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DeepLearning.AI

Probability and Statistics for Machine Learning and Data Science

Week 3: Sampling and Point Estimates

W3 Lesson 1



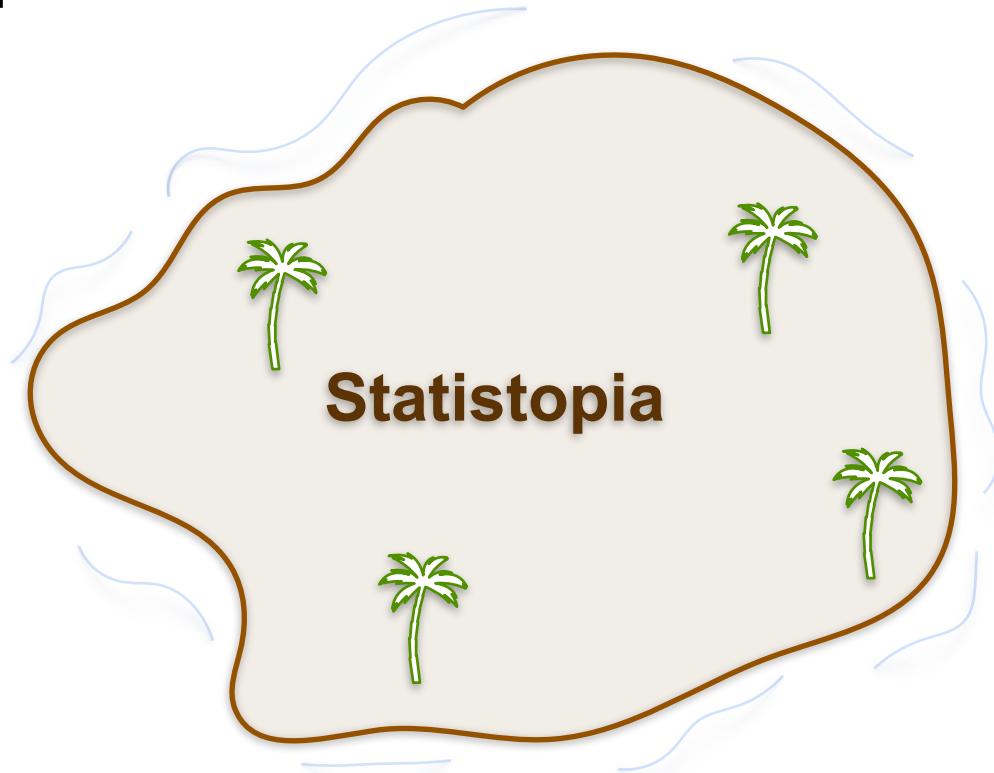
DeepLearning.AI

Sample and Population

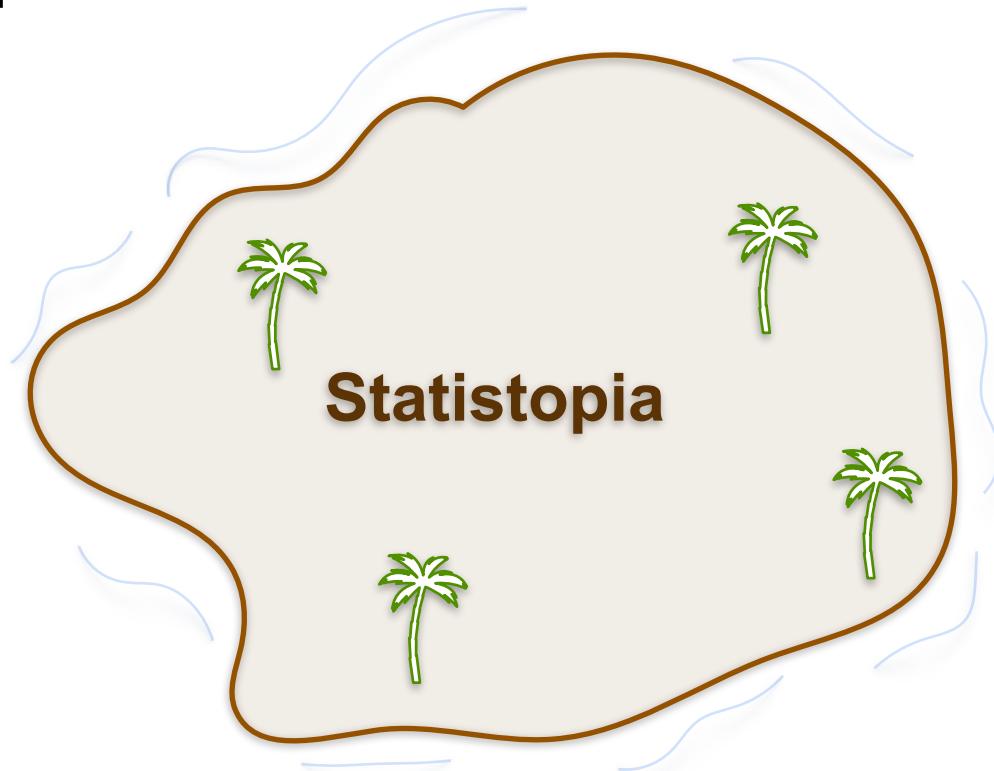
Population and Sample

Population and Sample

Population and Sample



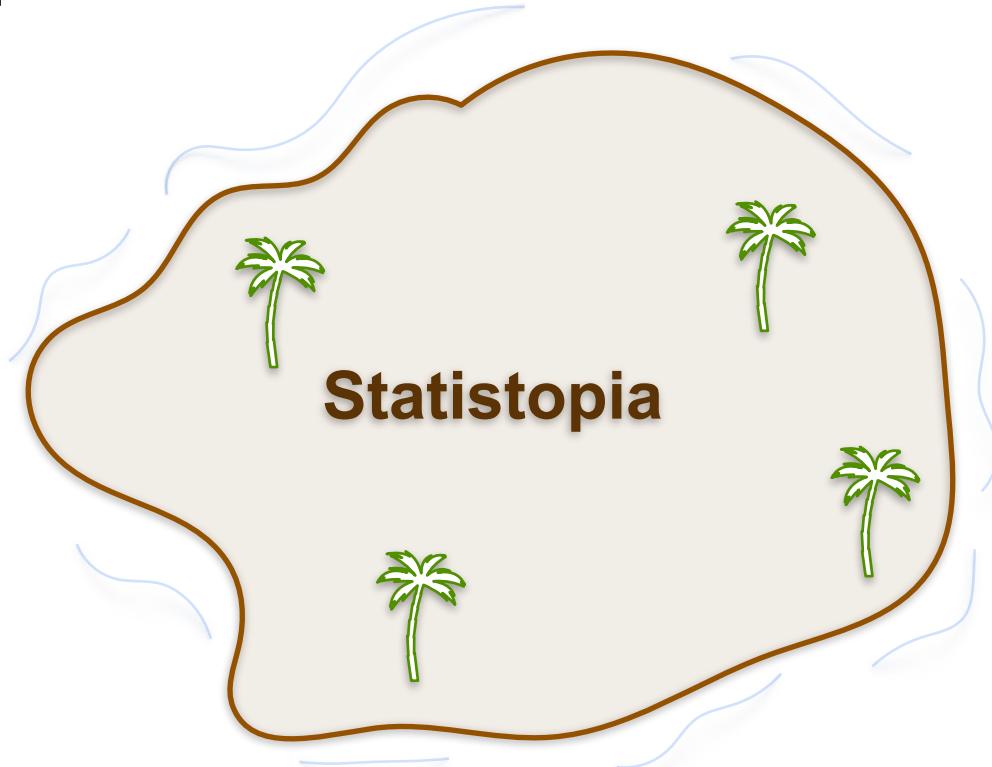
Population and Sample



Population and Sample



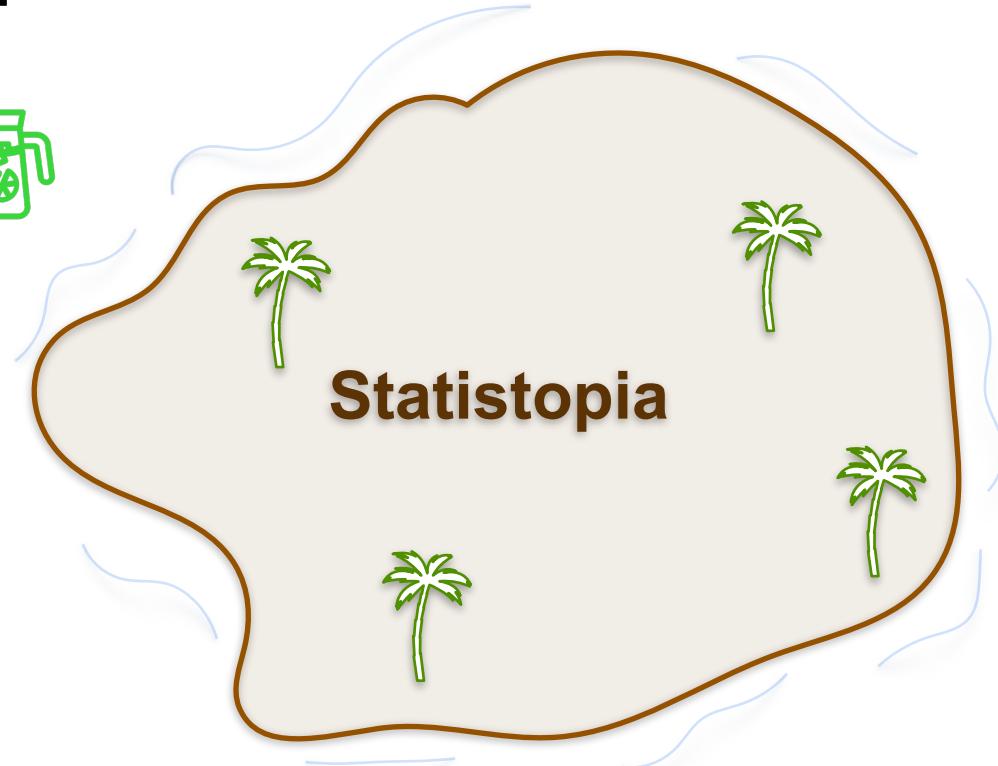
Find the **average height** of
the people living on
Statistopia



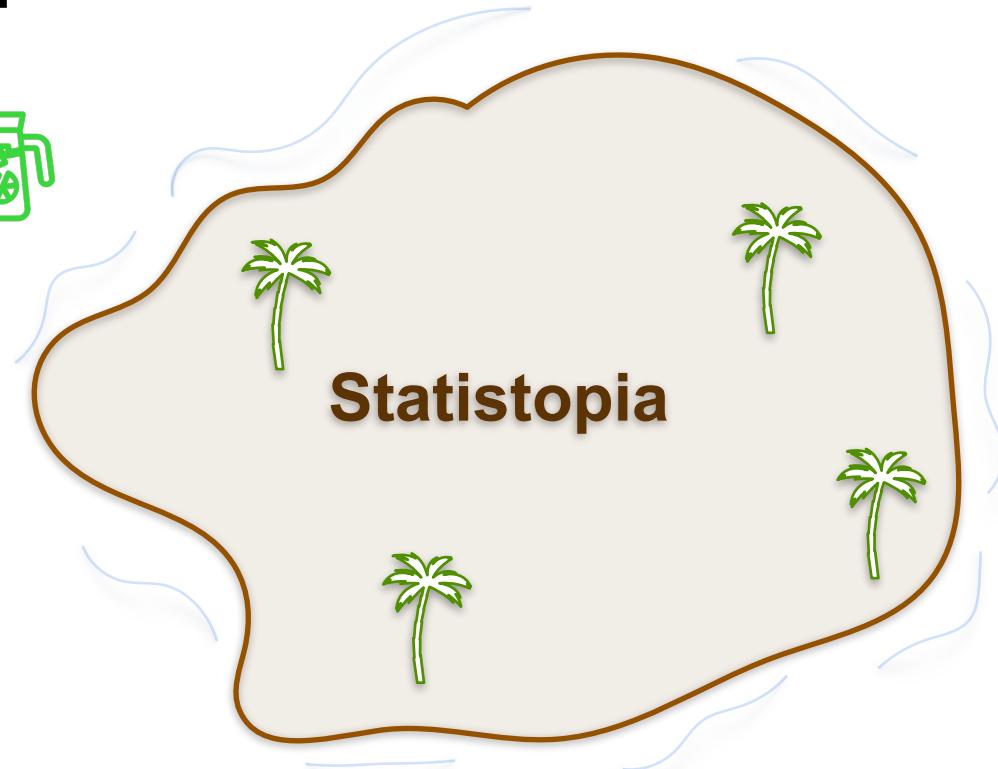
Population and Sample



Find the **average height** of
the people living on
Statistopia



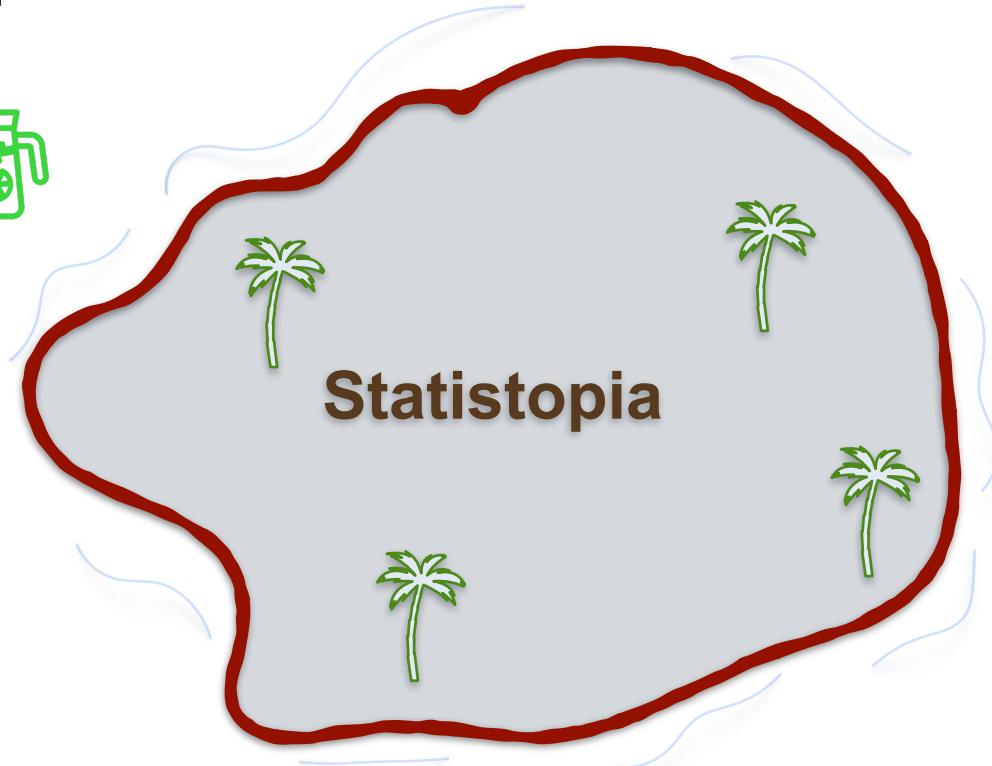
Population and Sample



Population and Sample



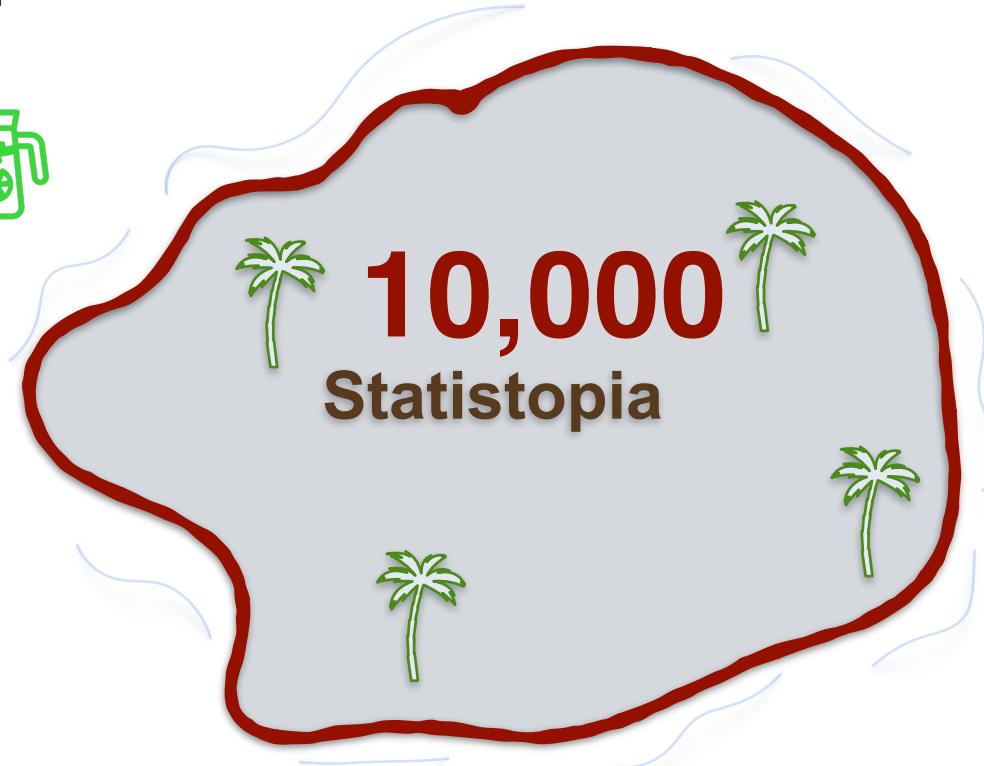
- Ask everyone on the island for their height.
- Divide by the total number



Population and Sample



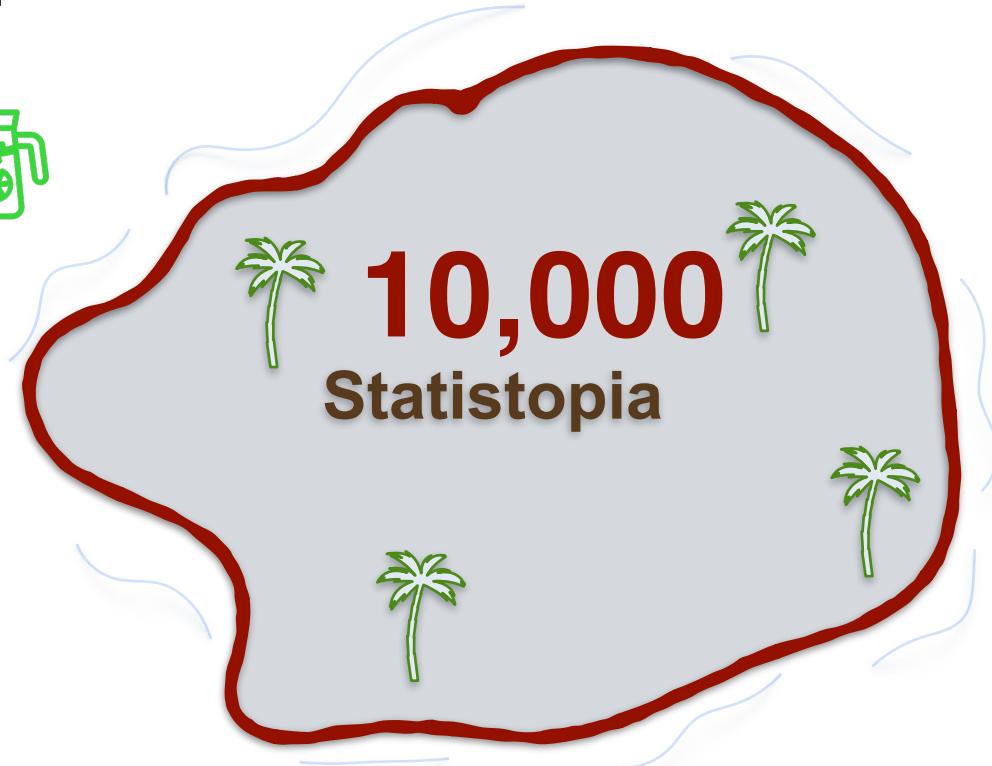
- Ask everyone on the island for their height.
- Divide by the total number



Population and Sample



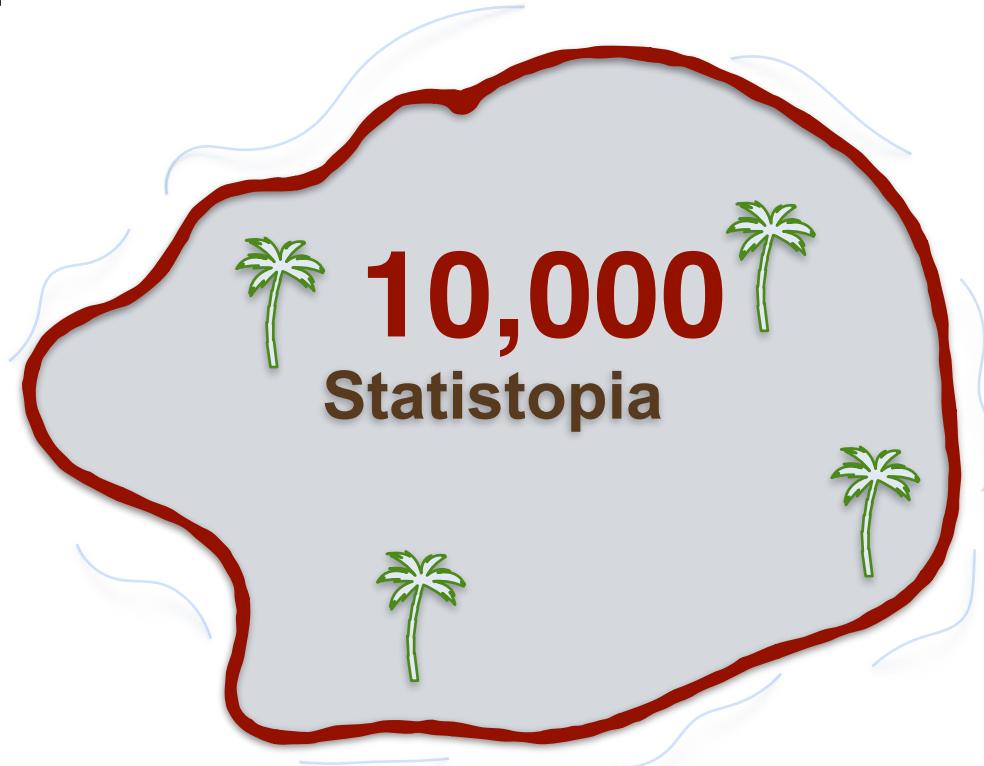
- Ask everyone on the island for their height.
- Divide by the total number



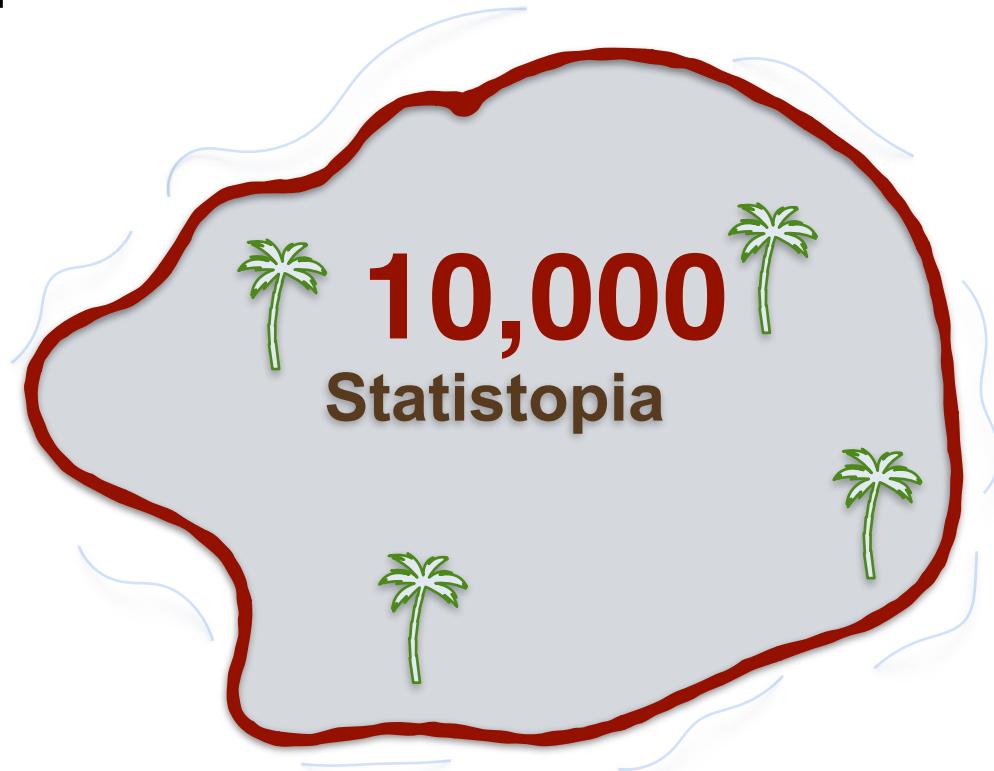
Population and Sample



- Ask everyone on the island for their height.
- Divide by the total number



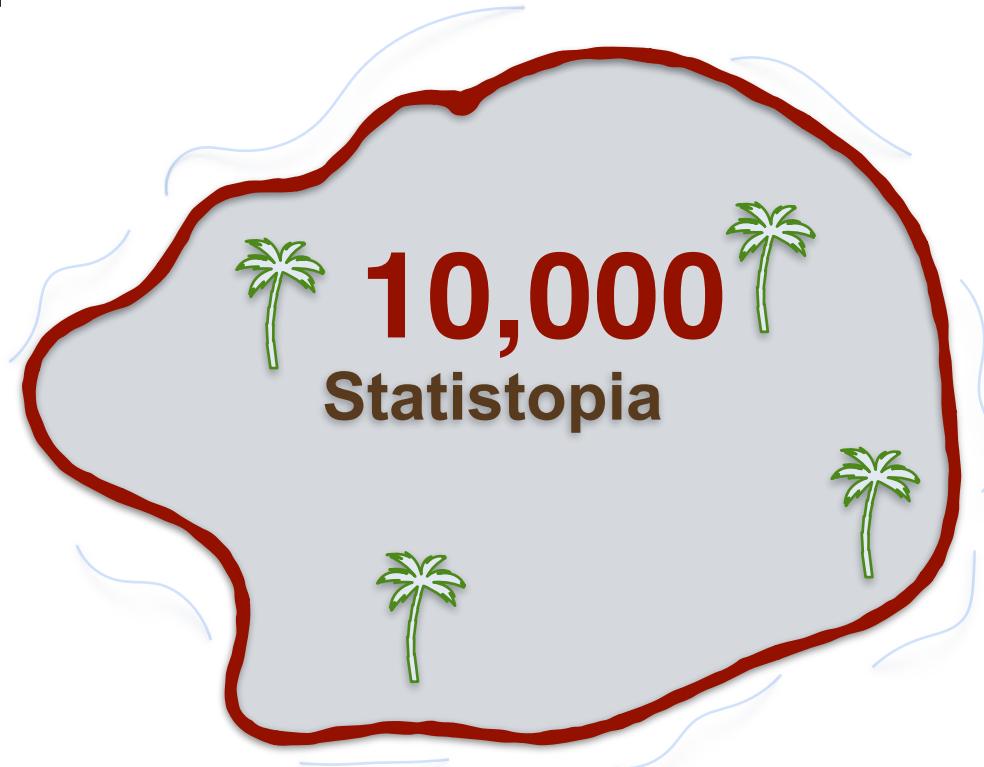
Population and Sample



Population and Sample



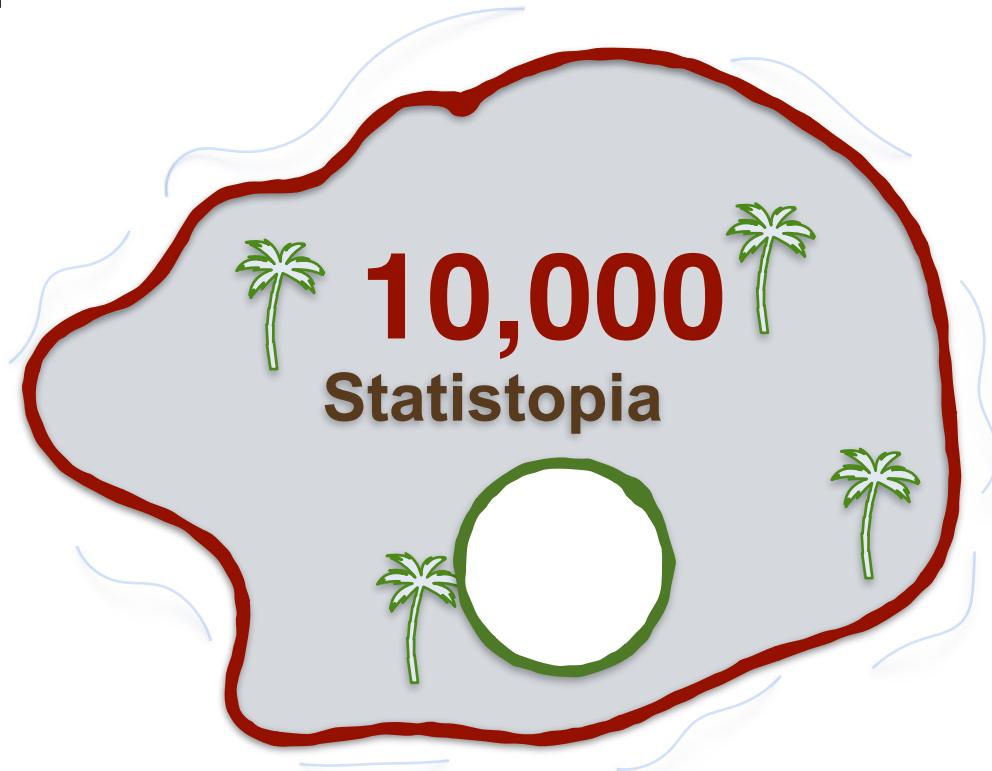
- Only ask a subset of the group to estimate the average height



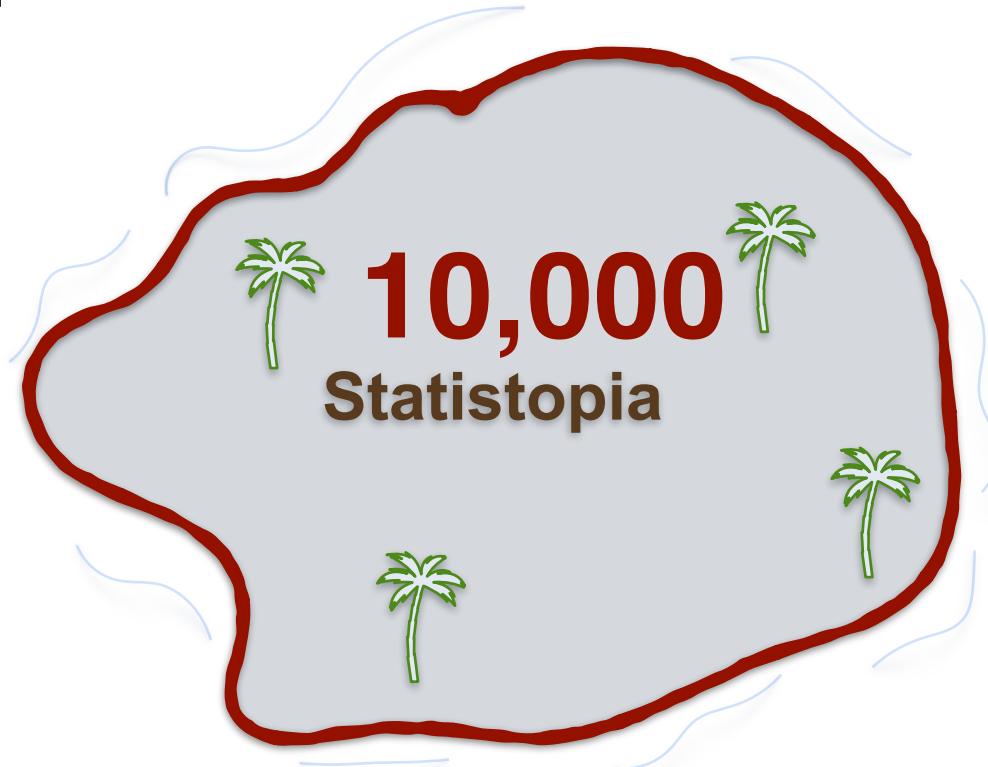
Population and Sample



- Only ask a subset of the group to estimate the average height



Population and Sample

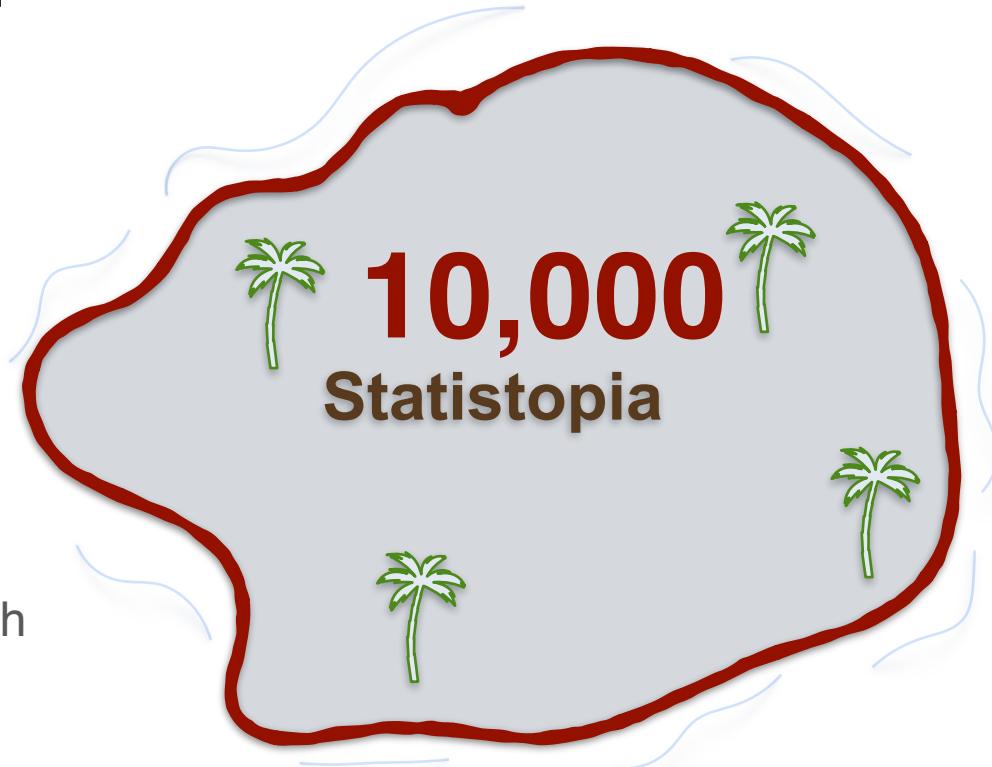


Population and Sample



Population:

the entire group of individuals or elements you want to study which share a common behaviour



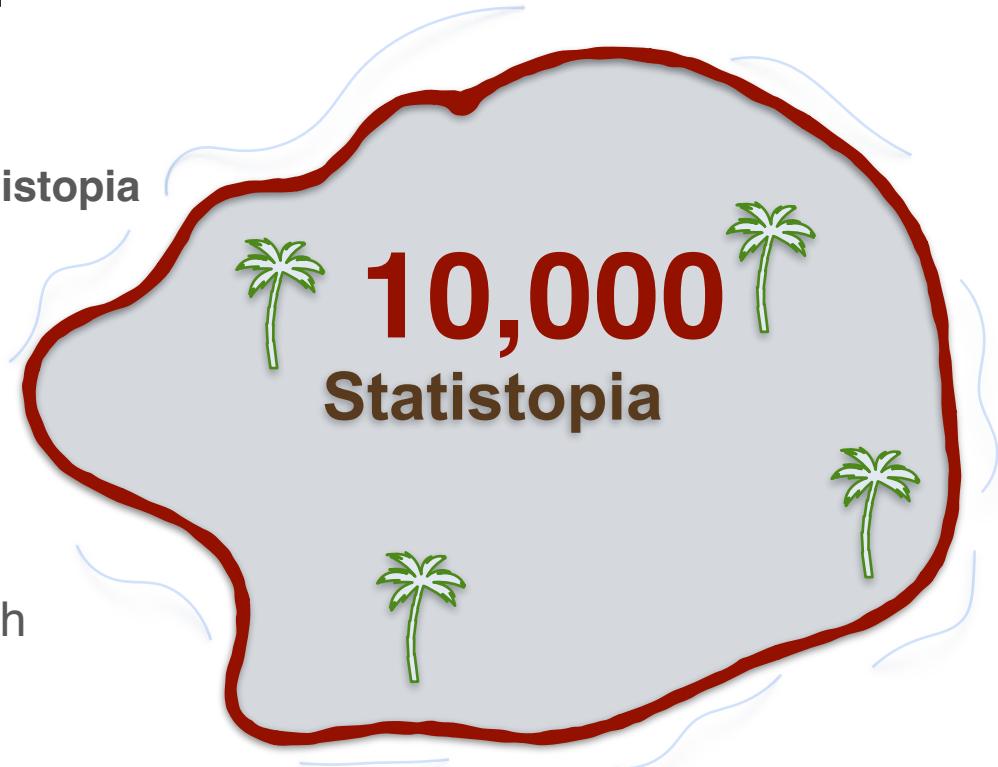
Population and Sample



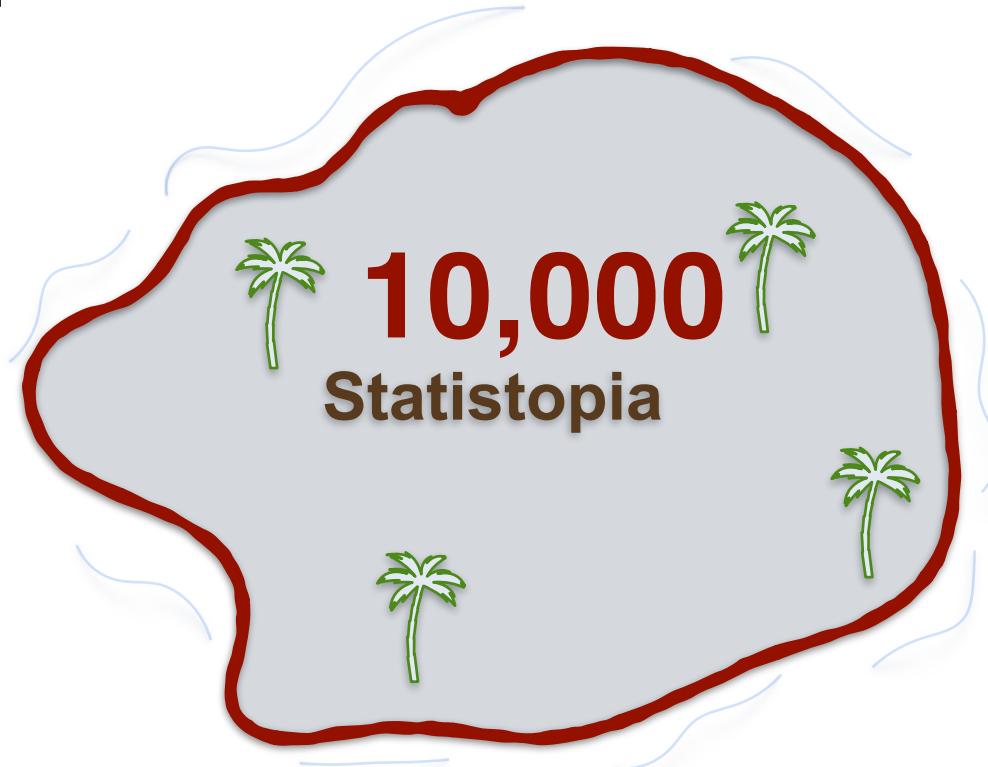
The people of statistopia

Population:

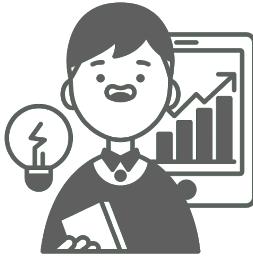
the entire group of individuals or elements you want to study which share a common behaviour



Population and Sample

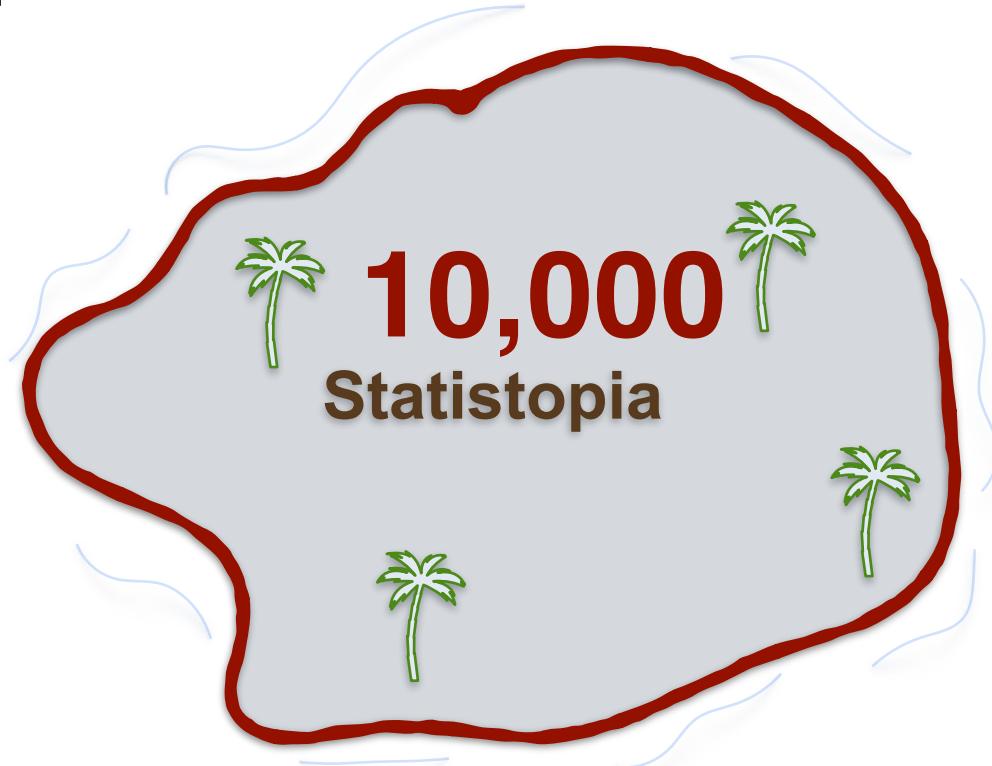


Population and Sample



Sample:

subset of the population you use
to draw conclusions about the
population as a whole



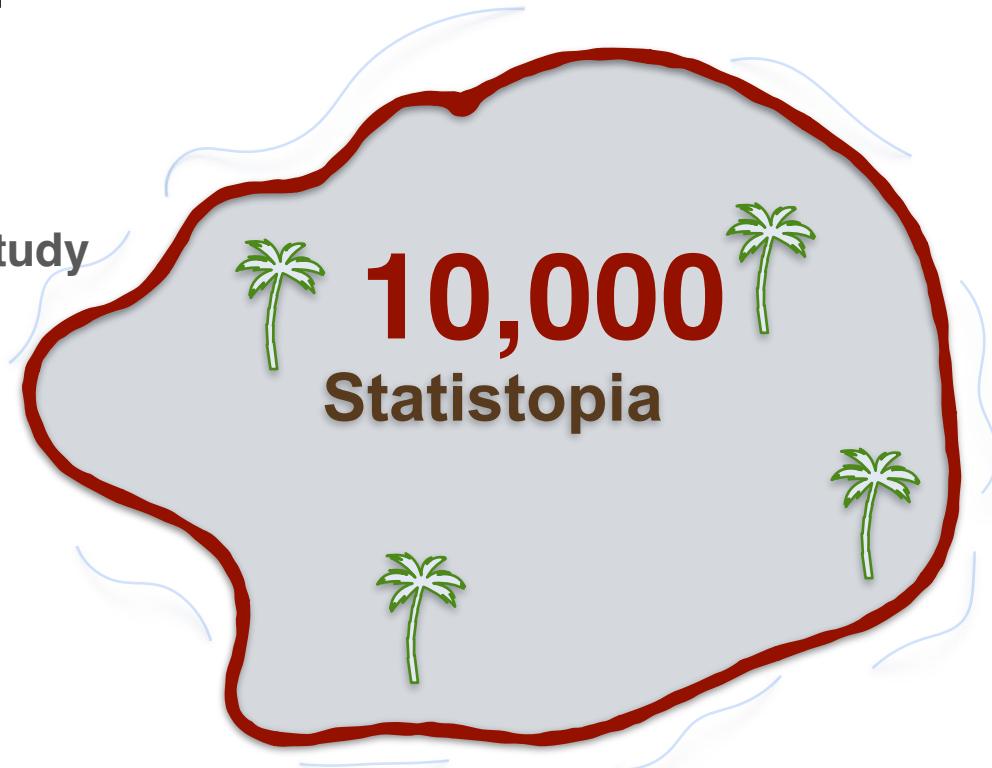
Population and Sample



The people you
select for your study

Sample:

subset of the population you use
to draw conclusions about the
population as a whole



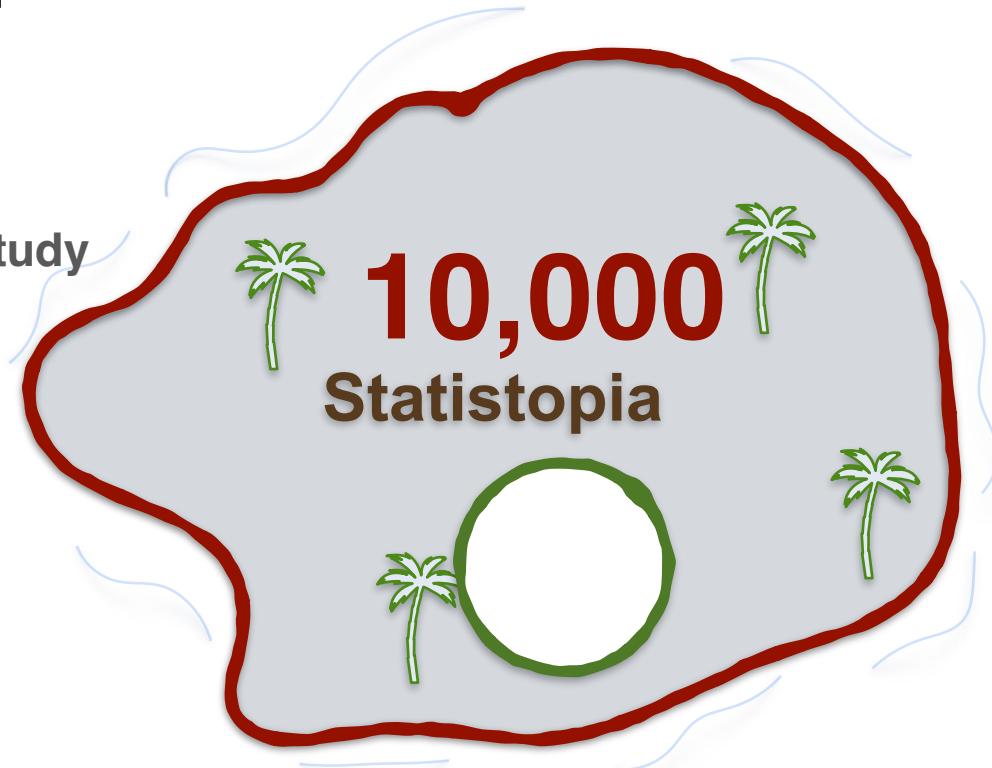
Population and Sample



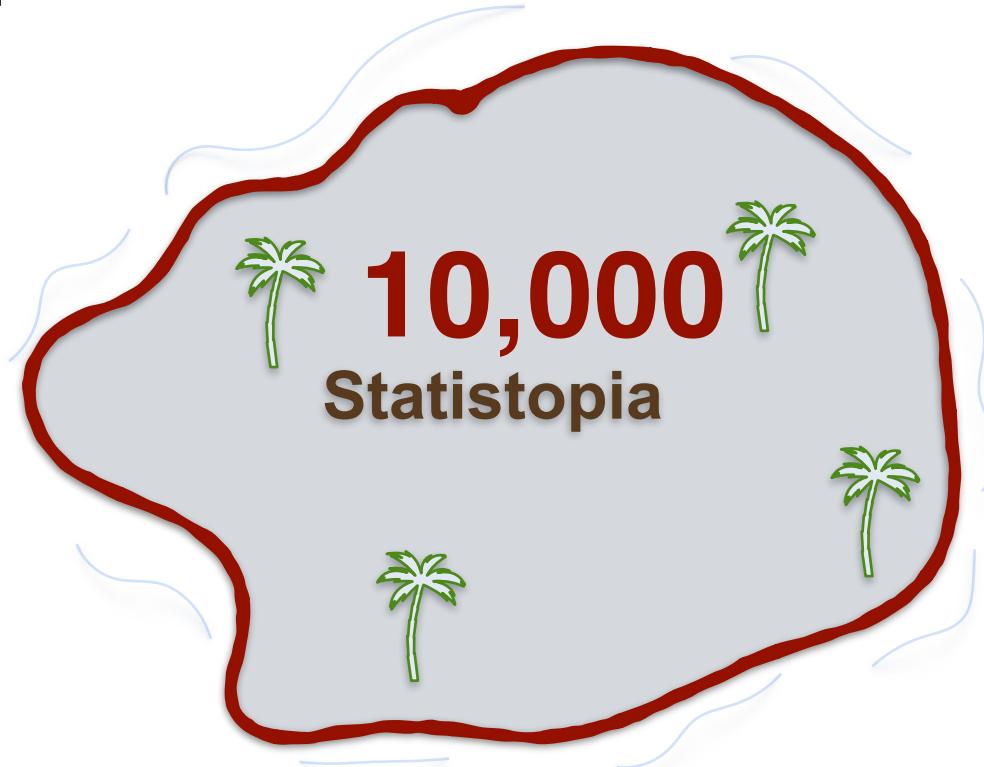
The people you
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Population and Sample

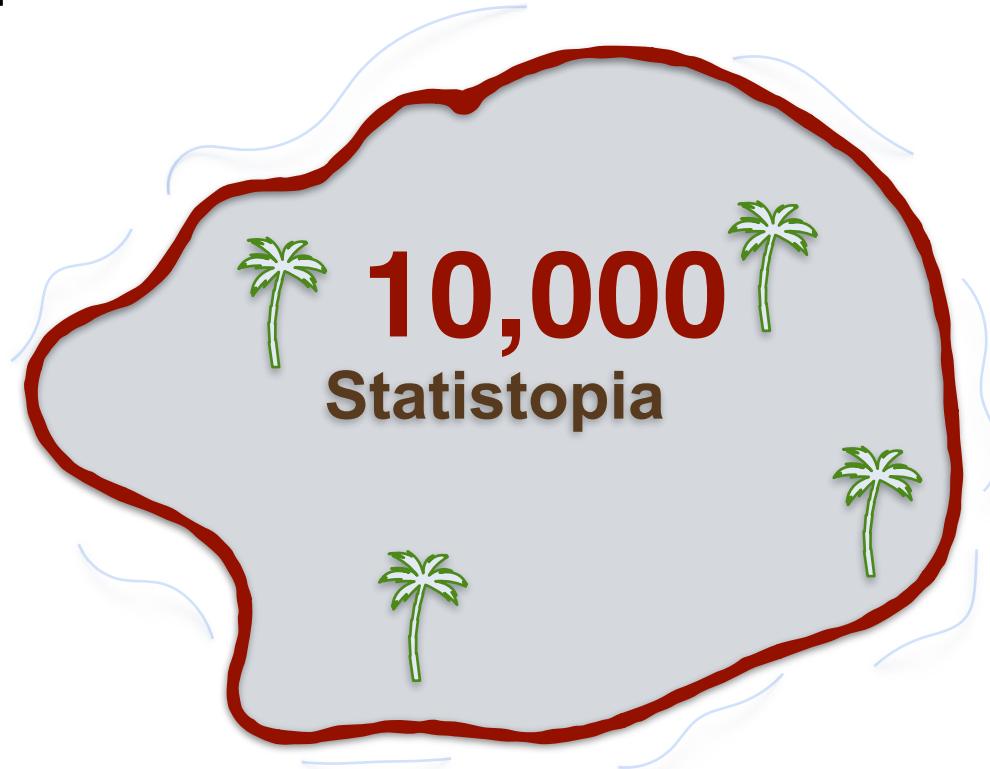


Population and Sample



Population Size (N)

10,000

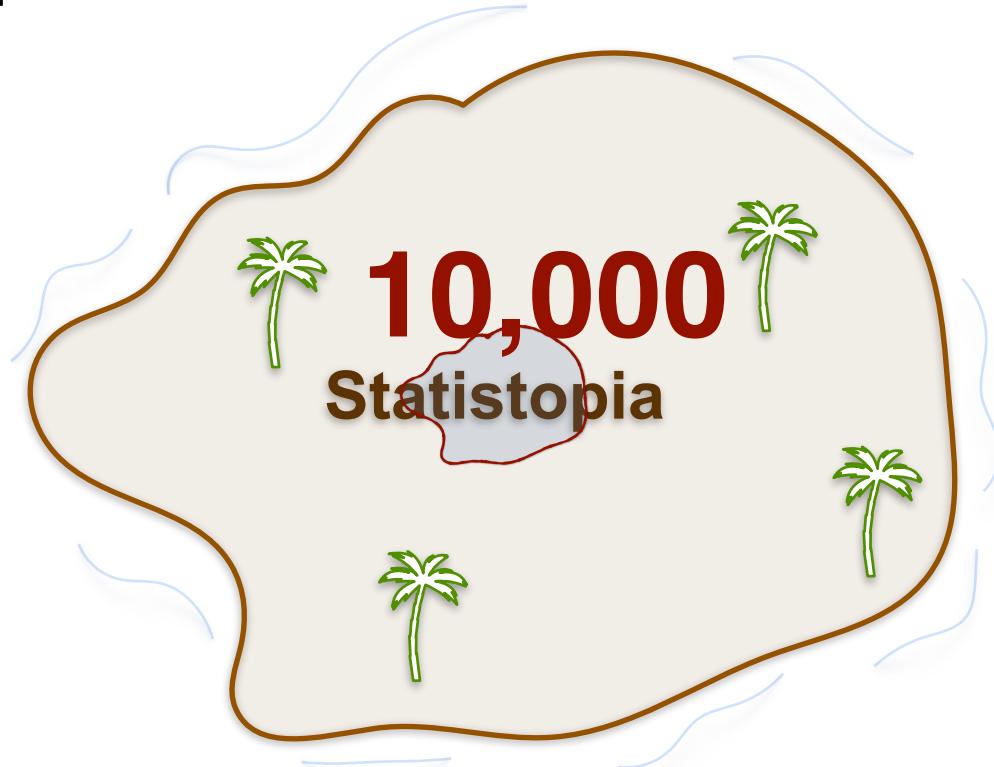


Population and Sample



Population Size (N)

10,000



Population and Sample



Population Size (N)

10,000

Sample Size (n)



Population and Sample

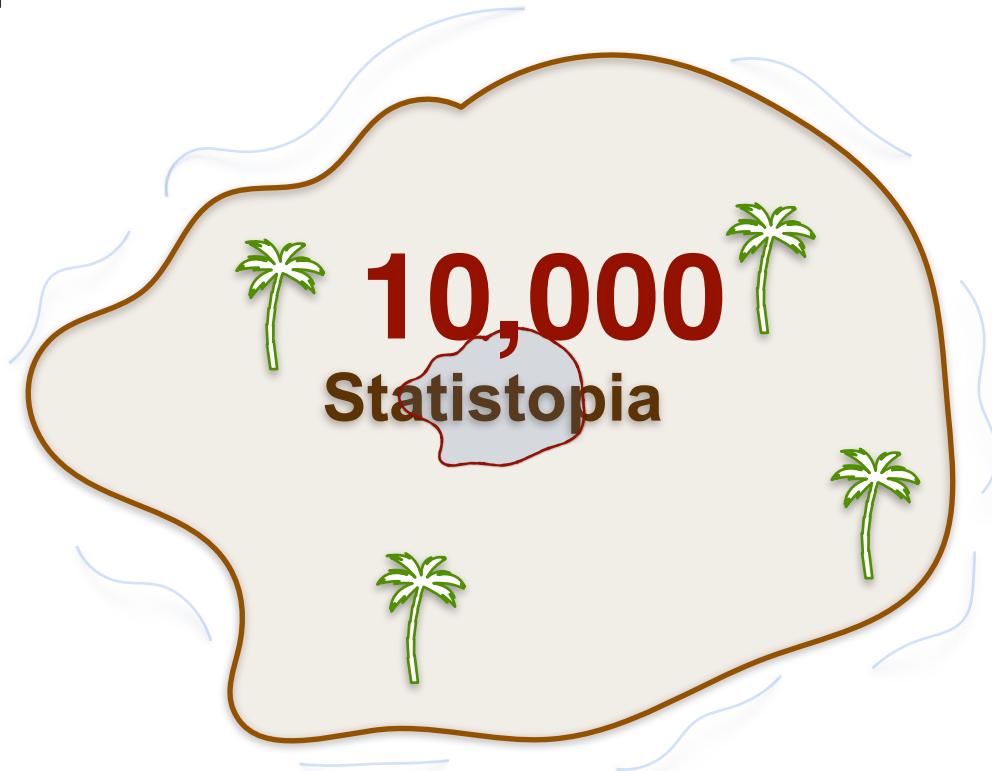


Population Size (N)

10,000

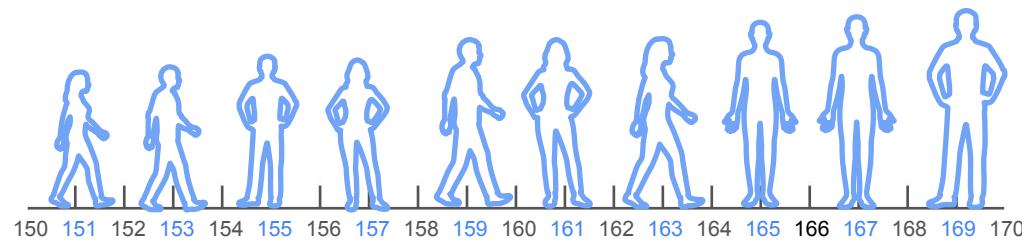
Sample Size (n)

1 - 9,999



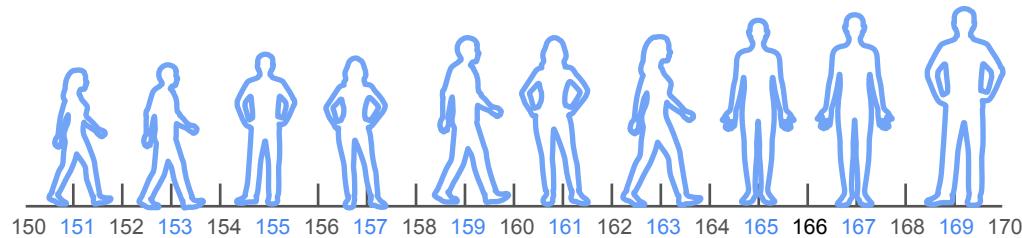
Population and Sample

Population and Sample

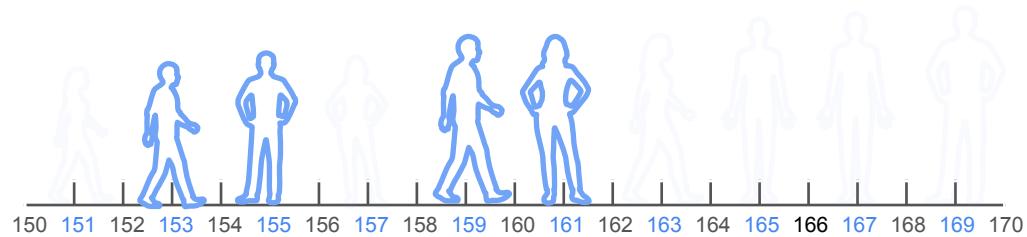


Population and Sample

$N = 10$

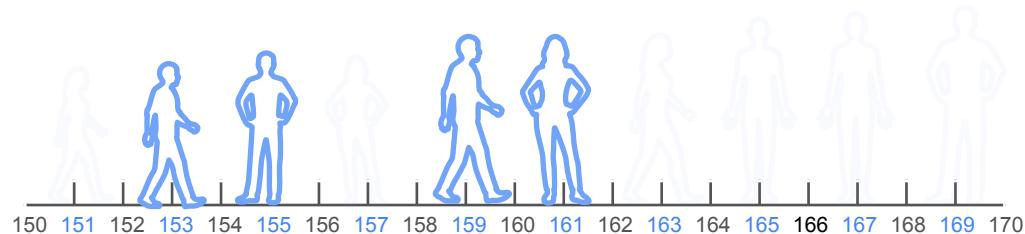


Random Sampling



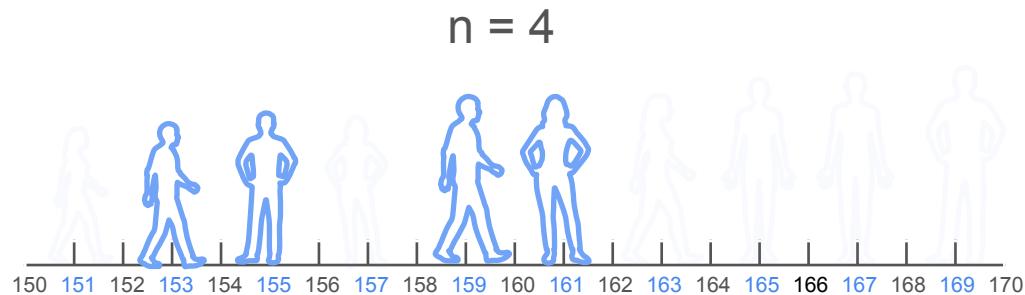
Random Sampling

$n = 4$



Random Sampling

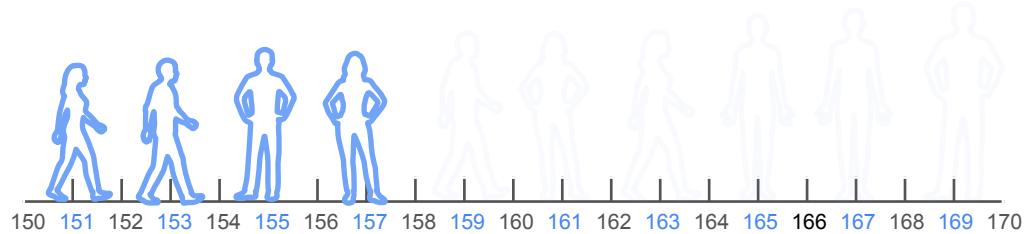
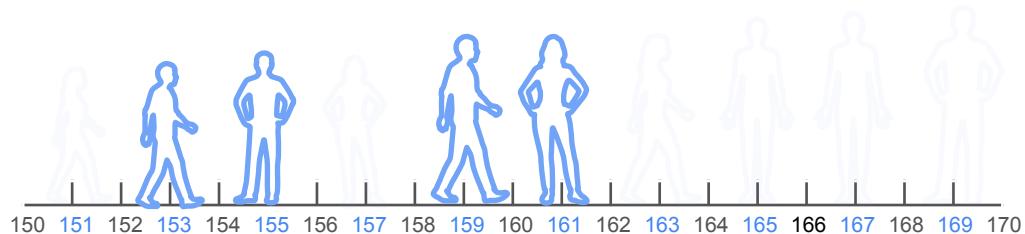
A



Random Sampling

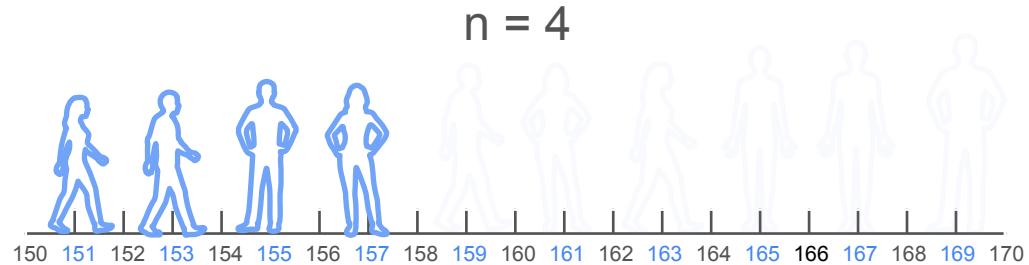
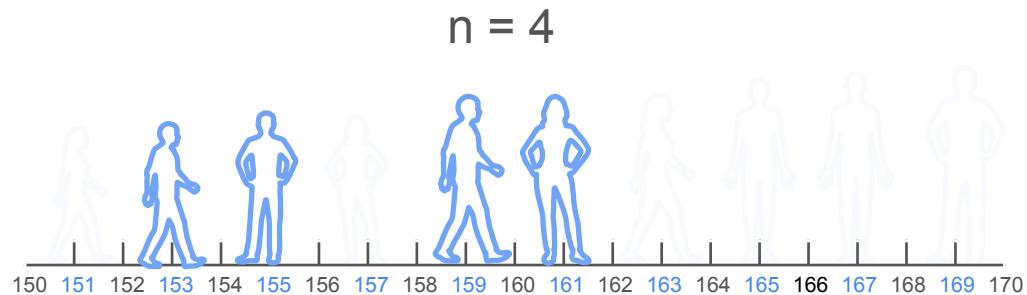
A

$n = 4$



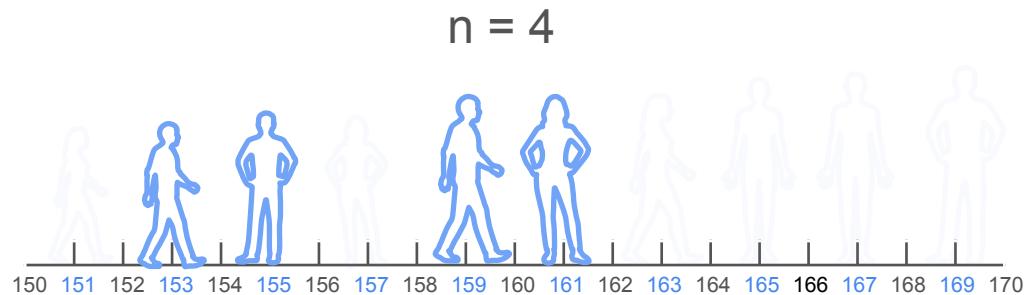
Random Sampling

A

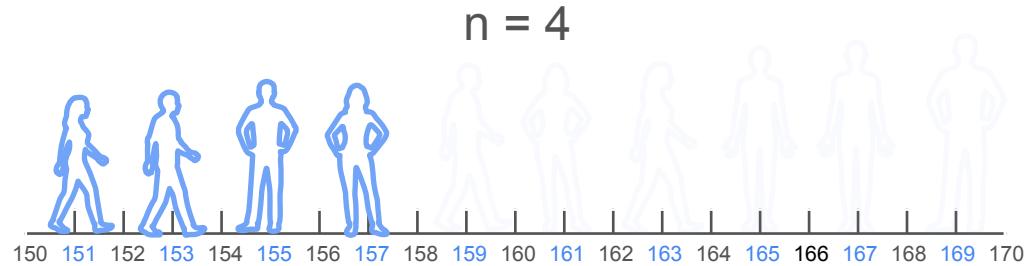


Random Sampling

A



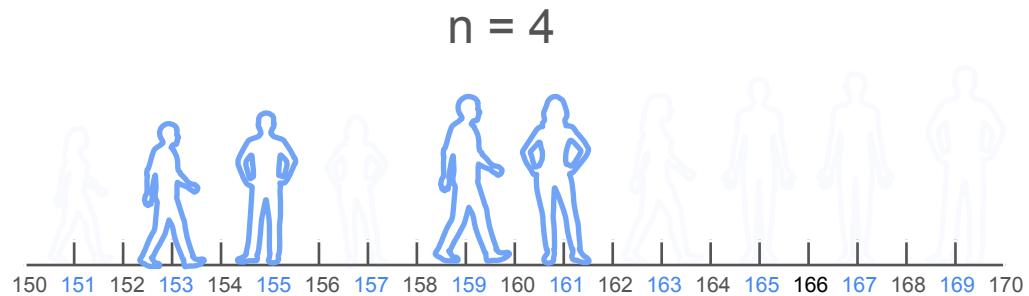
B



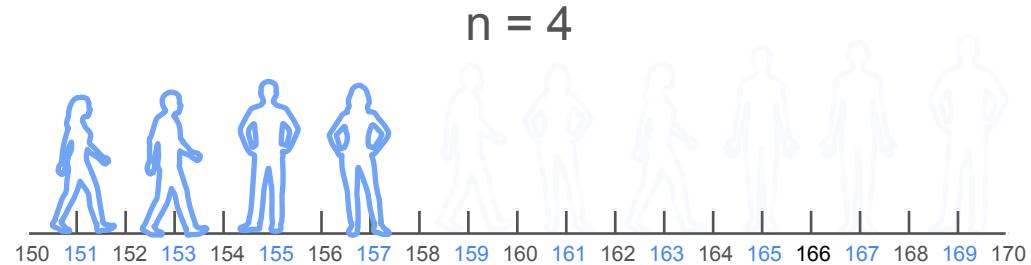
Random Sampling

A

Which is the better sample
to estimate the population
mean height?

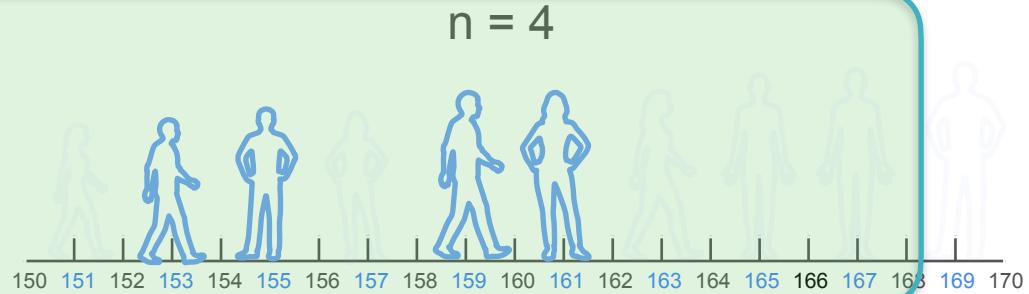


B



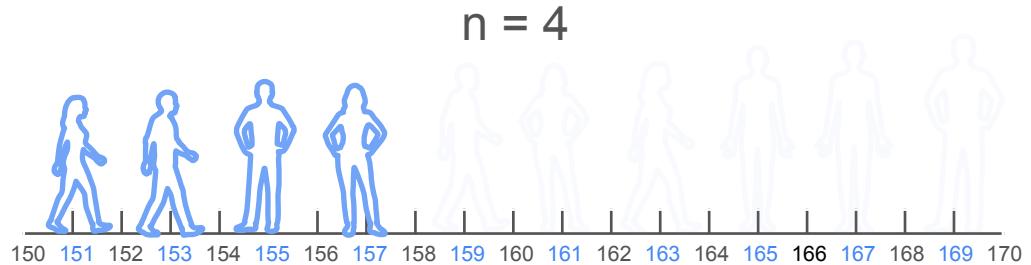
Random Sampling

A



Which is the better sample
to estimate the population
mean height?

B



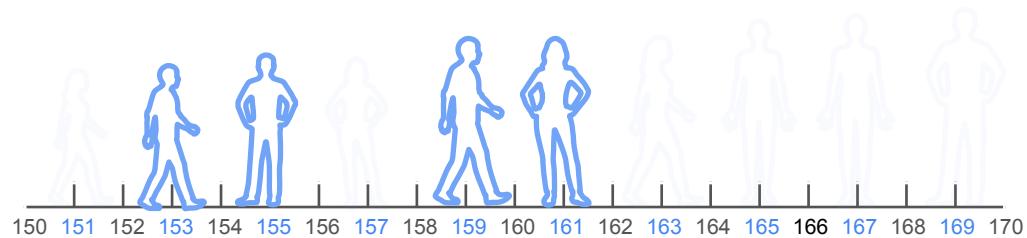
Independent Sample

Independent Sample

Example 1

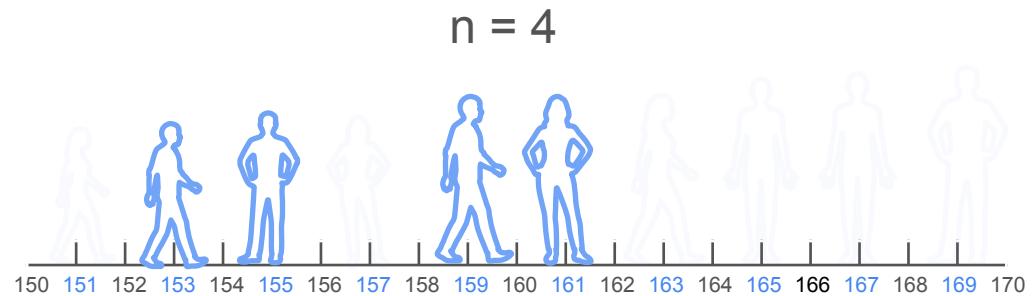
Independent Sample

Example 1



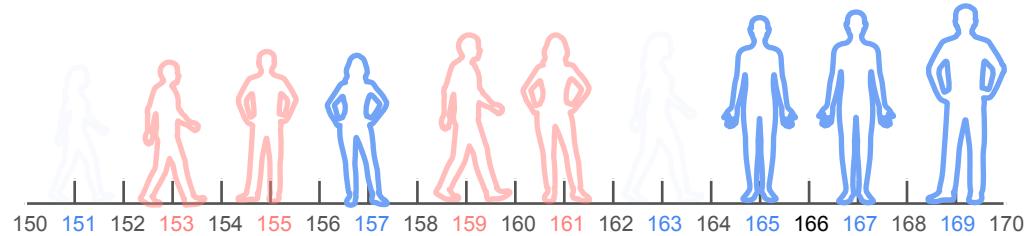
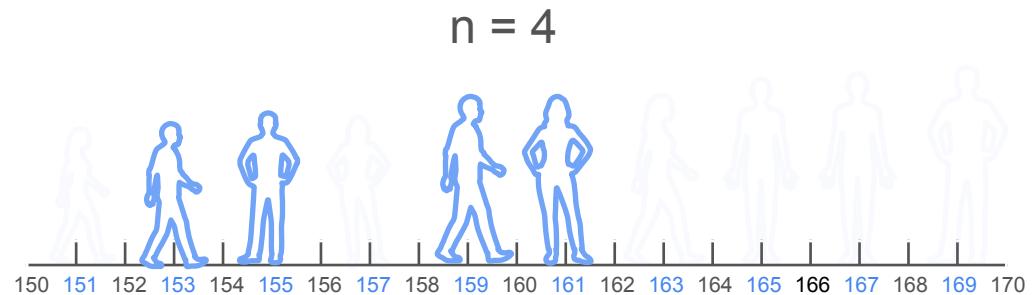
Independent Sample

Example 1



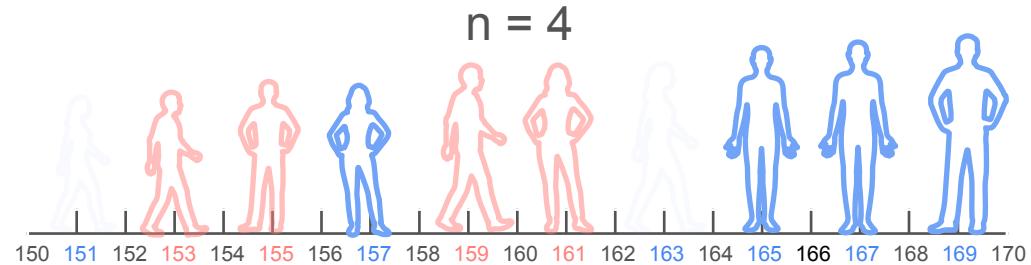
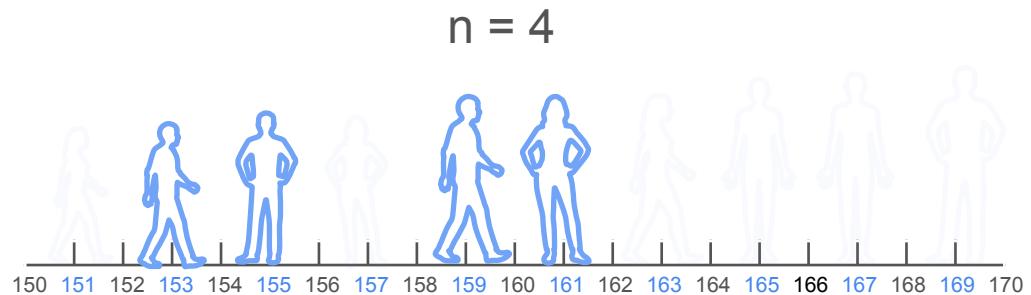
Independent Sample

Example 1



Independent Sample

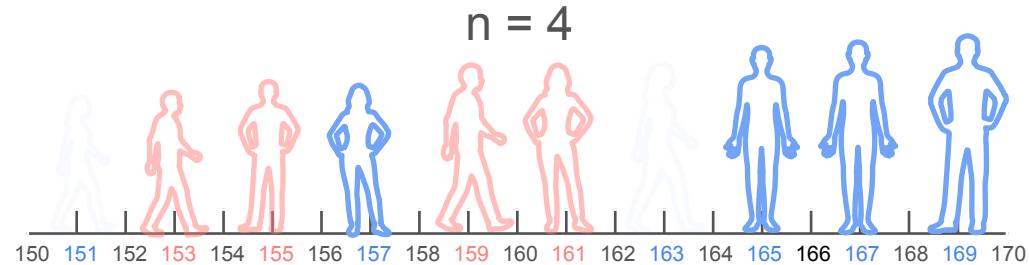
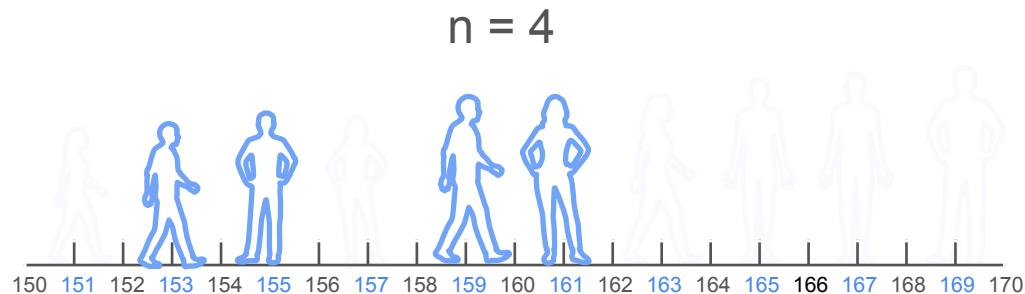
Example 1



Independent Sample

Example 1

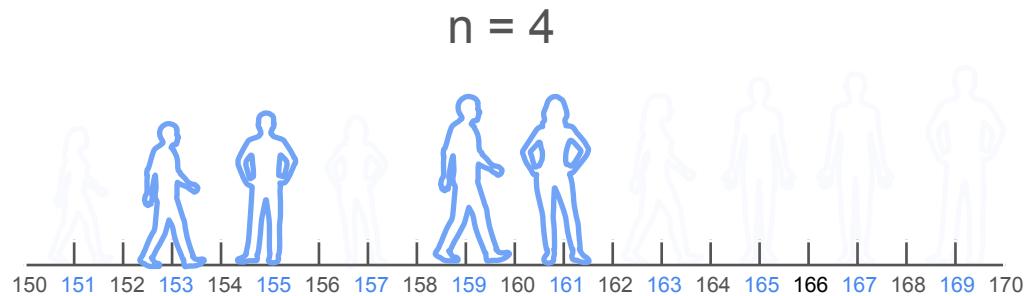
1st sample set



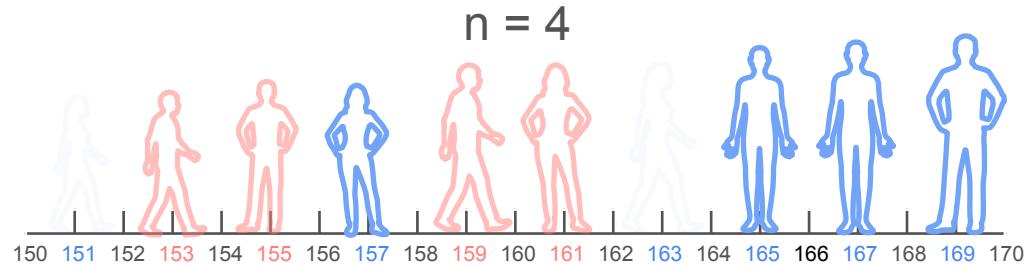
Independent Sample

Example 1

1st sample set



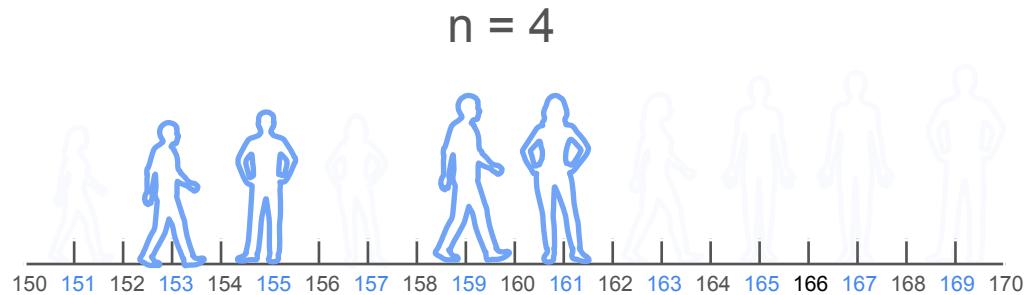
2nd sample set



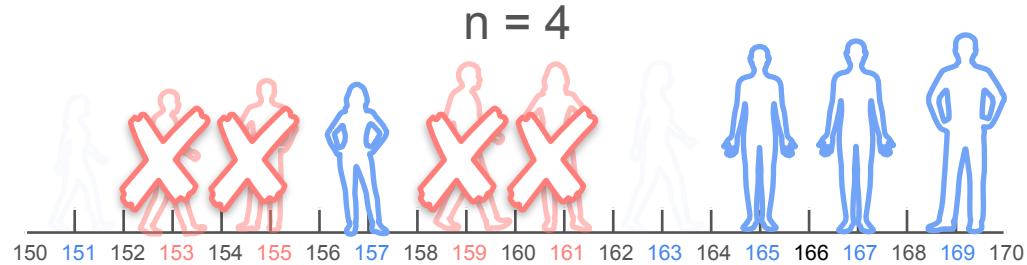
Independent Sample

Example 1

1st sample set



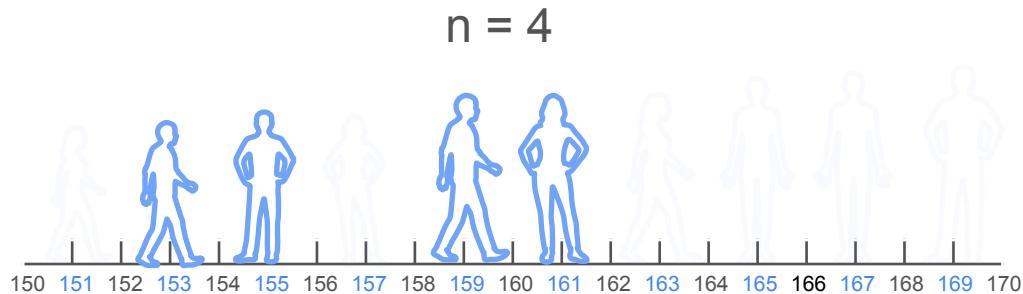
2nd sample set



Independent Sample

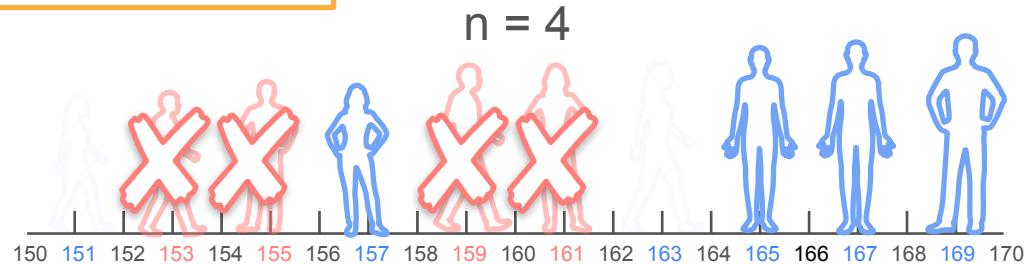
Example 1

1st sample set



Why is sample set two not a good sample?

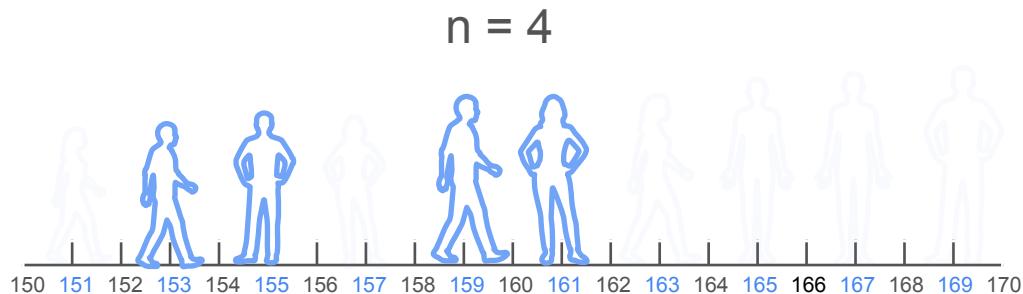
2nd sample set



Independent Sample

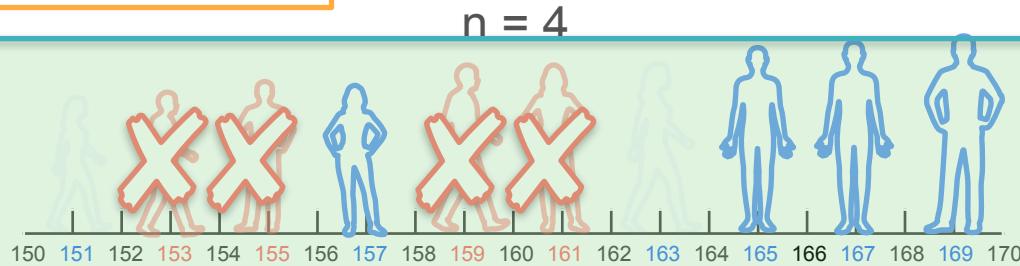
Example 1

1st sample set



Why is sample set two not a good sample?

