___/___/202   Day:________

Assignment #1

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Q# 1

Sol

Distribution of output from

	Goods	services	Purchased by	
output	↓	↓		
	.2	.7	→ Goods	(i)
	.8	.3	→ Services	(ii)

The decimal fraction in each column sum to 1.

Let donate total annual output by P_G and P_s

So first equation (i) will be

$$P_G = .2P_G + .7P_s$$

from row (ii)

$$P_s = .8P_G + .3P_s$$

so move all variables in leftside

$$.8P_G - .7P_s = 0$$

$$-.8P_G + .7P_s = 0$$

argumented matrix

$$\begin{bmatrix} .8 & -.7 & 0 \\ -.8 & .7 & 0 \end{bmatrix}$$

Date:

M T W T F S

Replace R_2 with $R_1 + R_2$

$$\begin{bmatrix} .8 & -.7 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

R_1 by $1/8$

$$\begin{bmatrix} 1 & -.875 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

So
The ratio of Prices is $P_G = .875 P_S$
P_S is free
General solution is $\{ P_G = (7/8) P_S \}$

Q #2

Take some values for P_S 200 million dollars with P_S is free. The other equilibrium prices are then
$P_C = 188$ million

$P_E = 170$ million
Any constant non-negative multiple of these prices is set of equilibrium prices because solution set of system of equation consists of all multiples of one vector. By changing the currency does not effect equilibrium radians price remains the same no matter what currency is used.

Q # 3

Sol:

(a)

Fuel and Power	manufacturing	services	Purchased by
.10	.10	.20	→ Fuel and Power
.80	.10	.40	→ manufacturing
.10	.80	.40	→ services

(b) Donate total annual output of sector by P_F, P_M and P_S.

From ~~one~~ 1st row

$$P_F = .1P_F + .1P_M + .2P_S$$

2nd row

$$P_M = .8P_F + .1P_M + .4P_S$$

From row (iii)

$$P_S = .1P_F + .8P_M + .4P_S$$

Move all variables to left side and combine like terms

$$.9P_F - .1P_M - .2P_S = 0$$
$$-.8P_F + .9P_M - .4P_S = 0$$
$$-.1P_F - .8P_M + .6P_S = 0$$

Argumented matrix

$$\begin{bmatrix} .9 & -.1 & -.2 & 0 \\ -.8 & .9 & -.4 & 0 \\ -.1 & -.8 & .6 & 0 \end{bmatrix}$$

(C)

$$\begin{bmatrix} .9 & -.1 & -.2 & 0 \\ -.8 & .9 & -.4 & 0 \\ -.1 & -.8 & .6 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -.301 & 0 \\ 0 & 1 & -.712 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The general solution is

$P_r = .301 P_s$

$P_m = .712 P_s$

P_s is free variable.

Q#4

Sol:-

Mining	Lumber	Energy	Transportation	Purchased by
.30	.15	.20	.20	→ mining
.10	.15	.15	.10	→ Lumber
.60	.50	.45	.50	→ Energy
0	.20	.20	.20	→ Transportation

Let Donate total Annual output by P_M, P_L, P_E and P_T

From both 4 above equations

$P_M = .30 P_M + .15 P_L + .20 P_E + .20 P_T$

M T W T F S

Date:

$P_L = .10 P_M + .15 P_L + .15 P_E + .10 P_T$

P_M

$P_E = .60 P_M + .50 P_L + .45 P_E + .50 P_T$

$P_T = 0 P_M + .20 P_L + .20 P_E + .20 P_T$

Move all variables to left

$$\begin{aligned}
.70 P_M - .15 P_L - .20 P_E - .20 P_T &= 0 \\
-.10 P_M + .85 P_L - .15 P_E - .10 P_T &= 0 \\
-.60 P_M - .50 P_L + .55 P_E - .50 P_T &= 0 \\
0 P_M - .20 P_L - .20 P_E + .80 P_T &= 0
\end{aligned}$$

Argumented matrix

$$\begin{bmatrix} .70 & -.15 & -.20 & -.20 & 0 \\ -.10 & .85 & -.15 & -.10 & 0 \\ -.60 & -.50 & .55 & -.50 & 0 \\ 0 & -.20 & -.20 & .80 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & -1.37 & 0 \\ 0 & 1 & 0 & -.84 & 0 \\ 0 & 0 & 1 & -3.16 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

P_T is free variable and $P_M = 1.37 P_T$

$P_L = .84 P_T$ and $P_E = 3.16 P_T$.

