DeepMind

Deep Learning with Low Resources

An algorithmic approach

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Will make slides available after the talk

UCL

Outline

- **Small data: Meta-Learning**
    - **Overview**
    - Optimisation-based
    - Black-box
- Faster training and inference: Sparsity
    - Overview
    - Explicit Sparsity
    - Implicit Sparsity

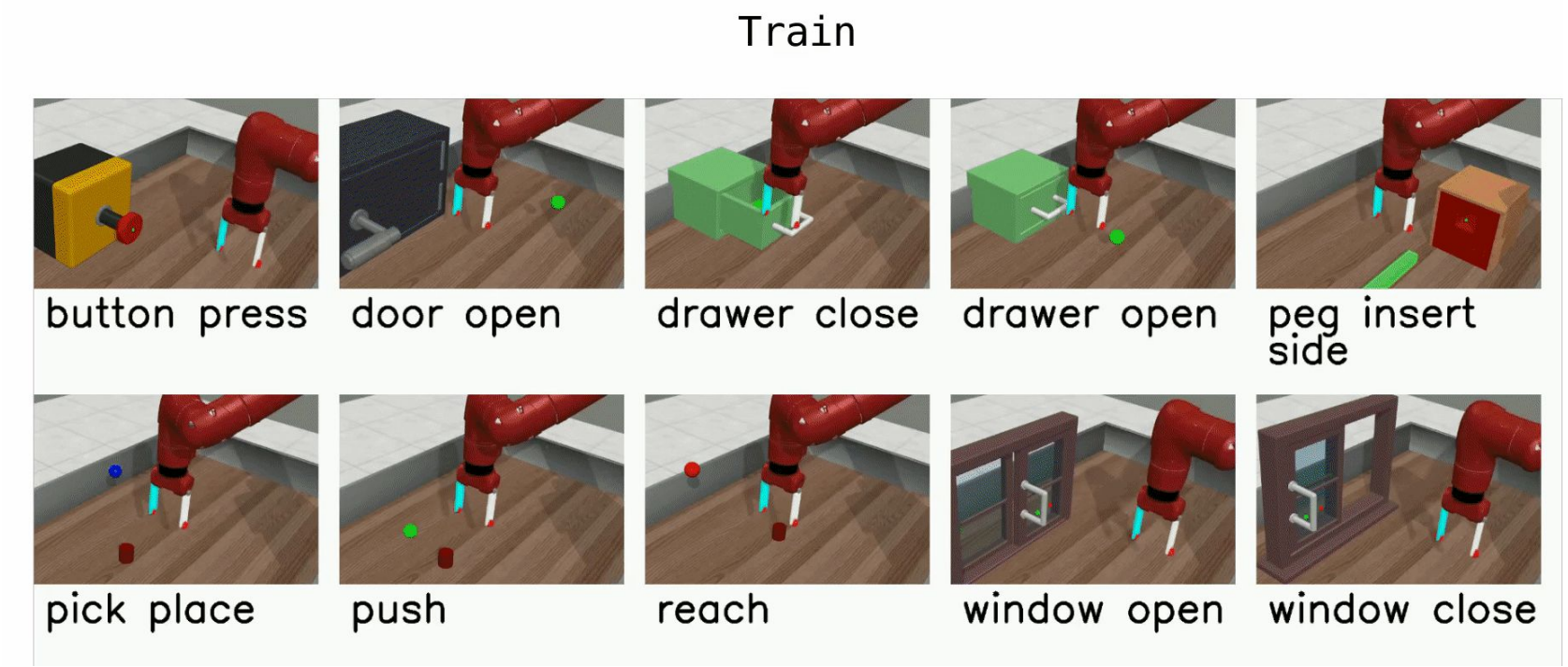
- Discussion & Looking forward

Small data problems - Few shot learning

[Andrei Rusu, ImageNet]

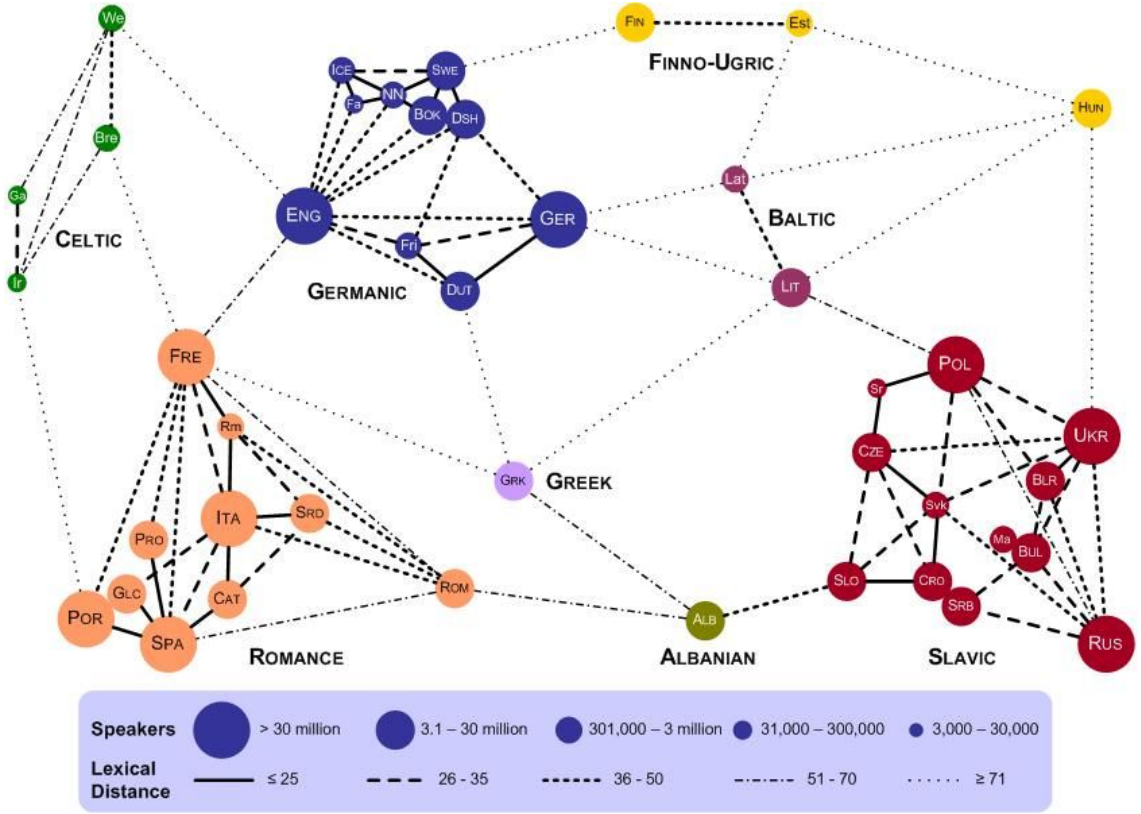
Small data problems - Recommender Systems

Small data problems - Robotics (e.g. demonstrations)

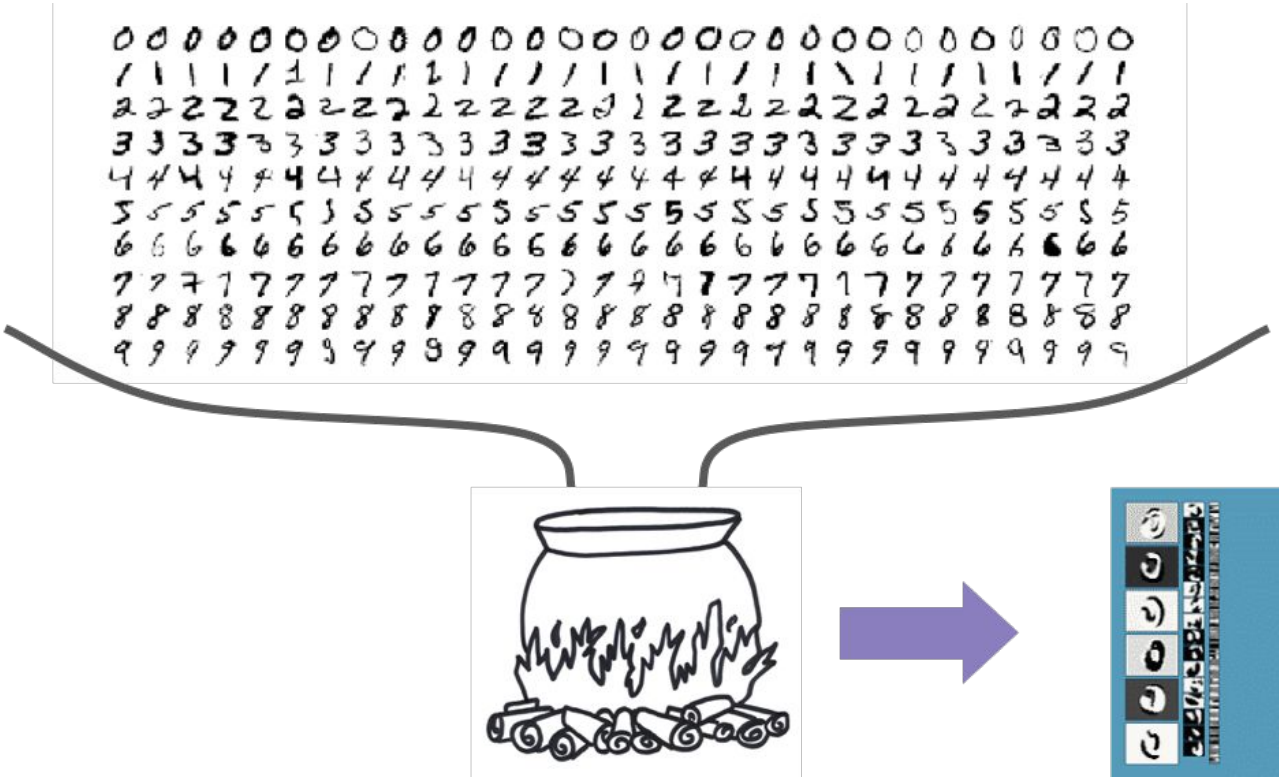


[MetaWorld]

Small data problems - Language

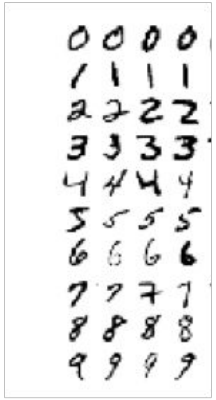


Machine Learning with big data



[Yann LeCun, MNIST]

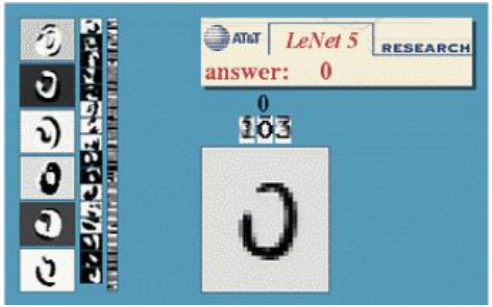
Machine Learning with small data



Inductive biases:

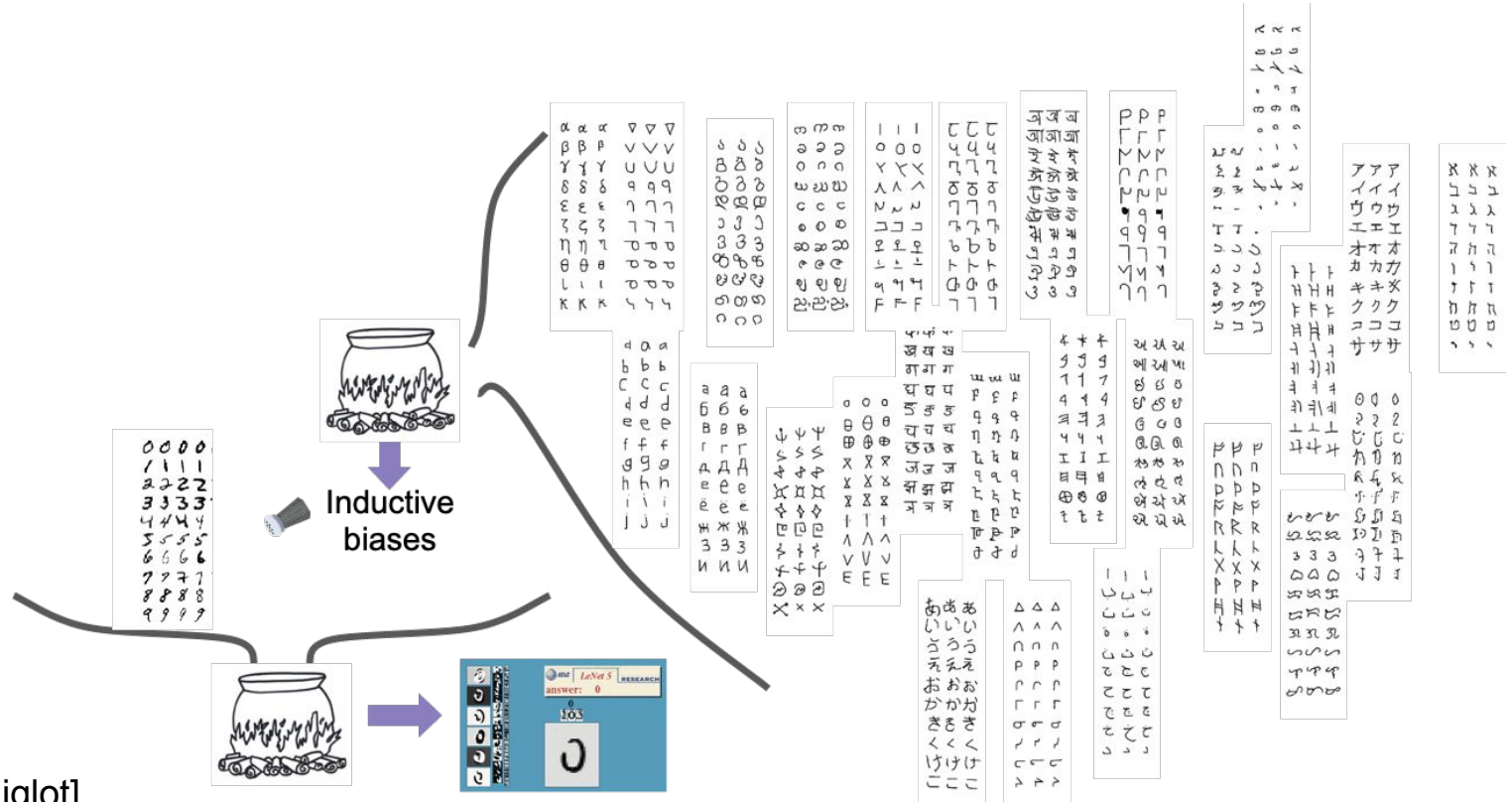
- Preprocessing/Feature engineering
- Data augmentation
- Regularisation
- Model architectures

→ **Time intensive!**



[Yann LeCun, MNIST]

