{Sheet6} ← {H}

GCD → Greatest Common Divisor

gcD(20, 15) → 5, gcD(5, 5) → 5, gcD(5, 0) → 5

0 Divisable by any number

30 = 2 × 3 × 5
45 = 3 × 3 × 5 → gcD(30, 45) = 3×5 = 15

⇒ Take Prime factorization of both and multiply the Common.

Euclidean Algorithm

gcD(20, 15)

first step → 20 15
5 (15-5)
5 10
5 (10-5)
5-5 5
0 5 → so gcD is 5

another ex: gcD(12, 8) →
12-8 8
4, 8-4
4, 0
This the GCD

gcD(30, 60)
30, 60-30
30, 30-30
30, 0
GCD

12 = 2 × 3 × 2
8 = 2 × 2 × 2
Common numbers (2, 2)

Before Understanding Euclidean → We need to understand modular arithmatic

14 % 3 → ايه اصغر رقم مش طالب انقصه من ال 14 علشان تقبل القسمة على 3

14 % 3 → 2, 14 - 2 = 12 / 3 ⇒ Can be divisable.

$X > y$

$23 - 5 = 18$

$X < y$

$X \% y = m$, $4 * 3 + 2 = 14$, $23 \% 6 = 5$, $4 \% 5 = 4$

$q * y + m = X$, $2 \% 2 = 0$

$X = y$

Because 0 can be divisable by any number.

$1 \% 0$ = Error , $12 \% 6 = 0$

Get modulas of each of the Following ⟹ 0 1 2 3 4 5 6 7 8 9 % 0 1 2 3 4 0 1 2 3 4

modular arithmatic Properties :
→ our mod Range is 0 to X-1 for num % X
→ cyclic

لان الارقام زى المثال اللى فات تلف وتتكرر
مثال و مطرر انقص منه 4 ثم 8 ← 3 لو جبنا مثال
أكبر زى 14 ← هننقص 4 كمان ثم و ثم ...

$(a+b) \% c = ((a \% c) + (b \% c)) \% c$

$(a * b) \% c = ((a \% c) * (b \% c)) \%$

بنجزئ احتساباً للأرقام الكبيرة وتجنباً لظهور errors

$a \% c \rightarrow 0 \rightarrow c-1$

$b \% c \rightarrow 0 \rightarrow c-$

Worst Case $(2c - 2)$

Negative example

→ $-8 \% 3 = (\underline{-8 + (3 * 3)}) \% \underline{3} \rightarrow 1 \% 3 = 1$

علشان نعرف فلا لازم تكون موجب

→ فمفروض ف الحالة ماتبقى موجب او $0 <$

⇒ $(a - b) \% c = ((a \% c) - (b \% c) + c) \% c$

we add c in worst case

$a \% c = 0$

$b \% c = c - 1$

$(a \% c) - (b \% c) = -(c-1)$

Back to Euclidean $A = B * Q + m$, $m = A - B * Q$

بنطرح عدد مم ال B لحد منوصل الواحد 0

$GCD(A, B) \longrightarrow GCD(B, A\%B) \longrightarrow$ Rule

$GCD(12, 8) \longrightarrow GCD(8, 12\%8)$

$GCD(8, 4) \longrightarrow GCD(4, 8\%4)$

We stop when $\longleftarrow GCD(4, 0)$

$B = 0$ This the Result

Worst Case Time $O(\log(\max(a, b)))$

$\Downarrow$

example Turn it into Python

1- input $\longrightarrow$ 12 , 18

Output $\longrightarrow$ 6 , 36 $\longrightarrow$ Without Lcm

GCD(12, 18)

```
def GCD(A, B):
    First number = min(A, B)   # We should ensure that A is the Max
    Second number = max(A, B)    and b is the min

    S.N = Second number % First number , Return
      GCD(Second number, First number)
```

```
def GCD(A, B):
    if B = 0:
        Return A
    Return GCD(B, A%B)   -> Recursivly
```

Least Common multiple → اصغر رقم ممكن اقسمه على A and B

$a = 3$, $b = 8$ → $Lcm(a,b) = 24$

$a = 6$, $b = 18$ → $lcm(a,b)$ → 18

$Lcm \rightarrow (a,b) \Rightarrow (3,8) \rightarrow 3 = 3$, $8 = 2 \times 2 \times 2$ → Prime factorizatic
$Lcm(3,8) = 2 \times 2 \times 2 \times 3 = 24$

$Lcm(6,18) \rightarrow 6 = 2 \times 3$, $18 = 2 \times 3 \times 3$, $Lcm(6,18) = 2 \times 3 \times 3 = 18$
2^1 3^1 2 3^2 → بناخذ أكبر عدد من التكرار لكل رقم

$2^2, 3^2, 5^1, 7^1$
$a = 2*2*3*3*5*7$
$b = 2*3*5*5*7*7$
$2^1, 3^1, 5^2, 7^2$
→ in Lcm We Take Most Repeted

$lcm(a,b) = 2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 7 \times 7$
$a \times b = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 5 \times 5 \times 7 \times 7 \times 7$ → We take all except
$2 \times 3 \times 5 \times 7$ → minmum of Repetition → gcd
Lcm → we take "الاكثر تكراراً"
We Conclude $Lcm = a \times b / GCD(a,b)$