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Date:

M T W T F S

Q # 6

The following vectors list the numbers of atoms of aluminum (Al), oxygen (O) and carbon (C)

$$Al_2O_3:\begin{bmatrix}2\\3\\0\end{bmatrix},\ C:\begin{bmatrix}0\\0\\1\end{bmatrix},\ Al:\begin{bmatrix}1\\0\\0\end{bmatrix}\ CO_2:\begin{bmatrix}0\\2\\1\end{bmatrix}\begin{array}{l}\text{aluminum}\\\text{oxygen}\\\text{carbon}\end{array}$$

The equation is

$$x_1 \cdot Al_2O_3 + x_2 C \longrightarrow x_3 Al + x_4 CO_2$$

$$x_1\begin{bmatrix}2\\3\\0\end{bmatrix} + x_2\begin{bmatrix}0\\0\\1\end{bmatrix} = x_3\begin{bmatrix}1\\0\\0\end{bmatrix} + x_4\begin{bmatrix}0\\2\\1\end{bmatrix}$$

Argumented Matrix

$$= \begin{bmatrix} 2 & 0 & -1 & 0 & 0 \\ 3 & 0 & 0 & -2 & 0 \\ 0 & 1 & 0 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & -1/2 & 0 & 0 \\ 3 & 0 & 0 & -2 & 0 \\ 0 & 1 & 0 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & -1/2 & 0 & 0 \\ 0 & 0 & 3/2 & -2 & 0 \\ 0 & 1 & 0 & -1 & 0 \end{bmatrix}$$

___/___/202 Day:______

$$= \begin{bmatrix} 1 & 0 & -1/2 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 3/2 & -2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & -1/2 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & -4/3 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & -2/3 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & -4/3 & 0 \end{bmatrix}$$

General solution is $x_1 = (2/3)x_4$, $x_2 = x_4$, $x_3 = (4/3)x_4$ with x_4 is free.

Then $x_1 = 2$, $x_2 = 3$ $x_3 = 4$ The balance equation is

$$2Al_2O_3 + 3C \rightarrow 4Al + 3CO_2$$

Q#7

Number of atoms of Na, H, C and O

$NaHCO_3 \begin{bmatrix} 1 \\ 1 \\ 1 \\ 3 \end{bmatrix}$, $H_3C_6H_5O_7 : \begin{bmatrix} 0 \\ 8 \\ 6 \\ 7 \end{bmatrix}$, $Na_3C_6H_5O_7 \begin{bmatrix} 3 \\ 5 \\ 6 \\ 7 \end{bmatrix}$

$H_2O \begin{bmatrix} 0 \\ 2 \\ 0 \\ 1 \end{bmatrix}$ $CO_2 \begin{bmatrix} 0 \\ 0 \\ 1 \\ 2 \end{bmatrix}$ sodium
hydrogen
carbon
oxygen

So equation is

$$x_1 . NaHCO_3 + x_2 . H_3C_6H_5O_7 \rightarrow x_3 . Na_3{}^{6}H_5O_7 + x_4 . H_2O + x_5 . CO_2$$

$$x_1 \begin{bmatrix} 1 \\ 1 \\ 1 \\ 3 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 8 \\ 6 \\ 7 \end{bmatrix} = x_3 \begin{bmatrix} 3 \\ 5 \\ 6 \\ 7 \end{bmatrix} + x_4 \begin{bmatrix} 0 \\ 2 \\ 0 \\ 1 \end{bmatrix} + x_5 \begin{bmatrix} 0 \\ 0 \\ 1 \\ 2 \end{bmatrix}$$

Argumented matrix is

$$\begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 0 \\ 1 & 8 & -5 & -2 & 0 & 0 \\ 1 & 6 & -6 & 0 & -1 & 0 \\ 3 & 7 & -7 & -1 & -2 & 0 \end{bmatrix}$$

M T W T F S

Date:

Reduced argumented matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & -1/3 & 0 \\ 0 & 0 & 1 & 0 & -1/3 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix}$$

General Solution
$$\begin{cases} x_1 = x_5 \\ x_2 = (1/3)x_5 \\ x_3 = (1/3)x_5 \\ x_4 = x_5 \\ x_5 \text{ is free} \end{cases}$$