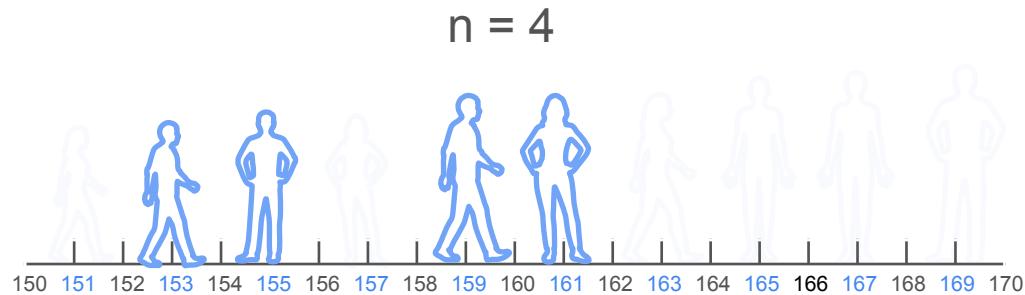
2nd sample set



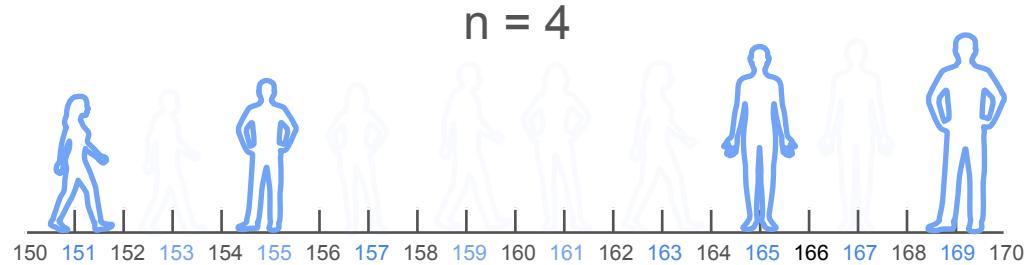
Independent Sample

Example 2

1st sample set



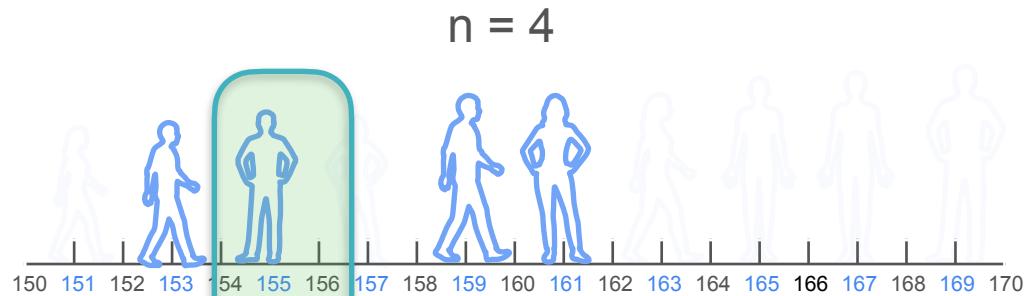
2nd sample set



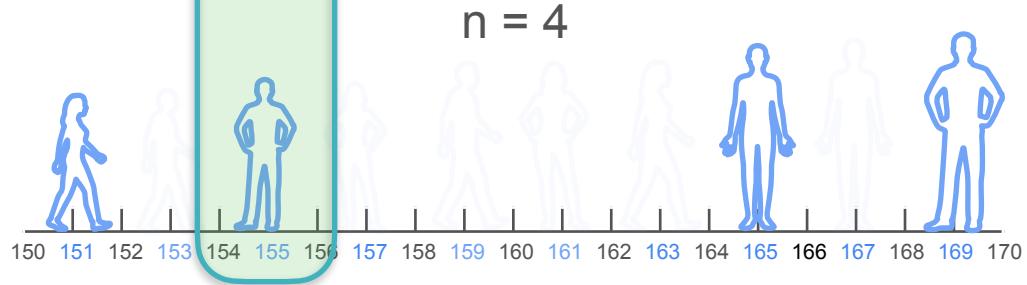
Independent Sample

Example 2

1st sample set



2nd sample set



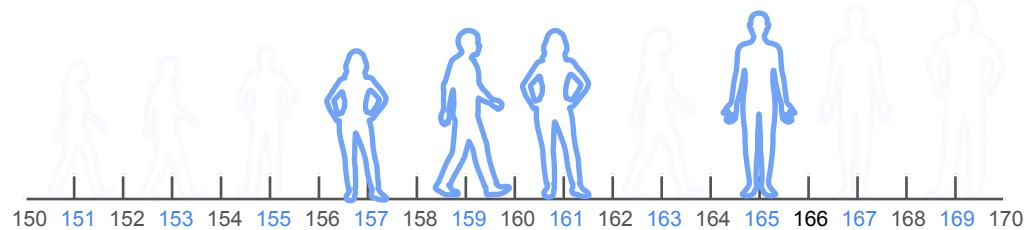
Identically Distributed Samples

Identically Distributed Samples

Example 1

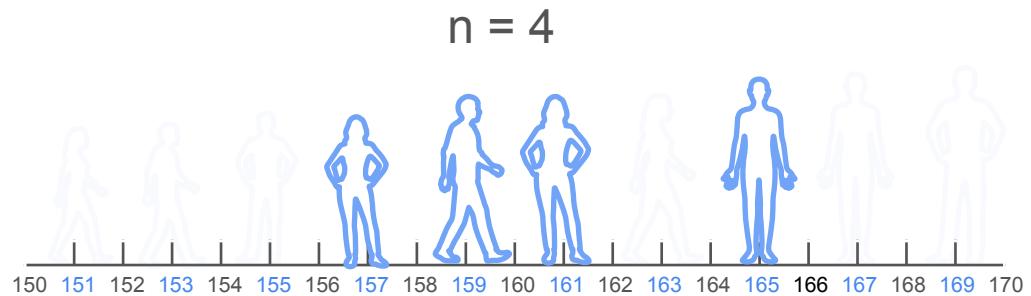
Identically Distributed Samples

Example 1



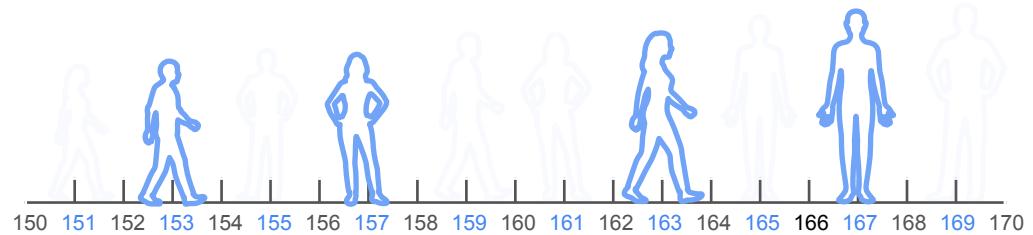
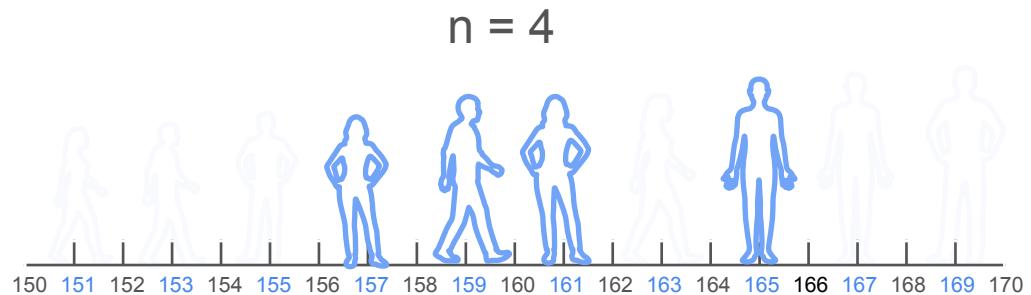
Identically Distributed Samples

Example 1



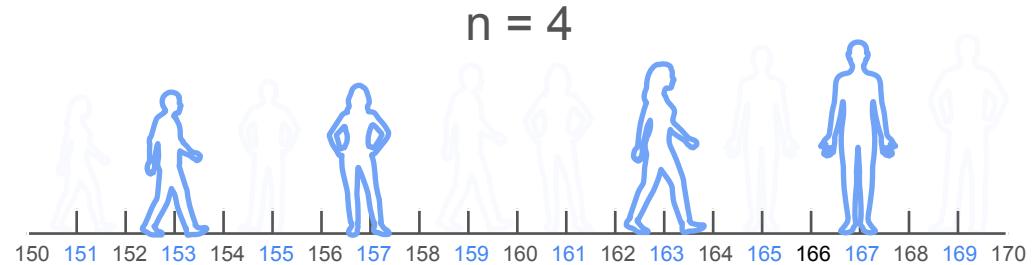
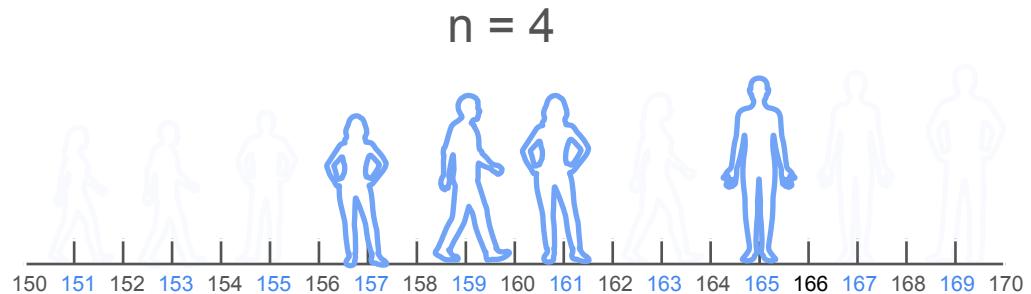
Identically Distributed Samples

Example 1



Identically Distributed Samples

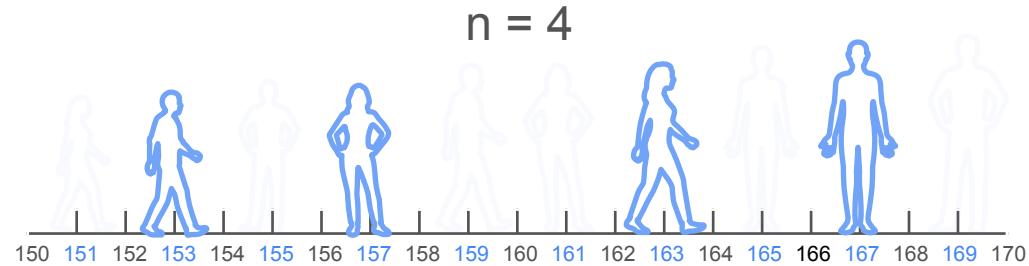
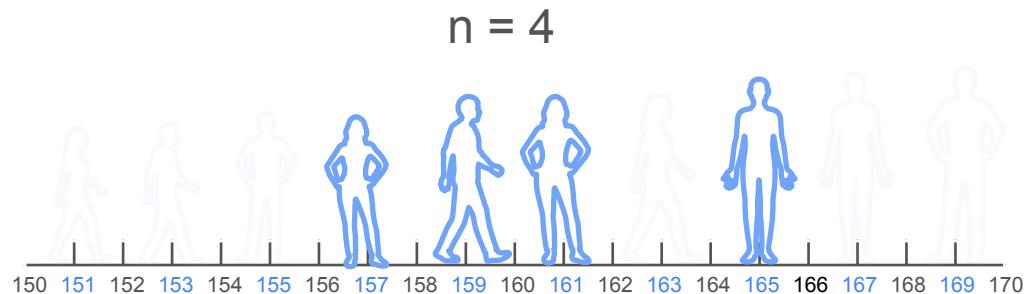
Example 1



Identically Distributed Samples

Example 1

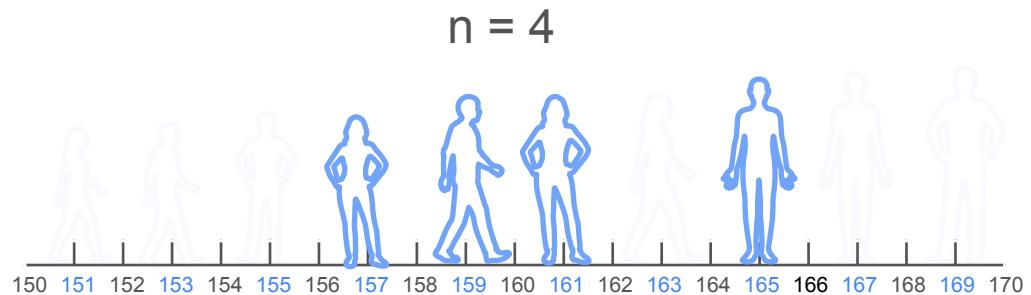
A



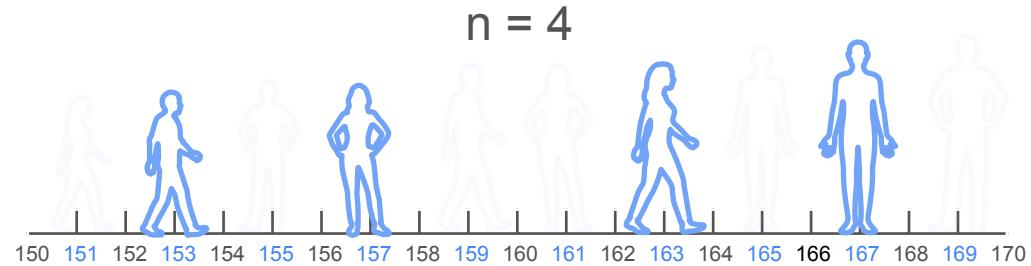
Identically Distributed Samples

Example 1

A



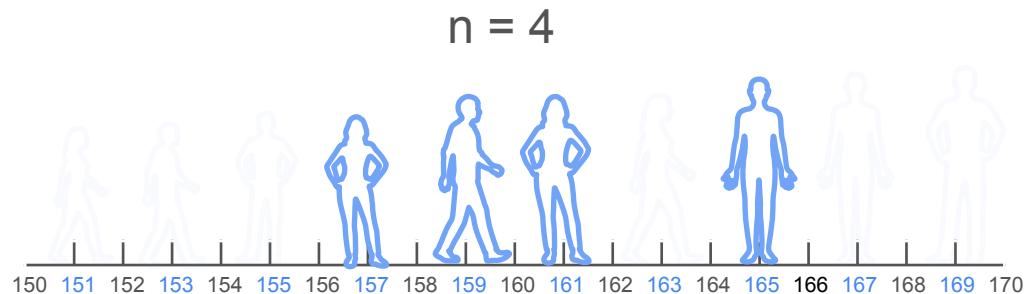
B



Identically Distributed Samples

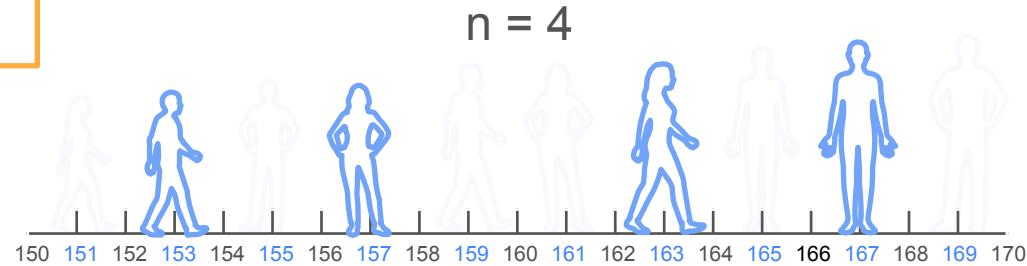
Example 1

A



Which of the following samples
are identically distributed?

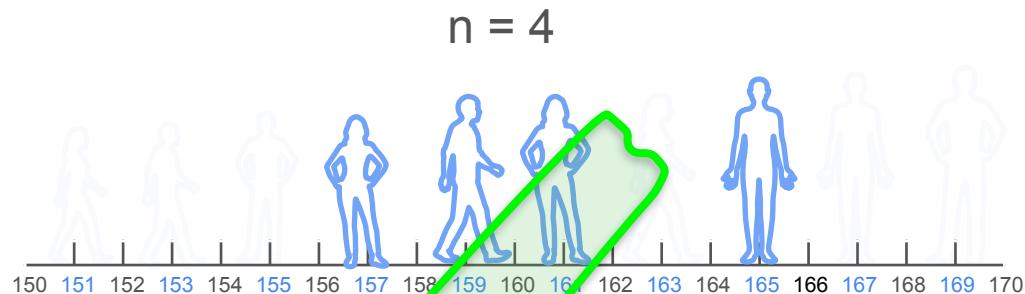
B



Identically Distributed Samples

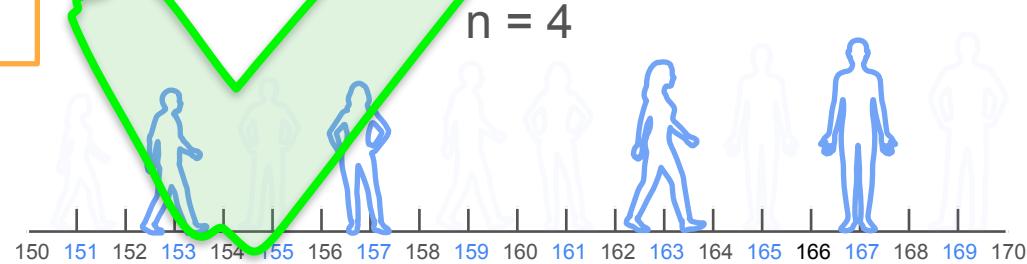
Example 1

A



Which of the following samples
are identically distributed?

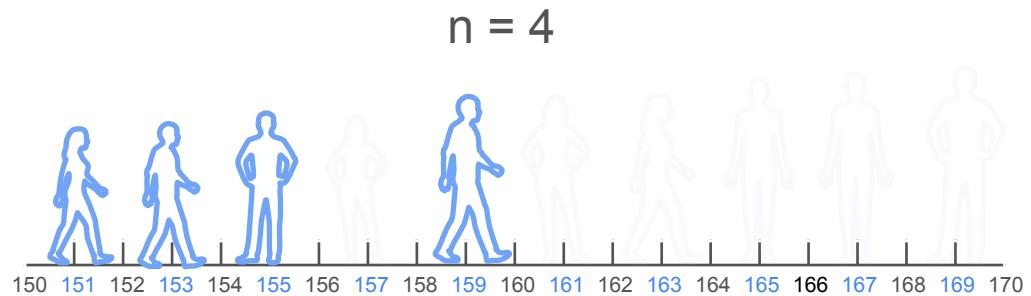
B



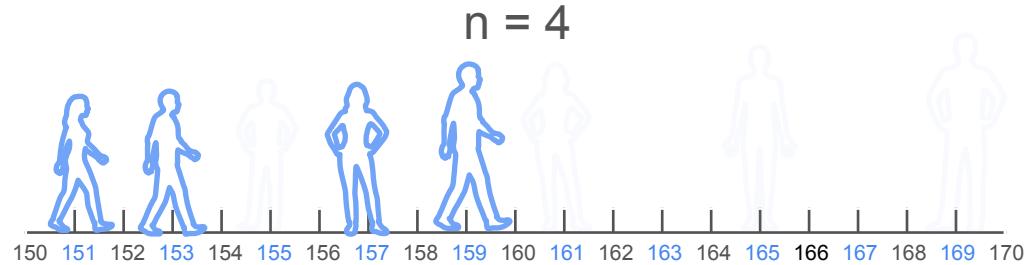
Identically Distributed Samples

Example 2

A



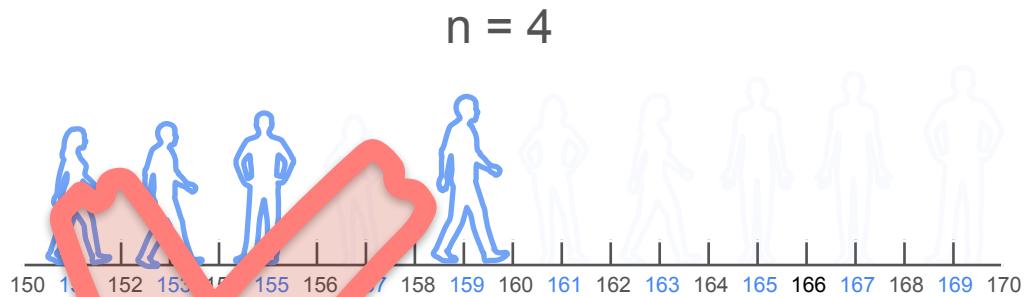
B



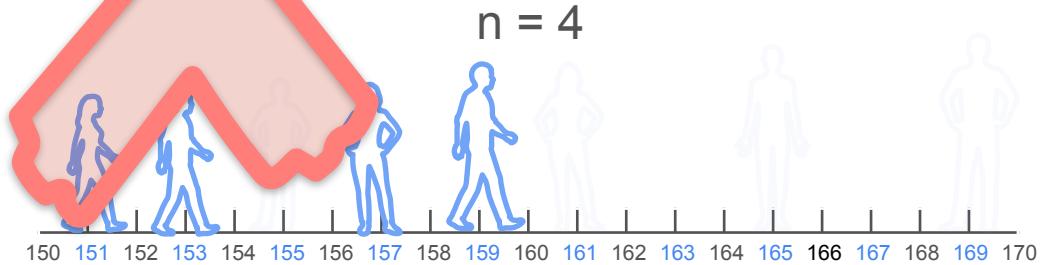
Identically Distributed Samples

Example 2

A

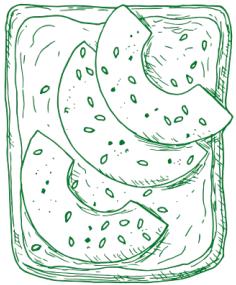


B

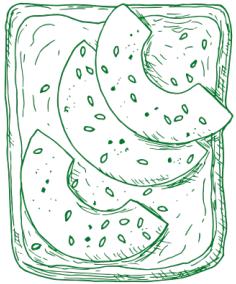


The Avocado Toast Trend

The Avocado Toast Trend



The Avocado Toast Trend



Study the price of avocados
in the United States



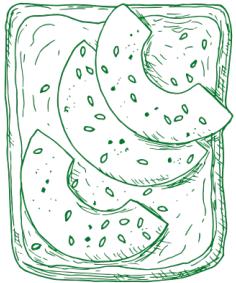
The Avocado Toast Trend



Study the price of avocados
in the United States



The Avocado Toast Trend

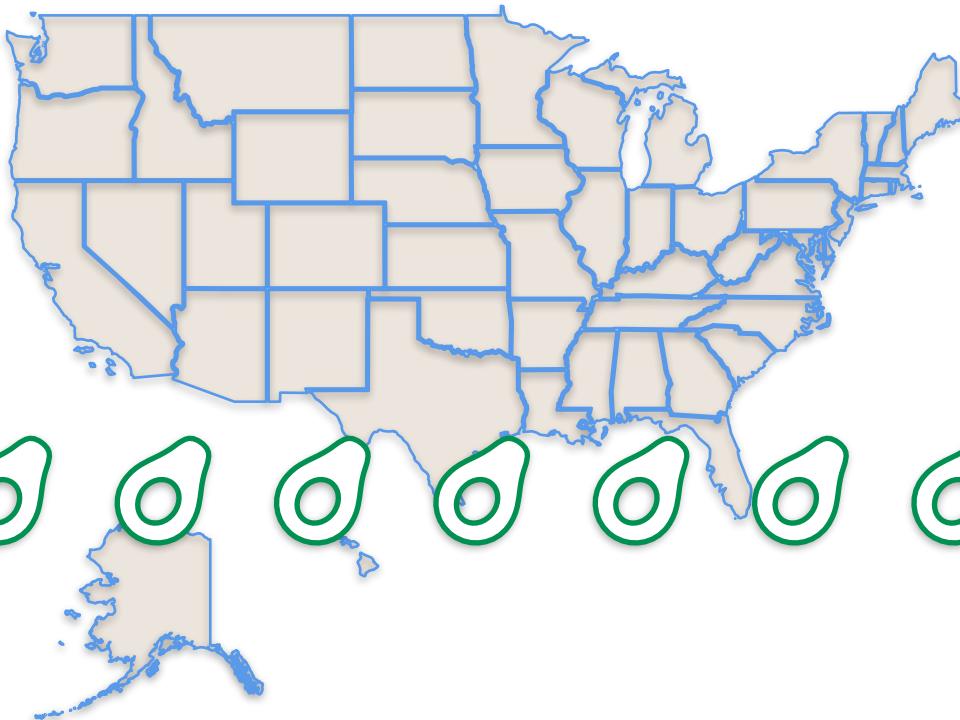


Study the price of avocados
in the United States



What is the population of your study?

The Avocado Toast Trend



All avocados
sold in the US

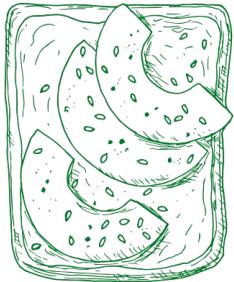
The Avocado Toast Trend



Study the price of avocados
in the United States



The Avocado Toast Trend



Study the price of avocados
in the United States



What is the sample of your study?

The Avocado Toast Trend

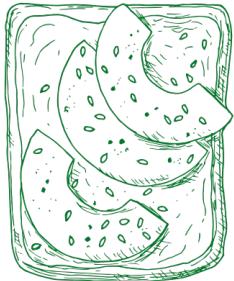


Study the price of avocados
in the United States



What is the sample of your study?

The Avocado Toast Trend



Study the price of avocados
in the United States



What is the sample of your study?

**Avocados sold
in the 4 stores
you selected**

Population and Sample in Machine Learning

Population and Sample in Machine Learning

Every dataset you work with in machine learning is a sample
NOT the population

Cats



Not cats

Population and Sample in Machine Learning

Every dataset you work with in machine learning is a sample
NOT the population

Cats



Not cats



Population and Sample in Machine Learning

Every dataset you work with in machine learning is a sample
NOT the population

Cats



Not cats



Population and Sample in Machine Learning

Every dataset you work with in machine learning is a sample
NOT the population

Cats



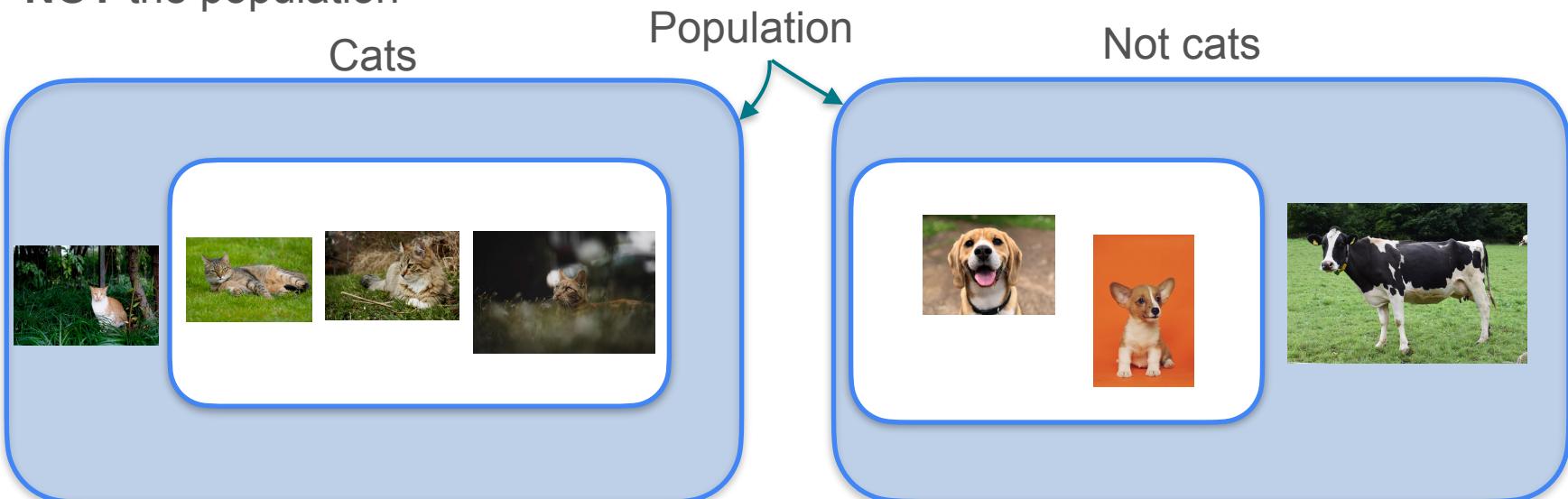
Not cats



Population and Sample in Machine Learning

Every dataset you work with in machine learning is a sample

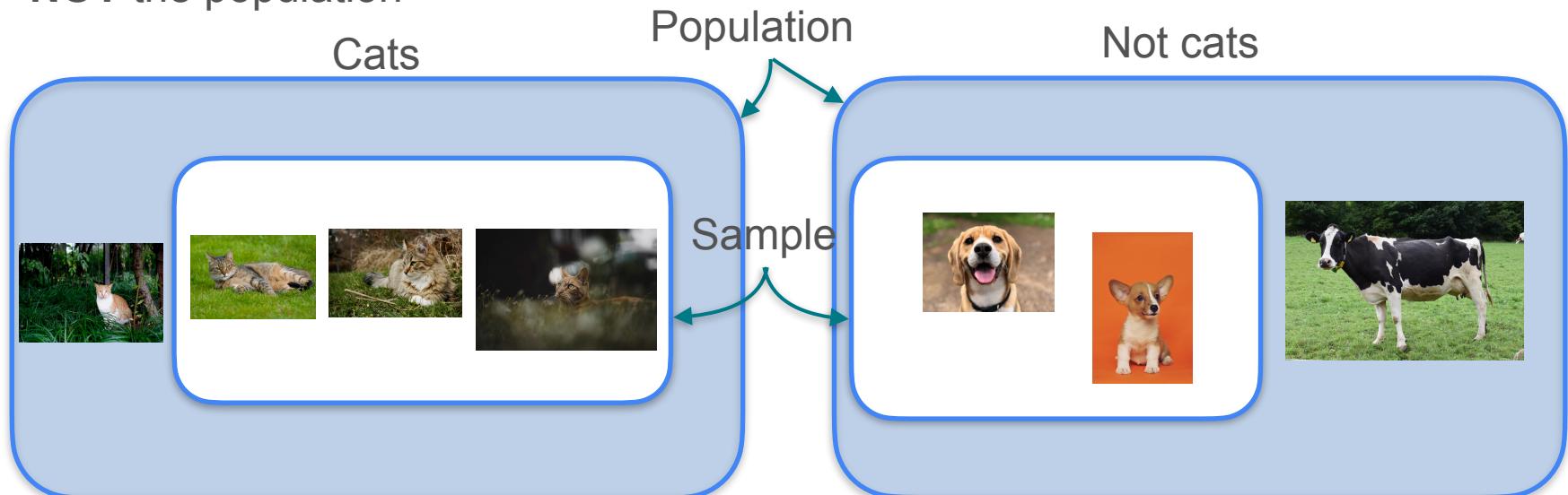
NOT the population



Population and Sample in Machine Learning

Every dataset you work with in machine learning is a sample

NOT the population



Recap

Population

the entire group of individuals or elements you want to study which share a common behaviour



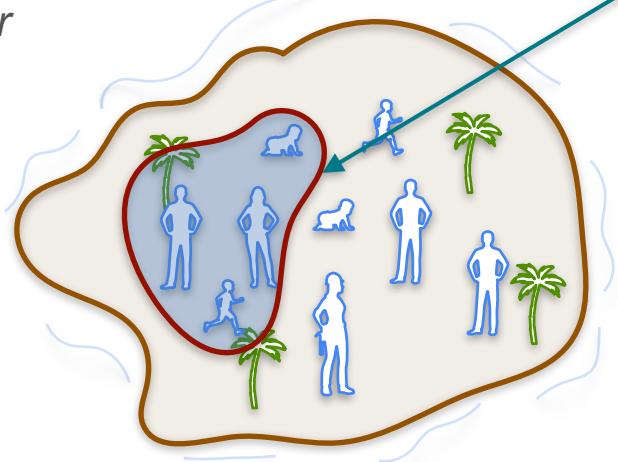
Sample

subset of the population you use to draw conclusions about the population as a whole

Recap

Population

the entire group of individuals or elements you want to study which share a common behaviour



Sample

subset of the population you use to draw conclusions about the population as a whole

Population Size:

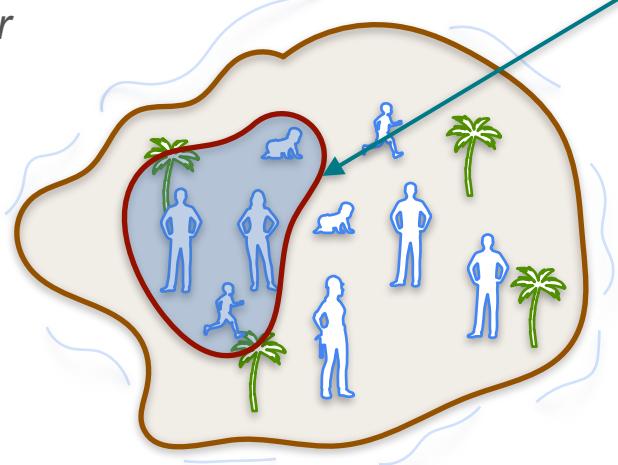
N

Sample Size:

Recap

Population

the entire group of individuals or elements you want to study which share a common behaviour



Sample

subset of the population you use to draw conclusions about the population as a whole

Population Size: N

Sample Size: n



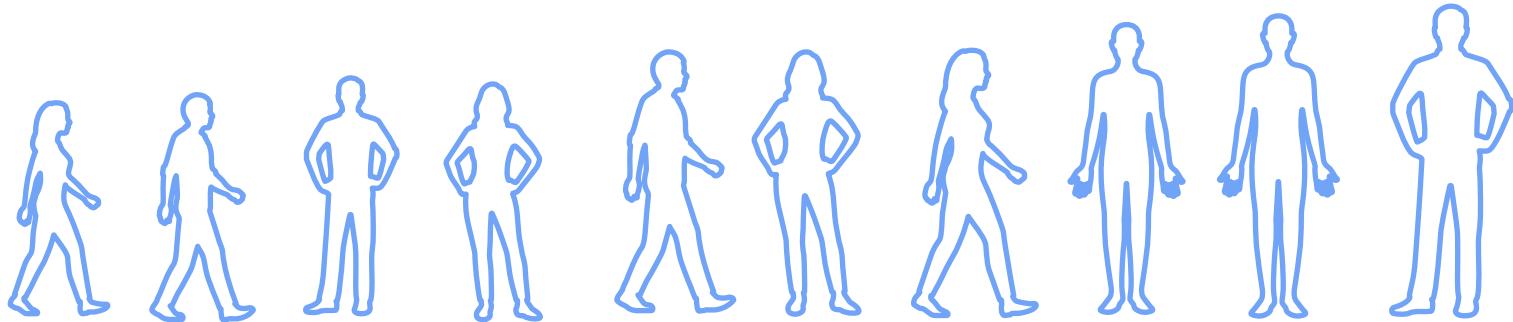
DeepLearning.AI

Sample and Population

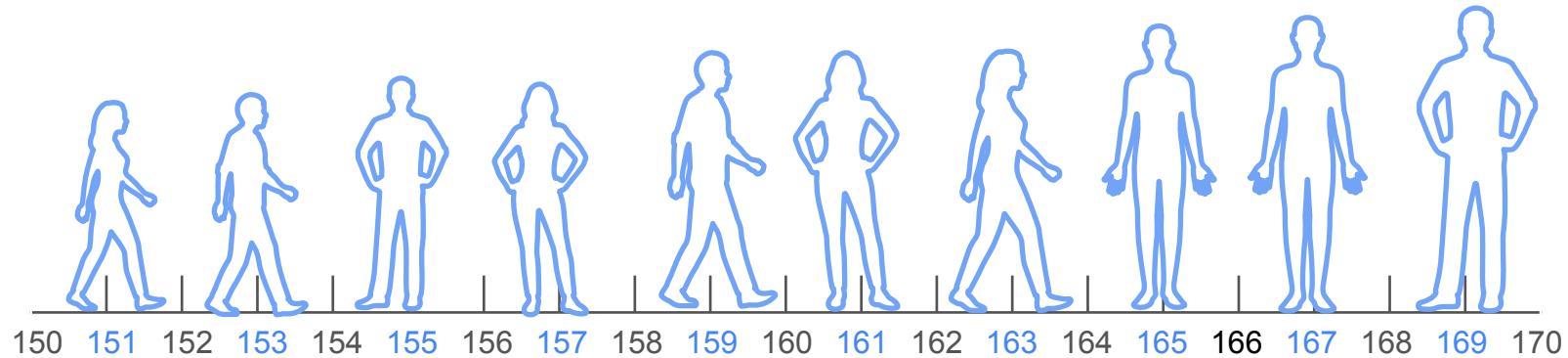
**Sample Mean, Proportion,
and Variance**

Population and Sample Mean

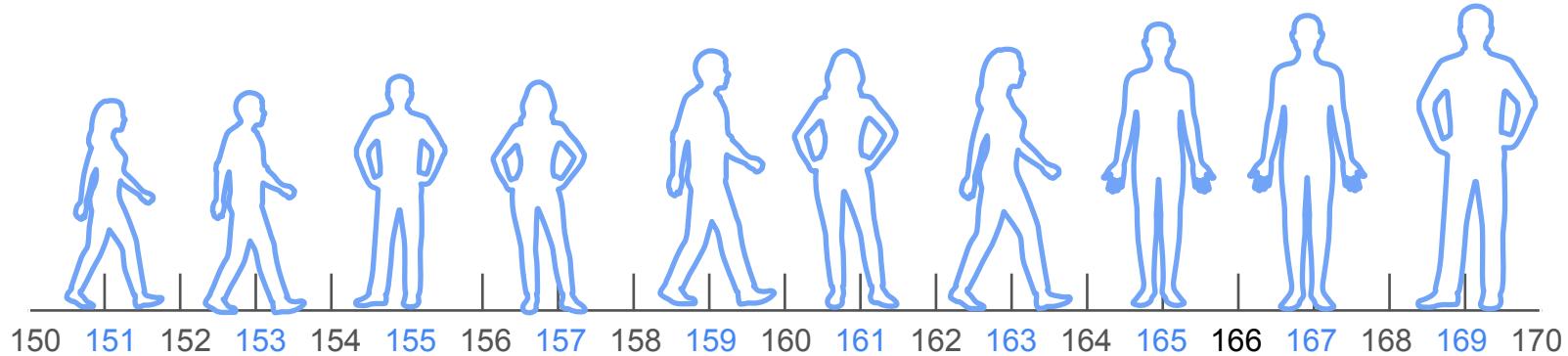
Population and Sample Mean



Population and Sample Mean

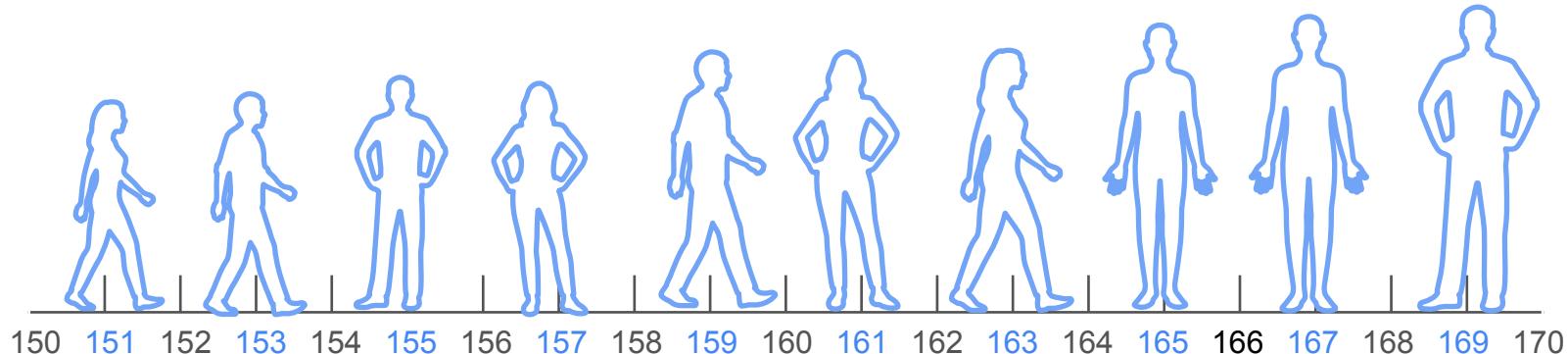


Population and Sample Mean



What is the average
height in statistopia?

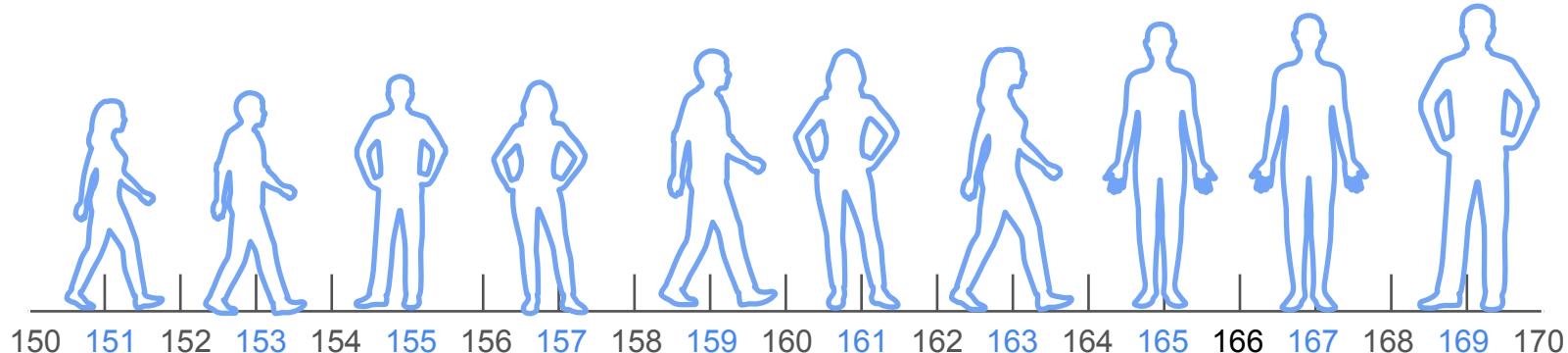
Population and Sample Mean



What is the average
height in statistopia?

$$\frac{151 + 153 + 155 + 157 + 159 + 161 + 163 + 165 + 167 + 169}{10}$$

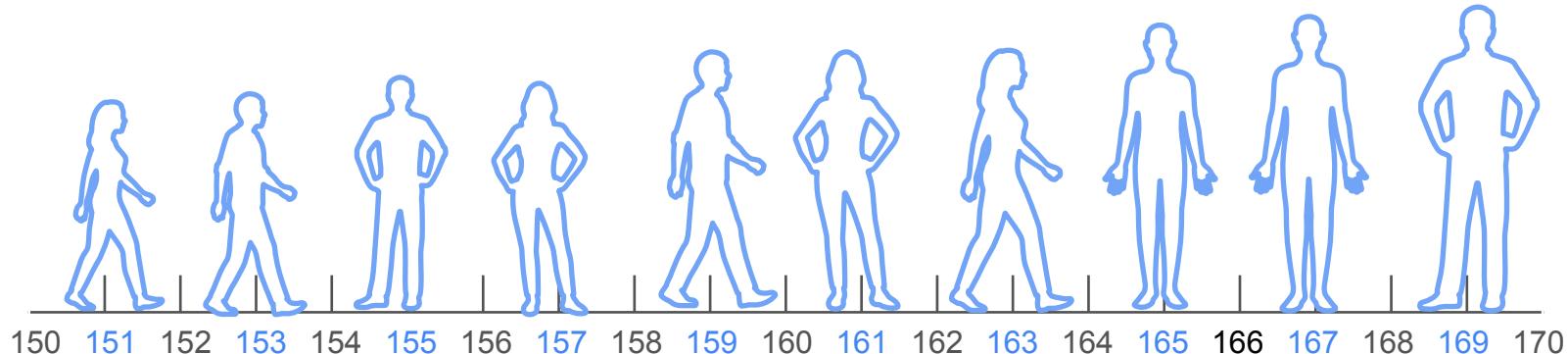
Population and Sample Mean



What is the average height in statistopia?

$$\frac{151 + 153 + 155 + 157 + 159 + 161 + 163 + 165 + 167 + 169}{10} = \frac{1600}{10} = 160\text{cm}$$

Population and Sample Mean



What is the average height in statistopia?

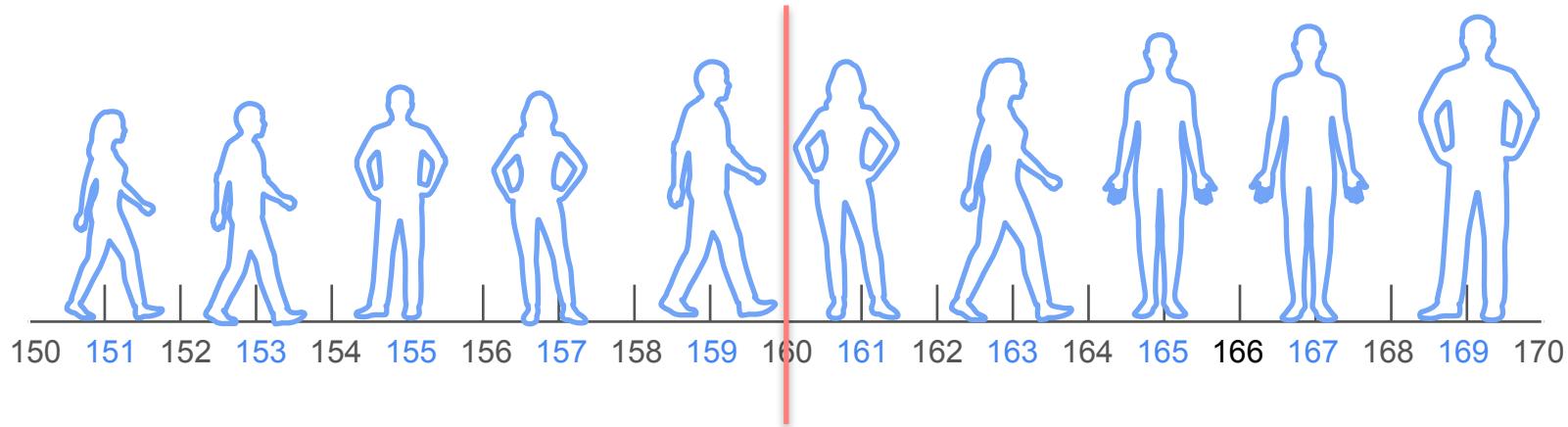
$$\frac{151 + 153 + 155 + 157 + 159 + 161 + 163 + 165 + 167 + 169}{10}$$

$$= \frac{1600}{10} = 160\text{cm}$$

Population mean

μ

Population and Sample Mean



What is the average height in statistopia?

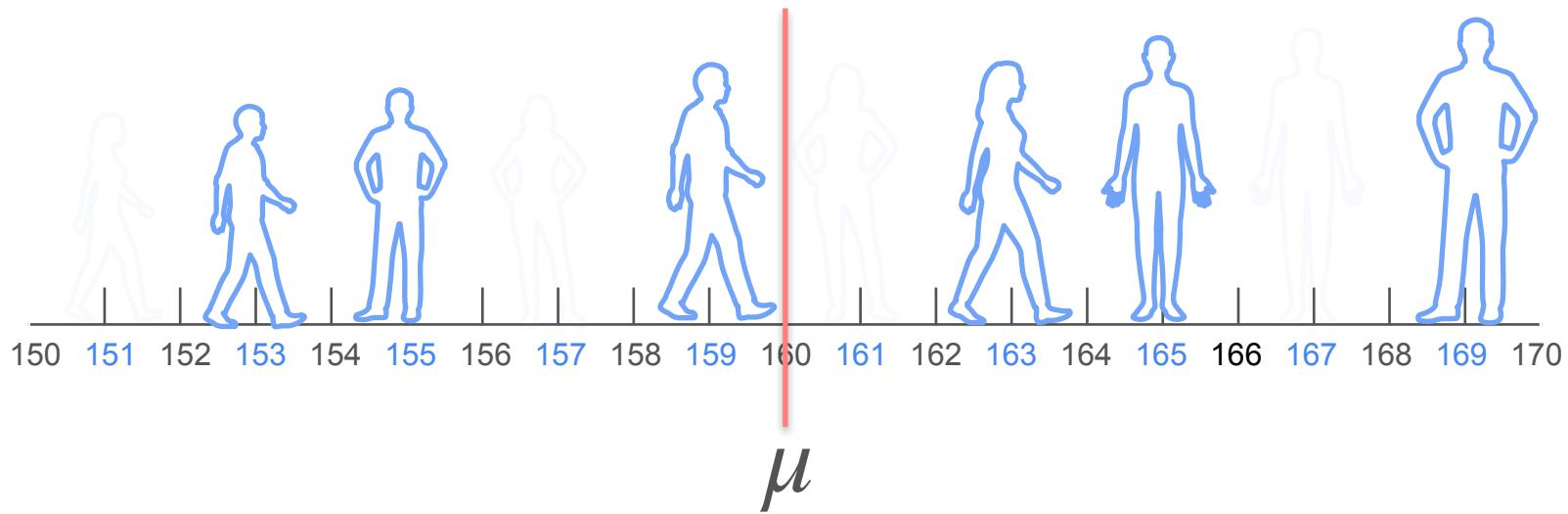
$$\frac{151 + 153 + 155 + 157 + 159 + 161 + 163 + 165 + 167 + 169}{10}$$

$$= \frac{1600}{10} = 160\text{cm}$$

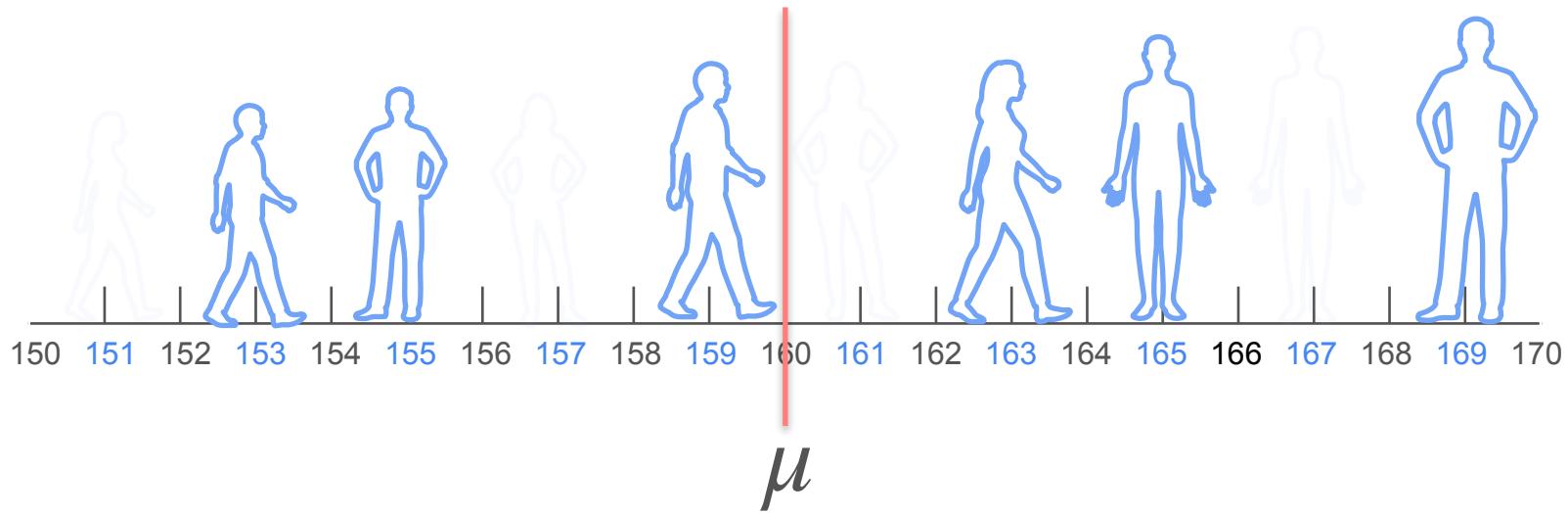
Population mean

μ

Population and Sample Mean

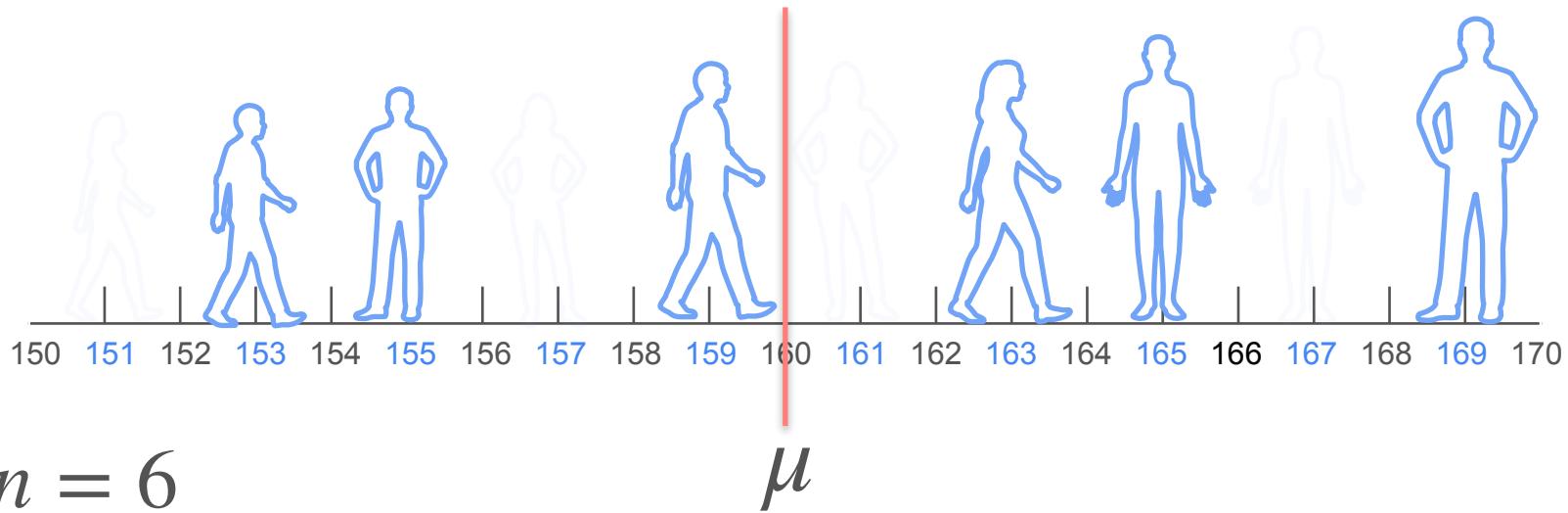


Population and Sample Mean



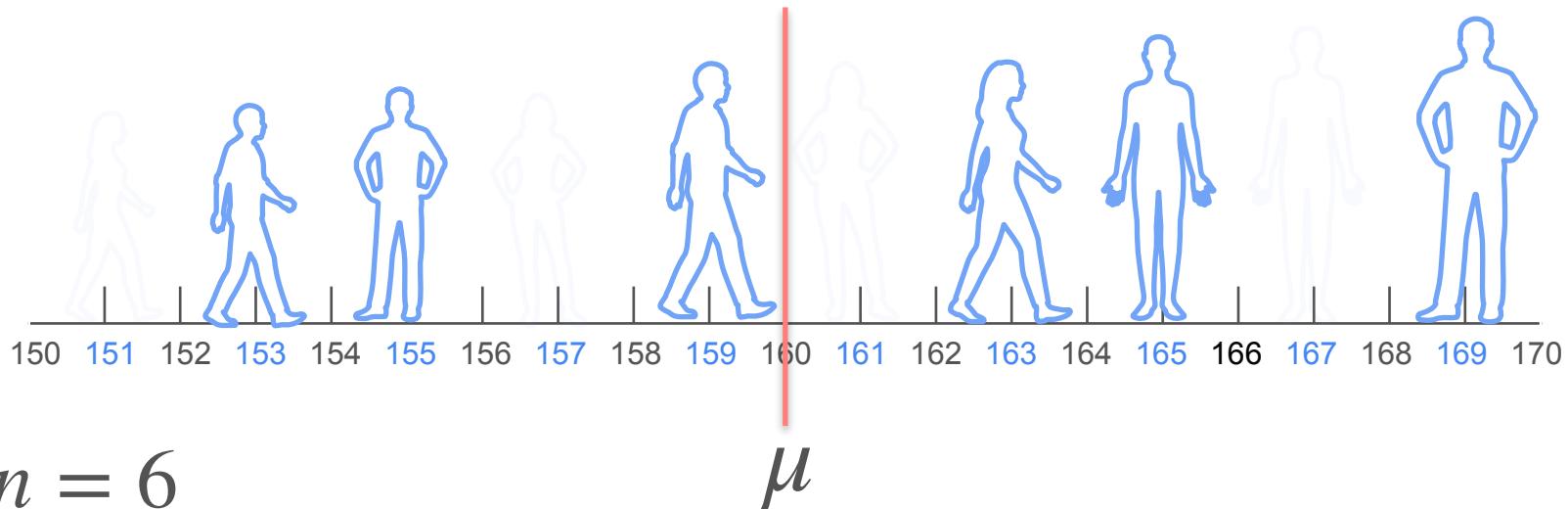
What is the average
height in statistopia?

Population and Sample Mean



What is the average
height in statistopia?

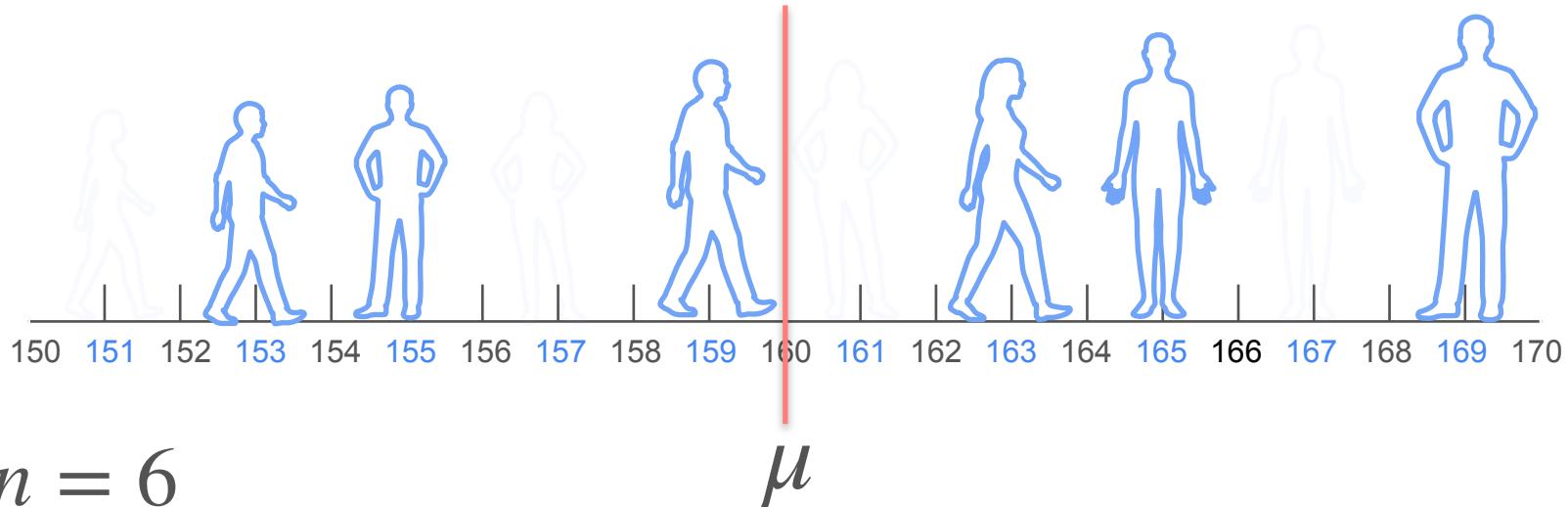
Population and Sample Mean



What is the average height in statistopia?

$$\frac{153 + 155 + 159 + 163 + 165 + 169}{6}$$

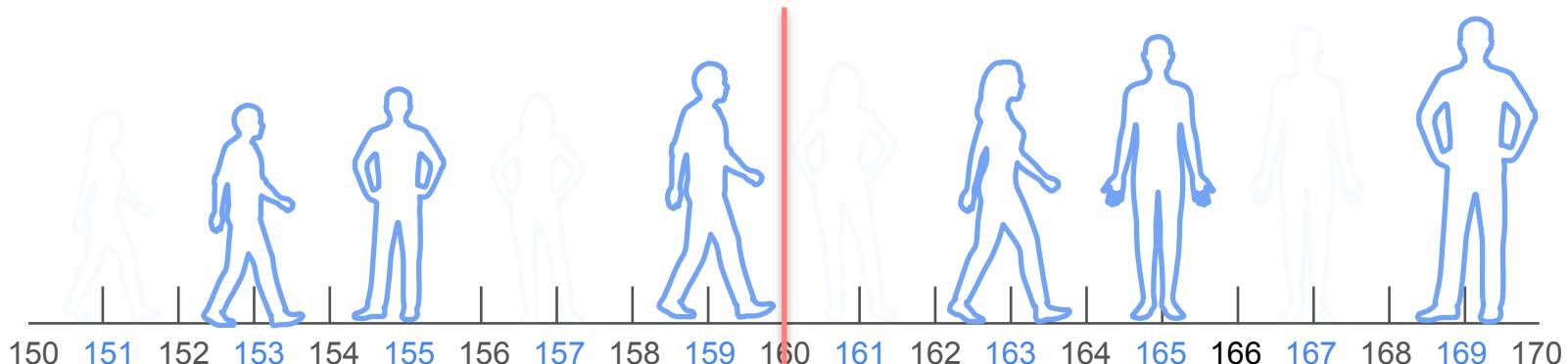
Population and Sample Mean



What is the average height in statistopia?

$$\frac{153 + 155 + 159 + 163 + 165 + 169}{6} = \frac{964}{6} = 160.97$$

Population and Sample Mean



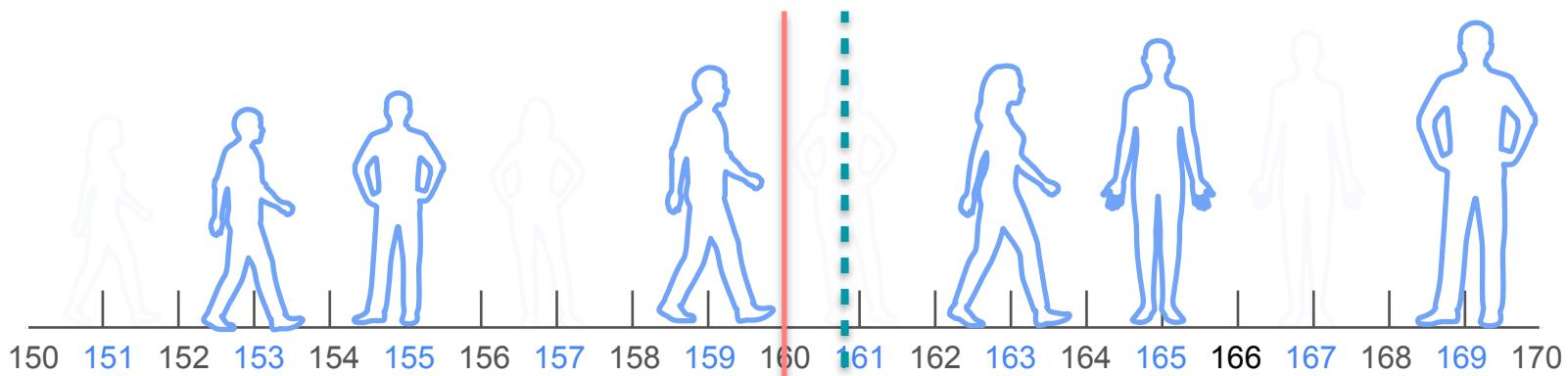
$$n = 6$$

What is the average height in statistopia?

$$\text{Sample mean } \bar{x}_1 = \frac{153 + 155 + 159 + 163 + 165 + 169}{6} = \frac{964}{6} = 160.97$$

\bar{x}

Population and Sample Mean



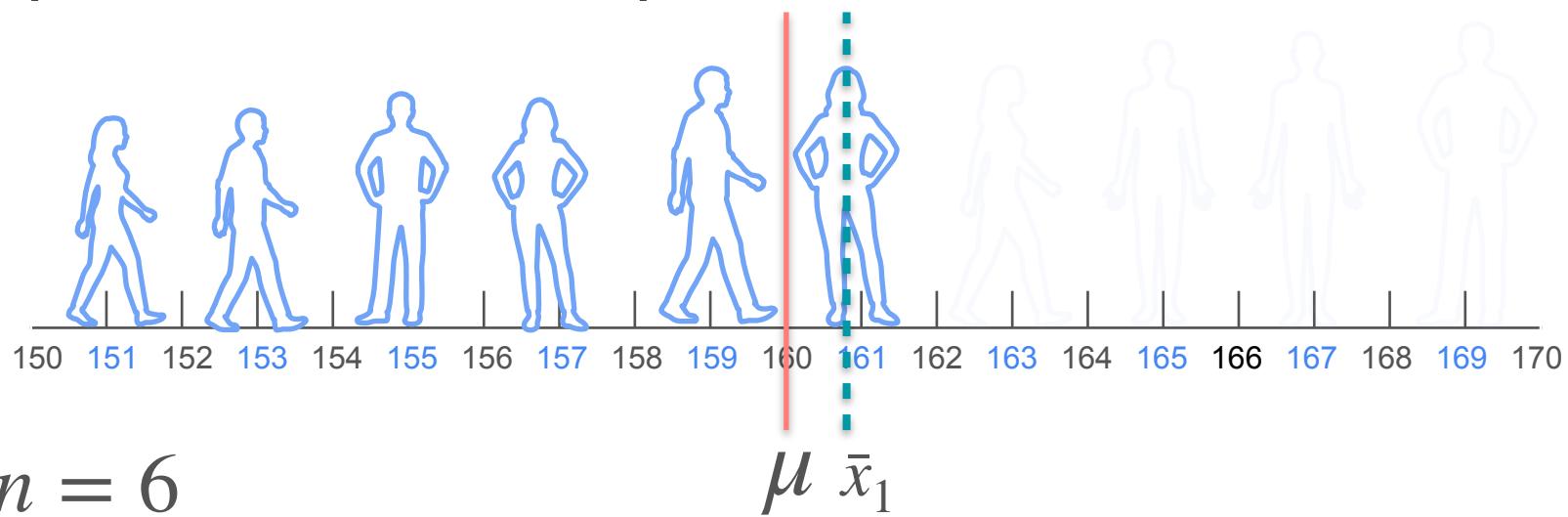
$$n = 6$$

What is the average height in statistopia?

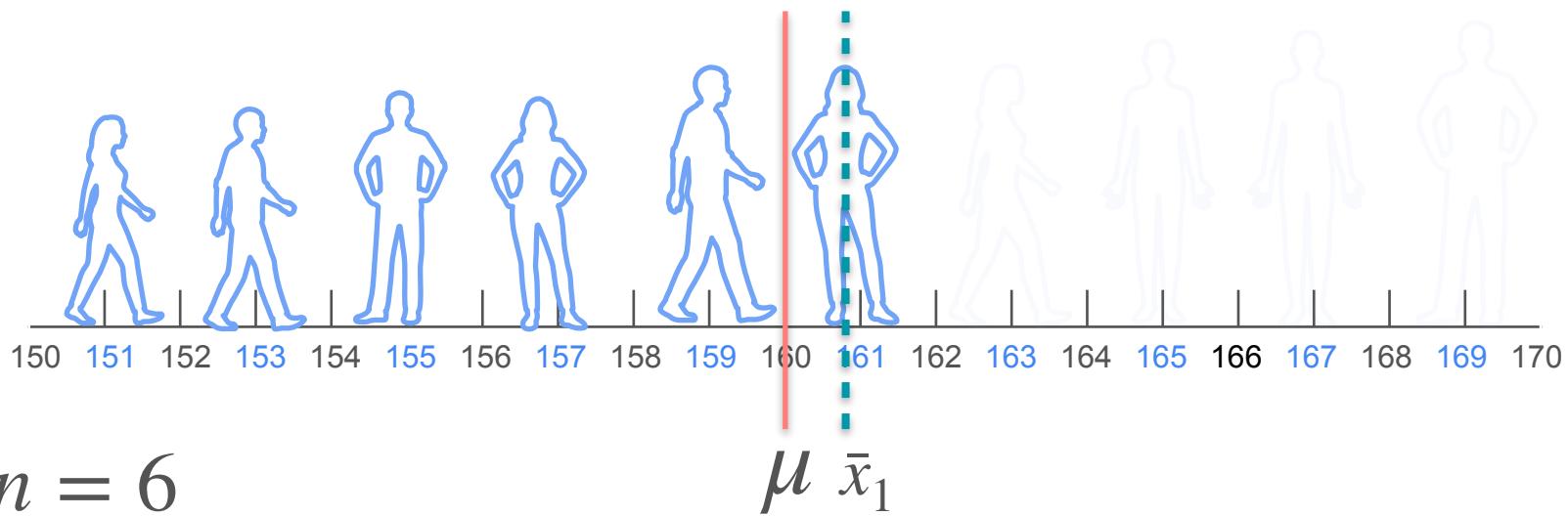
$$\frac{153 + 155 + 159 + 163 + 165 + 169}{6} = \frac{964}{6} = 160.97$$

\bar{x}

Population and Sample Mean

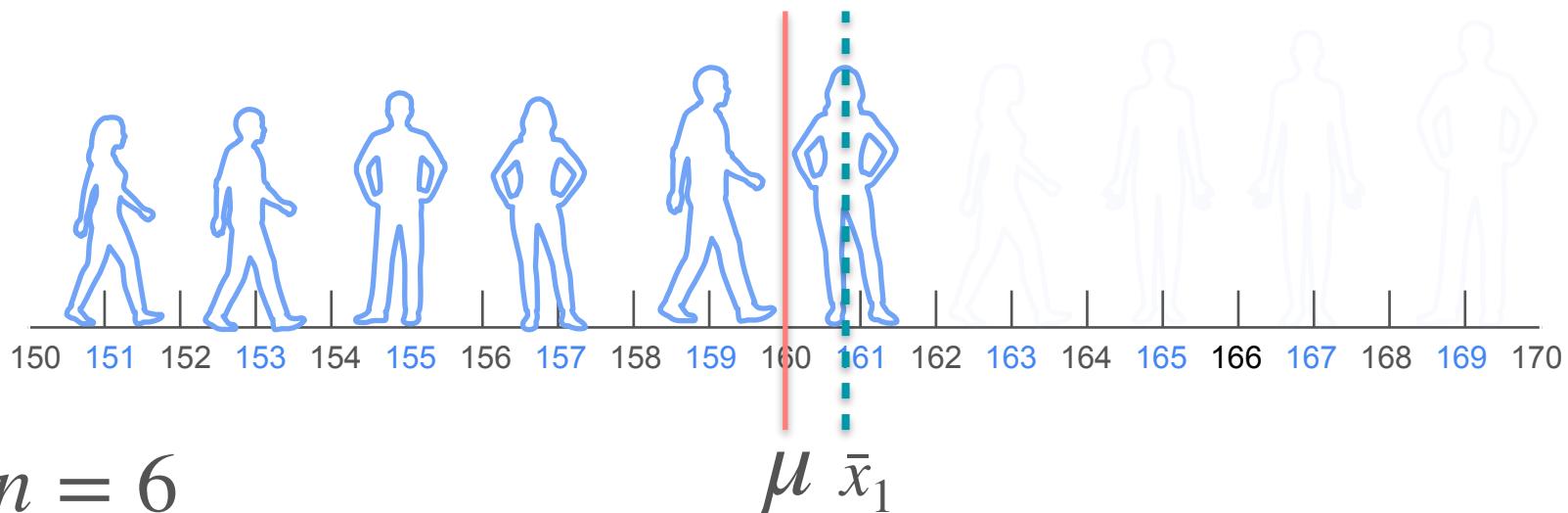


Population and Sample Mean



What is the average height in statistopia?

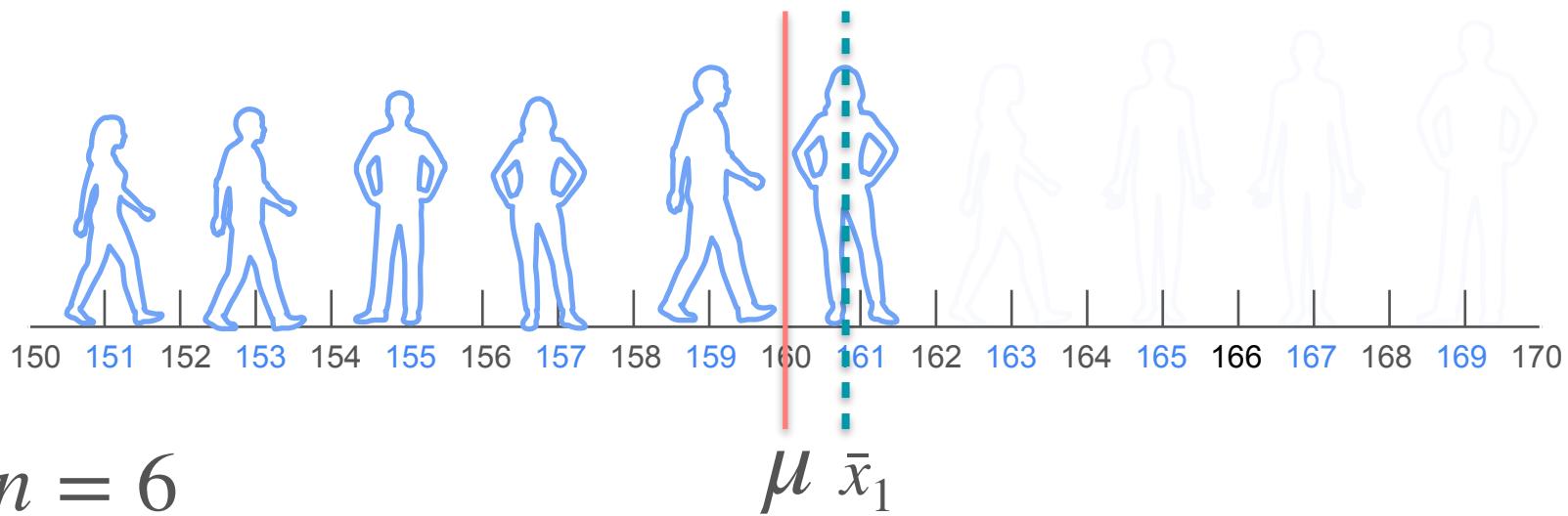
Population and Sample Mean



What is the average height in statistopia?

$$\frac{151 + 153 + 155 + 157 + 159 + 161}{6}$$

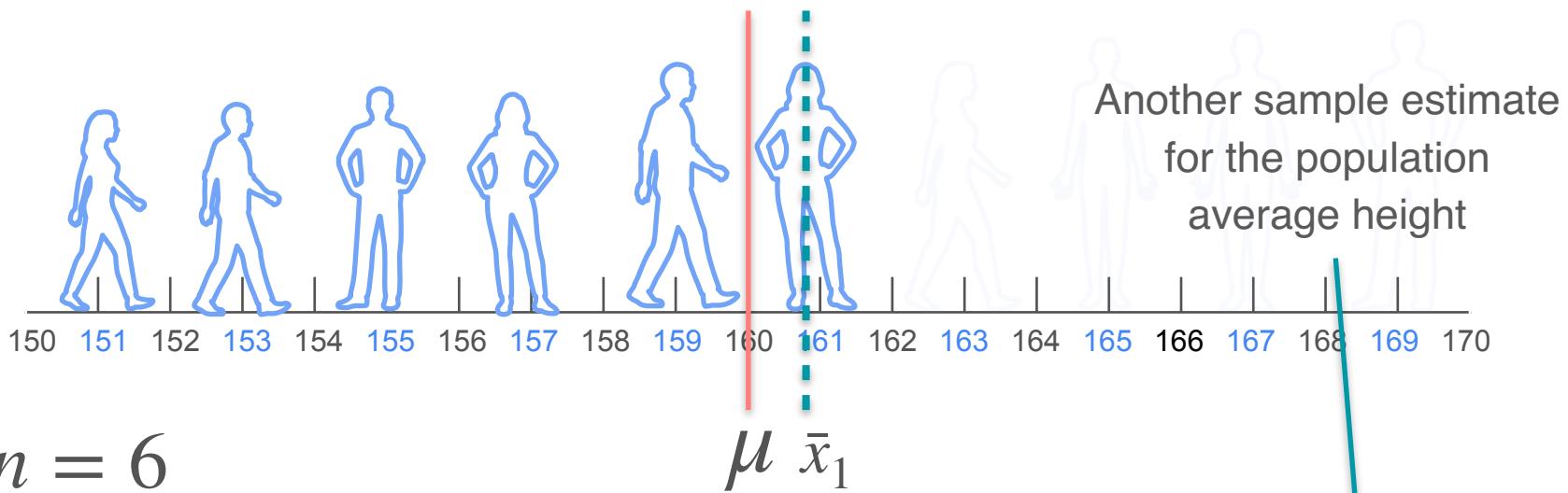
Population and Sample Mean



What is the average height in statistopia?

$$\frac{151 + 153 + 155 + 157 + 159 + 161}{6} = \frac{936}{6} = 156\text{cm}$$

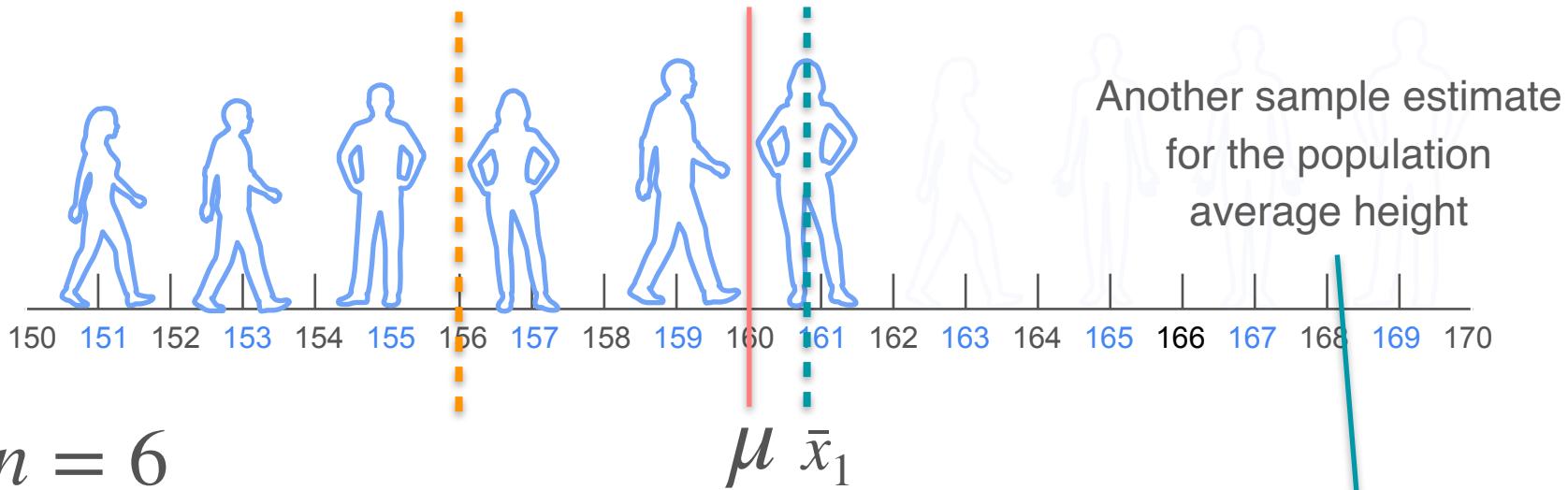
Population and Sample Mean



What is the average height in statistopia?

$$\frac{151 + 153 + 155 + 157 + 159 + 161}{6} = \frac{936}{6} = 156\text{cm}$$

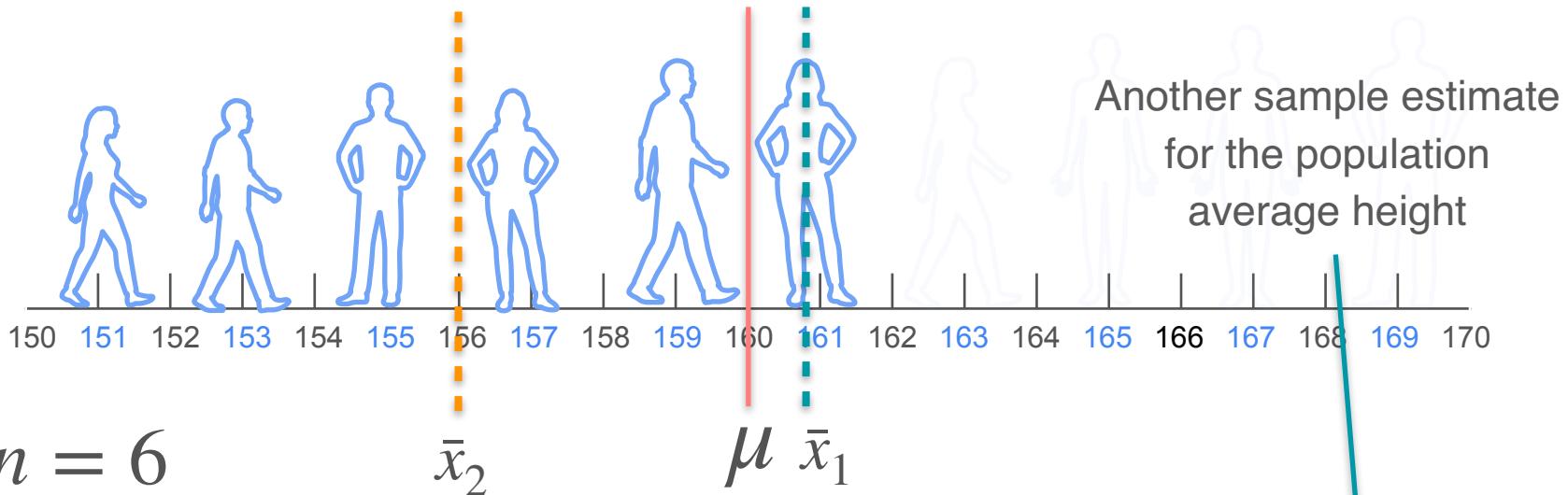
Population and Sample Mean



What is the average height in statistopia?

$$\frac{151 + 153 + 155 + 157 + 159 + 161}{6} = \frac{936}{6} = 156\text{cm}$$

Population and Sample Mean

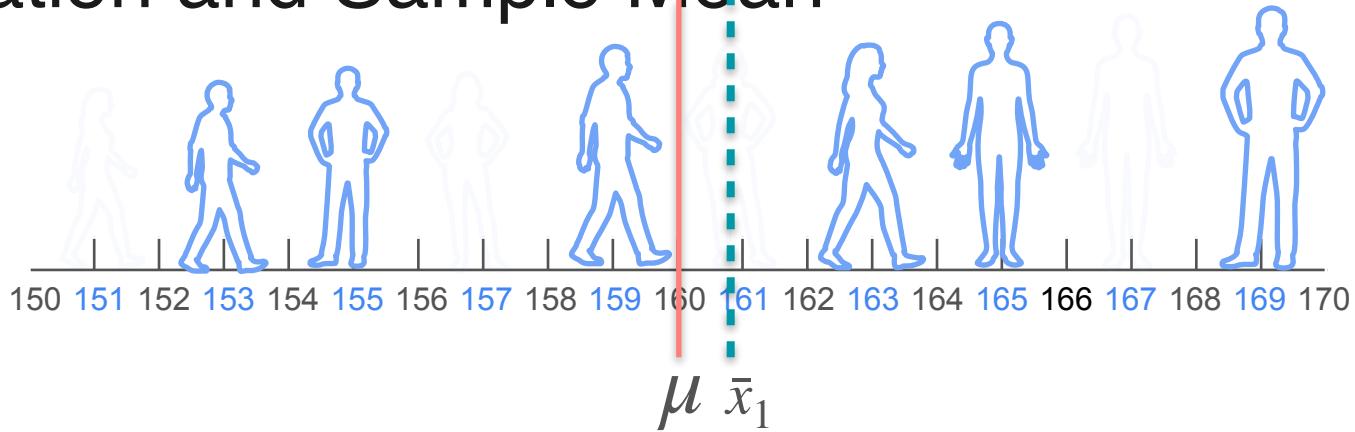


What is the average height in statistopia?

