

Assignment #1

Names:-

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A # 1

Soln

Distribution of output from

Goods output ↓	services ↓	Purchased by
2	.7	→ Goods — (i)
8	.3	→ Services — (ii)

The decimal fraction in each column
sum to 1.

Let denote total annual output by
 P_G and P_S .

So first equation (i) will be

$$P_G = .2P_G + .7P_S$$

From (ii)

$$P_S = .8P_G + .3P_S$$

so move all variables in left side

$$.8P_G - .7P_S = 0$$

$$-.8P_G + .7P_S = 0$$

augmented matrix

$$\left[\begin{array}{ccc} .8 & -.7 & 0 \\ -.8 & .7 & 0 \end{array} \right]$$

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Replace R_2 with $R_1 + R_2$

$$\begin{bmatrix} 8 & -7 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

R_1 by $1/8$

$$\begin{bmatrix} 1 & -875 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

so

The ratio of P_1 and P_2 is $P_1 = 875 P_2$

P_2 is free

General solution is $\{P_1 = (7/8)P_2\}$

Q #2

Take some values for P_2 200 million dollars with P_2 is free. The other equilibrium prices are then

$$P_1 = 188 \text{ million}$$

$$P_E = 170 \text{ million}$$

Any constant non-negative multiple of these prices is set of equilibrium prices because solution set of system of equation consists of all multiples of one vector. By changing the currency does not effect equilibrium ^{addition} prices remains the same no matter what currency is used.

Q # 3

Gol.

(a) Fuel and manufacturing services purchased by
Power

.10	.10	.20 \rightarrow Fuel and Power
.80	.10	.40 \rightarrow manufacturing
.10	.80	.40 \rightarrow services

(b) Calculate total annual output of sectors
by P_F , P_m and P_s .
From one 1st year

$$P_F = .1P_F + .1P_m + .2P_s$$

2nd year

$$P_m = .8P_F + .1P_m + .4P_s$$

From year (iii)

$$P_s = .1P_F + .8P_m + .4P_s$$

Move all variables to left side and
combine like terms

$$.9P_F - .1P_m - 2P_s = 0$$

$$-8P_F + .9P_m - .4P_s = 0$$

$$.1P_F - .8P_m + .6P_s = 0$$

Augmented matrix

$$\begin{bmatrix} .9 & -.1 & -.2 & 0 \\ -.8 & .9 & -.4 & 0 \\ -.1 & -.8 & .6 & 0 \end{bmatrix}$$

(C)

$$\begin{bmatrix} .9 & .1 & -.2 & 0 \\ -.8 & .9 & -.4 & 0 \\ .1 & -.8 & .6 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -301 & 0 \\ 0 & 1 & -712 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The general solution is

$$P_E = 301 P_S$$

$$P_M = 712 P_S$$

P_S is free variable.

Only

SD:-

mining Lumber Energy Transportation Purchased
by

.30	.15	.20	.20	\rightarrow mining
.10	.15	.15	.10	\rightarrow Lumber
.60	.50	.45	.50	\rightarrow Energy
0	.20	.20	.20	\rightarrow Transportation

Let Denote total Annual output by
 P_M, P_L, P_E and P_T
 from both u above equations

$$P_M = .30 P_M + .15 P_L + .20 P_E + .20 P_T$$

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$$P_L = .10 P_M + .15 P_L + .15 P_E + .10 P_T$$

$$P_E = .60 P_M + .50 P_L + .45 P_E + .50 P_T$$

$$P_T = 0 P_M + .20 P_L + .20 P_E + .20 P_T$$

Move all variables to left

$$.70 P_M - .15 P_L - .20 P_E - .20 P_T = 0$$

$$-.10 P_M + .85 P_L - .15 P_E - .10 P_T = 0$$

$$-.60 P_M - .50 P_L + .55 P_E + .50 P_T = 0$$

$$0 P_M - .20 P_L - .20 P_E + .80 P_T = 0$$

Augmented matrix

$$\left[\begin{array}{ccccc} .70 & -.15 & -.20 & -.20 & 0 \\ -.10 & .85 & -.15 & -.10 & 0 \\ -.60 & -.50 & .55 & .50 & 0 \\ 0 & -.20 & -.20 & .80 & 0 \end{array} \right]$$

$$\left[\begin{array}{ccccc} 1 & 0 & 0 & -1.37 & 0 \\ 0 & 1 & 0 & .84 & 0 \\ 0 & 0 & 1 & -3.18 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