Take $x_5 = 3$ $x_1 = x_4 = 3$ and $x_2 = x_3 = 1$
The balance equation is

$3NaCHO_3 + H_3C_6H_5O_7 \rightarrow Na_3C_6H_5O_7 + 3H_2O + 3CO_2$

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QUESTION : 8

Limestone, $CaCO_3$ neutralizes the acid, H_3O, in acid rain by the following unbalanced equation.

$$H_3O + CaCO_3 \longrightarrow H_2O + Ca + CO_2$$

Solution:

The following vectors list the number of atoms of hydrogen (H), oxygen (O), cal (ca) and carbon.

$$H_3O\begin{bmatrix}3\\1\\0\\0\end{bmatrix}, CaCO_3: \begin{bmatrix}0\\3\\1\\1\end{bmatrix}, H_2O\begin{bmatrix}2\\1\\0\\0\end{bmatrix}, Ca: \begin{bmatrix}0\\0\\1\\0\end{bmatrix}, CO_2\begin{bmatrix}0\\2\\0\\1\end{bmatrix}\begin{matrix}\text{hydrogen}\\\text{oxygen}\\\text{cal}\\\text{carbon}\end{matrix}$$

The coefficients in the chemical equation

$$x_1 H_3O + x_2 CaCO_3 \rightarrow H_2O + x_4 Ca + x_5 CO_2$$

satisfy the vector equation:-

$$x_1\begin{bmatrix}3\\1\\0\\0\end{bmatrix} + x_2\begin{bmatrix}0\\3\\1\\1\end{bmatrix} = x_2\begin{bmatrix}2\\1\\0\\0\end{bmatrix} + x_4\begin{bmatrix}0\\0\\1\\0\end{bmatrix} + x_5\begin{bmatrix}0\\2\\0\\1\end{bmatrix}$$

Reduce the augmented matrix

$$\begin{bmatrix}3 & 0 & -2 & 0 & 0 & 0\\1 & 3 & -1 & 0 & -2 & 0\\0 & 1 & 0 & -1 & 0 & 0\\0 & 1 & 0 & 0 & -1 & 0\end{bmatrix} \sim \begin{bmatrix}1 & 0 & 0 & 0 & -2 & 0\\0 & 1 & 0 & 0 & -1 & 0\\0 & 0 & 1 & 0 & -3 & 0\\0 & 0 & 0 & 1 & -1 & 0\end{bmatrix}$$

$\Rightarrow$ The general solution is,

8 : [illegible]

$x_1 = 2x_5, x_2 = x_5, x_3 = 3x_5$ and x_5 is free.

Take $x_5 = 1$, then $x_1 = 2$ and $x_2 = x_4 = 1$ and $x_3 = 3$. The balanced equation is:

$$2H_3O + CaCO_3 \rightarrow 3H_2O + Ca + CO_2$$ [illegible]

QUESTION-09:

Boron sulfide reacts violently with water to form boric acid and hydrogen gas. The unbalanced equation is:-

$$B_2S_3 + H_2O \rightarrow H_3BO_3 + H_2S$$

Solution:

The following vector list the number of atoms of boron (B), sulfer (S), hydrogen (H), and the oxygen (O).

$$B_2S_3 \begin{bmatrix} 2 \\ 3 \\ 0 \\ 0 \end{bmatrix}, H_2O \begin{bmatrix} 0 \\ 0 \\ 2 \\ 1 \end{bmatrix}, H_3BO_3 \begin{bmatrix} 1 \\ 0 \\ 3 \\ 3 \end{bmatrix}, H_2S \begin{bmatrix} 0 \\ 1 \\ 2 \\ 0 \end{bmatrix}$$

The coefficient in the equation

$$x_1 \cdot B_2S_3 + x_2 H_2O \rightarrow x_3 \cdot H_3BO_3 + x_4 H_2S$$

satisfy:

$$x_1\begin{bmatrix}2\\3\\0\\0\end{bmatrix}+x_2\begin{bmatrix}0\\0\\2\\1\end{bmatrix}=x_3\begin{bmatrix}1\\0\\3\\3\end{bmatrix}+x_4\begin{bmatrix}0\\1\\2\\0\end{bmatrix}$$