Learning to learn / Meta-Learning

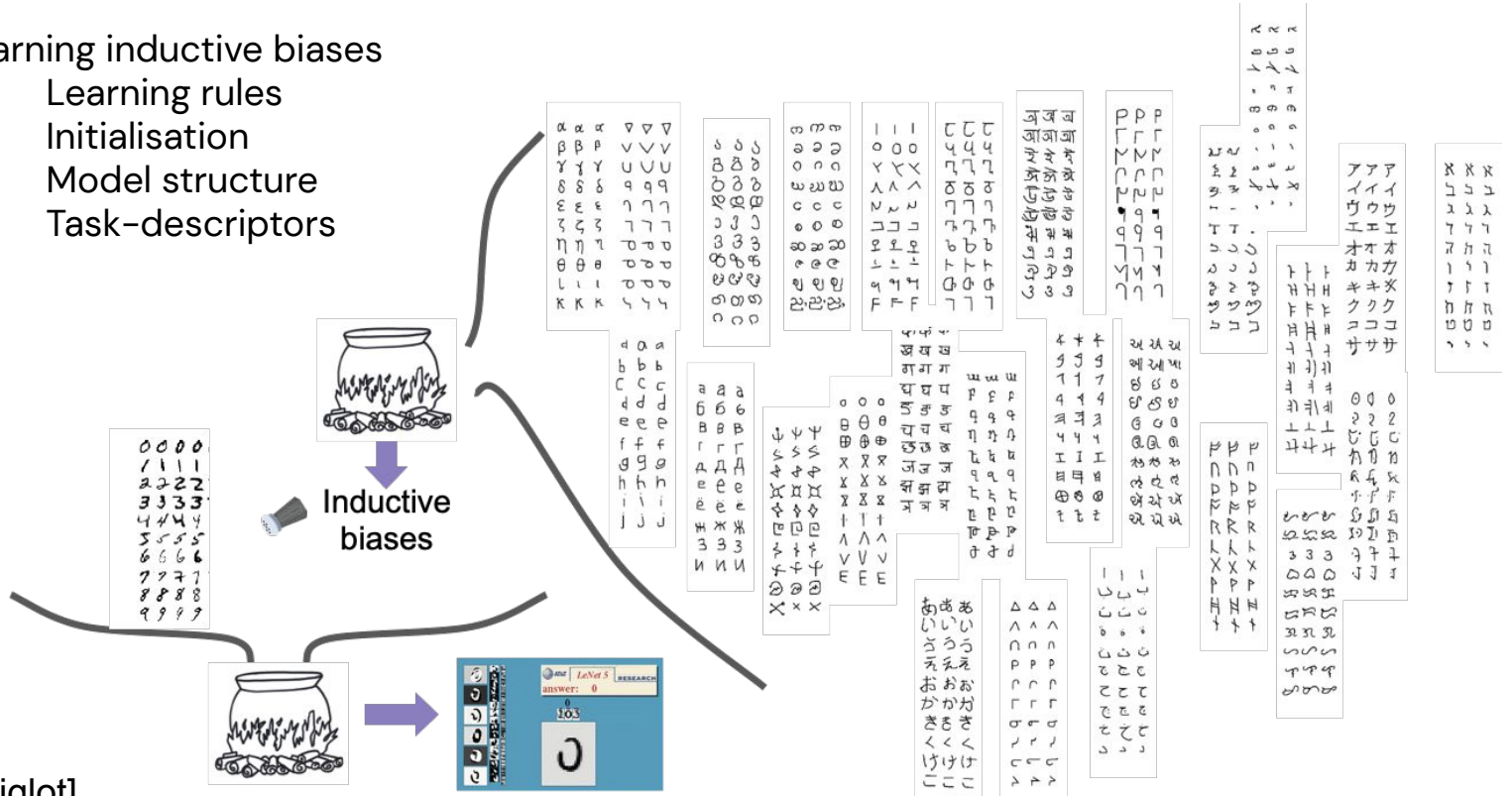


[Omniglot]

Learning to learn / Meta-Learning

Learning inductive biases

- Learning rules
- Initialisation
- Model structure
- Task-descriptors



[Omniglot]

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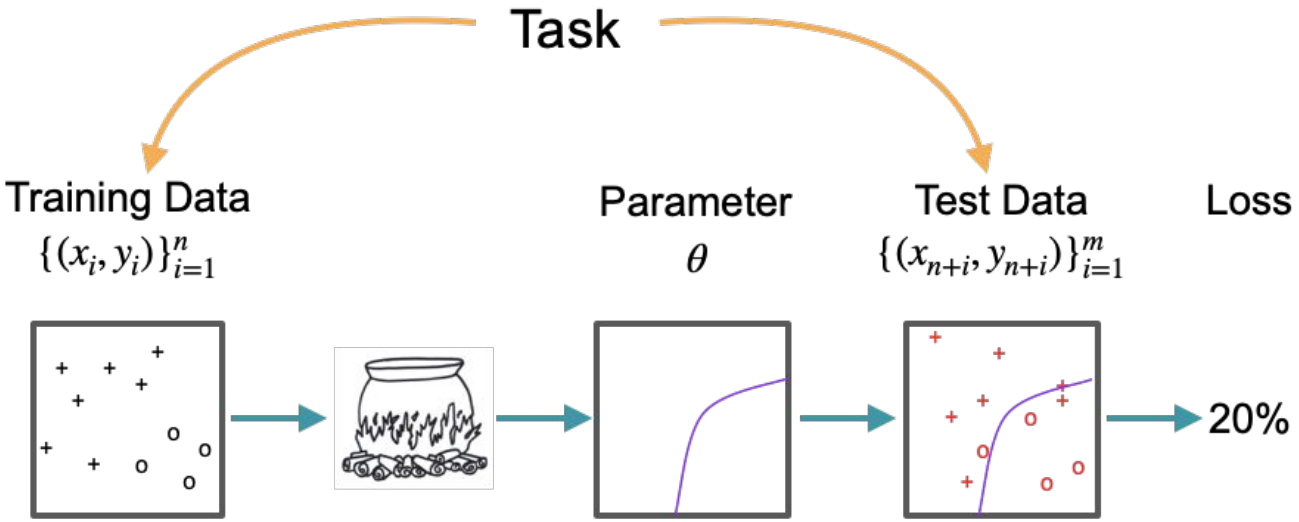
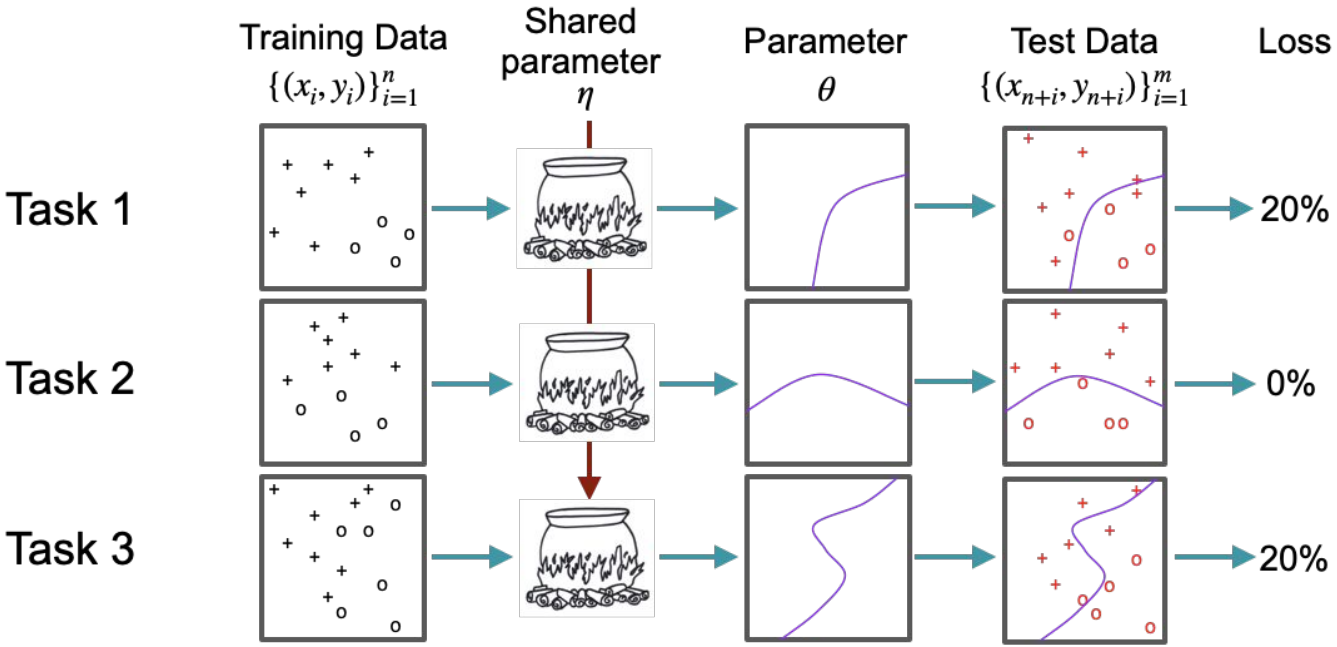
Single Task Learning



Empirical Risk Minimisation

$$\arg\min_{\theta} \sum_{i=1}^{n} \text{Loss}(f_{\eta,\theta}(x_i), y_i) + \text{Regulariser}(\theta)$$

Multi Task Learning



$$\arg\min_{\eta,\{\theta_j\}} \sum_{j=1}^{T} \sum_{i=1}^{n} \text{Loss}(f_{\eta,\theta_j}(x_{ji}), y_{ji}) + \text{Regulariser}(\theta_j)$$

Meta-Learning

- Is it possible to learn to generalise well from small data?
  - Learn:

$$\theta_j = \arg\min_{\theta_j} \sum_{i=1}^{n} \text{Loss}(f_{\eta,\theta_j}(x_{ji}), y_{ji}) =: \text{Learner}(\eta, \text{TrainData}_j)$$

  - Test performance

$$\sum_{i=n+1}^{n+m} \text{Loss}(f_{\eta,\theta_j}(x_{ji}), y_{ji}) =: \mathscr{L}(\eta, \theta_j, \text{TestData}_j)$$

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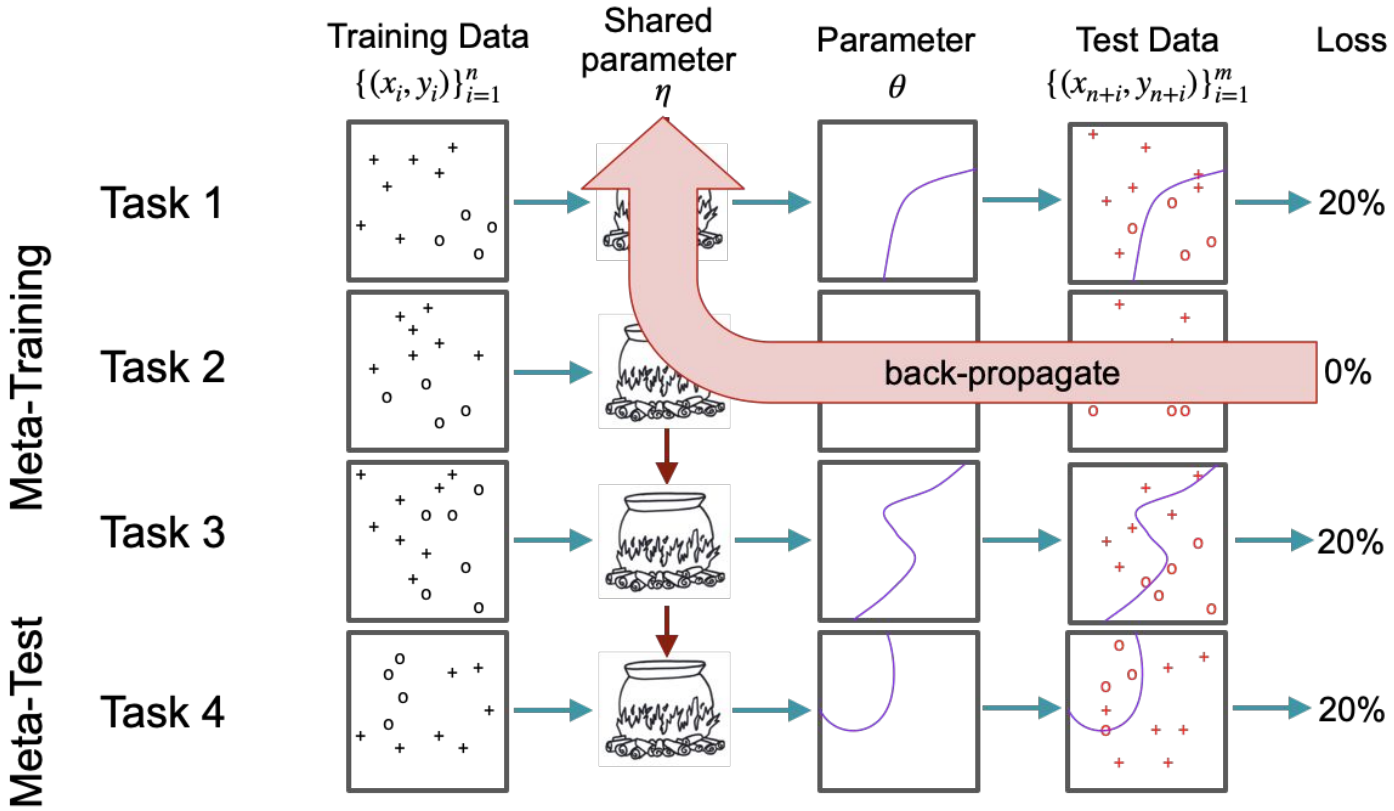
$$\sum_{i=n+1}^{n+m} \text{Loss}(f_{\eta,\theta_j}(x_{ji}), y_{ji}) =: \mathscr{L}(\eta, \theta_j, \text{TestData}_j)$$