P_T is free variable and $P_M = 1.37 P_T$
 $P_L = .84 P_T$ and $P_E = -3.18 P_T$

M T W T F S

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Q # 6

The following vectors list the numbers of atoms of aluminum (Al), oxygen (O) and carbon (C)

$$\text{Al}_2\text{O}_3 : \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}, \text{C} : \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \text{Al} : \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \text{CO}_2 : \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} \begin{array}{l} \text{aluminum} \\ \text{oxygen} \\ \text{carbon} \end{array}$$

The equation is



$$x_1 \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = x_3 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$$

Augmented Matrix

$$\left[\begin{array}{ccccc} 2 & 0 & -1 & 0 & 0 \\ 3 & 0 & 0 & -2 & 0 \\ 0 & 1 & 0 & -1 & 0 \end{array} \right]$$

$$\left[\begin{array}{ccccc} 1 & 0 & -1/2 & 0 & 0 \\ 3 & 0 & 0 & -2 & 0 \\ 0 & 1 & 0 & -1 & 0 \end{array} \right]$$

$$\left[\begin{array}{ccccc} 1 & 0 & -1/2 & 0 & 0 \\ 0 & 0 & 3/2 & -2 & 0 \\ 0 & 1 & 0 & -1 & 0 \end{array} \right]$$

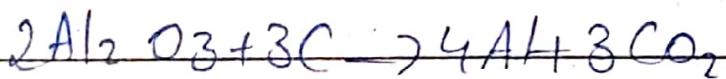
$$= \begin{bmatrix} 1 & 0 & -1/2 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 3/2 & -2 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & -1/2 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & -4/3 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & -2/3 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & -4/3 & 0 \end{bmatrix}$$

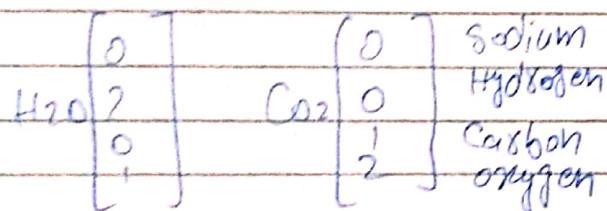
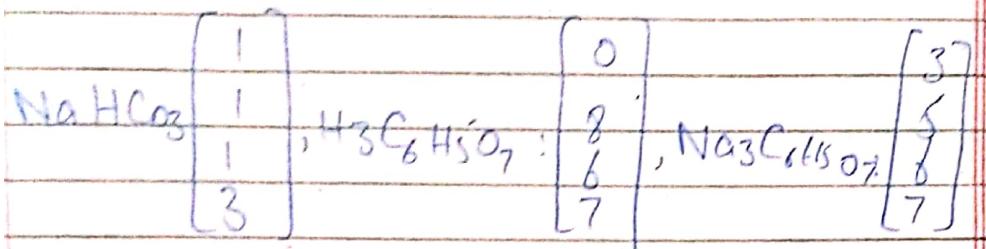
General solution is $x_1 = (2/3)x_4$, $x_2 = x_4$, $x_3 = (4/3)x_4$ with x_4 is free.

Then $x_1 = 2$, $x_2 = 3$, $x_3 = 4$. The balance equation is

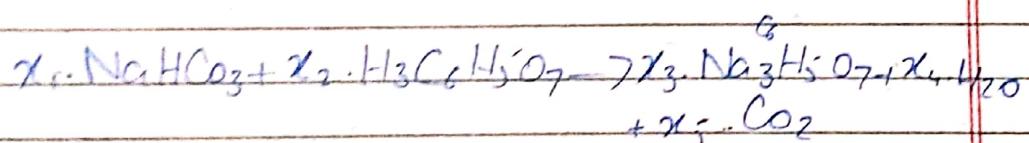


Q#7

Number of atoms of Na, H, C
and O



so equation is



$$x_1 \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 8 \\ 7 \end{bmatrix} = x_3 \begin{bmatrix} 3 \\ 5 \\ 7 \end{bmatrix} + x_4 \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} + x_5 \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$$