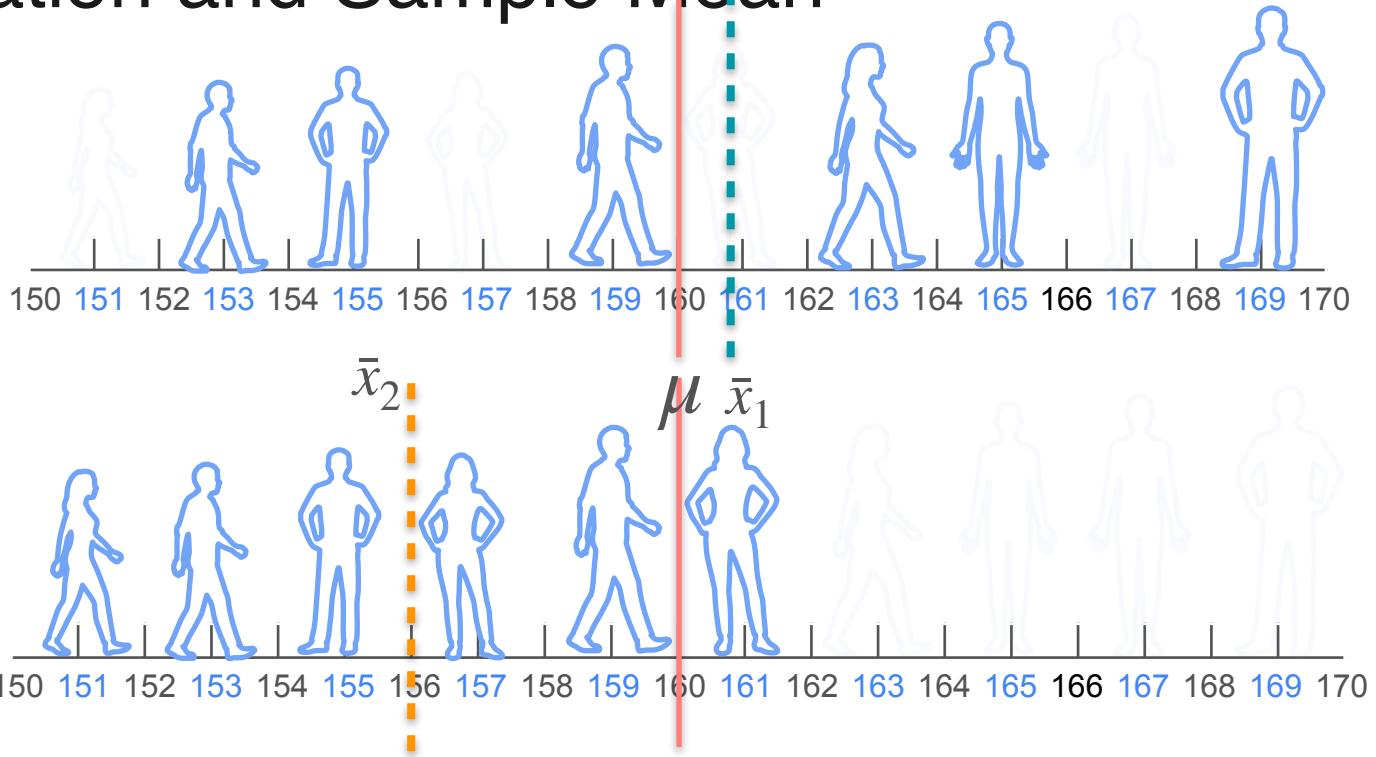
$$\frac{151 + 153 + 155 + 157 + 159 + 161}{6} = \frac{936}{6} = 156\text{cm}$$

Population and Sample Mean

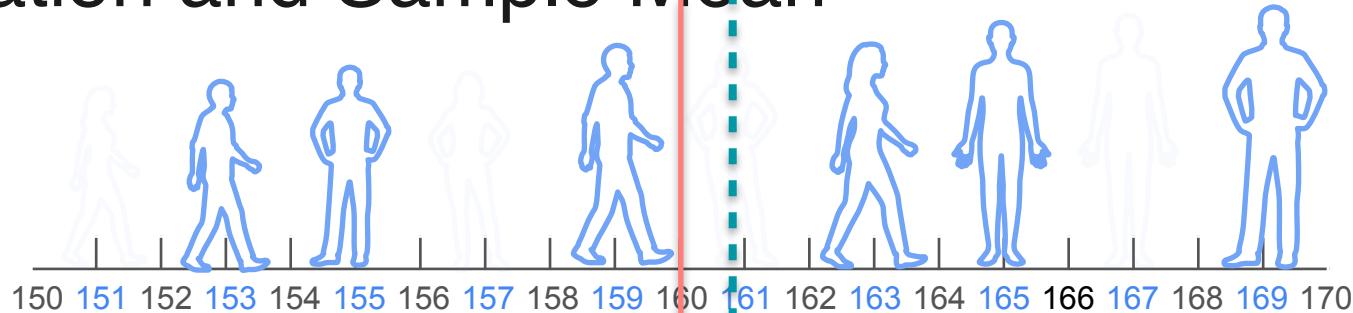
Population and Sample Mean



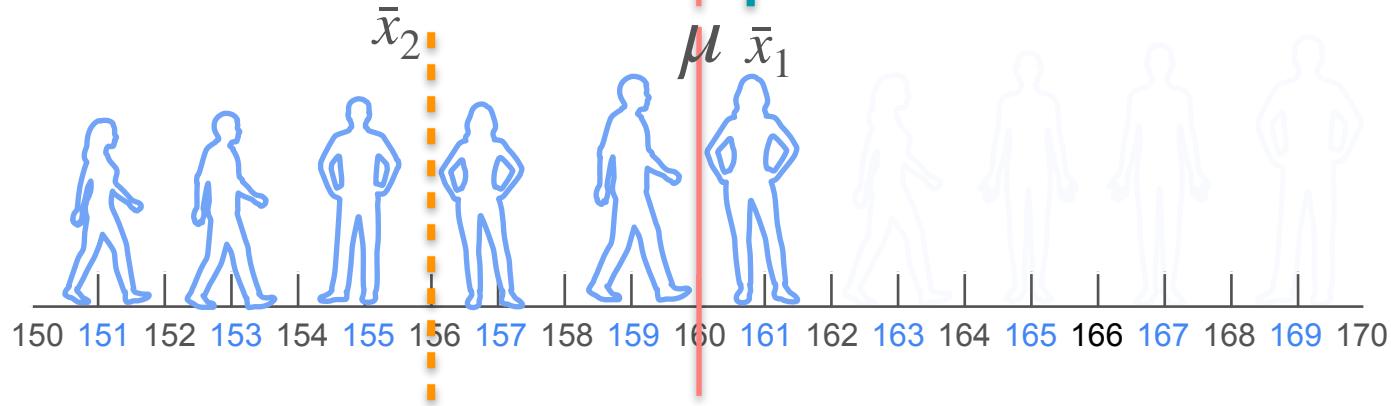
Population and Sample Mean



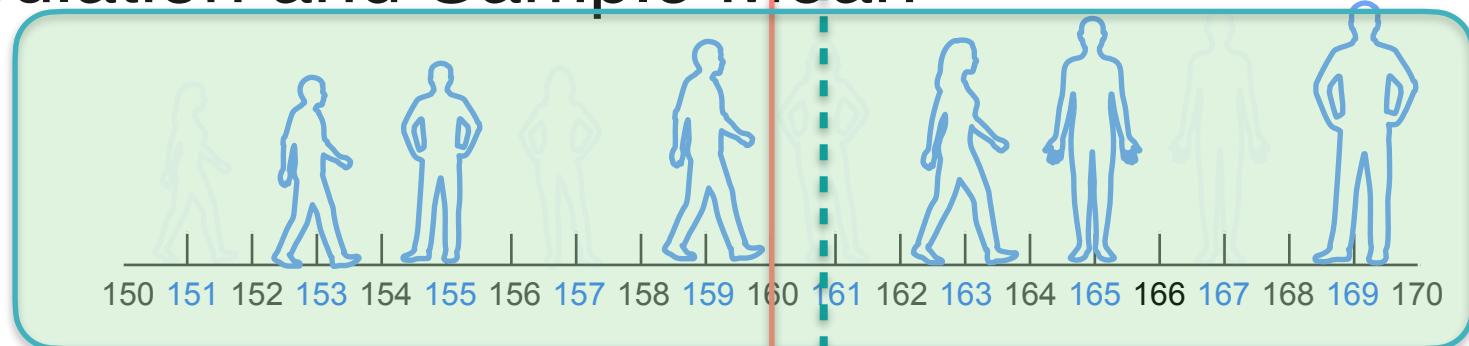
Population and Sample Mean



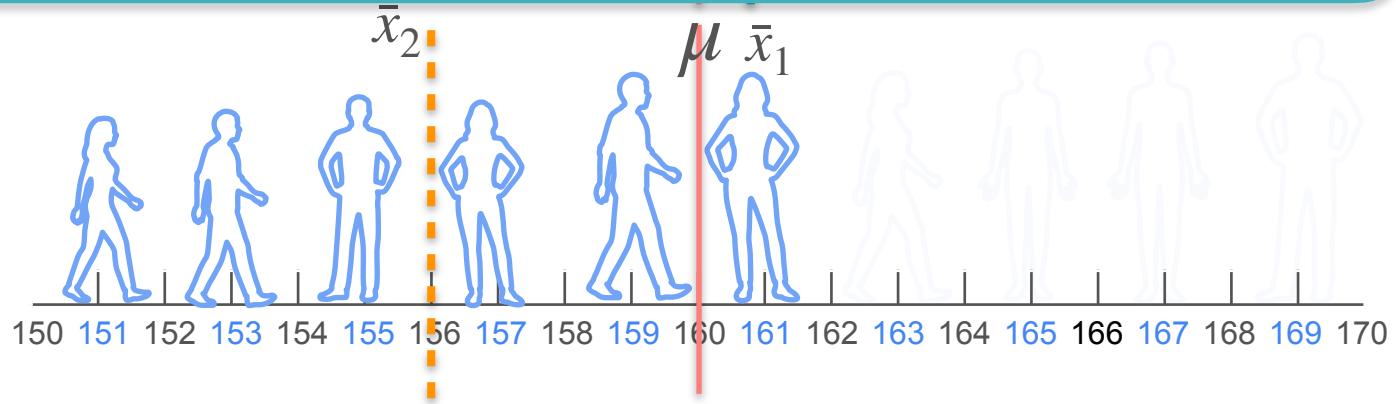
$$n = 6$$



Population and Sample Mean



$$n = 6$$

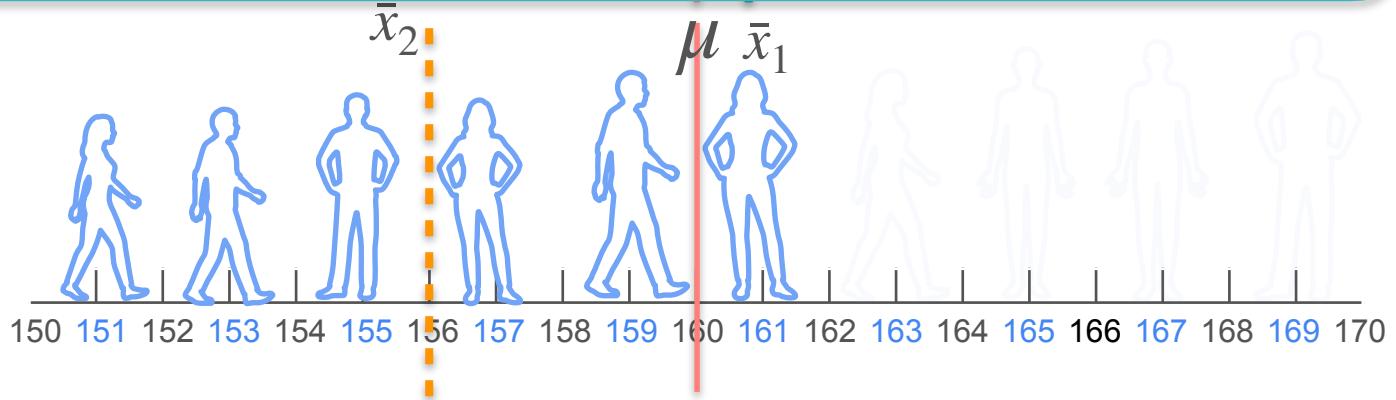


Population and Sample Mean

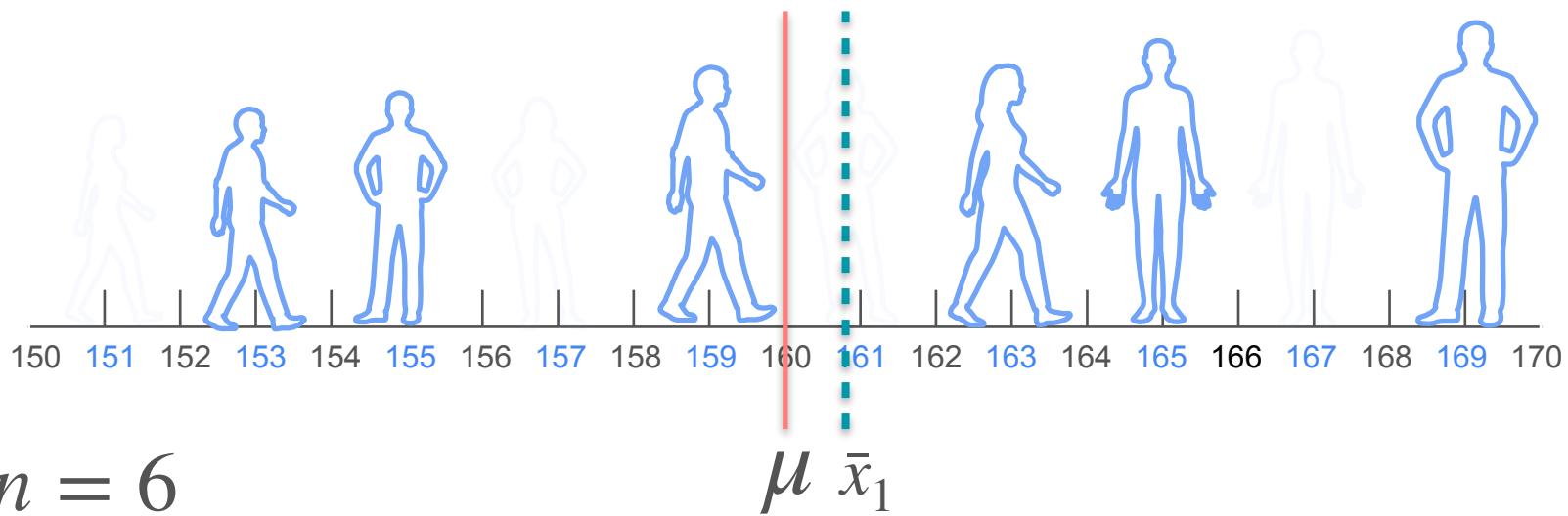
Better estimate of the population mean height

150 151 152 153 154 155 156 157 158 159 160 161 162 163 164 165 166 167 168 169 170

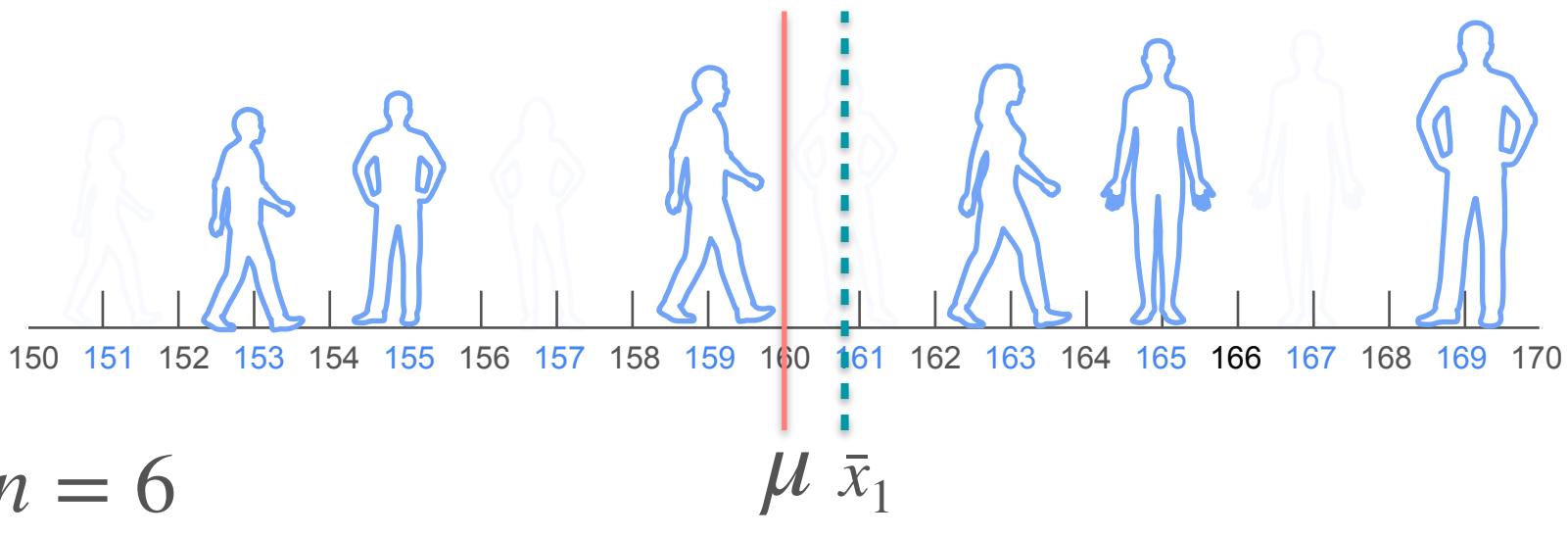
$$n = 6$$



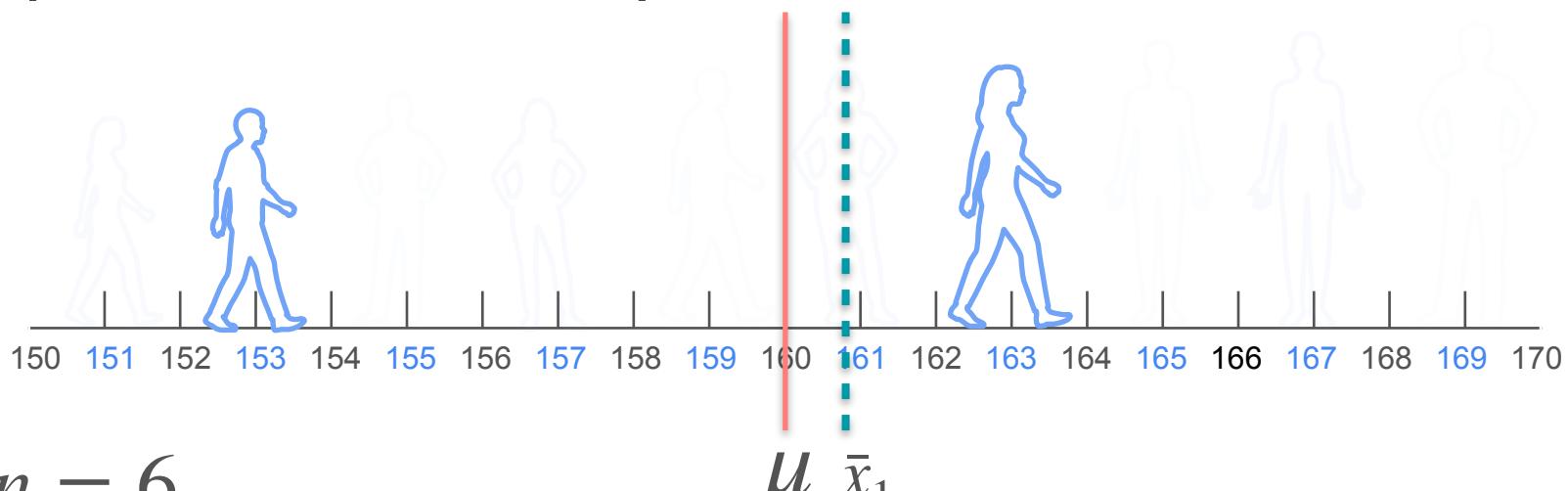
Population and Sample Mean



Population and Sample Mean

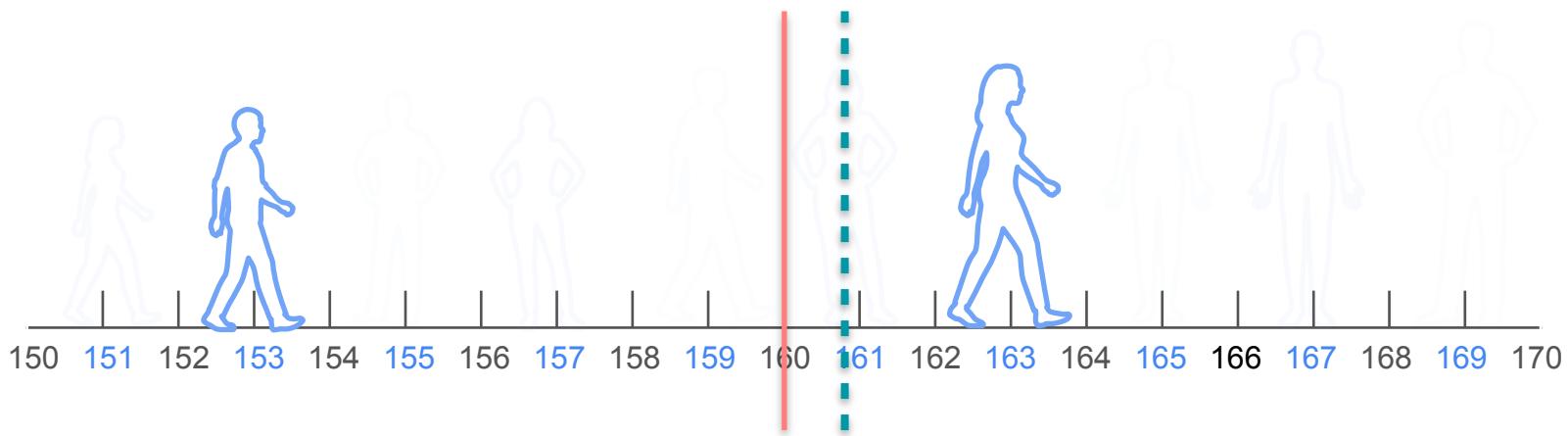


Population and Sample Mean



$$n = 2$$

Population and Sample Mean



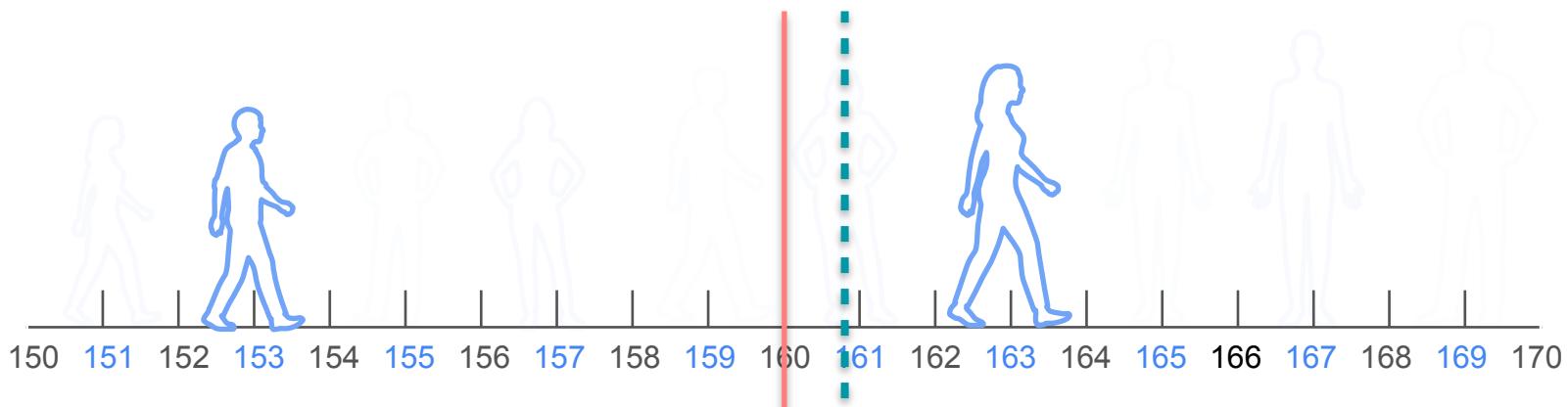
$$n = 6$$

$$n = 2$$

$$\mu \bar{x}_1$$

What is the average height in statistopia?

Population and Sample Mean



$$n = 6$$

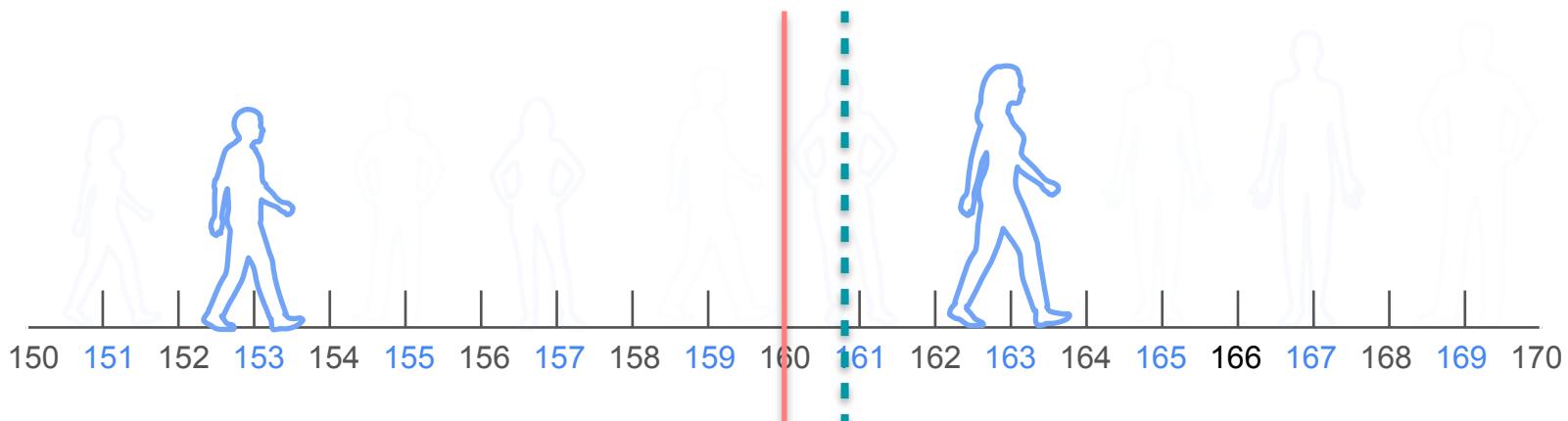
$$n = 2$$

What is the average height in statistopia?

$$\frac{153 + 163}{2}$$

$$\mu \bar{x}_1$$

Population and Sample Mean



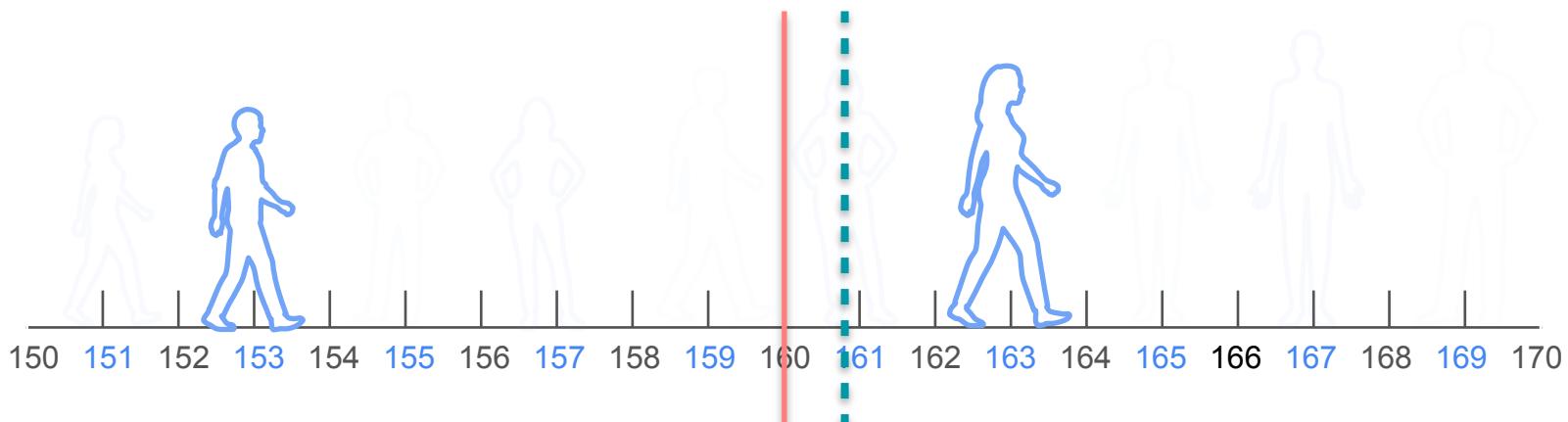
$$n = 6$$

$$n = 2$$

What is the average height in statistopia?

$$\frac{153 + 163}{2} = \frac{316}{2} = 158\text{cm}$$

Population and Sample Mean



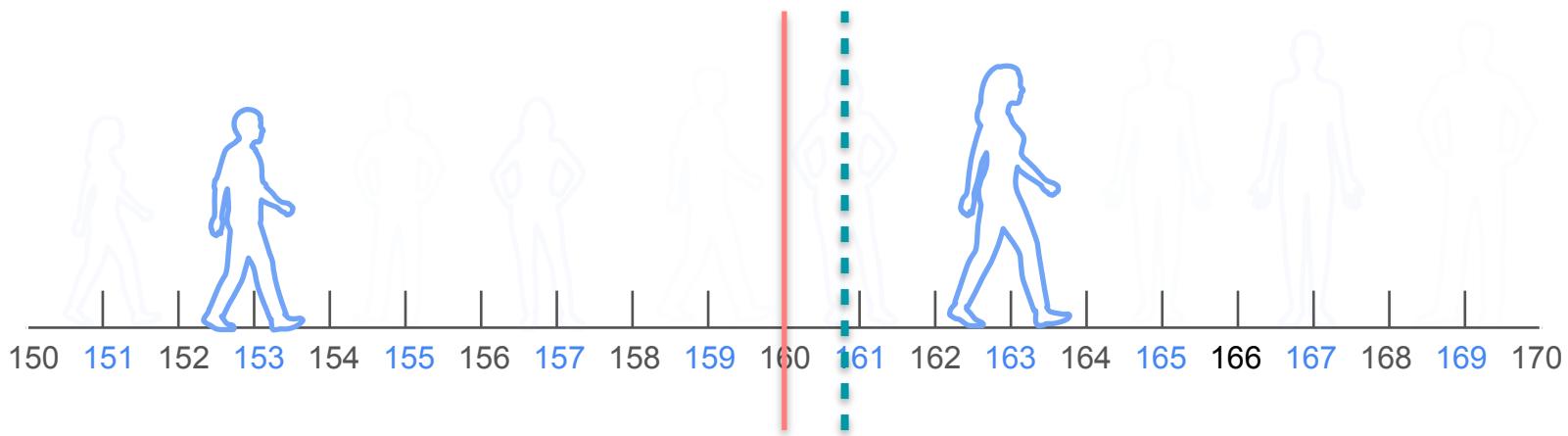
$$n = 6$$

$$n = 2$$

What is the average height in statistopia?

$$\frac{153 + 163}{2} = \frac{316}{2} = 158\text{cm}$$

Population and Sample Mean



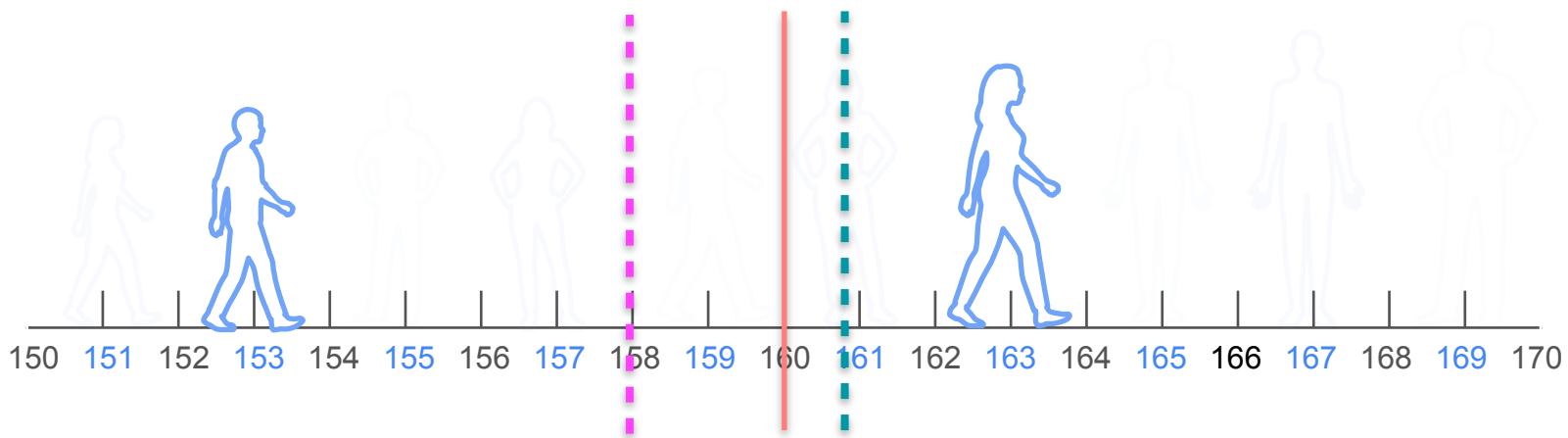
$$n = 6$$

$$n = 2$$

What is the average height in statistopia?

$$\frac{153 + 163}{2} = \frac{316}{2} = 158\text{cm}$$

Population and Sample Mean



$$n = 6$$

$$n = 2$$

What is the average height in statistopia?

$$\frac{153 + 163}{2} = \frac{316}{2} = 158\text{cm}$$

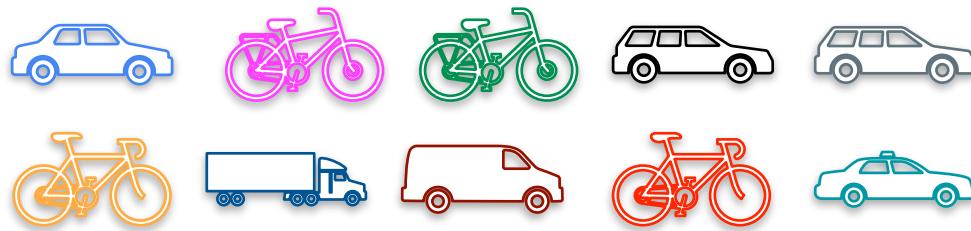
Proportion

Proportion

Population size: 10

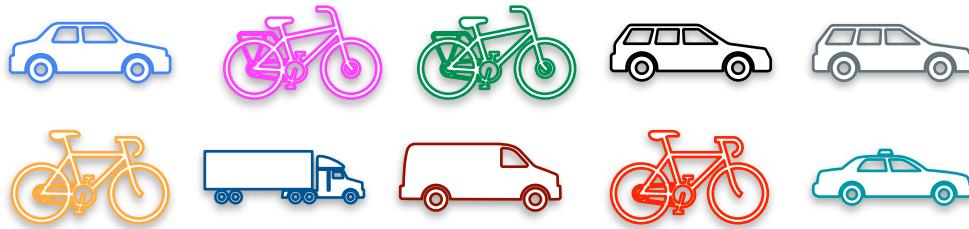
Proportion

Population size: 10



Proportion

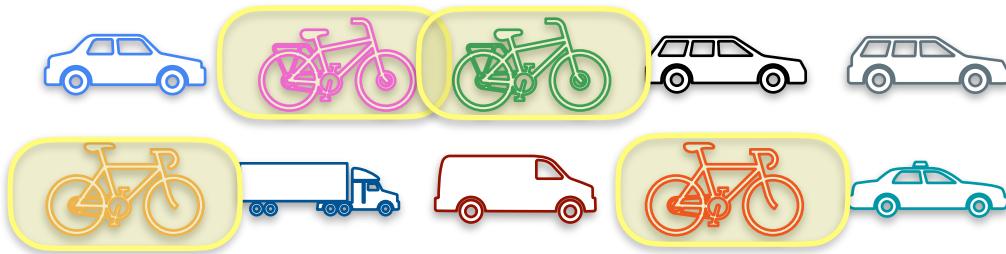
Population size: 10



What proportion of people own a bicycle?

Proportion

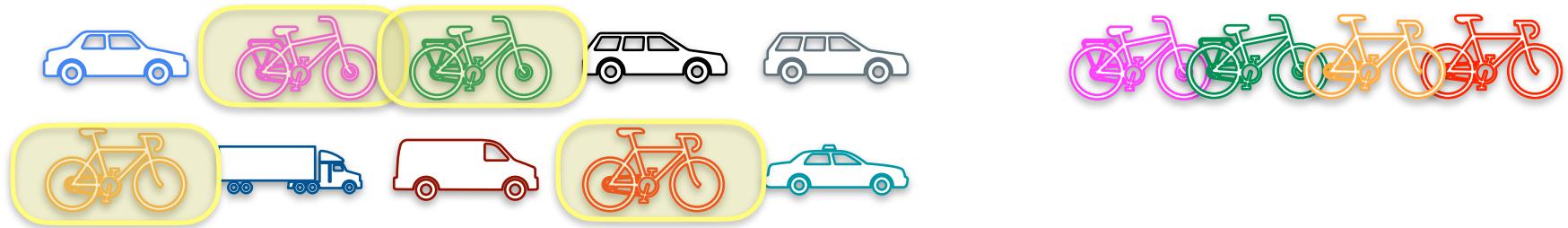
Population size: 10



What proportion of people own a bicycle?

Proportion

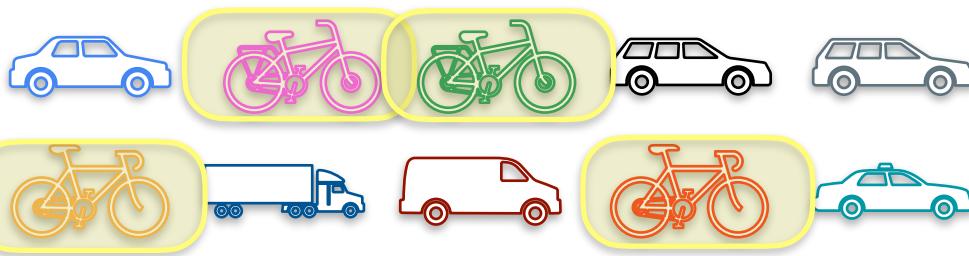
Population size: 10



What proportion of people own a bicycle?

Proportion

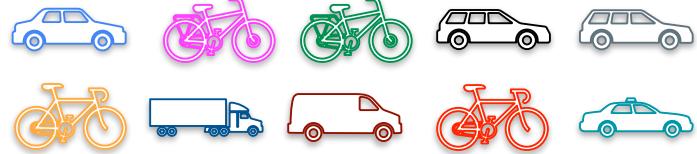
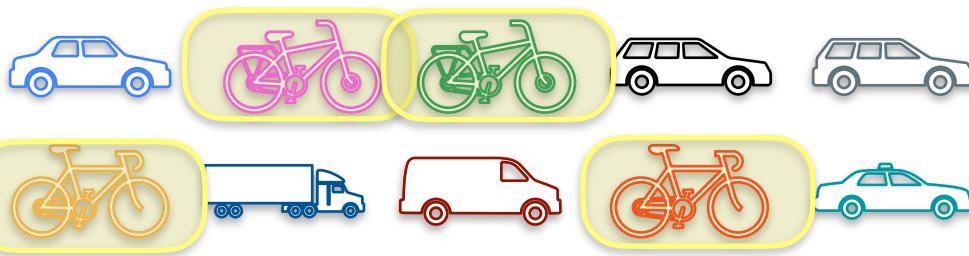
Population size: 10



What proportion of people own a bicycle?

Proportion

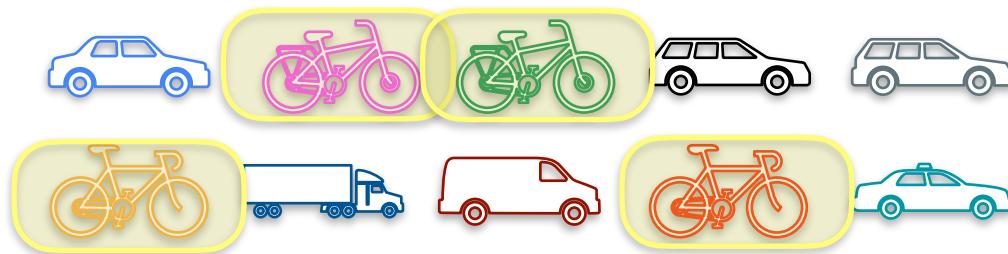
Population size: 10



What proportion of people own a bicycle?

Proportion

Population size: 10

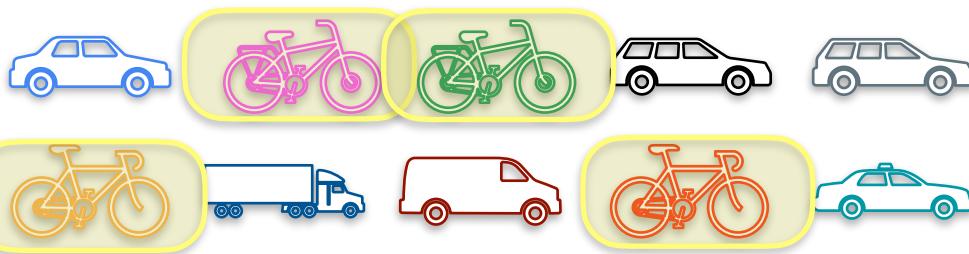


What proportion of people own a bicycle?

$$= \frac{4}{10} = 0.4 = 40\%$$

Proportion

Population size: 10

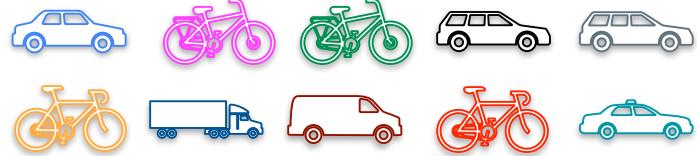
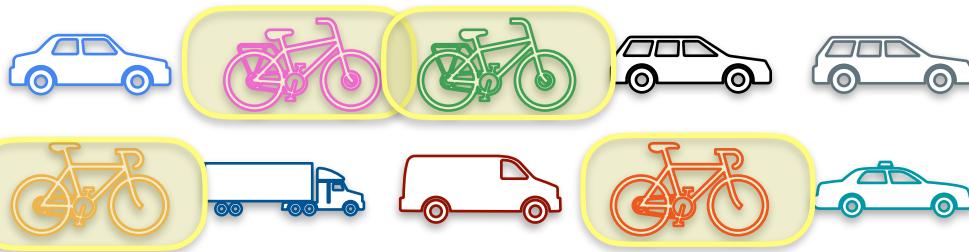


What proportion of people own a bicycle?

$$\text{population proportion} = \frac{4}{10} = 0.4 = 40\%$$

Proportion

Population size: 10



What proportion of people own a bicycle?

p

$$\text{population proportion} = \frac{4}{10} = 0.4 = 40\%$$

Proportion

$$P = \frac{\text{number of items with a given characteristic (}x\text{)}}{\text{population (}n\text{)}}$$

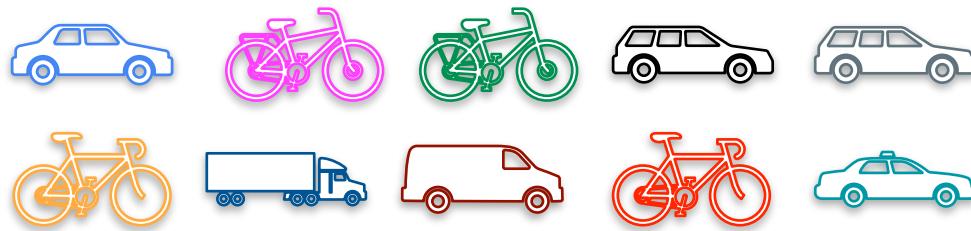
Proportion

population proportion

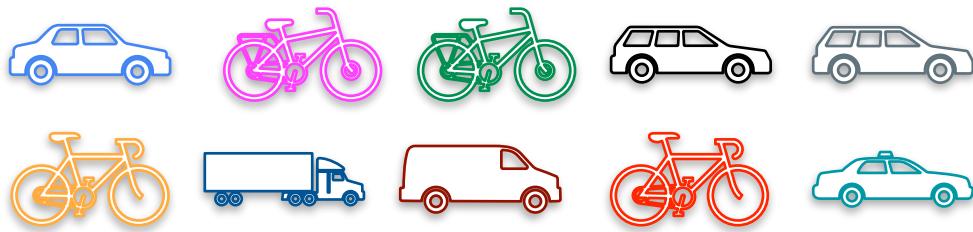
$$P = \frac{\text{number of items with a given characteristic } (x)}{\text{population } (n)}$$

Proportion

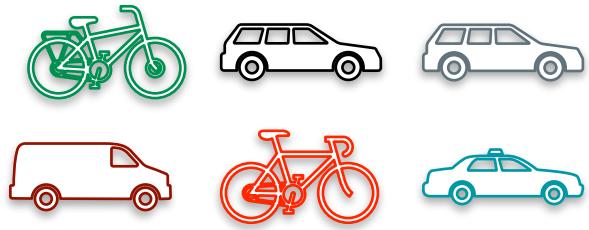
Population size: 10



Proportion

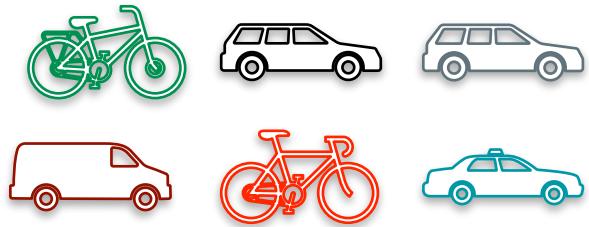


Sample Proportion



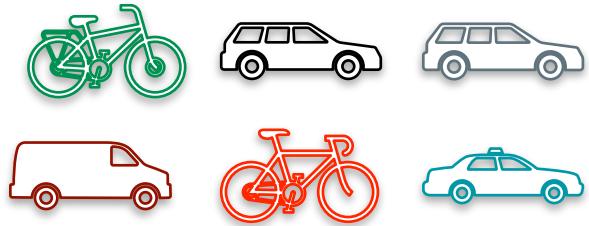
Sample Proportion

Sample size: 6



Sample Proportion

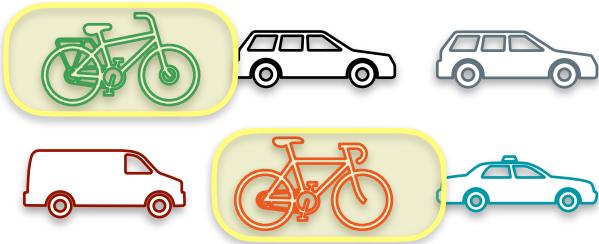
Sample size: 6



What proportion of people own a bicycle?

Sample Proportion

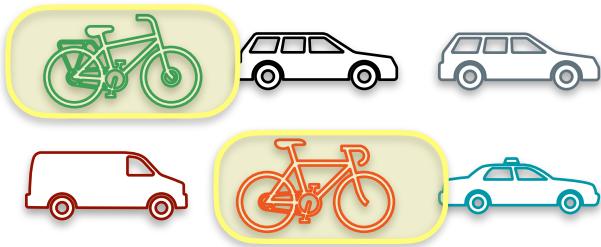
Sample size: 6



What proportion of people own a bicycle?

Sample Proportion

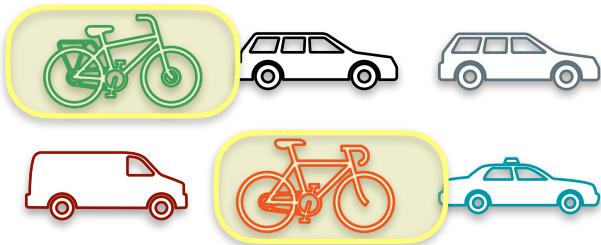
Sample size: 6



What proportion of people own a bicycle?

Sample Proportion

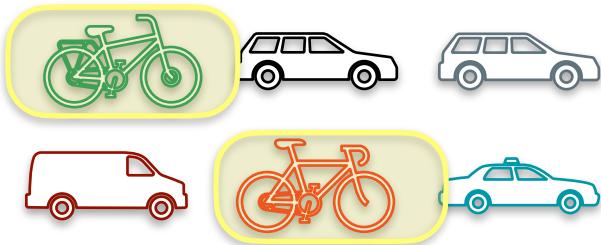
Sample size: 6



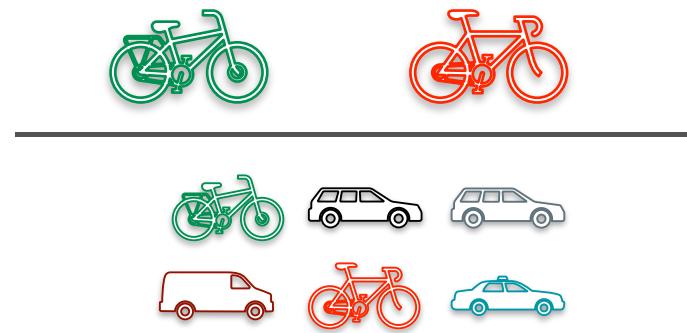
What proportion of people own a bicycle?

Sample Proportion

Sample size: 6

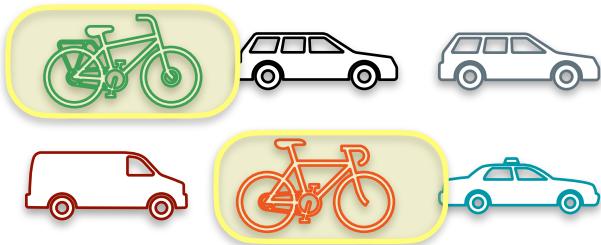


What proportion of people own a bicycle?

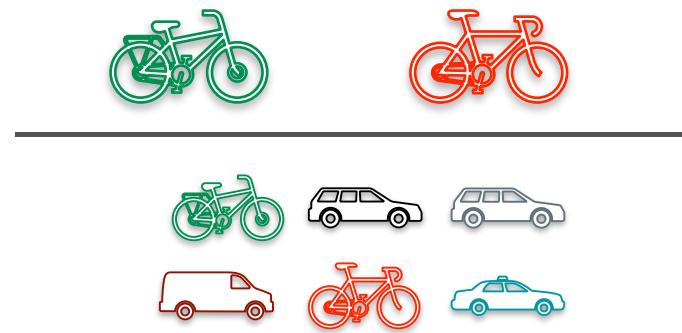


Sample Proportion

Sample size: 6



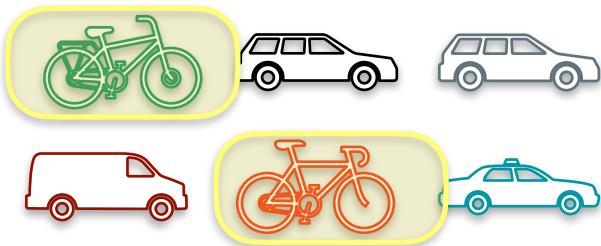
What proportion of people own a bicycle?



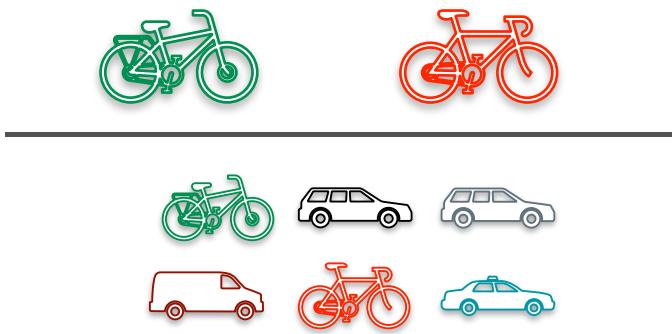
$$= \frac{2}{6} = 0.333 = 33.3\%$$

Sample Proportion

Sample size: 6



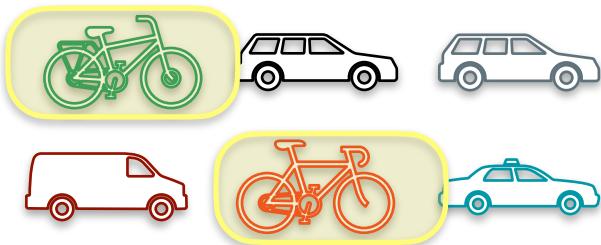
What proportion of people own a bicycle?



$$\text{sample proportion} = \frac{2}{6} = 0.333 = 33.3\%$$

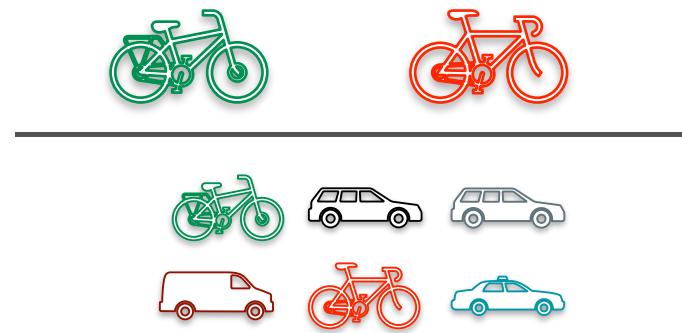
Sample Proportion

Sample size: 6



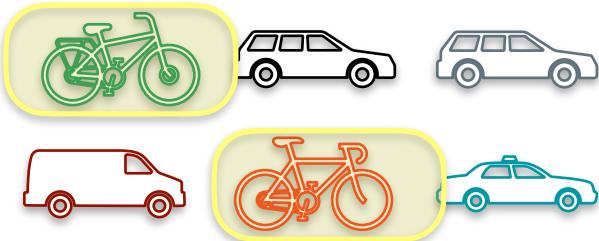
What proportion of people own a bicycle?

$$\hat{p} \text{ sample proportion} = \frac{2}{6} = 0.333 = 33.3\%$$

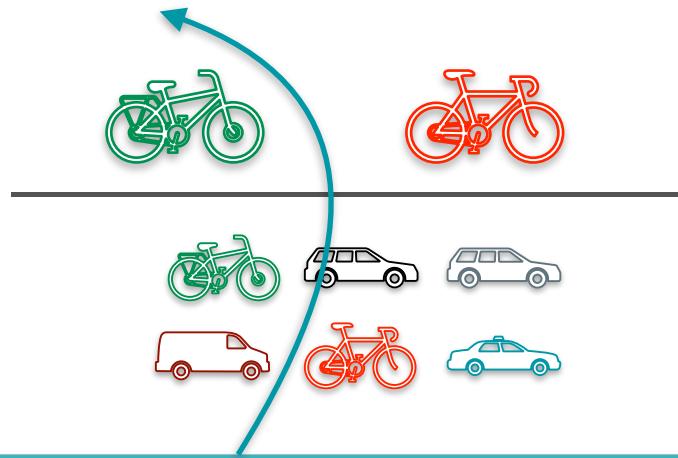


Sample Proportion

Sample size: 6



What proportion of people own a bicycle?

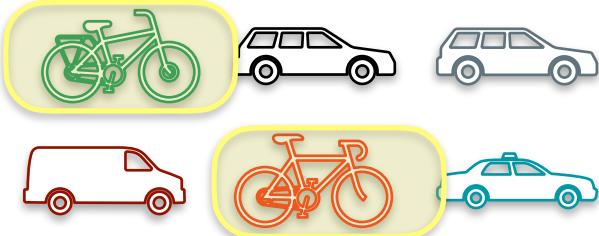


$$\hat{p} \text{ sample proportion} = \frac{2}{6} = 0.333 = 33.3\%$$

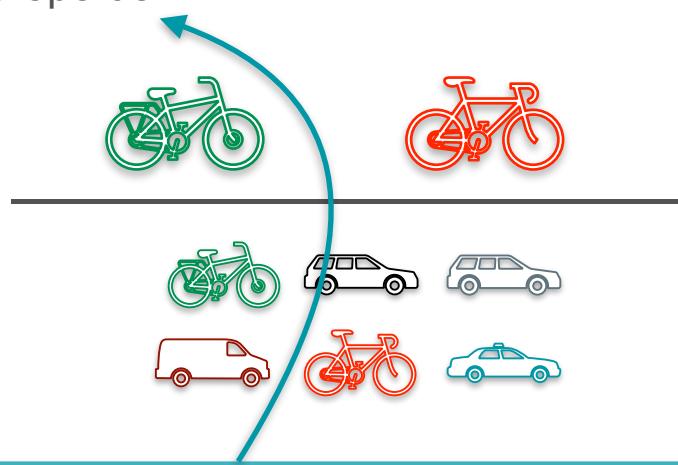
Sample Proportion

Sample size: 6

estimate of the population proportion



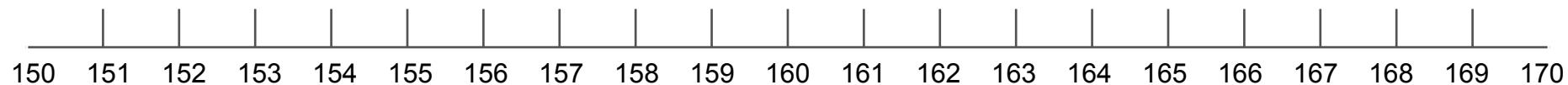
What proportion of people own a bicycle?



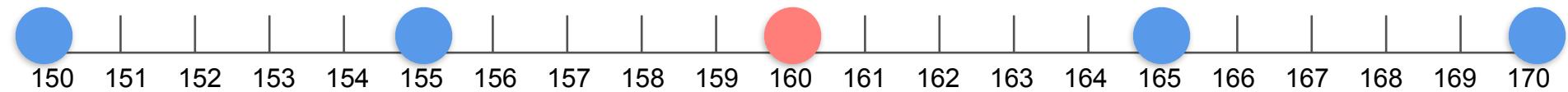
$$\hat{p} \text{ sample proportion } = \frac{2}{6} = 0.333 = 33.3\%$$

Sample Variance

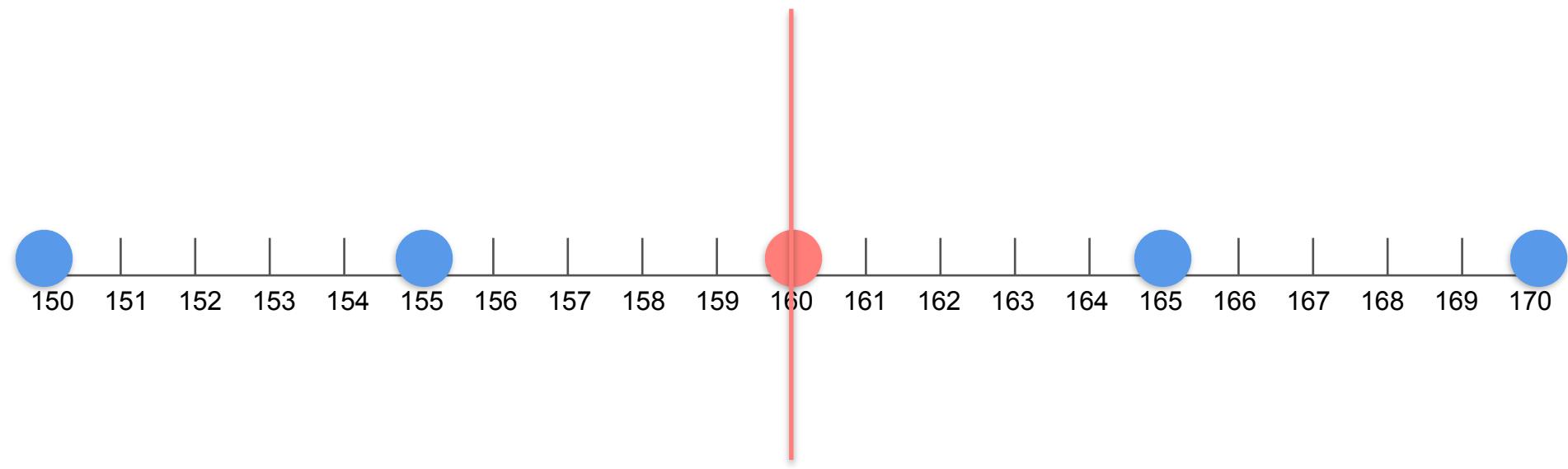
Sample Variance



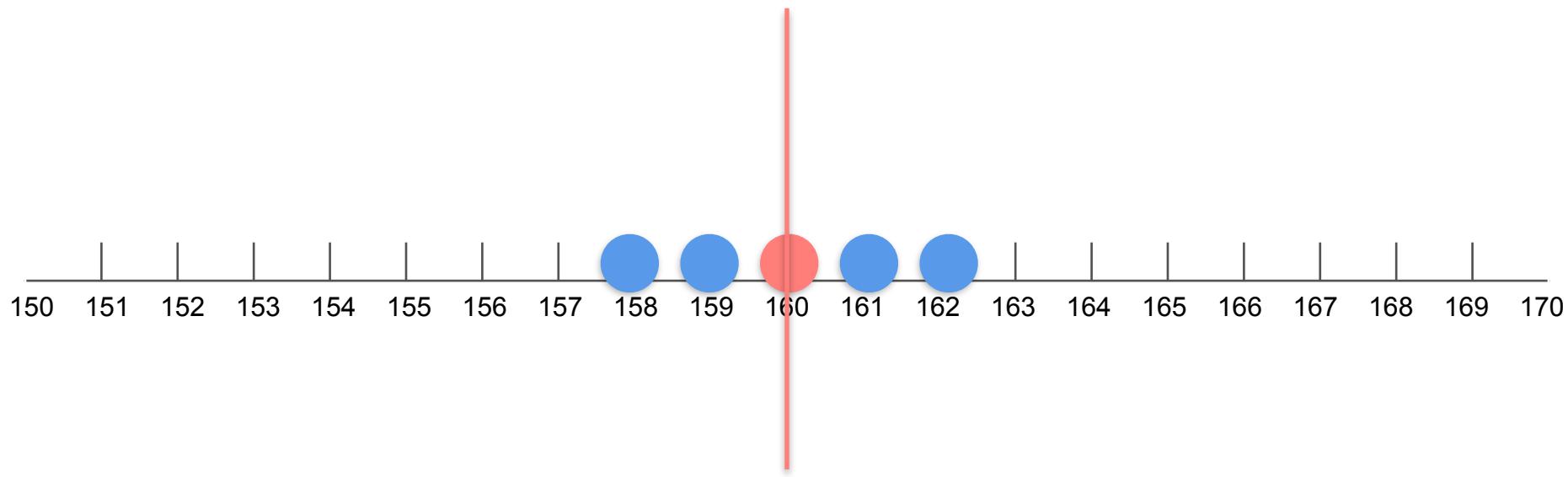
Sample Variance



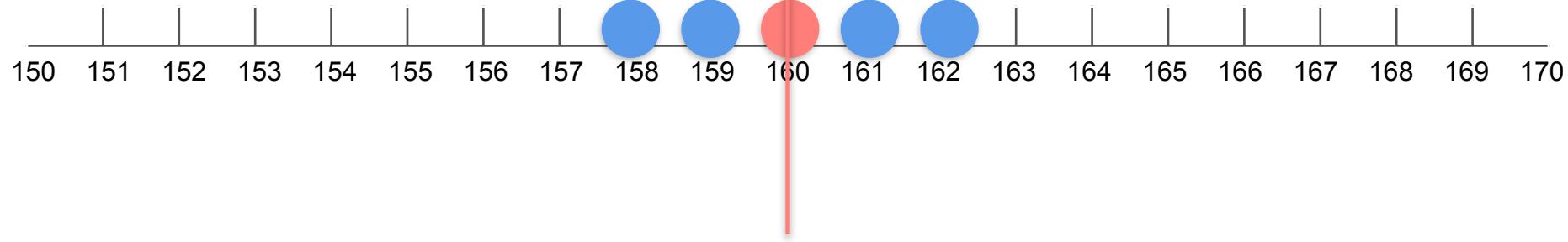
Sample Variance



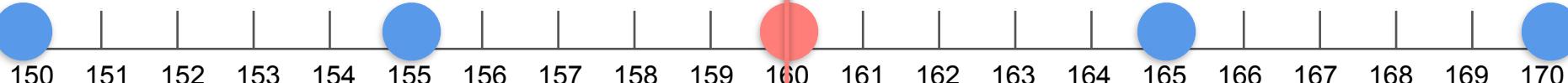
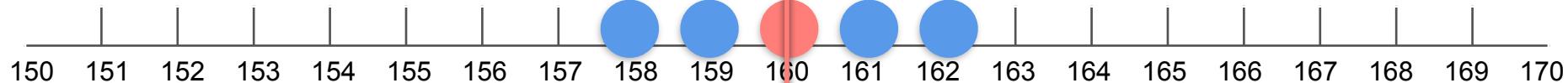
Sample Variance



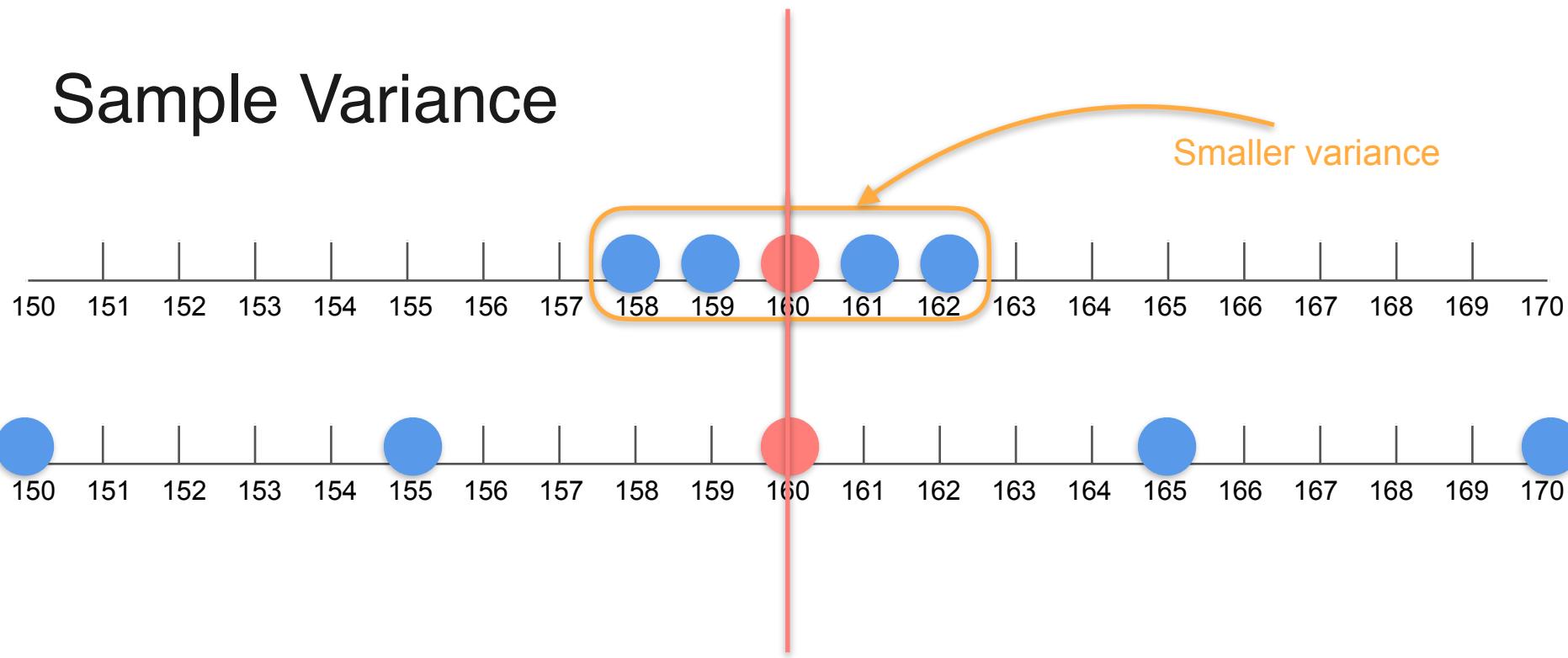
Sample Variance



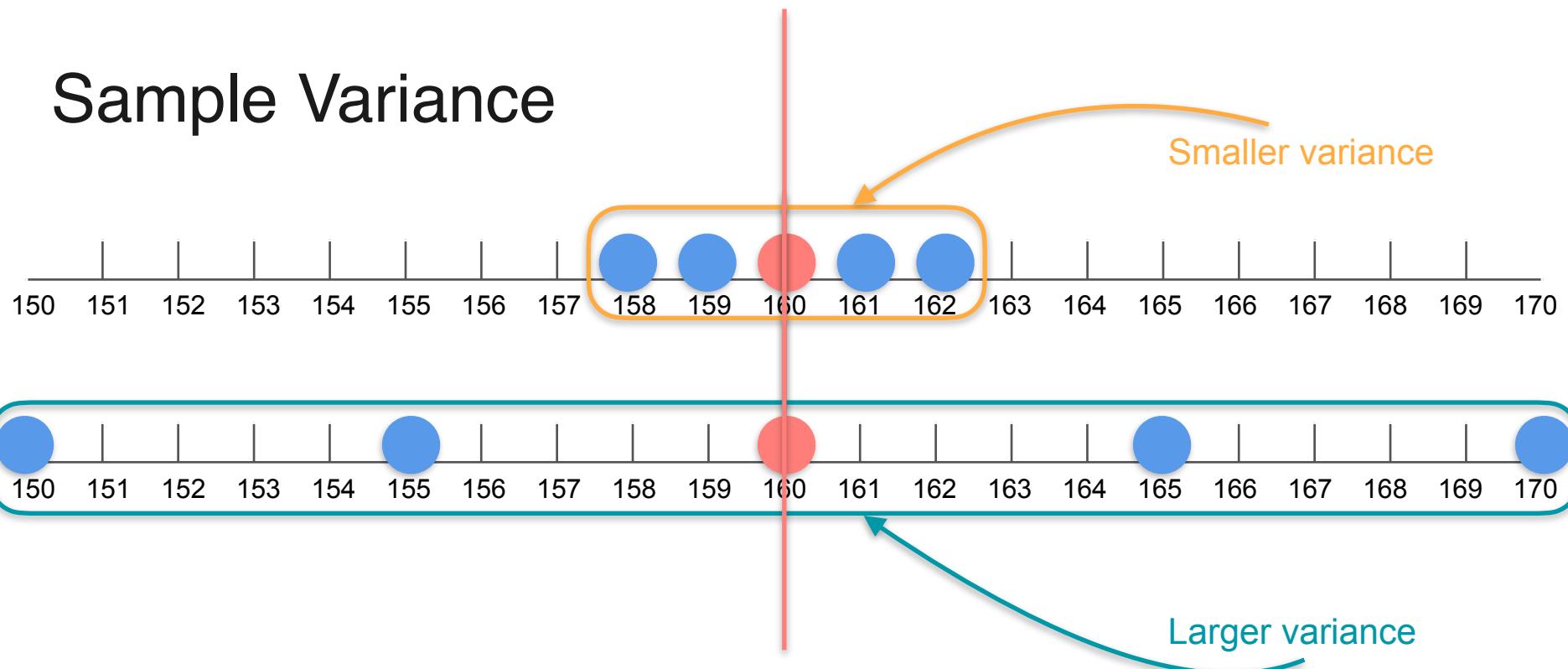
Sample Variance



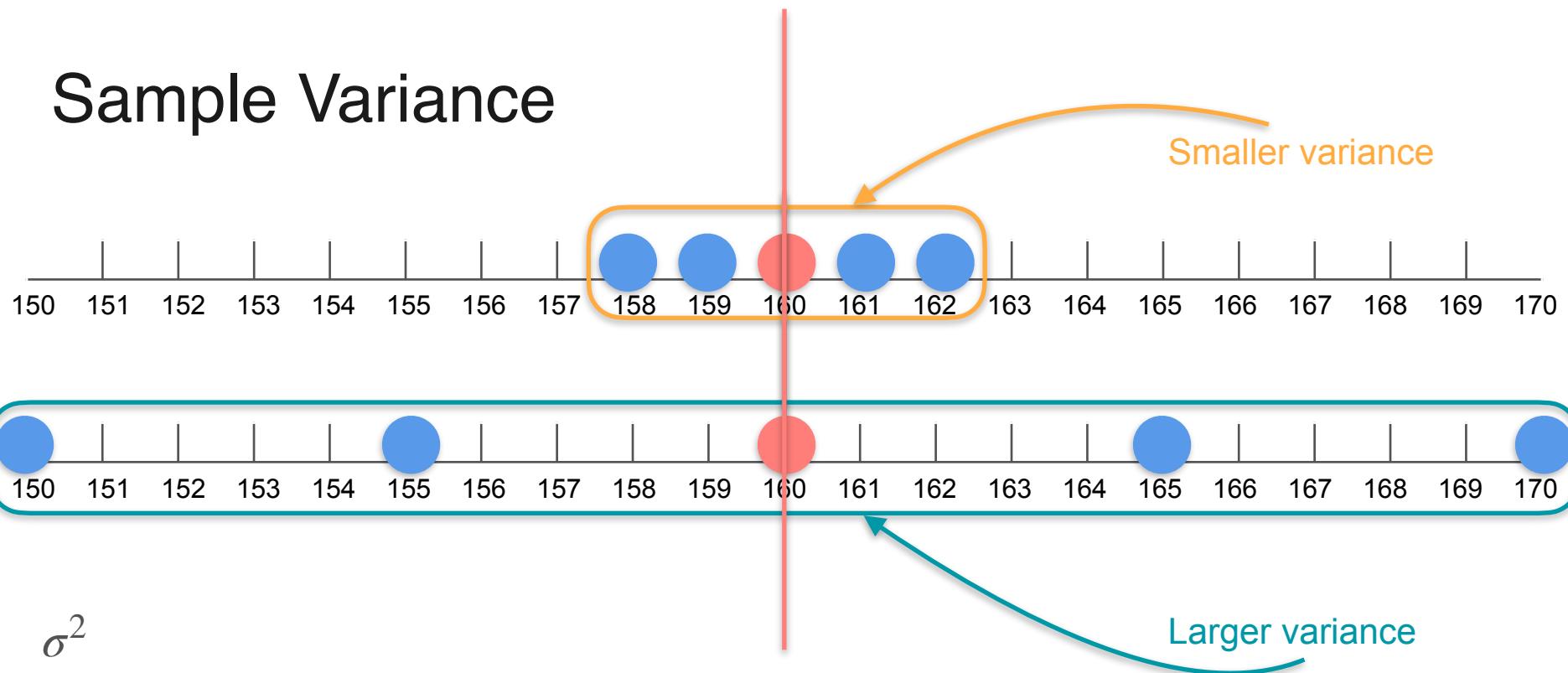
Sample Variance



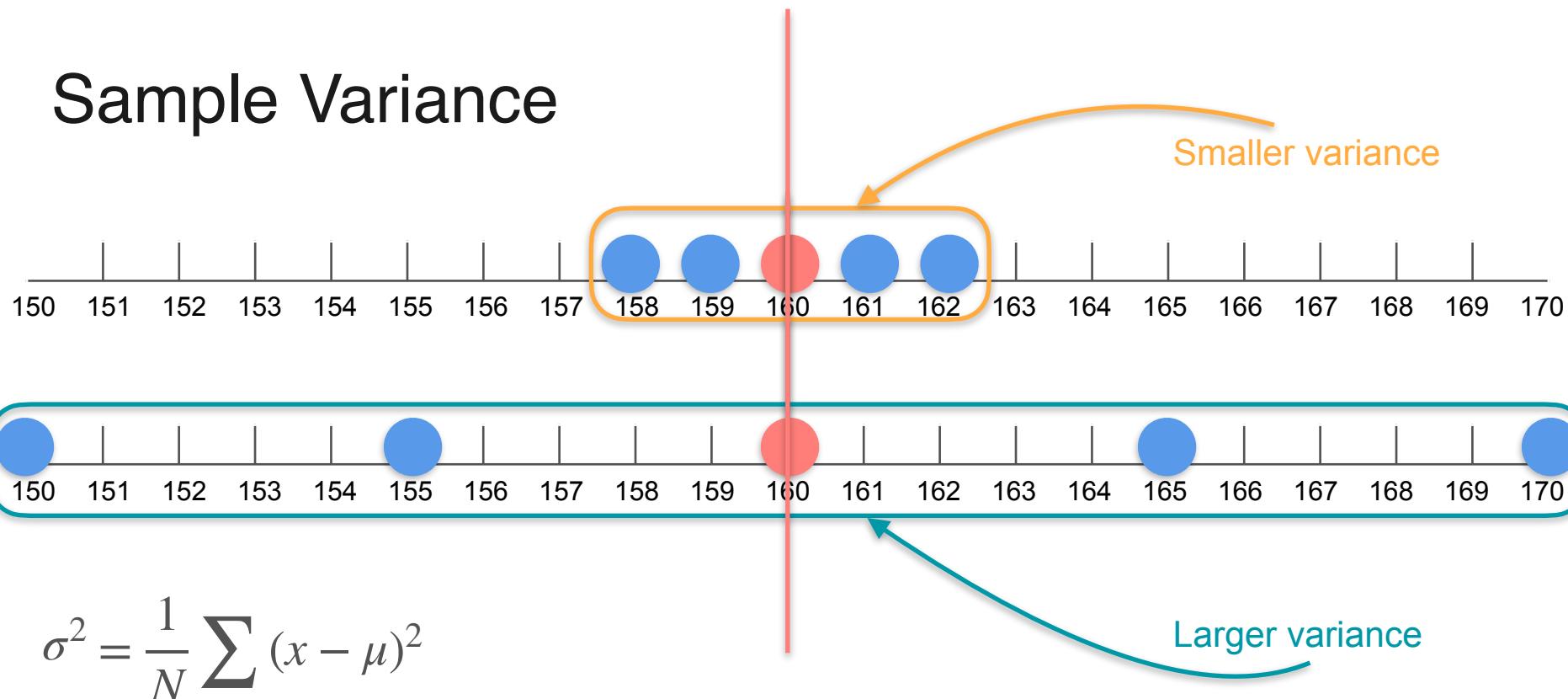
Sample Variance



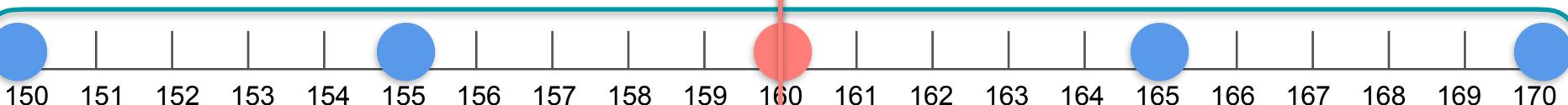
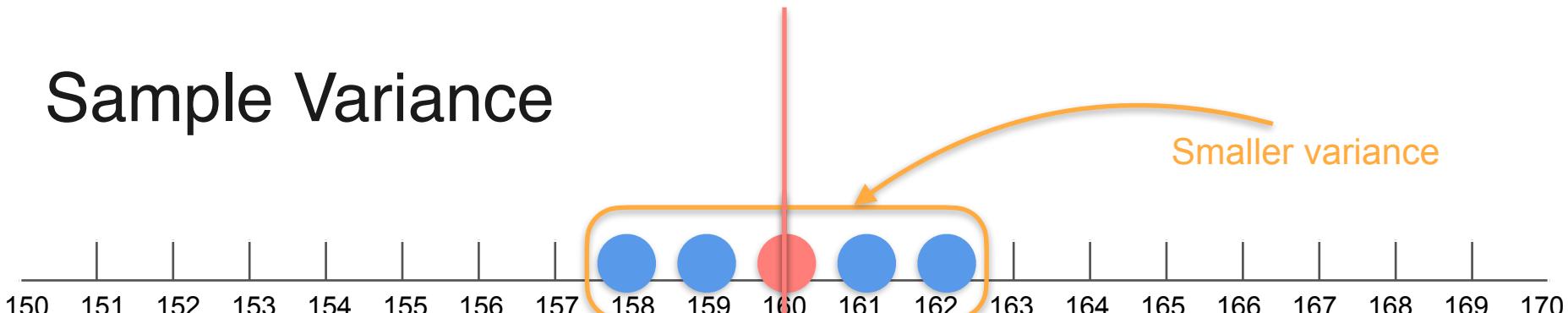
Sample Variance



Sample Variance



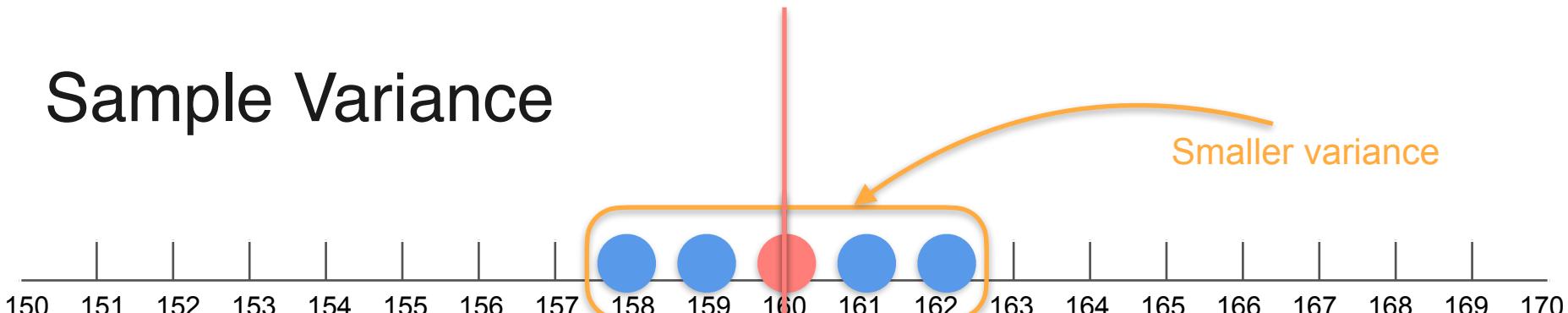
Sample Variance



$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

population size

Sample Variance



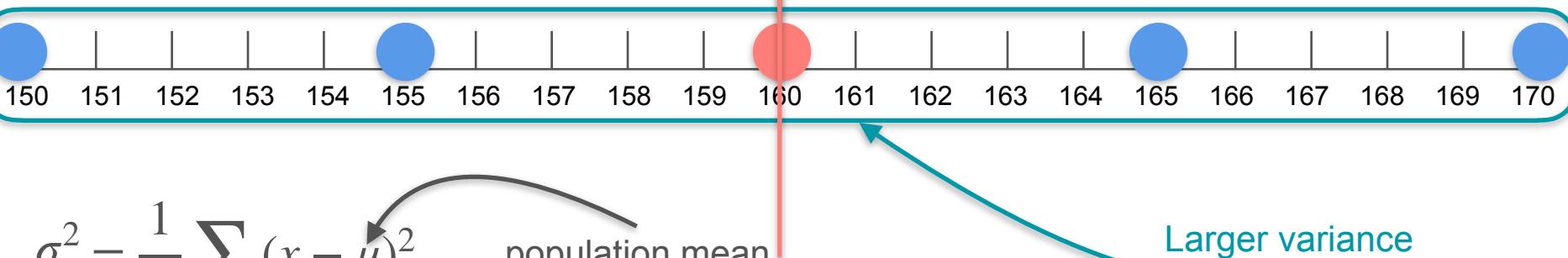
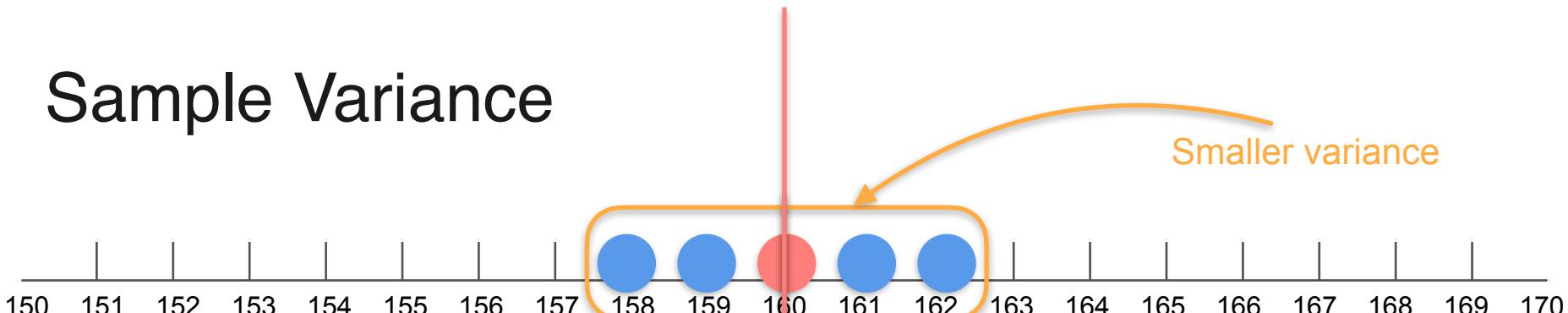
$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

population mean

population size

The equation for sample variance is shown: $\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$. Arrows point from the variables to their definitions: N is labeled "population size" and μ is labeled "population mean".

Sample Variance



$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

population size population mean

How to estimate population variance with the sample?