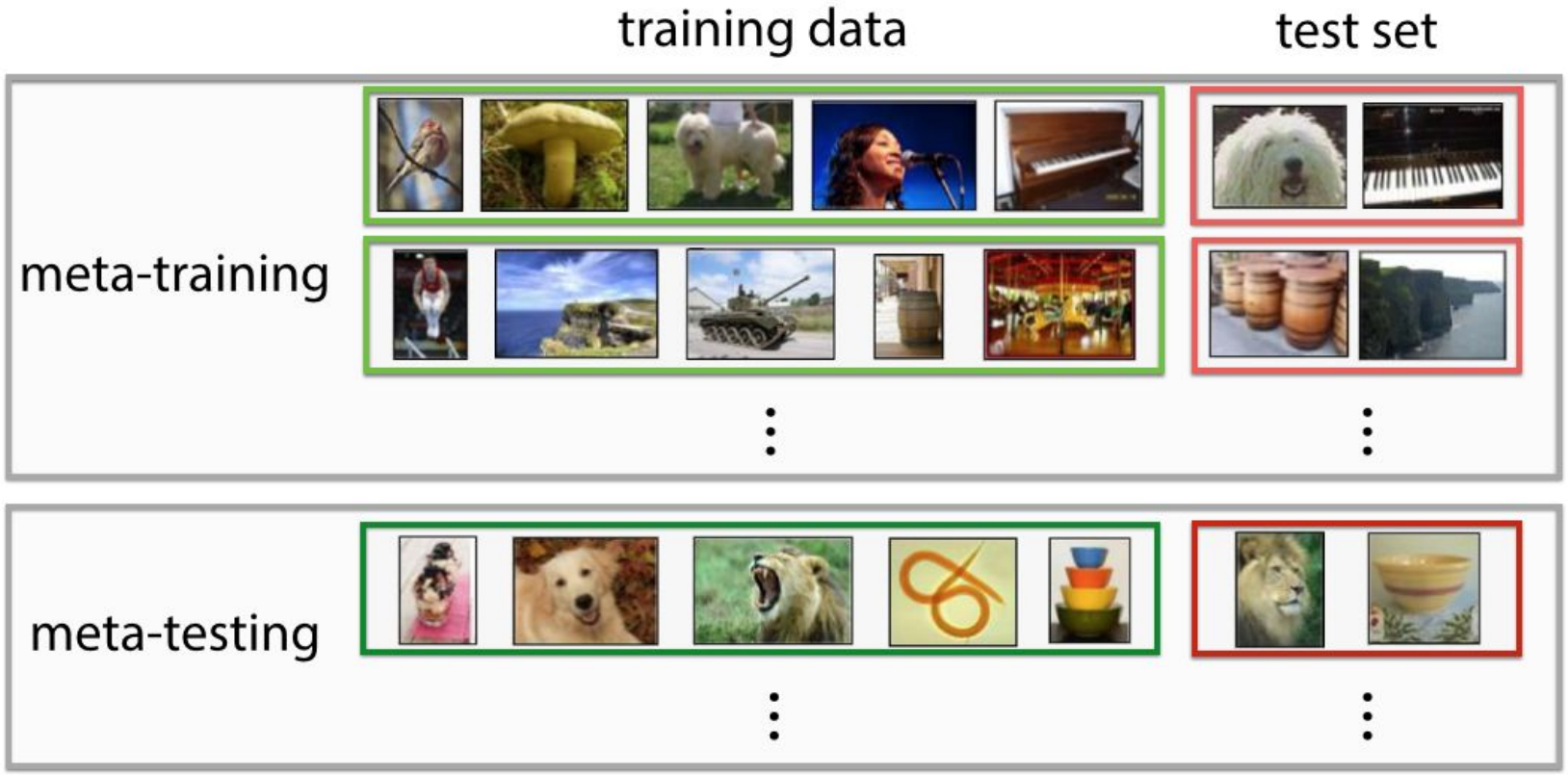
  - Optimise shared parameters for test performance

$$\arg\min_{\eta} \sum_{j=1}^{T} \mathscr{L}(\eta, \theta_j, \text{TestData}_j) = \arg\min_{\eta} \sum_{j=1}^{T} \sum_{i=n+1}^{n+m} \text{Loss}(f_{\eta,\text{Learner}(\eta,\text{TrainData}_j)}(x_{ji}), y_{ji})$$

Meta Learning



Training/Test?



[Chelsea Finn]

Meta-Learning Pseudocode

- For each iteration:
  - Pick a (minibatch of) training tasks (here j)
  - Base Learner

$$\theta_j = \text{Learner}(\eta, \text{TrainData}_j) = \arg\min_{\theta_j} \sum_{i=1}^{n} L(f_{\eta,\theta_j}(x_{ji}), y_{ji})$$

  - Test performance

$$\mathscr{L}(\eta, \theta_j, \text{TestData}_j) = \sum_{i=n+1}^{n+m} \text{Loss}(f_{\eta,\theta_j}(x_{ji}), y_{ji})$$

Meta-Learning Pseudocode

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  - Test performance

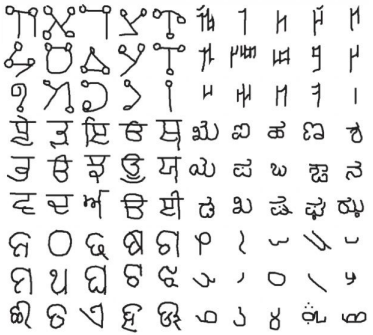
$$\mathscr{L}(\eta, \theta_j, \text{TestData}_j) = \sum_{i=n+1}^{n+m} \text{Loss}(f_{\eta,\theta_j}(x_{ji}), y_{ji})$$

  - Meta-Learner: Optimise shared parameters for test performance

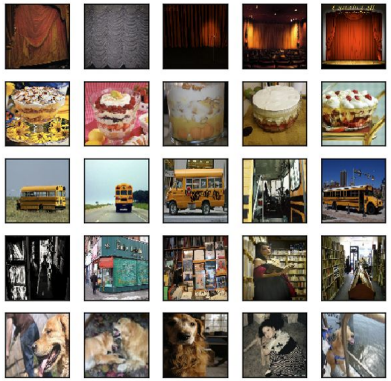
$$\eta \leftarrow \eta - \epsilon \frac{d}{d\eta} \sum_{i=n+1}^{n+m} \text{Loss}(f_{\eta,\text{Learner}(\eta,\text{TrainData}_j)}(x_{ji}), y_{ji})$$

Meta-Learning Datasets

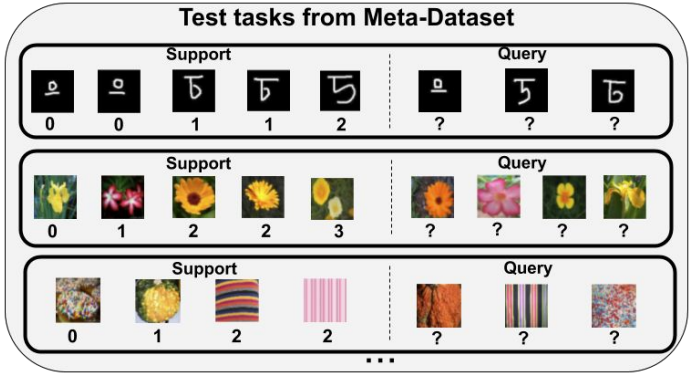
Omniglot



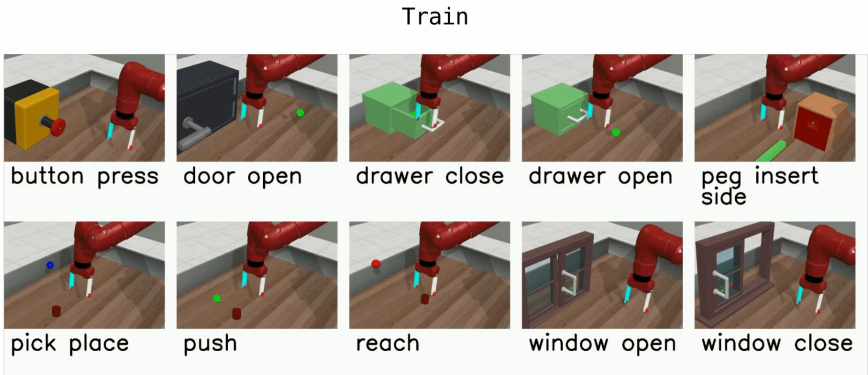
mini-Imagenet



Meta-Dataset

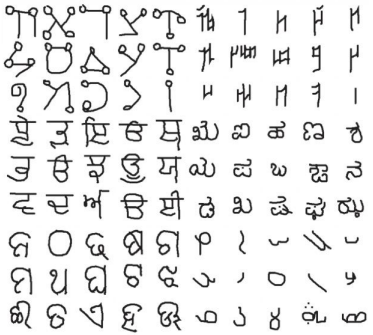


MetaWorld

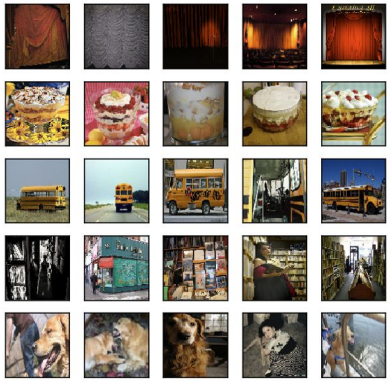


Meta-Learning Datasets

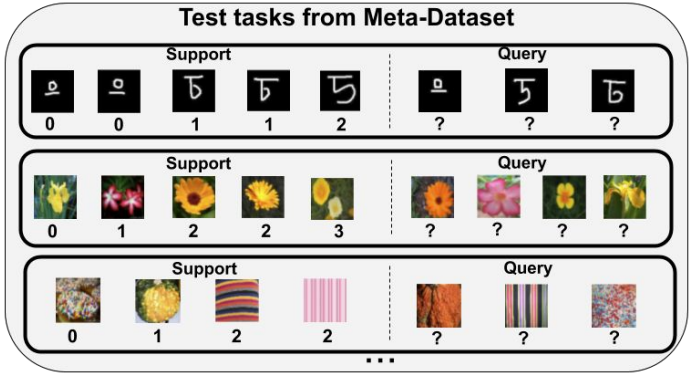
Omniglot



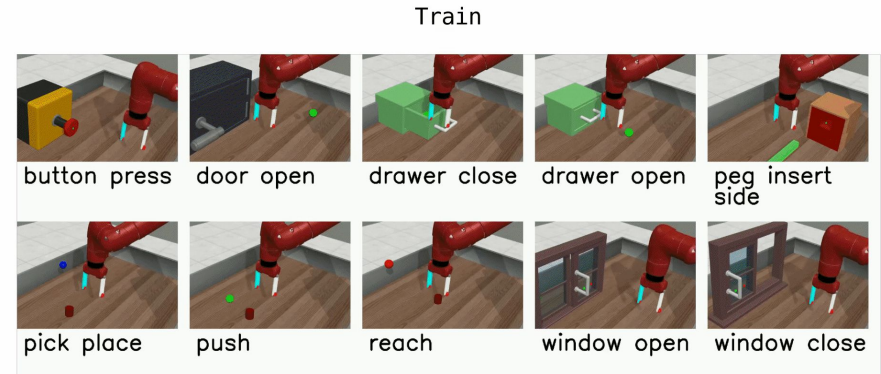
mini-Imagenet



Meta-Dataset



MetaWorld



Design of such datasets still an empirical question

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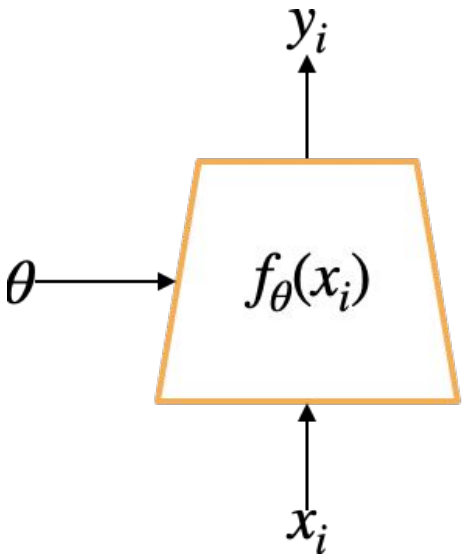
Optimisation-based Meta-Learning

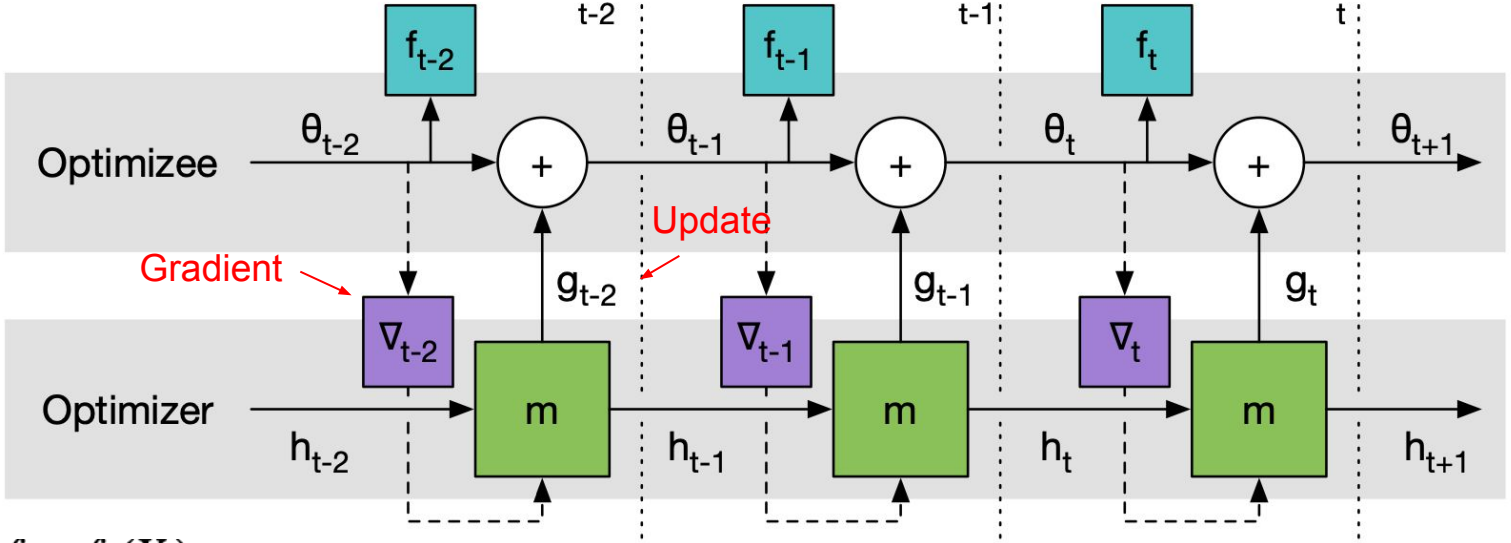


- Optimisation-based base learner (for each task j)

$$\theta_0$$
$$\theta_1 \leftarrow \theta_0 + \text{Update}_\eta(\nabla_{\theta_0} L(f_{\theta_0}(X_1), Y_1))$$
$$\theta_2 \leftarrow \theta_1 + \text{Update}_\eta(\nabla_{\theta_1} L(f_{\theta_1}(X_2), Y_2))$$
$$\vdots$$

