User Manual of the Packages Polyopt and PoolOpt

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Chapter 1

Introduction

Polyopt is the implementation of the generalized BSOS and sparse-BSOS hierarchies [2]. PoolOpt is the package that uses Polyopt to solve pooling problems. These packages are written in Julia 0.5.
To use Polyopt and PoolOpt, you need to install two packages:

- MOSEK by using julia> Pkg.add("MOSEK");
- Combinatorics by using julia> Pkg.add("Combinatorics");

Also, you can install the packages using the following codes:

- julia> Pkg.clone("https://github.com/MOSEK/Polyopt.jl.git");
- julia> Pkg.clone("https://github.com/AhmadrezaMarandi/PoolOpt.git");

In order to use them, you can use "using Polyopt", and "using PoolOpt". In the next chapters we define the available functions in the packages.

Chapter 2

Polyopt

As it was mentioned, Polyopt uses the generalization of the BSOS and sparse-BSOS hierarchy. The codes are based on the results in [2]. In this chapter, we present the available functions in this package.

2.1 variable(string, int)

To solve your problem, you first need to define the decision variables and constraints. This is possible by use of the function "variables". This function gets two arguments as the input: a string and an integer. The string denotes the name of the variable, and the integer is the dimension of it.

Example 1 *julia> y=variables("y",5)*
5-element ArrayPolyopt.PolyInt64,1:
y1
y2
y3
y4
y5

Example 2

$julia > x = variables(``x", 4)$

$4 - element\ ArrayPolyopt.PolyInt64, 1 :$

$x1$

$x2$

$x3$

$x4$

$julia > f = x[1]^2 - x[2]^2 + x[3]^2 - x[4]^2 + x[1] - x[2]$

$-x4^2 + x3^2 - x2 - x2^2 + x1 + x1^2$

$julia > g = [2 * x[1]^2 + 3 * x[2]^2 + 2 * x[1] * x[2] + 2 * x[3]^2 + 3 * x[4]^2 + 2 * x[3] * x[4],$

$3 * x[1]^2 + 2 * x[2]^2 - 4 * x[1] * x[2] + 3 * x[3]^2 + 2 * x[4]^2 - 4 * x[3] * x[4],$

$x[1]^2 + 6 * x[2]^2 - 4 * x[1] * x[2] + x[3]^2 + 6 * x[4]^2 - 4 * x[3] * x[4],$

$x[1]^2 + 4 * x[2]^2 - 3 * x[1] * x[2] + x[3]^2 + 4 * x[4]^2 - 3 * x[3] * x[4],$

$2 * x[1]^2 + 5 * x[2]^2 + 3 * x[1] * x[2] + 2 * x[3]^2 + 5 * x[4]^2 + 3 * x[3] * x[4],$

$x[1], x[2], x[3], x[4]]$

$9 - element\ ArrayPolyopt.PolyInt64, 1 :$

$3 * x4^2 + 2 * x3 * x4 + 2 * x3^2 + 3 * x2^2 + 2 * x1 * x2 + 2 * x1^2$

$2 * x4^2 - 4 * x3 * x4 + 3 * x3^2 + 2 * x2^2 - 4 * x1 * x2 + 3 * x1^2$

$6 * x4^2 - 4 * x3 * x4 + x3^2 + 6 * x2^2 - 4 * x1 * x2 + x1^2$

$4 * x4^2 - 3 * x3 * x4 + x3^2 + 4 * x2^2 - 3 * x1 * x2 + x1^2$

$5 * x4^2 + 3 * x3 * x4 + 2 * x3^2 + 5 * x2^2 + 3 * x1 * x2 + 2 * x1^2$

$x1$

$x2$

$x3$

$x4$

2.2 Polyopt.correlative_sparsity()

After constructing the problem, you can see the adjacent matrix of its graph using Polyopt.correlative_sparsity().