Row reduce the augumented matrix of the homogeneous system:

$$\begin{bmatrix}2&0&-1&0&0\\3&0&0&-1&0\\0&2&-3&-2&0\\0&1&-3&0&0\end{bmatrix}\sim\begin{bmatrix}1&0&-1/3&0&0\\0&1&-2&0&0\\0&0&-2/3&1&0\\0&0&0&0&0\end{bmatrix}$$

The general solution is:

$x_1=(1/3)x_4$, $x_2=2x_4$, $x_3=(2/3)x_4$

with x_4 free. Take ($x_4=3$), Then $x_1=1$ $x_2=6$ and $x_3=2$. The balanced equation is:-

$$B_2S_3+6H_2O\rightarrow 2H_3BO_3+3H_2S$$

QUESTION: 10:

If possible, use exact arithmetic or a rational format for calculation is balancing the following chemical reactions:-

$PbN_6+CrMn_2O_8\rightarrow Pb_3O_4+Cr_2O_3+MnO_2+NO$

Solution:

Set up vectors that list the atoms per molecule. Using the order lead (Pb), nitrogen (N), chromium (Cr), maganese (MN) and oxegen (O_2), the vector equation to be solved is:

$$x_1\begin{bmatrix}1\\6\\0\\0\\0\end{bmatrix}+x_2\begin{bmatrix}0\\0\\1\\2\\8\end{bmatrix}=x_3\begin{bmatrix}3\\0\\0\\0\\4\end{bmatrix}+x_4\begin{bmatrix}0\\0\\2\\0\\3\end{bmatrix}+x_5\begin{bmatrix}0\\0\\0\\1\\2\end{bmatrix}+x_6\begin{bmatrix}0\\1\\0\\0\\1\end{bmatrix}$$

⇒ The general solution is $x_1 = (1/6)x_6$, $x_2 = (22/45)x_6$, $x_3 = (1/18)x_6$, $x_4 = (11/45)x_6$, $x_5 = (44/45)x_6$, and x_6 is free.

⇒ Take $x_6 = 90$. Then $x_1 = 15$, $x_2 = 44$, $x_3 = 5$, $x_4 = 22$ and $x_5 = 88$.

The balanced equation is:

$$15PbN_6 + 44CrMn_2O_8 \rightarrow 5Pb_3O_4 + 22Cr_2O_3 + 88MnO_2 + 90NO.$$

Date:

QUESTION-011:

The chemical reactions below can be used in some industrial processes, such as the production of the arsene (AsH_3). Use exact arithmetic or a rational format for calculations to balance this equation.

$$MnS + As_2Cr_{10}O_{35} + H_2SO_4 \rightarrow HMnO_4 + AsH_3 + CrS_3O_{12} + H_2O$$

Solution:

Set up vectors that list the atoms per molecule. Using the order manganese (Mn), sulfur (S) arsenic (As), chromium (Cr), oxygen (O) and Hydrogen (H), the vector equation to be solved is:-

$$x_1\begin{bmatrix}1\\1\\0\\0\\0\\0\end{bmatrix}+x_2\begin{bmatrix}0\\0\\2\\10\\35\\0\end{bmatrix}+x_3\begin{bmatrix}0\\1\\0\\0\\4\\2\end{bmatrix}=x_4\begin{bmatrix}1\\0\\0\\0\\4\\1\end{bmatrix}+x_5\begin{bmatrix}0\\0\\1\\0\\0\\3\end{bmatrix}+x_6\begin{bmatrix}0\\3\\0\\1\\12\\0\end{bmatrix}+x_7\begin{bmatrix}0\\0\\0\\0\\1\\2\end{bmatrix}$$

Date:

In rational format, the general solution is $x_1 = (16/327)x_7$, $x_2 = (17/327)x_7$, $x_3 = (374/327)x_7$, $x_4 = (16/327)x_7$, $x_5 = (26/327)x_7$, $x_6 = (130/327)x_7$, and x_7 is free. Take $x_7 = 327$ to make the other variable whole numbers. The balanced equation is:-

$$16MnS + 13As_2Cr_{10}O_{35} + 374H_2SO_4 \rightarrow 16HMnSO_4 + 26AsH_3 + 130CrS_3O_{12} + 327H_2O$$

QUESTION.12:

Solution:-

Write the equation for each intersection:

Intersection	Flow in		Flow out
A	$x_1 + x_4$	=	x_2
B	x_2	=	$x_3 + 100$
C	$x_3 + 80$	=	x_4