Augmented matrix is

$$\left[\begin{array}{cccccc} 1 & 0 & -3 & 0 & 0 & 0 \\ 1 & 8 & -5 & -2 & 0 & 0 \\ 1 & 6 & -6 & 0 & -1 & 0 \\ 3 & 7 & -7 & -1 & 2 & 6 \end{array} \right]$$

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Reduced augmented matrix

$$\left[\begin{array}{cccccc} 1 & 0 & 2 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & -1/3 & 0 \\ 0 & 0 & 1 & 0 & -1/3 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{array} \right]$$

General Solution

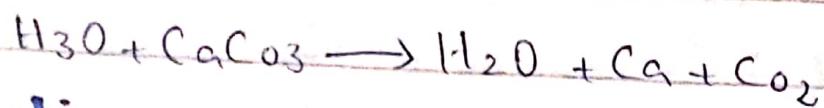
$$\left\{ \begin{array}{l} x_1 = x_5 \\ x_2 = (1/3)x_5 \\ x_3 = (1/3)x_5 \\ x_4 = x_5 \\ x_5 \text{ is free} \end{array} \right.$$

Take $x_5 = 3$ $x_1 = x_4 = 3$ and $x_2 = x_3 = 1$
 The balance equation is



QUESTION : 8

Limestone, CaCO_3 neutralizes the acid, H_3O , in acid rain by the following unbalanced equation.



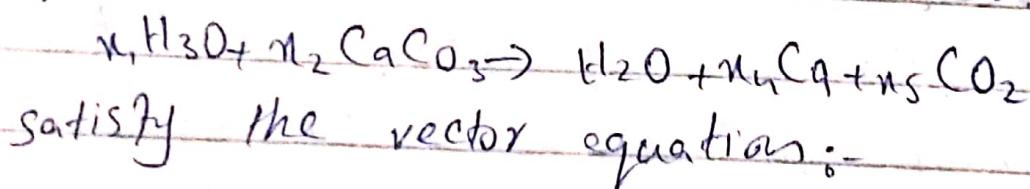
Solution:

The following vectors list the number of atoms of hydrogen (H), oxygen (O), calcium (Ca) and carbon.

$$\begin{array}{c} \text{H}_3\text{O} \\ \left[\begin{matrix} 3 \\ 1 \\ 0 \\ 0 \end{matrix} \right] \end{array}, \begin{array}{c} \text{CaCO}_3 \\ \left[\begin{matrix} 0 \\ 3 \\ 1 \\ 1 \end{matrix} \right] \end{array}, \begin{array}{c} \text{H}_2\text{O} \\ \left[\begin{matrix} 2 \\ 1 \\ 0 \\ 0 \end{matrix} \right] \end{array}, \begin{array}{c} \text{Ca} \\ \left[\begin{matrix} 0 \\ 0 \\ 1 \\ 0 \end{matrix} \right] \end{array}, \begin{array}{c} \text{CO}_2 \\ \left[\begin{matrix} 0 \\ 2 \\ 0 \\ 1 \end{matrix} \right] \end{array}$$

hydrogen
oxygen
calcium
carbon

The coefficients in the chemical equation



$$n_1 \left[\begin{matrix} 3 \\ 1 \\ 0 \\ 0 \end{matrix} \right] + n_2 \left[\begin{matrix} 0 \\ 3 \\ 1 \\ 1 \end{matrix} \right] = n_3 \left[\begin{matrix} 2 \\ 1 \\ 0 \\ 0 \end{matrix} \right] + n_4 \left[\begin{matrix} 0 \\ 0 \\ 1 \\ 0 \end{matrix} \right] + n_5 \left[\begin{matrix} 0 \\ 2 \\ 0 \\ 1 \end{matrix} \right]$$

Reduce the augmented matrix

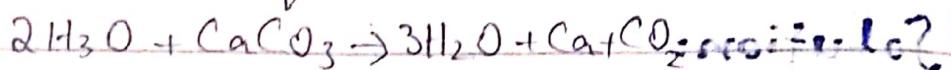
$$\left[\begin{array}{cccc|ccccc} 3 & 0 & -2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -2 & 0 \\ 1 & 3 & -1 & 0 & -2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & -3 & 0 \end{array} \right]$$

\Rightarrow The general solution is,

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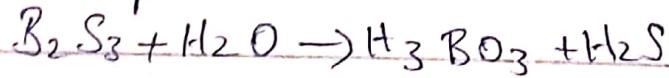
$n_1 = 2n_2$, $n_4 = n_5$, $n_3 = 3n_5$ and n_5 is free.

