-07  1.9e-06 2.1e-11
medv    1.6e-34 5.1e-88 1.6e-18 4.9e-31  7.1e-24 1.2e-19  5.5e-19 5.6e-29
zn      5.3e-20 2.9e-22 7.6e-45 1.3e-38  7.2e-36 5.5e-06    7e-13 4.4e-13
dis     1.2e-07 6.4e-33 9.9e-92 3.6e-78 4.2e-100 8.5e-19  1.4e-32   1e-38
```

```
           chas   black      rm    medv      zn dis
ptratio
lstat
age
indus
nox
crim
rad
tax
chas          0
black      0.27       0
rm         0.04  0.0039       0
medv    7.4e-05 1.3e-14 2.5e-74       0
zn         0.34 7.2e-05 6.9e-13 5.7e-17       0
dis       0.026 2.3e-11 3.2e-06 1.2e-08 9.7e-66   0

$sym
        ptratio lstat age indus nox crim rad tax chas black rm medv zn dis
ptratio 1
lstat   .       1
age             .     1
indus   .       .     ,   1
nox             .     ,   ,     1
crim            .     .   .     .   1
rad     .       .     .   .     ,   ,    1
tax     .       .     .   ,     ,   .    *   1
chas                                           1
black           .         .     .   .    .   .        1
rm      .       ,         .                                  1
medv    .       ,     .   .     .   .    .   .        .     ,  1
zn      .       .     .   .     .       .   .              .  .    1
dis             .     ,   ,     ,   .    .   .                      ,  1
attr(,"legend")
[1] 0 ‘ ’ 0.3 ‘.’ 0.6 ‘,’ 0.8 ‘+’ 0.9 ‘*’ 0.95 ‘B’ 1
```



Interprete the results. *Your interpretation of the results goes here!*

1.2.5 Use the multiple linear regression model

Predict the `medv` response values for some selected predictor values. Calculate the prediction intervals for these values.

[93]:
```
lstatC=c(5,10,15)
rmC=c(5,6.5,8)
selected_predictor_values = expand.grid(lstat = lstatC, rm = rmC)
selected_predictor_values
```

A data.frame: 9 × 2

lstat <dbl>	rm <dbl>
5	5.0
10	5.0
15	5.0
5	6.5
10	6.5
15	6.5
5	8.0
10	8.0
15	8.0

Predict the `medv` response values for some selected predictor values. Calculate the prediction intervals for these values.

[94]:
```
lm.fit=lm(medv~lstat+rm ,data=Boston)
predict(lm.fit, selected_predictor_values, interval ="prediction")
```

A matrix: 9 × 3 of type dbl

	fit	lwr	upr
1	20.90388	9.889729	31.91802
2	17.69208	6.722152	28.66202
3	14.48029	3.537875	25.42271
4	28.54606	17.635923	39.45619
5	25.33427	14.437027	36.23150
6	22.12247	11.221204	33.02374
7	36.18824	25.225479	47.15100
8	32.97645	21.995024	43.95787
9	29.76466	18.747835	40.78148

Interprete the results. *Your interpretation of the results goes here!*