- Meta-Learn parameters θ_0, η

Learning to Learn by Gradient Descent by Gradient Descent, LSTM Meta-Learner



$$f_t = f_{\theta_t}(X_t)$$

$$\nabla_t = \frac{d}{d\theta_t} L(f_t, Y_t)$$

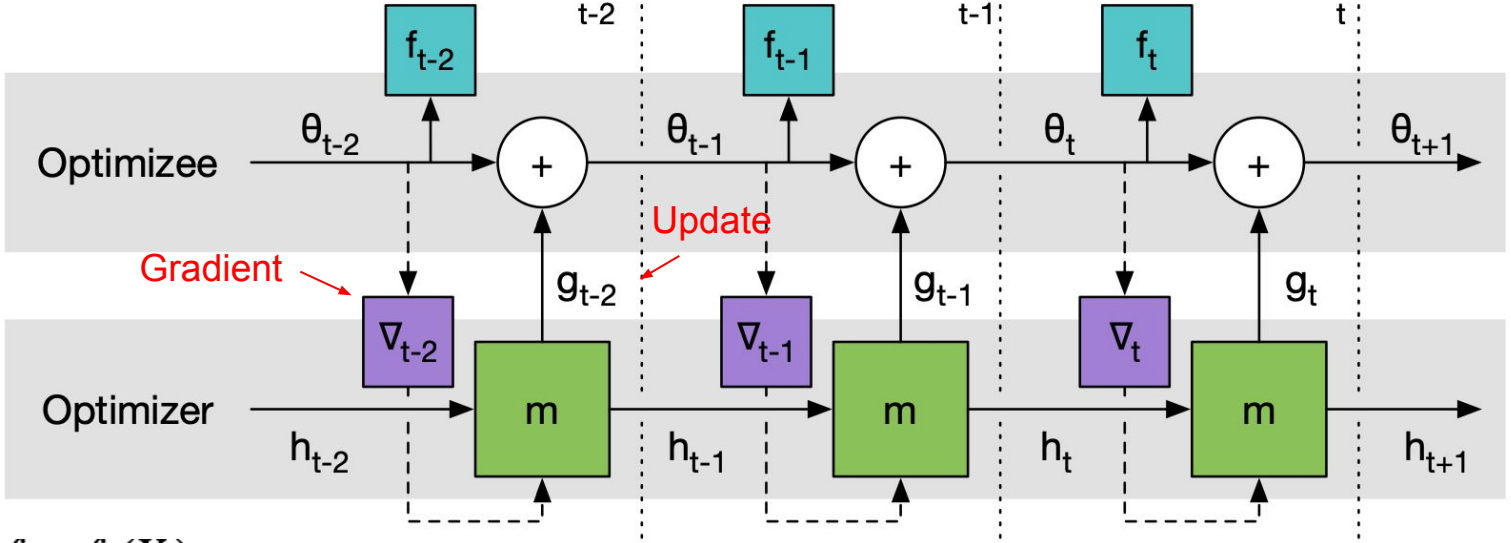
X_t : training inputs in iter t

Y_t : training targets in iter t

$$\theta_{t+1} = \theta_t + g_t$$

[Andrychowicz NeurIPS 2016, Ravi & Larochelle ICLR 2017]

Learning to Learn by Gradient Descent by Gradient Descent, LSTM Meta-Learner



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$$\nabla_t = \frac{d}{d\theta_t} L(f_t, Y_t)$$

X_t : training inputs in iter t

Y_t : training targets in iter t

ADAM

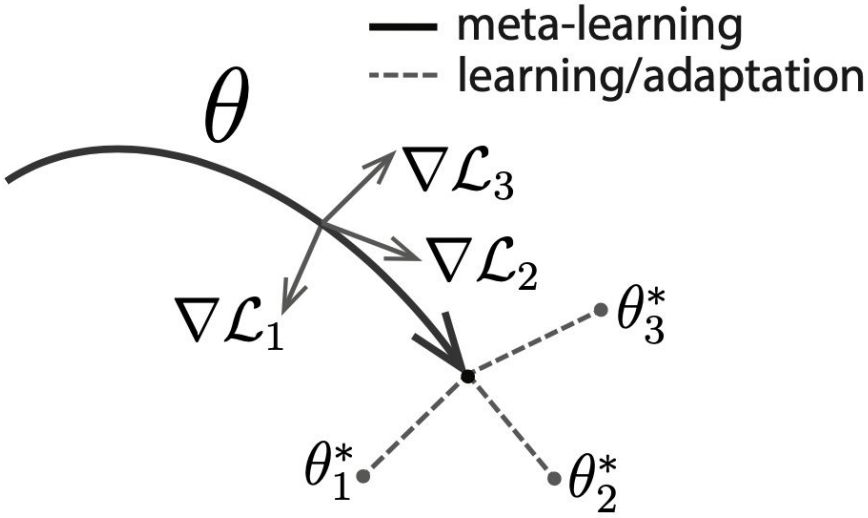
$$\mu_t = \beta_1 \mu_{t-1} + (1-\beta_1)\nabla_t$$

$$V_t = \beta_2 V_{t-1} + (1-\beta_2)\nabla_t^2$$

$$g_t = -\frac{\epsilon}{\sqrt{V_t} + \delta}\mu_t$$

[Andrychowicz NeurIPS 2016, Ravi & Larochelle ICLR 2017]

Model-Agnostic Meta-Learning (MAML)



- One (or few) Gradient step Learner:

$$\theta_j = \eta - \epsilon \nabla_\eta L(f_\eta(X_j), Y_j)$$

- Meta-Learning shared initialisation

$$\eta \leftarrow \eta - \epsilon_0 \frac{d}{d\eta} \sum_j L(f_{\theta_j}(X_{jt}), Y_{jt})$$

- Compute Gradient through inner optimisation step (Second order)

[Finn et al, ICML 2017]

Model-Agnostic Meta-Learning (MAML)

$$\theta_j = \eta - \epsilon \nabla_\eta L(f_\eta(X_j), Y_j)$$

$$\frac{d}{d\eta} L(f_{\theta_j}(X_{jt}), Y_{jt}) = \nabla_{\theta_j} L(f_{\theta_j}(X_{jt}), Y_{jt}) \frac{d\theta_j}{d\eta}$$

$$= \nabla_{\theta_j} L(f_{\theta_j}(X_{jt}), Y_{jt}) \left(I - \epsilon \nabla_\eta^2 L(f_\eta(X_{jt}), Y_{jt}) \right)$$

- Difficulty in computing second term
- Vector-Hessian products can be automatically computed in linear time

[Pearlmutter NECO 1994]

Optimisation-based Meta-Learning

- Two-level optimisation problem:
  - Flexible and applicable to wide range of neural architectures (in theory)
  - Positive inductive bias

- Challenges
  - Sensitive to neural architecture
  - Can be expensive and unstable
  - Difficult to learn for long inner optimisation processes

Outline

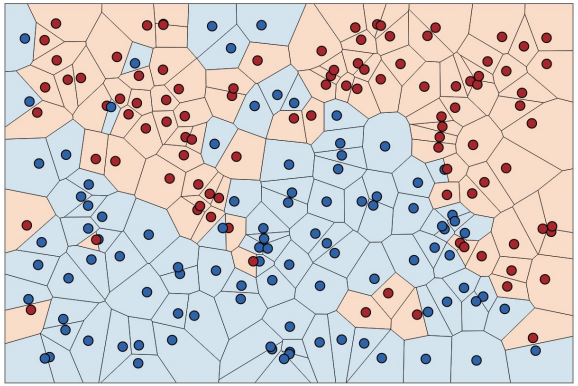
- **Meta-Learning**
  - Overview
  - Optimisation-based
  - **Black-box**
- Continual Learning
  - Overview
  - Regularisation-based
  - Rehearsal/Pseudo-rehearsal based methods

- Discussion & Looking forward

Black-box Meta-Learning

- So far, the base learner is an **optimisation-based** learner
- Not all learning algorithms are optimisation-based

- For example, in k-nearest neighbours, learner simply memorises training data and compares test inputs to make predictions



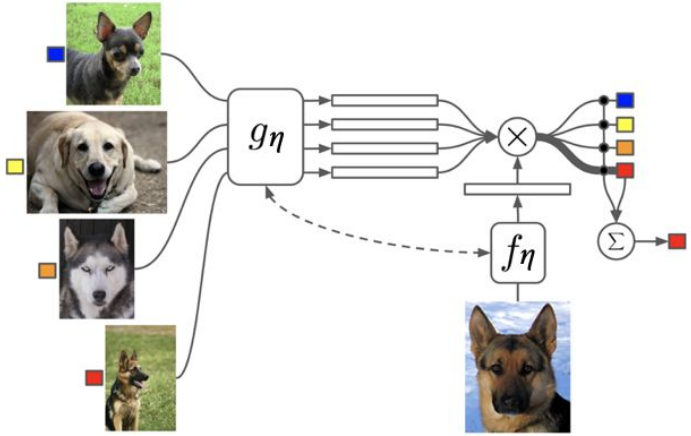
Matching Networks

- Embed each training input $x_i \rightarrow g_\eta(x_i)$
- Embed each test input $x_i \rightarrow f_\eta(x_i)$

- "Softened" 1-NN classifier

$$p(\cdot \mid x_{n+j}, \text{TrainData}) = \sum_{i=1}^{n} \frac{e^{g_\eta(x_i)^\top f_\eta(x_{n+j})}}{\sum_{l=1}^{n} e^{g_\eta(x_l)^\top f_\eta(x_{n+j})}} y_i$$

- Memory-based Meta-Learning
- Metric-based Meta-Learning



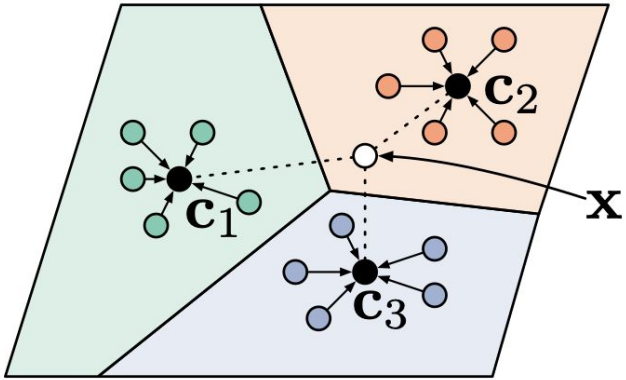
[Vinyals et. al, NeurIPS 2016]

Prototypical Networks

- Embed each input
- Form “prototypes”

$$x_i \rightarrow f_\eta(x_i)$$

$$c_k = \frac{\sum_{i:y_i=k} f_\eta(x_i)}{\sum_{i:y_i=k} 1}$$



- Predict

$$p(y_{n_j} = k \,|\, x_{n+j}, \text{TrainData}) = \frac{e^{-\|f_\eta(x_{n+j}) - c_k\|^2/\sigma^2}}{\sum_{l=1}^{K} e^{-\|f_\eta(x_{n+j}) - c_l\|^2/\sigma^2}}$$

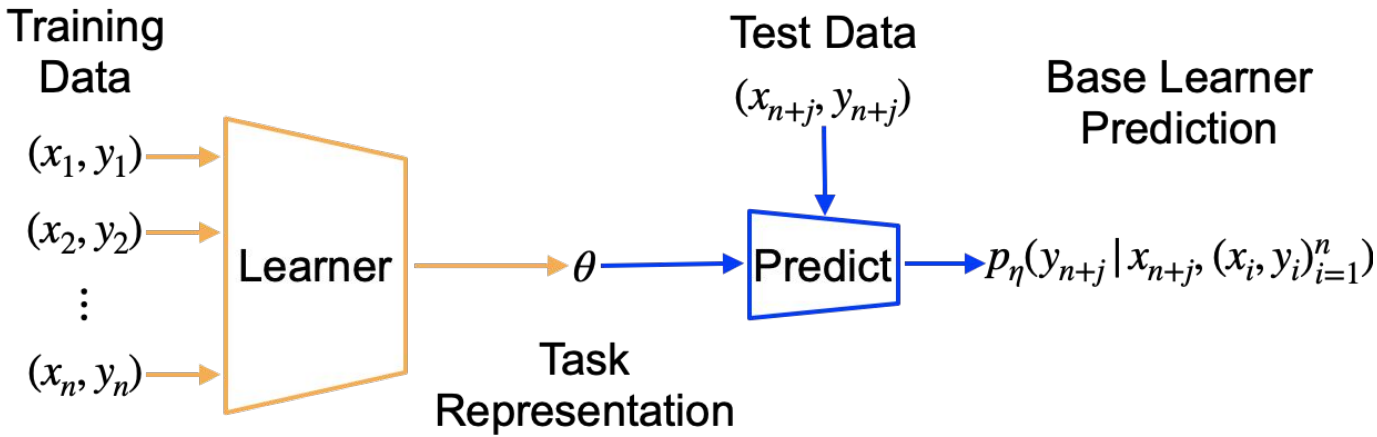
[Snell et al, NeurIPS 2017]

Black-box Meta-Learning

- Learn a differentiable function to map from training data to test predictions
  - Simple: Reduces Meta-Learning to Supervised Learning
  - Broad and flexible

- Challenges
  - Less able to generalise out of meta-training distribution

Task-inference



- Examples:
  - A vector describing preferences of a user in recommender systems
  - A physical description of the properties of a control system with unknown variation (different robot arms)

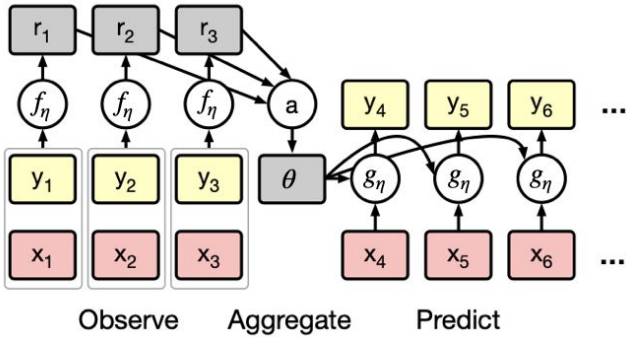
Permutation Invariance

- Assumed that data instances within each task are iid.
- Learner function should be invariant to permutation of the training data

$$\text{Learner}(\eta, \{(x_1, y_1), \ldots, (x_n, y_n)\}) = \text{Learner}(\eta, \{(x_{\pi(1)}, y_{\pi(1)}), \ldots, (x_{\pi(n)}, y_{\pi(n)})\})$$

- MAML, Prototypical Nets, Matching Nets are
- LSTM Meta-Learner are not
- Permutation invariance is an inductive bias

Conditional Neural Processes



- Embed input/output pairs

$$(x_i, y_i) \rightarrow f_\eta(x_i, y_i)$$

- Aggregate embeddings

$$\theta \leftarrow \frac{1}{N} \sum_{i=1}^{N} f_\eta(x_n, y_n)$$

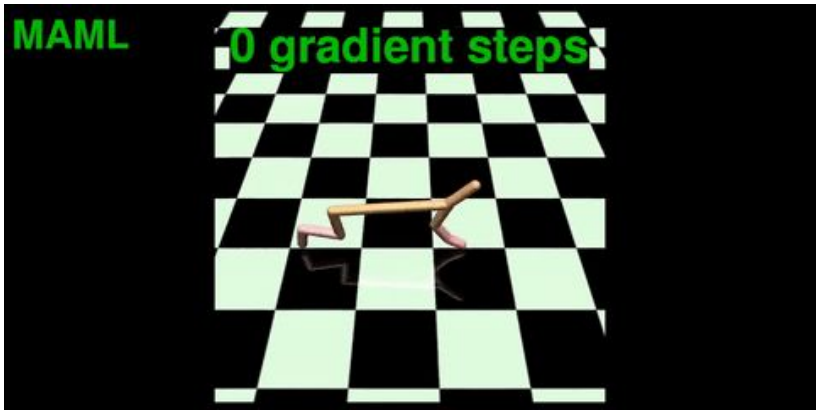
- Predict

$$p(y_t | x_t) = g_\theta(x_t, \theta)$$

- Permutation invariant by design
- Interpret θ as task representation

[Garnelo et al, ICML 2018]