Take $n_5 = 1$, then $n_1 = 2$ and $n_2 = n_4 = 1$ and $n_3 = 3$. The balanced equation is:



QUESTION.09:

Boron sulfide reacts violently with water to form boric acid and hydrogen gas. The unbalanced equation is:-

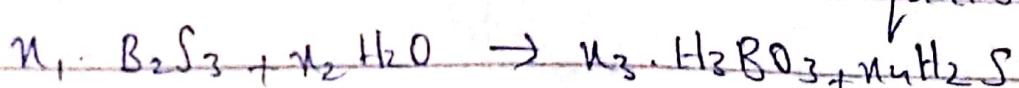


Solution:

The following vector list the number of atoms of boron (B), sulfur (S), hydrogen (H), and the oxygen (O).

$$\begin{bmatrix} B_2S_3 \\ 3 \\ 8 \end{bmatrix}, H_2O \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}, H_3BO_3 \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}, H_2S \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$$

The coefficient in the equation



satisfy:

$$\therefore n_1 \begin{pmatrix} 2 \\ 3 \\ 0 \\ 0 \end{pmatrix} + n_2 \begin{pmatrix} 0 \\ 0 \\ 2 \\ 1 \end{pmatrix} = n_3 \begin{pmatrix} 1 \\ 0 \\ 3 \\ 3 \end{pmatrix} + n_4 \begin{pmatrix} 0 \\ 1 \\ 2 \\ 0 \end{pmatrix}$$

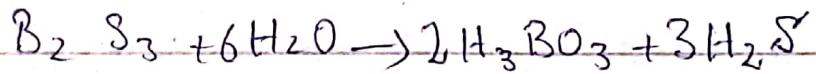
Row reduce the augmented matrix of the homogeneous system:

$$\left[\begin{array}{cccc|ccccc} 2 & 0 & -1 & 0 & 0 & 1 & 0 & -1/3 & 0 & 0 \\ 3 & 0 & 0 & -1 & 0 & 0 & 1 & -2 & 0 & 0 \\ 0 & 2 & -3 & -2 & 0 & 0 & 0 & -2/3 & 1 & 0 \\ 0 & 1 & -3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right] \xrightarrow{\text{Row operations}}$$

The general solution is:

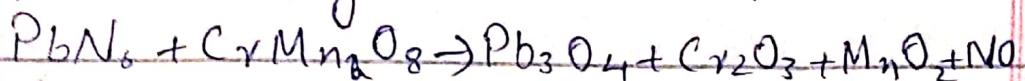
$$n_1 = (1/3)n_4, n_2 = 2n_4, n_3 = (-2/3)n_4$$

with n_4 free. Take ($n_4 = 3$), Then $n_1 = 1$
 $n_2 = 6$ and $n_3 = 2$. The balanced equation is:-



QUESTION: 10:

If possible, use exact arithmetic or a rational format for calculation in balancing the following chemical reactions:-



Solution:

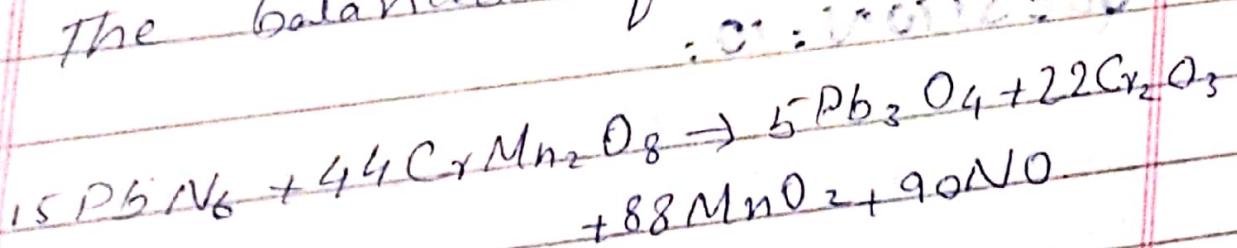
Set up vectors that list the atoms per molecule. Using