Example 3

$$julia > Polyopt.correlative\_sparsity(f, g)$$
$$4x4\ sparse\ matrix\ with\ 16\ Int64\ nonzero\ entries:$$
$$[1, 1] = 1$$
$$[2, 1] = 1$$
$$[3, 1] = 1$$
$$[4, 1] = 1$$
$$[1, 2] = 1$$
$$[2, 2] = 1$$
$$[3, 2] = 1$$
$$[4, 2] = 1$$
$$[1, 3] = 1$$
$$[2, 3] = 1$$
$$[3, 3] = 1$$
$$[4, 3] = 1$$
$$[1, 4] = 1$$
$$[2, 4] = 1$$
$$[3, 4] = 1$$
$$[4, 4] = 1$$

2.3 Polyopt.chordal_embedding()

Using this function, you can find the sparsity pattern of the associated graph:

Example 4

$$julia > Polyopt.chordal\_embedding(Polyopt.correlative\_sparsity(f, g))$$
$$1 - element\ Array\{Array\{Int64, 1\}, 1\}:$$
$$[1, 2, 3, 4]$$

2.4 bsosprob_chordal(d, k, I, f, g,eq)

This function will generate the level of the hierarchy. The arguments of this functions are:

d : level of the hierarchy;

k : the SOS polynomial in the hierarchy is with degree $2k$;

I : the sparsity pattern (maximal cliques of the graph associated to the problem) or any appropriate pattern;

f : objective function;

g : inequality constraints;

eq : equality constraints.

2.5 solve_mosek(prob, tolrelgap)

This function solves the constructed level of the hierarchy using MOSEK.

prob : problem that is constructed for the level of the hierarchy;

tolrelgap : an optional argument to specify the preferred error gap.

Chapter 3

PoolOpt

This package solves pooling problems using Polyopt package. This package contains many well-known pooling problem instances such as Havely1-3, Foulds1-4,..., and DeyGupte4 that is constructed in [1]. These instances can be reached by adding () after their name, like Haverly_1().

3.1 elimination_equality(data)

This function constructs the P-formulation of the pooling problem after elimination of equality constraints, which is introduced in [1]. “data” is the information of the pooling problem.

Example 5

```
julia > f, g = elimination_equality(Haverly_1());
Data was read.
 ================== Input − output variables are build! ===================
 ================== Pool − output variables are build! ===================
 ================== Concentration variables are build! ===================
 ========== Input − pool variables are build, considering elimination
of the equality constraints! ==============

 ========== The model is being constructed... ==============

 ========== Objective function is constructed! ==============

 ===================== Constraints are constructed! ==============

julia > f
 − 1000.0 ∗ y4 + 200.0 ∗ y3 + 200.00000000000142 ∗ y2 − 2000.000000000002 ∗ y2 ∗ y5+
1400.0000000000014 ∗ y1 − 2000.000000000002 ∗ y1 ∗ y5

julia > g
11 − elementArrayPolyopt.PolyFloat64, 1 :
1.0 − 2.0 ∗ y3 − 2.0 ∗ y1
1.0 − y4 − y2
0.05 ∗ y3 + 0.15000000000000002 ∗ y1 − 0.2 ∗ y1 ∗ y5
 − 0.08333333333333333 ∗ y4 + 0.08333333333333333 ∗ y2 − 0.3333333333333333 ∗ y2 ∗ y5
y5
0.2500000000000001 ∗ y2 ∗ y5 + 0.2500000000000001 ∗ y1 ∗ y5
0.2500000000000001 ∗ y2 − 0.2500000000000002 ∗ y2 ∗ y5 + 0.2500000000000001 ∗ y1 − 0.25000000
00000002 ∗ y1 ∗ y5
0.25 ∗ y1
0.25 ∗ y2
y3
y4
```

3.2 with_equality(data)

This function returns the P-formulation.

