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Assignment # 2

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What is matrix determinant?

Defination:-

The determinant is a scalar value that is a function of entries of a square matrix. In Particular, the determinant is non-zero if and only if the matrix is invertible. The determinant of a matrix A is denoted det(A), det A or |A|.

In case of a 2x2 matrix the determinant can be defined as

$$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

Properties of determinant:.

These are some Properties of determinant, which are commonly used

Property 1:

The value of the determinant remains unchanged if it's rows and columns are interchanged.

i.e

$$|A^T| = |A|$$

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Example:-

$$A = \begin{bmatrix} 2 & 1 \\ 3 & 5 \end{bmatrix} \qquad A^T = \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & 1 \\ 3 & 5 \end{vmatrix} = (2)(5) - (1)(3) = 10-3 = 7$$

$$|A^T| = \begin{vmatrix} 2 & 3 \\ 1 & 5 \end{vmatrix} = (2)(5) - (3)(1) = 10-3 = 7$$

As

$$|A| = |A^T|$$

Property 2:

If any two row (or columns) of a determinant are interchanged then sign of determinant changes

$$|A| = \begin{vmatrix} 1 & 2 & 4 \\ 3 & 2 & 4 \\ 5 & 2 & 1 \end{vmatrix}$$

$$|A| = 1(2-8) - 2(3-20) + 4(6-10)$$

$$= -6 + 34 - 16$$

$$|A| = 12$$

$$|A| = \begin{vmatrix} 1 & 2 & 4 \\ 3 & 2 & 4 \\ 5 & 2 & 1 \end{vmatrix}$$

interchange R_2 with R_3

$$|A'| = \begin{vmatrix} 1 & 2 & 4 \\ 5 & 2 & 1 \\ 3 & 2 & 4 \end{vmatrix}$$

$$= 1(8-2) - 2(20-3) + 4(10-6)$$

$$= 6 - 34 + 16$$

$$= -12$$

So its Proved

$$|A| = -|A'|$$

Property 3:

If all elements of a row (or Column) are zero then determinant is 0.

Example:-

$$|A| = \begin{vmatrix} 0 & 0 & 0 \\ 3 & 4 & 5 \\ 1 & 2 & 1 \end{vmatrix}$$

$|A| = 0(6-4) - 0(3-5) + 0(6-4)$

$|A| = (0)(2) - (0)(-2) + 0(2)$

$|A| = 0$

Hence Proved. a matrix of determinant with zero row or column is zero.

Property 4:-

If any two rows (or columns) of a determinant are identical the value of determinant is zero.

Example:-

$$|A| = \begin{vmatrix} 1 & 2 & 3 \\ 1 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$$

As R_1 and R_3 has same values or same.

$|A| = 1(0-2) - 2(3-1) + 3(2-0)$

$|A| = -2 - 4 + 6$

$|A| = -6 + 6$

$|A| = 0$

So Proved with same rows th

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determinant of this matrix is zero.

Property 5:

If each element of a row (or a column) of a determinant is multiplied by a constant K, then determinant's value get multiplied by K

Example:

As

$$L.H.S = \begin{vmatrix} 2 & 4 & 6 \\ 1 & 0 & 2 \\ 2 & 3 & 5 \end{vmatrix} = |A|$$

$$= 2(0-6) - 4(5-4) + 6(3-0)$$
$$= -12 - 4 + 18$$
$$= -16 + 18$$
$$= 2$$

Same

$$|A| = \begin{vmatrix} 2 & 4 & 6 \\ 1 & 0 & 2 \\ 2 & 3 & 5 \end{vmatrix} =$$

$$|A| = 2 \begin{vmatrix} 1 & 2 & 3 \\ 1 & 0 & 2 \\ 2 & 3 & 5 \end{vmatrix} = R.H.S$$

$$|A| = 2[1(0-6)-2(5-4)+3(3-0)]$$

$$|A| = 2[-6-2+9]$$

$$|A| = 2[-8+9]$$

$$|A| = 2(1) \Rightarrow 2$$

Soo Proved

$$\begin{vmatrix} 2 & 4 & 6 \\ 1 & 0 & 2 \\ 2 & 3 & 5 \end{vmatrix} = 2\begin{vmatrix} 1 & 2 & 3 \\ 1 & 0 & 2 \\ 2 & 3 & 5 \end{vmatrix}$$

L.H.S = R.H.S

Property 6:

If elements of a row or column of a determinant are expressed as sum of two (or more) terms, then the determinant can be expressed a sum of two (or more) determinants.

Example:-

$$|A| = \begin{vmatrix} a & b & c \\ a+2x & b+2y & c+2z \\ x & y & z \end{vmatrix}$$

As according to Property

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$$|A| = \begin{vmatrix} a & b & c \\ a & b & c \\ x & y & z \end{vmatrix} + \begin{vmatrix} a & b & c \\ 2x & 2y & 2z \\ x & y & z \end{vmatrix}$$

$$|A| = \begin{vmatrix} a & b & c \\ a & b & c \\ x & y & z \end{vmatrix} + 2\begin{vmatrix} a & b & c \\ x & y & z \\ x & y & z \end{vmatrix}$$

As we see in above Property if two rows are columns are same then determinant will be zero. So according to this Property

$|A| = 0 + (2)(0)$

$|A| = 0$

Property 7:

If in a determinant all the elements above or below the diagonal is zero, then value of determinant is equal to Product of diagonal elements.

Example:

$$\begin{vmatrix} a & b & c \\ 0 & e & f \\ 0 & 0 & i \end{vmatrix} = \begin{vmatrix} a & 0 & 0 \\ d & e & 0 \\ g & h & i \end{vmatrix} = a \times e \times i$$