What Meta-Learning can achieve - Reinforcement Learning

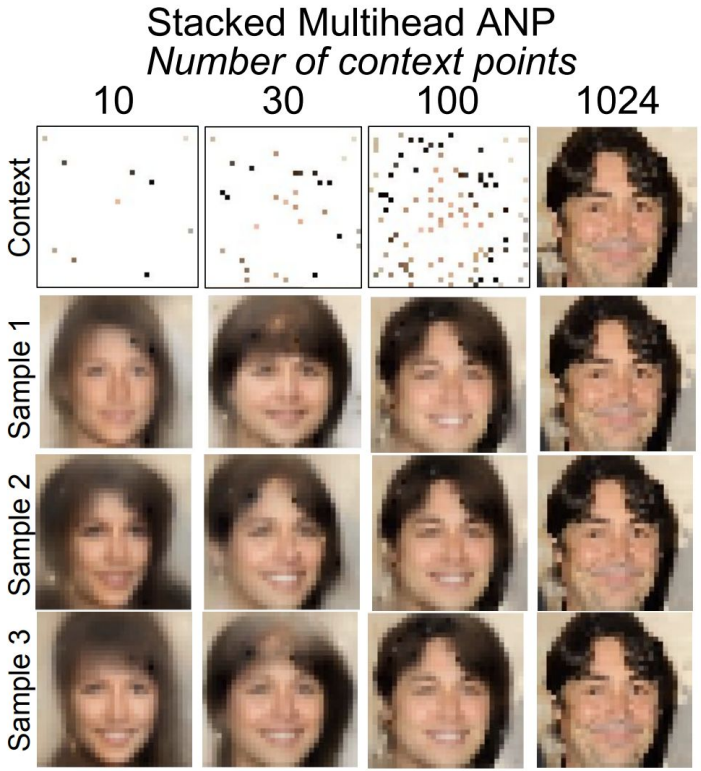


[Finn et al, ICML 2017]

What Meta-Learning can achieve - Reinforcement Learning

[Finn et al, ICML 2017]

What Meta-Learning can achieve - Image Inpainting



[Kim et al, ICRL 2018]

What Meta-Learning can achieve - Recommender Systems

MovieLens 100k

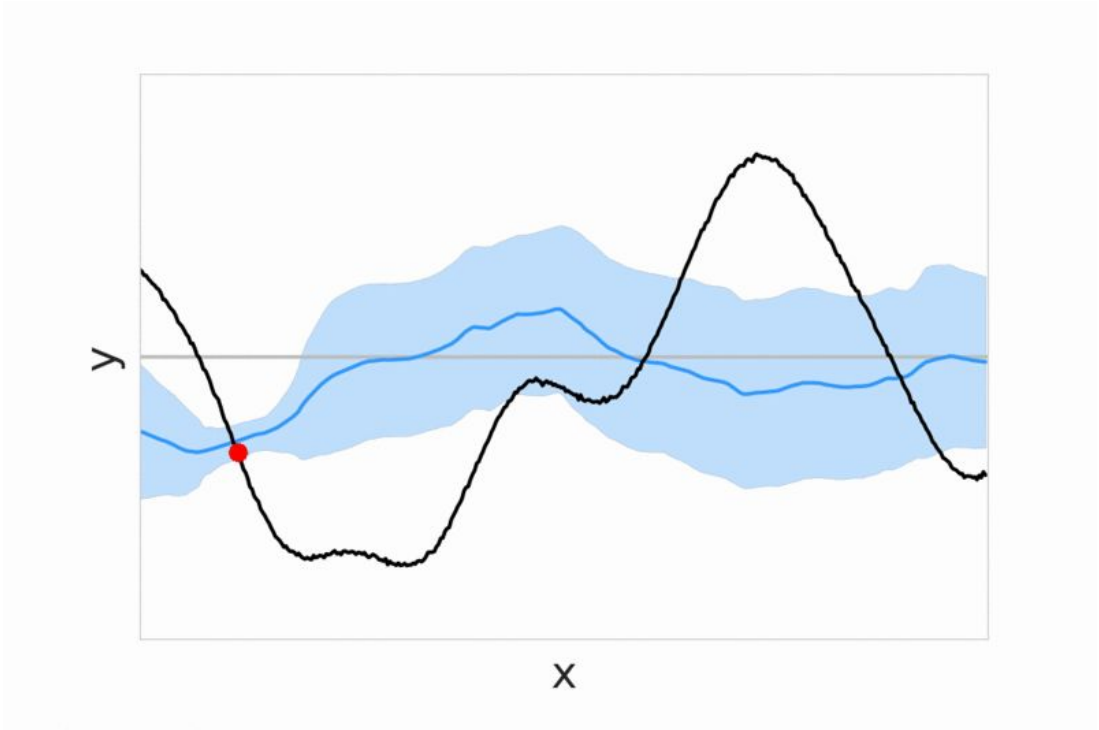
Model	20% of user data	50%	80%
SVD++ (SELF)	1.0517	1.0217	1.0124
Multitask MLP	0.9831	0.9679	0.9507
MAML	0.9593	0.9441	0.9295
NP (random)	$0.9359_{\pm 0.0017}$	$0.9215_{\pm 0.0055}$	$0.9151_{\pm 0.0085}$
NP (Info gain)	$0.9288_{\pm 0.0011}$	$0.8829_{\pm 0.0020}$	$0.8557_{\pm 0.0084}$

MovieLens 20m

Model	20% of user data	50%	80%
SVD++ (MIXED)	$0.9454_{\pm 0.0002}$	$0.9454_{\pm 0.0005}$	$0.9452_{\pm 0.0010}$
Multitask MLP	$0.8570_{\pm 0.0003}$	$0.8401_{\pm 0.0003}$	$0.8348_{\pm 0.0008}$
MAML	$0.8142_{\pm 0.0043}$	$0.7852_{\pm 0.0019}$	$0.7780_{\pm 0.0048}$
NP (random)	$0.7982_{\pm 0.0002}$	$0.7684_{\pm 0.0003}$	$0.7570_{\pm 0.0006}$
NP (Info gain)	$0.7926_{\pm 0.0003}$	$0.7362_{\pm 0.0005}$	$0.6859_{\pm 0.0006}$

[Schwarz et al, arxiv 2019]

What Meta-Learning can achieve - Learning Uncertainty



[Garnelo et al, ICML 2018]

DeepMind

Meta-Learning Q&A

UCL

DeepMind

10min Break

UCL

Outline

- Small data: Meta-Learning
    - Overview
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    - Black-box
- **Faster training and inference: Sparsity**
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    - Implicit Sparsity

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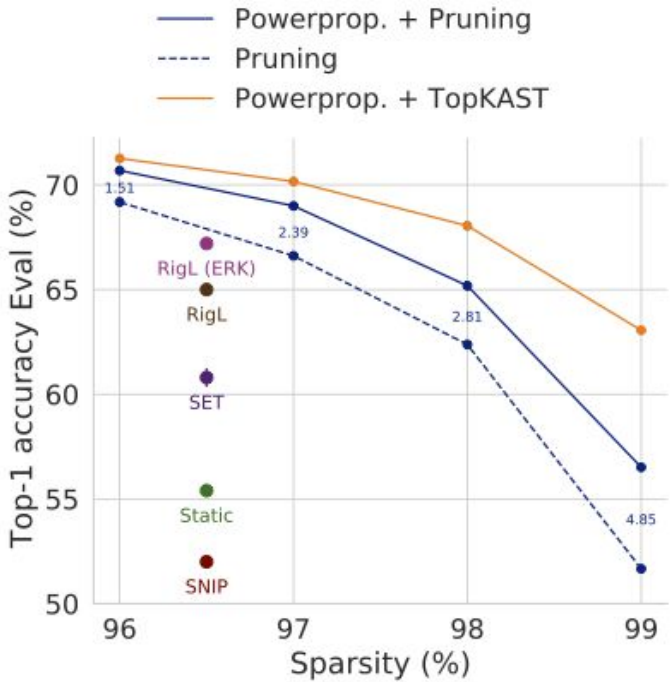
Overview

Neural networks are incredibly powerful learning tools. But they are massively overparameterised:
We can train incredibly sparse networks at little performance decrease:



Big potential to cut training and inference cost!

ResNet 50 - Dense: ~76.8% Top-1 accuracy

The Lottery Ticket Hypothesis

So why is this possible? The LTH:

> *"Dense, randomly-initialized, feed-forward networks contain subnetworks (winning tickets) that - when trained in isolation - reach test accuracy comparable to the original network in a similar number of iterations. The winning tickets we find have won the initialization lottery: their connections have initial weights that make training particularly effective."*

[Frankle & Carbin, ICLR 2019]

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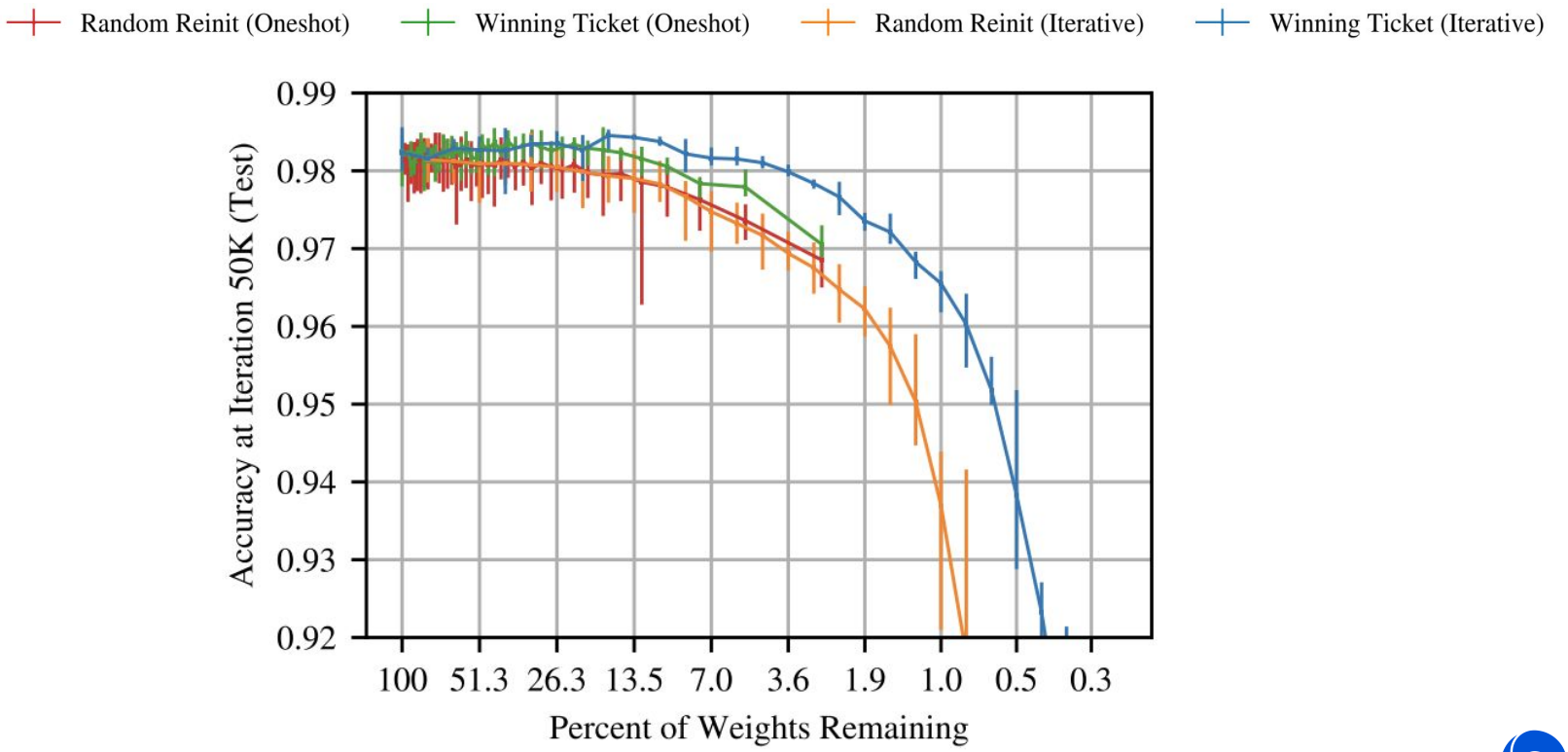
Finding winning tickets

1. Randomly initialize a neural network $f(x; \theta_0)$ (where $\theta_0 \sim \mathcal{D}_\theta$).
2. Train the network for j iterations, arriving at parameters θ_j.
3. Prune $p\%$ of the parameters in θ_j, creating a mask m.
4. Reset the remaining parameters to their values in θ_0, creating the winning ticket $f(x; m \odot \theta_0)$

[Frankle & Carbin, ICLR 2019]

The Lottery Ticket Hypothesis

Reinit: Same structure, different initialisation



[Frankle & Carbin, ICLR 2019]

LeNet MLP (300-100-10) on MNIST

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How do we train networks that are *explicitly sparse*?

- Explicit sparsity = Solution encoded by reduced capacity, resulting from an external (to the learning process) force used to impose frugality (e.g.. masking, regularisation, etc.)

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- *One-shot pruning:* Train a dense network to convergence, p% of weights are pruned, and the surviving weights remain unchanged.
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Which parameters to remove? Magnitude-based Pruning

$$f(x,\theta) : f(x,\theta_s) \approx f(\theta,x) + g^T(\theta_s - \theta) + \frac{1}{2}(\theta_s - \theta)^T H(\theta_s - \theta) + \ldots$$

[Magnitude based pruning: Frankle & Carbin, 1995, Iterative pruning: Ström 1997]

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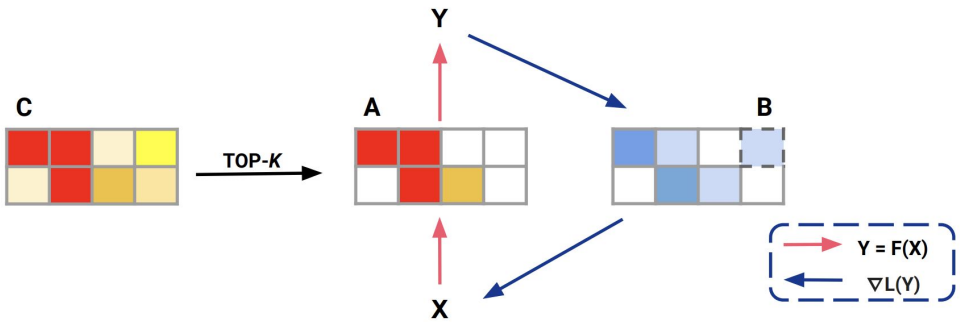
- Higher-order derivatives useful but computationally intractable to do so for very large modern networks. However, as error scales with $(\theta_s - \theta)$, minimise that norm instead

[Magnitude based pruning: Frankle & Carbin, 1995, Iterative pruning: Ström 1997]

TopKAST: Top-K Always Sparse Training

One-shot and iterative pruning sparsify a dense network. Can we stay sparse throughout training?