Variance Estimation

Variance Estimation

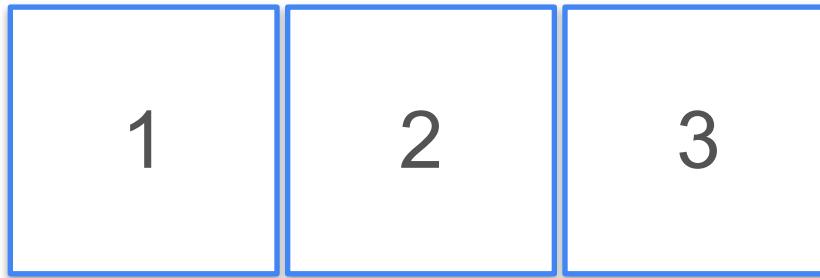


Variance Estimation



μ

Variance Estimation



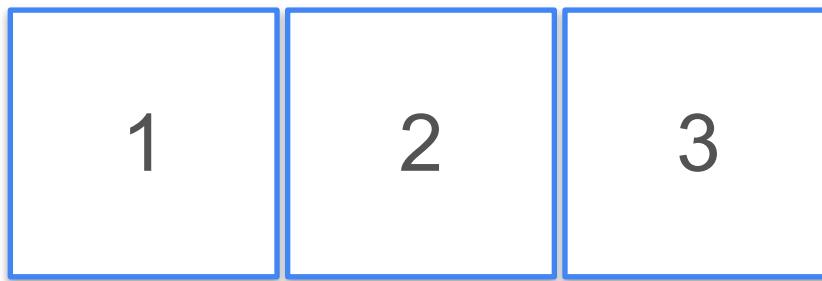
$$\mu = \frac{1 + 2 + 3}{3}$$

Variance Estimation



$$\mu = \frac{1 + 2 + 3}{3} = \frac{6}{3}$$

Variance Estimation



$$\mu = \frac{1 + 2 + 3}{3} = \frac{6}{3} = 2$$

Variance Estimation

1 2 3

$$\mu = 2$$

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

Variance Estimation

1	2	3
---	---	---

$$\mu = 2$$

$$x \quad x - \mu \quad (x - \mu)^2$$

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

Variance Estimation

1	2	3
---	---	---

$$\mu = 2$$

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

x
1
2
3

$$x - \mu \quad (x - \mu)^2$$

Variance Estimation

1	2	3
---	---	---

$$\mu = 2$$

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

x
1
2
3

$$x - 2$$

$$x - \mu$$

$$(x - \mu)^2$$

Variance Estimation

1	2	3
---	---	---

$$\mu = 2$$

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

x	$x - \mu$	$(x - \mu)^2$
1	-1	1
2	0	0
3	1	1

Variance Estimation

1	2	3
---	---	---

$$\mu = 2$$

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

x	$x - \mu$	$(x - \mu)^2$
1	-1	1
2	0	0
3	1	1

Variance Estimation

1	2	3
---	---	---

$$\mu = 2$$

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

x	$x - \mu$	$(x - \mu)^2$
1	-1	1
2	0	0
3	1	1

$$\sum (x - \mu)^2$$

Variance Estimation

1	2	3
---	---	---

$$\mu = 2$$

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

x	$x - \mu$	$(x - \mu)^2$
1	-1	1
2	0	0
3	1	1

$$\sum (x - \mu)^2 = 2$$

Variance Estimation

1	2	3
---	---	---

$$\mu = 2$$

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

x	$x - \mu$	$(x - \mu)^2$
1	-1	1
2	0	0
3	1	1

$$\frac{\sum (x - \mu)^2}{N} = 2$$

Variance Estimation

1	2	3
---	---	---

$$\mu = 2$$

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

x	$x - \mu$	$(x - \mu)^2$
1	-1	1
2	0	0
3	1	1

$$\frac{\sum (x - \mu)^2}{N} = \frac{2}{3}$$

Variance Estimation

1	2	3
---	---	---

$$\mu = 2$$

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

x	$x - \mu$	$(x - \mu)^2$
1	-1	1
2	0	0
3	1	1

$$\frac{\sum (x - \mu)^2}{N} = \frac{2}{3}$$

σ^2
Population variance

Variance Estimation

1	2	3
---	---	---

$$\mu = 2$$

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

$$\sigma^2 = \frac{2}{3}$$

Variance Estimation

1 2 3

$$n = 2$$

Samples

$$\mu = 2$$

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

$$\sigma^2 = \frac{2}{3}$$

Variance Estimation

1	2	3
---	---	---

$$\mu = 2$$

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

$$\sigma^2 = \frac{2}{3}$$

$n = 2$
Samples

1	1
1	2
1	3
2	1
2	2
2	3
3	1
3	2
3	3

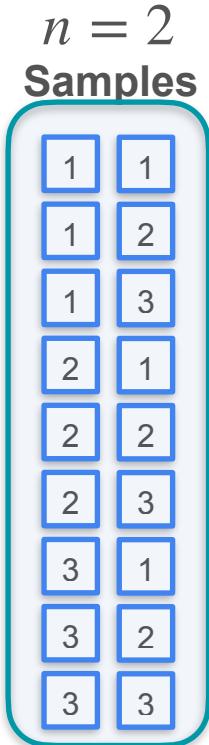
Variance Estimation

1 2 3

$$\mu = 2$$

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

$$\sigma^2 = \frac{2}{3}$$



Variance Estimation

1 2 3

$$\mu = 2$$

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

$$\sigma^2 = \frac{2}{3}$$

$n = 2$	Samples
1	1
1	2
1	3
2	1
2	2
2	3
3	1
3	2
3	3

Variance Estimation

1 2 3

$$\mu = 2$$

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

$$\sigma^2 = \frac{2}{3}$$

$$n = 2$$

Samples

1	1
1	2
1	3
2	1
2	2
2	3
3	1
3	2
3	3

Variance Estimation

1	2	3
---	---	---

$$\mu = 2$$

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

$$\sigma^2 = \frac{2}{3}$$

$n = 2$
Samples

1	1
1	2
1	3
2	1
2	2
2	3
3	1
3	2
3	3

Variance Estimation

1	2	3
---	---	---

$$\mu = 2$$

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

$$\sigma^2 = \frac{2}{3}$$

$n = 2$
Samples

1	1
1	2
1	3
2	1
2	2
2	3
3	1
3	2
3	3

Variance Estimation

1	2	3
---	---	---

$$\mu = 2$$

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

$$\sigma^2 = \frac{2}{3}$$

$n = 2$	Samples	\bar{x}
	1 1	
	1 2	
	1 3	
	2 1	
	2 2	
	2 3	
	3 1	
	3 2	
	3 3	

Variance Estimation

1	2	3
---	---	---

$$\mu = 2$$

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

$$\sigma^2 = \frac{2}{3}$$

$n = 2$ Samples	\bar{x}
1 1	1
1 2	1,5
1 3	2
2 1	2,5
2 2	2
2 3	2,5
3 1	2
3 2	2,5
3 3	3

Variance Estimation

1	2	3
---	---	---

$$\mu = 2$$

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

$$\sigma^2 = \frac{2}{3}$$

$n = 2$ Samples	\bar{x}
1 1	1
1 2	1,5
1 3	2
2 1	1,5
2 2	2
2 3	2,5
3 1	2
3 2	2,5
3 3	3

Variance Estimation

1 2 3

$$\mu = 2$$

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

$$\sigma^2 = \frac{2}{3}$$

$n = 2$ Samples	\bar{x}
1 1	1
1 2	1,5
1 3	2
2 1	2
2 2	2,5
2 3	2
3 1	2,5
3 2	3
3 3	3

Variance Estimation

1	2	3
---	---	---

$$\mu = 2$$

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

$$\sigma^2 = \frac{2}{3}$$

$n = 2$
Samples

1	1
1	2
1	3
2	1
2	2
2	3
3	1
3	2
3	3

$$\bar{x}$$

$$Var(x) = \frac{\sum (x - \bar{x})^2}{n}$$

1
1,5
2
1,5
2
2,5
2
2,5
3

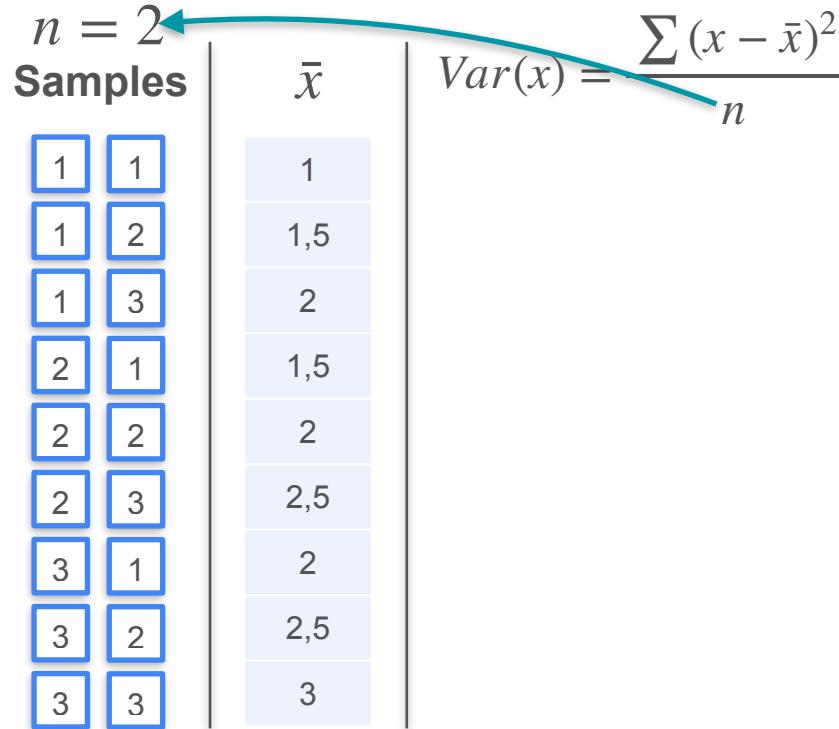
Variance Estimation

1	2	3
---	---	---

$$\mu = 2$$

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

$$\sigma^2 = \frac{2}{3}$$



Variance Estimation

1	2	3
---	---	---

$$\mu = 2$$

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

$$\sigma^2 = \frac{2}{3}$$

$n = 2$ Samples	\bar{x}	$Var(x) = \frac{\sum (x - \bar{x})^2}{n}$
1 1	1	0
1 2	1,5	0,25
1 3	2	1
2 1	1,5	0,25
2 2	2	0
2 3	2,5	0,25
3 1	2	1
3 2	2,5	0,25
3 3	3	0

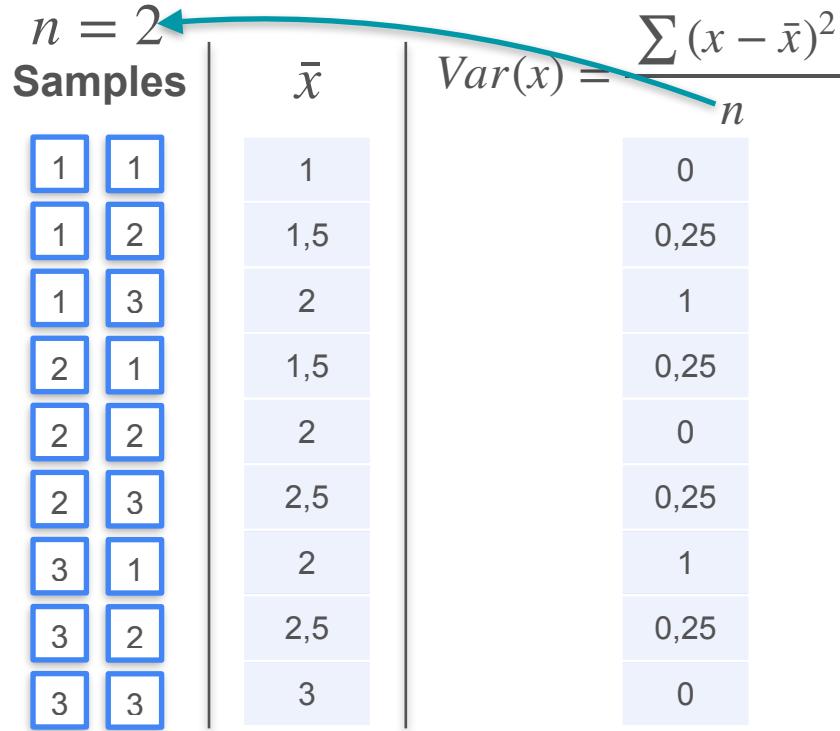
Variance Estimation

1	2	3
---	---	---

$$\mu = 2$$

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

$$\sigma^2 = \frac{2}{3}$$



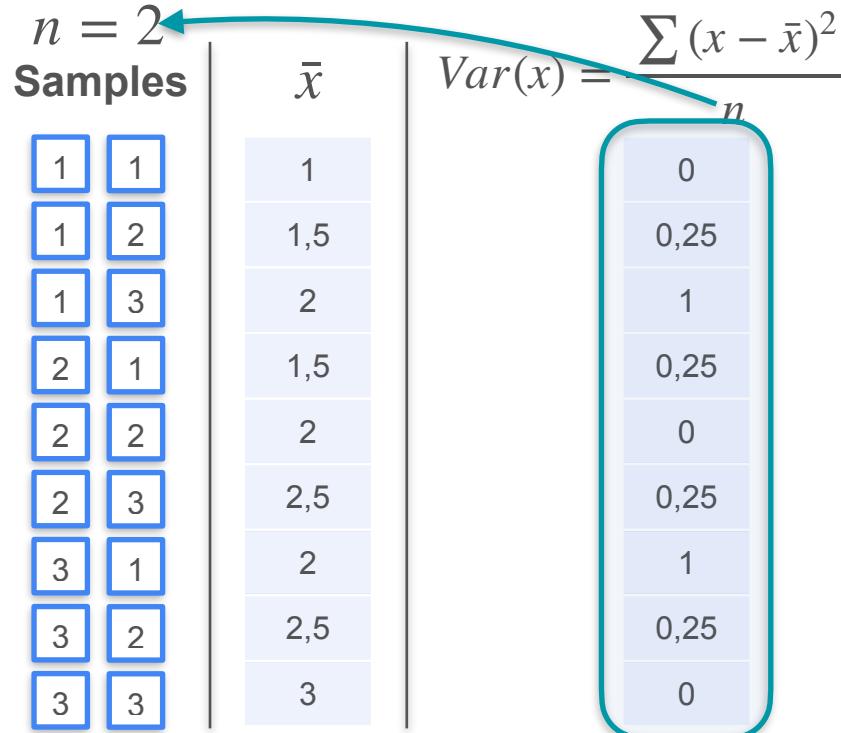
Variance Estimation

1 2 3

$$\mu = 2$$

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

$$\sigma^2 = \frac{2}{3}$$



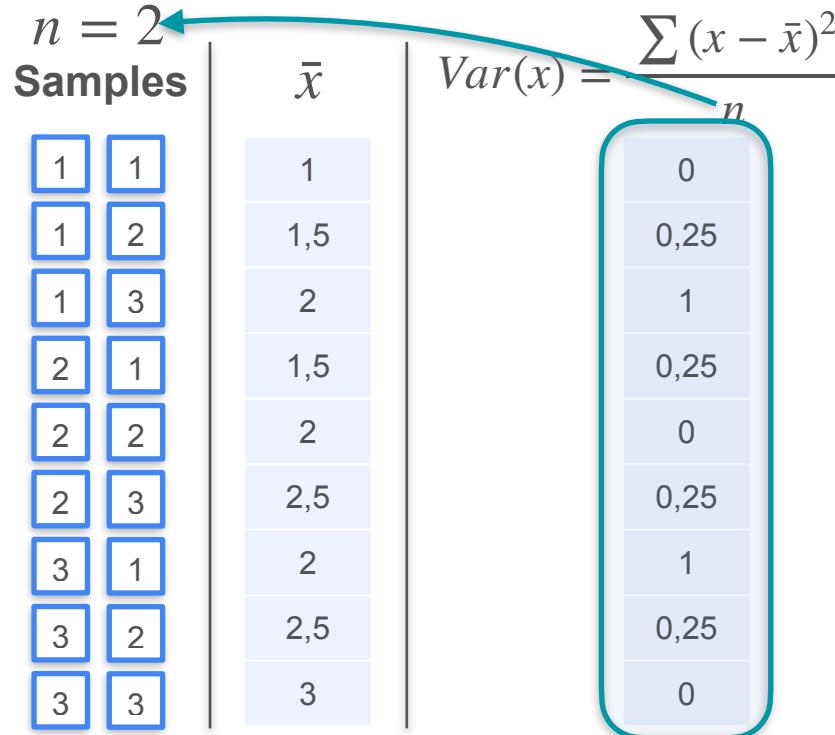
Variance Estimation

1 2 3

$$\mu = 2$$

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

$$\sigma^2 = \frac{2}{3}$$



estimated variance

$$= 0.333$$

$$= \frac{1}{3}$$

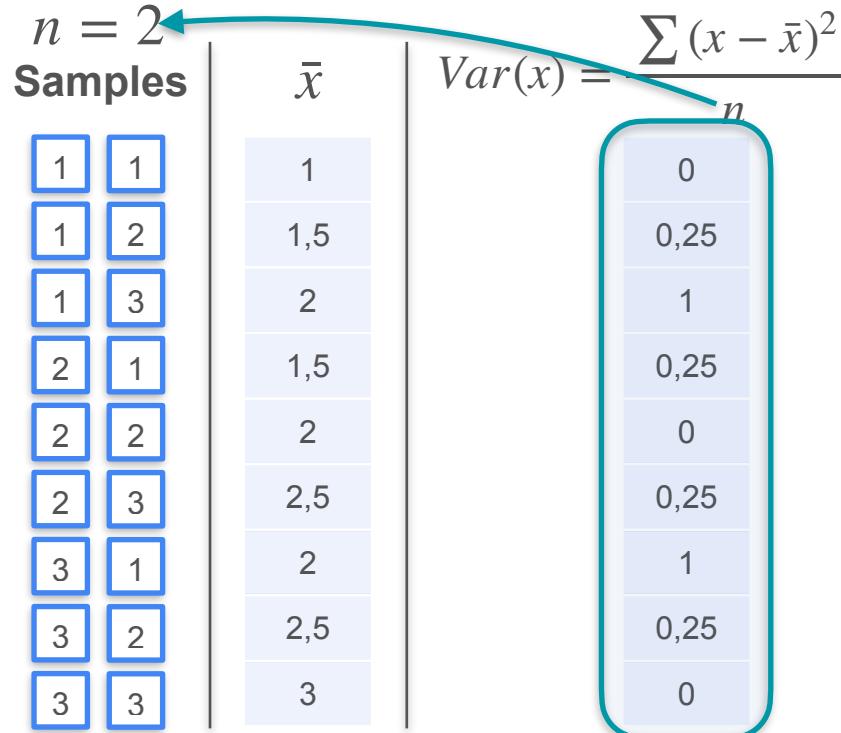
Variance Estimation

1 2 3

$$\mu = 2$$

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

$$\sigma^2 = \frac{2}{3}$$



estimated variance

$$= 0.333$$

$$= \frac{1}{3}$$

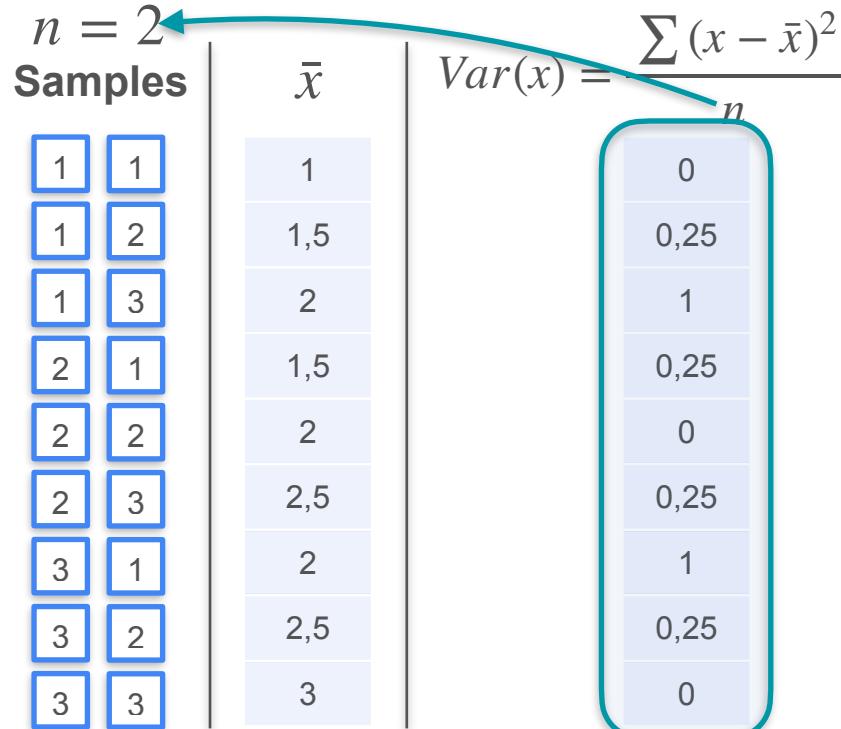
Variance Estimation

1 2 3

$$\mu = 2$$

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

$$\sigma^2 = \frac{2}{3}$$



estimated
variance

$$= 0.333$$

$$= \frac{1}{3}$$

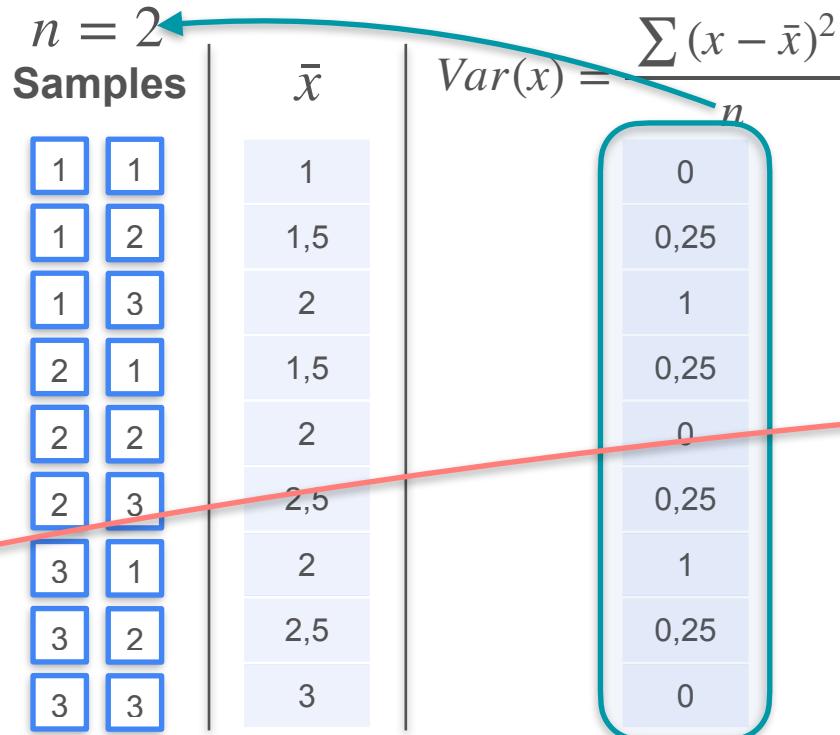
Variance Estimation

1 2 3

$$\mu = 2$$

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

$$\sigma^2 = \frac{2}{3}$$



estimated variance

$$= 0.333$$

$$= \frac{1}{3}$$

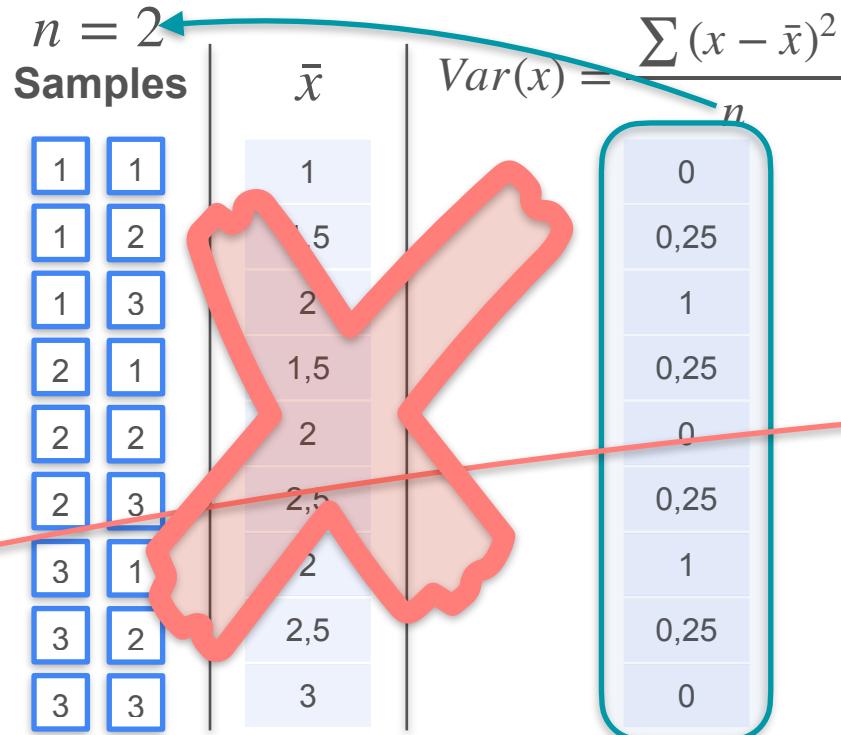
Variance Estimation

1 2 3

$$\mu = 2$$

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

$$\sigma^2 = \frac{2}{3}$$



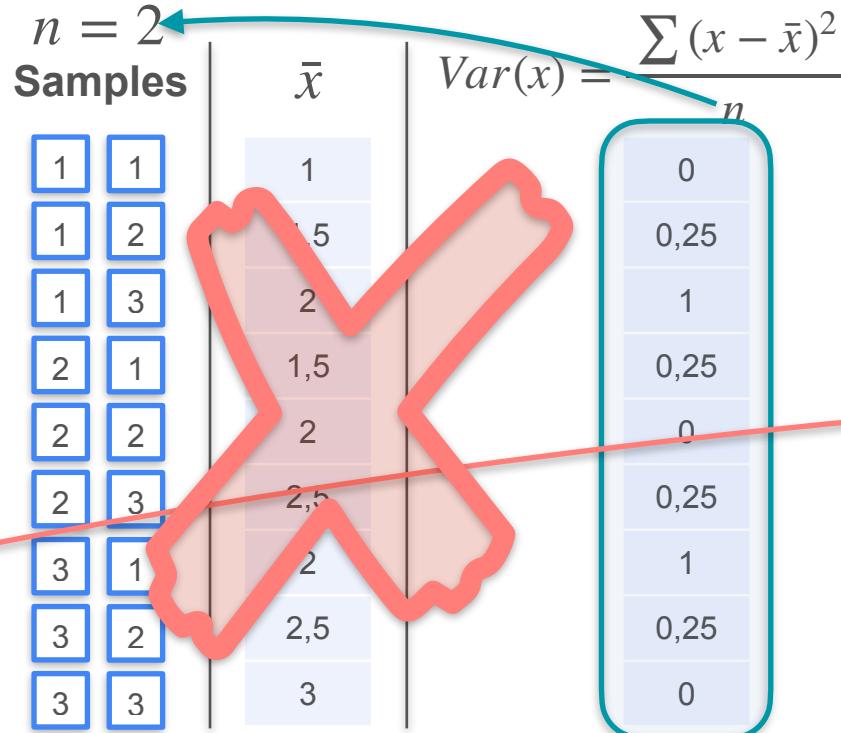
Variance Estimation

1 2 3

$$\mu = 2$$

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

$$\sigma^2 = \frac{2}{3}$$



estimated
variance

$$= 0.333$$

$$= \frac{1}{3}$$

Variance Estimation

1	2	3
---	---	---

$$\mu = 2$$

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

$$\sigma^2 = \frac{2}{3}$$

$n = 2$
Samples

1	1
1	2
1	3
2	1
2	2
2	3
3	1
3	2
3	3

$$\bar{x}$$

1
1,5
2
1,5
2
2,5
2
2,5
3

$$Var(x) = \frac{\sum (x - \bar{x})^2}{n}$$

Variance Estimation

1	2	3
---	---	---

$$\mu = 2$$

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

$$\sigma^2 = \frac{2}{3}$$

$n = 2$
Samples

1	1
1	2
1	3
2	1
2	2
2	3
3	1
3	2
3	3

$$\bar{x}$$

1
1,5
2
1,5
2
2,5
2
2,5
3

$$Var(x) = \frac{\sum (x - \bar{x})^2}{n - 1}$$

Variance Estimation

1	2	3
---	---	---

$$\mu = 2$$

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

$$\sigma^2 = \frac{2}{3}$$

$n = 2$
Samples

1	1
1	2
1	3
2	1
2	2
2	3
3	1
3	2
3	3

$$\bar{x}$$

1
1,5
2
1,5
2
2,5
2
2,5
3

$$Var(x) = \frac{\sum (x - \bar{x})^2}{n - 1}$$

Variance Estimation

1	2	3
---	---	---

$$\mu = 2$$

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

$$\sigma^2 = \frac{2}{3}$$

$n = 2$
Samples

1	1
1	2
1	3
2	1
2	2
2	3
3	1
3	2
3	3

$$\bar{x}$$

1
1,5
2
1,5
2
2,5
2
2,5
3

$$Var(x) = \frac{\sum (x - \bar{x})^2}{n - 1}$$

0
0,5
2
0,5
0
0,5
2
0,5
0

Variance Estimation

1	2	3
---	---	---

$$\mu = 2$$

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

$$\sigma^2 = \frac{2}{3}$$

$n = 2$
Samples

1	1
1	2
1	3
2	1
2	2
2	3
3	1
3	2
3	3

$$\bar{x}$$

1
1,5
2
1,5
2
2,5
2
2,5
3

$$Var(x) = \frac{\sum (x - \bar{x})^2}{n - 1}$$

0
0,5
2
0,5
0
0,5
2
0,5
0

Variance Estimation

1 2 3

$$\mu = 2$$

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

$$\sigma^2 = \frac{2}{3}$$

$n = 2$
Samples

1	1
1	2
1	3
2	1
2	2
2	3
3	1
3	2
3	3

$$\bar{x}$$

1
1,5
2
1,5
2
2,5
2
2,5
3

$$Var(x) = \frac{\sum (x - \bar{x})^2}{n - 1}$$

0
0,5
2
0,5
0
0,5
2
0,5
0

estimated
variance

$$= 0.667$$

$$= \frac{2}{3}$$

Variance Estimation

1 2 3

$$\mu = 2$$

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

$$\sigma^2 = \frac{2}{3}$$

$n = 2$
Samples

1	1
1	2
1	3
2	1
2	2
2	3
3	1
3	2
3	3

$$\bar{x}$$

1
1,5
2
1,5
2
2,5
2
2,5
3

$$Var(x) = \frac{\sum (x - \bar{x})^2}{n - 1}$$

0
0,5
2
0,5
0
0,5
2
0,5
0

estimated variance

$$= 0.667$$

$$= \frac{2}{3}$$

Variance Estimation

1 2 3

$$\mu = 2$$

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

$$\sigma^2 = \frac{2}{3}$$

$n = 2$
Samples

1	1
1	2
1	3
2	1
2	2
2	3
3	1
3	2
3	3

$$\bar{x}$$

1
1,5
2
1,5
2
2,5
2
2,5
3

$$Var(x) = \frac{\sum (x - \bar{x})^2}{n - 1}$$

0
0,5
2
0,5
0
0,5
2
0,5
0

estimated variance

$$= 0.667$$

$$= \frac{2}{3}$$

Variance Estimation

Population Variance Formula

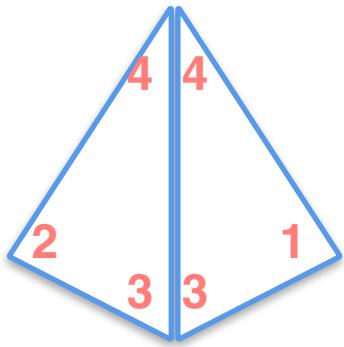
$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

Sample Variance Formula

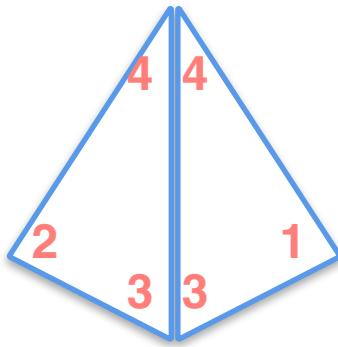
$$Var(x) = \frac{1}{n - 1} \sum (x - \bar{x})^2$$

Variance Estimation

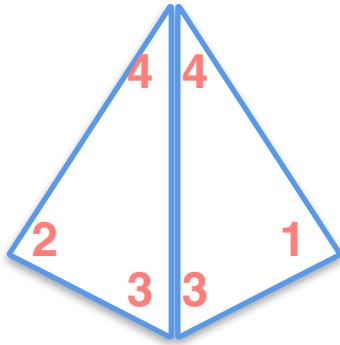
Variance Estimation



Variance Estimation

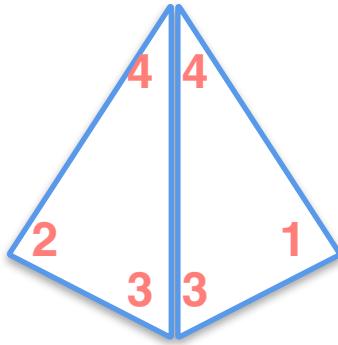


Variance Estimation



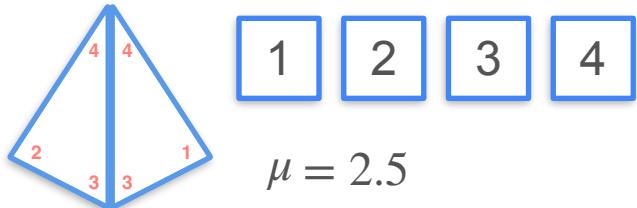
$$\mu = \frac{1 + 2 + 3 + 4}{4}$$

Variance Estimation

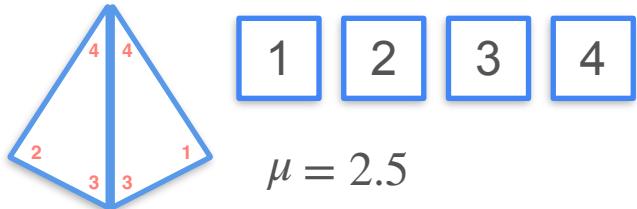


$$\mu = \frac{1 + 2 + 3 + 4}{4} = 2.5$$

Variance Estimation

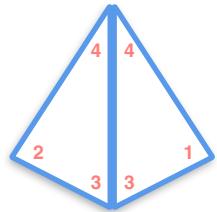


Variance Estimation



$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

Variance Estimation



$$\mu = 2.5$$

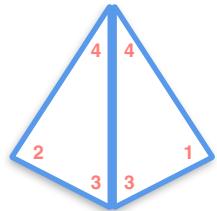
x

$x - \mu$

$(x - \mu)^2$

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

Variance Estimation



$$\mu = 2.5$$

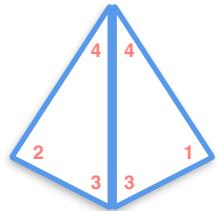
$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

x
1
2
3
4

$$x - \mu$$

$$(x - \mu)^2$$

Variance Estimation



$$\mu = 2.5$$

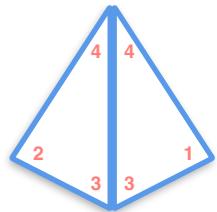
$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

x
1
2
3
4

$$x - 2.5$$

$$x - \mu \quad (x - \mu)^2$$

Variance Estimation



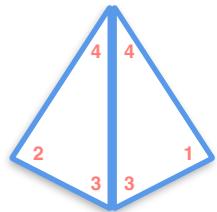
$$\mu = 2.5$$

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

x	$x - \mu$	$(x - \mu)^2$
1	-1,5	
2	-0,5	
3	0,5	
4	1,5	

$$x - 2.5$$

Variance Estimation



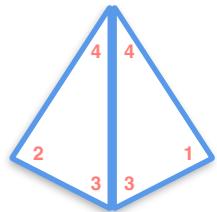
$$\mu = 2.5$$

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

$$x - 2.5$$

x	$x - \mu$	$(x - \mu)^2$
1	-1,5	2,25
2	-0,5	0,25
3	0,5	0,25
4	1,5	2,25

Variance Estimation



$$\mu = 2.5$$

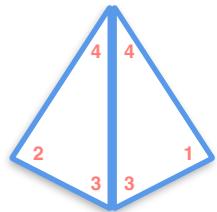
$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

$$x - 2.5$$

x	$x - \mu$	$(x - \mu)^2$
1	-1,5	2,25
2	-0,5	0,25
3	0,5	0,25
4	1,5	2,25

$$\sum (x - \mu)^2$$

Variance Estimation



$$\mu = 2.5$$

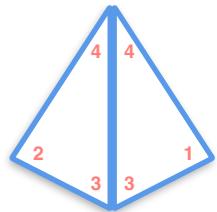
$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

x	$x - \mu$	$(x - \mu)^2$
1	-1,5	2,25
2	-0,5	0,25
3	0,5	0,25
4	1,5	2,25

$$x - 2.5$$

$$\sum (x - \mu)^2 = 5$$

Variance Estimation



$$\mu = 2.5$$

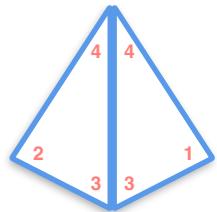
$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

x	$x - \mu$	$(x - \mu)^2$
1	-1,5	2,25
2	-0,5	0,25
3	0,5	0,25
4	1,5	2,25

$$x - 2.5$$

$$\frac{\sum (x - \mu)^2}{N} = 5$$

Variance Estimation



$$\mu = 2.5$$

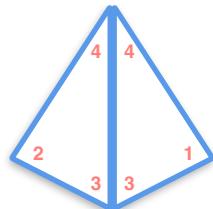
$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

x	$x - \mu$	$(x - \mu)^2$
1	-1,5	2,25
2	-0,5	0,25
3	0,5	0,25
4	1,5	2,25

$$x - 2.5$$

$$\frac{\sum (x - \mu)^2}{N} = \frac{5}{4}$$

Variance Estimation



$$\mu = 2.5$$

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

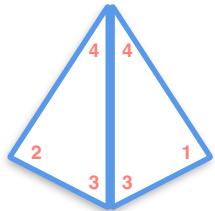
x	$x - \mu$	$(x - \mu)^2$
1	-1,5	2,25
2	-0,5	0,25
3	0,5	0,25
4	1,5	2,25

$$x - 2.5$$

$$\frac{\sum (x - \mu)^2}{N} = \frac{5}{4}$$

Population variance
 σ^2

Variance Estimation

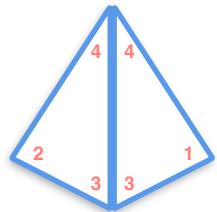


$$\mu = 2.5$$

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

$$\sigma^2 = \frac{5}{4}$$

Variance Estimation



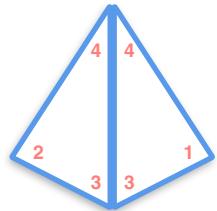
$$\mu = 2.5$$

$n = 2$
Samples

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

$$\sigma^2 = \frac{5}{4}$$

Variance Estimation



$$\mu = 2.5$$

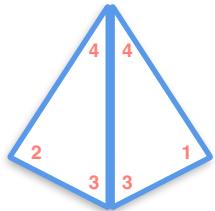
$n = 2$
Samples

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

$$\sigma^2 = \frac{5}{4}$$

x

Variance Estimation



$$\mu = 2.5$$

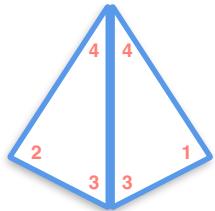
$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

$$\sigma^2 = \frac{5}{4}$$

$n = 2$
Samples

x	1,1	1,2	1,3	1,4	2,1	2,2	2,3	2,4	3,1	3,2	3,3	3,4	4,1	4,2	4,3	4,4
-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----

Variance Estimation



$$\mu = 2.5$$

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

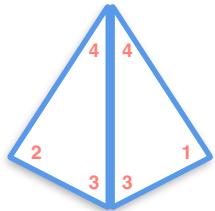
$$\sigma^2 = \frac{5}{4}$$

$n = 2$
Samples

x	1,1	1,2	1,3	1,4	2,1	2,2	2,3	2,4	3,1	3,2	3,3	3,4	4,1	4,2	4,3	4,4
-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----

$$\bar{x}$$

Variance Estimation



1 2 3 4

$$\mu = 2.5$$

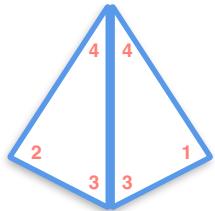
$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

$$\sigma^2 = \frac{5}{4}$$

$n = 2$
Samples

x	1,1	1,2	1,3	1,4	2,1	2,2	2,3	2,4	3,1	3,2	3,3	3,4	4,1	4,2	4,3	4,4
\bar{x}	1	1.5	2	2.5	1.5	2	2.5	3	2	2.5	3	3.5	2.5	3	3.5	4

Variance Estimation



1 2 3 4

$$\mu = 2.5$$

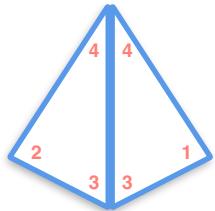
$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

$$\sigma^2 = \frac{5}{4}$$

$n = 2$
Samples

x	1,1	1,2	1,3	1,4	2,1	2,2	2,3	2,4	3,1	3,2	3,3	3,4	4,1	4,2	4,3	4,4
\bar{x}	1	1.5	2	2.5	1.5	2	2.5	3	2	2.5	3	3.5	2.5	3	3.5	4

Variance Estimation



1 2 3 4

$$\mu = 2.5$$

$$n = 2 \\ \text{Samples}$$

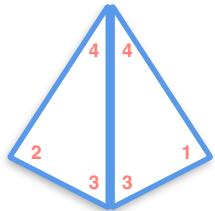
$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

$$Var(x) = \frac{\sum (x - \bar{x})^2}{n}$$

$$\sigma^2 = \frac{5}{4}$$

x	1,1	1,2	1,3	1,4	2,1	2,2	2,3	2,4	3,1	3,2	3,3	3,4	4,1	4,2	4,3	4,4
\bar{x}	1	1.5	2	2.5	1.5	2	2.5	3	2	2.5	3	3.5	2.5	3	3.5	4

Variance Estimation



1 2 3 4

$$\mu = 2.5$$

$$n = 2 \\ \text{Samples}$$

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

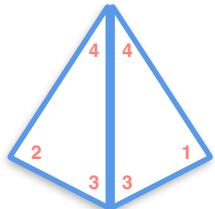
$$Var(x) = \frac{\sum (x - \bar{x})^2}{n}$$

$$\sigma^2 = \frac{5}{4}$$

x	1,1	1,2	1,3	1,4	2,1	2,2	2,3	2,4	3,1	3,2	3,3	3,4	4,1	4,2	4,3	4,4
\bar{x}	1	1.5	2	2.5	1.5	2	2.5	3	2	2.5	3	3.5	2.5	3	3.5	4

$$var(x)$$

Variance Estimation



1 2 3 4

$$\mu = 2.5$$

$$n = 2 \\ \text{Samples}$$

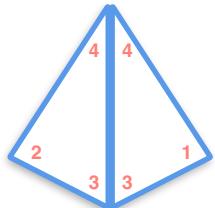
$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

$$Var(x) = \frac{\sum (x - \bar{x})^2}{n}$$

$$\sigma^2 = \frac{5}{4}$$

x	1,1	1,2	1,3	1,4	2,1	2,2	2,3	2,4	3,1	3,2	3,3	3,4	4,1	4,2	4,3	4,4
\bar{x}	1	1,5	2	2,5	1,5	2	2,5	3	2	2,5	3	3,5	2,5	3	3,5	4
$var(x)$	0	0,25	1	2,25	0,25	0	0,25	1	1	0,25	0	0,25	2,25	1	0,25	0

Variance Estimation



1 2 3 4

$$\mu = 2.5$$

$$n = 2 \\ \text{Samples}$$

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

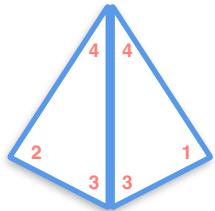
$$Var(x) = \frac{\sum (x - \bar{x})^2}{n}$$

$$\sigma^2 = \frac{5}{4}$$

$$var(x) = \frac{5}{8}$$

x	1,1	1,2	1,3	1,4	2,1	2,2	2,3	2,4	3,1	3,2	3,3	3,4	4,1	4,2	4,3	4,4
\bar{x}	1	1,5	2	2,5	1,5	2	2,5	3	2	2,5	3	3,5	2,5	3	3,5	4
$var(x)$	0	0,25	1	2,25	0,25	0	0,25	1	1	0,25	0	0,25	2,25	1	0,25	0

Variance Estimation



1 2 3 4

$$\mu = 2.5$$

$$n = 2 \\ \text{Samples}$$

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

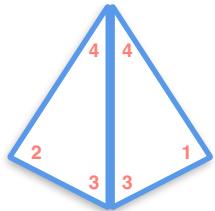
$$Var(x) = \frac{\sum (x - \bar{x})^2}{n}$$

$$\sigma^2 = \frac{5}{4}$$

$$var(x) = \frac{5}{8}$$

x	1,1	1,2	1,3	1,4	2,1	2,2	2,3	2,4	3,1	3,2	3,3	3,4	4,1	4,2	4,3	4,4
\bar{x}	1	1,5	2	2,5	1,5	2	2,5	3	2	2,5	3	3,5	2,5	3	3,5	4
$var(x)$	0	0,25	1	2,25	0,25	0	0,25	1	1	0,25	0	0,25	2,25	1	0,25	0

Variance Estimation



$$\mu = 2.5$$

$n = 2$
Samples

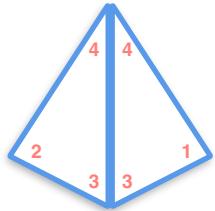
$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

$$Var(x) = \frac{\sum (x - \bar{x})^2}{n}$$

$$\sigma^2 = \frac{5}{4}$$

x	1,1	1,2	1,3	1,4	2,1	2,2	2,3	2,4	3,1	3,2	3,3	3,4	4,1	4,2	4,3	4,4
\bar{x}	1	1.5	2	2.5	1.5	2	2.5	3	2	2.5	3	3.5	2.5	3	3.5	4

Variance Estimation



$$\mu = 2.5$$

$$n = 2 \\ \text{Samples}$$

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

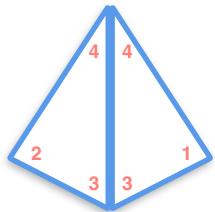
$$Var(x) = \frac{\sum (x - \bar{x})^2}{n}$$

$$\sigma^2 = \frac{5}{4}$$

$$Var(x) = \frac{\sum (x - \bar{x})^2}{n - 1}$$

x	1,1	1,2	1,3	1,4	2,1	2,2	2,3	2,4	3,1	3,2	3,3	3,4	4,1	4,2	4,3	4,4
\bar{x}	1	1.5	2	2.5	1.5	2	2.5	3	2	2.5	3	3.5	2.5	3	3.5	4

Variance Estimation



1 2 3 4

$$\mu = 2.5$$

$$n = 2 \\ \text{Samples}$$

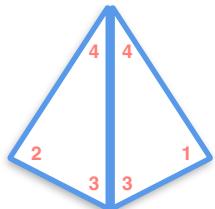
$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

$$Var(x) = \frac{\sum (x - \bar{x})^2}{n - 1}$$

$$\sigma^2 = \frac{5}{4}$$

x	1,1	1,2	1,3	1,4	2,1	2,2	2,3	2,4	3,1	3,2	3,3	3,4	4,1	4,2	4,3	4,4
\bar{x}	1	1.5	2	2.5	1.5	2	2.5	3	2	2.5	3	3.5	2.5	3	3.5	4

Variance Estimation



1 2 3 4

$$\mu = 2.5$$

$$n = 2 \\ \text{Samples}$$

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

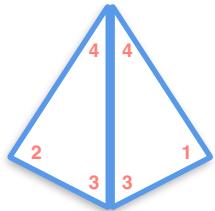
$$Var(x) = \frac{\sum (x - \bar{x})^2}{n - 1}$$

$$\sigma^2 = \frac{5}{4}$$

x	1,1	1,2	1,3	1,4	2,1	2,2	2,3	2,4	3,1	3,2	3,3	3,4	4,1	4,2	4,3	4,4
\bar{x}	1	1.5	2	2.5	1.5	2	2.5	3	2	2.5	3	3.5	2.5	3	3.5	4

$$var(x)$$

Variance Estimation



1 2 3 4

$$\mu = 2.5$$

$$n = 2 \\ \text{Samples}$$

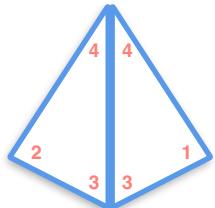
$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

$$Var(x) = \frac{\sum (x - \bar{x})^2}{n - 1}$$

$$\sigma^2 = \frac{5}{4}$$

x	1,1	1,2	1,3	1,4	2,1	2,2	2,3	2,4	3,1	3,2	3,3	3,4	4,1	4,2	4,3	4,4
\bar{x}	1	1,5	2	2,5	1,5	2	2,5	3	2	2,5	3	3,5	2,5	3	3,5	4
$var(x)$	0	0,5	2	4,5	0,5	0	0,5	2	2	0,5	0	0,5	4,5	2	0,5	0

Variance Estimation



$$\mu = 2.5$$

$$n = 2 \\ \text{Samples}$$

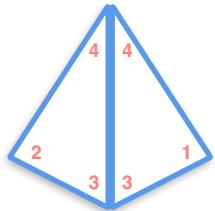
$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

$$Var(x) = \frac{\sum (x - \bar{x})^2}{n - 1}$$

$$\sigma^2 = \frac{5}{4}$$

x	1,1	1,2	1,3	1,4	2,1	2,2	2,3	2,4	3,1	3,2	3,3	3,4	4,1	4,2	4,3	4,4
\bar{x}	1	1,5	2	2,5	1,5	2	2,5	3	2	2,5	3	3,5	2,5	3	3,5	4
$var(x)$	0	0,5	2	4,5	0,5	0	0,5	2	2	0,5	0	0,5	4,5	2	0,5	0

Variance Estimation



$$\mu = 2.5$$

$$n = 2 \\ \text{Samples}$$

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

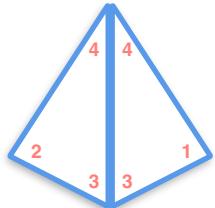
$$Var(x) = \frac{\sum (x - \bar{x})^2}{n - 1}$$

$$\sigma^2 = \frac{5}{4}$$

$$Var(x) = \frac{5}{4}$$

x	1,1	1,2	1,3	1,4	2,1	2,2	2,3	2,4	3,1	3,2	3,3	3,4	4,1	4,2	4,3	4,4
\bar{x}	1	1,5	2	2,5	1,5	2	2,5	3	2	2,5	3	3,5	2,5	3	3,5	4
$var(x)$	0	0,5	2	4,5	0,5	0	0,5	2	2	0,5	0	0,5	4,5	2	0,5	0

Variance Estimation



$$\mu = 2.5$$

$n = 2$
Samples

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

$$Var(x) = \frac{\sum (x - \bar{x})^2}{n - 1}$$

$$\sigma^2 = \frac{5}{4}$$

$$Var(x) = \frac{5}{4}$$

x	1,1	1,2	1,3	1,4	2,1	2,2	2,3	2,4	3,1	3,2	3,3	3,4	4,1	4,2	4,3	4,4
\bar{x}	1	1,5	2	2,5	1,5	2	2,5	3	2	2,5	3	3,5	2,5	3	3,5	4
$var(x)$	0	0,5	2	4,5	0,5	0	0,5	2	2	0,5	0	0,5	4,5	2	0,5	0

Variance Estimation

Population Variance Formula

$$\sigma^2 = \frac{1}{N} \sum (x - \mu)^2$$

Sample Variance Formula

$$Var(x) = \frac{\sum (x - \bar{x})^2}{n - 1}$$

$$s^2 = \frac{\sum (x - \bar{x})^2}{n - 1}$$



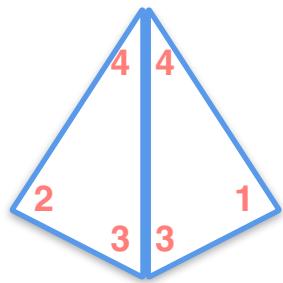
DeepLearning.AI

Sample and Population

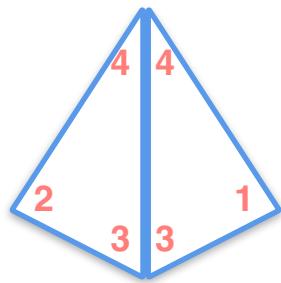
Law of Large Numbers

Law of Large Numbers

Law of Large Numbers

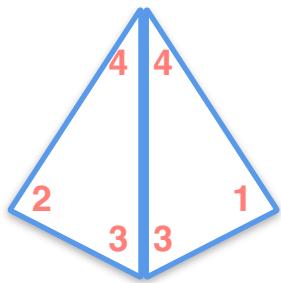


Law of Large Numbers



1 2 3 4

Law of Large Numbers

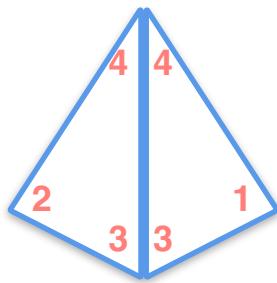


$$\begin{array}{cccc} 1 & 2 & 3 & 4 \\ \mu = 2.5 \end{array}$$

Experiment:

Toss the 4-sided dice twice and record the average of your outcomes

Law of Large Numbers



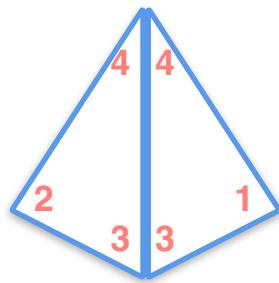
$$\begin{array}{cccc} 1 & 2 & 3 & 4 \\ \mu = 2.5 \end{array}$$

	1	2	3	4
1	1,1	1,2	1,3	1,4
2	2,1	2,2	2,3	2,4
3	3,1	3,2	3,3	3,4
4	4,1	4,2	4,3	4,4

Experiment:

Toss the 4-sided dice twice and record the average of your outcomes

Law of Large Numbers



1 2 3 4

$$\mu = 2.5$$

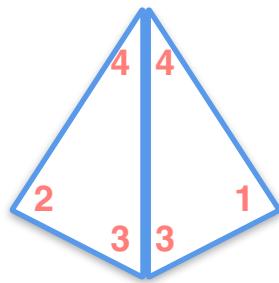
	1	2	3	4
1	1,1	1,2	1,3	1,4
2	2,1	2,2	2,3	2,4
3	3,1	3,2	3,3	3,4
4	4,1	4,2	4,3	4,4

	1	2	3	4
1	1	1.5	2	2.5
2	1.5	2	2.5	3
3	2	2.5	3	3.5
4	2.5	3	3.5	4

Experiment:

Toss the 4-sided dice twice and record the average of your outcomes

Law of Large Numbers



1 2 3 4

$$\mu = 2.5$$

	1	2	3	4
1	1,1	1,2	1,3	1,4
2	2,1	2,2	2,3	2,4
3	3,1	3,2	3,3	3,4
4	4,1	4,2	4,3	4,4

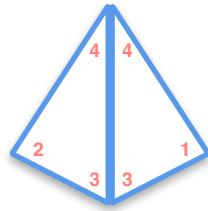
	1	2	3	4
1	1	1.5	2	2.5
2	1.5	2	2.5	3
3	2	2.5	3	3.5
4	2.5	3	3.5	4

	1	2	3	4
1				
2				
3				
4				

Experiment:

Toss the 4-sided dice twice and record the average of your outcomes

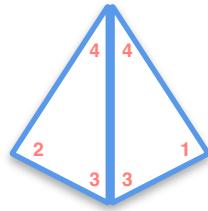
Law of Large Numbers



1 2 3 4
 $\mu = 2.5$

	1	2	3	4
1	1,1	1,2	1,3	1,4
2	2,1	2,2	2,3	2,4
3	3,1	3,2	3,3	3,4
4	4,1	4,2	4,3	4,4

Law of Large Numbers

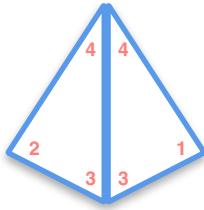


1 2 3 4
 $\mu = 2.5$

1 trial

	1	2	3	4
1	1,1	1,2	1,3	1,4
2	2,1	2,2	2,3	2,4
3	3,1	3,2	3,3	3,4
4	4,1	4,2	4,3	4,4

Law of Large Numbers



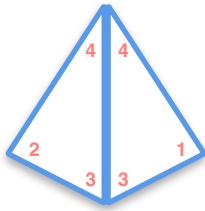
1 2 3 4
 $\mu = 2.5$

1 trial

4,3

	1	2	3	4
1	1,1	1,2	1,3	1,4
2	2,1	2,2	2,3	2,4
3	3,1	3,2	3,3	3,4
4	4,1	4,2	4,3	4,4

Law of Large Numbers

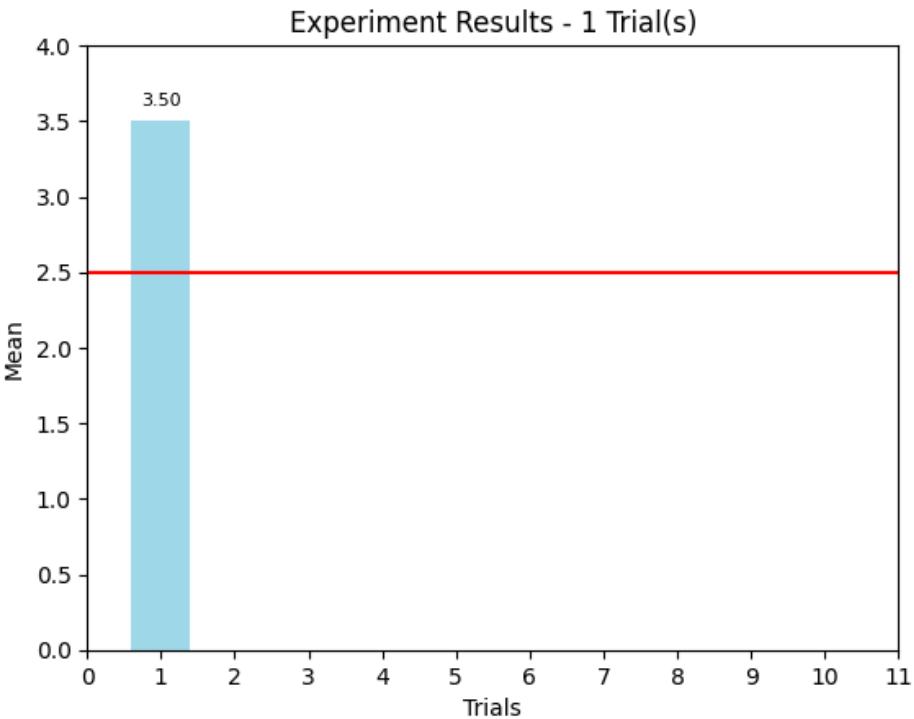


1 2 3 4
 $\mu = 2.5$

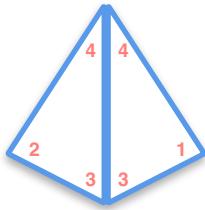
1 trial

4,3

	1	2	3	4
1	1,1	1,2	1,3	1,4
2	2,1	2,2	2,3	2,4
3	3,1	3,2	3,3	3,4
4	4,1	4,2	4,3	4,4



Law of Large Numbers

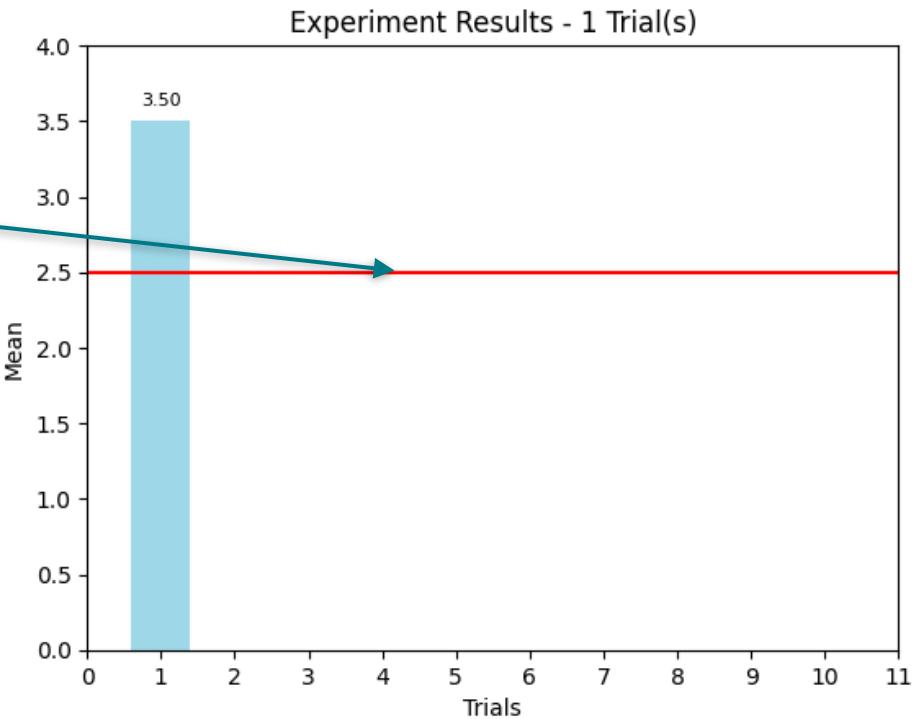


1 2 3 4
 $\mu = 2.5$

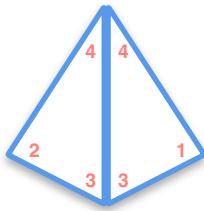
1 trial

4,3

	1	2	3	4
1	1,1	1,2	1,3	1,4
2	2,1	2,2	2,3	2,4
3	3,1	3,2	3,3	3,4
4	4,1	4,2	4,3	4,4



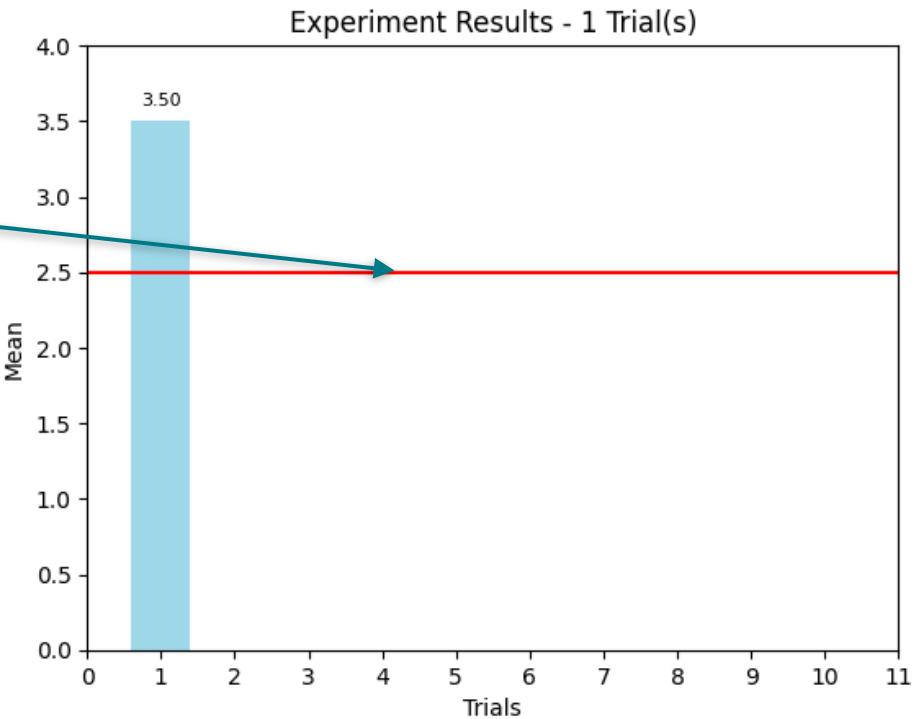
Law of Large Numbers



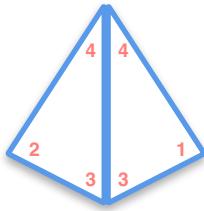
$$1 \ 2 \ 3 \ 4$$
$$\mu = 2.5$$

1 trial
4,3 \bar{x}_1

	1	2	3	4
1	1,1	1,2	1,3	1,4
2	2,1	2,2	2,3	2,4
3	3,1	3,2	3,3	3,4
4	4,1	4,2	4,3	4,4



Law of Large Numbers



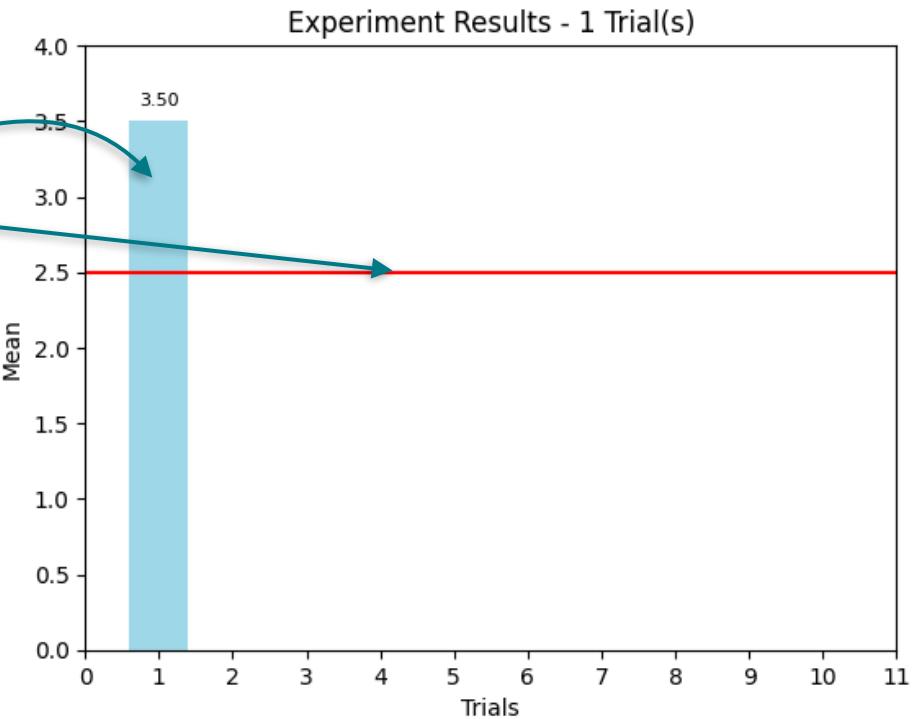
$$1 \ 2 \ 3 \ 4$$
$$\mu = 2.5$$

1 trial

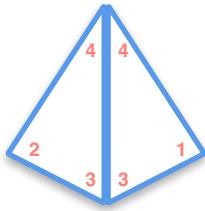
4,3

$$\bar{x}_1$$

	1	2	3	4
1	1,1	1,2	1,3	1,4
2	2,1	2,2	2,3	2,4
3	3,1	3,2	3,3	3,4
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Law of Large Numbers



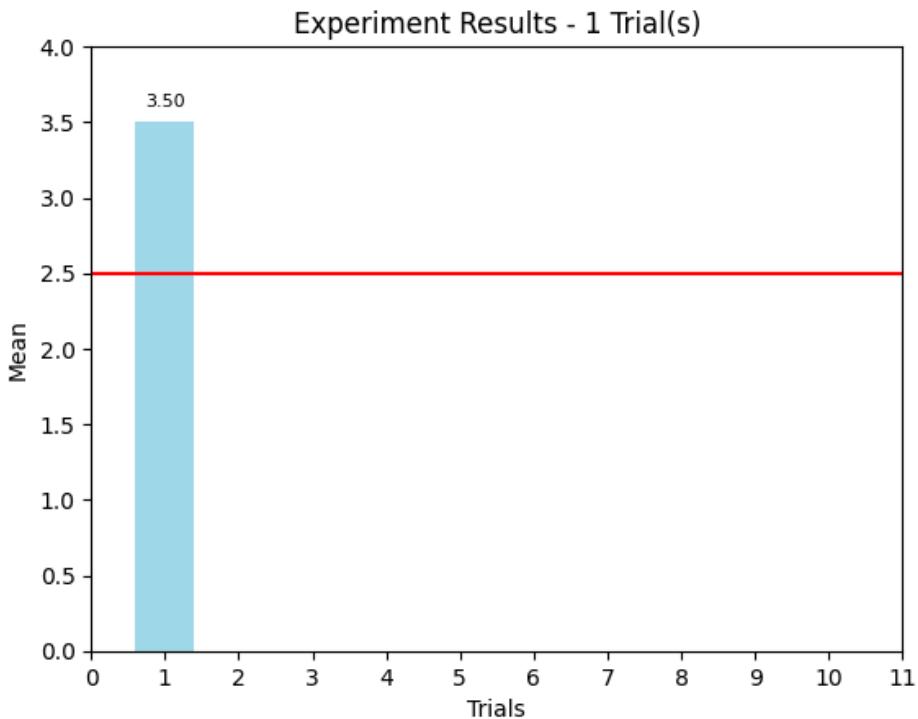
1 2 3 4
 $\mu = 2.5$

	1	2	3	4
1	1,1	1,2	1,3	1,4
2	2,1	2,2	2,3	2,4
3	3,1	3,2	3,3	3,4
4	4,1	4,2	4,3	4,4

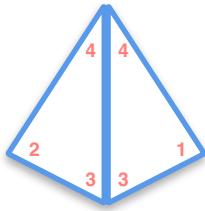
1 trial

4,3

2 trials



Law of Large Numbers



1 2 3 4
 $\mu = 2.5$

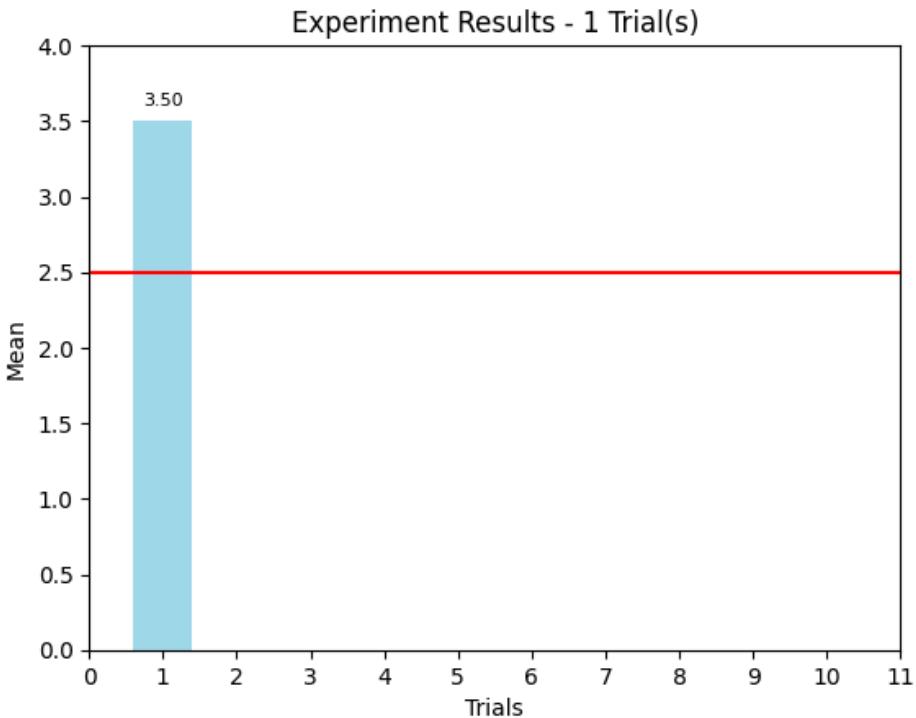
	1	2	3	4
1	1,1	1,2	1,3	1,4
2	2,1	2,2	2,3	2,4
3	3,1	3,2	3,3	3,4
4	4,1	4,2	4,3	4,4

1 trial

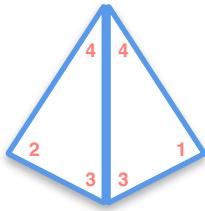
4,3

2 trials

3,4
1,3



Law of Large Numbers



1 2 3 4
 $\mu = 2.5$

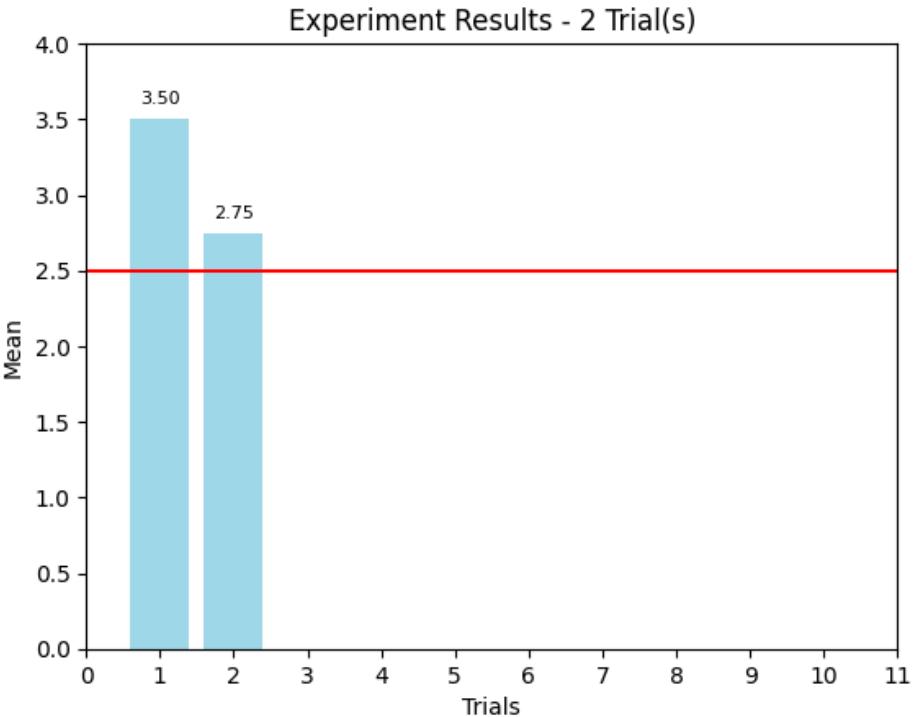
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3	3,1	3,2	3,3	3,4
4	4,1	4,2	4,3	4,4

1 trial

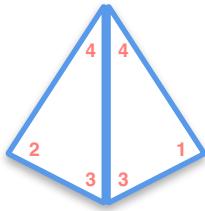
4,3

2 trials

3,4
1,3



Law of Large Numbers



1 2 3 4
 $\mu = 2.5$

	1	2	3	4
1	1,1	1,2	1,3	1,4
2	2,1	2,2	2,3	2,4
3	3,1	3,2	3,3	3,4
4	4,1	4,2	4,3	4,4

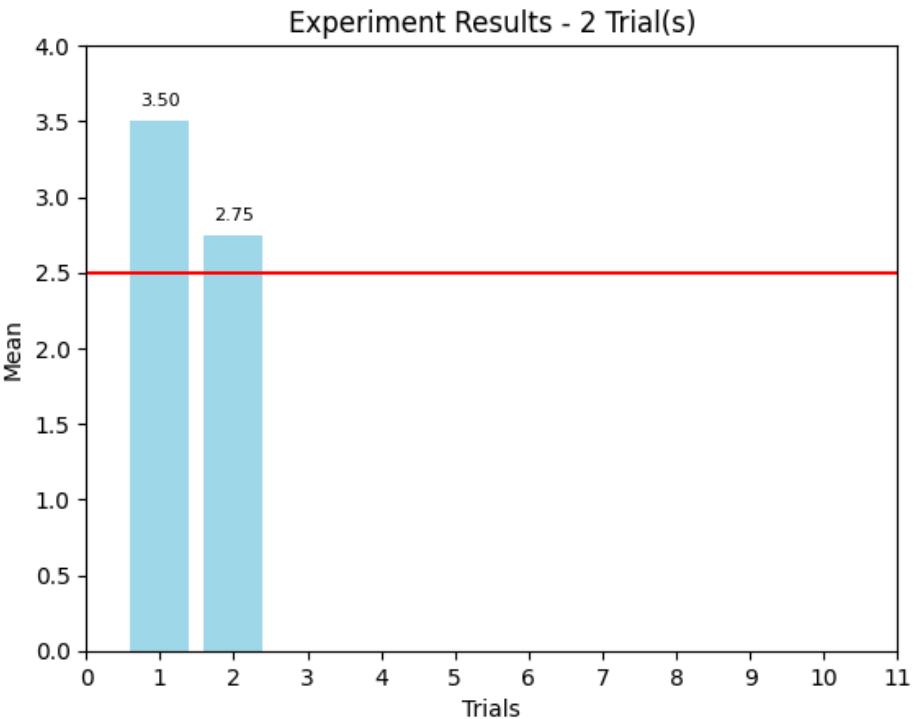
1 trial

4,3

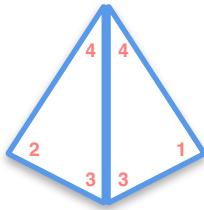
2 trials

3,4
1,3

3 trials



Law of Large Numbers



1 2 3 4
 $\mu = 2.5$

	1	2	3	4
1	1,1	1,2	1,3	1,4
2	2,1	2,2	2,3	2,4
3	3,1	3,2	3,3	3,4
4	4,1	4,2	4,3	4,4

1 trial

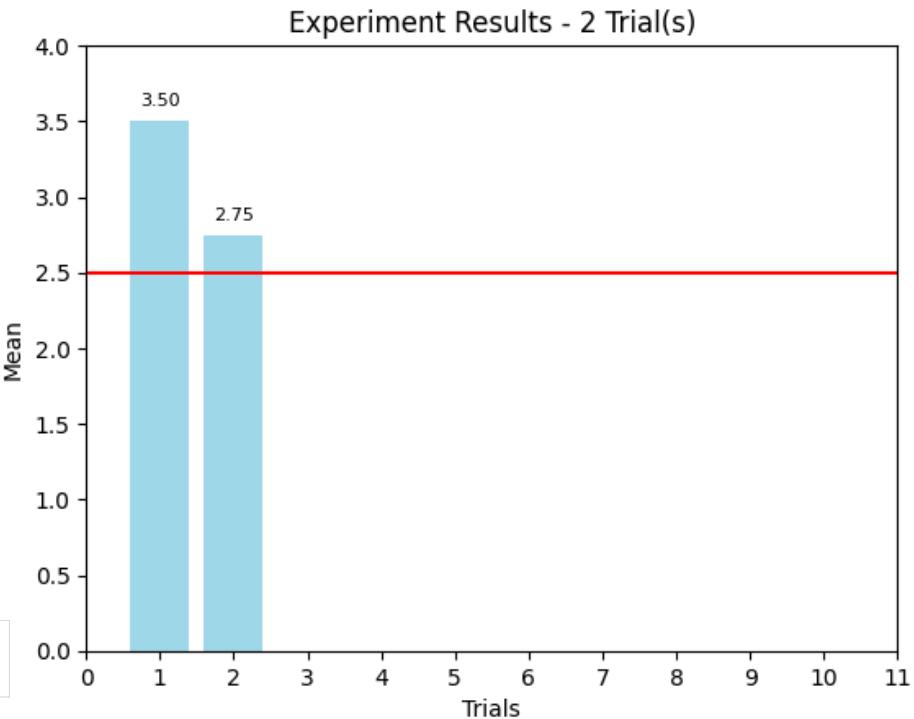
4,3

2 trials

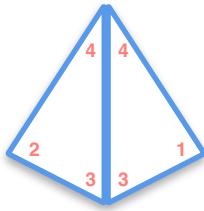
3,4
1,3

3 trials

3,1 1,4 1,1



Law of Large Numbers



1 2 3 4
 $\mu = 2.5$

	1	2	3	4
1	1,1	1,2	1,3	1,4
2	2,1	2,2	2,3	2,4
3	3,1	3,2	3,3	3,4
4	4,1	4,2	4,3	4,4

1 trial

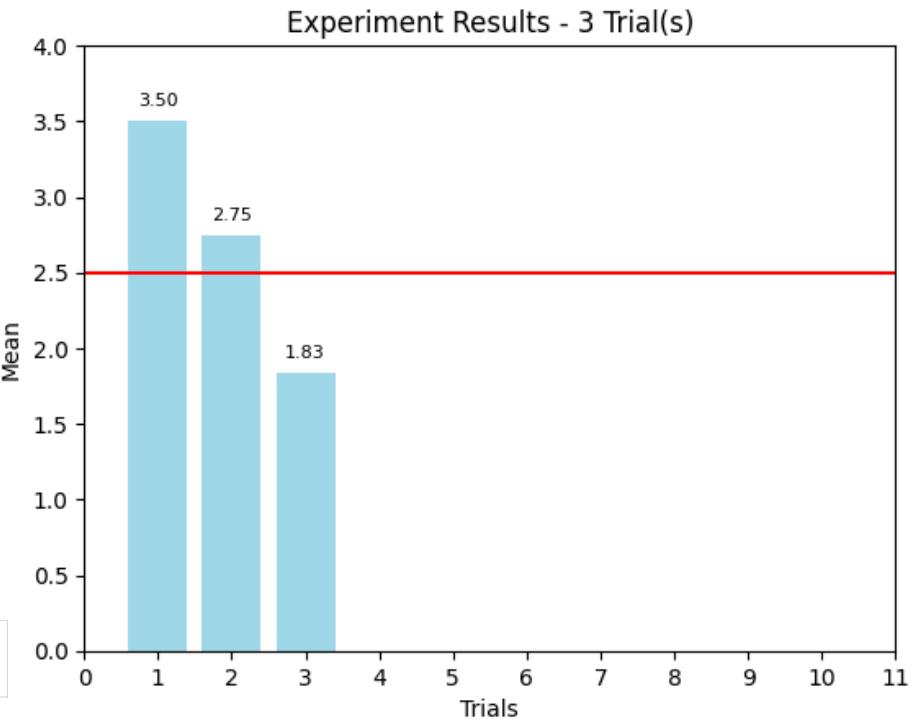
4,3

2 trials

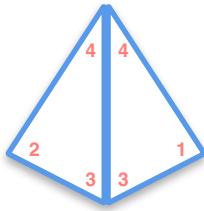
3,4
1,3

3 trials

3,1 1,4 1,1



Law of Large Numbers



1 2 3 4
 $\mu = 2.5$

	1	2	3	4
1	1,1	1,2	1,3	1,4
2	2,1	2,2	2,3	2,4
3	3,1	3,2	3,3	3,4
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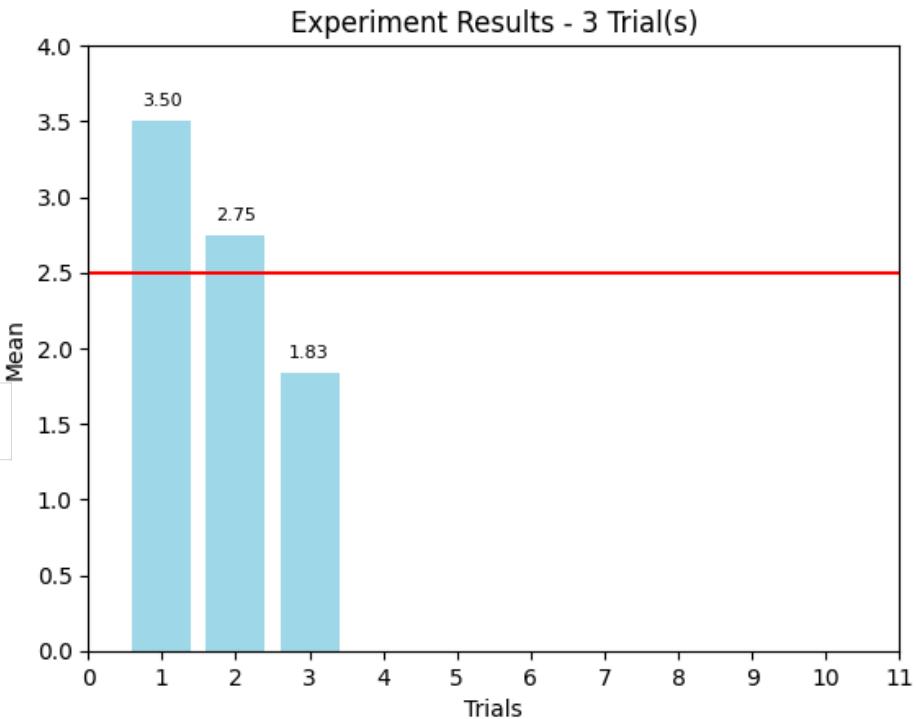
2 trials