the order lead (Pb), nitrogen (N), chromium (Cr), manganese (Mn) and oxygen (O₂), the vector equation to be solved is:

$$n_1 \begin{bmatrix} 1 \\ 6 \\ 0 \\ 0 \\ 0 \end{bmatrix} + n_2 \begin{bmatrix} 0 \\ 0 \\ 1 \\ 2 \\ 8 \end{bmatrix} = n_3 \begin{bmatrix} 3 \\ 0 \\ 0 \\ 0 \\ 4 \end{bmatrix} + n_4 \begin{bmatrix} 0 \\ 0 \\ 2 \\ 0 \\ 3 \end{bmatrix} + n_5 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 2 \end{bmatrix} + n_6 \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

⇒ The general solution is
 $n_1 = (1/6)n_6, n_2 = (2/15)n_6, n_3 = (1/18)n_6, n_4 = (11/45)n_6, n_5 = (44/15)n_6$, and n_6 is free.

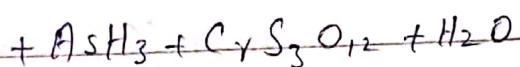
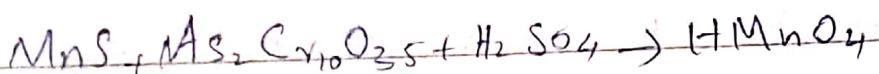
⇒ Take $n_6 = 90$. Then $n_1 = 15, n_2 = 64, n_3 = 5, n_4 = 22$ and $n_5 = 88$.
 The balanced equation is:



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QUESTION-011:

The chemical reactions below can be used in some industrial processes, such as the production of the arsene (AsH_3). Use exact arithmetic or a rational formal for calculations to balance this equation.



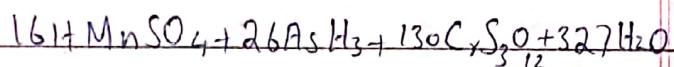
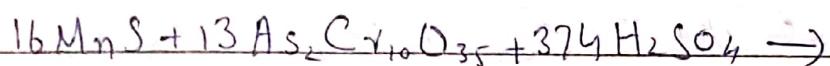
Solution:

Set up vectors that list the atoms per molecule. Using the order manganese (Mn), sulfur (S), arsenic (As), chromium (Cr), oxygen (O) and hydrogen (H), the vector equation to be solved is:-

$$\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 2 \\ 10 \\ 35 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 4 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 4 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 3 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 12 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 2 \\ 2 \end{bmatrix}$$

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In rational format, the general solution is $n_1 = (16/327)n_7$,
 $n_2 = (17/327)n_7$, $n_3 = (374/327)n_7$,
 $n_4 = (16/327)n_7$, $n_5 = (26/327)n_7$, $n_6 = (130/327)n_7$, and n_7 is free. Take $n_7 = 327$ to make the other variables whole numbers. The balanced equation is:-



QUESTION. 12:

Solution:-

Write the equations for each intersection:

Intersection Flow in Flow out

$$A \quad n_1 + n_4 = n_2$$

$$B \quad n_2 = n_3 + 100$$

$$C \quad n_3 + 80 = n_4$$

Re-arrange the equations:-

$$n_1 - n_2 + n_4 = 0$$

$$n_2 - n_3 = 100$$

$$n_3 - n_4 = -80$$

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Reduce the Augmented matrix:

$$\left[\begin{array}{ccccc} 1 & -1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 & 100 \\ 0 & 0 & 1 & -1 & -80 \end{array} \right] \xrightarrow{\text{Row operations}} \left[\begin{array}{ccccc} 1 & 0 & 0 & 0 & 20 \\ 0 & 1 & 0 & -1 & 20 \\ 0 & 0 & 1 & -1 & -80 \end{array} \right]$$

The general solution is:-

$$\begin{cases} n_1 = 20 \\ n_2 = 20 + n_4 \\ n_3 = -80 + n_4 \end{cases}$$

and n_4 is free

Since n_3 cannot be negative

, the maximum value of n_4 is
80.