- Sparse forward mask: $A^t = \{i | \theta_i^t \in \texttt{TopK}(\theta^t, D)\}$ $\quad \alpha_i^t = \begin{cases} \theta_i^t & \texttt{if } i \in A^t \\ 0 & \texttt{otherwise} \end{cases}$



[Jayakumar et al., NeurIPS 2020]

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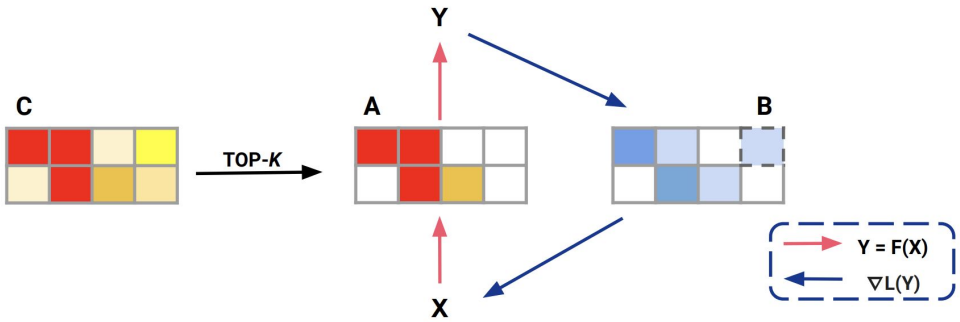
- Sparse backward mask: $B^t = \{i | \theta_i^t \in \texttt{TopK}(\theta^t, D + M)\}$ $\Delta_{\theta_i^t} = \begin{cases} -\eta \nabla_{\alpha^t} L(y, x, \alpha^t)_i & \texttt{if } i \in B \\ 0 & \texttt{otherwise} \end{cases}$



[Jayakumar et al., NeurIPS 2020]

TopKAST: Exploration

- *Exploratory stage:* At each iteration we select a different active set A, and its corresponding α, and perform one update step on θ using gradients obtained from the loss on f(α, x) and the regularizer.

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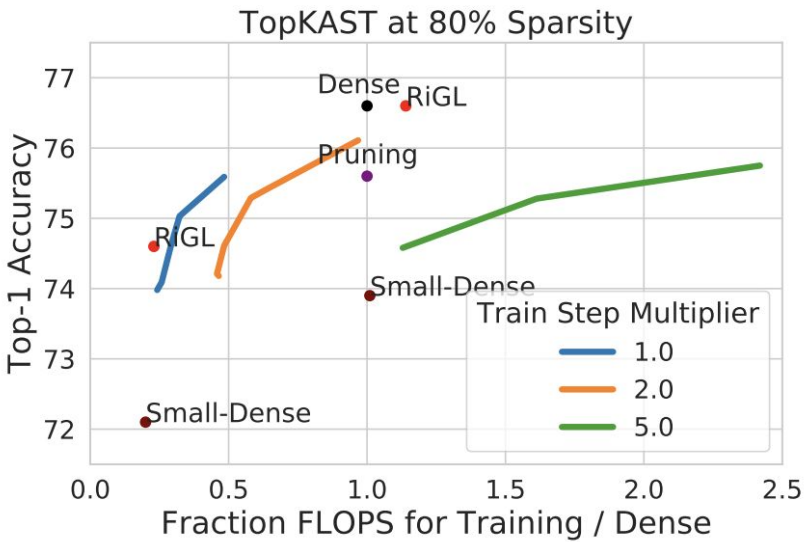
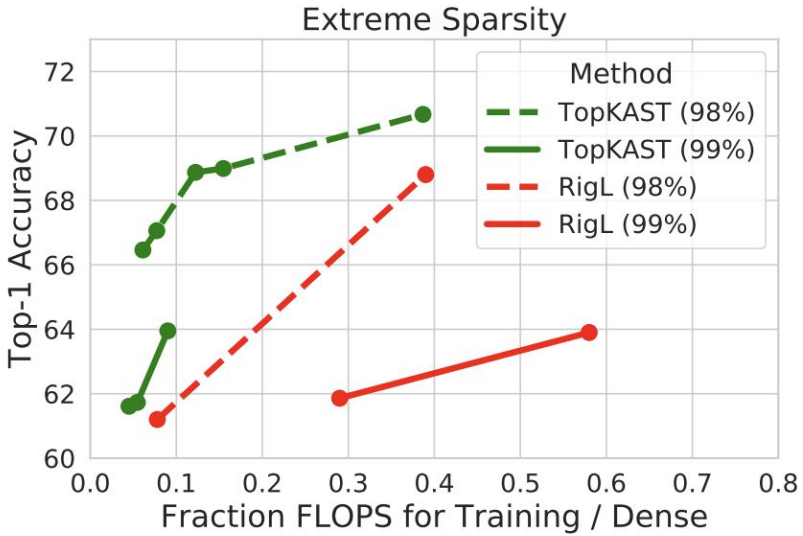
Could this lead to a rich-get-richer scheme? Exploration loss:

$$Loss_R(\alpha_i^t) = \begin{cases} |\theta_i^t| & \texttt{if } i \in A^t \\ \frac{|\theta_i^t|}{D} & \texttt{if } i \in B^t \setminus A^t \\ 0 & else \end{cases}$$

[Jayakumar et al., NeurIPS 2020]

TopKAST: FLOPS trade-off





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Training/Inference FLOPs savings

Method	Top-1 Accuracy	FLOPs (Train)	FLOPs (Test)	Top-1 Accuracy	FLOPs (Train)	FLOPs (Test)
Dense	76.8±0.09	1x (3.2e18)	1x (8.2e9)			
	S=0.95			S=0.965		
Static	59.5+-0.11	0.23x	0.08x	55.4+-0.06	0.13x	0.07x
Snip	57.8+-0.40	0.23x	0.08x	52.0+-0.20	0.13x	0.07x
SET	64.4+-0.77	0.23x	0.08x	60.8+-0.45	0.13x	0.07x
RigL	67.5+-0.10	0.23x	0.08x	65.0+-0.28	0.13x	0.07x
$\text{RigL}_{5\times}$	73.1+-0.12	1.14x	0.08x	71.1+-0.20	0.66x	0.07x
Static (ERK)	72.1±0.04	0.42x	0.42x	67.7±0.12	0.24x	0.24x
RigL (ERK)	69.7+-0.17	0.42x	0.12x	67.2+-0.06	0.25x	0.11x
$\text{RigL}_{5\times}$ (ERK)	74.5+-0.09	2.09x	0.12x	72.7+-0.02	1.23x	0.11x
SNFS (ERK)	70.0+-0.04	0.61x	0.12x	67.1+-0.72	0.50x	0.11x
Pruning* (Gale)	70.6	0.56x	0.08x	n/a	0.51x	0.07x
$\text{Pruning}_{1.5\times}$ (Gale)	72.7	0.84x	0.08x	69.26	0.76x	0.07x

[Evci et al., ICML 2020]

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- Motivation from (Vaškevičius et al, NeurIPS 2019: Implicit regularization for optimal sparse recovery). Sparsity for matrix factorisation: $w = v \odot v - u \odot u$

Powerpropagation

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Standard Update

Powerprop. scaling

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Properties:

- a=1 recovers the baseline, i.e. standard gradient descent
- Zero is a critical point of any weight for a>1.
- In addition, zero is surrounded by a plateau: Weights are less likely to change sign

Powerpropagation: Details

- How to ensure a good initialisation?
  - Set $\phi_i \leftarrow \text{sign}(\theta_i) \cdot \sqrt[\alpha]{|\theta_i|}$, $\theta_i \sim p(\theta)$
  - Preserves identical results at first forward pass

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  - Assume weights don't change: Each gradient is scaled by the same value: $\text{diag}(\alpha|\phi|^{\circ 2(\alpha-1)})$
  - This factors out in decaying average summation: Optimiser will undo the scaling

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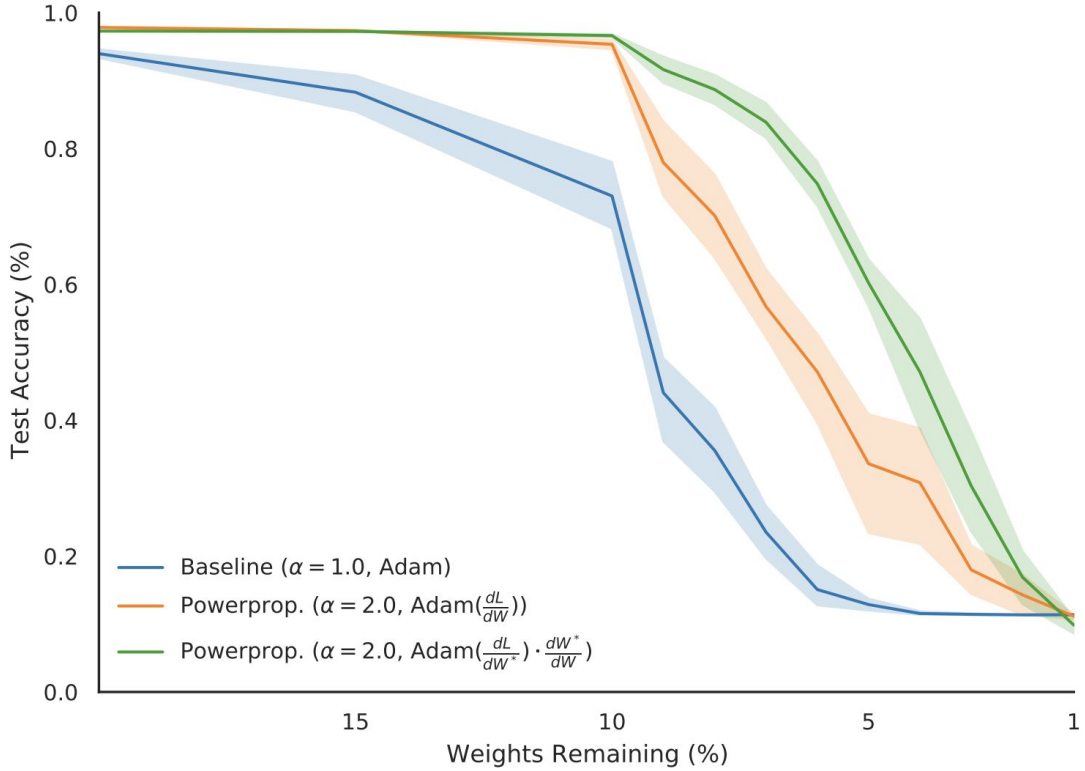
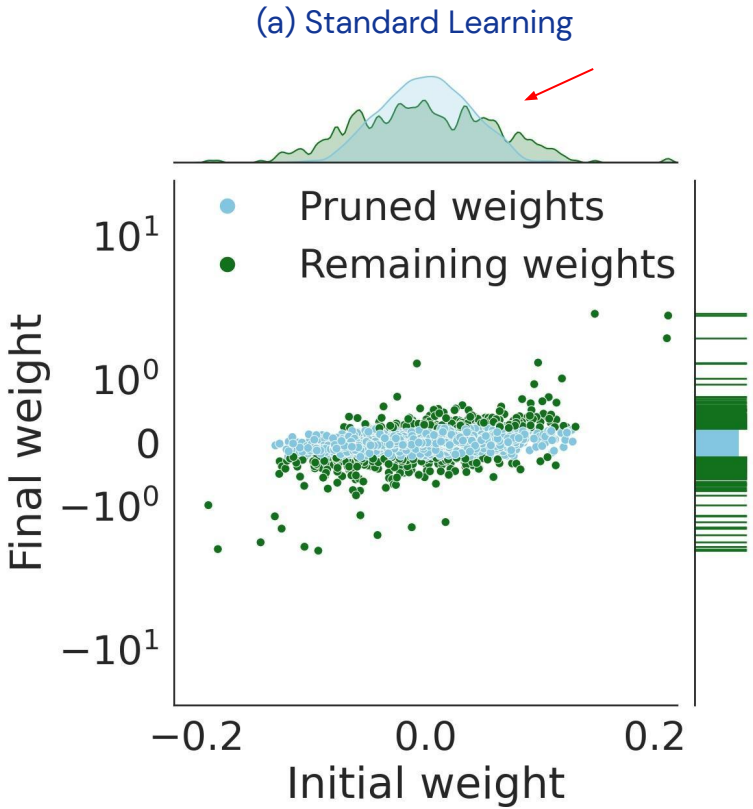
  - This will be approx. proportional to the powerpropagation scaling term, reducing the desired "rich get richer" effect.
  - **Solution:** Pretend that the exponentiated parameters are the de-facto parameters of the model. Compute update wrt. to this virtual target, multiply by partial derivative. This is a correct update (Follows typical proof steps taken in the target propagation literature) $\Delta\phi = optim\left(\frac{\partial \mathcal{L}}{\partial \Psi(\phi)}\right) \text{diag}(\alpha|\phi|^{\circ\alpha-1})$

Powerpropagation with modern optimisers



Effect on weight distribution



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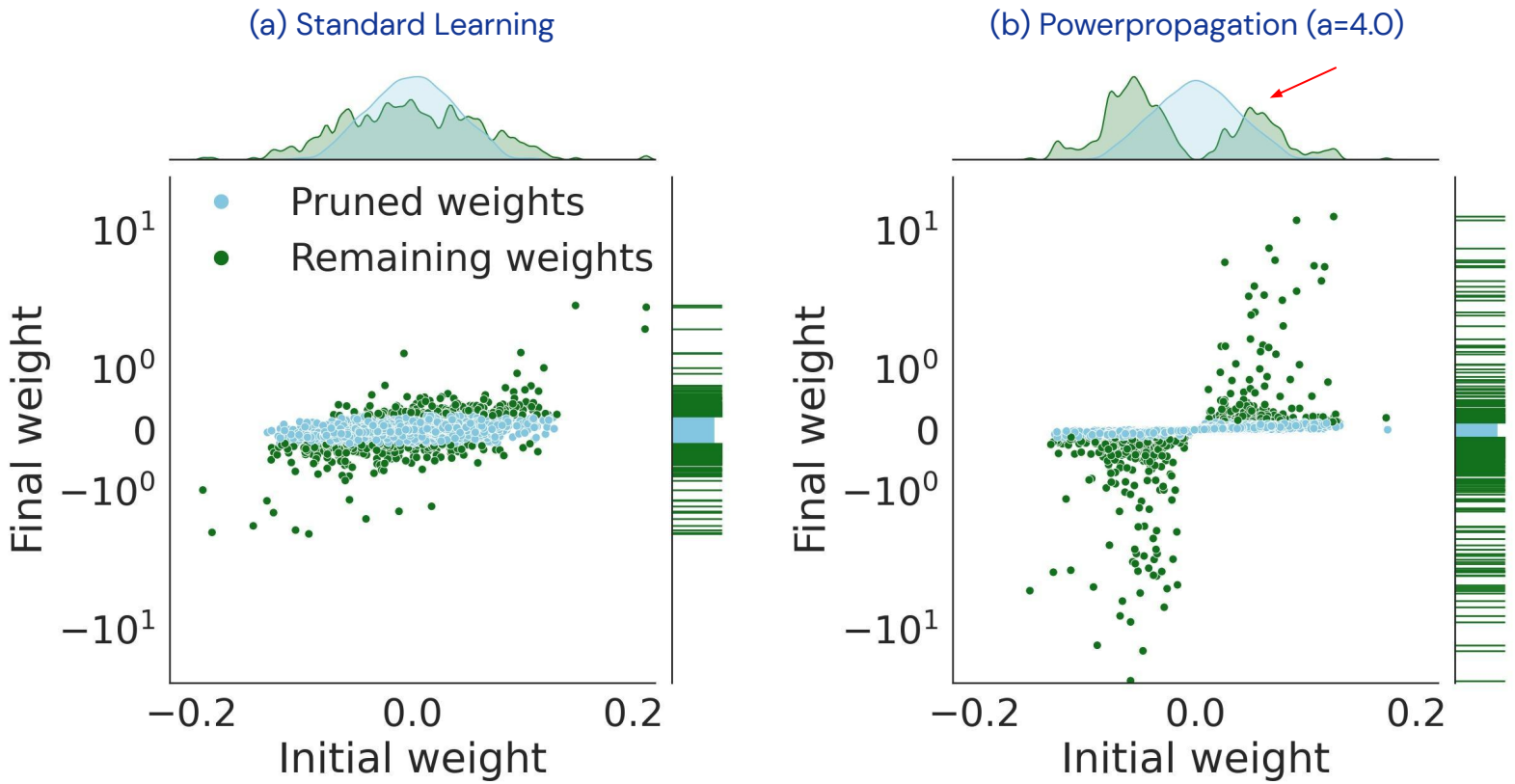