Example 6

```
julia > f, g, eq = with_equality(Haverly_1());
Datawasread.
 ================== Input − output variables are build! ===================
 ================== Pool − output variables are build! ===================
 ================== Input − Pool variables are build! ===================
 ================== Pool_specification variables are build! ===================
 =========== The model is being constructed ... ==============

 =========== Objective function is constructed! ==============




julia > f
3200.0 ∗ y7 + 1200.0 ∗ y6 − 1000.0 ∗ y4 + 200.0 ∗ y3 − 3000.0 ∗ y2 − 1800.0 ∗ y1

julia > g
11 − element ArrayPolyopt.PolyFloat64, 1 :
1.0 − 2.0 ∗ y3 − 2.0 ∗ y1
1.0 − y4 − y2
0.05 ∗ y3 + 0.15000000000000002 ∗ y1 − 0.2 ∗ y1 ∗ y5
 − 0.08333333333333333 ∗ y4 + 0.08333333333333333 ∗ y2 − 0.3333333333333333 ∗ y2 ∗ y5
y5
y6
y7
y1
y2
y3
y4
```

$julia > eq$

$4-elementArrayPolyopt.PolyFloat64,1:$

$0.3333333333333333*y7+0.3333333333333333*y6-0.3333333333333333*y2-0.33333333333 33333*y1$

$-0.5*y7-0.5*y6+0.5*y2+0.5*y1$

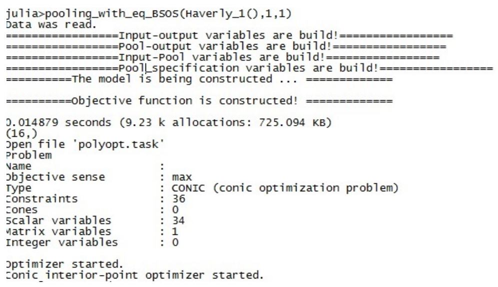
$0.1111111111111111*y7+0.3333333333333333*y6-0.1111111111111111*y2-0.22222222222 22222*y2*y5-0.1111111111111111*y1-0.2222222222222222*y1*y5$

$-0.16666666666666666*y7-0.5*y6+0.16666666666666666*y2+0.3333333333333333*y2*y5+ 0.16666666666666666*y1+0.3333333333333333*y1*y5$

3.3 pooling_with_eq_BSOS(data, d, k)

This function solves the dth level of the BSOS hierarchy when the SOS polynomial in it is with degree $2k$.