3,4
1,3

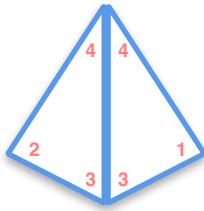
3 trials

3,1	1,4	1,1
-----	-----	-----

4 trials



Law of Large Numbers



1 2 3 4
 $\mu = 2.5$

	1	2	3	4
1	1,1	1,2	1,3	1,4
2	2,1	2,2	2,3	2,4
3	3,1	3,2	3,3	3,4
4	4,1	4,2	4,3	4,4

2 trials

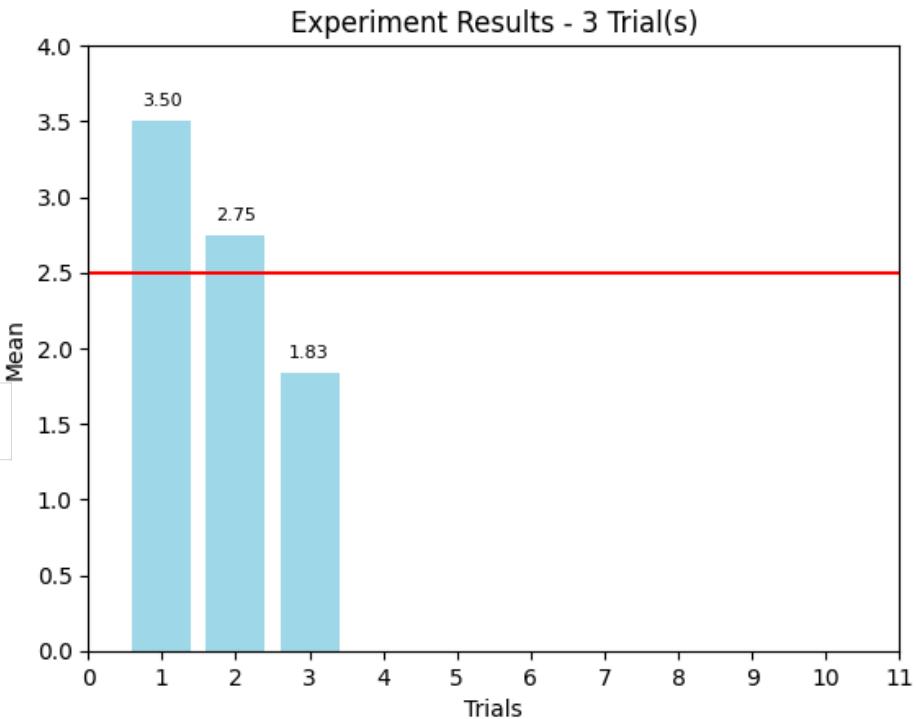
3,4
1,3

3 trials

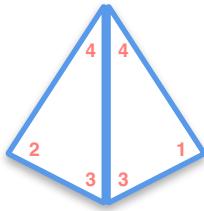
3,1	1,4	1,1
-----	-----	-----

4 trials

3,1	3,1
1,2	3,2



Law of Large Numbers



1 2 3 4
 $\mu = 2.5$

	1	2	3	4
1	1,1	1,2	1,3	1,4
2	2,1	2,2	2,3	2,4
3	3,1	3,2	3,3	3,4
4	4,1	4,2	4,3	4,4

2 trials

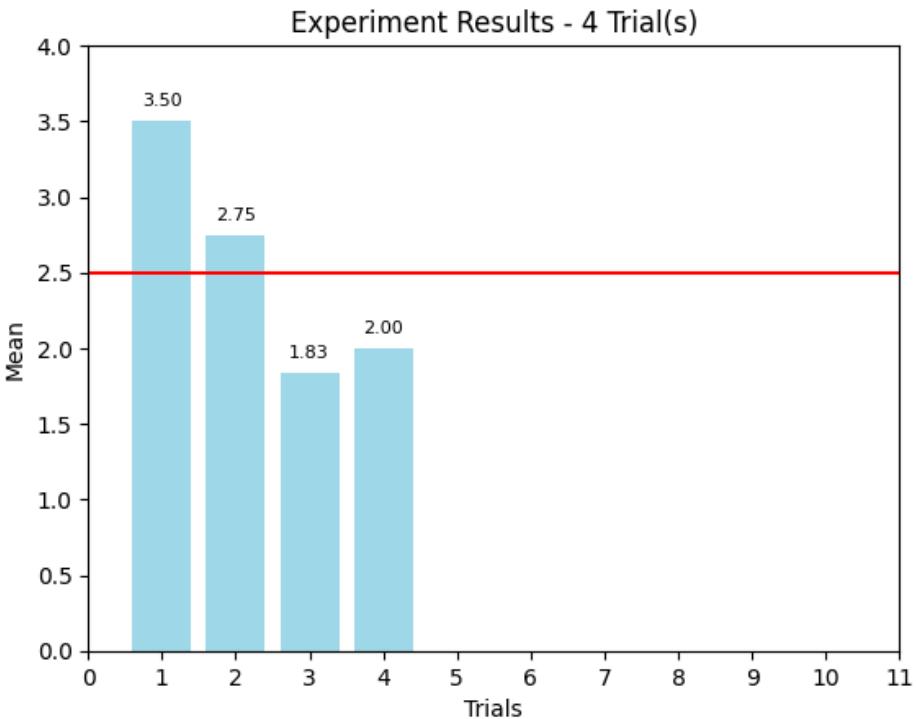
3,4
1,3

3 trials

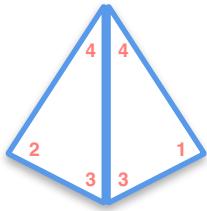
3,1	1,4	1,1
-----	-----	-----

4 trials

3,1	3,1
1,2	3,2

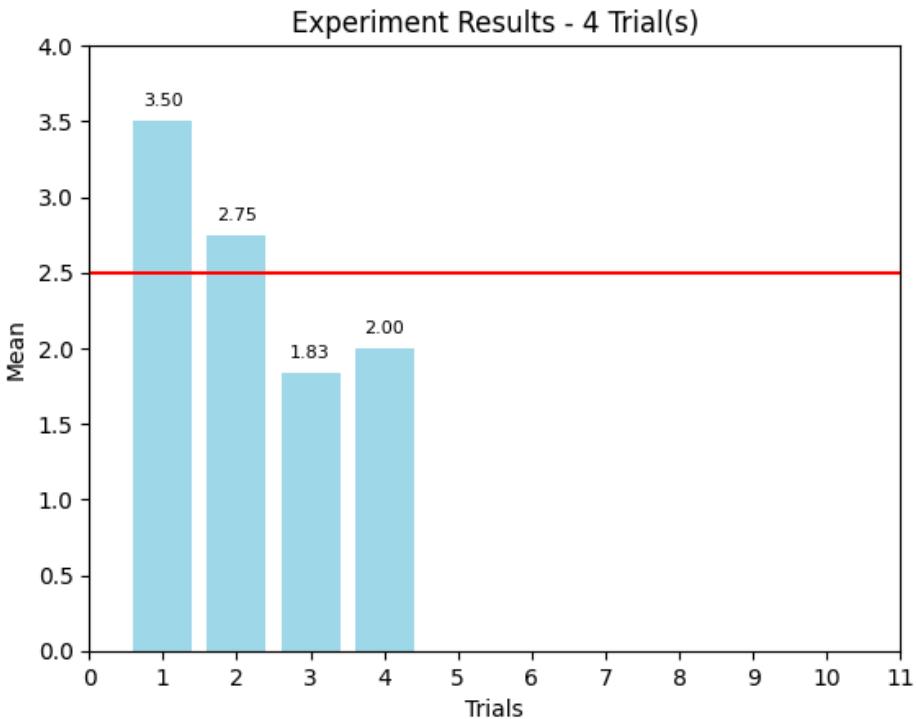


Law of Large Numbers

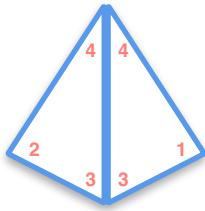


1 2 3 4
 $\mu = 2.5$

	1	2	3	4
1	1,1	1,2	1,3	1,4
2	2,1	2,2	2,3	2,4
3	3,1	3,2	3,3	3,4
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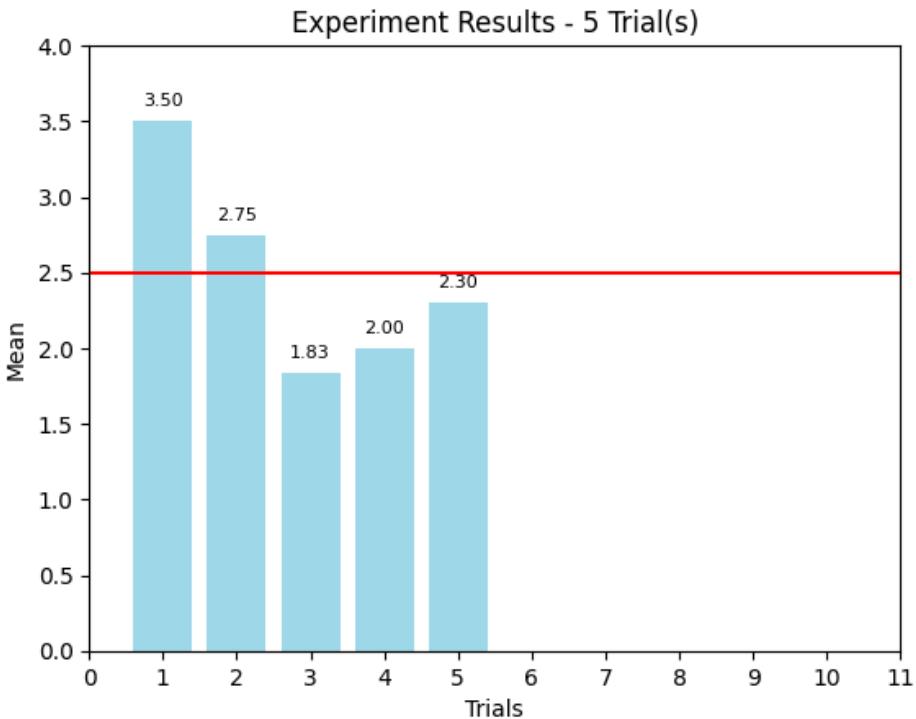


Law of Large Numbers

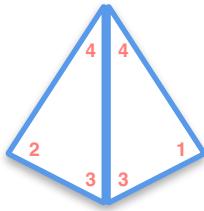


1 2 3 4
 $\mu = 2.5$

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2	2,1	2,2	2,3	2,4
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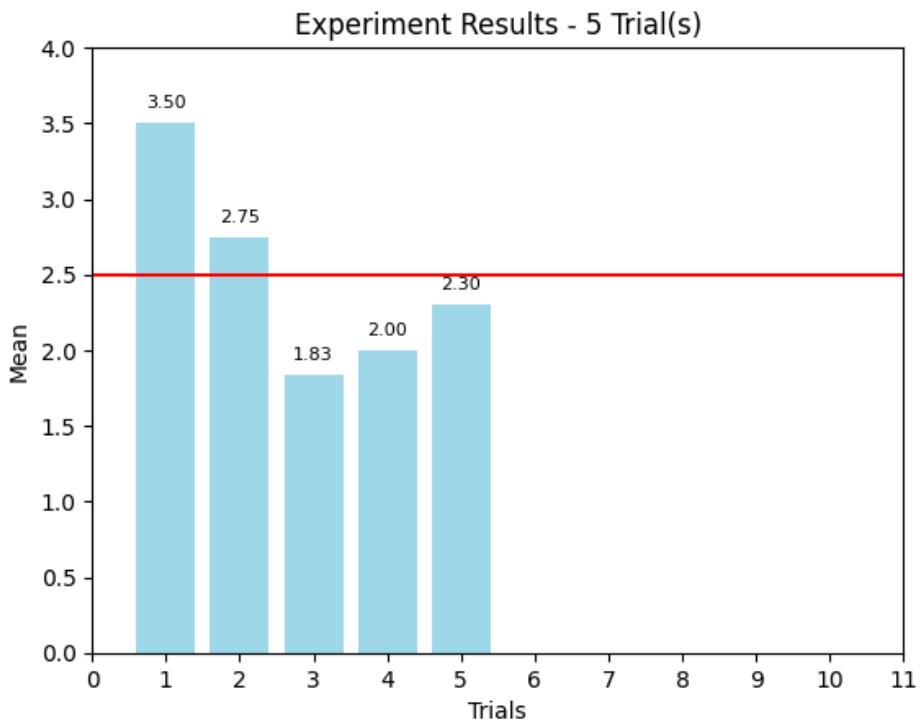


Law of Large Numbers

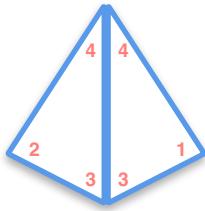


1 2 3 4
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3	3,1	3,2	3,3	3,4
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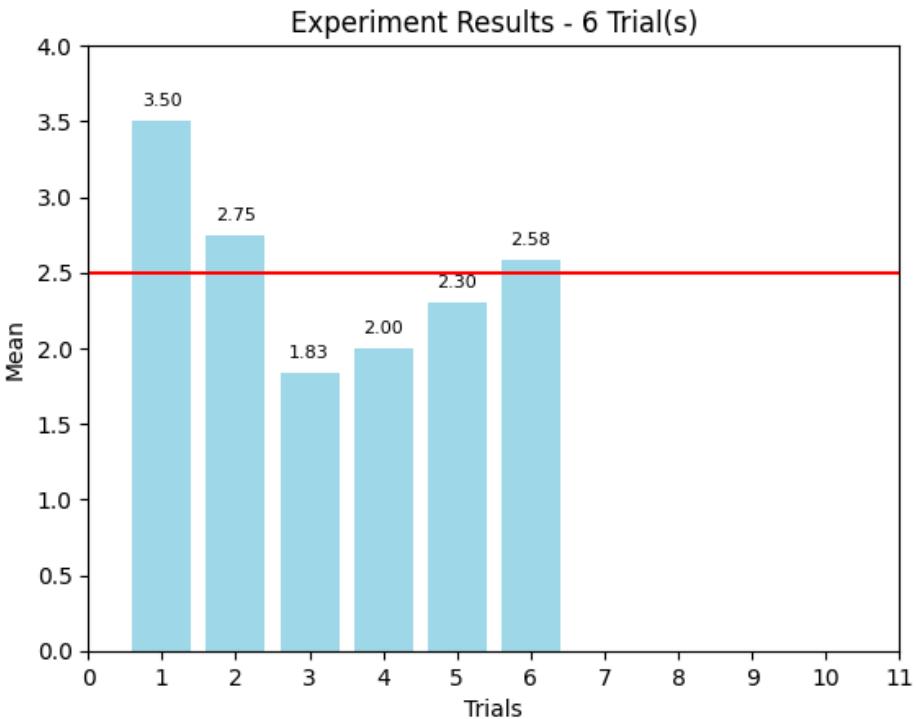


Law of Large Numbers

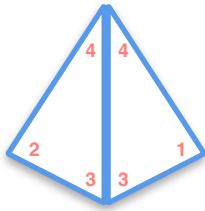


1 2 3 4
 $\mu = 2.5$

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4	4,1	4,2	4,3	4,4

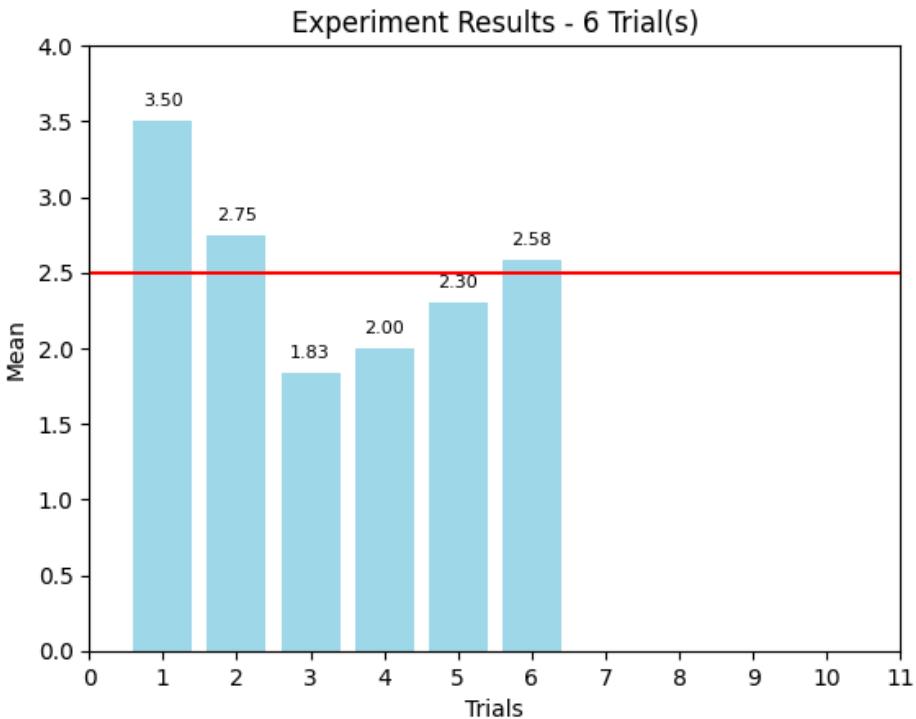


Law of Large Numbers

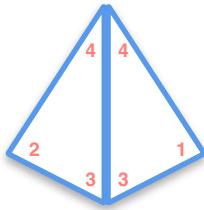


1 2 3 4
 $\mu = 2.5$

	1	2	3	4
1	1,1	1,2	1,3	1,4
2	2,1	2,2	2,3	2,4
3	3,1	3,2	3,3	3,4
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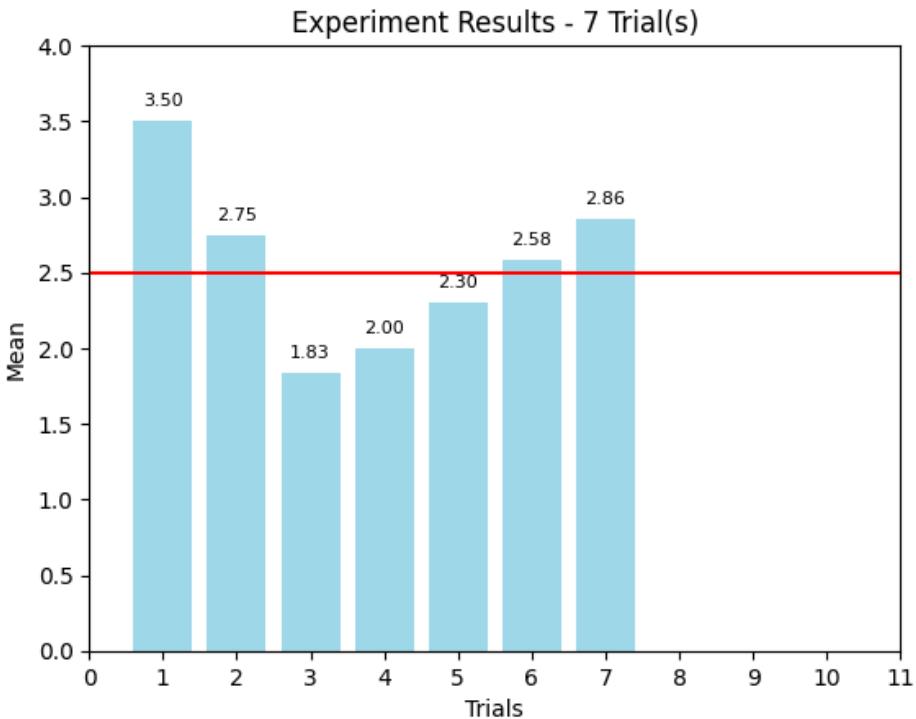


Law of Large Numbers

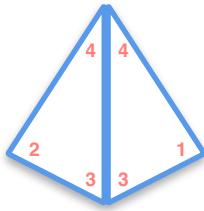


1 2 3 4
 $\mu = 2.5$

	1	2	3	4
1	1,1	1,2	1,3	1,4
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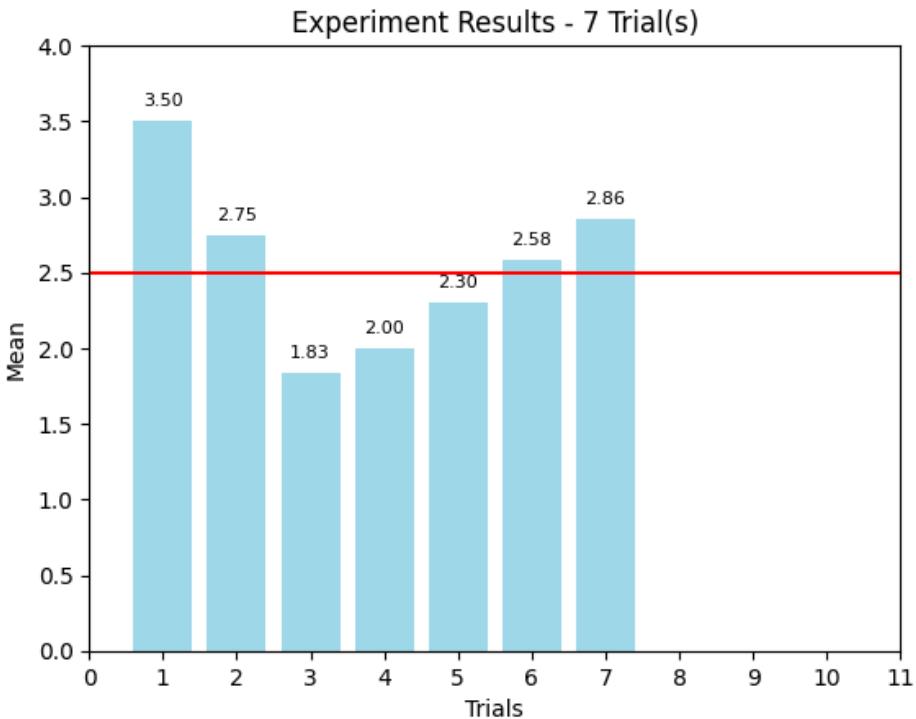


Law of Large Numbers

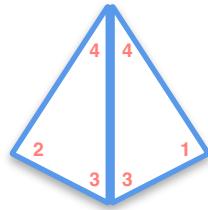


1 2 3 4
 $\mu = 2.5$

	1	2	3	4
1	1,1	1,2	1,3	1,4
2	2,1	2,2	2,3	2,4
3	3,1	3,2	3,3	3,4
4	4,1	4,2	4,3	4,4

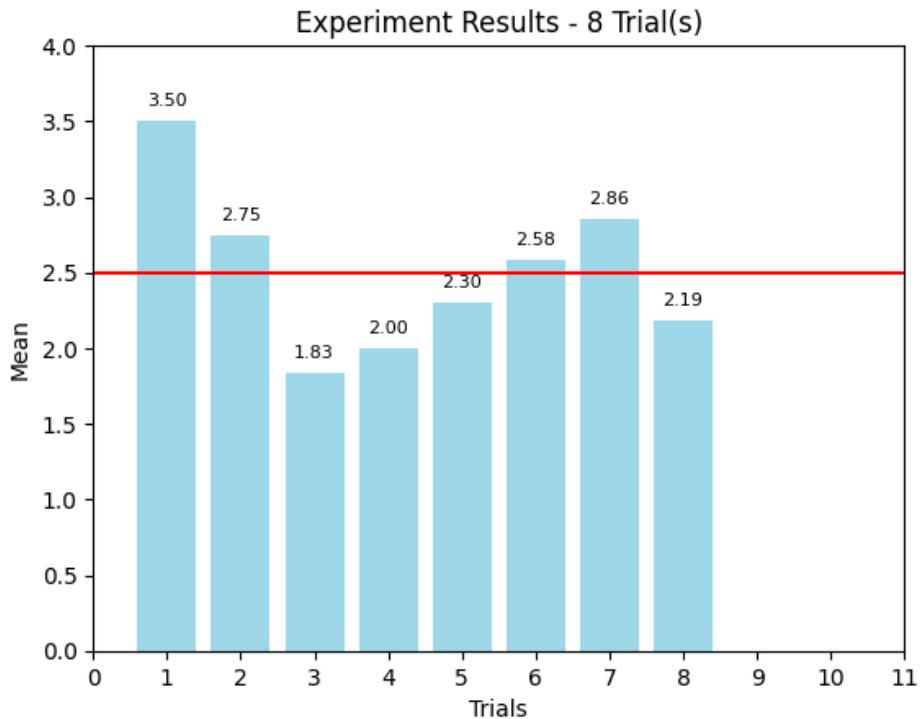


Law of Large Numbers

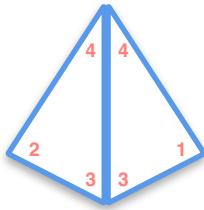


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 $\mu = 2.5$

	1	2	3	4
1	1,1	1,2	1,3	1,4
2	2,1	2,2	2,3	2,4
3	3,1	3,2	3,3	3,4
4	4,1	4,2	4,3	4,4

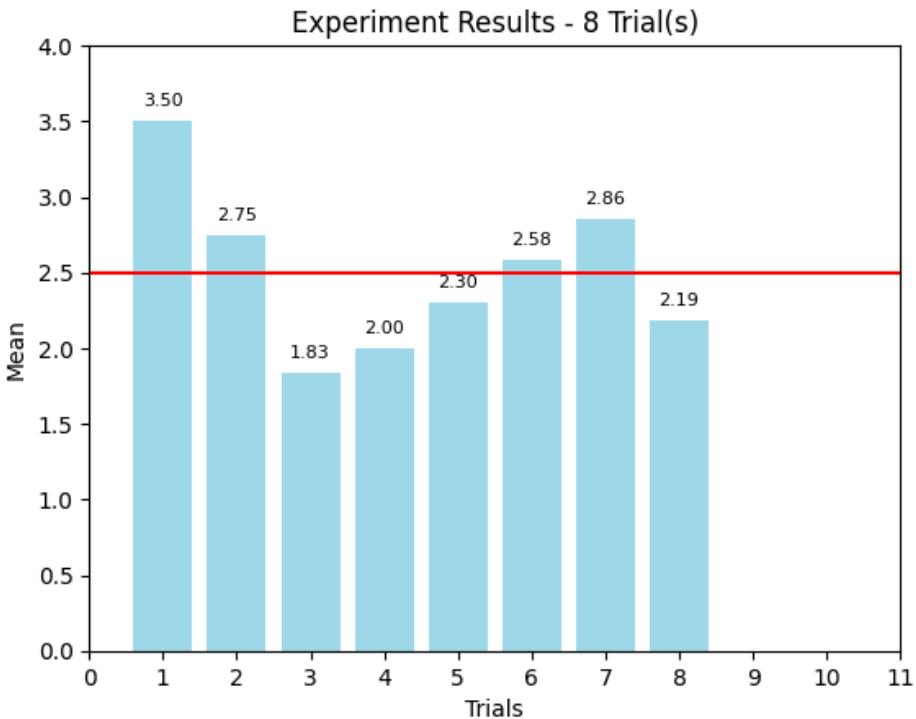


Law of Large Numbers

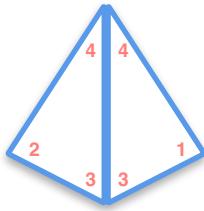


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2	2,1	2,2	2,3	2,4
3	3,1	3,2	3,3	3,4
4	4,1	4,2	4,3	4,4

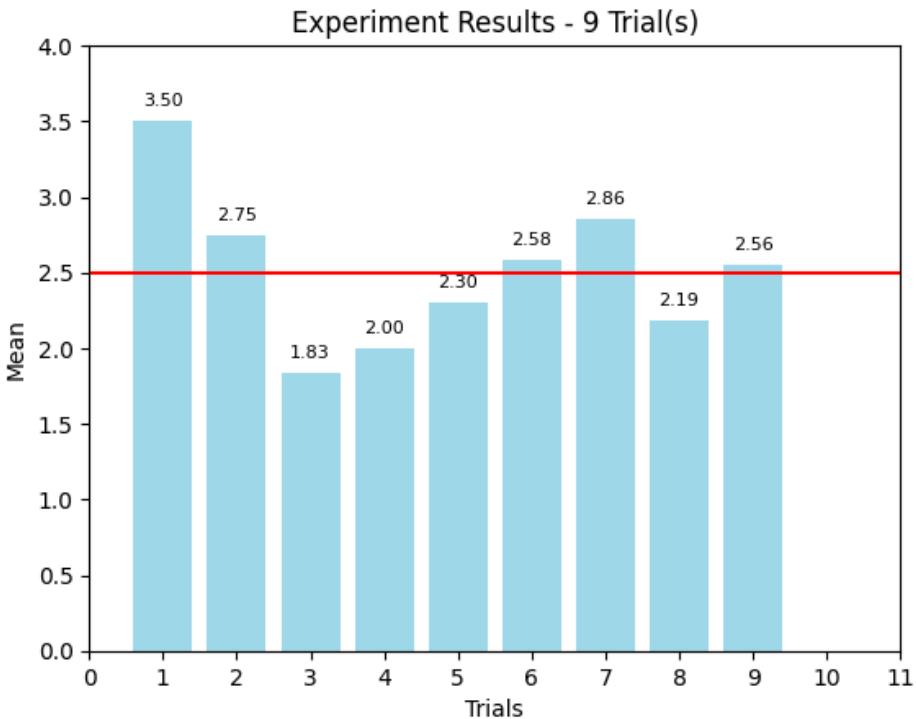


Law of Large Numbers

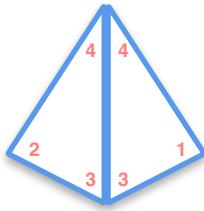


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4	4,1	4,2	4,3	4,4

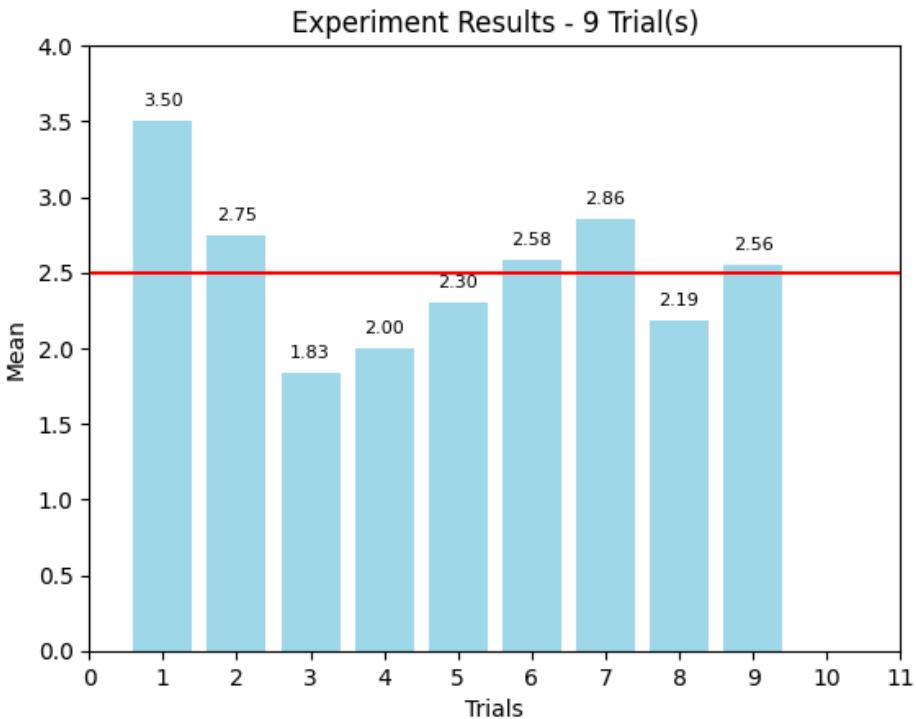


Law of Large Numbers

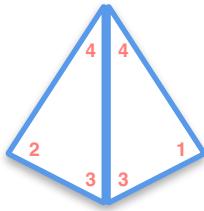


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4	4,1	4,2	4,3	4,4

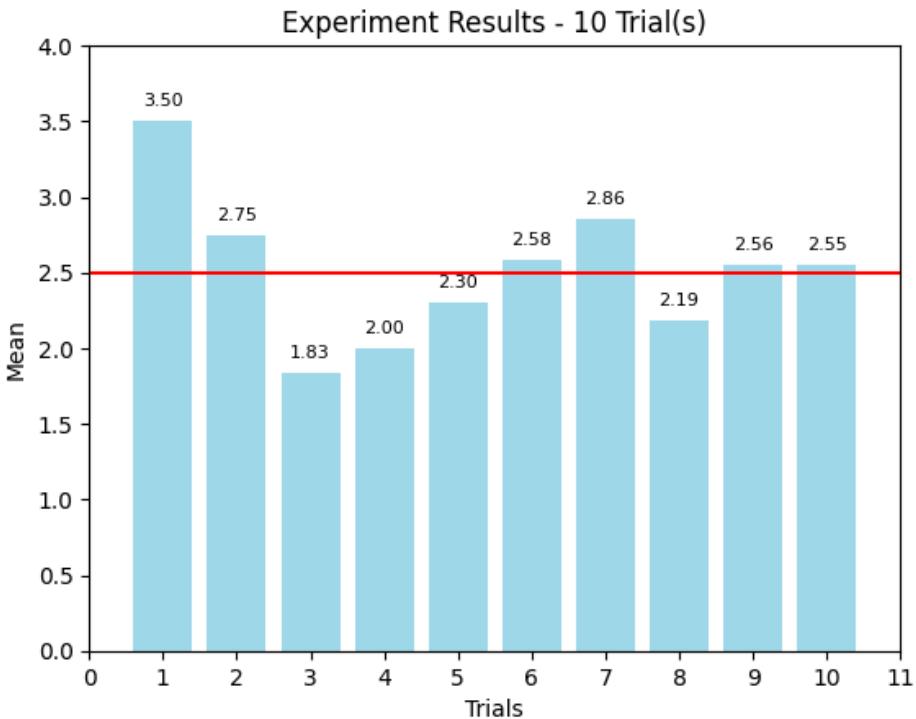


Law of Large Numbers

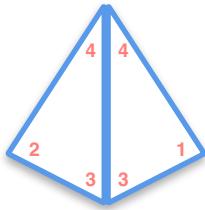


1 2 3 4
 $\mu = 2.5$

	1	2	3	4
1	1,1	1,2	1,3	1,4
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3	3,1	3,2	3,3	3,4
4	4,1	4,2	4,3	4,4

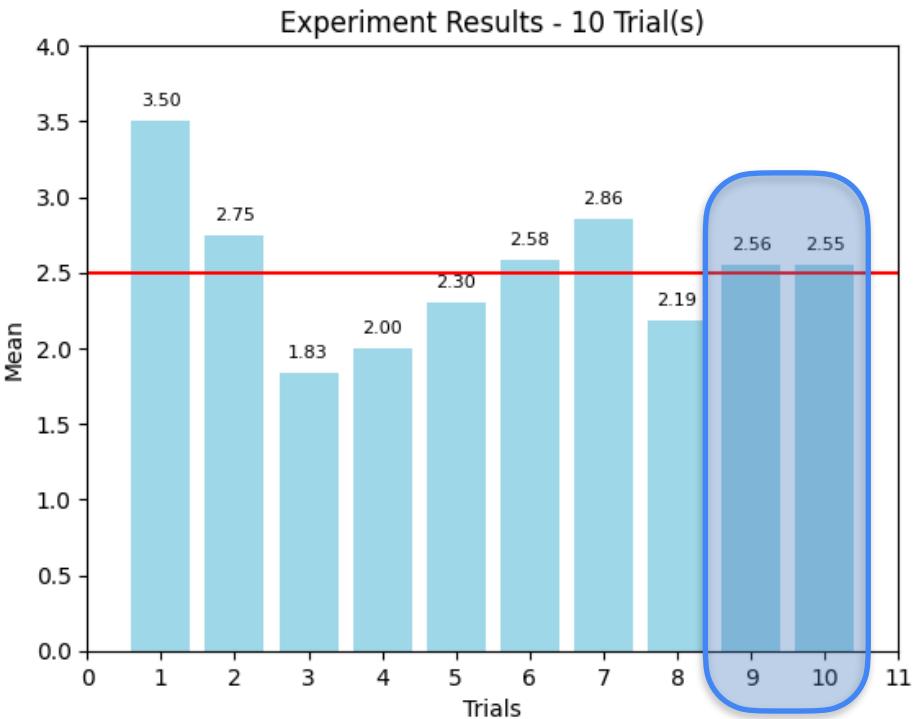


Law of Large Numbers

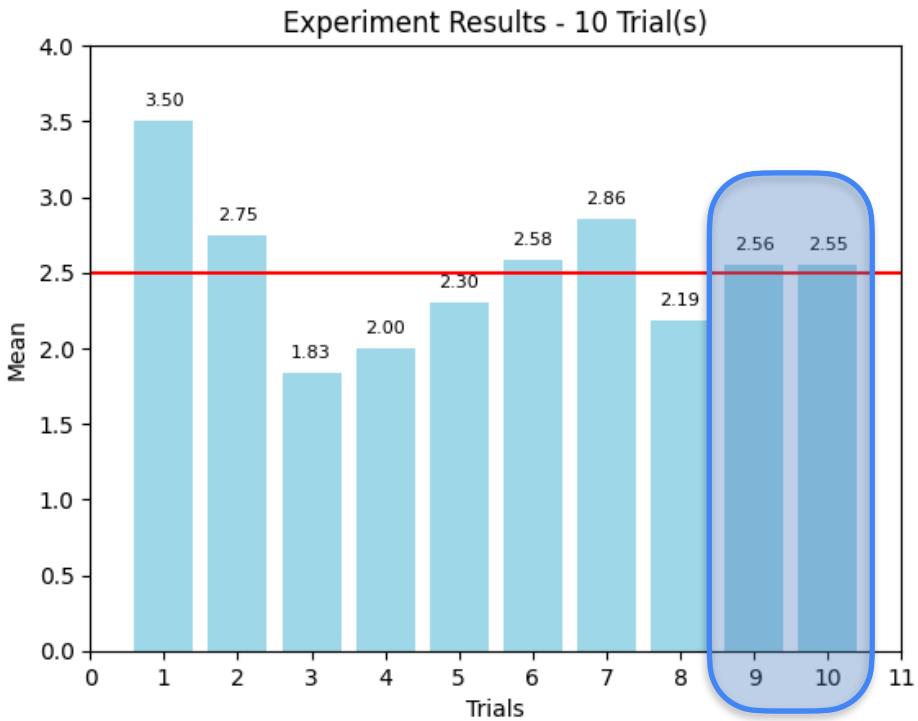


1 2 3 4
 $\mu = 2.5$

	1	2	3	4
1	1,1	1,2	1,3	1,4
2	2,1	2,2	2,3	2,4
3	3,1	3,2	3,3	3,4
4	4,1	4,2	4,3	4,4



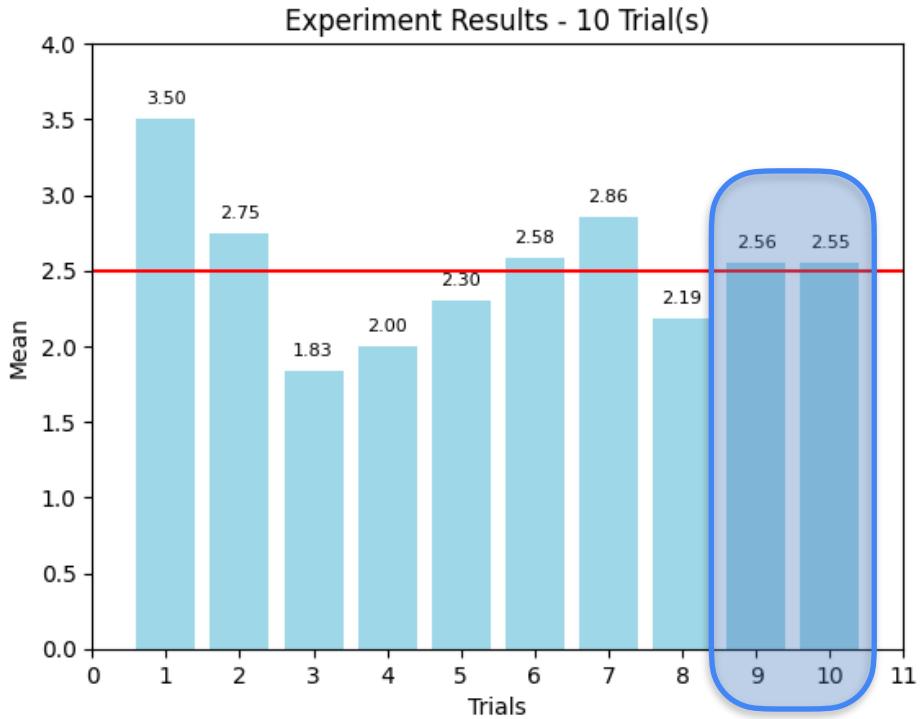
Law of Large Numbers



Law of Large Numbers

As the sample size increases, the average of the sample will tend to get closer to the average of the entire population.

Law of Large Numbers



Law of Large Numbers

Law of Large Numbers

Law of Large Numbers

Law of Large Numbers

n : number of samples

Law of Large Numbers

Law of Large Numbers

n : number of samples

X_i : some estimate X for a sample size i

Law of Large Numbers

Law of Large Numbers

n : number of samples

X_i : some estimate X for a sample size i

as $n \rightarrow \infty$

Law of Large Numbers

Law of Large Numbers

n : number of samples

X_i : some estimate X for a sample size i

as $n \rightarrow \infty$

$$\frac{1}{n} \sum_{i=1}^n X_i \rightarrow \mathbb{E}[X] = \mu_X$$

Law of Large Numbers

as $n \rightarrow \infty$

Law of Large Numbers

n : number of samples

X_i : some estimate X for a sample size i

$$\text{as } n \rightarrow \infty \quad \frac{1}{n} \sum_{i=1}^n X_i \rightarrow \mathbb{E}[X] = \mu_X$$

Law of Large Numbers

Law of Large Numbers

n : number of samples

X_i : some estimate X for a sample size i

as $n \rightarrow \infty$

$$\frac{X_1 + X_2 + X_3 + \dots + X_n}{n} \longrightarrow \mathbb{E}[X] = \mu_X$$

UNDER CERTAIN CONDITIONS

as $n \rightarrow \infty$

$$\frac{1}{n} \sum_{i=1}^n X_i \rightarrow \mathbb{E}[X] = \mu_X$$

Law of Large Numbers

UNDER CERTAIN CONDITIONS

Law of Large Numbers

UNDER CERTAIN CONDITIONS

- Sample is randomly drawn.

Law of Large Numbers

UNDER CERTAIN CONDITIONS

- Sample is randomly drawn.

Law of Large Numbers

UNDER CERTAIN CONDITIONS

- Sample is randomly drawn.
- Sample size must be sufficiently large.

Law of Large Numbers

UNDER CERTAIN CONDITIONS

- Sample is randomly drawn.
- Sample size must be sufficiently large.

Law of Large Numbers

UNDER CERTAIN CONDITIONS

- Sample is randomly drawn.
- Sample size must be sufficiently large.
- Independent observations.



DeepLearning.AI

Sample and Population

Central Limit Theorem

Central Limit Theorem (CLT) - Example 1

If $n = 1$

Central Limit Theorem (CLT) - Example 1

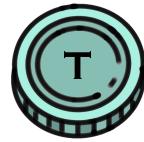


If $n = 1$

Central Limit Theorem (CLT) - Example 1



$$\mathbf{P}(H) = 0.5$$



$$\mathbf{P}(T) = 0.5$$

If $n = 1$

Central Limit Theorem (CLT) - Example 1



$$\mathbf{P}(H) = 0.5$$



$$\mathbf{P}(T) = 0.5$$

Random
variable $\xrightarrow{\hspace{1cm}}$ X

If $n = 1$

Central Limit Theorem (CLT) - Example 1



$$\mathbf{P}(H) = 0.5$$

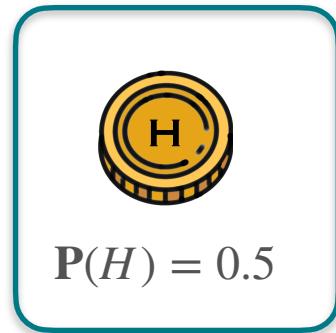


$$\mathbf{P}(T) = 0.5$$

Random variable $\rightarrow X$ number of heads when a coin is flipped n times

If $n = 1$

Central Limit Theorem (CLT) - Example 1



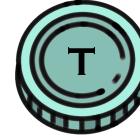
Random variable $\rightarrow X$ number of heads when a coin is flipped n times

If $n = 1$ $X = 1$

Central Limit Theorem (CLT) - Example 1



$$\mathbf{P}(H) = 0.5$$



$$\mathbf{P}(T) = 0.5$$

Random variable $\rightarrow X$ number of heads when a coin is flipped n times

$$\text{If } n = 1$$

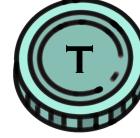
$$X = 1$$

$$X = 0$$

Central Limit Theorem (CLT) - Example 1



$$P(H) = 0.5$$



$$P(T) = 0.5$$

Random variable $\rightarrow X$ number of heads when a coin is flipped n times

$$\text{If } n = 1$$

$$X = 1$$

$$X = 0$$

Discrete Random Variable

Central Limit Theorem (CLT) - Example 1



$$P(H) = 0.5 \quad P(T) = 0.5$$

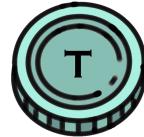
$$X = 1$$

$$X = 0$$

Central Limit Theorem (CLT) - Example 1



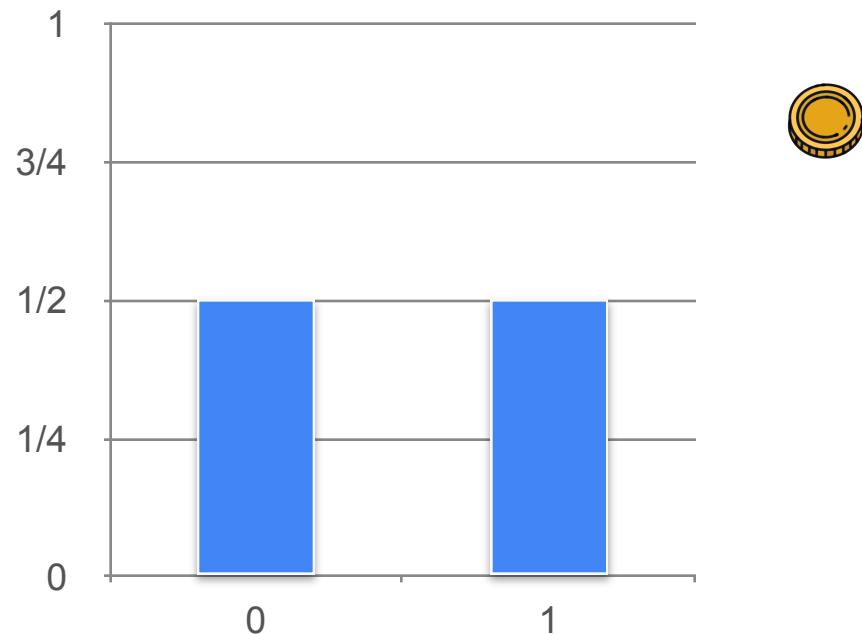
$$P(H) = 0.5$$



$$P(T) = 0.5$$

$$X = 1$$

$$X = 0$$



Central Limit Theorem (CLT) - Example 1



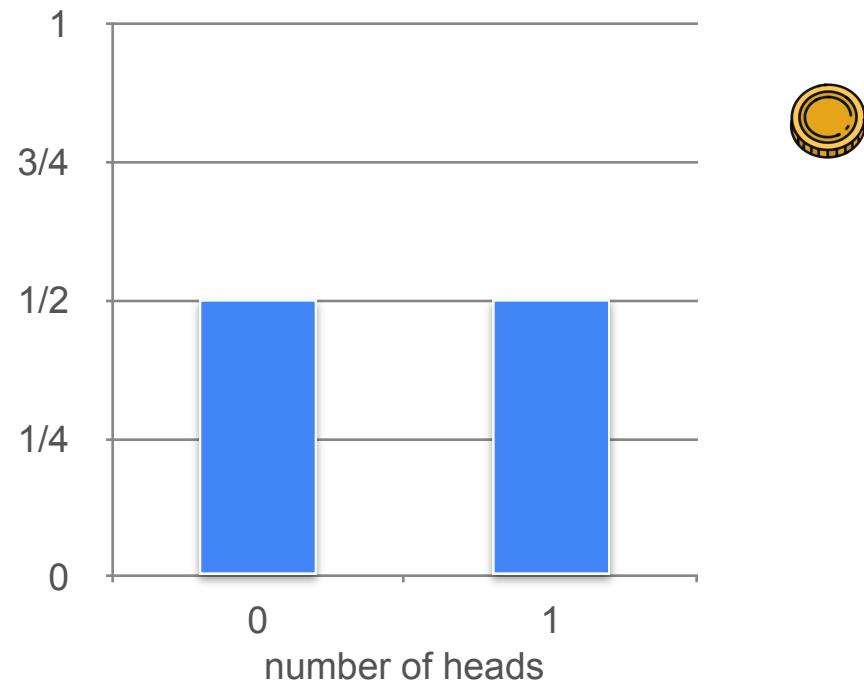
$$P(H) = 0.5$$



$$P(T) = 0.5$$

$$X = 1$$

$$X = 0$$



Central Limit Theorem (CLT) - Example 1



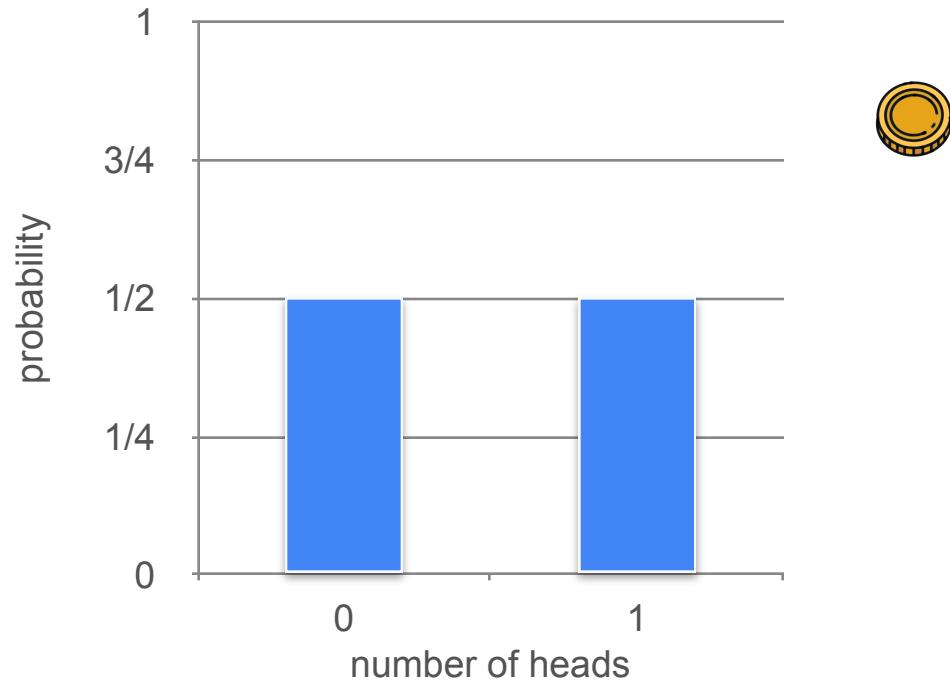
$$P(H) = 0.5$$

$$X = 1$$



$$P(T) = 0.5$$

$$X = 0$$



Central Limit Theorem (CLT) - Example 1



$$P(H) = 0.5$$

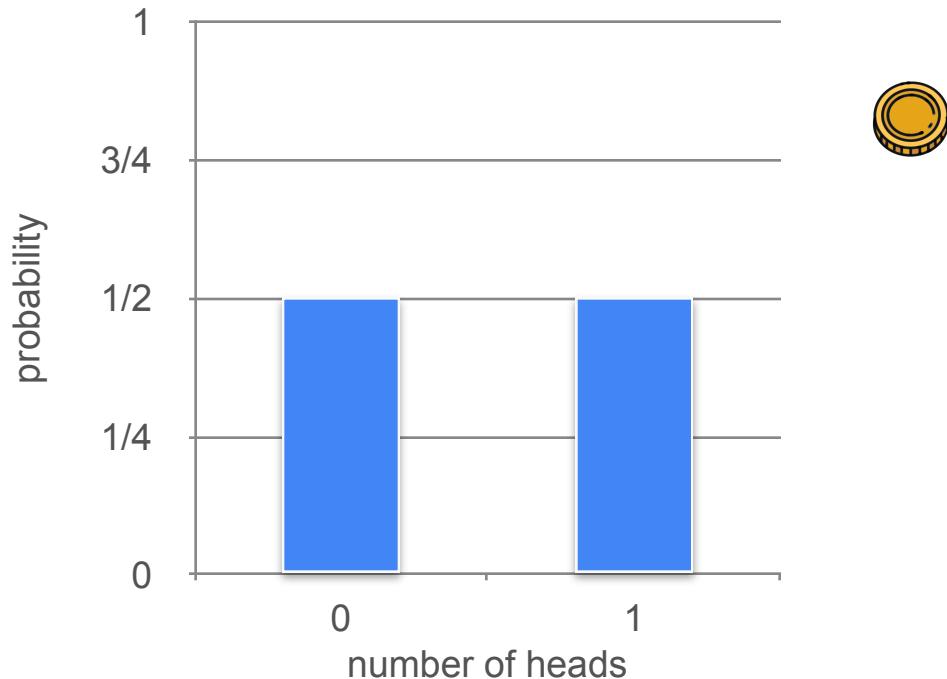


$$P(T) = 0.5$$

$$X = 1$$

$$X = 0$$

What can we say about the probability distribution when the number of coin flips increases?



Central Limit Theorem (CLT) - Example 1



$$P(H) = 0.5$$

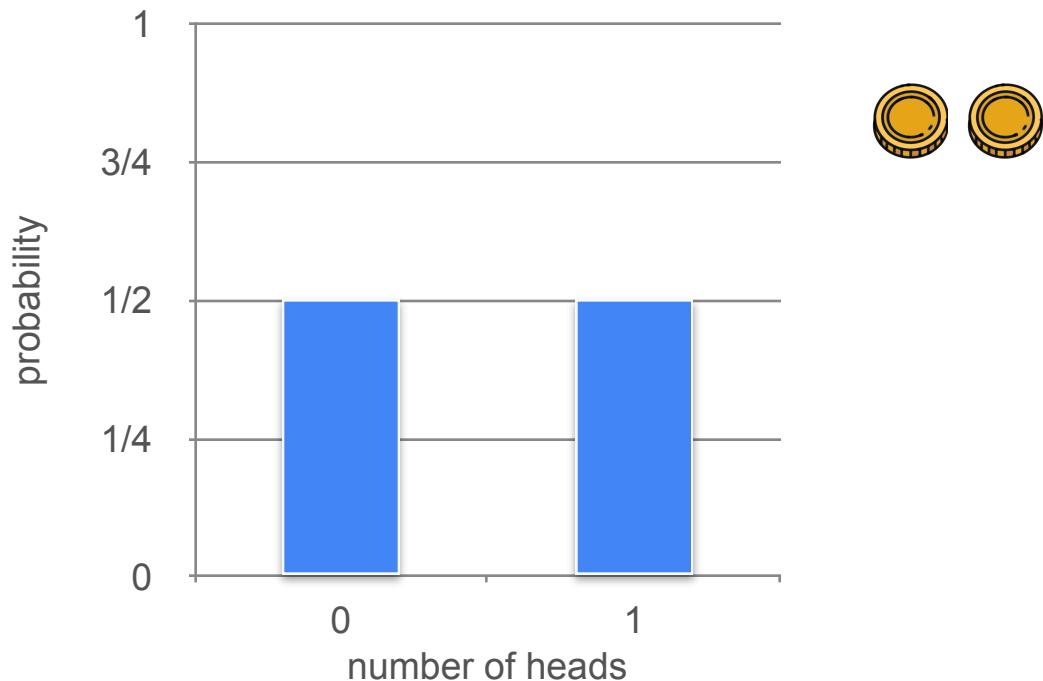


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Central Limit Theorem (CLT) - Example 1



$$P(H) = 0.5$$

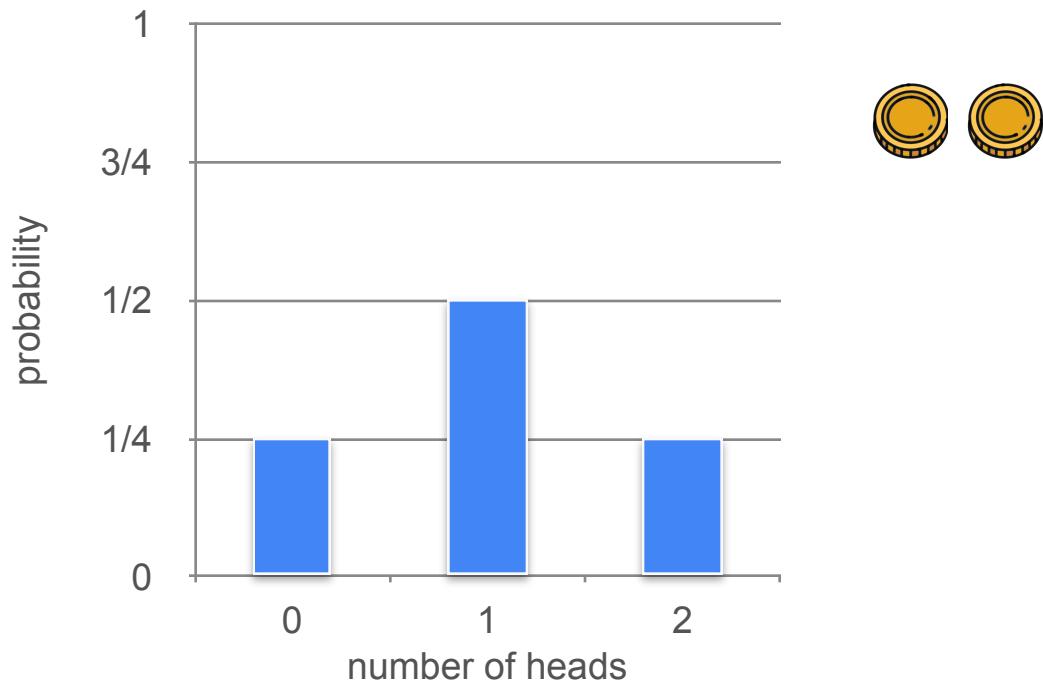


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Central Limit Theorem (CLT) - Example 1



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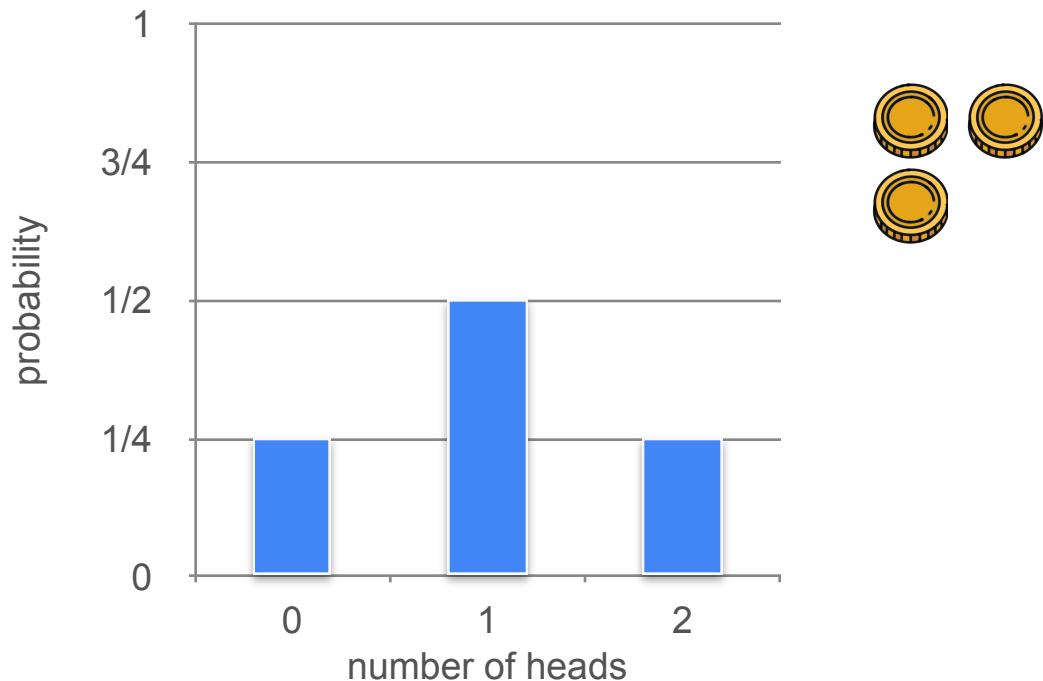


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Central Limit Theorem (CLT) - Example 1



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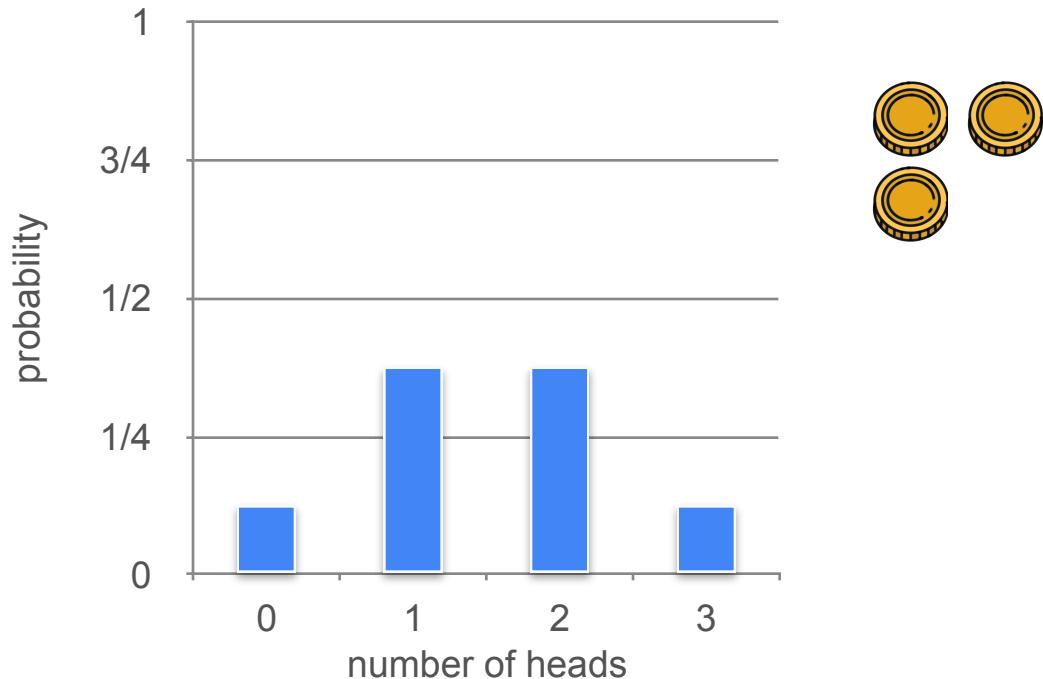


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Central Limit Theorem (CLT) - Example 1



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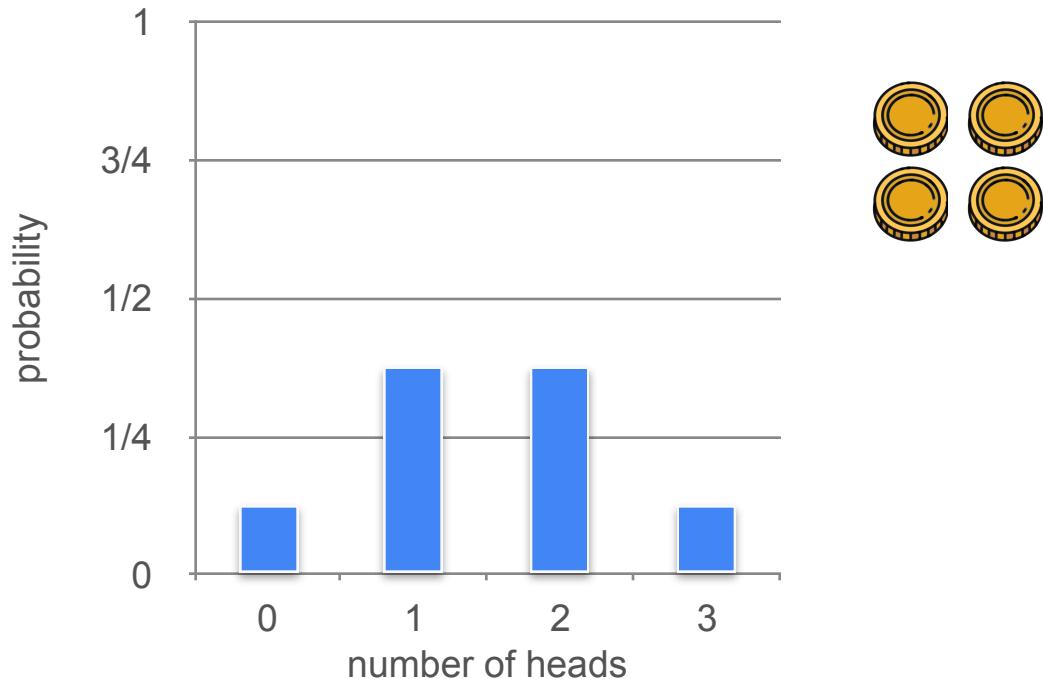


$$P(T) = 0.5$$

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Central Limit Theorem (CLT) - Example 1



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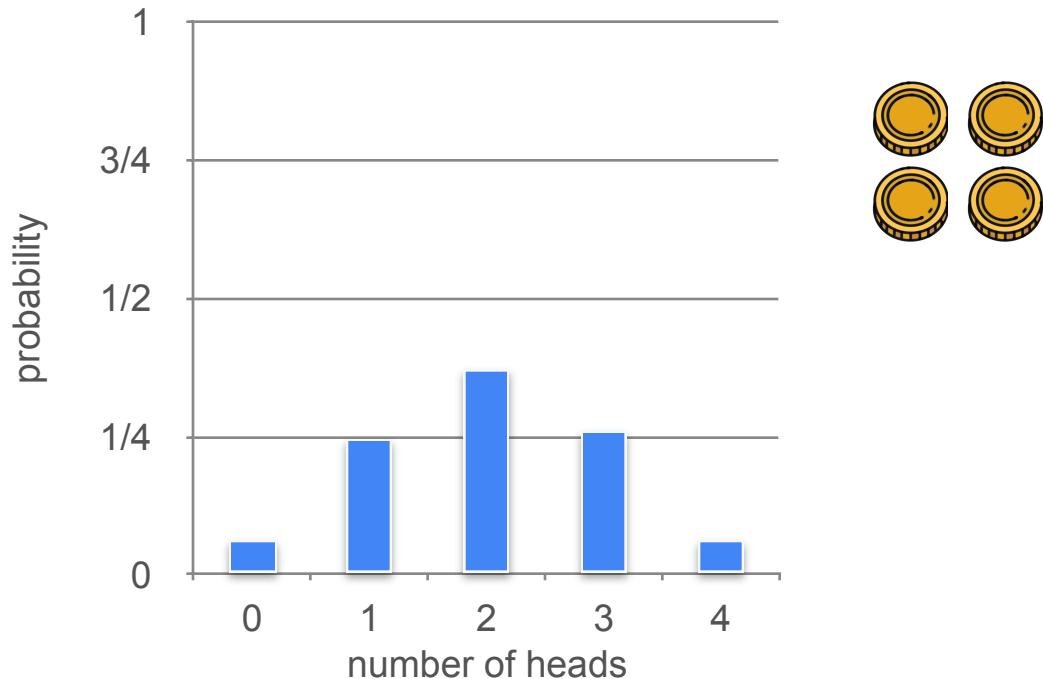


$$P(T) = 0.5$$

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What can we say about the probability distribution when the number of coin flips increases?



Central Limit Theorem (CLT) - Example 1



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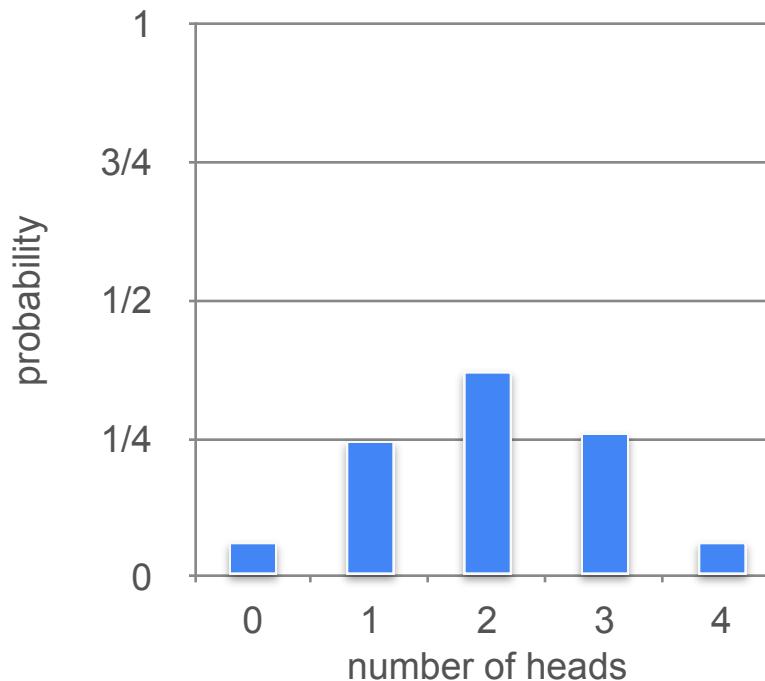


$$P(T) = 0.5$$

$$X = 1$$

$$X = 0$$

What can we say about the distribution when the number of coins we flip increases?



Central Limit Theorem (CLT) - Example 1



$$P(H) = 0.5$$

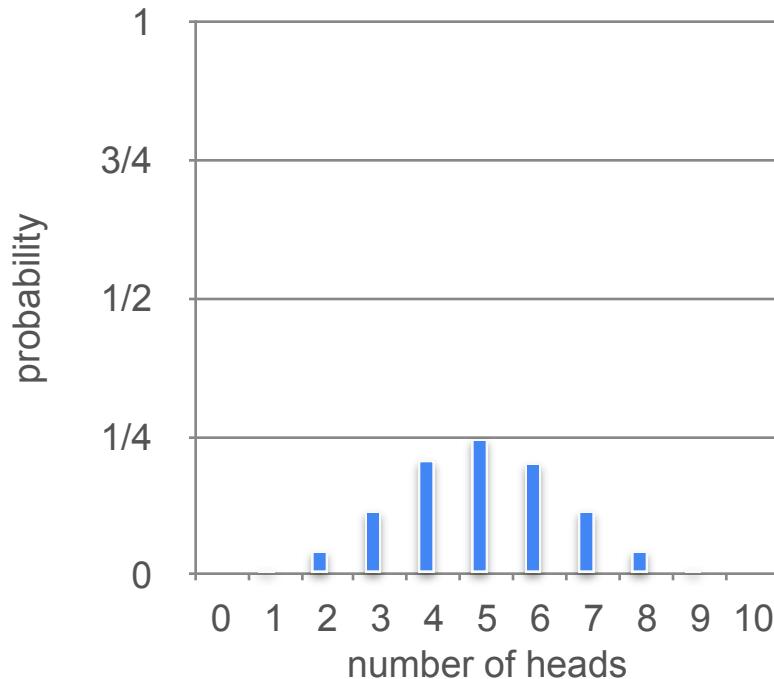


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What can we say about the distribution when the number of coins we flip increases?



Central Limit Theorem (CLT) - Example 1



$$P(H) = 0.5$$

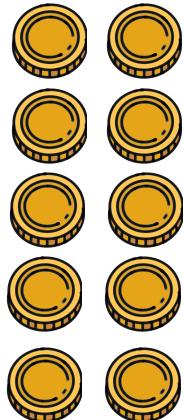
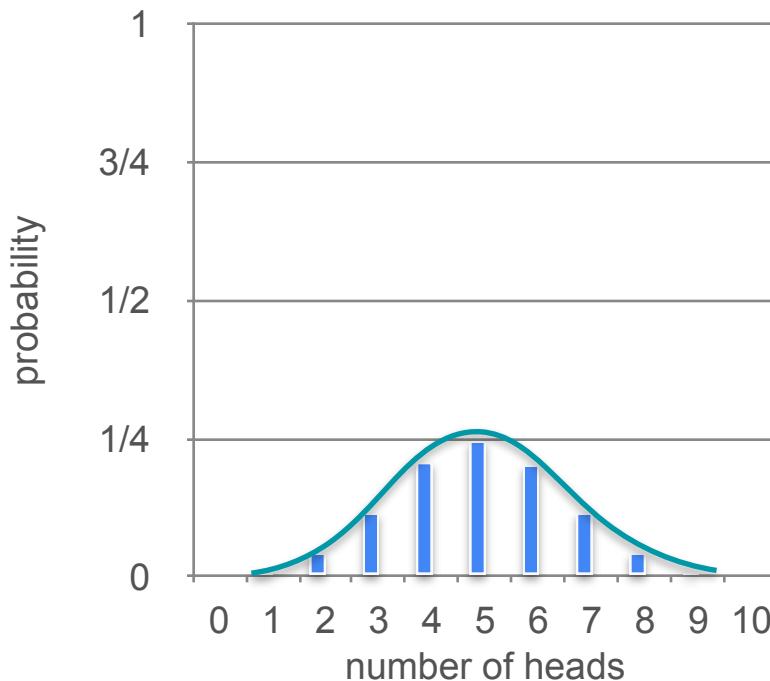


$$P(T) = 0.5$$

$$X = 1$$

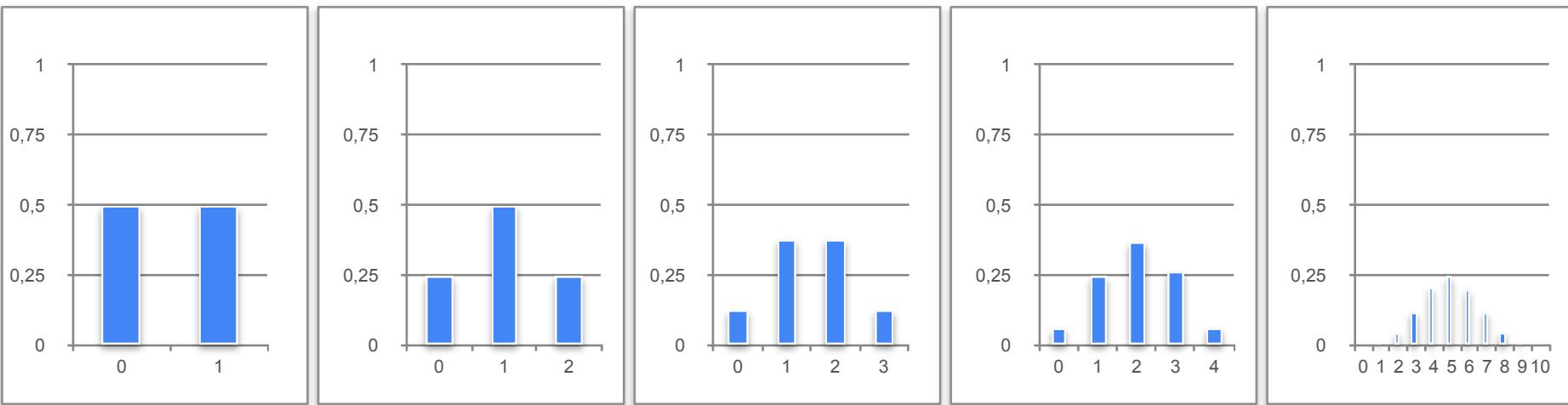
$$X = 0$$

What can we say about the distribution when the number of coins we flip increases?

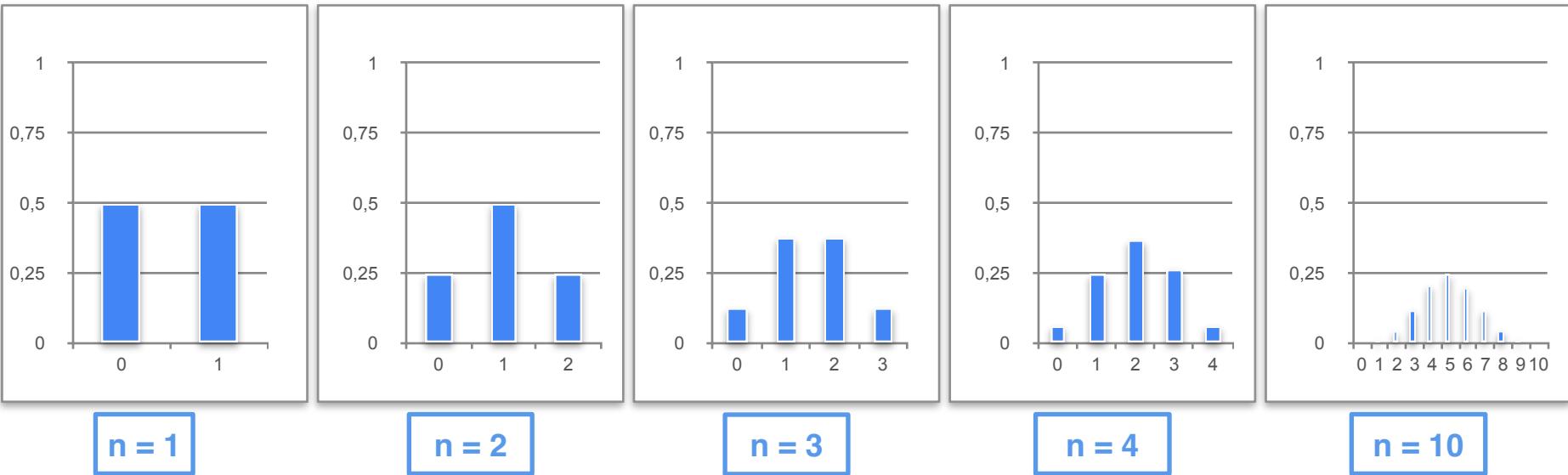


Central Limit Theorem (CLT) - Example 1

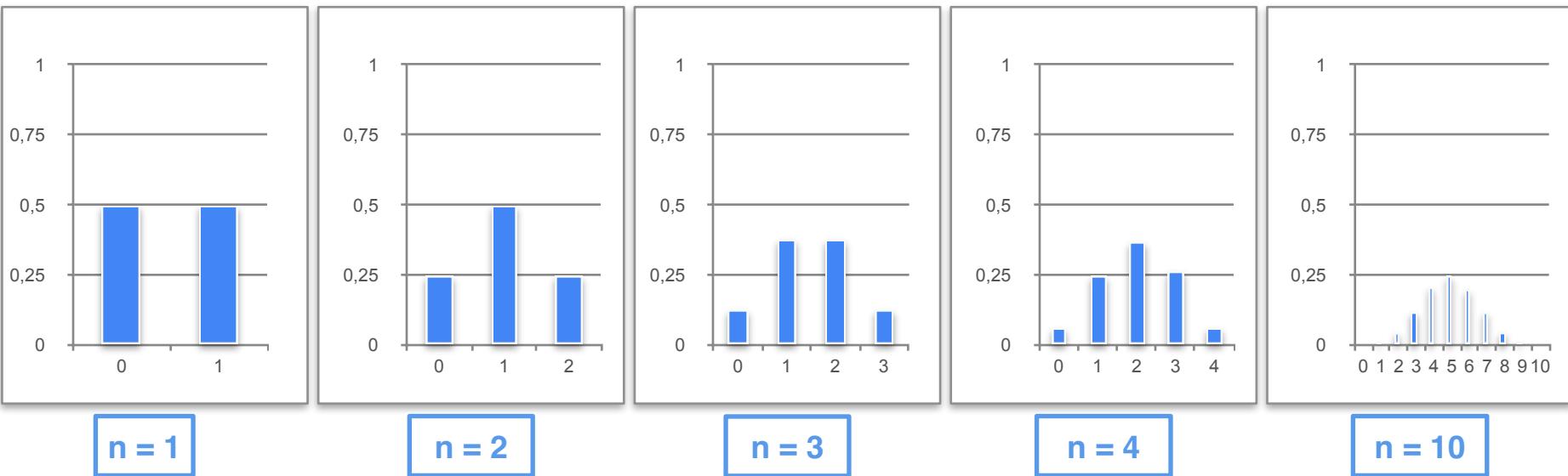
Central Limit Theorem (CLT) - Example 1



Central Limit Theorem (CLT) - Example 1



Central Limit Theorem (CLT) - Example 1



As n increases, the probability distribution becomes closer to a Gaussian distribution

Central Limit Theorem (CLT) - Example 1



$$\mathbf{P}(H) = 0.5$$



$$\mathbf{P}(T) = 0.5$$

Random
variable



X number of heads when a coin is flipped n times

Central Limit Theorem (CLT) - Example 1



$$\mathbf{P}(H) = 0.5$$



$$\mathbf{P}(T) = 0.5$$

Random variable

$$\rightarrow X$$

number of heads when a coin is flipped n times

$$\mu = np$$

Central Limit Theorem (CLT) - Example 1



$$\mathbf{P}(H) = 0.5$$



$$\mathbf{P}(T) = 0.5$$

Random variable



X number of heads when a coin is flipped n times

$$\mu = np = n\mathbf{P}(H)$$



Central Limit Theorem (CLT) - Example 1



$$\mathbf{P}(H) = 0.5$$



$$\mathbf{P}(T) = 0.5$$

Random variable



X number of heads when a coin is flipped n times

$$\mu = np = n\mathbf{P}(H)$$


$$\sigma^2 = np(1 - p)$$

Central Limit Theorem (CLT) - Example 1



$$\mathbf{P}(H) = 0.5$$



$$\mathbf{P}(T) = 0.5$$

Random variable



X number of heads when a coin is flipped n times

$$\mu = np = n\mathbf{P}(H)$$



$$\sigma^2 = np(1 - p) = n\mathbf{P}(H)\mathbf{P}(T)$$



Central Limit Theorem (CLT) - Example 1



$$\mathbf{P}(H) = 0.5 \quad \mathbf{P}(T) = 0.5$$

$$\mu = np = n\mathbf{P}(H)$$


$$\sigma^2 = np(1 - p) = n\mathbf{P}(H)\mathbf{P}(T)$$


Central Limit Theorem (CLT) - Example 1



$$\mathbf{P}(H) = 0.5 \quad \mathbf{P}(T) = 0.5$$

$$\mu = np = n\mathbf{P}(H)$$
A gold-colored coin with a black outline. The letter 'H' is printed in the center.

$$\sigma^2 = np(1 - p) = n\mathbf{P}(H)\mathbf{P}(T)$$
Two coins, one gold (H) and one teal (T), are shown side-by-side.

Central Limit Theorem (CLT) - Example 1



$n = 1$

$$\mathbf{P}(H) = 0.5 \quad \mathbf{P}(T) = 0.5$$

$$\mu = np = n\mathbf{P}(H)$$
A gold-colored coin with a black outline. The letter 'H' is printed in the center.

$$\sigma^2 = np(1 - p) = n\mathbf{P}(H)\mathbf{P}(T)$$
Two coins, one gold (H) and one teal (T), are shown side-by-side.

Central Limit Theorem (CLT) - Example 1



$$\mathbf{P}(H) = 0.5 \quad \mathbf{P}(T) = 0.5$$

$$\mu = np = n\mathbf{P}(H)$$
A gold-colored coin with a black outline and a central circle containing the letter 'H'.

$$\sigma^2 = np(1 - p) = n\mathbf{P}(H)\mathbf{P}(T)$$
Two coins, one gold (heads) and one teal (tails), side-by-side.



$n = 1$

$$\mu = np$$



Central Limit Theorem (CLT) - Example 1



$$\mathbf{P}(H) = 0.5 \quad \mathbf{P}(T) = 0.5$$

$$\mu = np = n\mathbf{P}(H)$$


$$\sigma^2 = np(1 - p) = n\mathbf{P}(H)\mathbf{P}(T)$$




$$n = 1$$

$$\mu = np$$



$$\mu = 1 \times 0.5$$

Central Limit Theorem (CLT) - Example 1



$$\mathbf{P}(H) = 0.5 \quad \mathbf{P}(T) = 0.5$$

$$\mu = np = n\mathbf{P}(H)$$


$$\sigma^2 = np(1 - p) = n\mathbf{P}(H)\mathbf{P}(T)$$




$$n = 1$$

$$\mu = np$$



$$\mu = 1 \times 0.5 = 0.5$$

Central Limit Theorem (CLT) - Example 1



$$\mathbf{P}(H) = 0.5 \quad \mathbf{P}(T) = 0.5$$

$$\mu = np = n\mathbf{P}(H)$$
Two gold-colored coins, one with 'H' and one with 'T'.

$$\sigma^2 = np(1 - p) = n\mathbf{P}(H)\mathbf{P}(T)$$
Two gold-colored coins, one with 'H' and one with 'T'.