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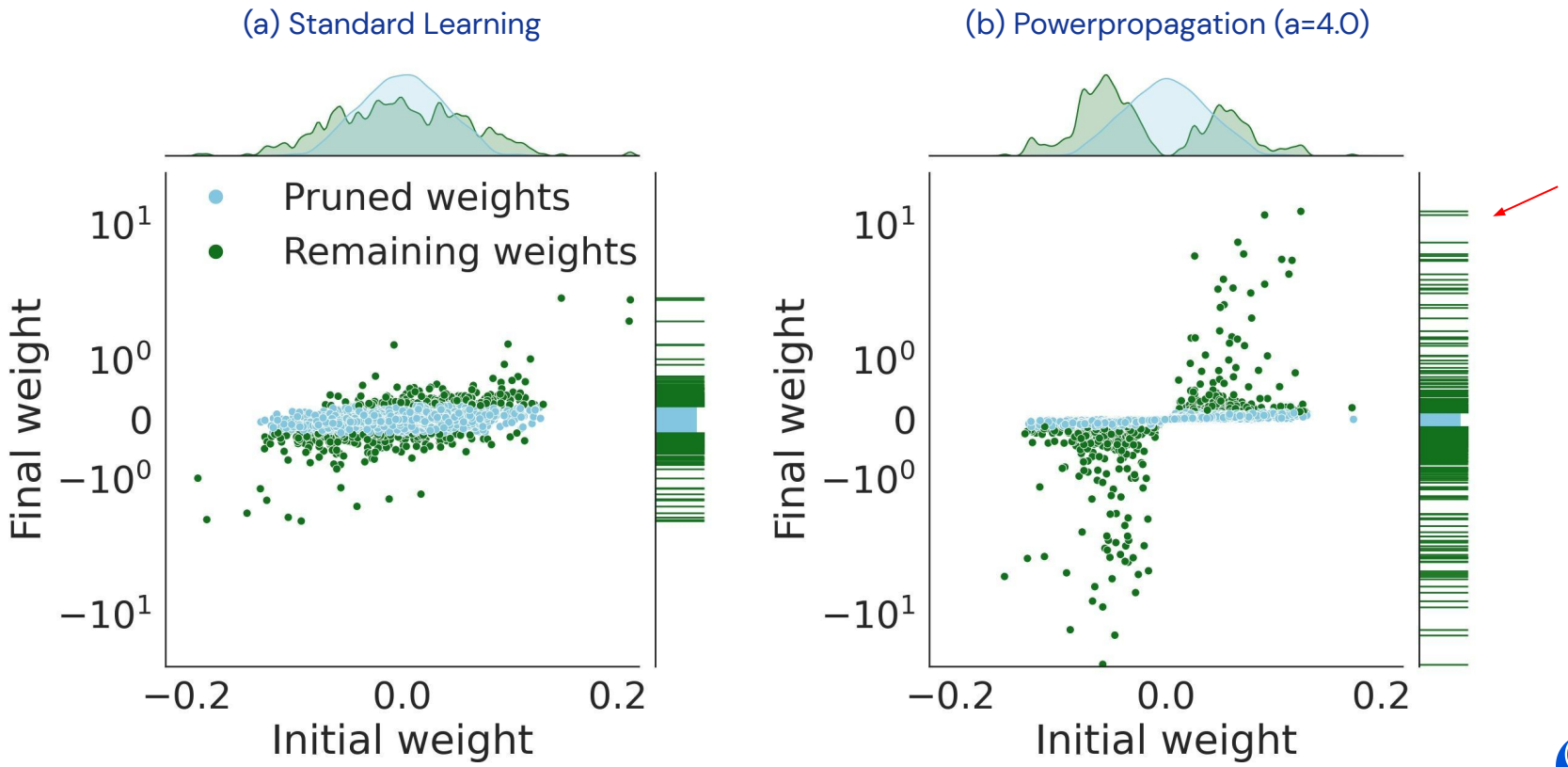

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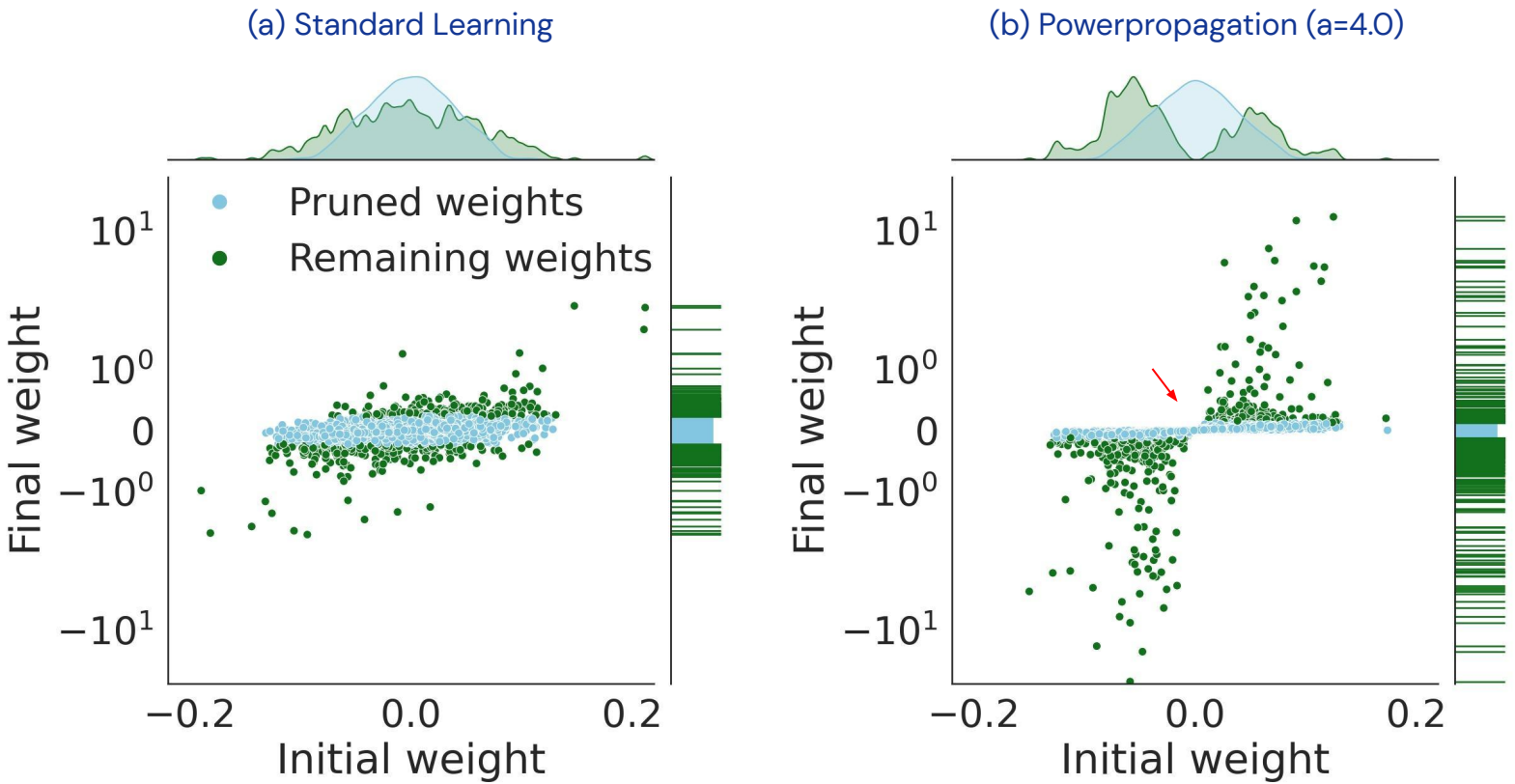
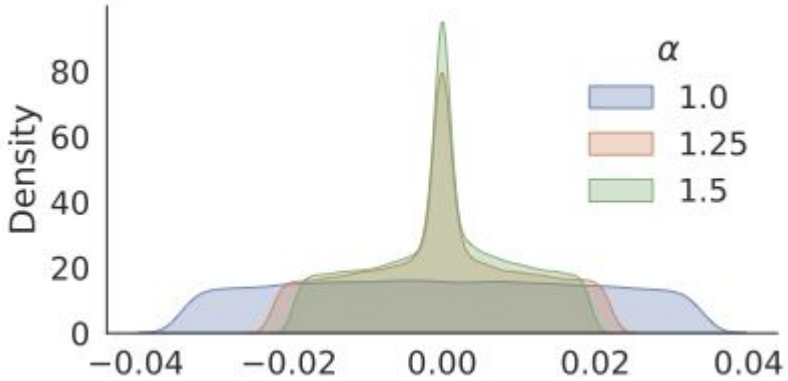


Effect on weight distribution

(a) 100k smallest weights



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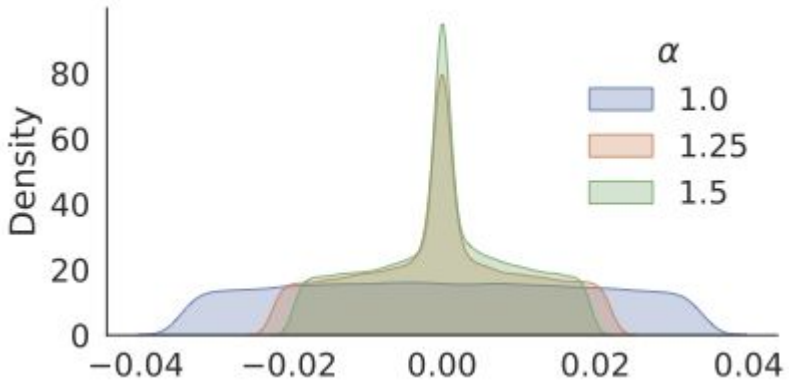
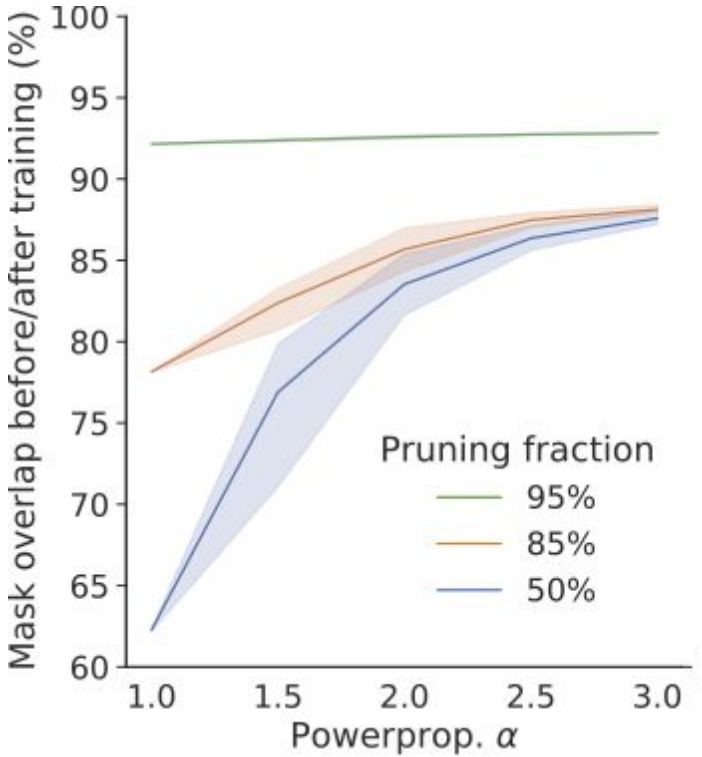


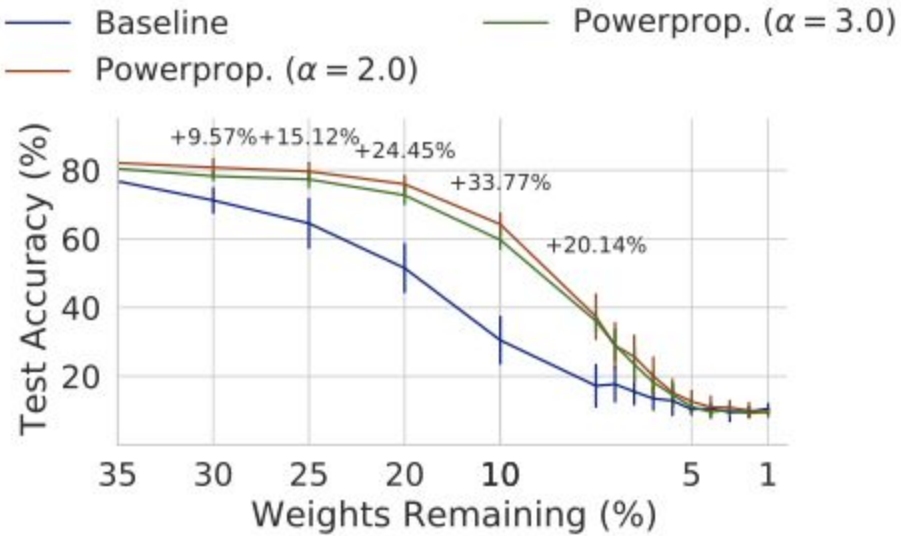
(b) Mask overlap



Inherent sparsity: One-shot pruning

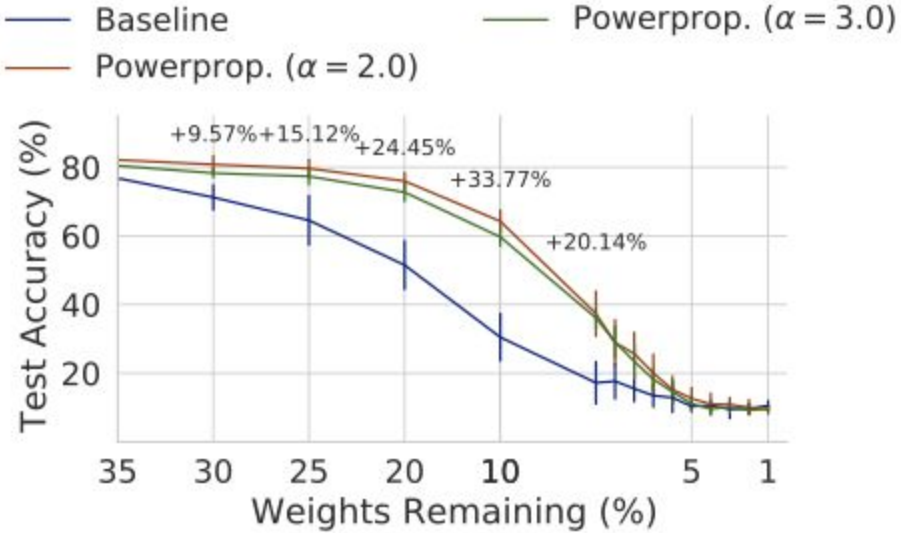
Inherent sparsity: One-shot pruning (AlexNet)