Re-arrange the equations:-

$$x_1 - x_2 + x_4 = 0$$

$$x_2 - x_3 = 100$$

$$x_3 - x_4 = -80$$

Date:

Reduce the Augmented matrix:

$$\begin{bmatrix} 1 & -1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 & 100 \\ 0 & 0 & 1 & -1 & -80 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 0 & 20 \\ 0 & 1 & 0 & -1 & 20 \\ 0 & 0 & 1 & -1 & -80 \end{bmatrix}$$

The general solution is:-

$$\begin{cases} x_1 = 20 \\ x_2 = 20 + x_4 \\ x_3 = -80 + x_4 \end{cases}$$

and x_4 is free

Since x_3 cannot be negative, the maximum value of x_4 is 80.

QUESTION.013

Solution:-

Intersection	Flow in		Flow out
A	x_2+30	=	x_1+80
B	x_2+x_5	=	x_2+x_4
C	x_6+100	=	x_5+40
D	x_4+40	=	x_6+90
E	x_1+60	=	x_3+20

Date:

Re arrange this equations:-

$$
\begin{aligned}
x_1 - x_2 &= 50 \\
x_2 - x_3 + x_4 - x_5 &= 0 \\
x_5 - x_6 &= 60 \\
x_4 - x_6 &= 50 \\
x_1 - x_3 &= -40
\end{aligned}
$$

Reduce the augumented matrix:-

$$
\begin{bmatrix}
1 & -1 & 0 & 0 & 0 & 0 & 50 \\
0 & 1 & -1 & 1 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & -1 & 60 \\
0 & 0 & 0 & 1 & 0 & -1 & 50 \\
1 & 0 & -1 & 0 & 0 & 0 & -40
\end{bmatrix}
\sim
\begin{bmatrix}
1 & 0 & -1 & 0 & 0 & 0 & -40 \\
0 & 1 & -1 & 0 & 0 & 0 & 10 \\
0 & 0 & 0 & 1 & 0 & -1 & 50 \\
0 & 0 & 0 & 0 & 1 & -1 & 60 \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

(a) The general solution is:-

$$
\begin{cases}
x_1 = x_3 - 40 \\
x_2 = x_3 + 10 \\
x_3 \text{ is free} \\
x_4 = x_6 + 50 \\
x_5 = x_6 + 60 \\
\quad \text{free}
\end{cases}
$$

Date:

QUESTION .13 (b).

Ans:

To find minimum flows, note that since x_1 cannot be negative, $x_3 \geq 40$. This implies that $x_2 \geq 50$. Also, since x_6 cannot be negative, $x_4 \geq 50$ and $x_5 \geq 60$. The minimum flows are $x_2 = 50, x_3 = 40$, $x_4 = 50$, $x_5 = 60$ (when $x_1 = 0$ and $x_6 = 0$)

QUESTION.014:

Write the equation for each intersection:-

Intersection	Flow in		Flow out
A	80	=	$x_1 + x_5$
B	$x_1 + x_2 + 100$	=	x_4
C	x_3	=	$x_2 + 90$
D	$x_4 + x_5$	=	$x_3 + 90$

Re-arrange the equations:-

$x_1 \quad + x_5 = 80$

$x_1 + x_2 - x_4 = -100$

$x_2 - x_3 \quad = -90$

90

Date:

Reduce the augumented matrix:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 80 \\ 1 & 1 & 0 & -1 & 0 & -100 \\ 0 & 1 & -1 & 0 & 0 & -90 \\ 0 & 0 & 1 & -1 & -1 & -90 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 80 \\ 0 & 1 & 0 & -1 & -1 & -180 \\ 0 & 0 & 1 & -1 & -1 & -90 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) The general Solution is:-

$$\begin{cases} x_1 = 80 - x_5 \\ x_2 = x_4 + x_5 - 180 \\ x_3 = x_4 + x_5 - 90 \\ x_4 \text{ is free} \\ x_5 \text{ is free} \end{cases}$$

b).

If $x_5 = 0$, then the general Solution is:

$$\begin{cases} x_1 = 80 \\ x_2 = x_4 - 180 \\ x_3 = x_4 - 90 \\ x_4 \text{ is free} \end{cases}$$

Date:

(c). Since x_2 cannot be a negative, the minimum value of x_4 when $x_5 = 0$ is 180.