$n = 1$

$$\mu = np$$



$$\mu = 1 \times 0.5 = 0.5$$

$$\sigma^2 = np(1 - p)$$



Central Limit Theorem (CLT) - Example 1



$$\mathbf{P}(H) = 0.5 \quad \mathbf{P}(T) = 0.5$$

$$\mu = np = n\mathbf{P}(H)$$
Two gold-colored coins, one with 'H' and one with 'T'.

$$\sigma^2 = np(1 - p) = n\mathbf{P}(H)\mathbf{P}(T)$$
Two gold-colored coins, one with 'H' and one with 'T'.



$$n = 1$$

$$\mu = np$$



$$\mu = 1 \times 0.5 = 0.5$$

$$\sigma^2 = np(1 - p)$$



$$\sigma^2 = (1 \times 0.5)(0.5)$$

Central Limit Theorem (CLT) - Example 1



$$\mathbf{P}(H) = 0.5 \quad \mathbf{P}(T) = 0.5$$

$$\mu = np = n\mathbf{P}(H)$$
Two gold-colored coins, one showing 'H' and the other showing 'T'.

$$\sigma^2 = np(1 - p) = n\mathbf{P}(H)\mathbf{P}(T)$$
Two gold-colored coins, one showing 'H' and the other showing 'T'.



$$n = 1$$

$$\mu = np$$



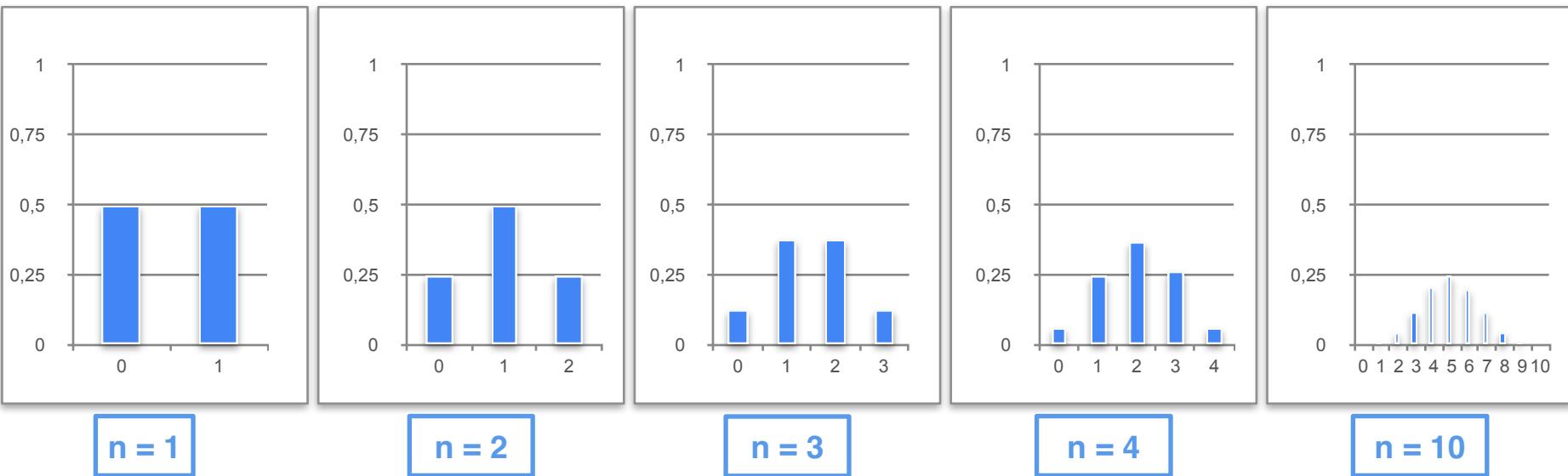
$$\mu = 1 \times 0.5 = 0.5$$

$$\sigma^2 = np(1 - p)$$



$$\sigma^2 = (1 \times 0.5)(0.5) = 0.25$$

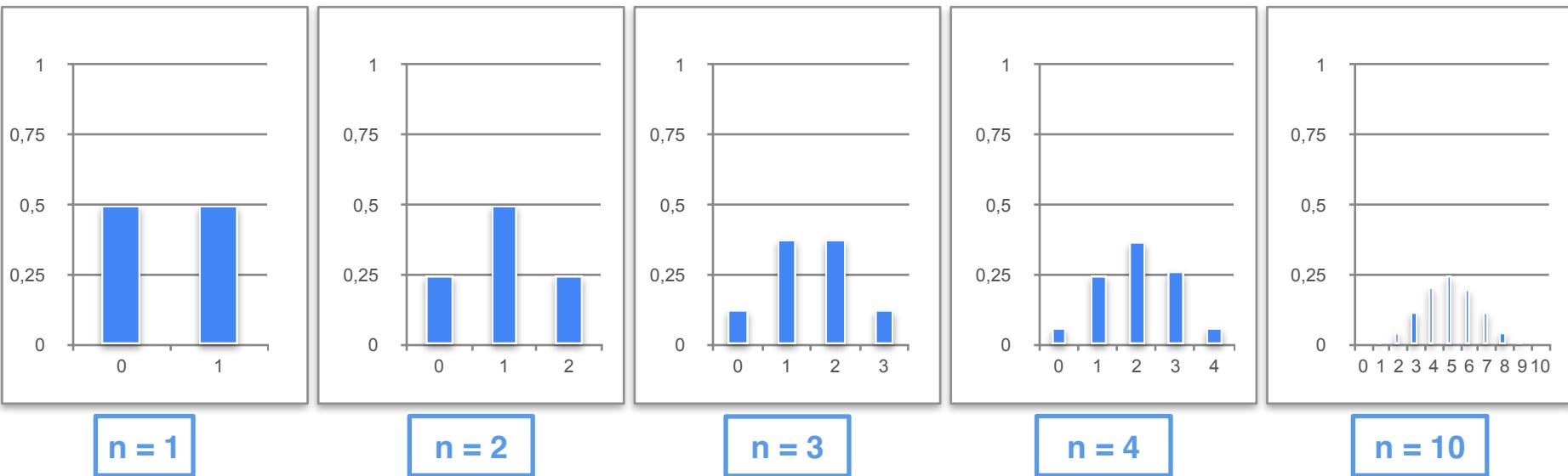
Central Limit Theorem (CLT) - Example 1



As n increases, the probability distribution becomes closer to a gaussian distribution

Central Limit Theorem (CLT) - Example 1

$$\mu = np$$

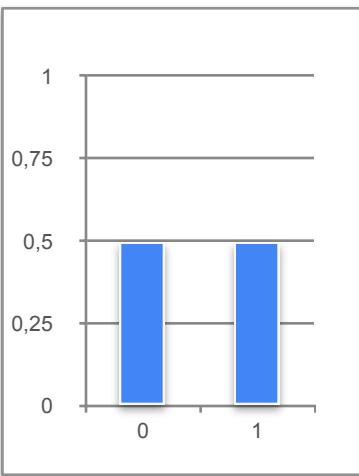


As n increases, the probability distribution becomes closer to a gaussian distribution

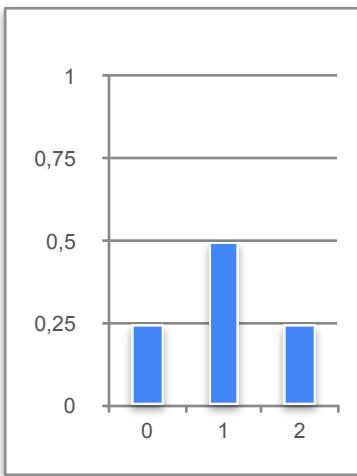
Central Limit Theorem (CLT) - Example 1

$$\mu = np$$

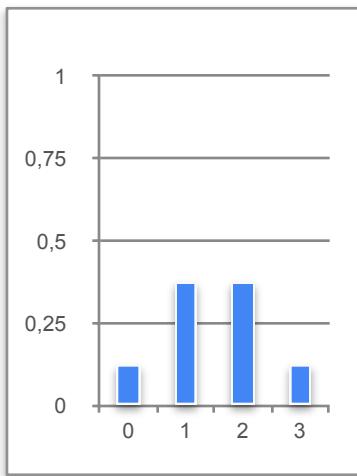
$$\sigma^2 = np(1 - p)$$



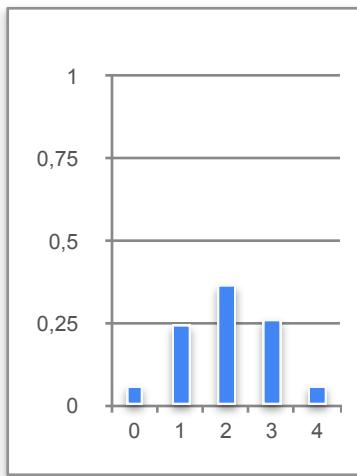
n = 1



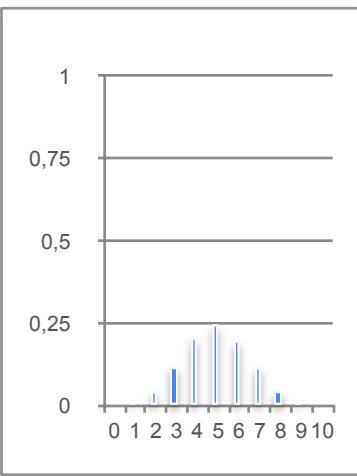
n = 2



n = 3



n = 4



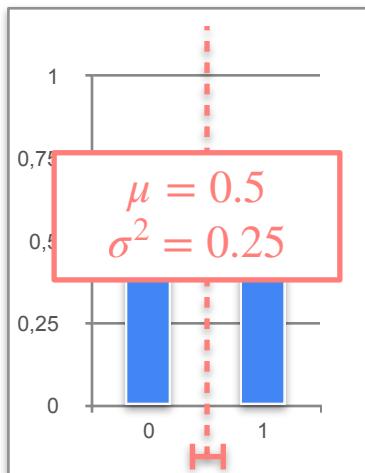
n = 10

As n increases, the probability distribution becomes closer to a gaussian distribution

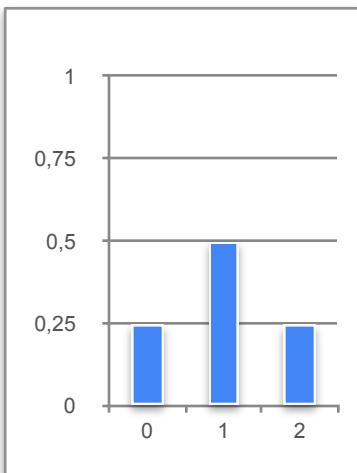
Central Limit Theorem (CLT) - Example 1

$$\mu = np$$

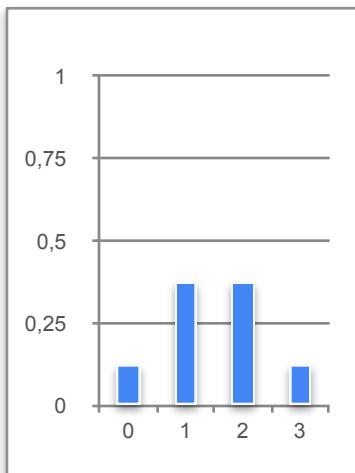
$$\sigma^2 = np(1 - p)$$



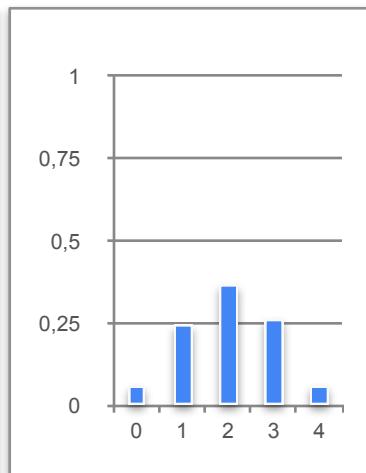
$n = 1$



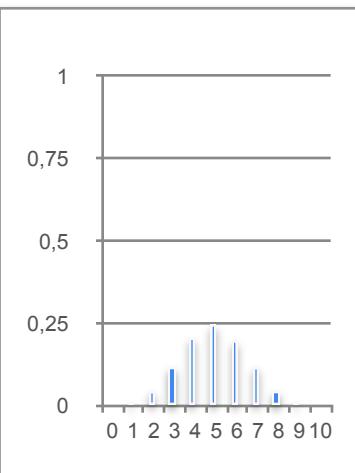
$n = 2$



$n = 3$



$n = 4$



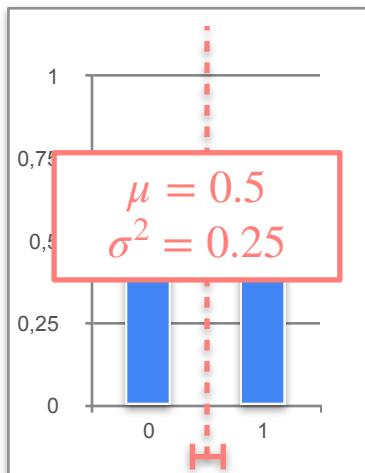
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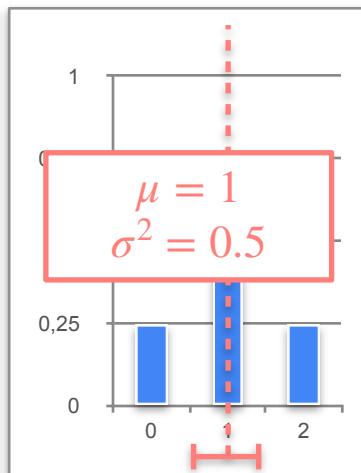
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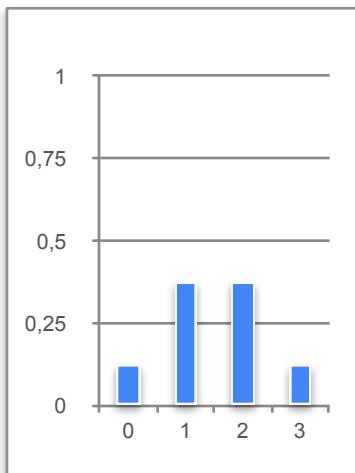
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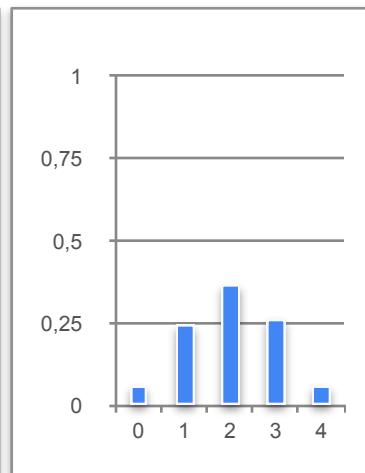
$n = 1$



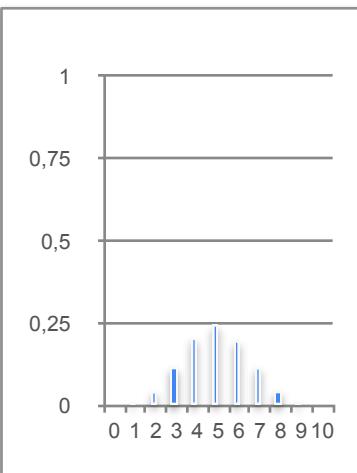
$n = 2$



$n = 3$



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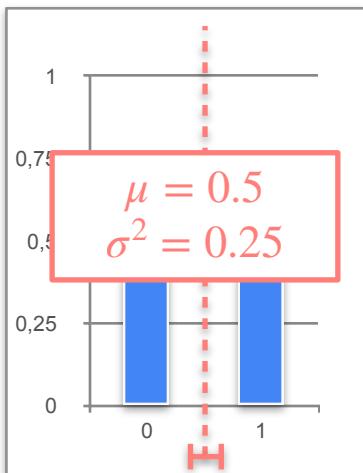
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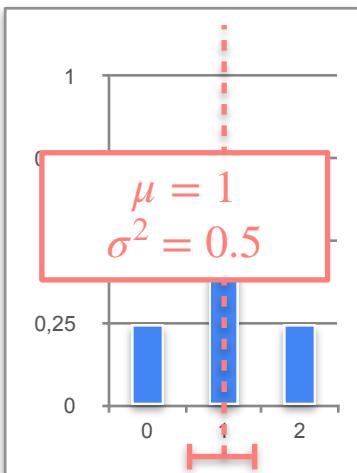
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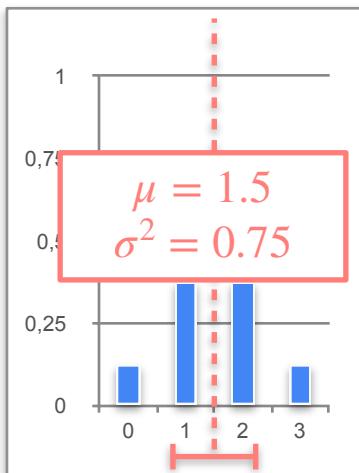
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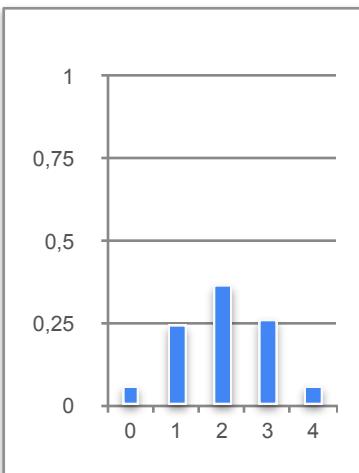
$n = 1$



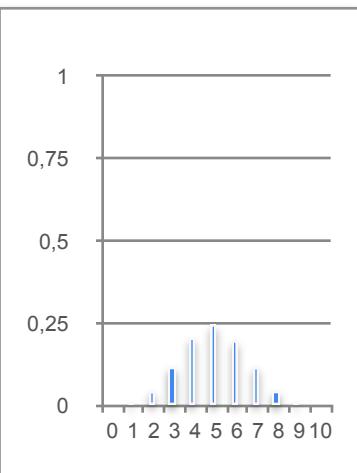
$n = 2$



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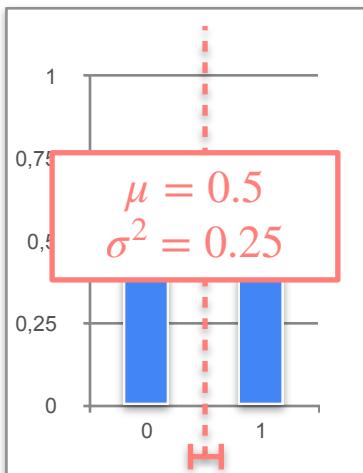
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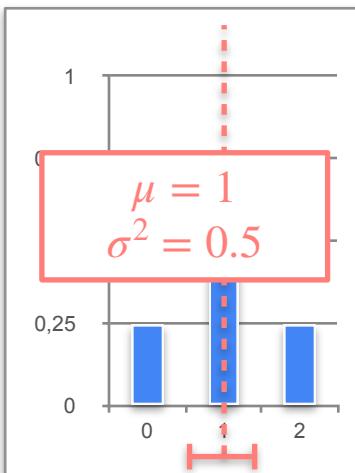
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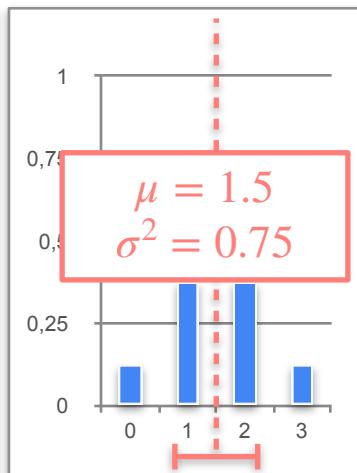
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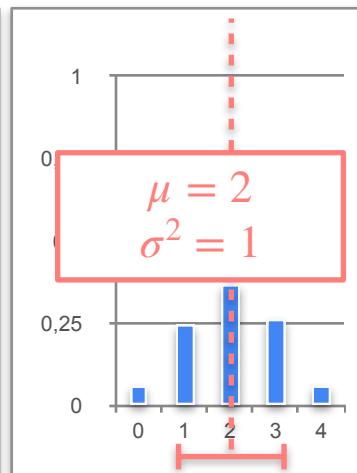
$n = 1$



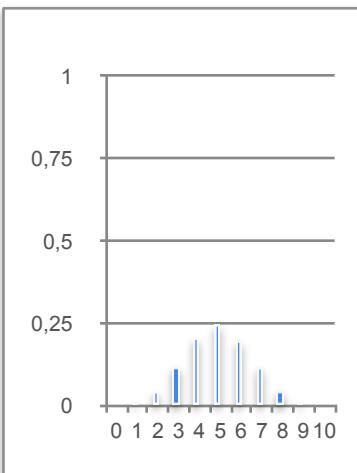
$n = 2$



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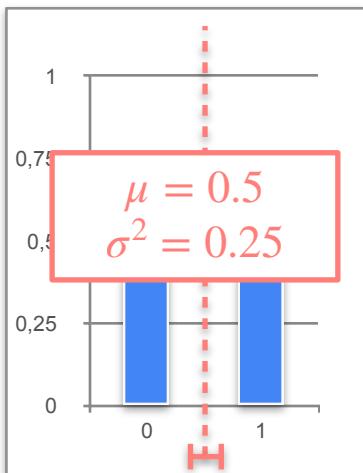
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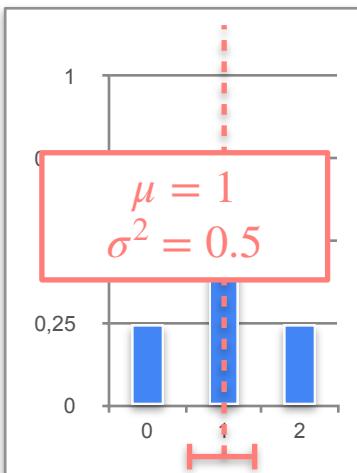
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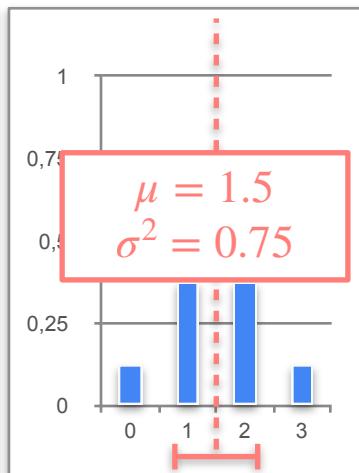
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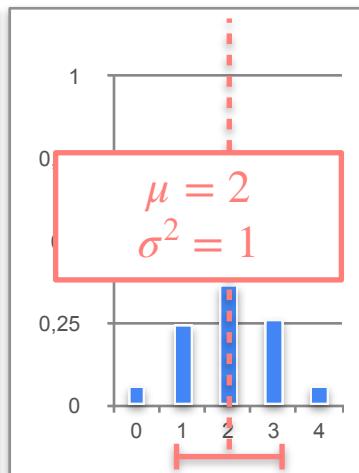
$n = 1$



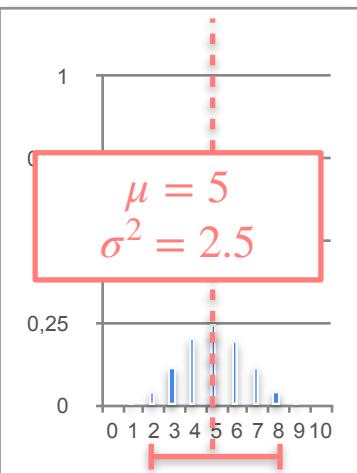
$n = 2$



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Central Limit Theorem (CLT) - Example 1

$$\begin{aligned}\mu &= 0.5 \\ \sigma^2 &= 0.25\end{aligned}$$

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$$\begin{aligned}\mu &= 1.5 \\ \sigma^2 &= 0.75\end{aligned}$$

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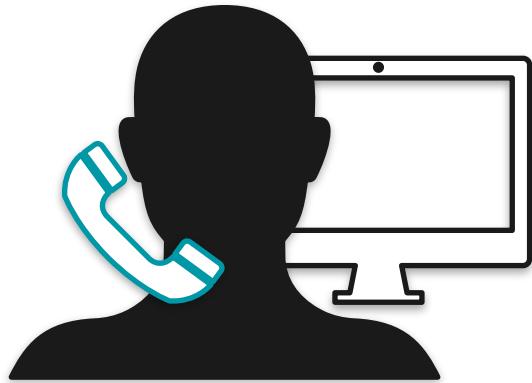
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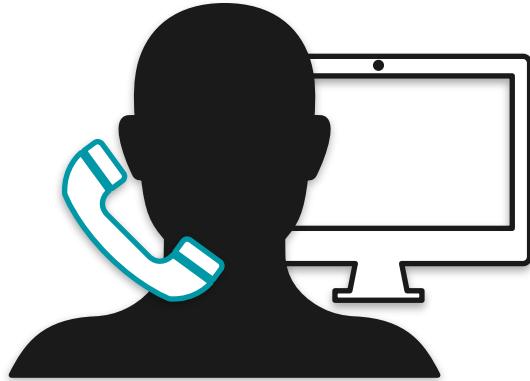
Video 4b: Central Limit Theorem

- Example 2: Continuous Random Variable (Sample mean of a sampling distribution)

Uniform Distribution: Motivation

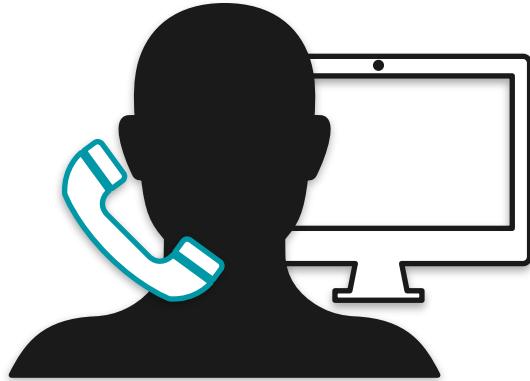


Uniform Distribution: Motivation



You're calling a tech support line. They can answer any time between zero and 15 minutes and if they don't answer in this time, the line is disconnected.

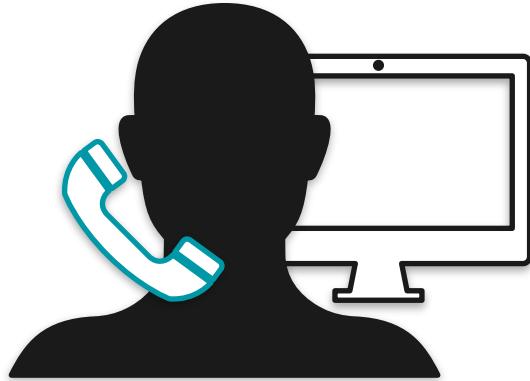
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$$X \sim \mathcal{U}(0,15)$$

Central Limit Theorem (CLT) - Example 2

$$n = 1 \quad Y_1 = \frac{X_1}{1}$$

Central Limit Theorem (CLT) - Example 2

$$n = 1 \quad Y_1 = \frac{X_1}{1}$$

$$n = 2 \quad Y_2 = \frac{X_1 + X_2}{2}$$

Central Limit Theorem (CLT) - Example 2

$$n = 1 \quad Y_1 = \frac{X_1}{1}$$

$$n = 2 \quad Y_2 = \frac{X_1 + X_2}{2}$$

$$n = 3 \quad Y_3 = \frac{X_1 + X_2 + X_3}{3}$$

⋮

⋮

Central Limit Theorem (CLT) - Example 2

$$n = 1$$

$$Y_1 = \frac{X_1}{1}$$

Record the average of all n experiments

$$n = 2$$

$$Y_2 = \frac{X_1 + X_2}{2}$$

$$n = 3$$

$$Y_3 = \frac{X_1 + X_2 + X_3}{3}$$

⋮

⋮

Central Limit Theorem (CLT) - Example 2

$$n = 1$$

$$Y_1 = \frac{X_1}{1}$$

Record the average of all n experiments

$$n = 2$$

$$Y_2 = \frac{X_1 + X_2}{2}$$

$$Y_n = \frac{1}{n} \sum_{i=1}^n X_i$$

$$n = 3$$

$$Y_3 = \frac{X_1 + X_2 + X_3}{3}$$

⋮

⋮

Central Limit Theorem (CLT) - Example 2

$$n = 1$$

$$Y_1 = \frac{X_1}{1}$$

Record the average of all n experiments

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$$n = 3$$

$$Y_3 = \frac{X_1 + X_2 + X_3}{3}$$

⋮

⋮

Can we say anything about the distribution of this average?

Central Limit Theorem (CLT) - Example 2

$$n = 1 \quad Y_1 = \frac{X_1}{1}$$

Central Limit Theorem (CLT) - Example 2

$$n = 1 \quad Y_1 = \frac{X_1}{1}$$

What happens to the distribution of these averages as n increases?

Central Limit Theorem (CLT) - Example 2

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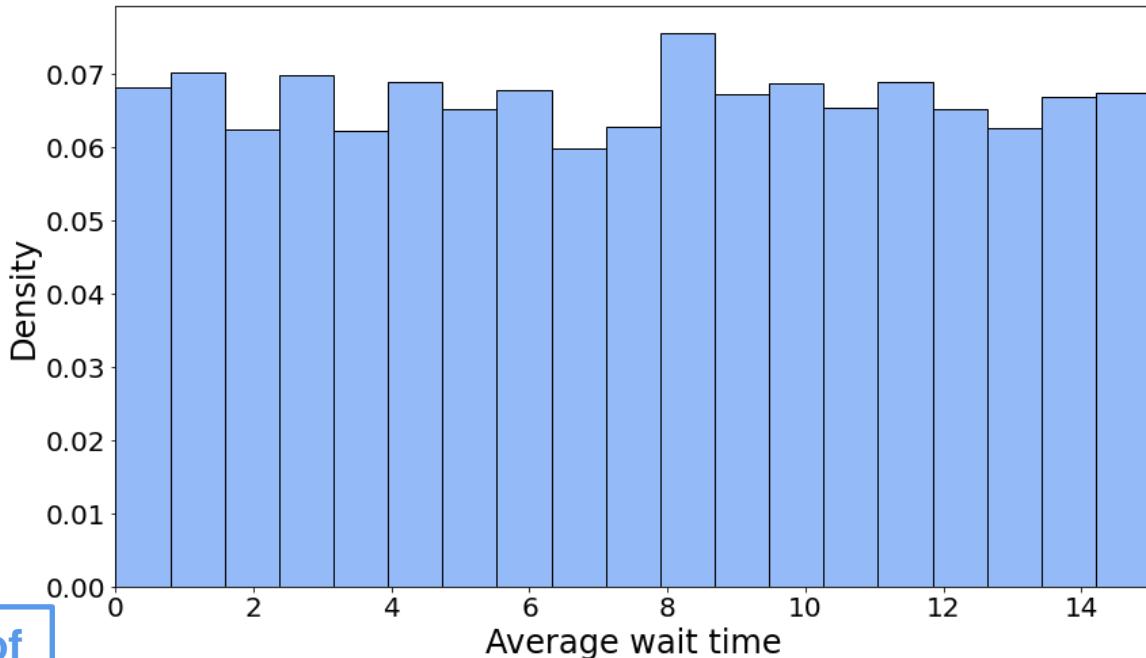
Create many samples of Y_1 so you can get a pretty histogram

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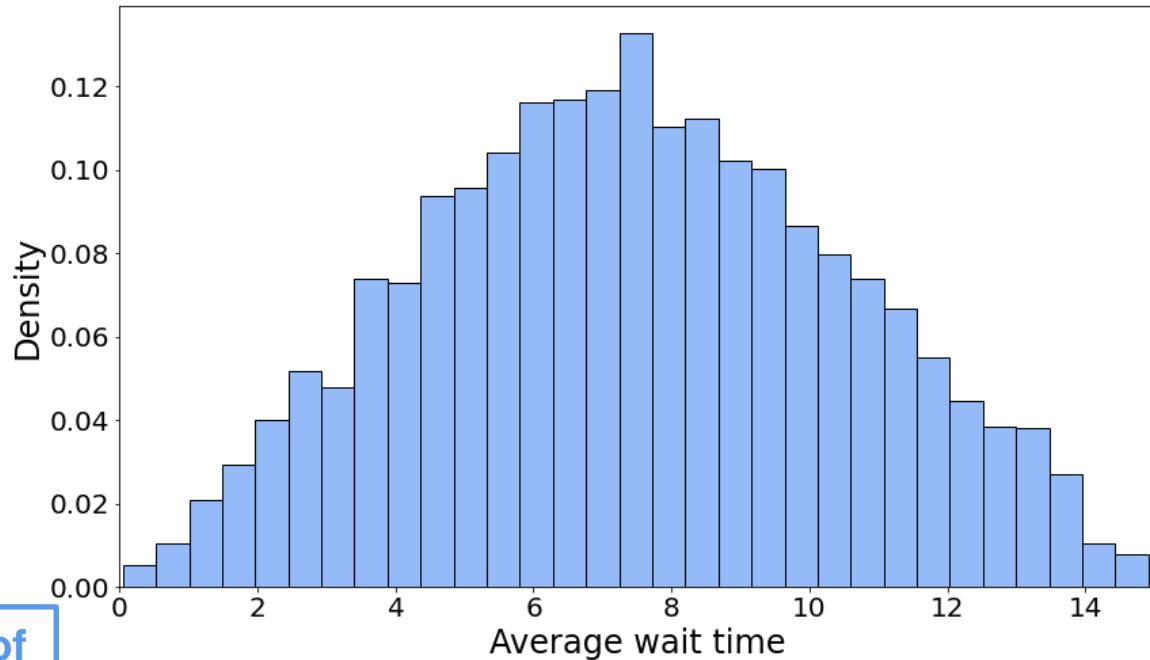


What happens to the distribution of these averages as n increases?

Central Limit Theorem (CLT) - Example 2

$$n = 2 \quad Y_2 = \frac{X_1 + X_2}{2}$$

Create many samples of Y_2 so you can get a pretty histogram

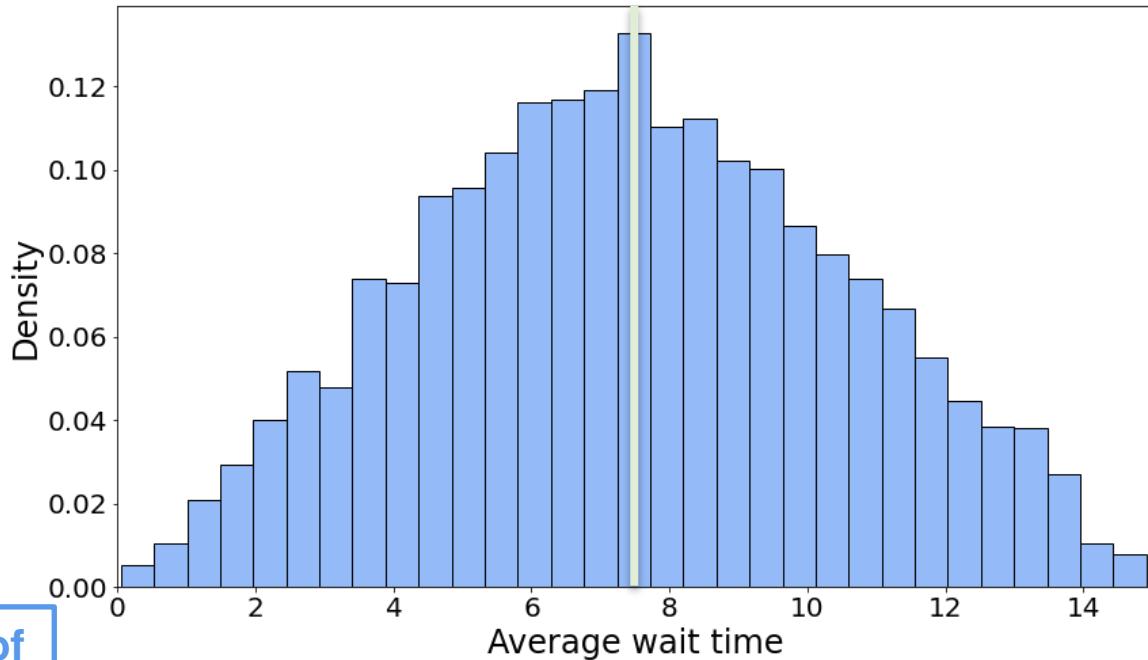


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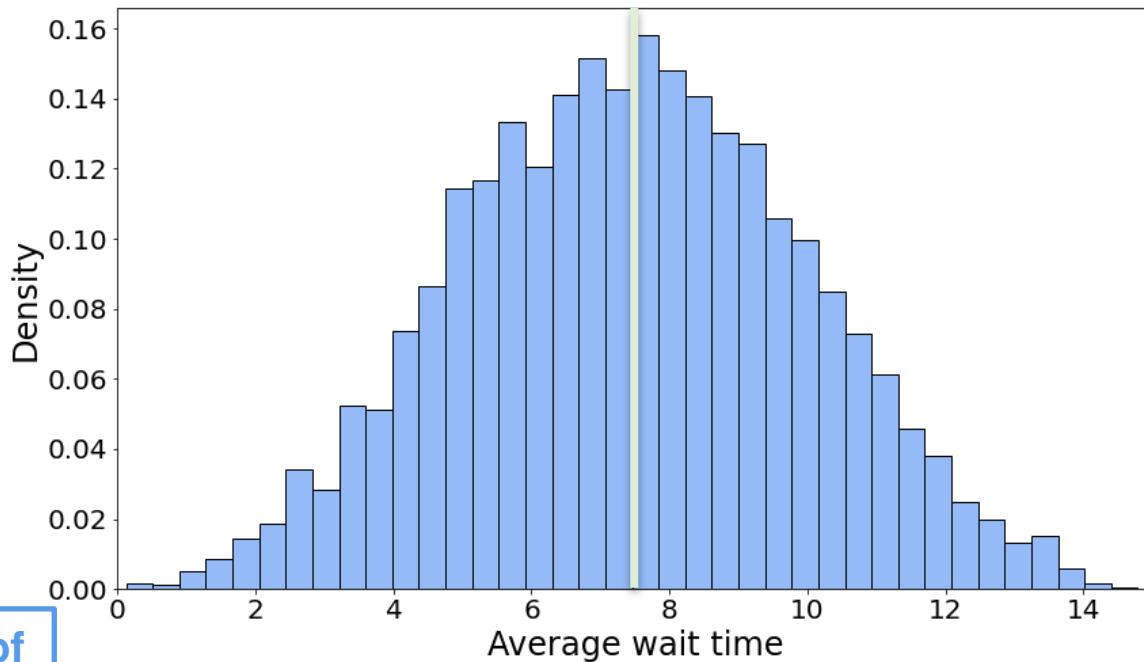
What happens to the distribution of these averages as n increases

Central Limit Theorem (CLT) - Example 2

$$n = 3 \quad Y_3 = \frac{X_1 + X_2 + X_3}{3}$$

Create many samples of Y_3 so you can get a pretty histogram

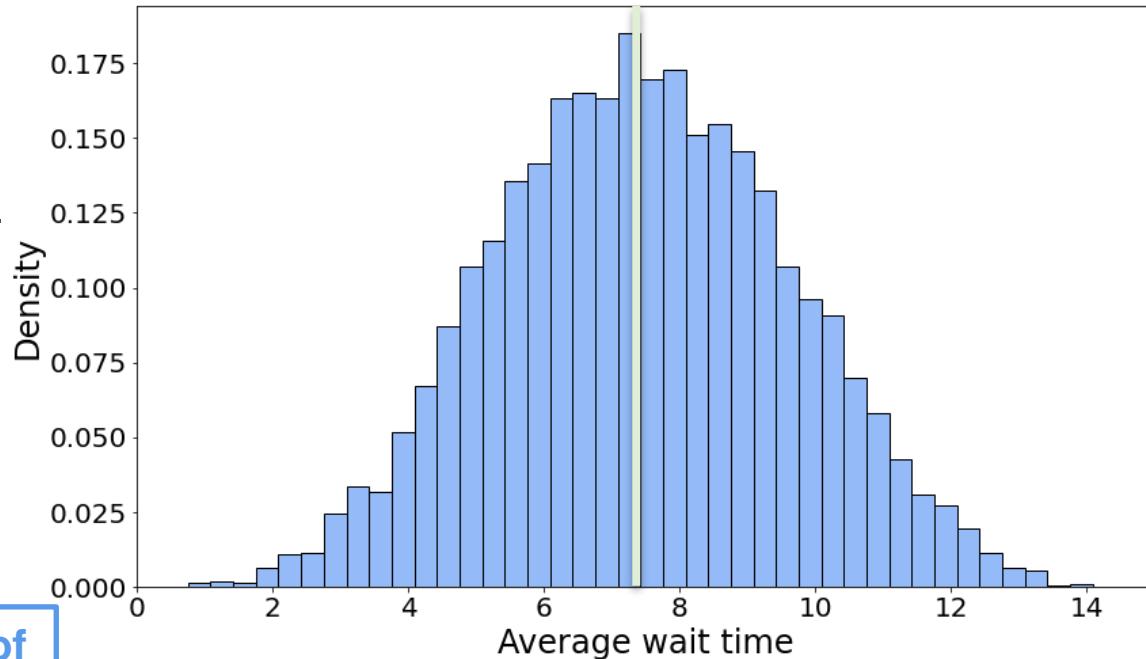
What happens to the distribution of these averages as n increases



Central Limit Theorem (CLT) - Example 2

$$n = 4 \quad Y_4 = \frac{X_1 + X_2 + X_3 + X_4}{4}$$

Create many samples of Y_4 so you can get a pretty histogram

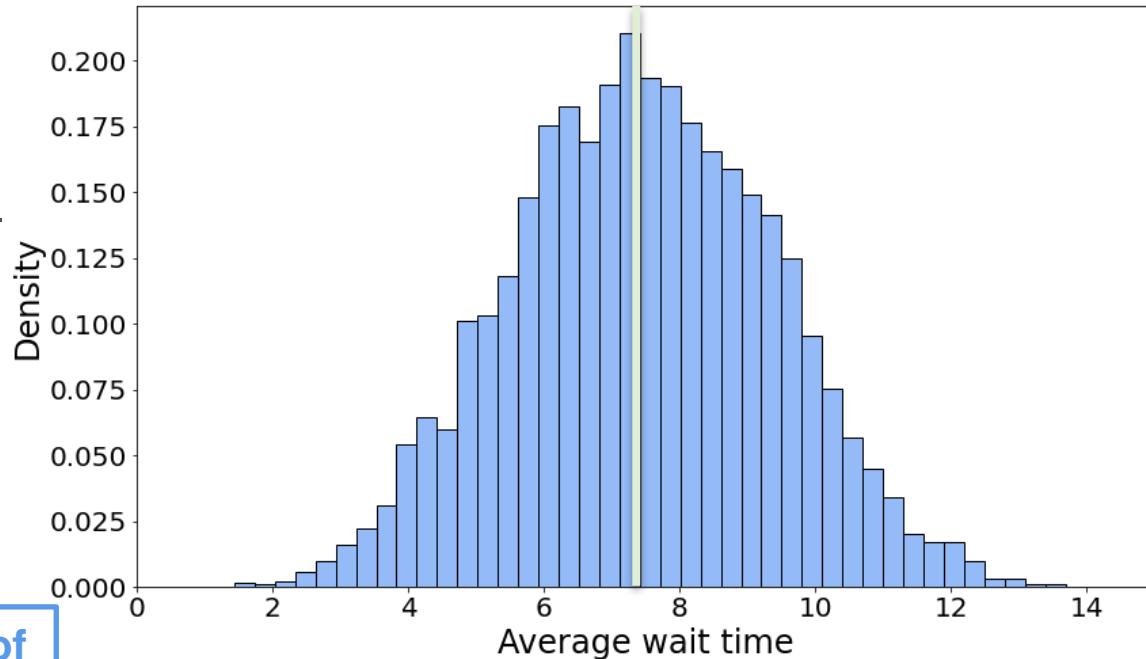


What happens to the distribution of these averages as n increases

Central Limit Theorem (CLT) - Example 2

$$n = 5 \quad Y_5 = \frac{X_1 + X_2 + X_3 + X_4 + X_5}{5}$$

Create many samples of Y_5 so you can get a pretty histogram

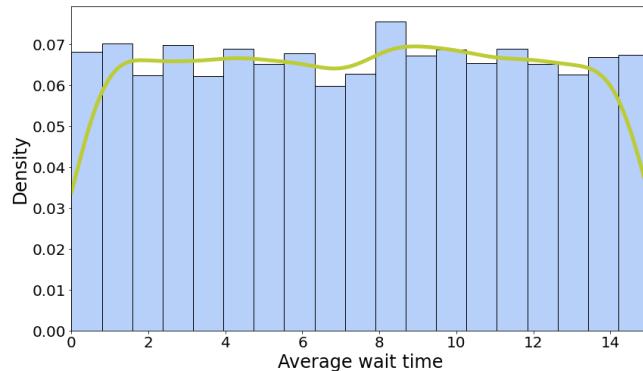


What happens to the distribution of these averages as n increases

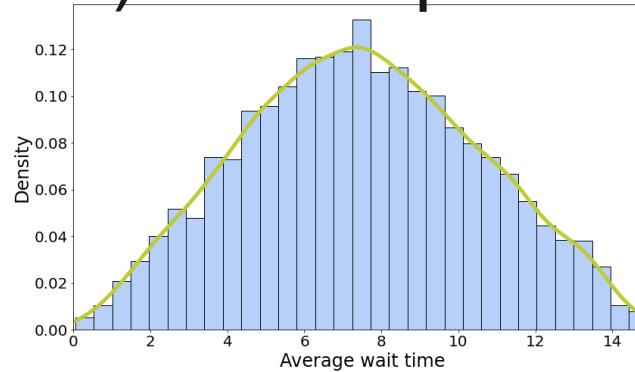
Central Limit Theorem (CLT) - Example 2

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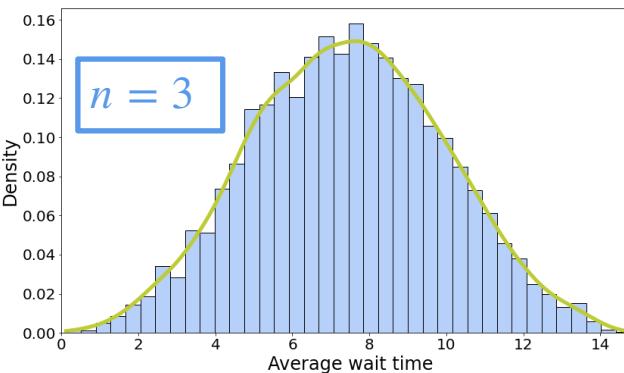
$n = 1$



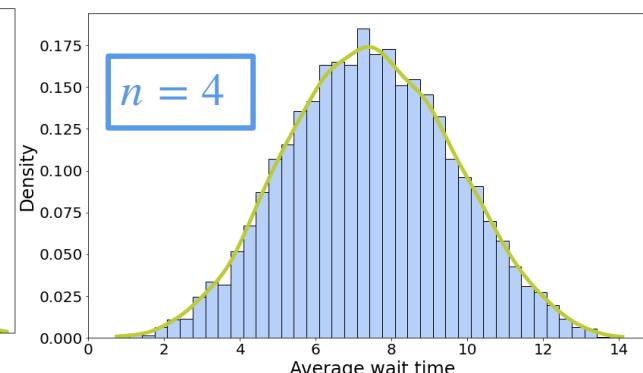
$n = 2$



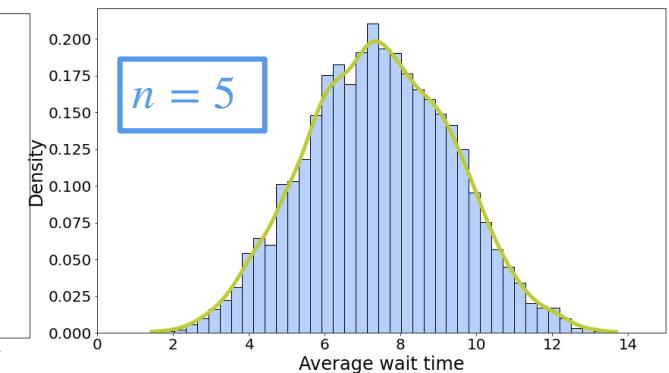
$n = 3$



$n = 4$



$n = 5$



Central Limit Theorem (CLT) - Example 2

$$= \frac{1}{n^2}$$

Central Limit Theorem (CLT) - Example 2

$$\mathbb{E}[Y_n] = \mathbb{E} \left[\frac{1}{n} \sum_{i=1}^n X_i \right]$$

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$$\mathbb{E}[Y_n] = \mathbb{E} \left[\frac{1}{n} \sum_{i=1}^n X_i \right] = \frac{1}{n} \sum_{i=1}^n \mathbb{E}[X_i]$$

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$$Var[Y_n] = Var \left(\frac{1}{n} \sum_{i=1}^n X_i \right) = \frac{1}{n^2}$$

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$$\mathbb{E}[Y_n] = \mathbb{E} \left[\frac{1}{n} \sum_{i=1}^n X_i \right] = \frac{1}{n} \sum_{i=1}^n \mathbb{E}[X_i] = \frac{1}{n} n \mathbb{E}[X] = \mathbb{E}[X] = 7.5$$

$$Var[Y_n] = Var \left(\frac{1}{n} \sum_{i=1}^n X_i \right) = \frac{1}{n^2} \sum_{i=1}^n Var(X_i)$$

Central Limit Theorem (CLT) - Example 2

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$$\begin{aligned} Var[Y_n] &= Var \left(\frac{1}{n} \sum_{i=1}^n X_i \right) = \frac{1}{n^2} \sum_{i=1}^n Var(X_i) \\ &= \frac{1}{n^2} n Var(X) = \frac{Var(X)}{n} \end{aligned}$$

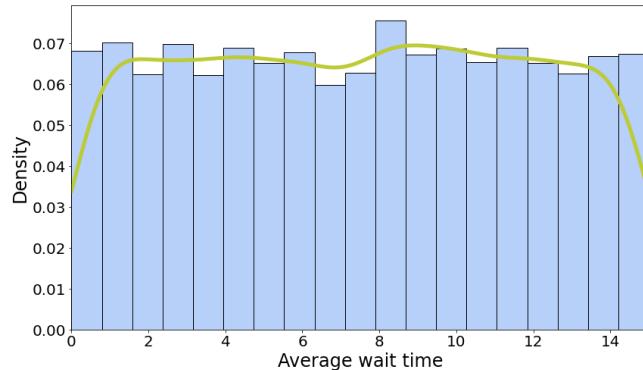
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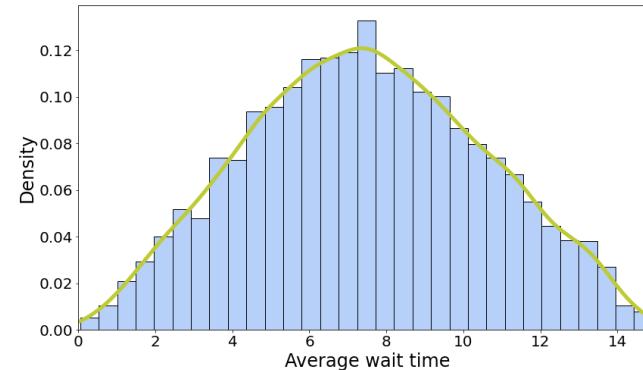
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Central Limit Theorem (CLT) - Example 2

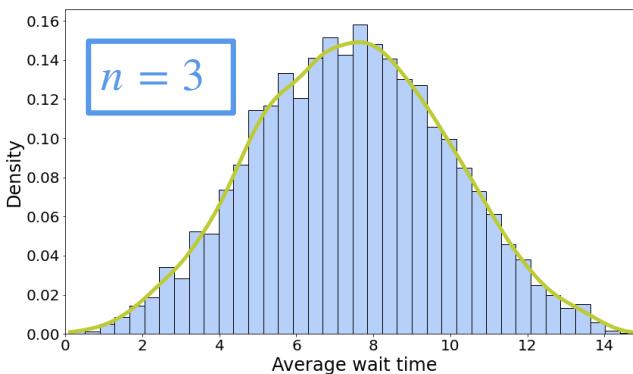
$n = 1$



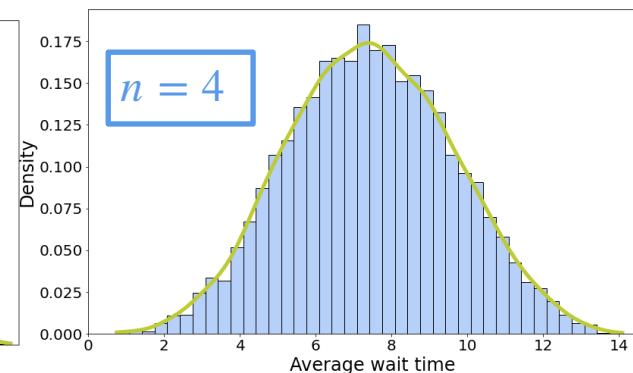
$n = 2$



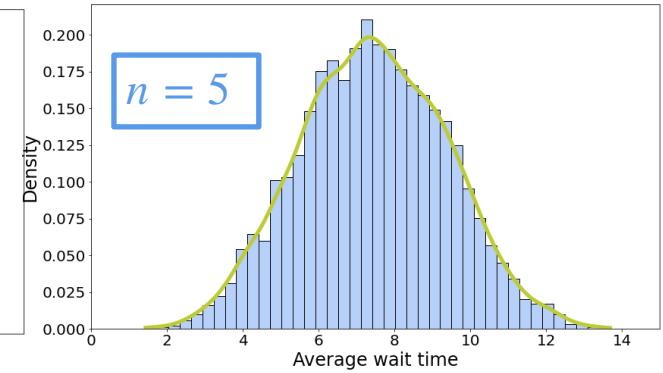
$n = 3$



$n = 4$

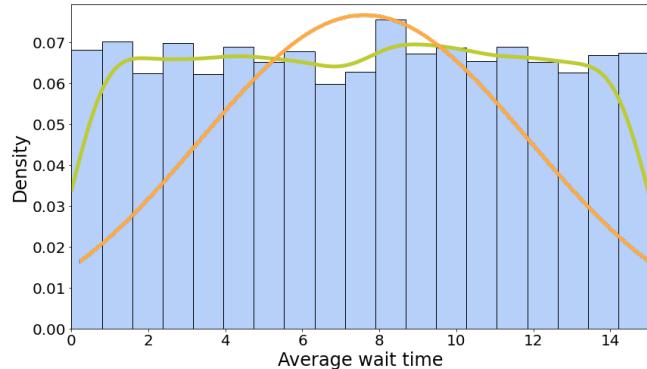


$n = 5$

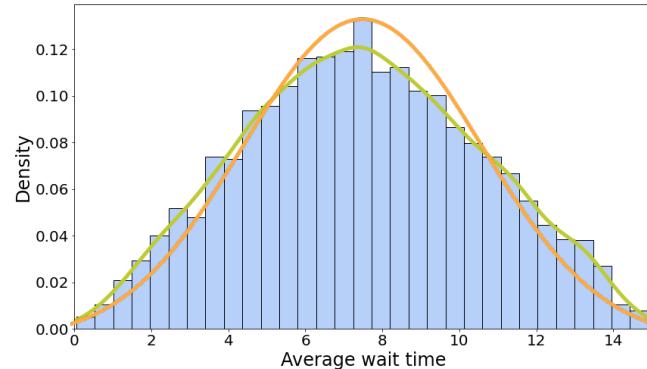


Central Limit Theorem (CLT) - Example 2

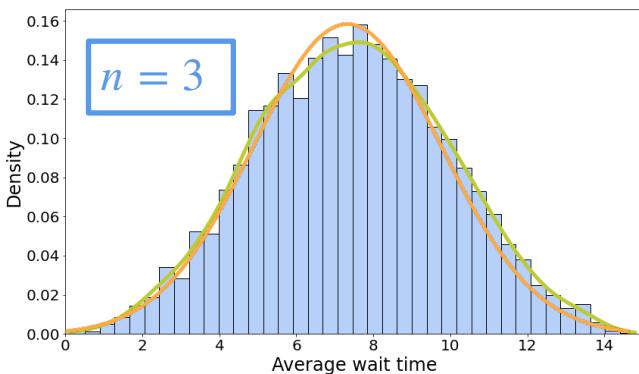
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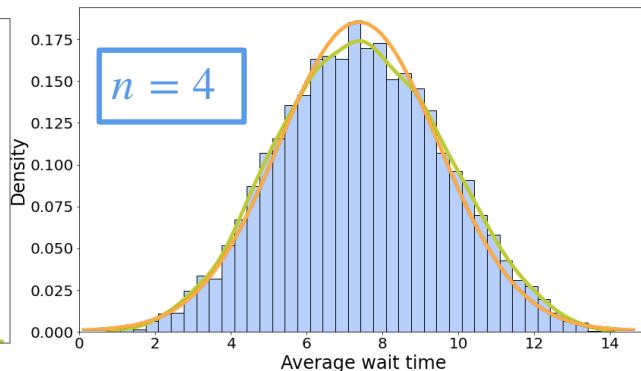
$n = 2$



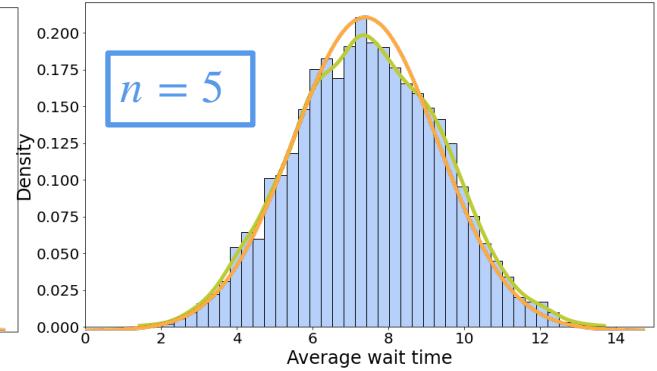
$n = 3$



$n = 4$

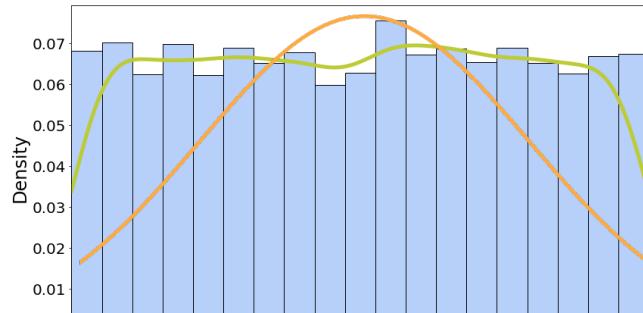


$n = 5$

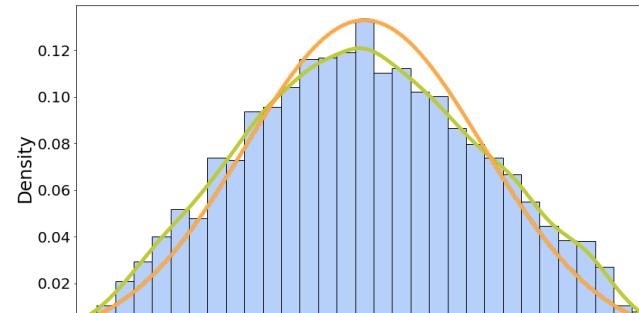


Central Limit Theorem (CLT) - Example 2

$n = 1$

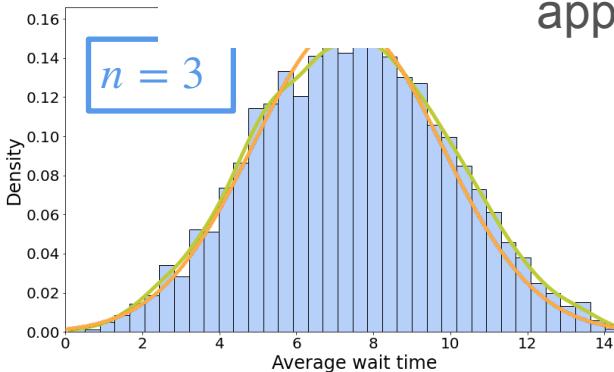


$n = 2$

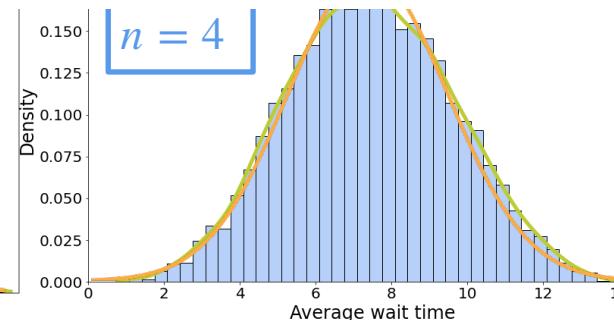


When you average a large enough number of variables, the distribution will approximately follow a normal distribution

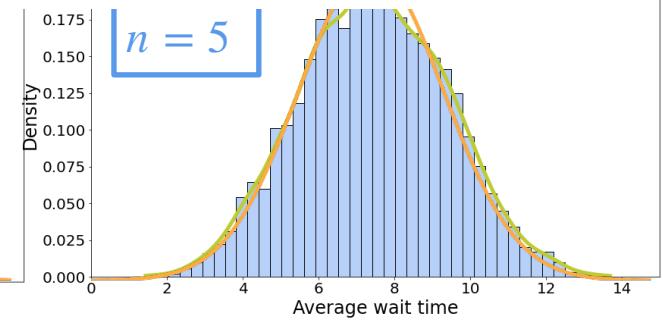
$n = 3$



$n = 4$

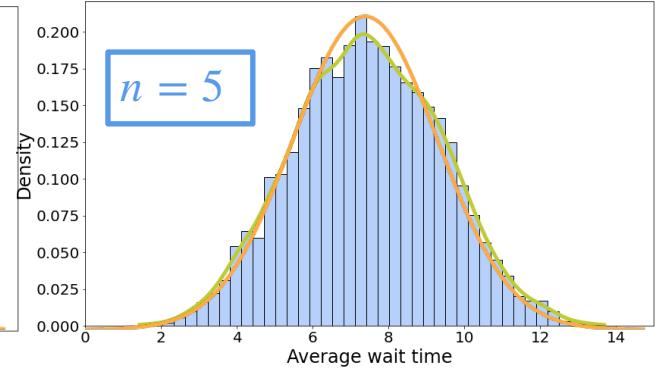
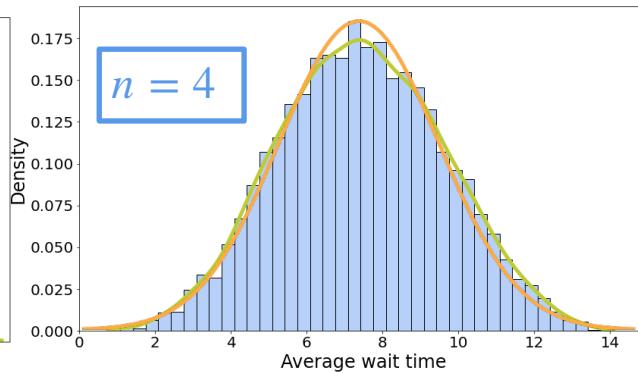
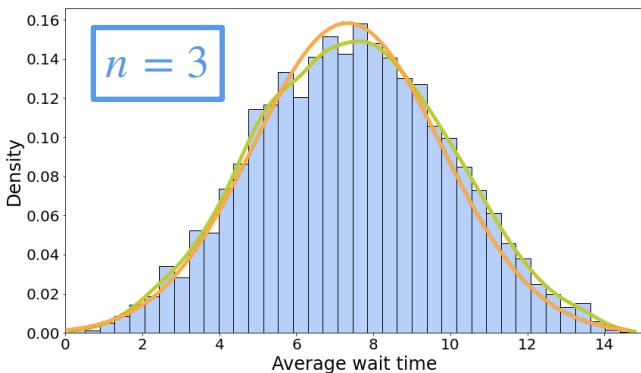


$n = 5$



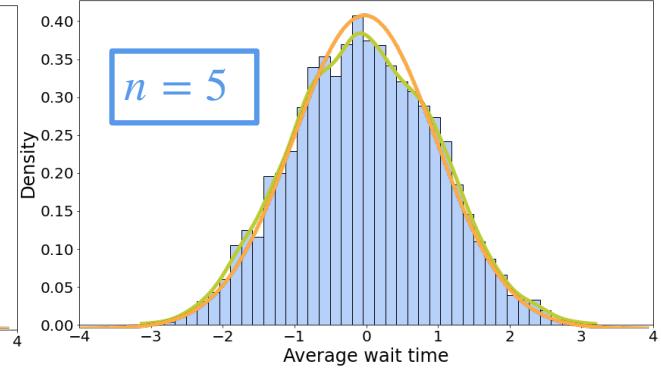
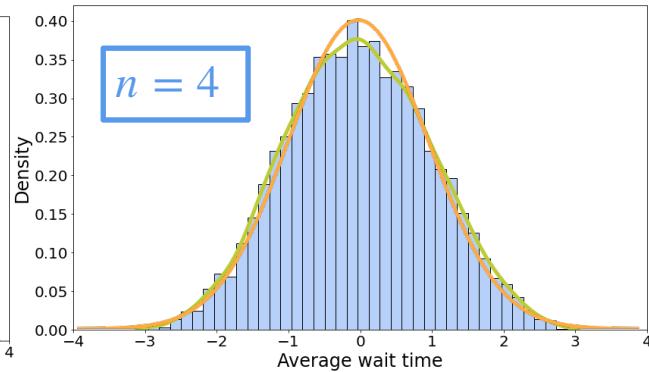
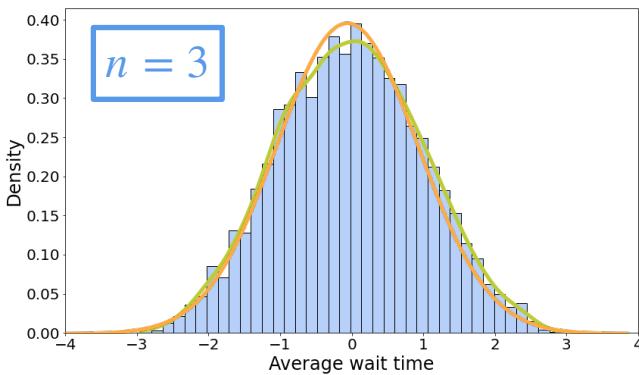
Central Limit Theorem (CLT) - Example 2

$$\frac{Y_n - 7.5}{\sqrt{18.75/n}}$$



Central Limit Theorem (CLT) - Example 2

$$\frac{Y_n - 7.5}{\sqrt{18.75/n}} \xrightarrow{n \uparrow} \mathcal{N}(0,1)$$



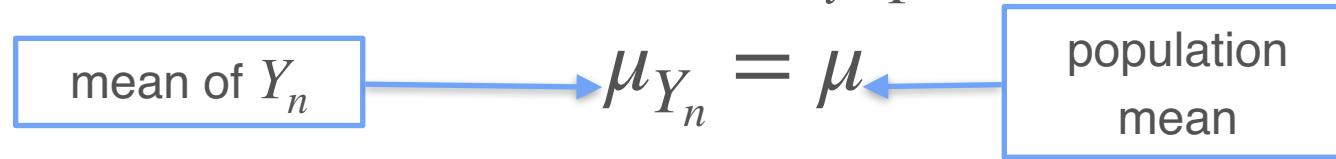
Central Limit Theorem (CLT) - Example 2

Central Limit Theorem (CLT) - Example 2

$$Y_n = \frac{1}{n} \sum_{i=1}^n X_i$$

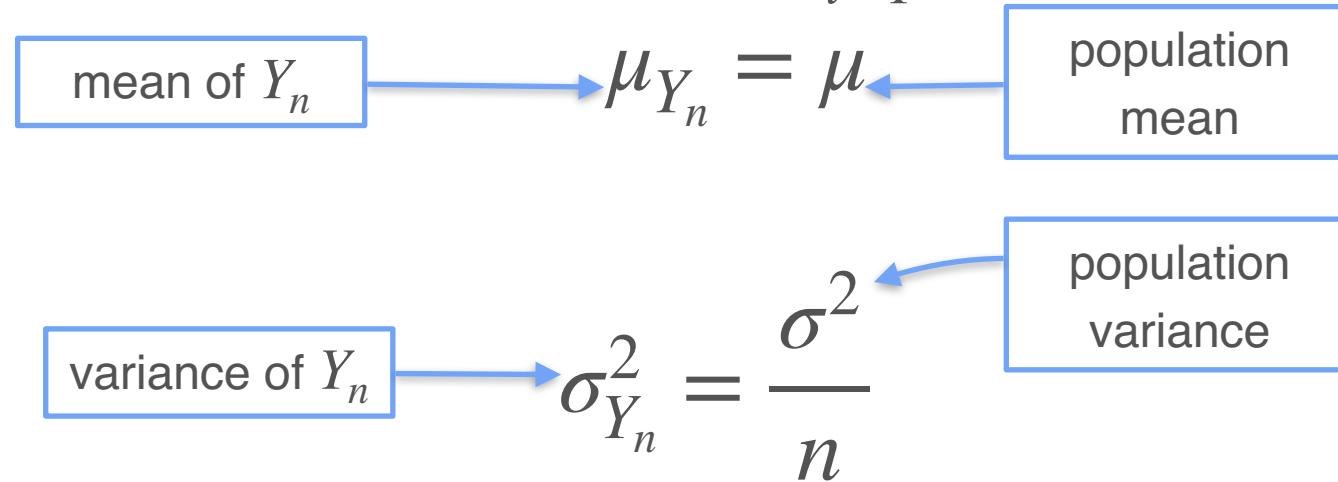
Central Limit Theorem (CLT) - Example 2

$$Y_n = \frac{1}{n} \sum_{i=1}^n X_i$$



Central Limit Theorem (CLT) - Example 2

$$Y_n = \frac{1}{n} \sum_{i=1}^n X_i$$



Central Limit Theorem (CLT) - Formal Definition

Central Limit Theorem (CLT) - Formal Definition

As $n \rightarrow \infty$

$$\frac{\frac{1}{n} \sum_{i=1}^n X_i - \mathbb{E}[X]}{\sigma_X} \sqrt{n} \sim \mathcal{N}(0, 1^2)$$

Central Limit Theorem (CLT) - Formal Definition

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$$\text{As } n \rightarrow \infty \quad \frac{1}{n} \left(\frac{\sum_{i=1}^n X_i - \frac{1}{n} n \mathbb{E}[X]}{\sigma_X} \right) \sqrt{n} \sim \mathcal{N}(0, 1^2)$$

Central Limit Theorem (CLT) - Formal Definition

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Central Limit Theorem (CLT) - Formal Definition

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$$\text{As } n \rightarrow \infty \quad \frac{1}{\cancel{\sqrt{n}}} \left(\frac{\sum_{i=1}^n X_i - \cancel{\frac{1}{n} n \mathbb{E}[X]}}{\sigma_X} \right) \cancel{\sqrt{n}} \sim \mathcal{N}(0, 1^2)$$

Central Limit Theorem (CLT) - Formal Definition

$$\text{As } n \rightarrow \infty \quad \frac{\frac{1}{n} \sum_{i=1}^n X_i - \mathbb{E}[X]}{\sigma_X} \sqrt{n} \sim \mathcal{N}(0, 1^2)$$

As $n \rightarrow \infty$

Central Limit Theorem (CLT) - Formal Definition

$$\text{As } n \rightarrow \infty \quad \frac{\frac{1}{n} \sum_{i=1}^n X_i - \mathbb{E}[X]}{\sigma_X} \sqrt{n} \sim \mathcal{N}(0, 1^2)$$

$$\text{As } n \rightarrow \infty \quad \frac{\sum_{i=1}^n X_i - n\mathbb{E}[X]}{\sqrt{n}\sigma_X} \sim \mathcal{N}(0, 1^2)$$

W3 Lesson 2



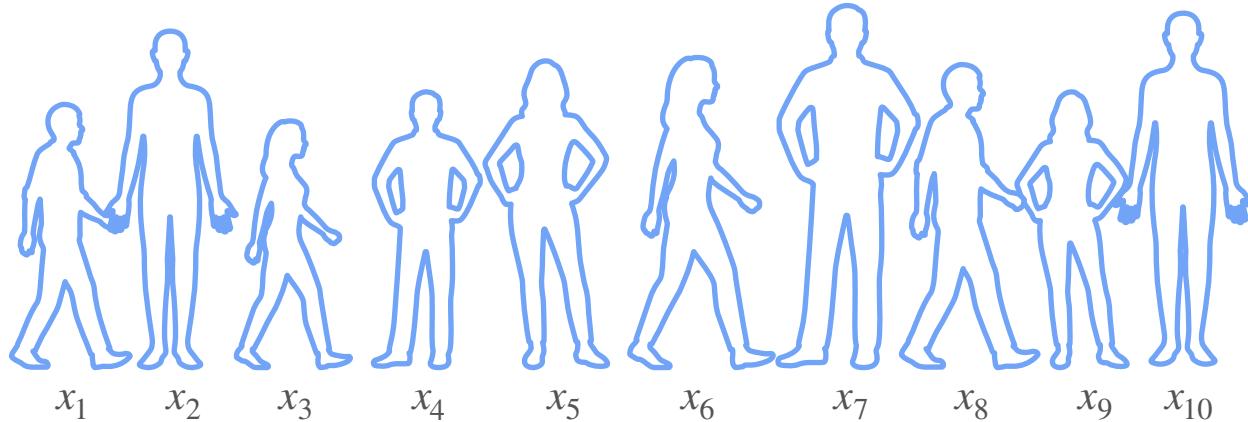
DeepLearning.AI

Point Estimation

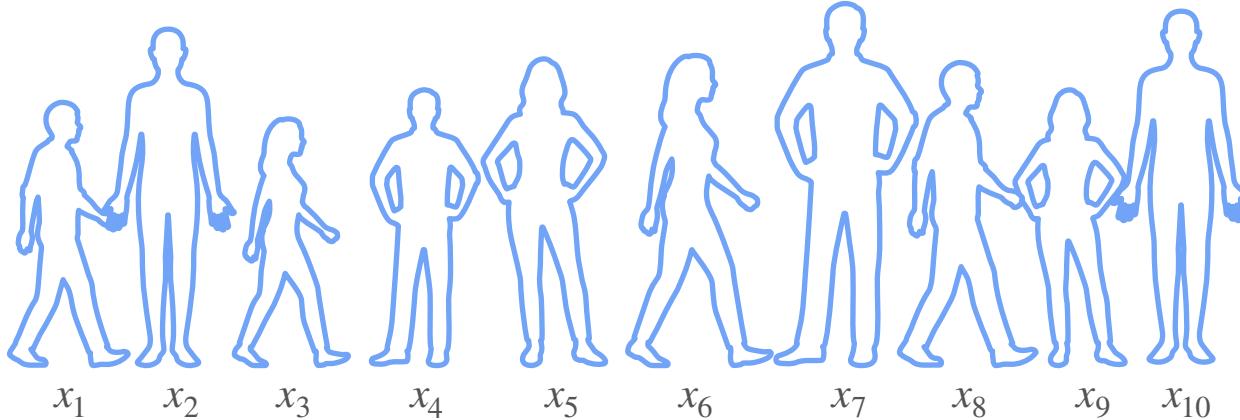
What is Point Estimation?

Point Estimates

Point Estimates



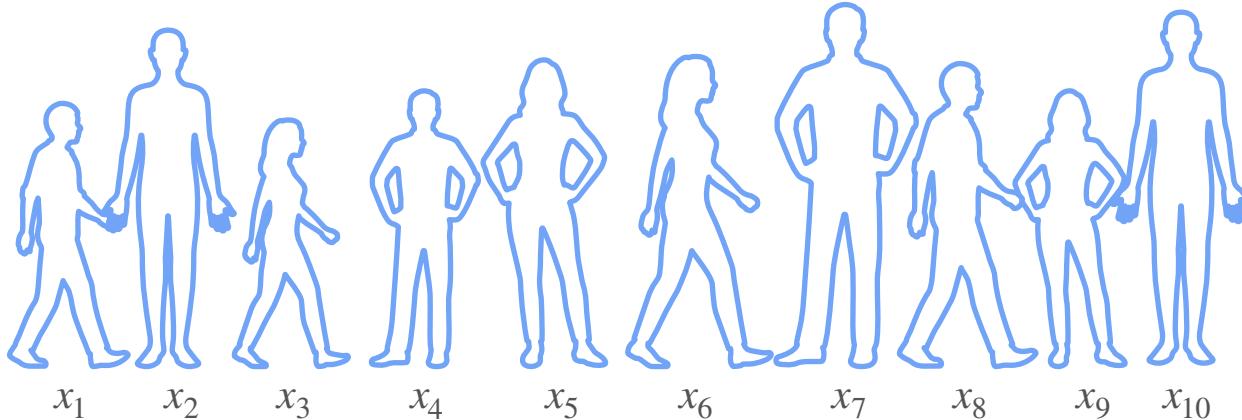
Point Estimates



Mean of the population?

$$\mu \approx \frac{1}{10} \sum_{i=1}^{10} x_i = \bar{x}$$

Point Estimates



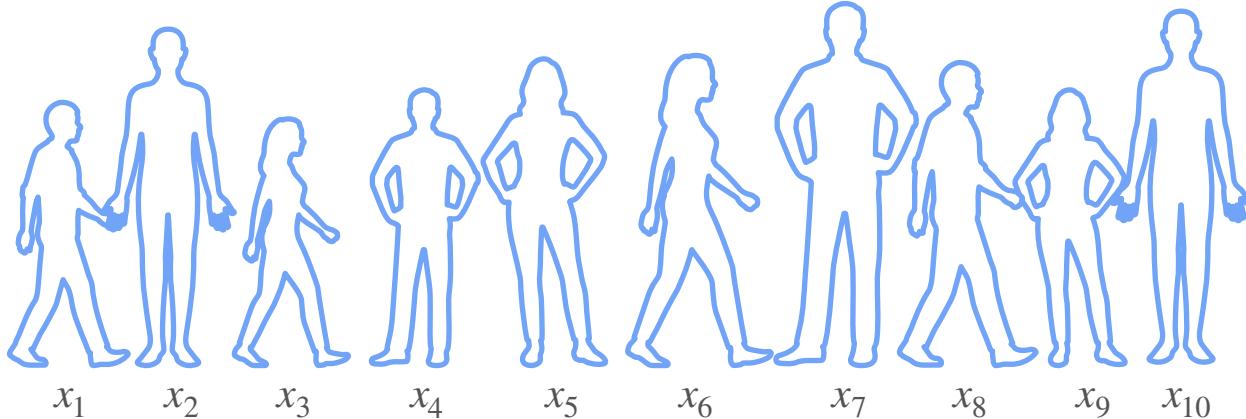
Mean of the population?

$$\mu \approx \frac{1}{10} \sum_{i=1}^{10} x_i = \bar{x}$$

Variance of the population?

$$\sigma^2 \approx \frac{1}{10 - 1} \sum_{i=1}^{10} (x_i - \bar{x})^2 = s^2$$

Point Estimates



Mean of the population?

$$\mu \approx \frac{1}{10} \sum_{i=1}^{10} x_i = \bar{x}$$

Variance of the population?

$$\sigma^2 \approx \frac{1}{10 - 1} \sum_{i=1}^{10} (x_i - \bar{x})^2 = s^2$$

\bar{x} and s^2 are point estimates

Point Estimates

Point Estimates

A **point estimate** is a **single numerical value** based on **sample data** that is used to **approximate an unknown parameter** of a population or model parameter.

Point Estimates

Point Estimates



$\mathbf{P}(H) ?$

Point Estimates



$P(H) ?$



Point Estimates



$P(H)$?

10 throws
7 heads
3 tails



Point Estimates



$P(H)$?

10 throws
7 heads
3 tails

$$P(H) = \frac{7}{10}$$



Point Estimates

	x	y
0		
1		
2		
\vdots		
50		

Point Estimates

	x	y
0		
1		
2		
\vdots		
50		



Point Estimates

	x	y
0		
1		
2		
\vdots		
50		

$$y = \beta_0 + \beta_1 x$$

$$\beta_0, \beta_1 = ??$$

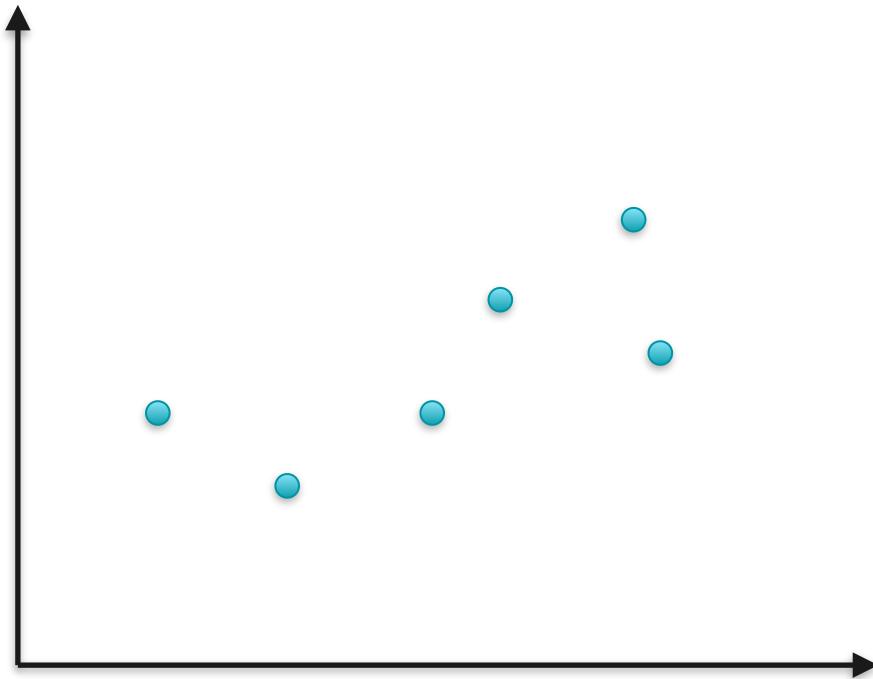


Point Estimates

	x	y
0		
1		
2		
\vdots		
50		

$$y = \beta_0 + \beta_1 x$$

$$\beta_0, \beta_1 = ??$$

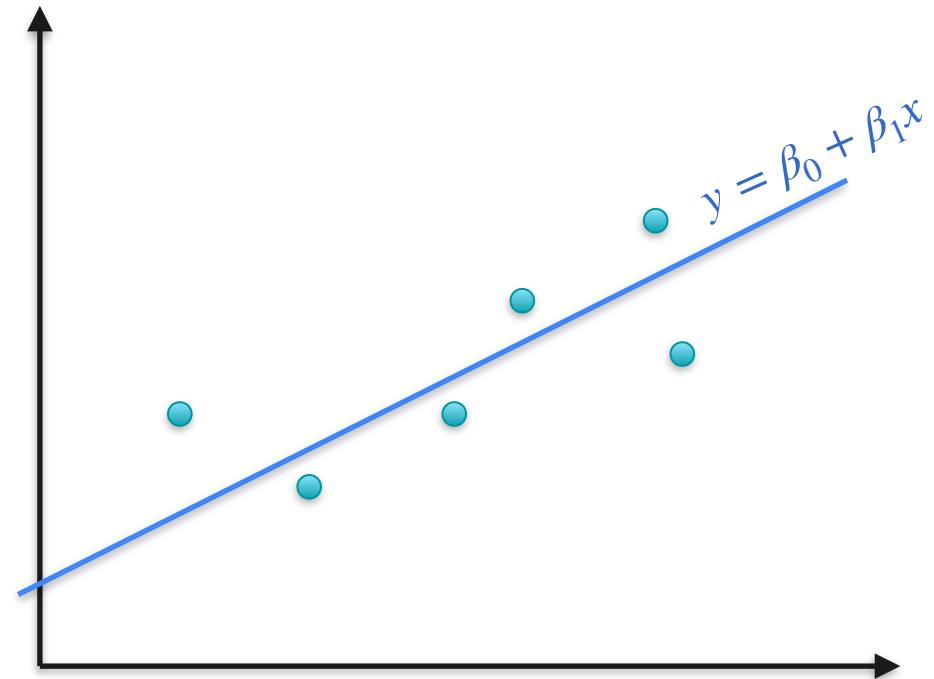


Point Estimates

	x	y
0		
1		
2		
\vdots		
50		

$$y = \beta_0 + \beta_1 x$$

$$\beta_0, \beta_1 = ??$$





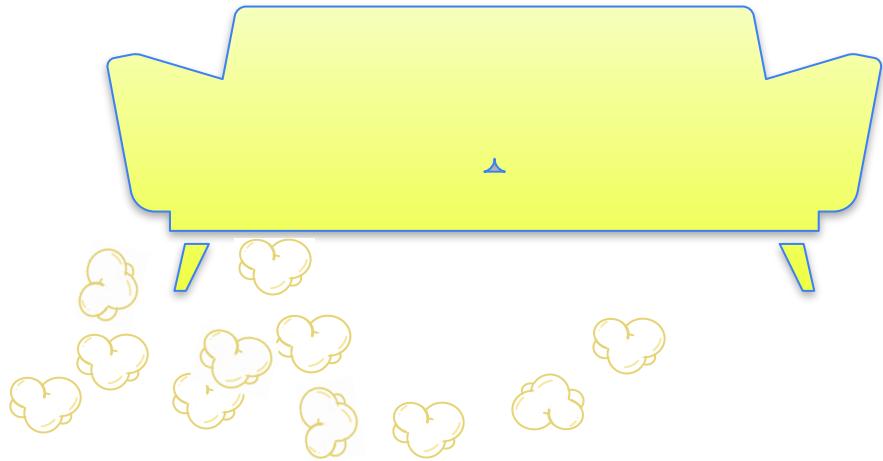
DeepLearning.AI

Point Estimation

**Maximum Likelihood
Estimation: Motivation**

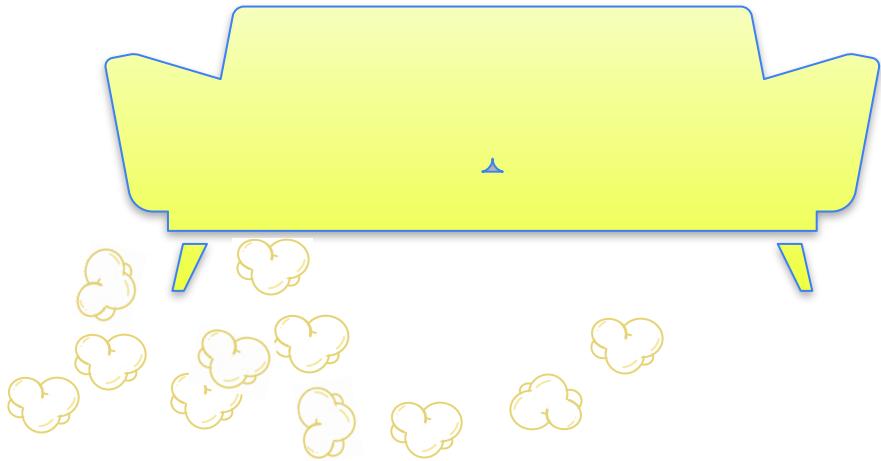
There's Popcorn on the Floor. What Happened?

There's Popcorn on the Floor. What Happened?

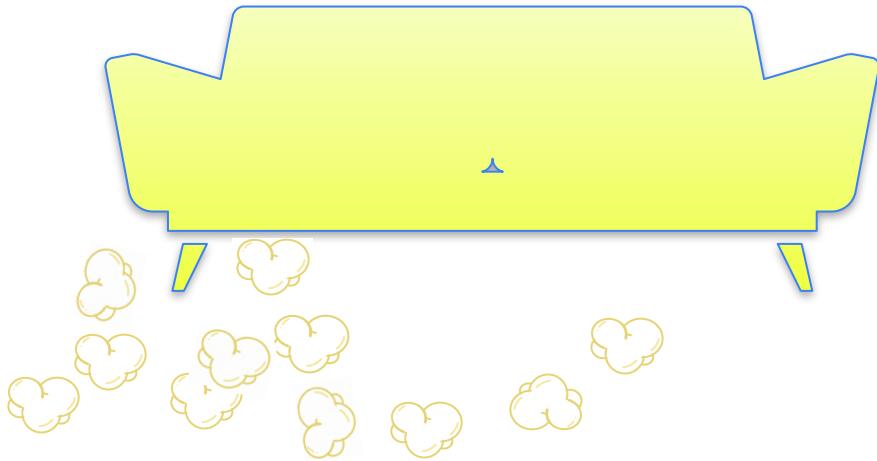


There's Popcorn on the Floor. What Happened?

Movies



There's Popcorn on the Floor. What Happened?



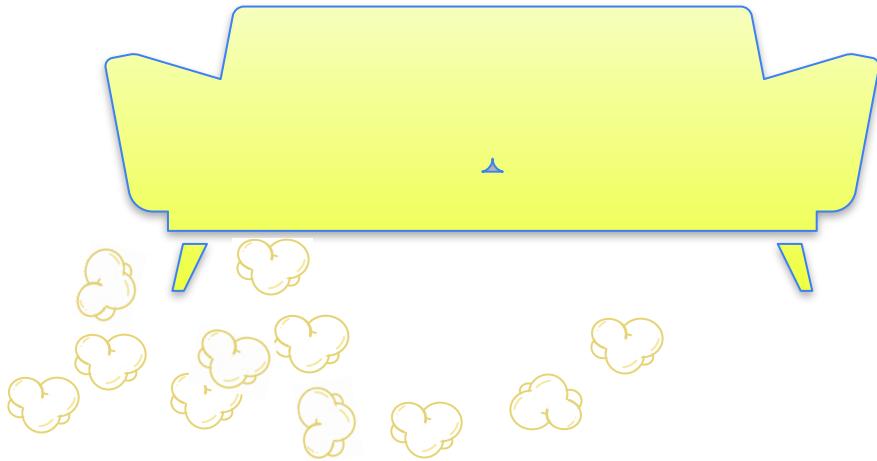
Movies



Board Games



There's Popcorn on the Floor. What Happened?



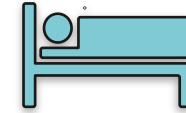
Movies



Board Games



Nap



Quiz

What do you think happened?

- A. People were watching a movie
- B. People were playing boardgames
- C. People were taking a nap

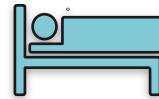
There's Popcorn on the Floor. What Happened?



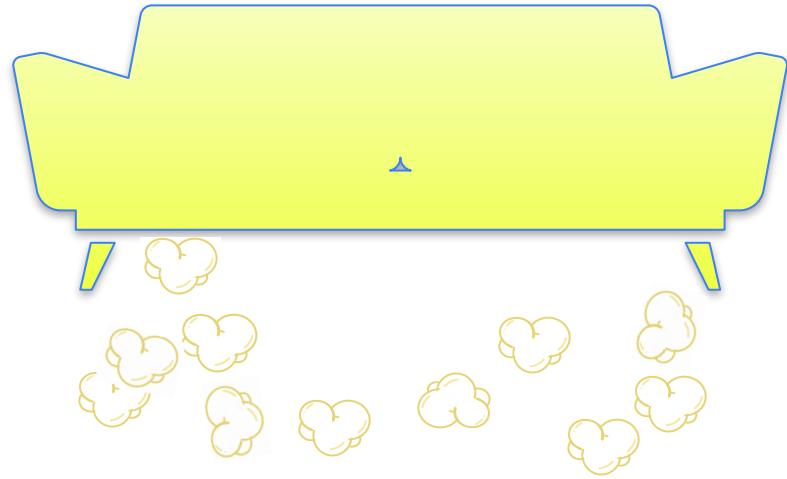
Movies



Board
Games



Nap



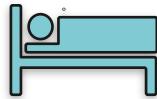
There's Popcorn on the Floor. What Happened?