(a) CIFAR-10

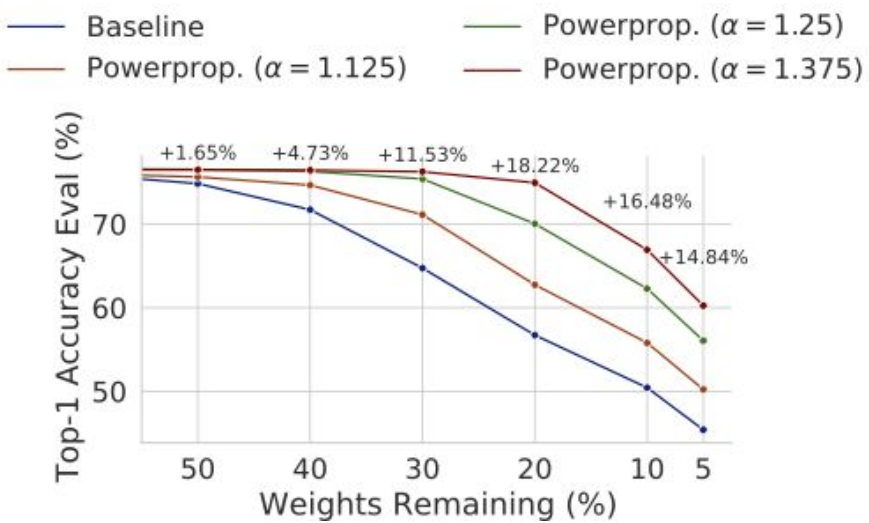


Inherent sparsity: One-shot pruning (ResNet50)

(a) CIFAR-10

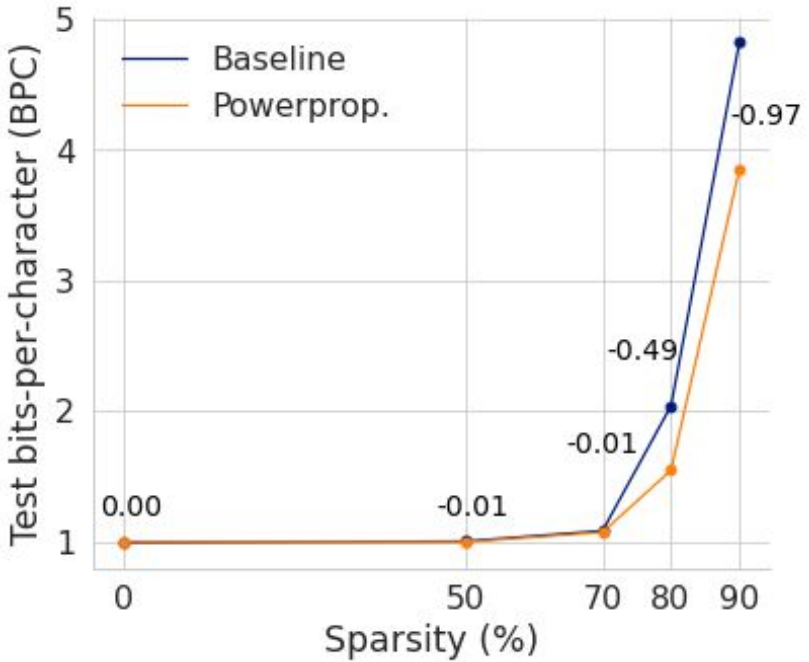


(b) ImageNet



Inherent sparsity: One-shot pruning (TransformerXL)

(a) enwik8



ImageNet w/ ResNet50 (Following RigL [Utku et al., ICML 2020])

Method	0%		
Dense	76.8 ± 0.09		
	80%	**90%**	**95%**
Static	70.6 ± 0.06	65.8 ± 0.04	59.5 ± 0.11
DSR	73.3	71.6	
SNFS	74.2	72.3	
SNIP	72.0 ± 0.10	67.2 ± 0.12	57.8 ± 0.40
RigL	74.6 ± 0.06	72.0 ± 0.05	67.5 ± 0.10
Iterative pruning	75.6	73.9	70.6

ImageNet w/ ResNet50 (Dense -> Sparse)

Method	0%		
Dense	76.8 ± 0.09		
	80%	**90%**	**95%**
Static	70.6 ± 0.06	65.8 ± 0.04	59.5 ± 0.11
DSR	73.3	71.6	
SNFS	74.2	72.3	
SNIP	72.0 ± 0.10	67.2 ± 0.12	57.8 ± 0.40
RigL	74.6 ± 0.06	72.0 ± 0.05	67.5 ± 0.10
Iterative pruning	75.6	73.9	70.6
Iterative pruning (ours)	75.3 ± 0.07	73.7 ± 0.14	70.6 ± 0.05
Powerprop. + Iter. Pruning	**75.7** ± 0.05	**74.4** ± 0.02	**72.1** ± 0.00

ImageNet w/ ResNet50 (Sparse -> Sparse)

Method	0%		
Dense	76.8 ± 0.09		
	80%	**90%**	**95%**
Static	70.6 ± 0.06	65.8 ± 0.04	59.5 ± 0.11
DSR	73.3	71.6	
SNFS	74.2	72.3	
SNIP	72.0 ± 0.10	67.2 ± 0.12	57.8 ± 0.40
RigL	74.6 ± 0.06	72.0 ± 0.05	67.5 ± 0.10
Iterative pruning	75.6	73.9	70.6
Iterative pruning (ours)	75.3 ± 0.07	73.7 ± 0.14	70.6 ± 0.05
Powerprop. + Iter. Pruning	**75.7** ± 0.05	**74.4** ± 0.02	**72.1** ± 0.00
TopKAST† [14]	75.47 ± 0.03	74.65 ± 0.03	**72.73** ± 0.10
Powerprop. + TopKAST†	**75.75** ± 0.05	**74.74** ± 0.04	**72.89** ± 0.10

@50% Backward sparsity

ImageNet w/ ResNet50 (Sparse -> Sparse)

Method	0%		
Dense	76.8 ± 0.09		
	80%	**90%**	**95%**
Static	70.6 ± 0.06	65.8 ± 0.04	59.5 ± 0.11
DSR	73.3	71.6	
SNFS	74.2	72.3	
SNIP	72.0 ± 0.10	67.2 ± 0.12	57.8 ± 0.40
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Powerprop. + TopKAST†	**75.75** ± 0.05	**74.74** ± 0.04	**72.89** ± 0.10
TopKAST* [14]	76.08 ± 0.02	75.13 ± 0.03	73.19 ± 0.02
Powerprop. + TopKAST*	**76.24** ± 0.07	**75.23** ± 0.02	**73.25** ± 0.02

@0% Backward sparsity

ImageNet w/ ResNet50 (Erdos-Renyi Kernel)

Method	0%				
Dense	76.8 ± 0.09				
	80%	**90%**	**95%**	**80% (ERK)**	**90% (ERK)**
Static	70.6 ± 0.06	65.8 ± 0.04	59.5 ± 0.11	72.1 ± 0.04	67.7 ± 0.12
DSR	73.3	71.6			
SNFS	74.2	72.3		75.2 ± 0.11	72.9 ± 0.06
SNIP	72.0 ± 0.10	67.2 ± 0.12	57.8 ± 0.40		
RigL	74.6 ± 0.06	72.0 ± 0.05	67.5 ± 0.10	75.1 ± 0.05	73.0 ± 0.04
Iterative pruning	75.6	73.9	70.6		
Iterative pruning (ours)	75.3 ± 0.07	73.7 ± 0.14	70.6 ± 0.05		
Powerprop. + Iter. Pruning	**75.7** ± 0.05	**74.4** ± 0.02	**72.1** ± 0.00		
TopKAST† [14]	75.47 ± 0.03	74.65 ± 0.03	**72.73** ± 0.10	75.71 ± 0.06	74.79 ± 0.05
Powerprop. + TopKAST†	**75.75** ± 0.05	**74.74** ± 0.04	**72.89** ± 0.10	**75.84 ± 0.01**	**74.98 ± 0.09**
TopKAST* [14]	76.08 ± 0.02	75.13 ± 0.03	73.19 ± 0.02	76.42 ± 0.03	75.51 ± 0.05
Powerprop. + TopKAST*	**76.24** ± 0.07	**75.23** ± 0.02	**73.25** ± 0.02	**76.76 ± 0.08**	**75.74 ± 0.08**

ImageNet w/ ResNet50: Extended Training & Extreme Sparsity

Method	80% Sparsity	90% Sparsity
Iter. Pruning $_{(1.5\times)}$	76.5 [0.84x]	75.2 [0.76x]
RigL $_{(5\times)}$	$76.6 \pm_{0.06}$ [1.14x]	$75.7 \pm_{0.06}$ [0.52x]
ERK		
RigL $_{(5\times)}$	$77.1 \pm_{0.06}$ [2.09x]	$76.4 \pm_{0.05}$ [1.23x]
Powerprop. + TopKAST* $_{(2\times)}$	$77.51 \pm_{0.03}$ [1.21x]	$76.94 \pm_{0.10}$ [0.97x]
Powerprop. + TopKAST* $_{(3\times)}$	$\mathbf{77.64} \pm_{0.05}$ [1.81x]	$\mathbf{77.16} \pm_{0.19}$ [1.46x]

ImageNet w/ ResNet50: Extended Training & Extreme Sparsity

Method	80% Sparsity	90% Sparsity
Iter. Pruning $_{(1.5\times)}$	76.5 [0.84x]	75.2 [0.76x]
RigL $_{(5\times)}$	76.6$\pm_{0.06}$ [1.14x]	75.7$\pm_{0.06}$ [0.52x]
ERK		
RigL $_{(5\times)}$	77.1$\pm_{0.06}$ [2.09x]	76.4$\pm_{0.05}$ [1.23x]
Powerprop. + TopKAST* $_{(2\times)}$	77.51$\pm_{0.03}$ [1.21x]	76.94$\pm_{0.10}$ [0.97x]
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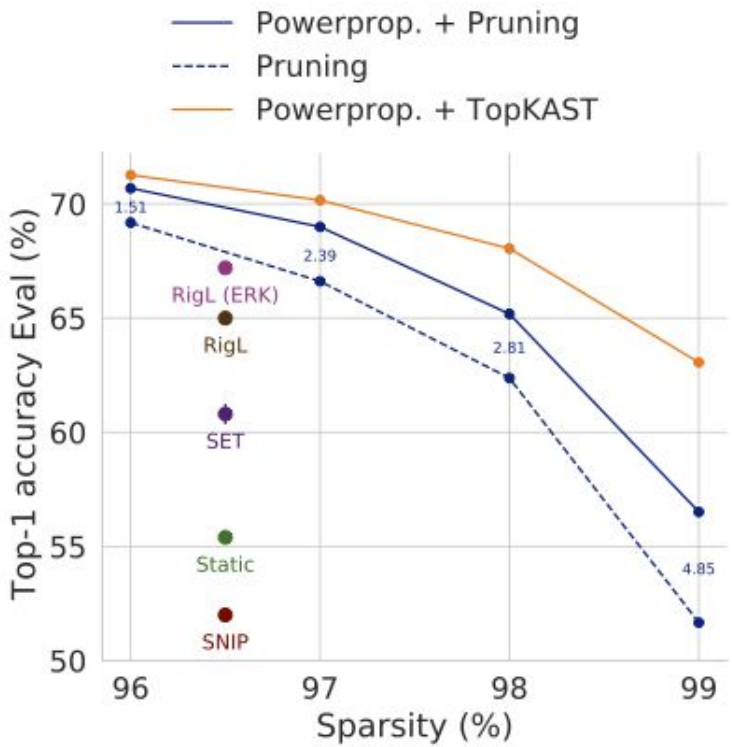
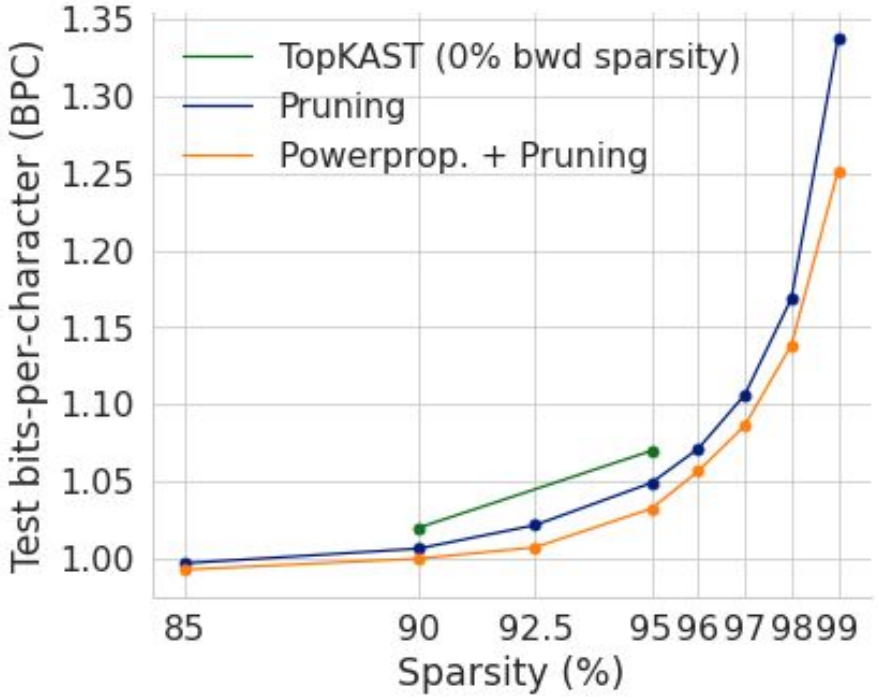


enwik8 w/ TransformerXL: Iterative Pruning



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Sparsity Q&A

UCL

Discussion & Looking forward

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- There are specific architectures optimised for e.g. mobile devices (e.g. MobileNet [Howard et al., CVPR 2017])

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- In practice, task design continues to be a mainly empirical (but very important problem)
- It can be difficult to understand when a test task is outside of the space of tasks seen at training time
- There are specific architectures optimised for e.g. mobile devices (e.g. MobileNet [Howard et al., CVPR 2017])
- Very active fields with dozens/hundreds of papers each year

DeepMind

Thank you!

Special thanks to:

Yee Whye Teh