Example 7

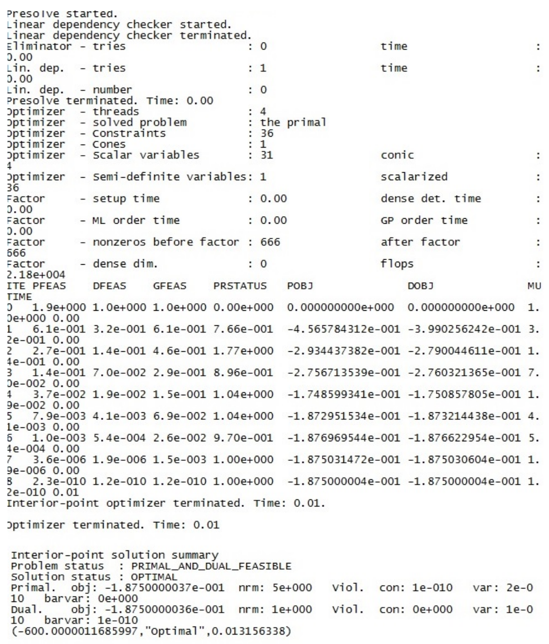


```
julia>pooling_with_eq_BSOS(Haverly_1(),1,1)
Data was read.
=================Input-output variables are build!=================
=================Pool-output variables are build!=================
=================Input-Pool variables are build!=================
=================Pool_specification variables are build!=================
==========The model is being constructed ... =============

==========Objective function is constructed! =============

0.014879 seconds (9.23 k allocations: 725.094 KB)
(16,)
Open file 'polyopt.task'
Problem
Name                   :
Objective sense        : max
Type                   : CONIC (conic optimization problem)
Constraints            : 36
Cones                  : 0
Scalar variables       : 34
Matrix variables       : 1
Integer variables      : 0

Optimizer started.
Conic interior-point optimizer started.
```



```
Presolve started.
Linear dependency checker started.
Linear dependency checker terminated.
Eliminator - tries                  : 0                 time                  :
0.00
Lin. dep.  - tries                  : 1                 time                  :
0.00
Lin. dep.  - number                 : 0
Presolve terminated. Time: 0.00
Optimizer  - threads                : 4
Optimizer  - solved problem         : the primal
Optimizer  - Constraints            : 36
Optimizer  - Cones                  : 1
Optimizer  - Scalar variables       : 31                conic                 :
4
Optimizer  - Semi-definite variables: 1                 scalarized            :
36
Factor     - setup time             : 0.00              dense det. time       :
0.00
Factor     - ML order time          : 0.00              GP order time         :
0.00
Factor     - nonzeros before factor : 666               after factor          :
666
Factor     - dense dim.             : 0                 flops                 :
2.18e+004
ITE PFEAS    DFEAS    GFEAS    PRSTATUS   POBJ               DOBJ               MU
TIME
0   1.9e+000 1.0e+000 1.0e+000 0.00e+000  0.000000000e+000  0.000000000e+000  1.
0e+000 0.00
1   6.1e-001 3.2e-001 6.1e-001 7.66e-001  -4.565784312e-001 -3.990256242e-001 3.
2e-001 0.00
2   2.7e-001 1.4e-001 4.6e-001 1.77e+000  -2.934437382e-001 -2.790044611e-001 1.
4e-001 0.00
3   1.4e-001 7.0e-002 2.9e-001 8.96e-001  -2.756713539e-001 -2.760321365e-001 7.
0e-002 0.00
4   3.7e-002 1.9e-002 1.5e-001 1.04e+000  -1.748599341e-001 -1.750857805e-001 1.
9e-002 0.00
5   7.9e-003 4.1e-003 6.9e-002 1.04e+000  -1.872951534e-001 -1.873214438e-001 4.
1e-003 0.00
6   1.0e-003 5.4e-004 2.6e-002 9.70e-001  -1.876969544e-001 -1.876622954e-001 5.
4e-004 0.00
7   3.6e-006 1.9e-006 1.5e-003 1.00e+000  -1.875031472e-001 -1.875030604e-001 1.
9e-006 0.00
8   2.3e-010 1.2e-010 1.2e-010 1.00e+000  -1.875000004e-001 -1.875000004e-001 1.
2e-010 0.01
Interior-point optimizer terminated. Time: 0.01.

Optimizer terminated. Time: 0.01


 Interior-point solution summary
 Problem status  : PRIMAL_AND_DUAL_FEASIBLE
 Solution status : OPTIMAL
 Primal.  obj: -1.8750000037e-001  nrm: 5e+000   Viol.  con: 1e-010   var: 2e-0
 10   barvar: 0e+000
 Dual.    obj: -1.8750000036e-001  nrm: 1e+000   Viol.  con: 0e+000   var: 1e-0
 10   barvar: 1e-010
 (-600.0000011685997,"Optimal",0.013156338)
```

Similarly, we have

- pooling_without_eq_BSOS(data, d, k)
- pooling_with_eq_Sparse_BSOS(data , d, k)
- pooling_without_eq_Sparse_BSOS(data , d, k)
- pooling_with_eq_Merge_Sparse_BSOS(data , d, k)
- pooling_without_eq_Merge_Sparse_BSOS(data , d, k)

where the last two function merge the cliques with high overlaps and then use sparse-BSOS hierarchy.

Bibliography

[1] A. Marandi, J. Dahl, and E. de Klerk. A numerical evaluation of the bounded degree sum-of-squares hierarchy of lasserre, toh, and yang on the pooling problem. *Annals of Operations Research*, pages 1–26, 2017.

[2] A. Marandi, E. De Klerk, and J. Dahl. Solving sparse polynomial optimization problems with chordal structure using the sparse, bounded-degree sum-of-squares hierarchy. *Optimization Online*, 2017.