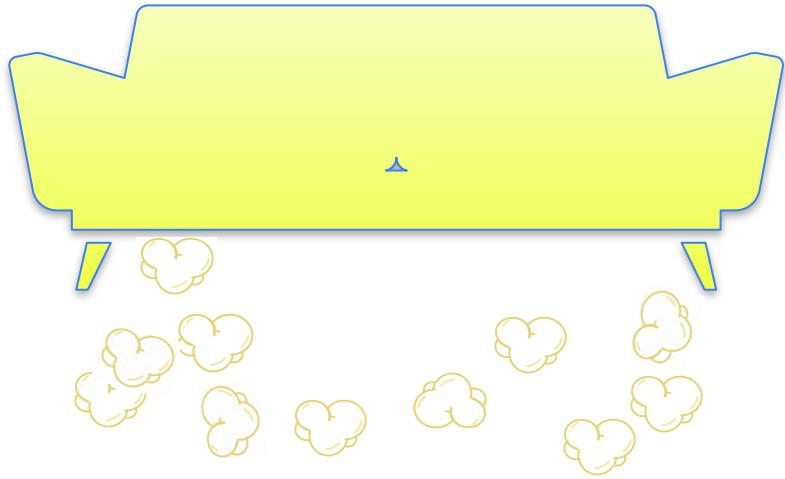
Movies *High*



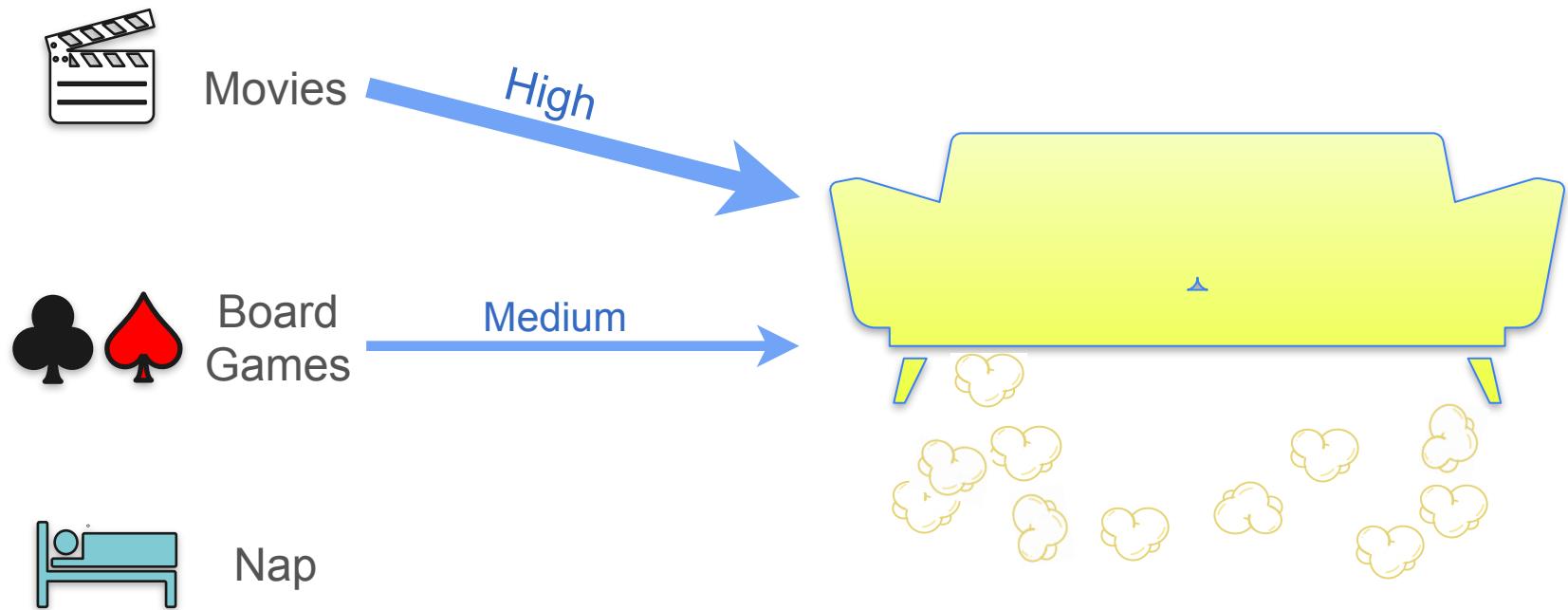
Board Games



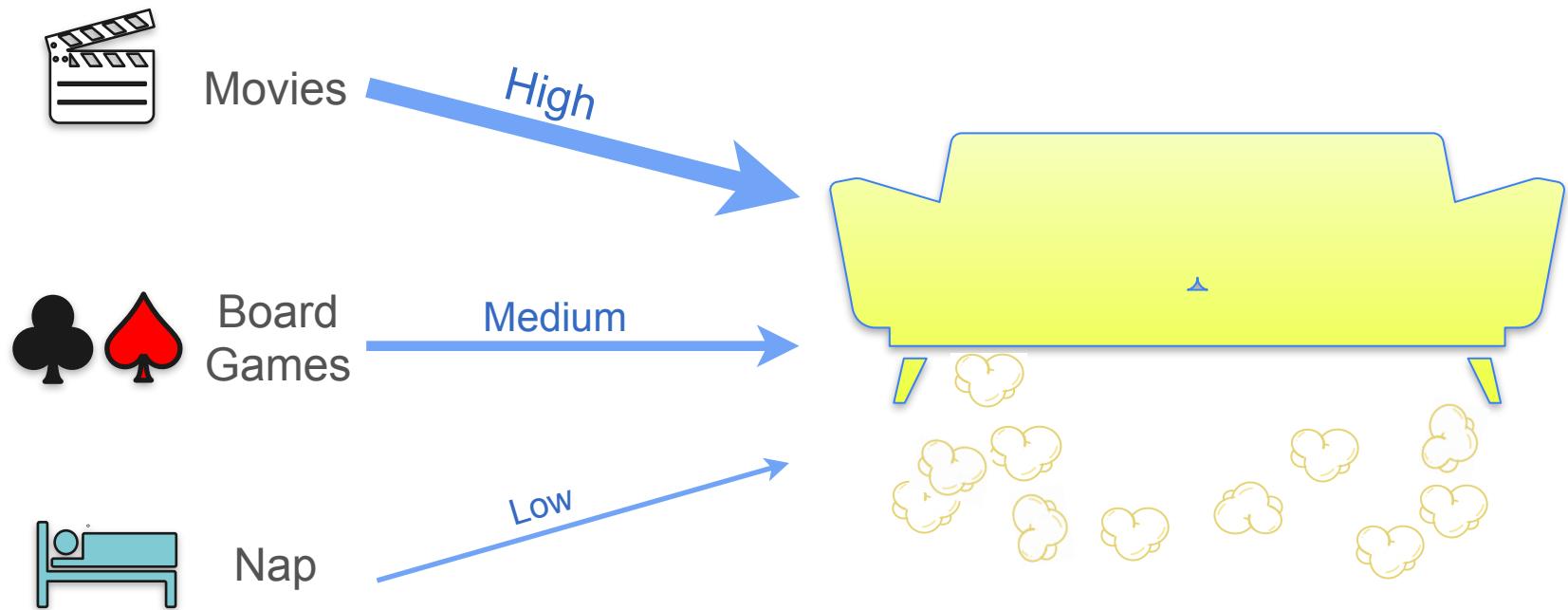
Nap



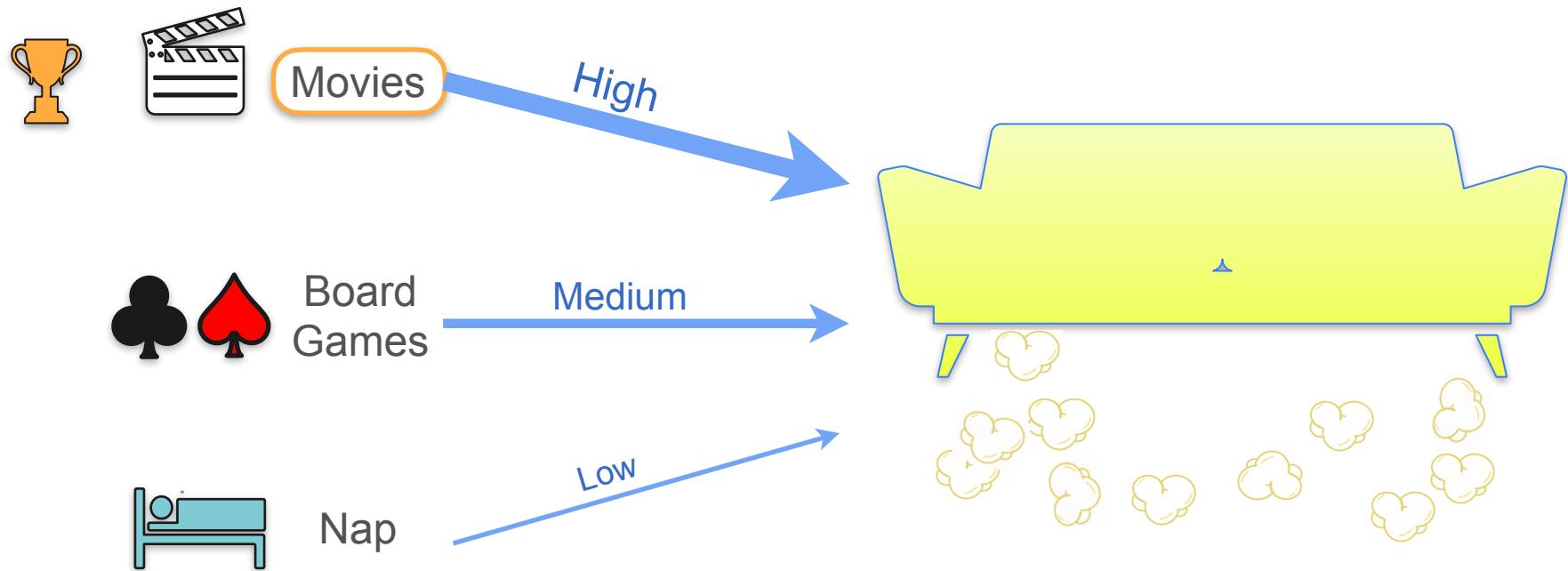
There's Popcorn on the Floor. What Happened?



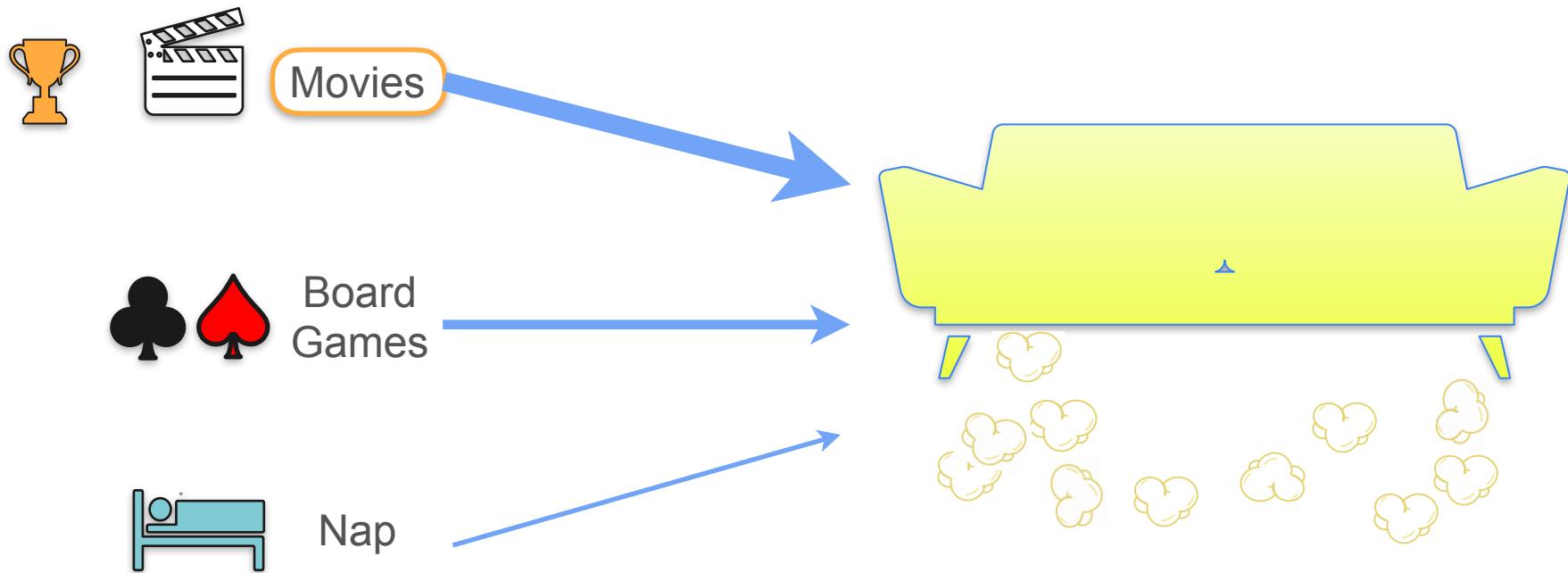
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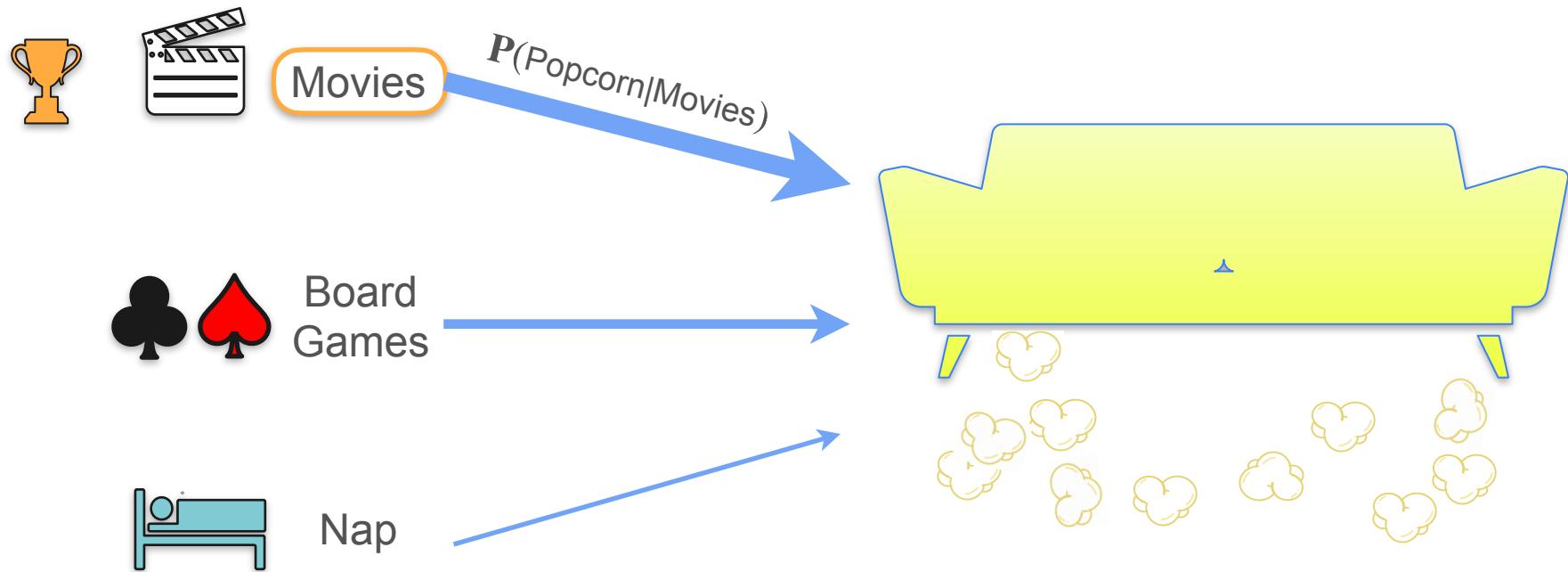
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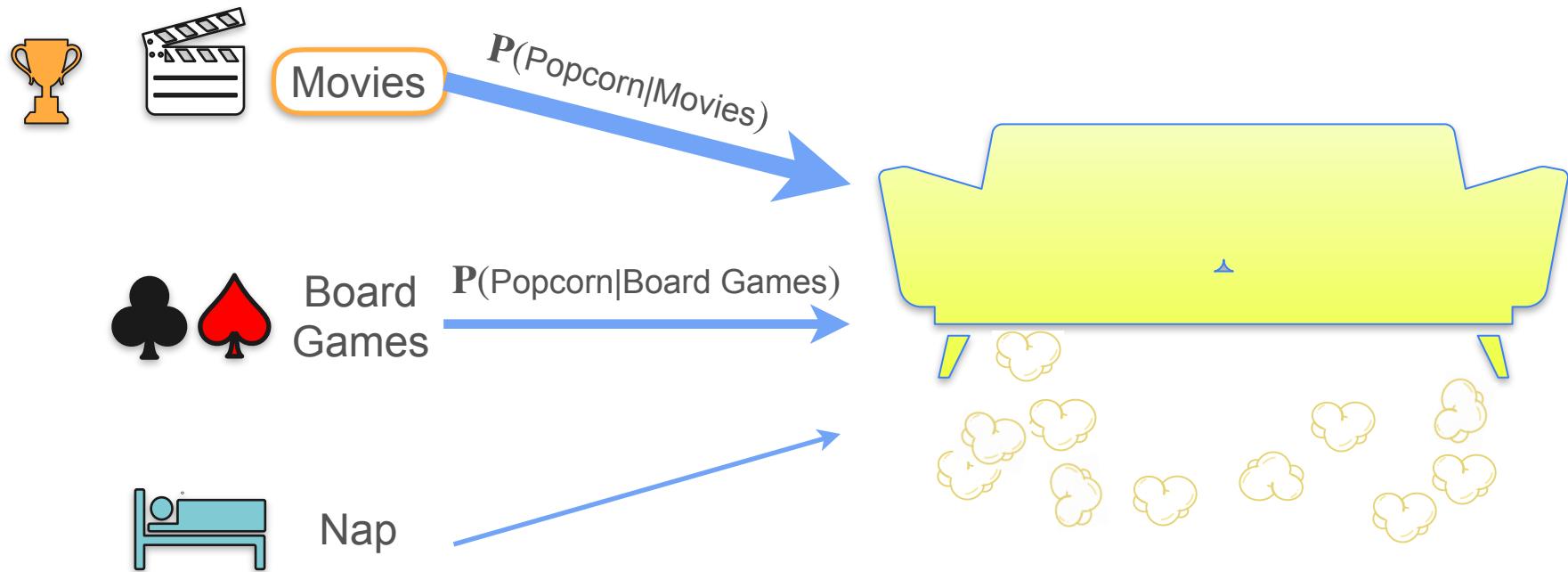
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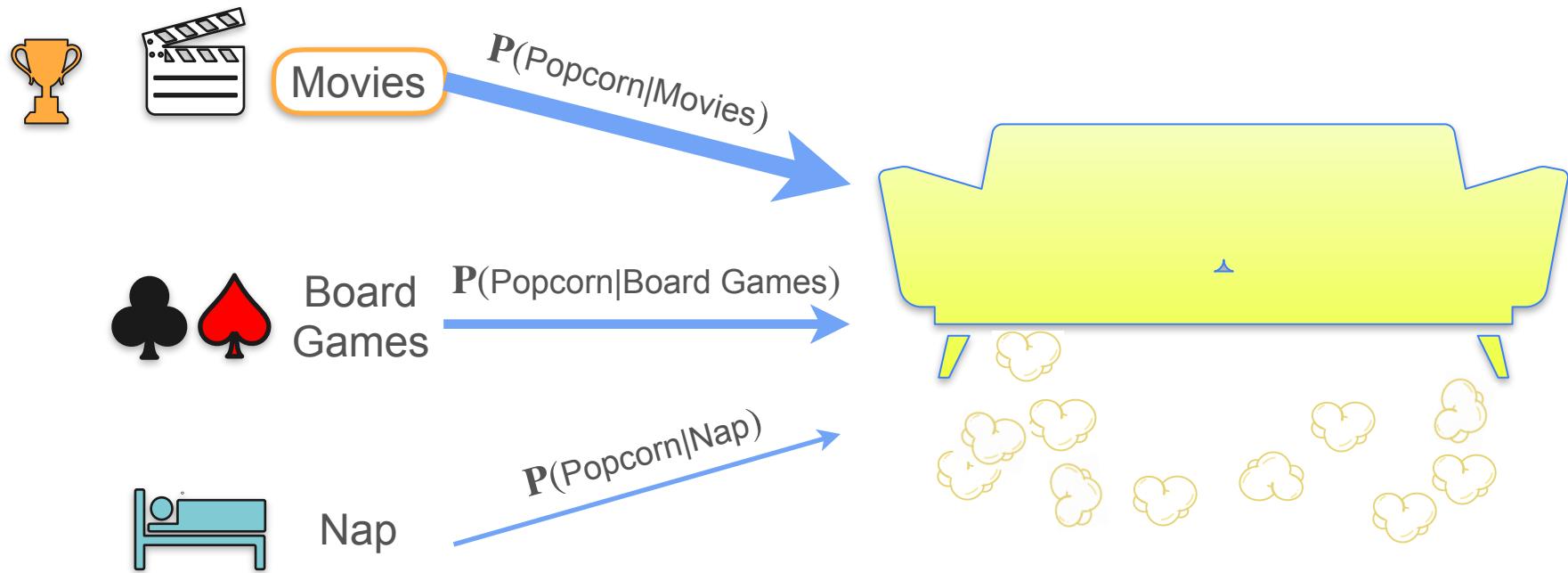
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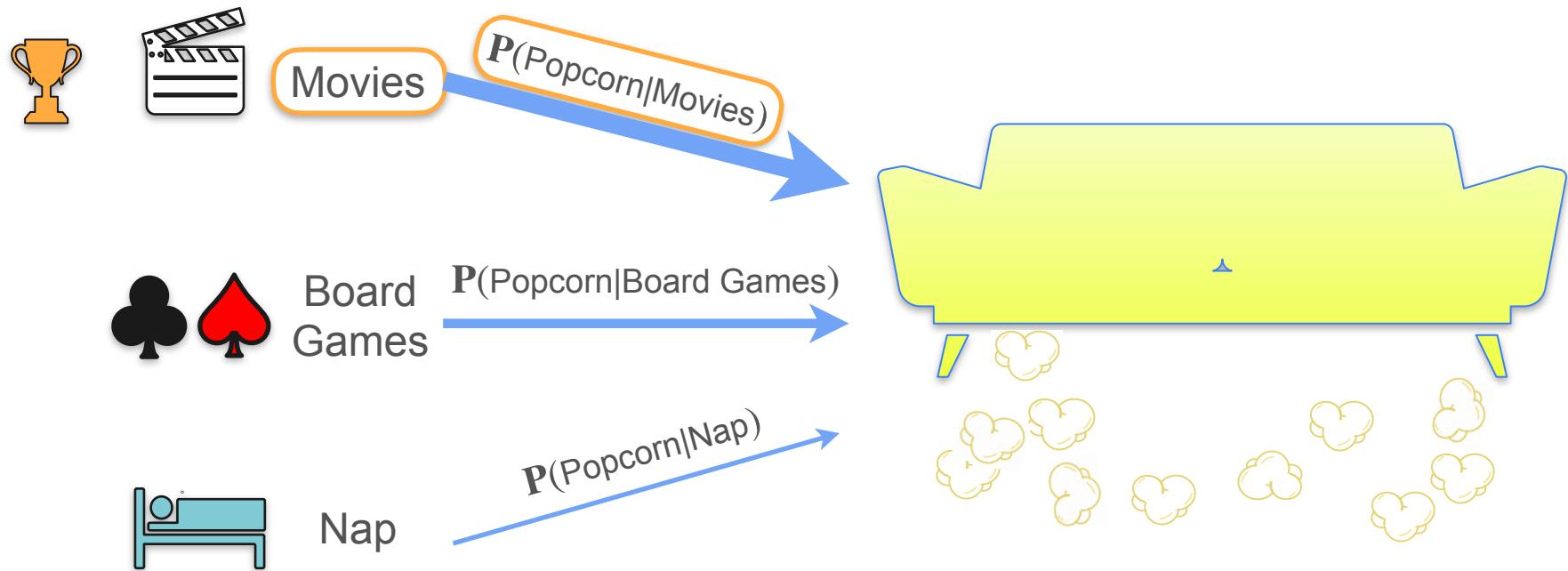
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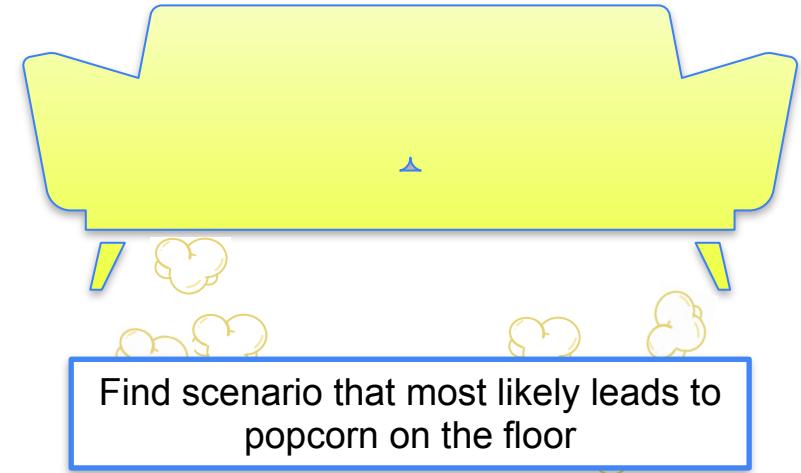
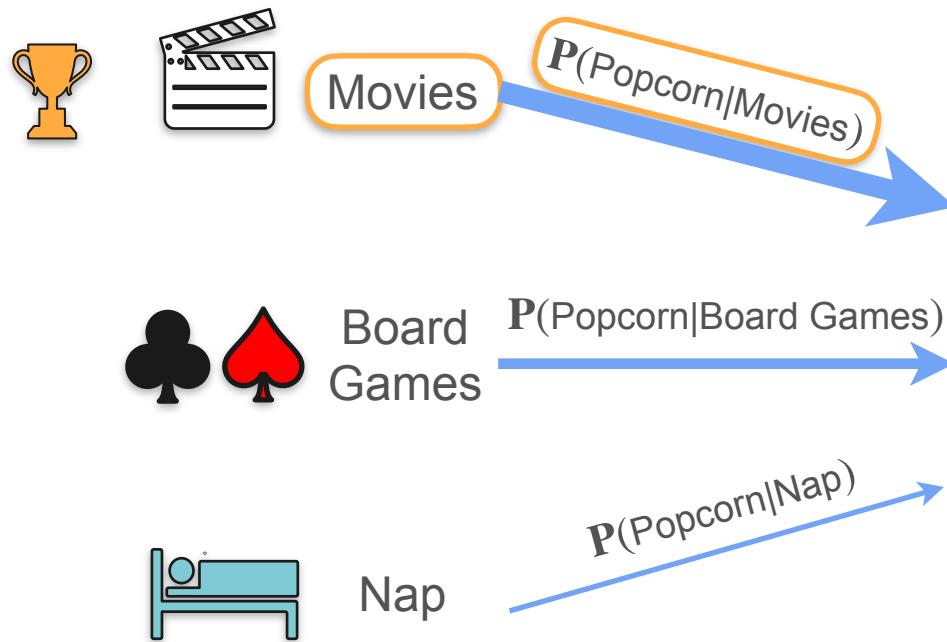
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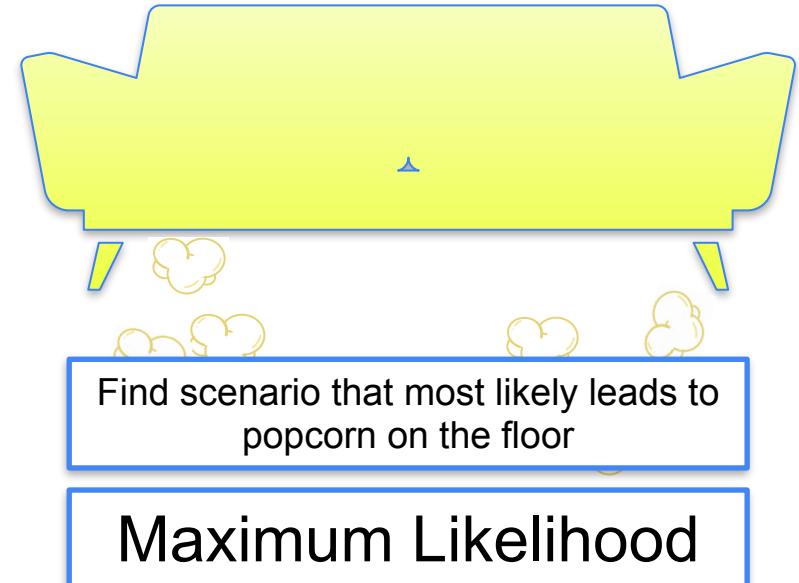
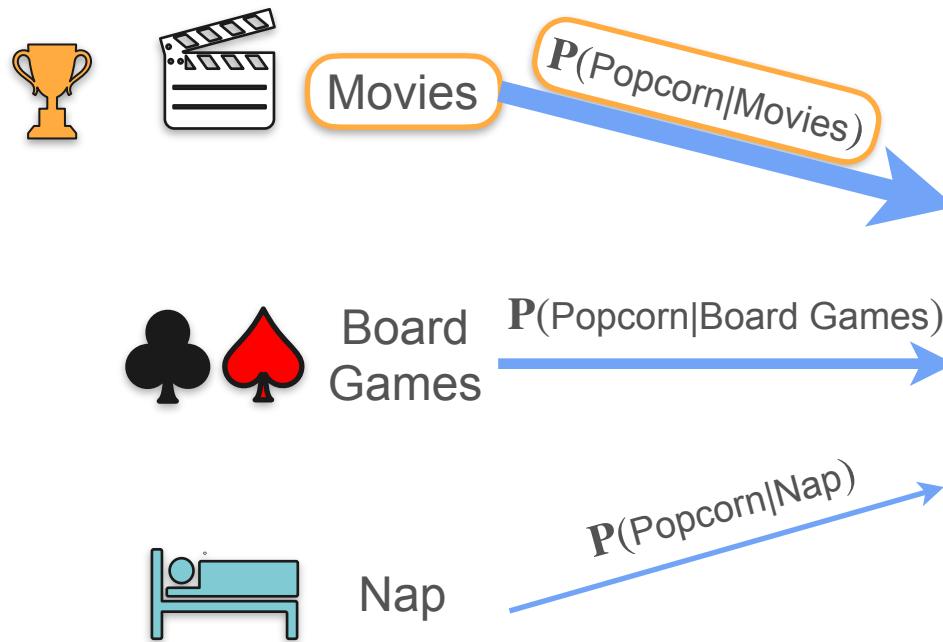
There's Popcorn on the Floor. What Happened?



There's Popcorn on the Floor. What Happened?



There's Popcorn on the Floor. What Happened?



Maximum Likelihood

Maximum Likelihood



Data

Maximum Likelihood



Model 1



Data

Maximum Likelihood



Model 1



Model 2



Data

Maximum Likelihood



Model 1



Model 2

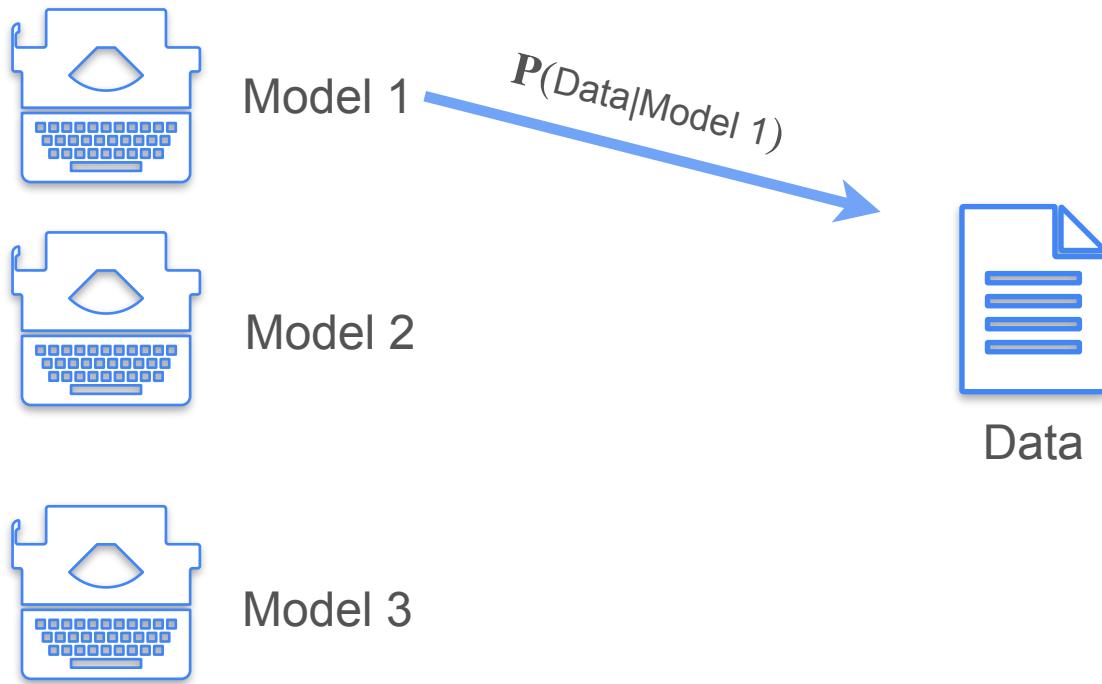


Model 3

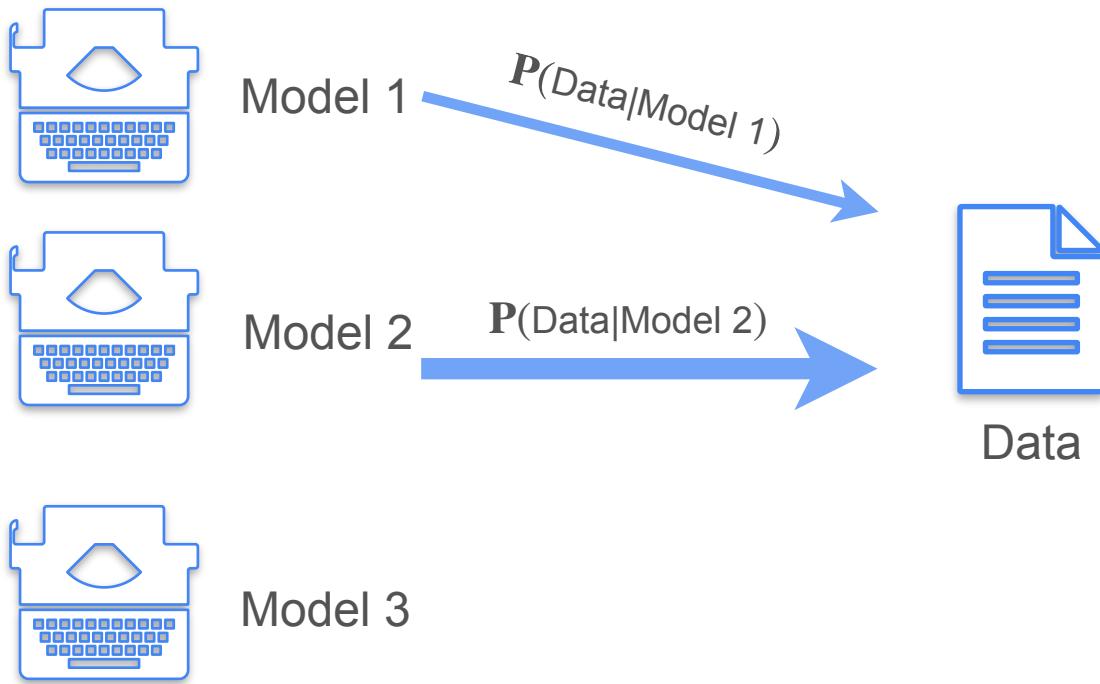


Data

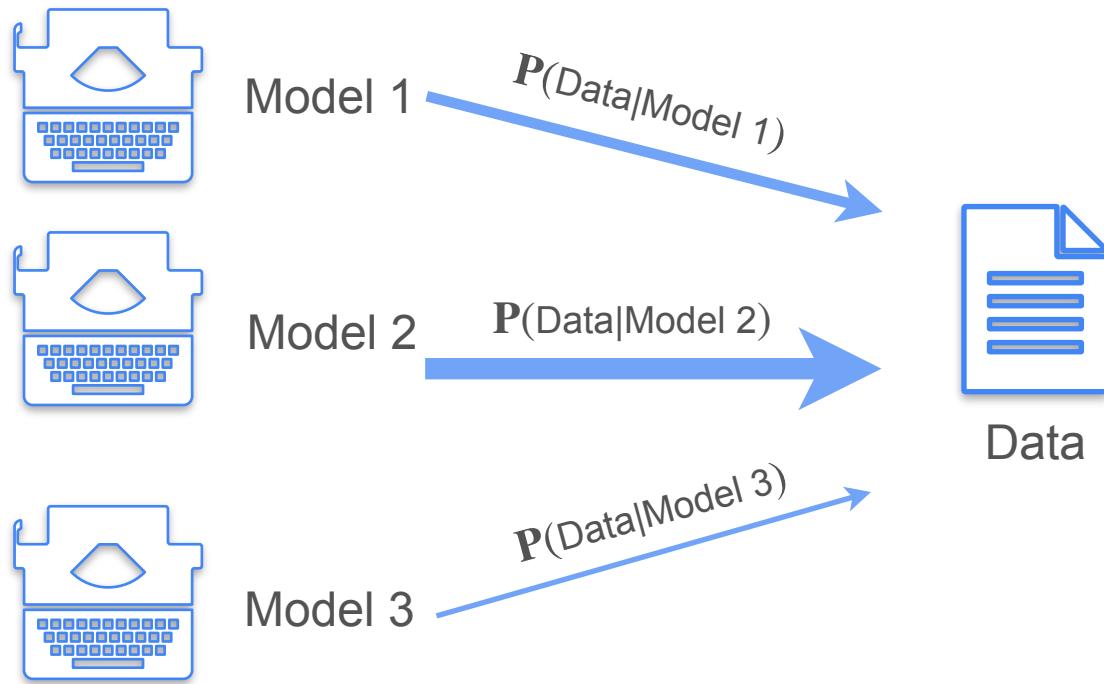
Maximum Likelihood



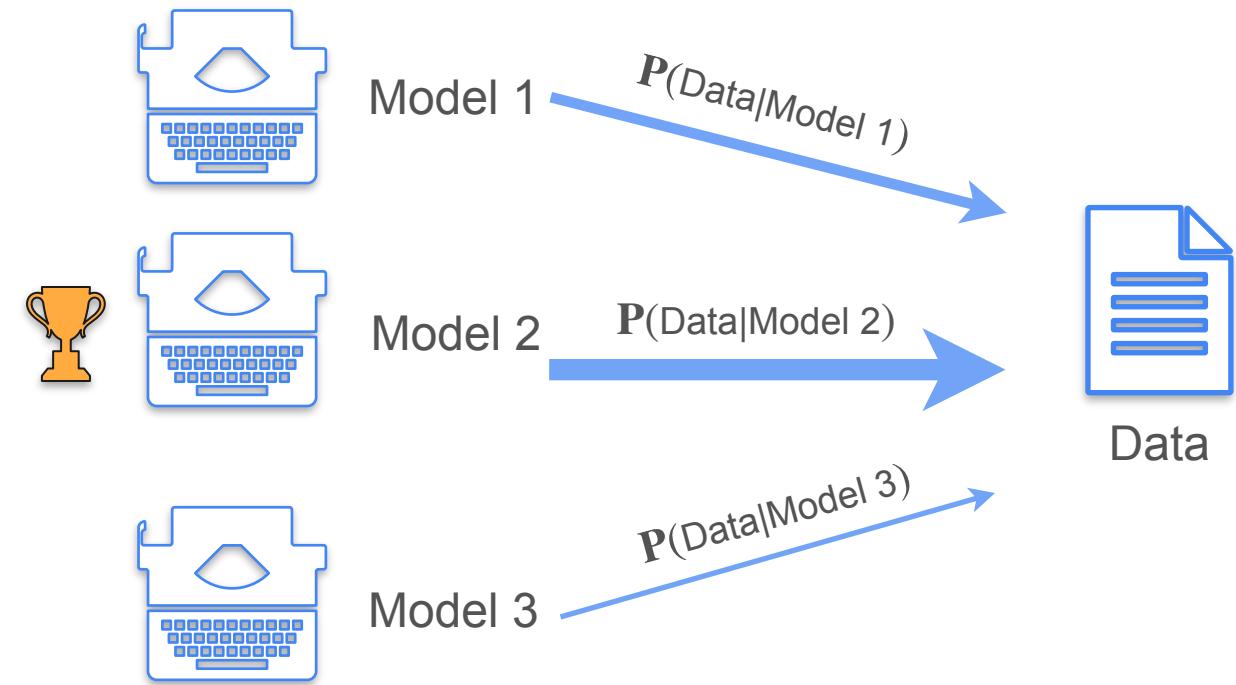
Maximum Likelihood



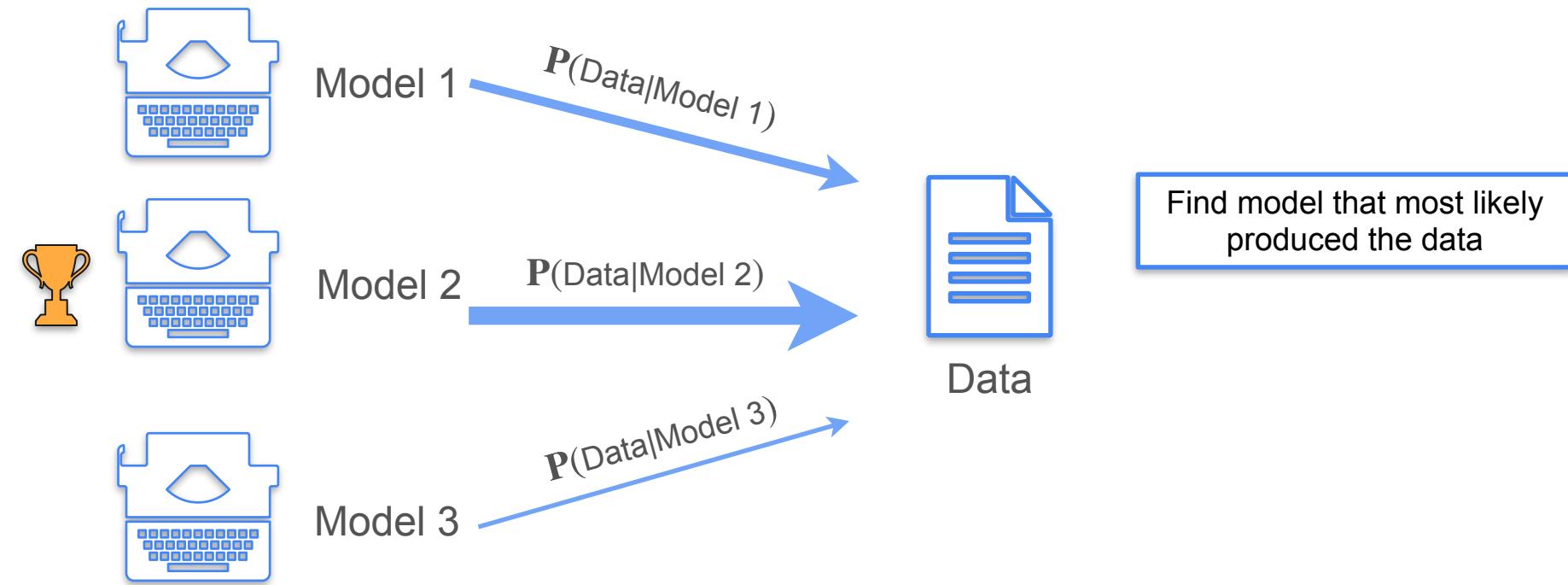
Maximum Likelihood



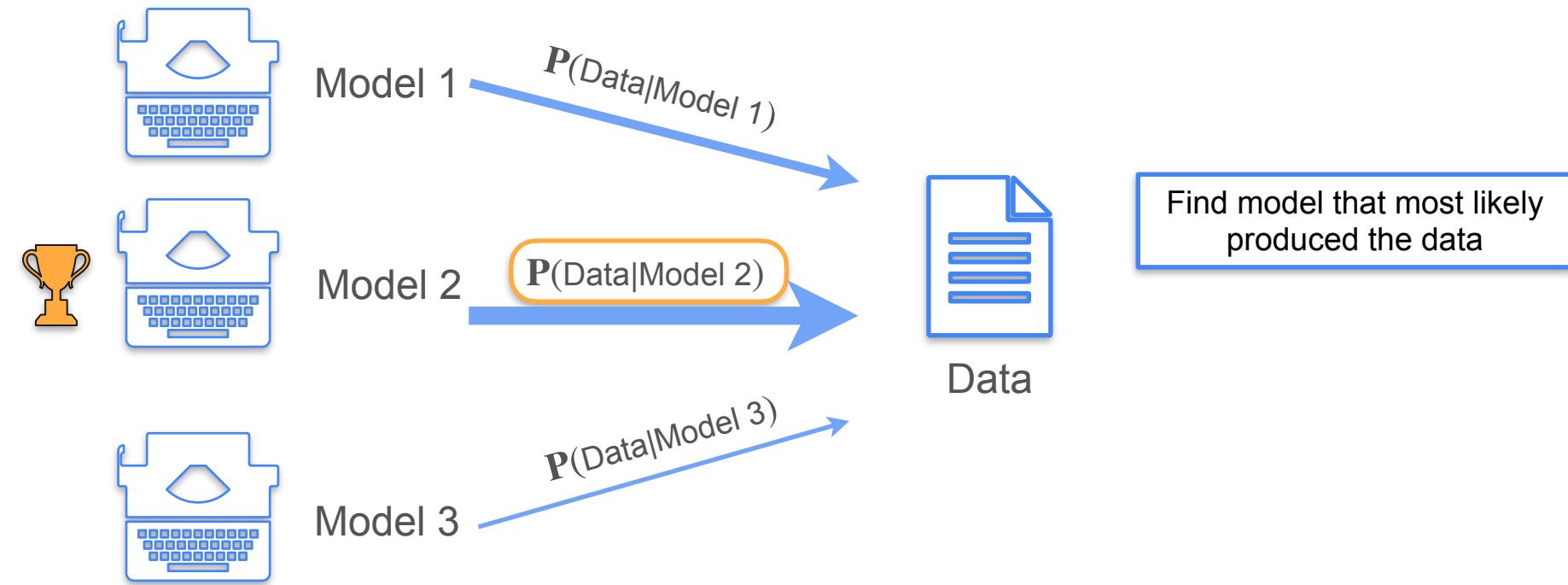
Maximum Likelihood



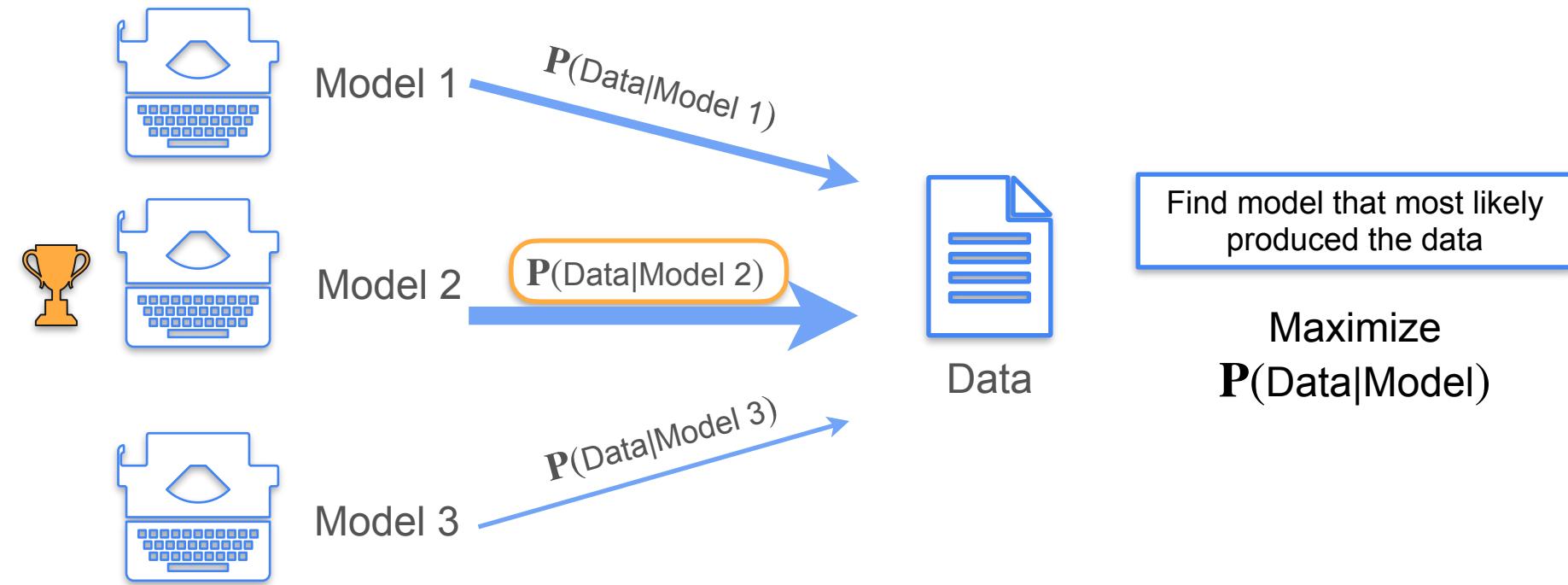
Maximum Likelihood



Maximum Likelihood

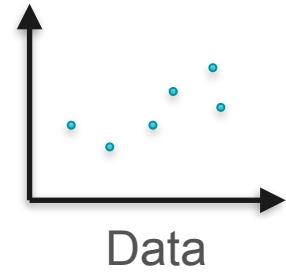


Maximum Likelihood

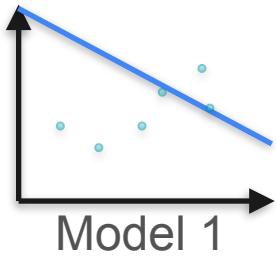


Example: Linear Regression

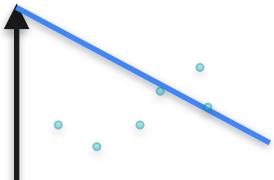
Example: Linear Regression



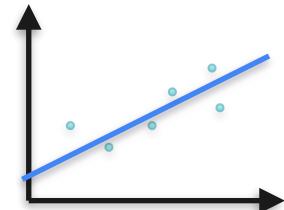
Example: Linear Regression



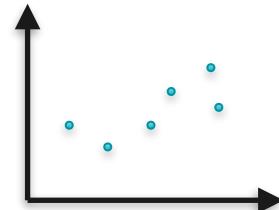
Example: Linear Regression



Model 1

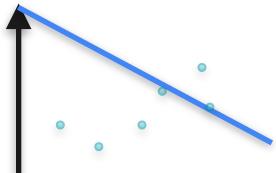


Model 2

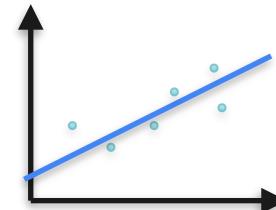


Data

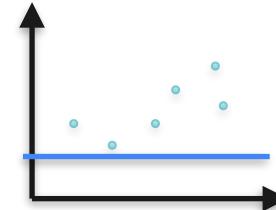
Example: Linear Regression



Model 1



Model 2

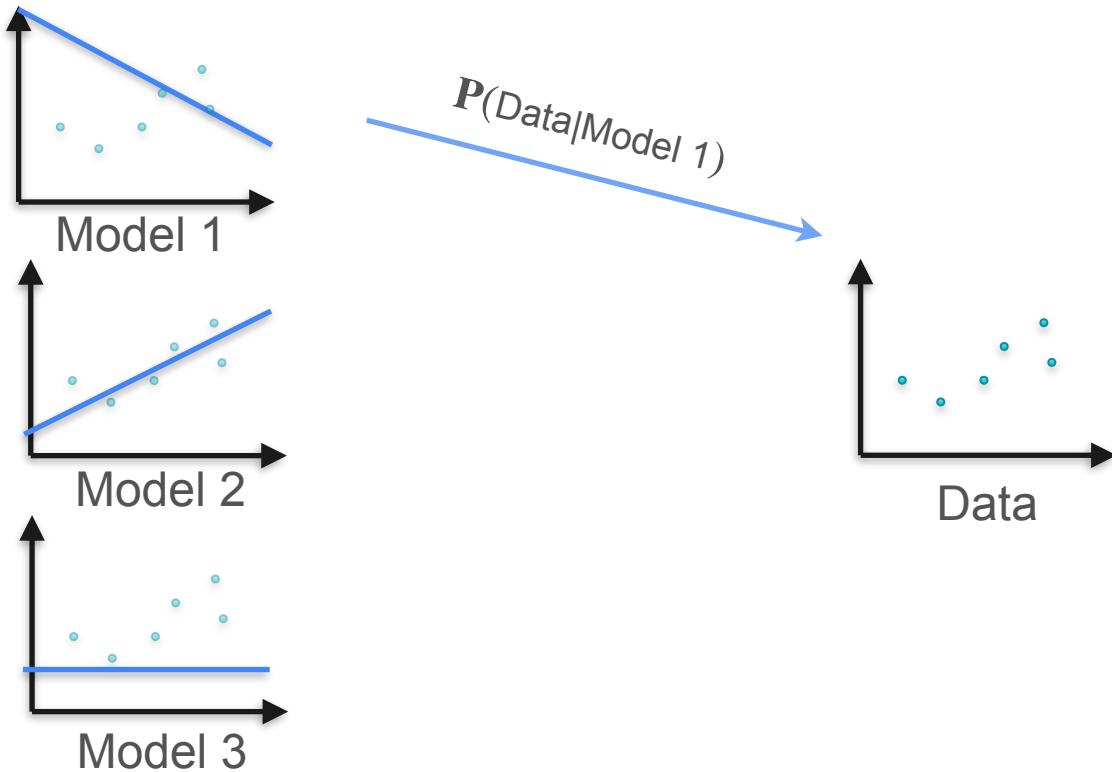


Model 3

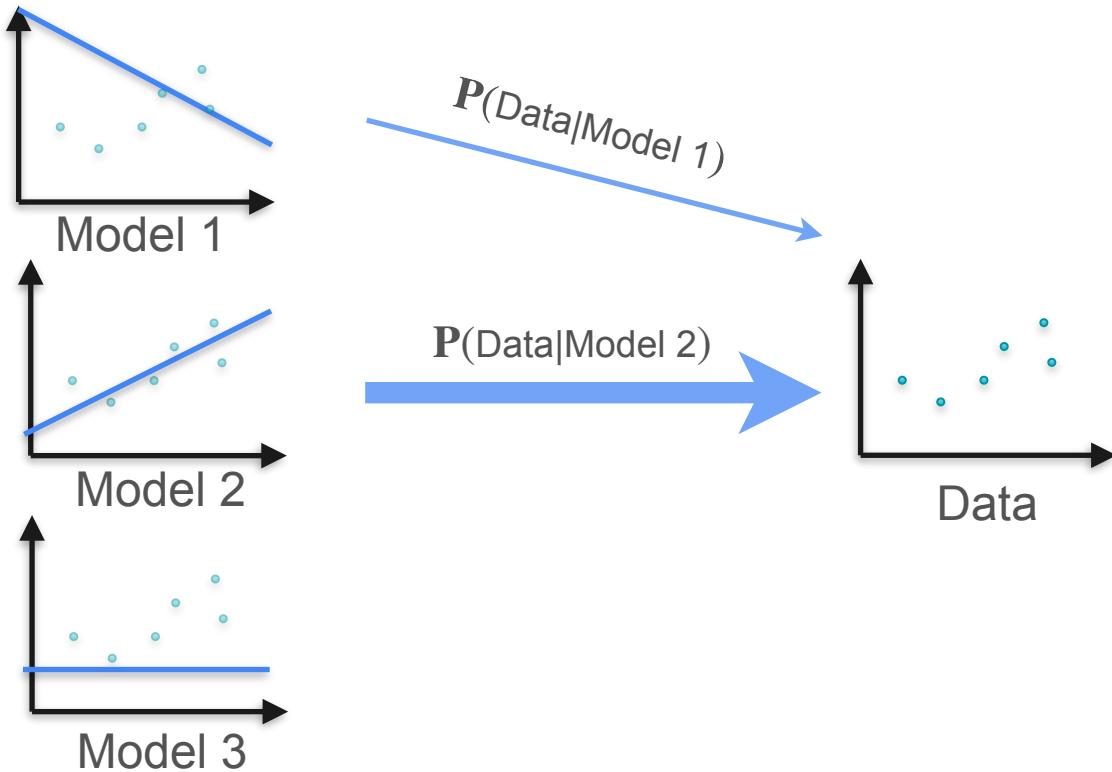


Data

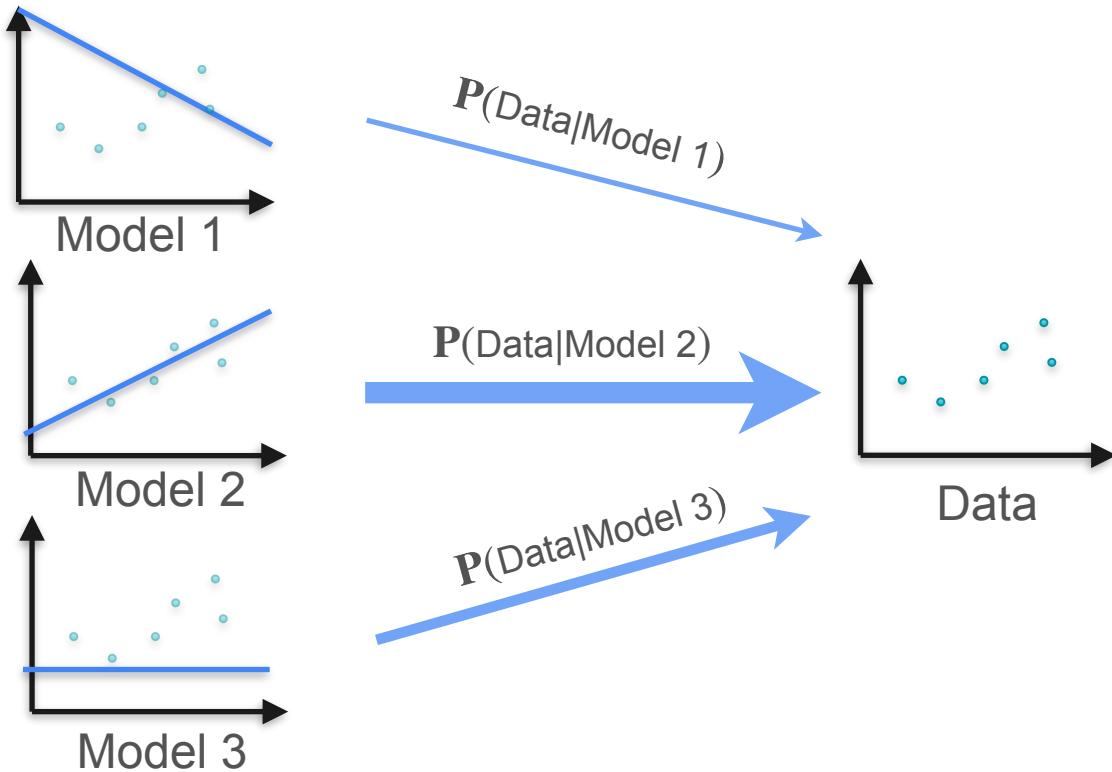
Example: Linear Regression



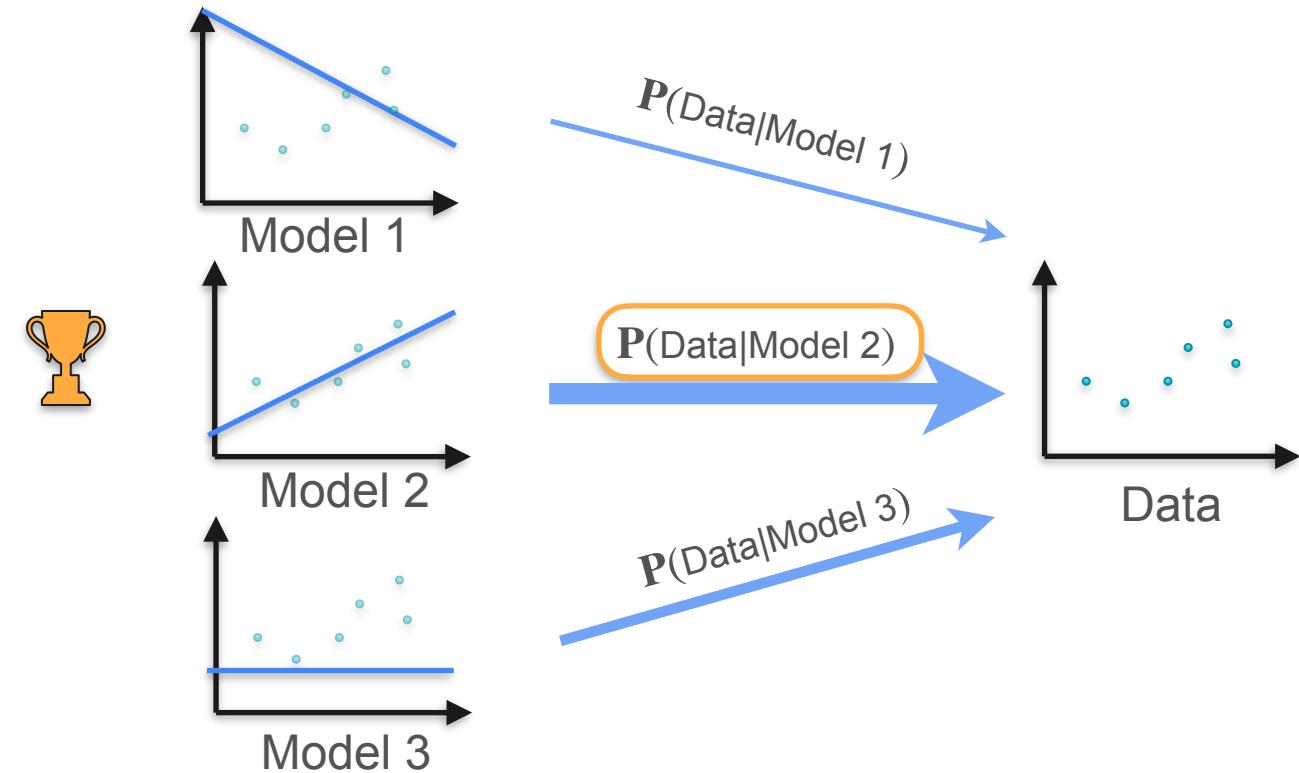
Example: Linear Regression



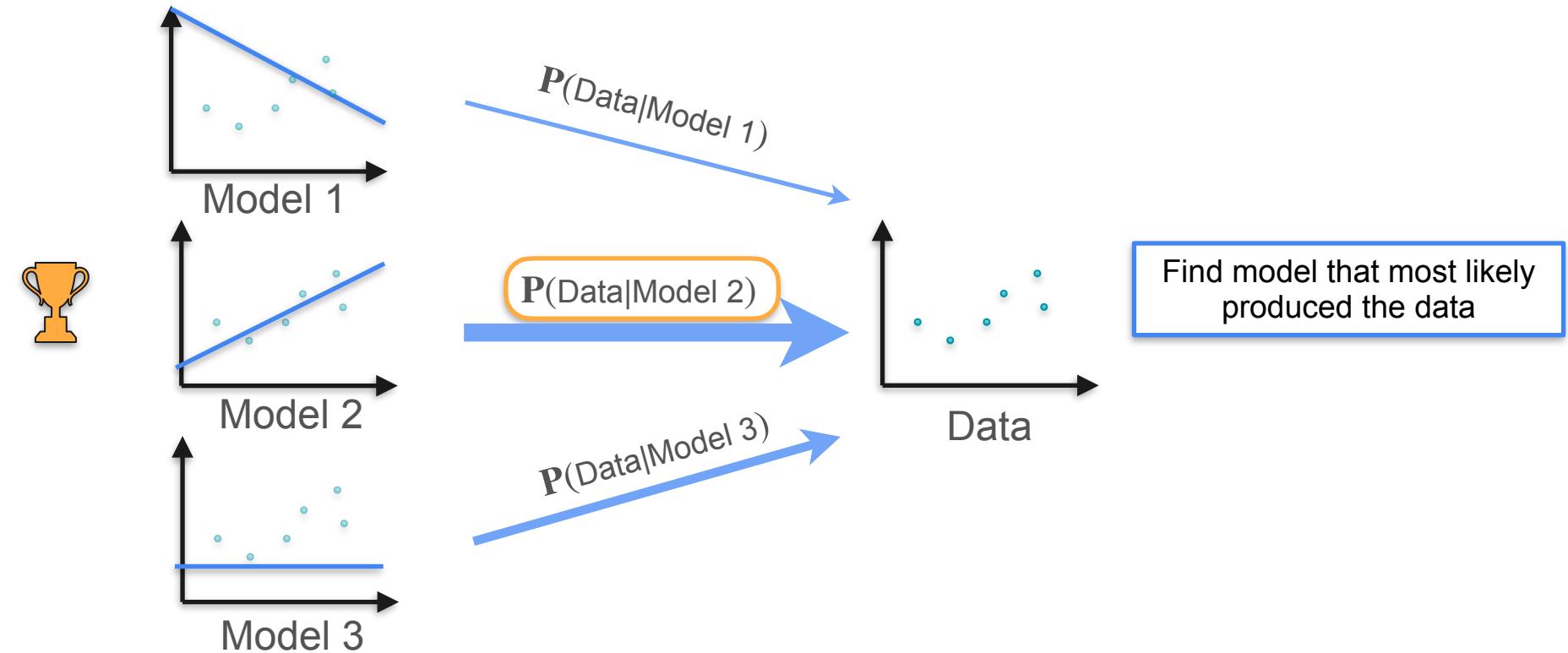
Example: Linear Regression



Example: Linear Regression



Example: Linear Regression





DeepLearning.AI

Point Estimation

MLE: Bernoulli Example

Maximum Likelihood: Bernoulli Example

Maximum Likelihood: Bernoulli Example



Maximum Likelihood: Bernoulli Example



Coin 1



$$P(H) = 0.7$$

$$P(T) = 0.3$$

Coin 2



$$P(H) = 0.5$$

$$P(T) = 0.5$$

Coin 3



$$P(H) = 0.3$$

$$P(T) = 0.7$$

Quiz

Which of the coins is more likely?

- A. Coin 1 ($P(H) = 0.7$)
- B. Coin 2 ($P(H) = 0.5$)
- C. Coin 3 ($P(H) = 0.3$)

Maximum Likelihood: Bernoulli Example



Maximum Likelihood: Bernoulli Example



Coin 1 0.7 0.7 0.7 0.7 0.7 0.7 0.7 0.7 0.3 0.3 = 0.0051

Maximum Likelihood: Bernoulli Example



Coin 1 0.7 0.7 0.7 0.7 0.7 0.7 0.7 0.7 0.7 0.3 = 0.0051

Coin 2 0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5 = 0.0010

Maximum Likelihood: Bernoulli Example



Coin 1 0.7 0.7 0.7 0.7 0.7 0.7 0.7 0.7 0.7 0.3 = 0.0051

Coin 2 0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5 = 0.0010

Coin 3 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.30 0.7 0.7 = 0.00003

Maximum Likelihood: Bernoulli Example



Coin 1	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.3	0.3 = 0.0051
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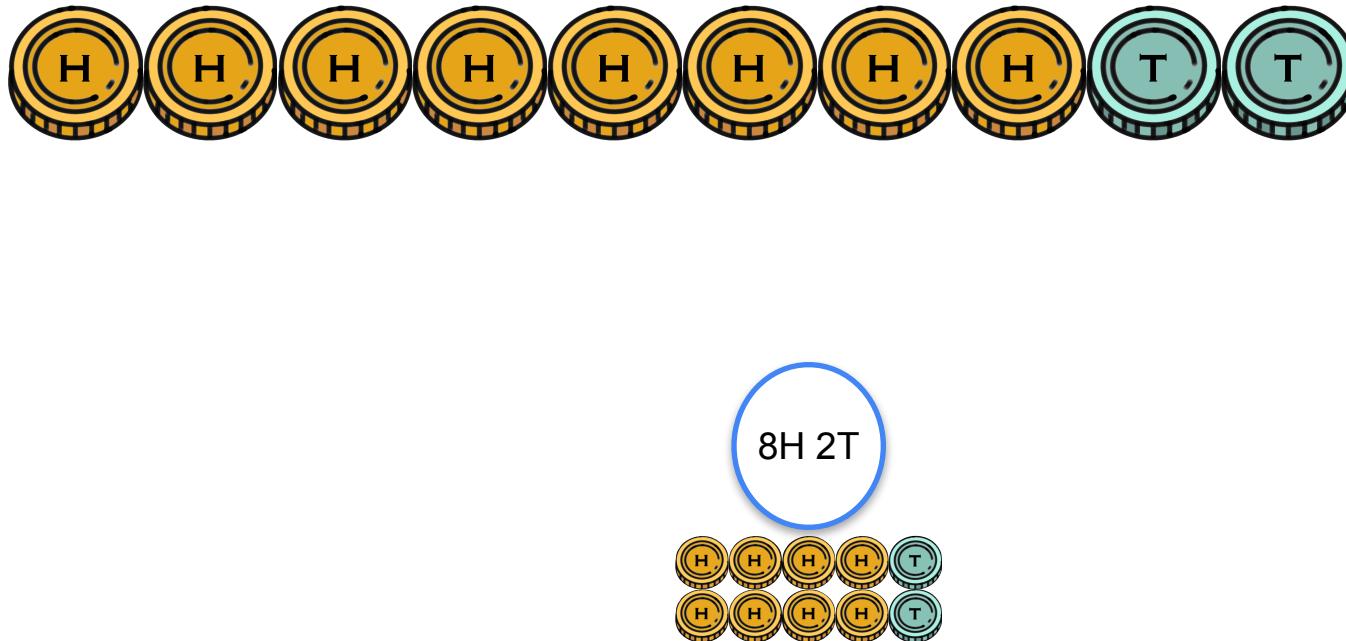
Coin 2	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5 = 0.0010
---------------	-----	-----	-----	-----	-----	-----	-----	-----	-----	--------------

Coin 3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.30	0.7	0.7 = 0.00003
---------------	-----	-----	-----	-----	-----	-----	-----	------	-----	---------------

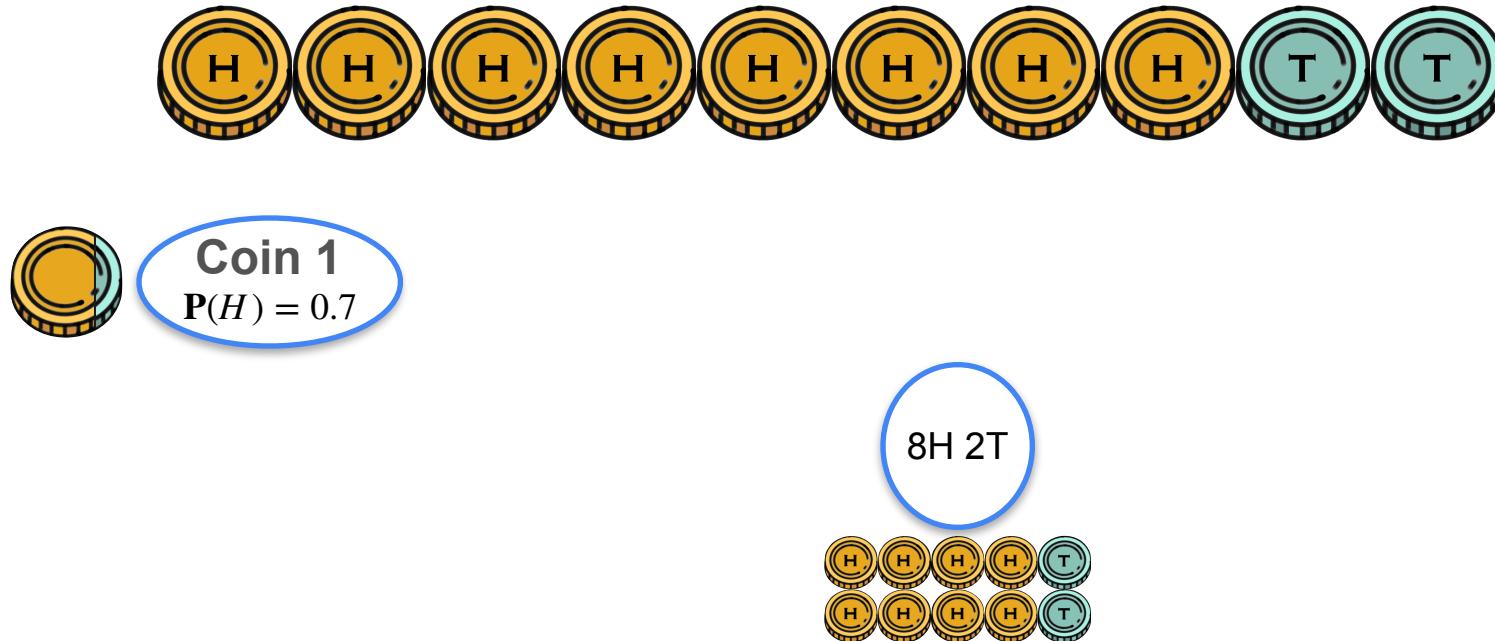
Maximum Likelihood: Bernoulli Example



Maximum Likelihood: Bernoulli Example



Maximum Likelihood: Bernoulli Example



Maximum Likelihood: Bernoulli Example



Coin 1

$$P(H) = 0.7$$



Coin 2

$$P(H) = 0.5$$

8H 2T



Maximum Likelihood: Bernoulli Example



Coin 1
 $P(H) = 0.7$



Coin 2
 $P(H) = 0.5$

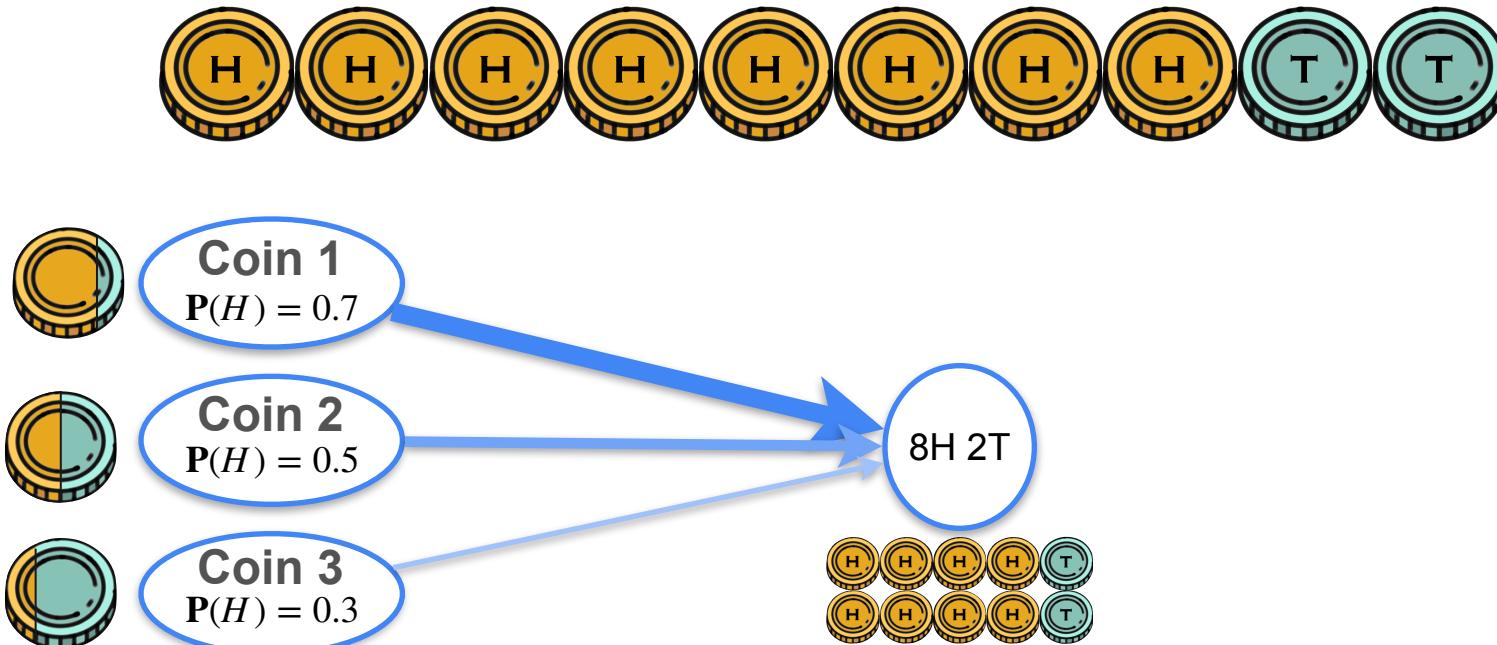


Coin 3
 $P(H) = 0.3$

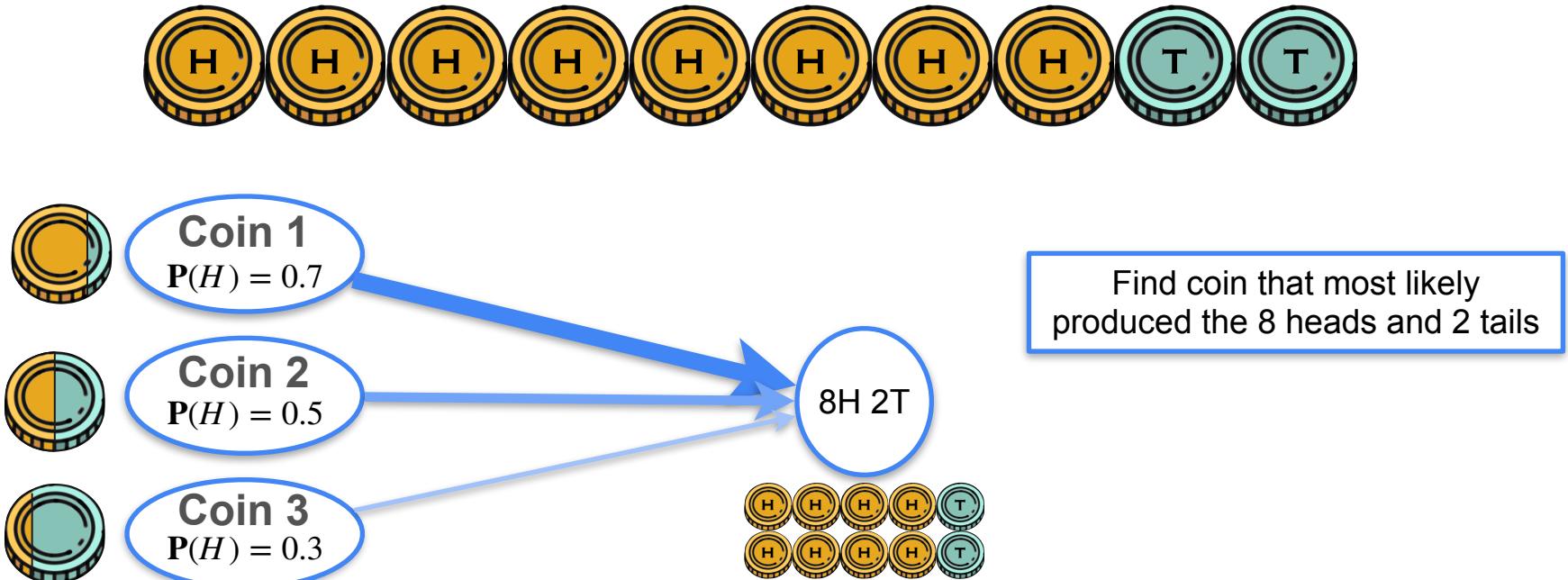
8H 2T



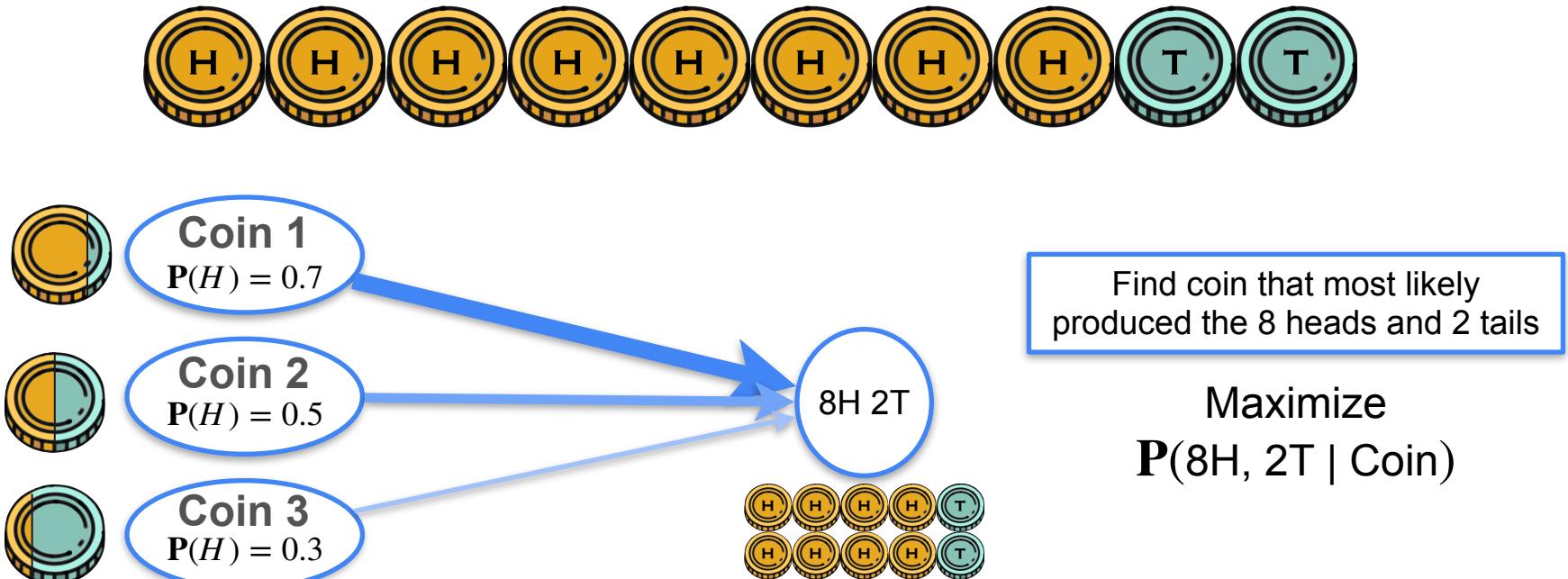
Maximum Likelihood: Bernoulli Example



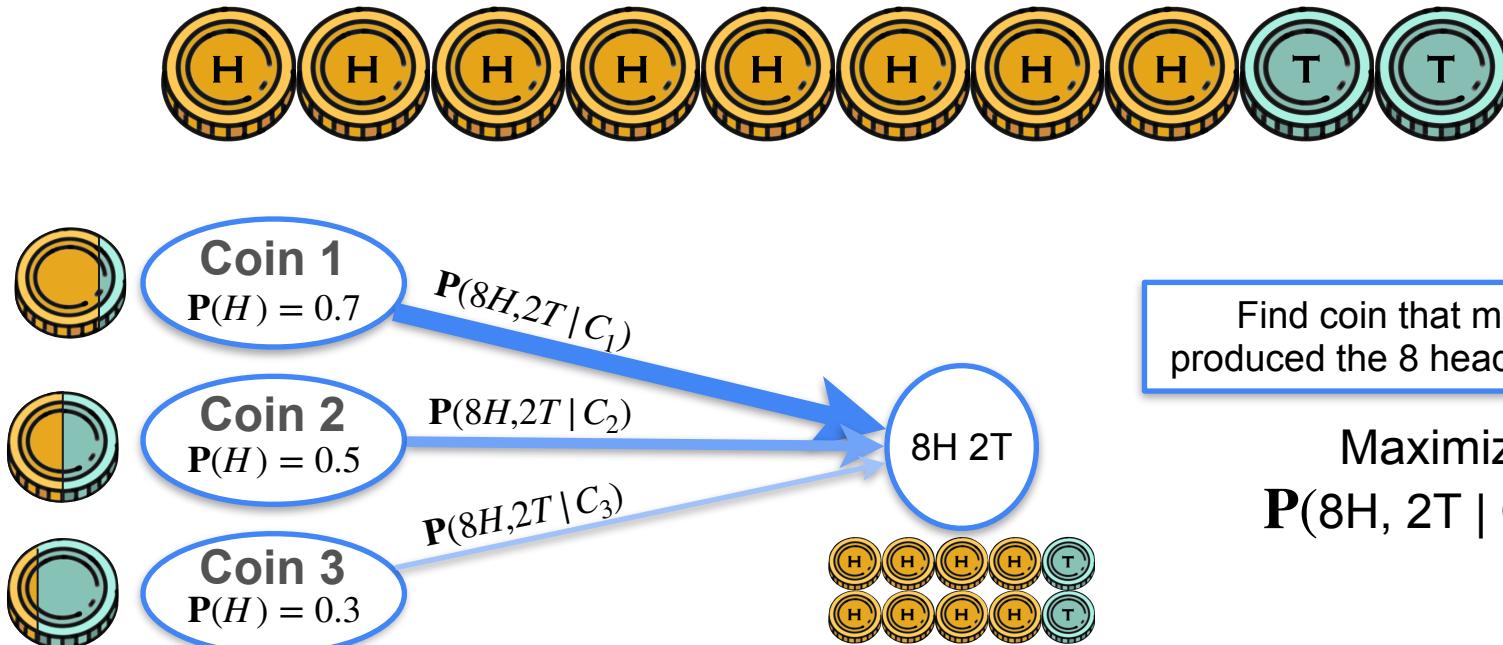
Maximum Likelihood: Bernoulli Example



Maximum Likelihood: Bernoulli Example



Maximum Likelihood: Bernoulli Example



Maximum Likelihood: Bernoulli Example



Coin 1
 $P(H) = 0.7$

$$P(8H, 2T | C_1) = 0.0051$$



Coin 2
 $P(H) = 0.5$

$$P(8H, 2T | C_2) = 0.0010$$



Coin 3
 $P(H) = 0.3$

$$P(8H, 2T | C_3) = 0.00003$$

8H 2T



Find coin that most likely produced the 8 heads and 2 tails

Maximize
 $P(8H, 2T | \text{Coin})$

Maximum Likelihood: Bernoulli Example



Coin 1
 $P(H) = 0.7$

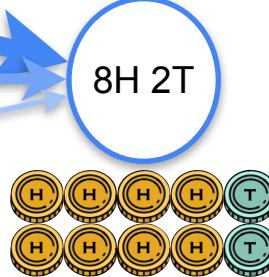
$$P(8H, 2T | C_1) = 0.0051$$

Coin 2
 $P(H) = 0.5$

$$P(8H, 2T | C_2) = 0.0010$$

Coin 3
 $P(H) = 0.3$

$$P(8H, 2T | C_3) = 0.00003$$



Find coin that most likely produced the 8 heads and 2 tails

Maximize
 $P(8H, 2T | \text{Coin})$

Maximum Likelihood: Bernoulli Example

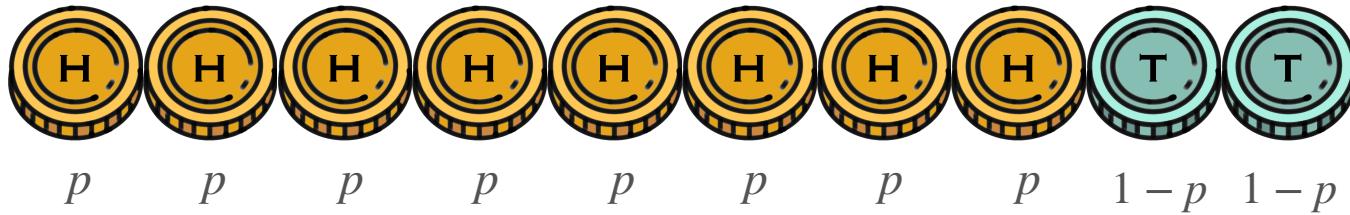


Maximum Likelihood: Bernoulli Example



Can you do any better?