QUESTION.013

Solution:-

Intersection Flow in Flow out

$$A \quad n_2 + 30 = n_1 + 80$$

$$B \quad n_2 + 75 = n_2 + n_4$$

$$C \quad n_6 + 100 = n_5 + 40$$

$$D \quad n_4 + 40 = n_6 + 90$$

$$E \quad n_1 + 60 = n_3 + 20$$

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Rearrange this equations:-

$$n_1 - n_2 = 50$$

$$n_2 - n_3 + n_4 - n_5 = 0$$

$$n_5 - n_6 = 60$$

$$n_4 - n_6 = 50$$

$$n_1 - n_3 = -40$$

Reduce the augmented matrix :-

$$\left[\begin{array}{ccccccc} 1 & -1 & 0 & 0 & 0 & 0 & -50 \\ 0 & 1 & -1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 60 \\ 0 & 0 & 0 & 1 & 0 & -1 & 50 \\ 1 & 0 & -1 & 0 & 0 & 0 & -40 \end{array} \right]$$

$$\sim \left[\begin{array}{ccccccc} 1 & 0 & -1 & 0 & 0 & 0 & -40 \\ 0 & 1 & -1 & 0 & 0 & 0 & 10 \\ 0 & 0 & 0 & 1 & 0 & -1 & 50 \\ 0 & 0 & 0 & 0 & 1 & -1 & 60 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

(a) The general solution is:-

$$\left\{ \begin{array}{l} n_1 = n_3 - 40 \\ n_2 = n_3 + 10 \\ n_3 \text{ is free} \\ n_4 = n_6 + 50 \\ n_5 = n_6 + 60 \end{array} \right.$$

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QUESTION .13 (b)

Ans:

To find minimum flows, note that since n_1 cannot be negative, $n_3 \geq 40$. This implies that $n_2 \geq 50$. Also, since n_6 cannot be negative, $n_4 \geq 50$ and $n_5 \geq 60$. The minimum flows are $n_2 = 50, n_3 = 40, n_4 = 50, n_5 = 60$ (when $n_1 = 0$ and $n_6 = 0$)

QUESTION .014:

Write the equation for each intersection:-

Intersection Flow in Flow out

$$A \quad 80 = n_1 + n_5$$

$$B \quad n_1 + n_2 + 100 = n_4$$

$$C \quad n_3 = n_2 + 90$$

$$D \quad n_4 + n_5 = n_3 + 90$$

Re-arrange the equations :-

$$n_1 + n_5 = 80$$

$$n_1 + n_2 - n_4 = -100$$

$$n_2 - n_3 = -90$$

Date:

Reduce the augmented matrix.

$$\left[\begin{array}{cccccc} 1 & 0 & 0 & 0 & 1 & 80 \\ 1 & 1 & 0 & -1 & 0 & -100 \\ 0 & 1 & -1 & 0 & 0 & -90 \\ 0 & 0 & 1 & -1 & -1 & -90 \end{array} \right]$$

$$\sim \left[\begin{array}{cccccc} 1 & 0 & 0 & 0 & 1 & 80 \\ 0 & 1 & 0 & -1 & -1 & -180 \\ 0 & 0 & 1 & -1 & -1 & -90 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

(a) The general Solution is :-

$$\begin{cases} x_1 = 80 - n_5 \\ x_2 = n_4 + n_5 - 180 \\ x_3 = n_4 + n_5 - 90 \\ x_4 \text{ is free} \\ x_5 \text{ is free} \end{cases}$$

b).

If $n = 0$, then the general
solution is :-

$$\begin{cases} x_1 = 80 \\ x_2 = n_4 - 180 \\ x_3 = n_4 - 90 \\ n_4 \text{ is free} \end{cases}$$

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(c) Since α_2 cannot be a negative, the minimum value of α_4 when $\alpha_5 = 0$ is 180.