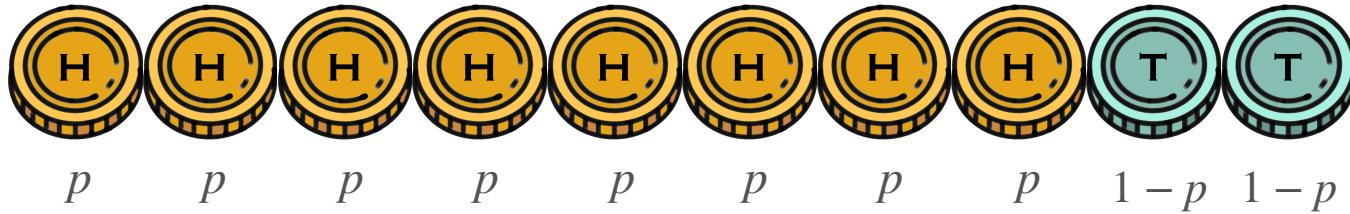
Maximum Likelihood: Bernoulli Example



Can you do any better?

$$p = \mathbf{P}(H)$$

Maximum Likelihood: Bernoulli Example

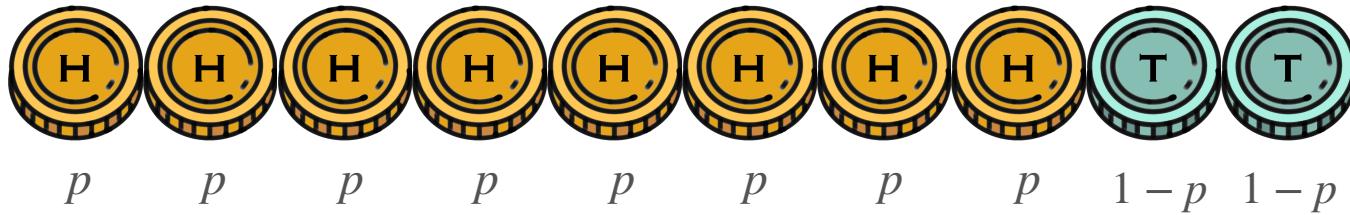


Can you do any better?

$$p = \mathbf{P}(H)$$

$$p^8(1 - p)^2$$

Maximum Likelihood: Bernoulli Example



Can you do any better?

$$p = \mathbf{P}(H)$$

$$p^8(1 - p)^2$$

You want p that maximizes the chances of seeing 8H

Maximum Likelihood: Bernoulli Example



$$p = \mathbf{P}(H)$$

$$p^8(1 - p)^2$$

You want p that maximizes the chances of seeing 8H

Maximum Likelihood: Bernoulli Example



$$p = \mathbf{P}(H) \quad \text{Likelihood} \quad L(p; 8H) = p^8(1 - p)^2$$

You want p that maximizes the chances of seeing 8H

Maximum Likelihood: Bernoulli Example



$$p = \mathbf{P}(H) \quad \text{Likelihood} \quad L(p; 8H) = p^8(1 - p)^2 \quad \text{Function of } p$$

You want p that maximizes the chances of seeing 8H

Maximum Likelihood: Bernoulli Example



$$p = \mathbf{P}(H) \quad \text{Likelihood} \quad L(p; 8H) = p^8(1 - p)^2 \quad \text{Function of } p$$

You want p that maximizes the chances of seeing 8H

$$\log((p^8(1 - p)^2))$$

Maximum Likelihood: Bernoulli Example



$$p = \mathbf{P}(H) \quad \text{Likelihood} \quad L(p; 8H) = p^8(1-p)^2 \quad \text{Function of } p$$

You want p that maximizes the chances of seeing 8H

$$\log((p^8(1-p)^2)) = 8\log(p) + 2\log(1-p)$$

Maximum Likelihood: Bernoulli Example



$$p = \mathbf{P}(H) \quad \text{Likelihood} \quad L(p; 8H) = p^8(1-p)^2 \quad \text{Function of } p$$

You want p that maximizes the chances of seeing 8H

$$\text{Log-likelihood} \quad \ell(p; 8H) = \log((p^8(1-p)^2)) = 8\log(p) + 2\log(1-p)$$

Maximum Likelihood: Bernoulli Example



$$p = \mathbf{P}(H) \quad \text{Likelihood} \quad L(p; 8H) = p^8(1-p)^2 \quad \text{Function of } p$$

You want p that maximizes the chances of seeing 8H

$$\text{Log-likelihood} \quad \ell(p; 8H) = \log((p^8(1-p)^2)) = 8\log(p) + 2\log(1-p)$$

$$\frac{d}{dp} (8\log(p) + 2\log(1-p))$$

Maximum Likelihood: Bernoulli Example



$$p = \mathbf{P}(H) \quad \text{Likelihood} \quad L(p; 8H) = p^8(1-p)^2 \quad \text{Function of } p$$

You want p that maximizes the chances of seeing 8H

$$\text{Log-likelihood} \quad \ell(p; 8H) = \log((p^8(1-p)^2)) = 8\log(p) + 2\log(1-p)$$

$$\frac{d}{dp} (8\log(p) + 2\log(1-p)) = \frac{8}{p} + \frac{2}{1-p}(-1)$$

Maximum Likelihood: Bernoulli Example



$$p = \mathbf{P}(H) \quad \text{Likelihood} \quad L(p; 8H) = p^8(1-p)^2 \quad \text{Function of } p$$

You want p that maximizes the chances of seeing 8H

$$\text{Log-likelihood} \quad \ell(p; 8H) = \log((p^8(1-p)^2)) = 8\log(p) + 2\log(1-p)$$

$$\frac{d}{dp} (8\log(p) + 2\log(1-p)) = \frac{8}{p} + \frac{2}{1-p}(-1) = 0 \rightarrow \hat{p} = \frac{8}{10}$$

The General Case

Maximum Likelihood: Bernoulli Example

n coins

k heads

Maximum Likelihood: Bernoulli Example

n coins

k heads

X_1

X_2

X_3

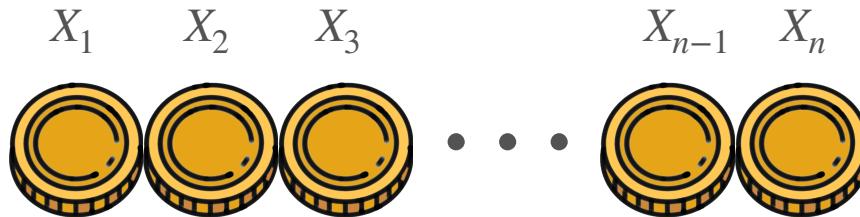
X_{n-1}

X_n



Maximum Likelihood: Bernoulli Example

n coins
 k heads

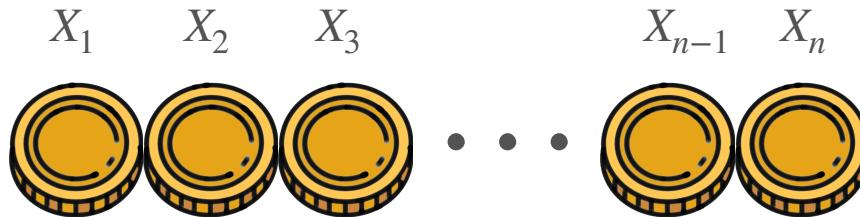


$$\mathbf{X} = (X_1, \dots, X_n)$$

$$X_i \stackrel{i.i.d}{\sim} \text{Bernoulli}(p)$$

Maximum Likelihood: Bernoulli Example

n coins
 k heads



$$\mathbf{X} = (X_1, \dots, X_n)$$

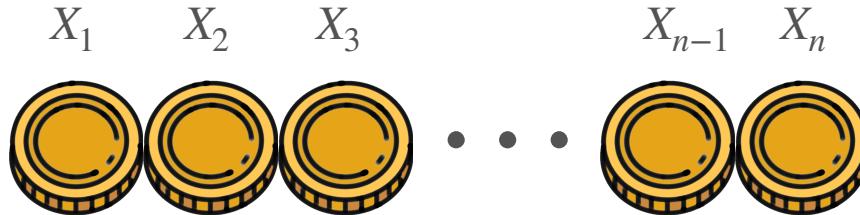
$$X_i \stackrel{i.i.d}{\sim} \text{Bernoulli}(p)$$

Likelihood

$$L(p; \mathbf{x}) = P_p(\mathbf{X} = \mathbf{x}) = \prod_{i=1}^n p_{X_i}(x_i)$$

Maximum Likelihood: Bernoulli Example

n coins
 k heads



$$\mathbf{X} = (X_1, \dots, X_n)$$

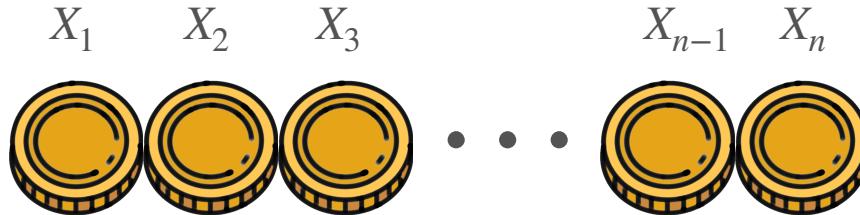
$$X_i \stackrel{i.i.d}{\sim} \text{Bernoulli}(p)$$

Likelihood

$$L(p; \mathbf{x}) = P_p(\mathbf{X} = \mathbf{x}) = \prod_{i=1}^n p_{X_i}(x_i) = \prod_{i=1}^n p^{x_i} (1-p)^{1-x_i}$$

Maximum Likelihood: Bernoulli Example

n coins
 k heads



$$\mathbf{X} = (X_1, \dots, X_n)$$

$$X_i \stackrel{i.i.d}{\sim} \text{Bernoulli}(p)$$

Likelihood

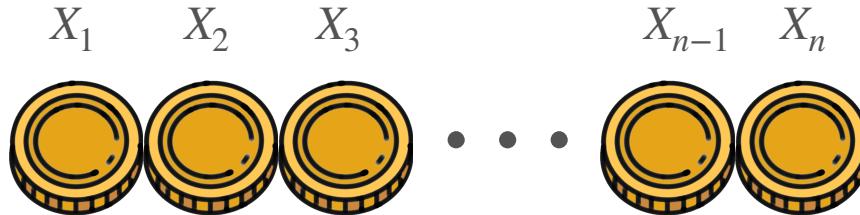
$$L(p; \mathbf{x}) = P_p(\mathbf{X} = \mathbf{x}) = \prod_{i=1}^n p_{X_i}(x_i) = \prod_{i=1}^n p^{x_i} (1-p)^{1-x_i}$$

If $x_i = 1$, $p^{[x_i]}(1-p)^{[1-x_i]} = p$
If $x_i = 0$, $p^{[x_i]}(1-p)^{[1-x_i]} = (1-p)$



Maximum Likelihood: Bernoulli Example

n coins
 k heads



$$\mathbf{X} = (X_1, \dots, X_n)$$

$$X_i \stackrel{i.i.d}{\sim} \text{Bernoulli}(p)$$

Likelihood

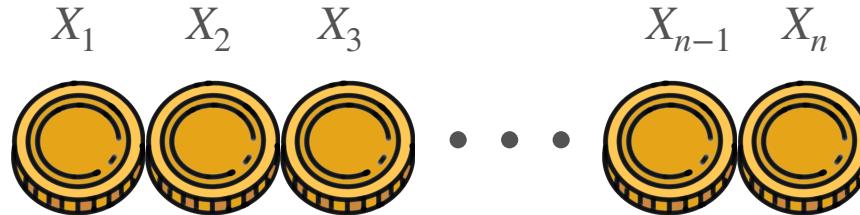
$$L(p; \mathbf{x}) = P_p(\mathbf{X} = \mathbf{x}) = \prod_{i=1}^n p_{X_i}(x_i) = \prod_{i=1}^n p^{x_i} (1-p)^{1-x_i}$$

If $x_i = 1$, $p^{[x_i]}(1-p)^{[1-x_i]} = p$
If $x_i = 0$, $p^{[x_i]}(1-p)^{[1-x_i]} = (1-p)$

$$\sum_{i=1}^n x_i = \# \text{ heads}$$

$$n - \sum_{i=1}^n x_i = \# \text{ tails}$$

Maximum Likelihood: Bernoulli Example



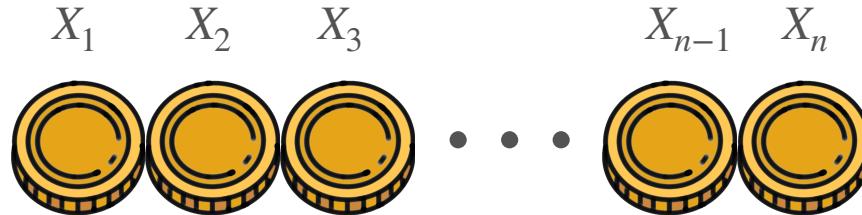
$$\mathbf{X} = (X_1, \dots, X_n)$$

$$X_i \stackrel{i.i.d}{\sim} \text{Bernoulli}(p)$$

Likelihood

$$L(p; \mathbf{x}) = P_p(\mathbf{X} = \mathbf{x}) = \prod_{i=1}^n p_{X_i}(x_i) = \prod_{i=1}^n p^{x_i} (1-p)^{1-x_i}$$

Maximum Likelihood: Bernoulli Example



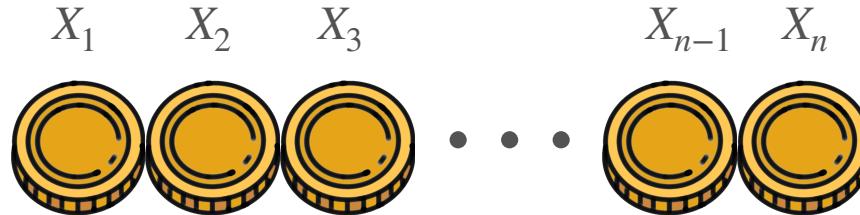
$$\mathbf{X} = (X_1, \dots, X_n)$$

$$X_i \stackrel{i.i.d}{\sim} \text{Bernoulli}(p)$$

Likelihood

$$L(p; \mathbf{x}) = P_p(\mathbf{X} = \mathbf{x}) = \prod_{i=1}^n p_{X_i}(x_i) = \prod_{i=1}^n p^{x_i} (1-p)^{1-x_i} = p^{\left(\sum_{i=1}^n x_i\right)} (1-p)^{\left(n - \sum_{i=1}^n x_i\right)}$$

Maximum Likelihood: Bernoulli Example



$$\mathbf{X} = (X_1, \dots, X_n)$$

$$X_i \stackrel{i.i.d}{\sim} \text{Bernoulli}(p)$$

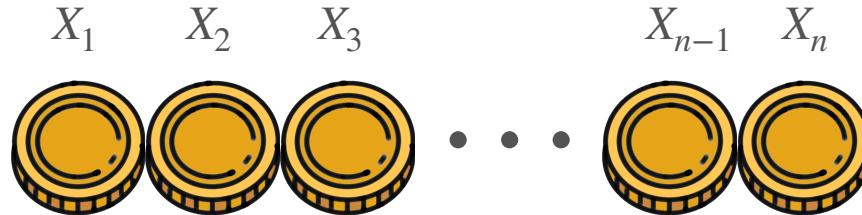
Likelihood

$$L(p; \mathbf{x}) = P_p(\mathbf{X} = \mathbf{x}) = \prod_{i=1}^n p_{X_i}(x_i) = \prod_{i=1}^n p^{x_i} (1-p)^{1-x_i} = p^{\left(\sum_{i=1}^n x_i\right)} (1-p)^{\left(n - \sum_{i=1}^n x_i\right)}$$

Log-likelihood

$$\ell(p; \mathbf{x}) = \log \left((p^{\sum_{i=1}^n x_i} (1-p)^{n - \sum_{i=1}^n x_i}) \right)$$

Maximum Likelihood: Bernoulli Example



$$\mathbf{X} = (X_1, \dots, X_n)$$

$$X_i \stackrel{i.i.d}{\sim} \text{Bernoulli}(p)$$

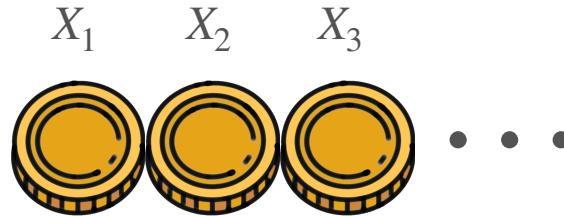
Likelihood

$$L(p; \mathbf{x}) = P_p(\mathbf{X} = \mathbf{x}) = \prod_{i=1}^n p_{X_i}(x_i) = \prod_{i=1}^n p^{x_i} (1-p)^{1-x_i} = p^{\left(\sum_{i=1}^n x_i\right)} (1-p)^{\left(n - \sum_{i=1}^n x_i\right)}$$

Log-likelihood

$$\ell(p; \mathbf{x}) = \log \left((p^{\sum_{i=1}^n x_i} (1-p)^{n - \sum_{i=1}^n x_i}) \right) = \left(\sum_{i=1}^n x_i \right) \log(p) + \left(n - \sum_{i=1}^n x_i \right) \log(1-p)$$

Maximum Likelihood: Bernoulli Example

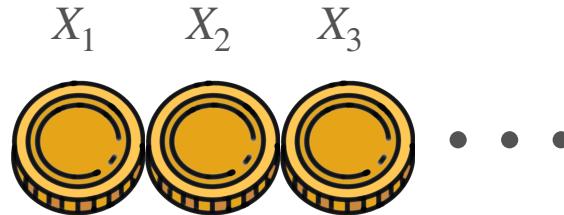


$$\mathbf{X} = (X_1, \dots, X_n)$$

$$X_i \stackrel{i.i.d}{\sim} \text{Bernoulli}(p)$$

$$\ell(p; \mathbf{x}) = \log \left((p^{\sum_{i=1}^n x_i} (1-p)^{n - \sum_{i=1}^n x_i}) \right) = \left(\sum_{i=1}^n x_i \right) \log(p) + \left(n - \sum_{i=1}^n x_i \right) \log(1-p)$$

Maximum Likelihood: Bernoulli Example



$$\mathbf{X} = (X_1, \dots, X_n)$$

$$X_i \stackrel{i.i.d}{\sim} \text{Bernoulli}(p)$$

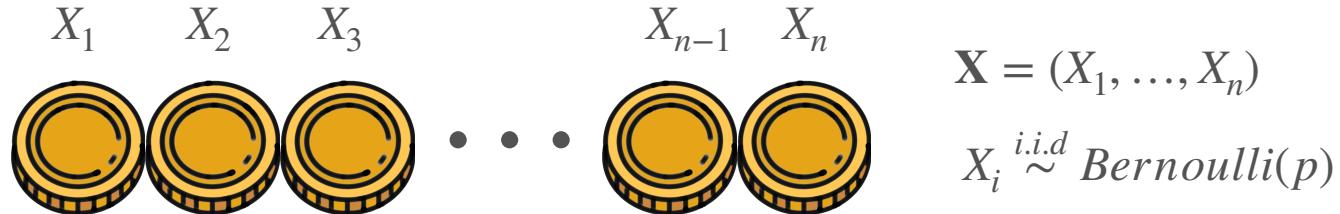
$$\ell(p; \mathbf{x}) = \log \left((p^{\sum_{i=1}^n x_i} (1-p)^{n - \sum_{i=1}^n x_i}) \right) = \left(\sum_{i=1}^n x_i \right) \log(p) + \left(n - \sum_{i=1}^n x_i \right) \log(1-p)$$

Find the maximum!

$$\frac{d}{dp} \ell(p; \mathbf{x}) = \frac{d}{dp} \left(\left(\sum_{i=1}^n x_i \right) \log(p) + \left(n - \sum_{i=1}^n x_i \right) \log(1-p) \right)$$

$$= \frac{\sum_{i=1}^n x_i}{p} + \frac{n - \sum_{i=1}^n x_i}{1-p} (-1) = 0$$

Maximum Likelihood: Bernoulli Example



$$\ell(p; \mathbf{x}) = \log \left((p^{\sum_{i=1}^n x_i} (1-p)^{n - \sum_{i=1}^n x_i}) \right) = \left(\sum_{i=1}^n x_i \right) \log(p) + \left(n - \sum_{i=1}^n x_i \right) \log(1-p)$$

Find the maximum!

$$\begin{aligned} \frac{d}{dp} \ell(p; \mathbf{x}) &= \frac{d}{dp} \left(\left(\sum_{i=1}^n x_i \right) \log(p) + \left(n - \sum_{i=1}^n x_i \right) \log(1-p) \right) \\ &= \frac{\sum_{i=1}^n x_i}{p} + \frac{n - \sum_{i=1}^n x_i}{1-p} (-1) = 0 \end{aligned} \quad \rightarrow \quad \hat{p} = \frac{\sum_{i=1}^n x_i}{n} = \bar{x}$$

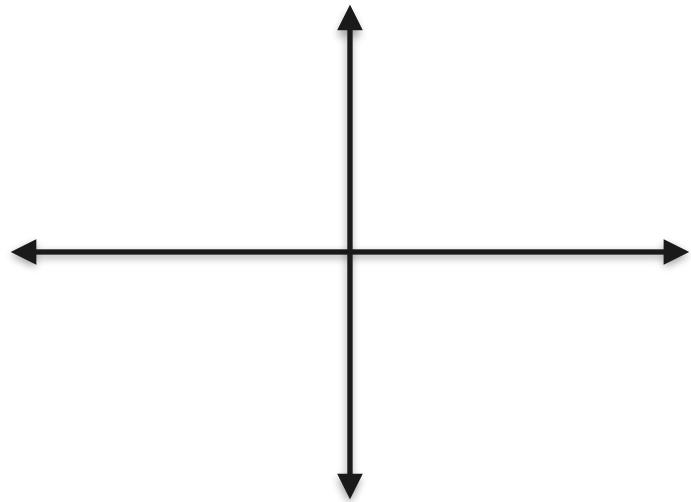


DeepLearning.AI

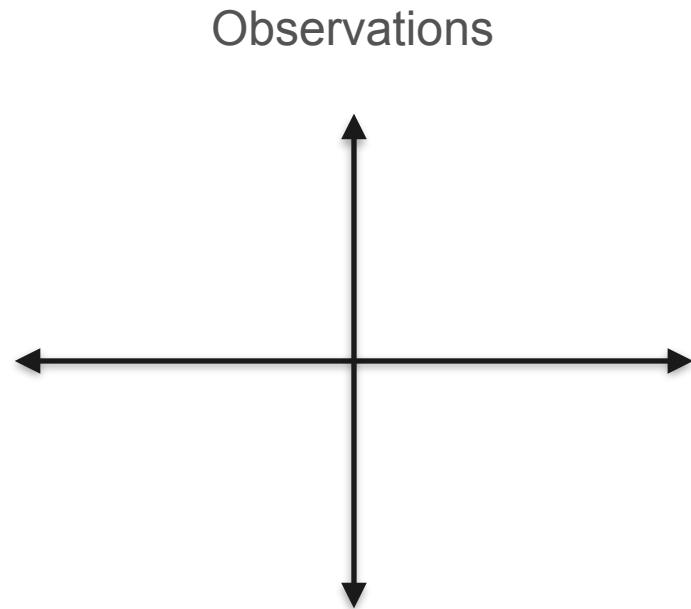
Point Estimation

MLE: Gaussian Example

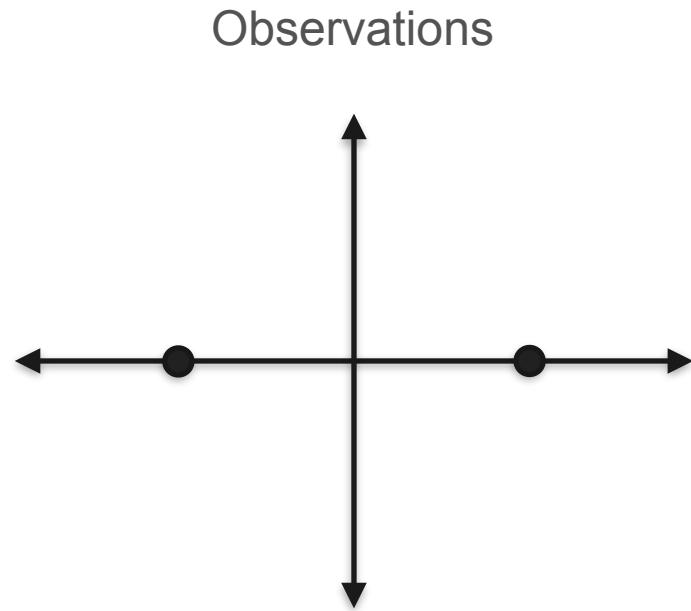
Maximum Likelihood: Gaussian Example



Maximum Likelihood: Gaussian Example



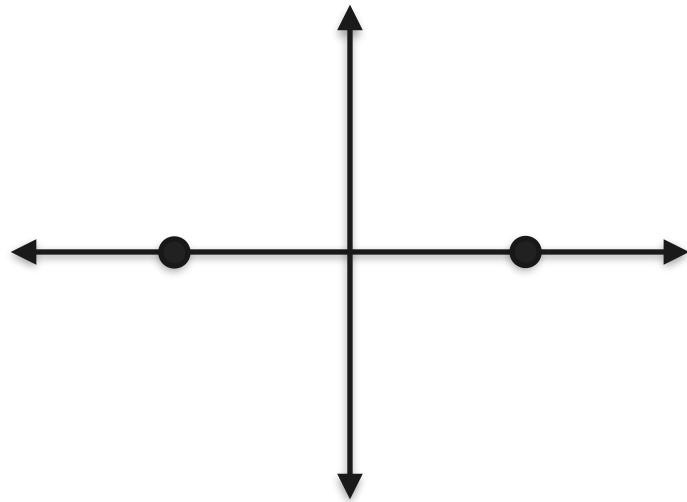
Maximum Likelihood: Gaussian Example



Maximum Likelihood: Gaussian Example

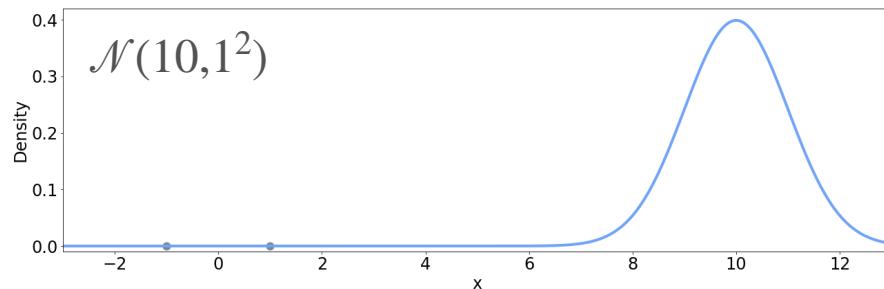
Candidates

Observations

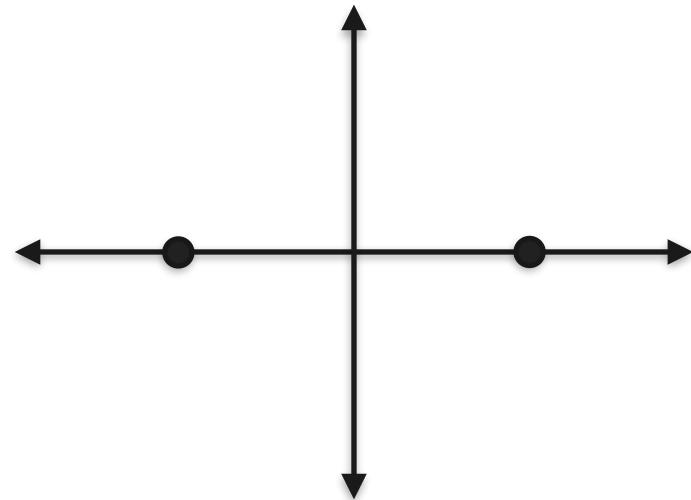


Maximum Likelihood: Gaussian Example

Candidates

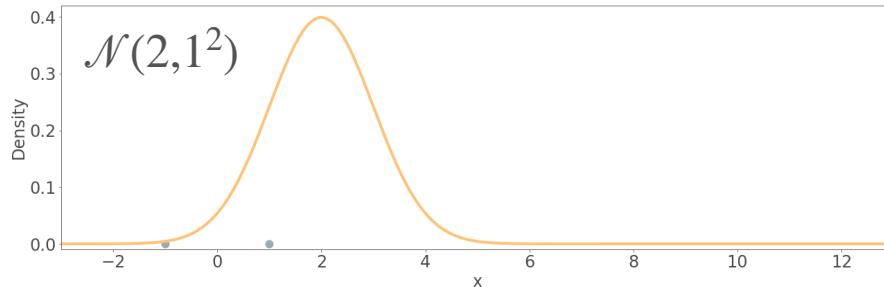
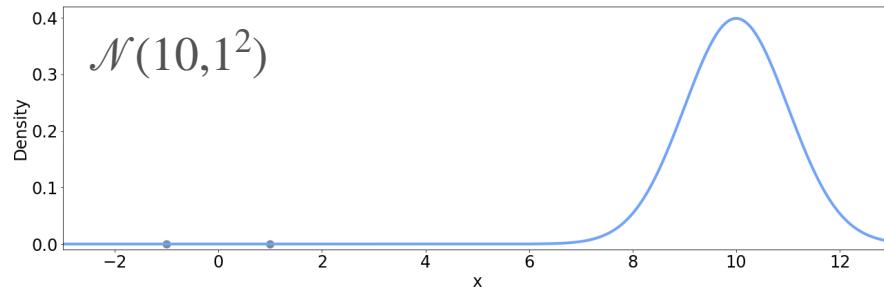


Observations

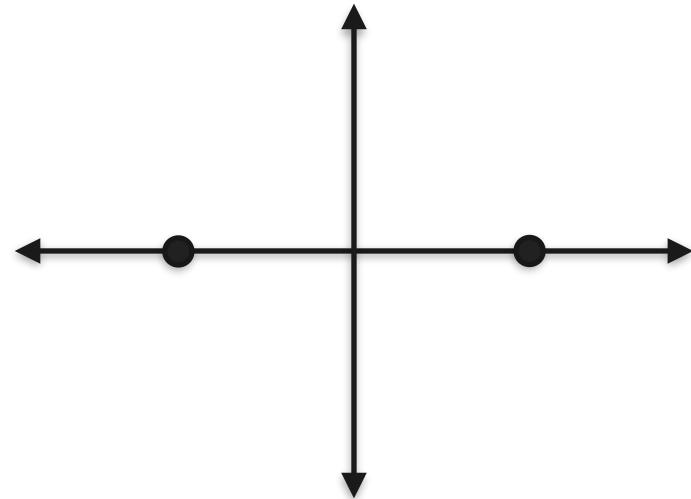


Maximum Likelihood: Gaussian Example

Candidates

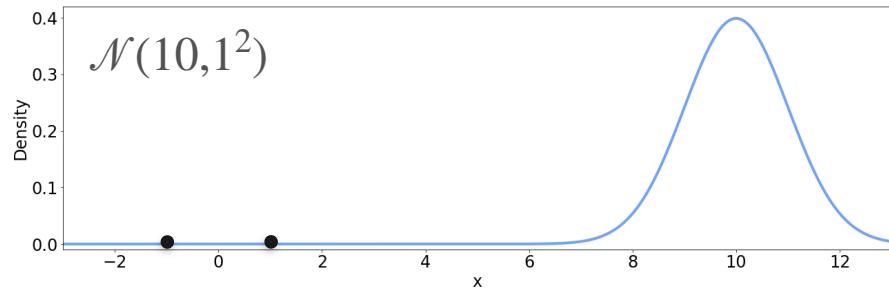


Observations

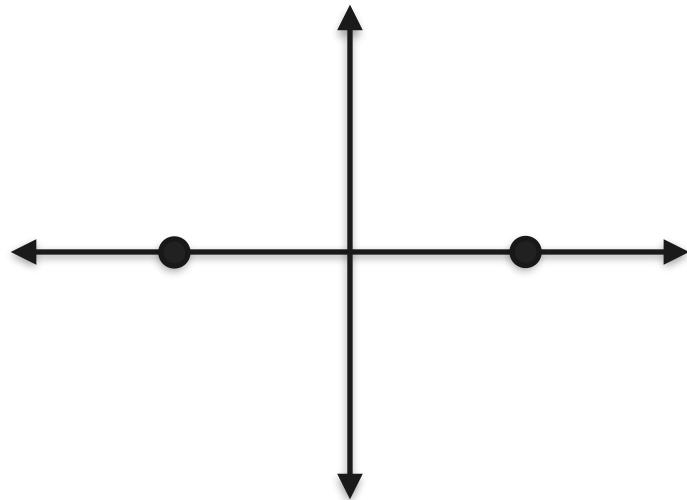


Maximum Likelihood: Gaussian Example

Candidates

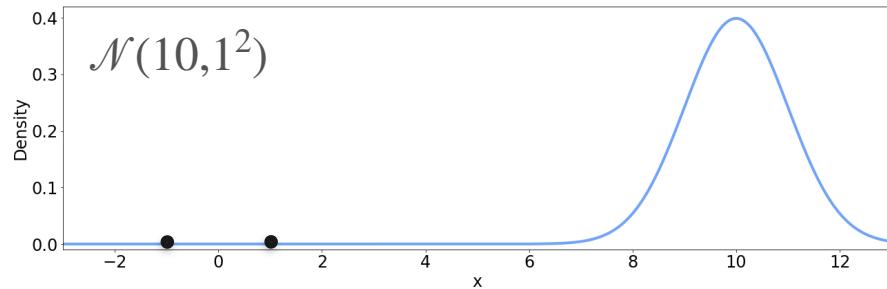


Observations

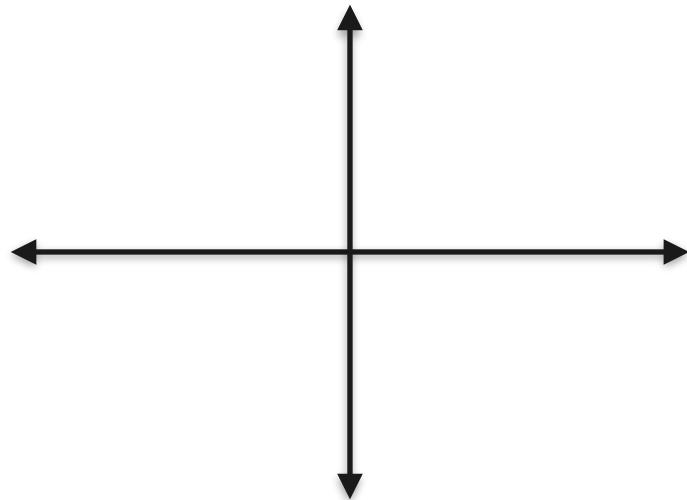


Maximum Likelihood: Gaussian Example

Candidates

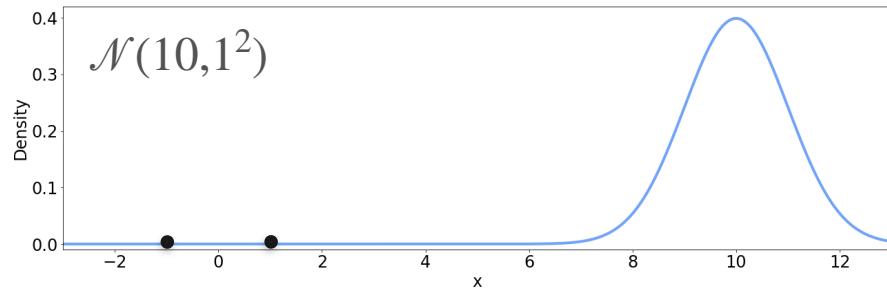


Observations

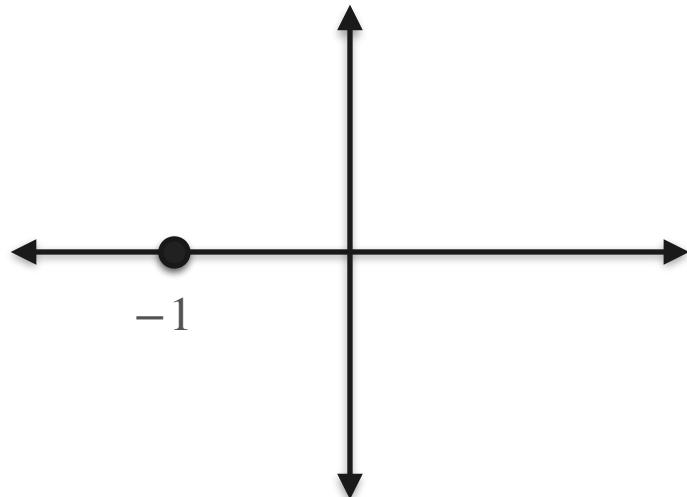


Maximum Likelihood: Gaussian Example

Candidates

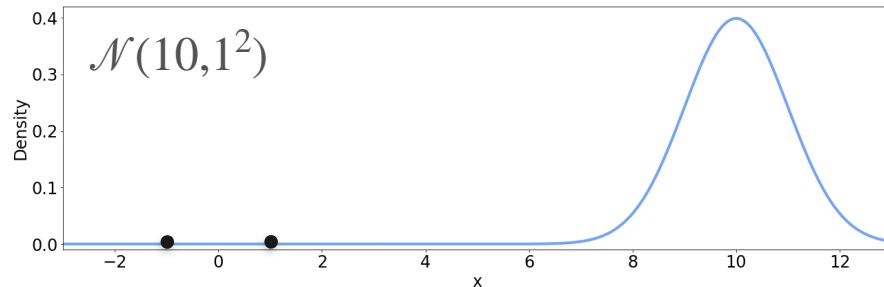


Observations

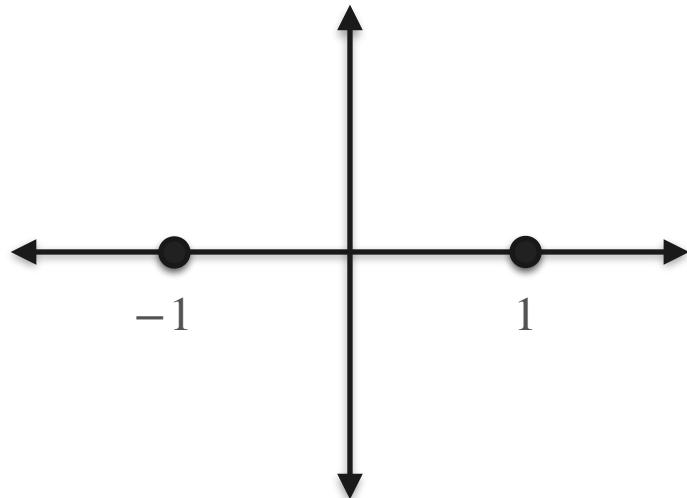


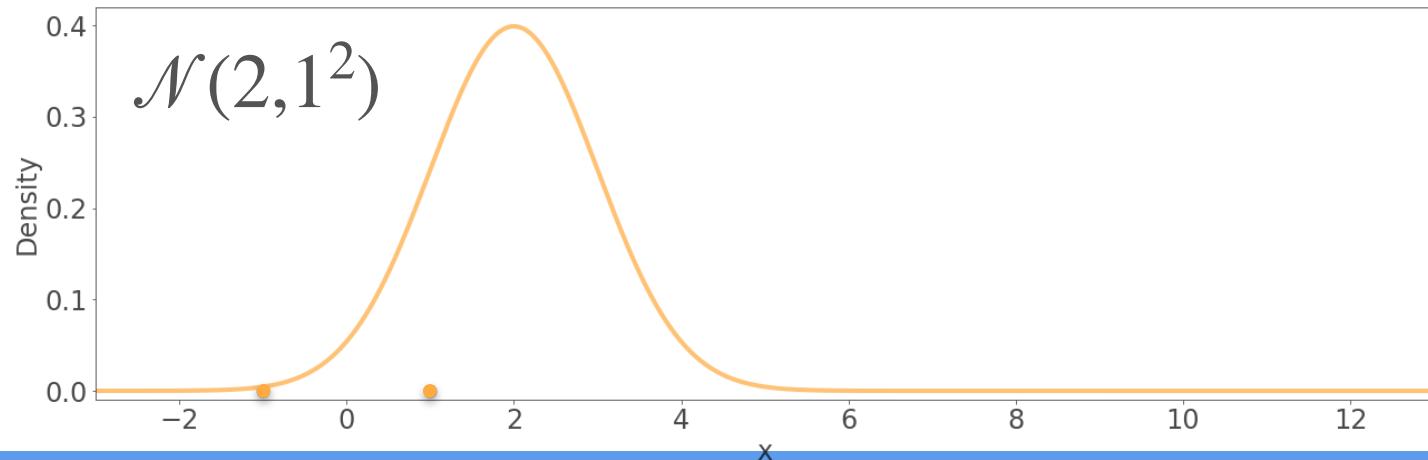
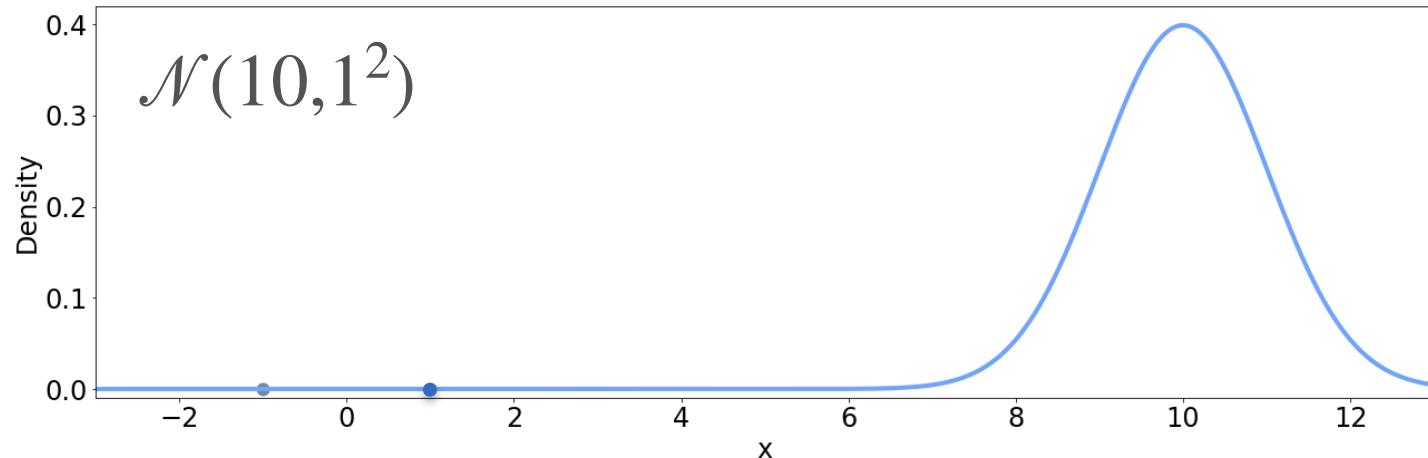
Maximum Likelihood: Gaussian Example

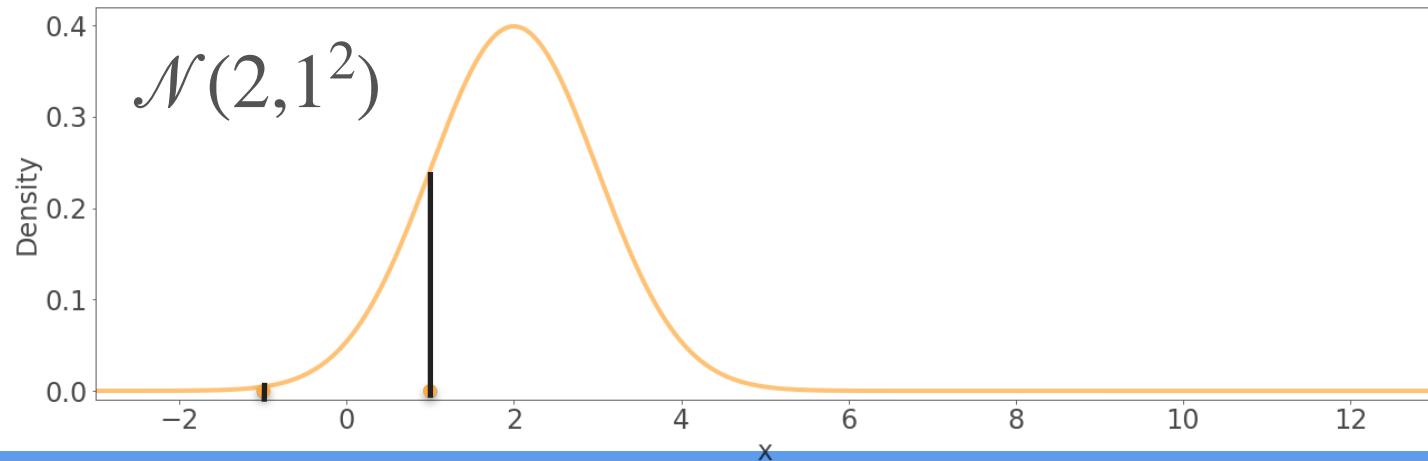
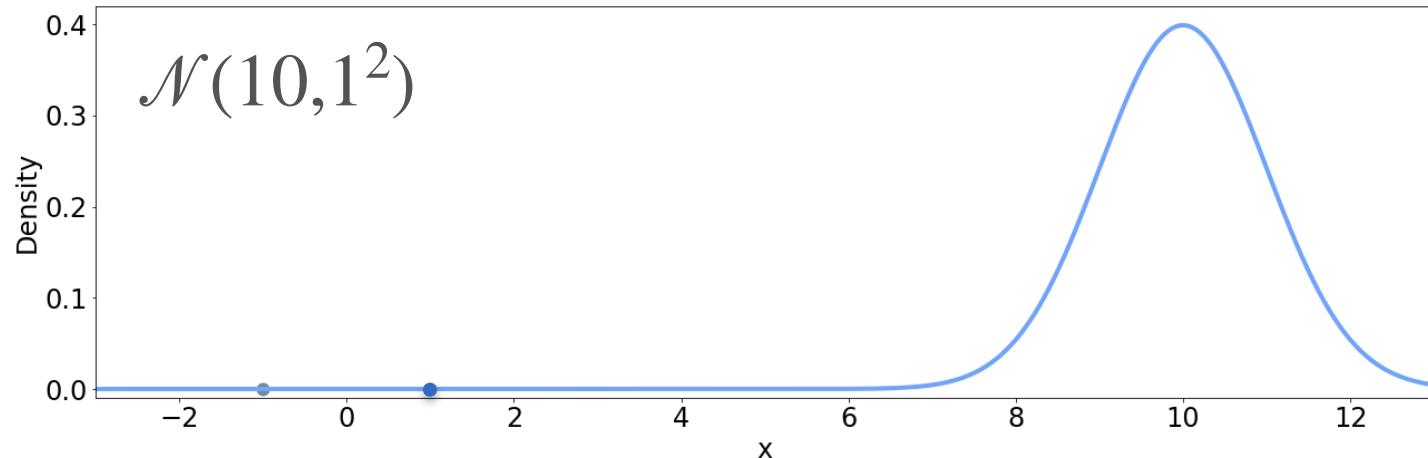
Candidates

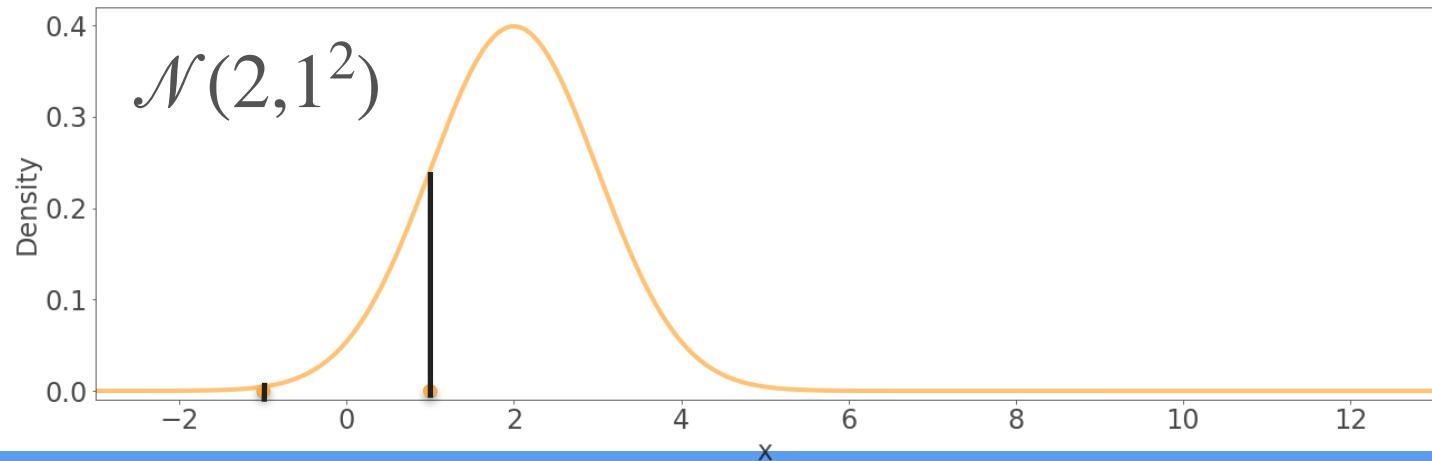
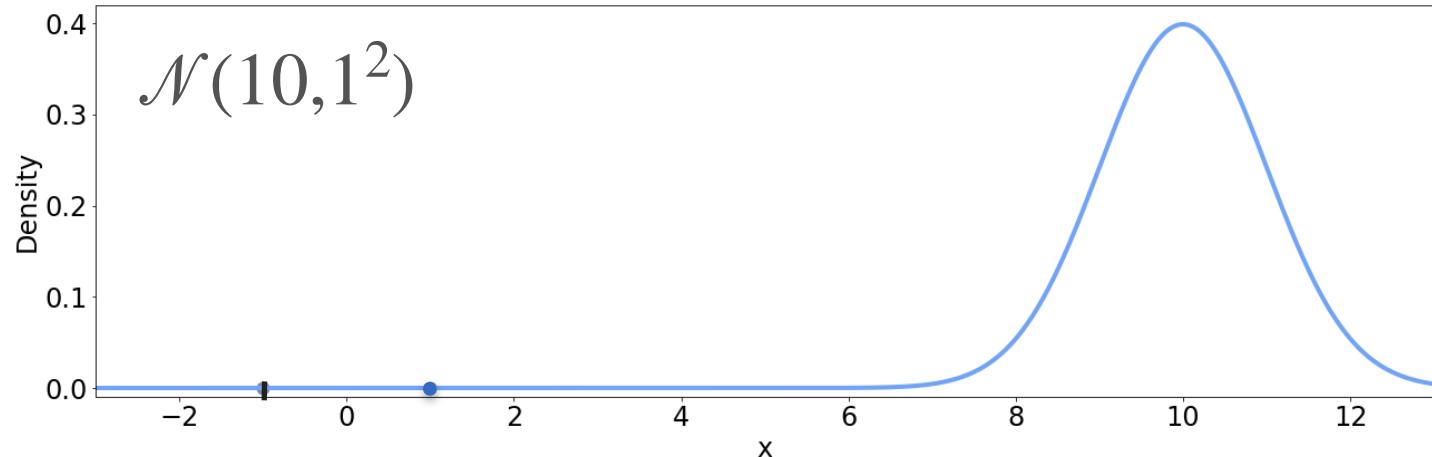


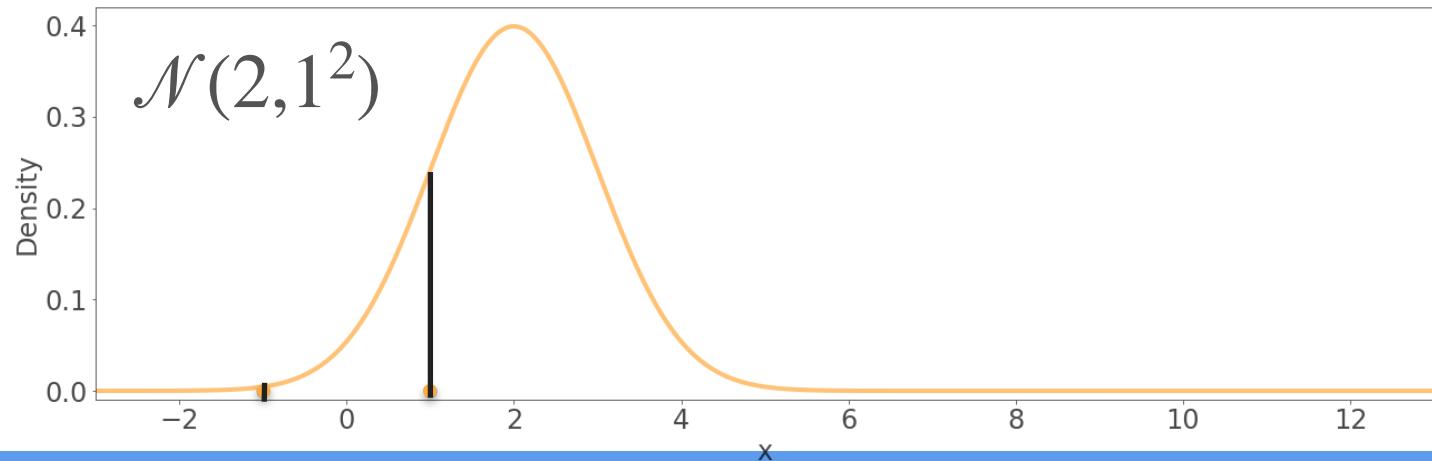
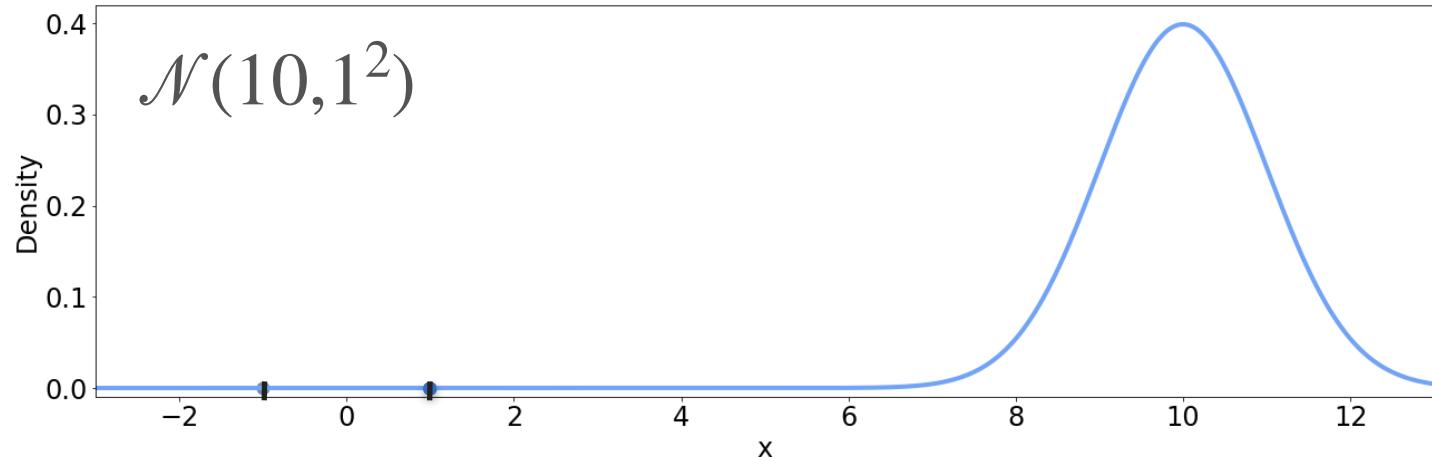
Observations

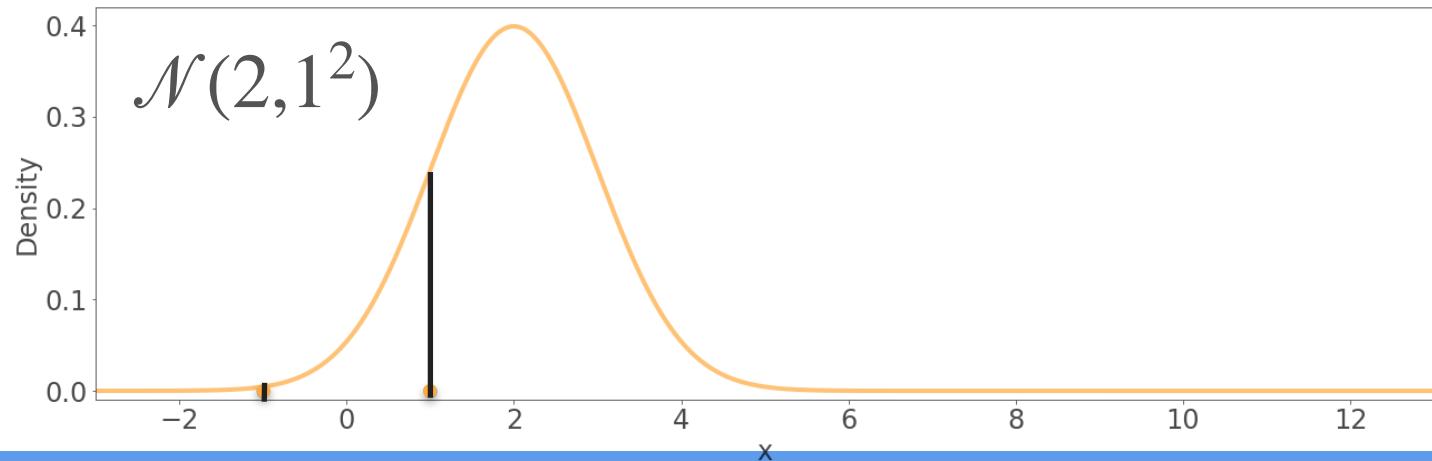
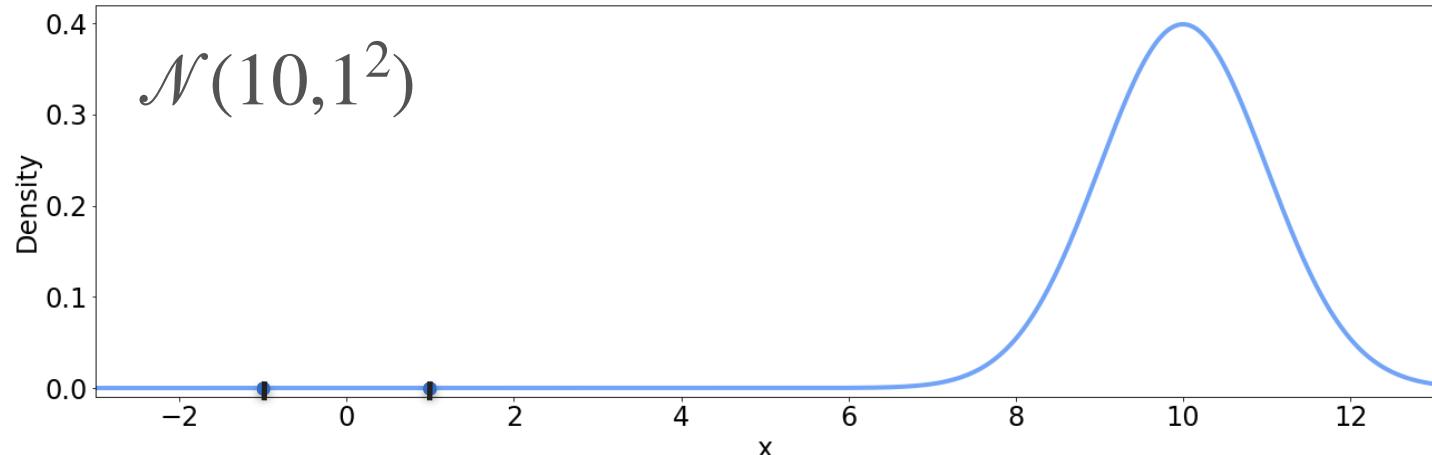


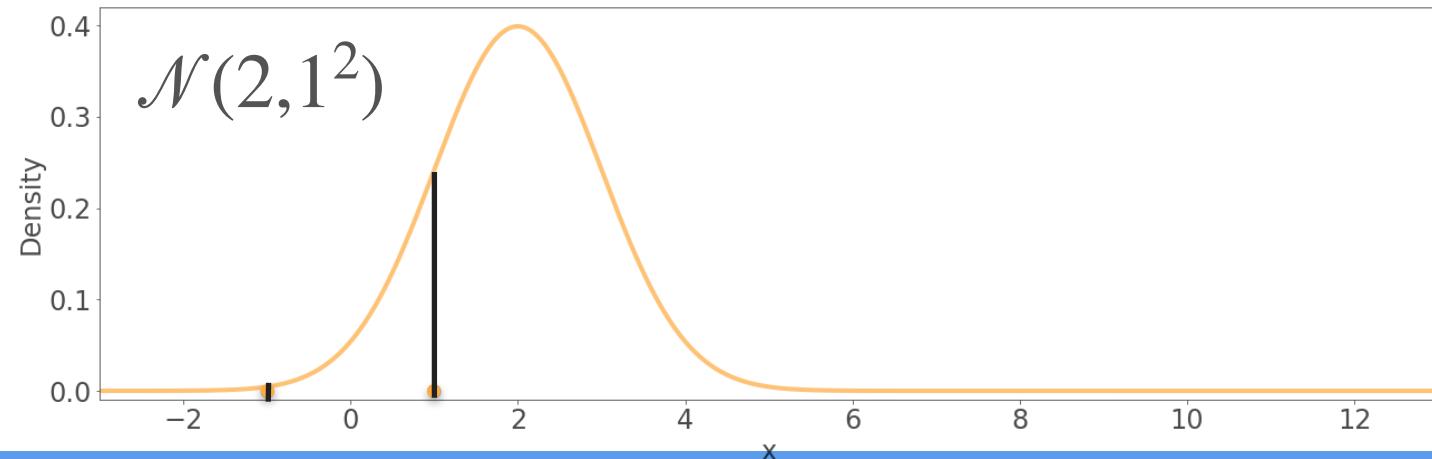
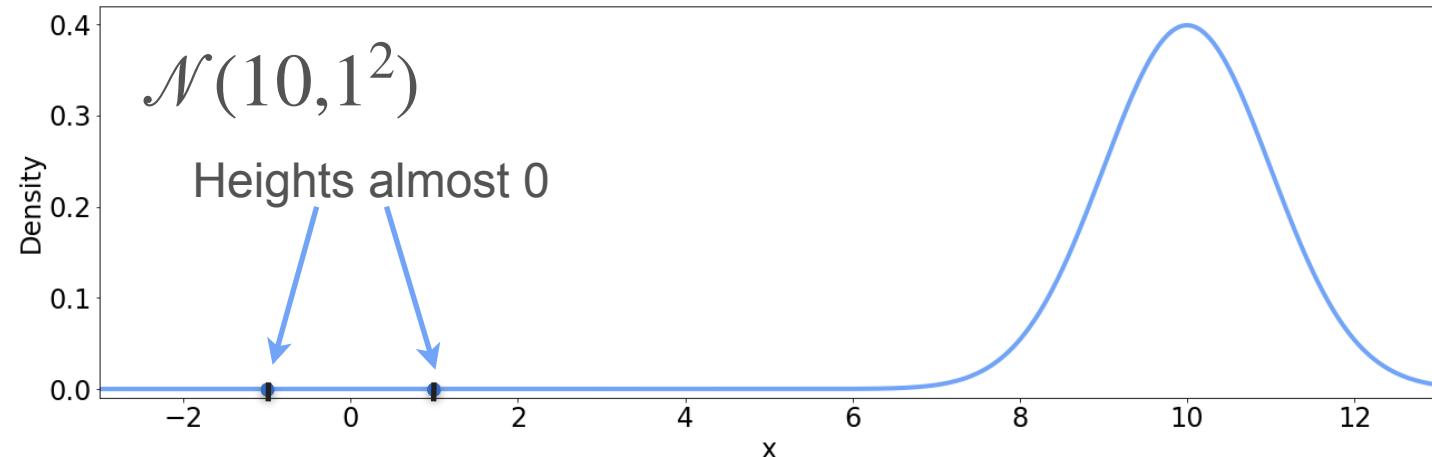


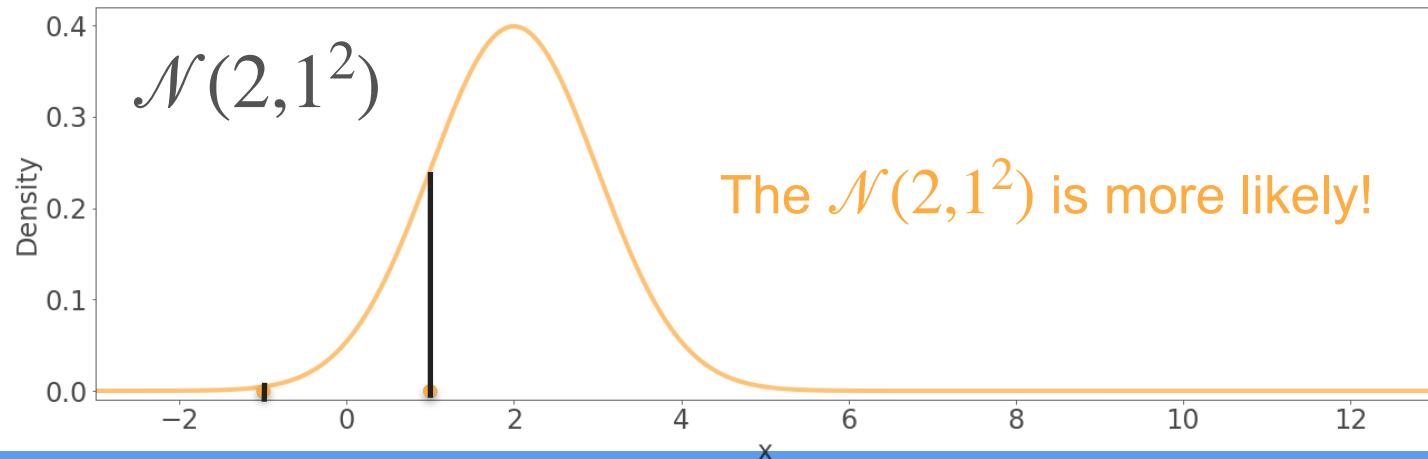
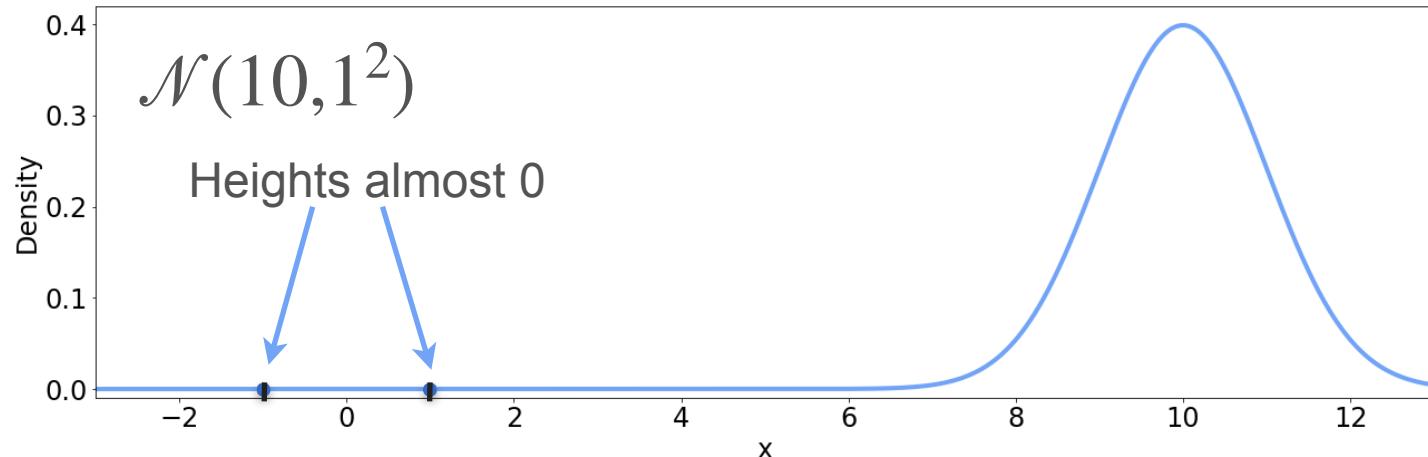


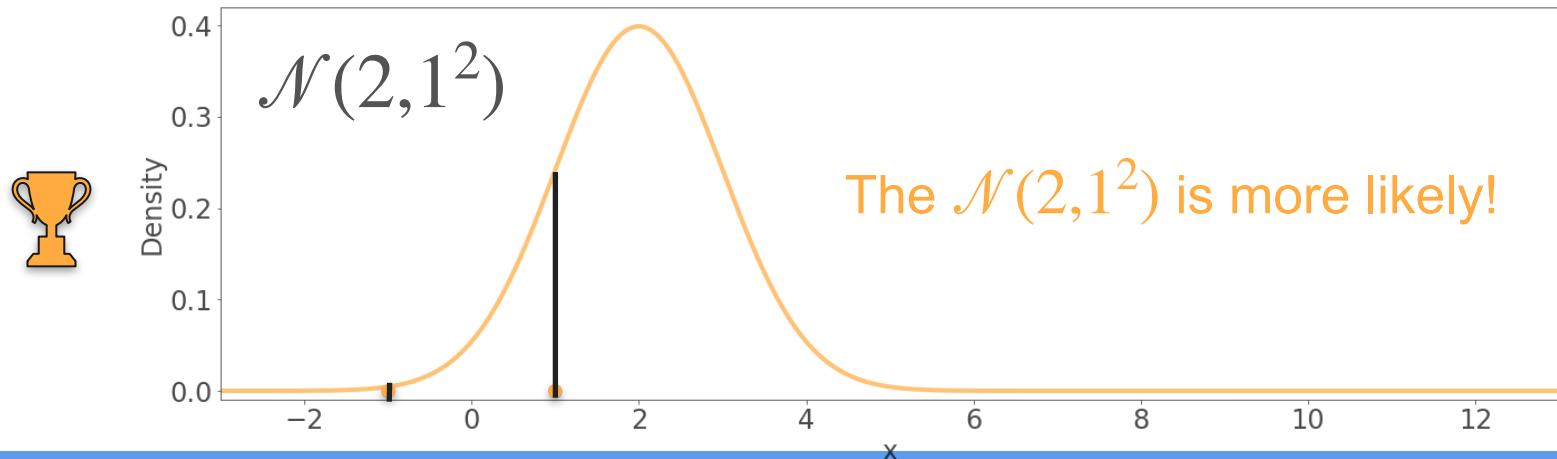
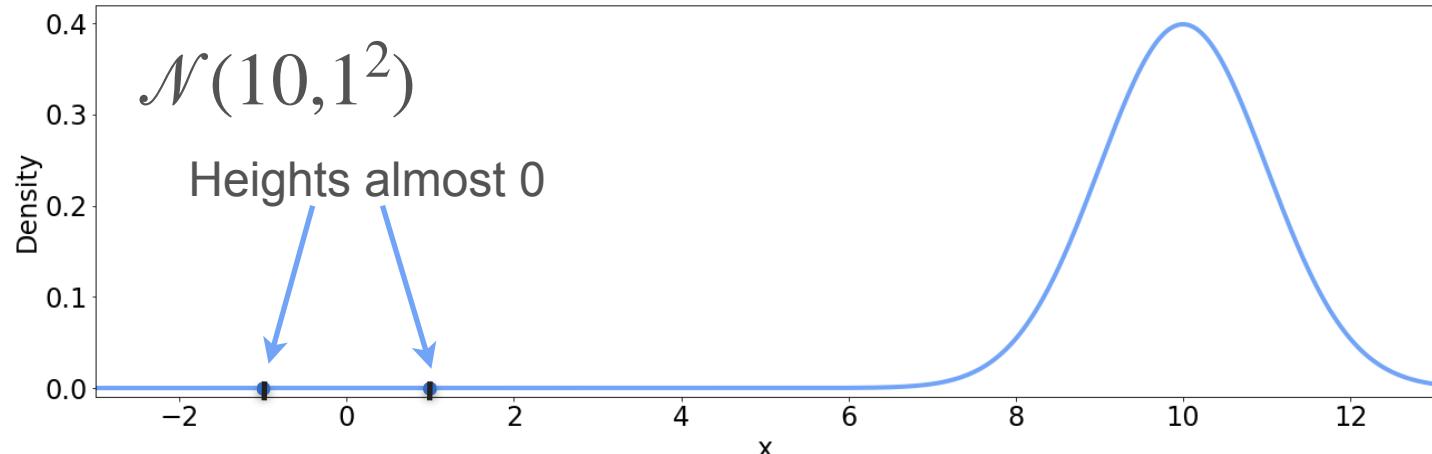






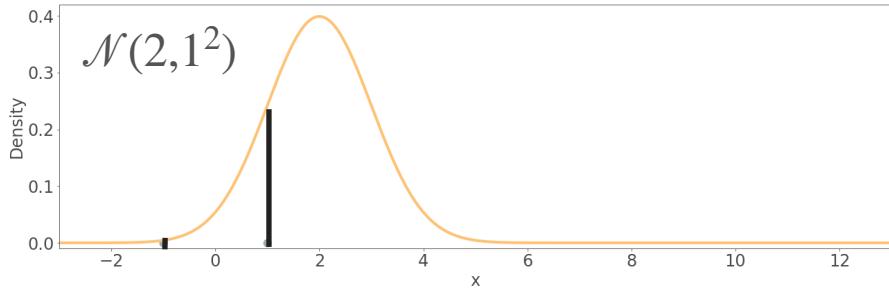
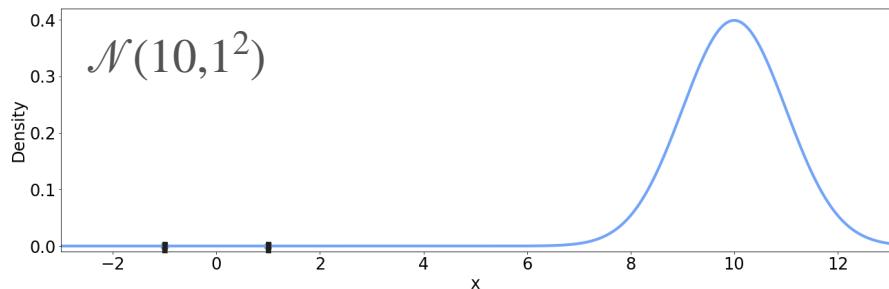




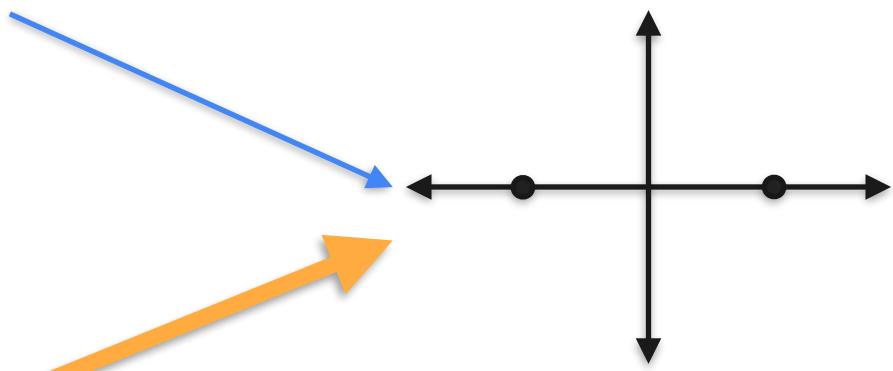


Maximum Likelihood: Gaussian Example

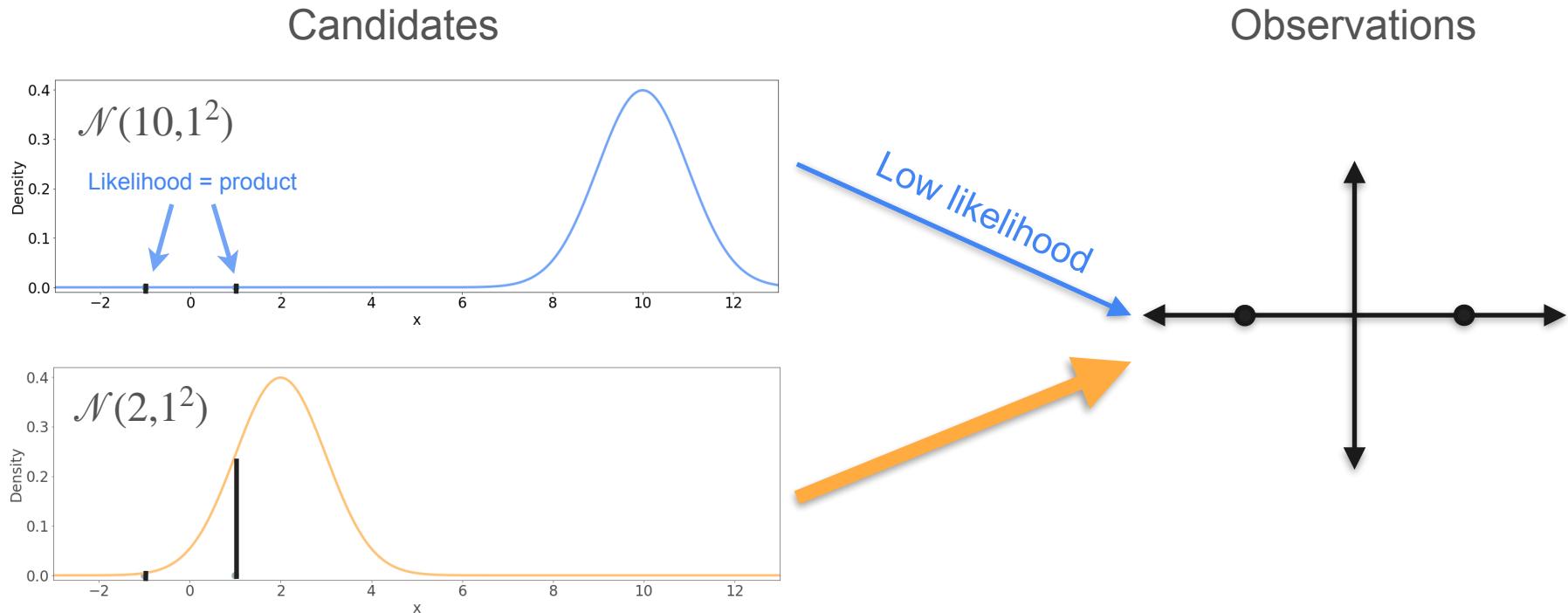
Candidates



Observations

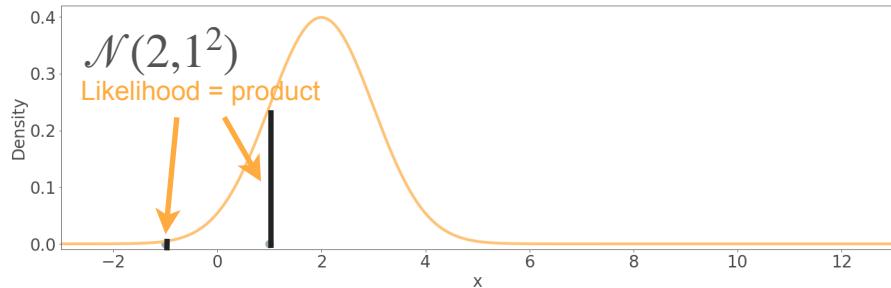
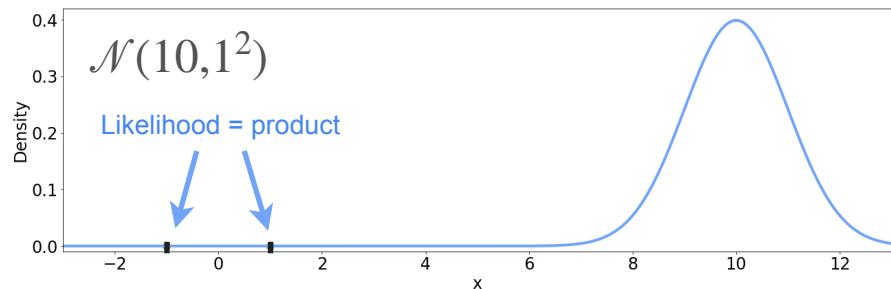


Maximum Likelihood: Gaussian Example

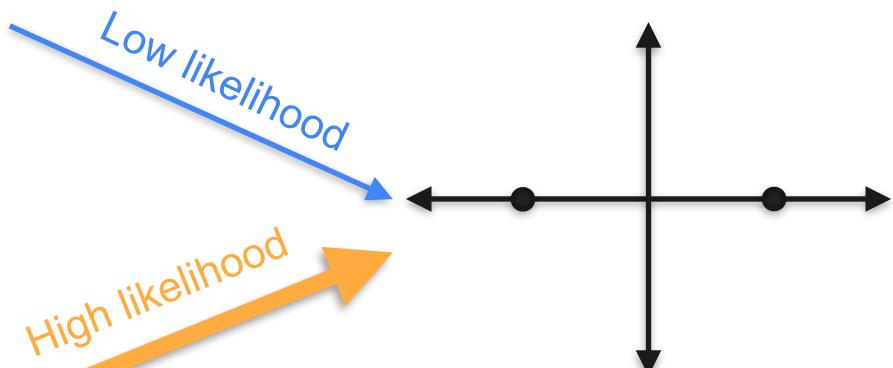


Maximum Likelihood: Gaussian Example

Candidates



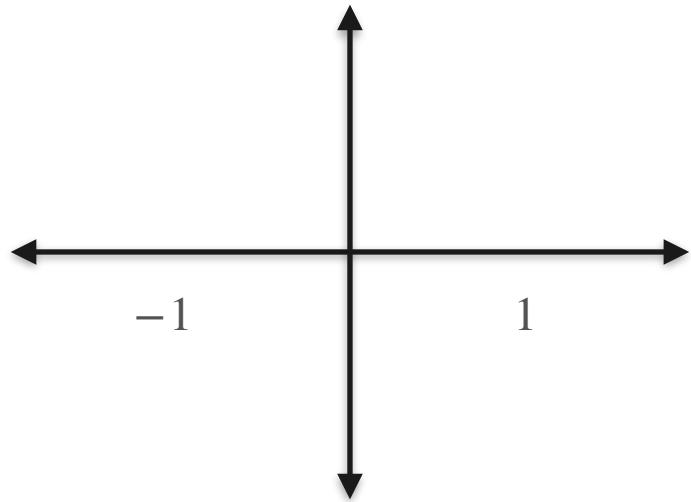
Observations



Gaussians With Three Different Means

Candidates

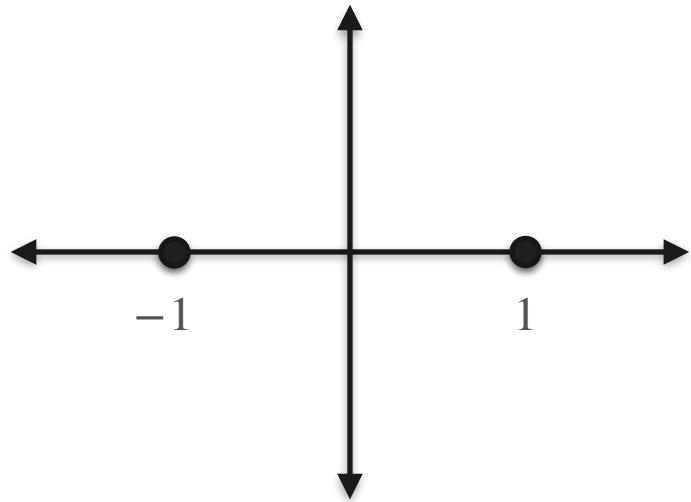
Observations



Gaussians With Three Different Means

Candidates

Observations

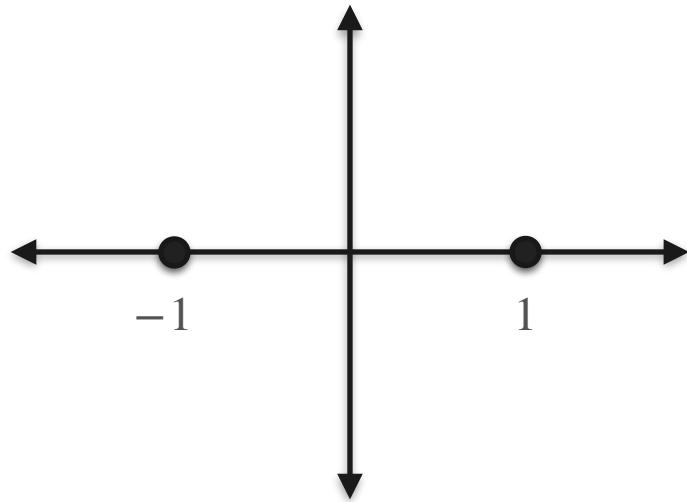


Gaussians With Three Different Means

Candidates

$$\mathcal{N}(-1, 1^2)$$

Observations



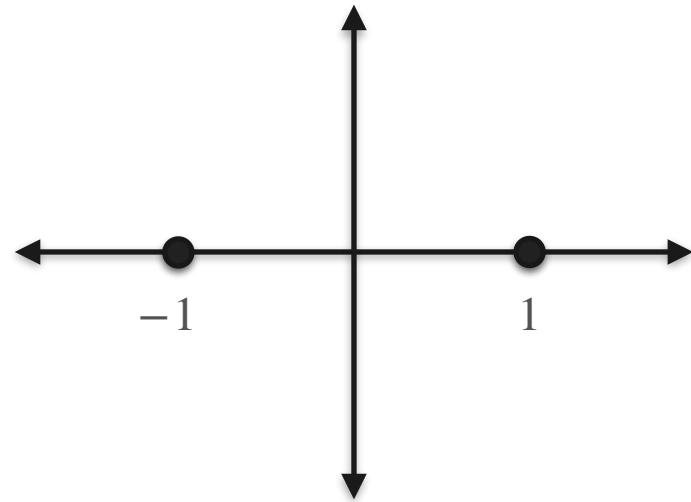
Gaussians With Three Different Means

Candidates

$$\mathcal{N}(-1, 1^2)$$

$$\mathcal{N}(0, 1^2)$$

Observations



Gaussians With Three Different Means

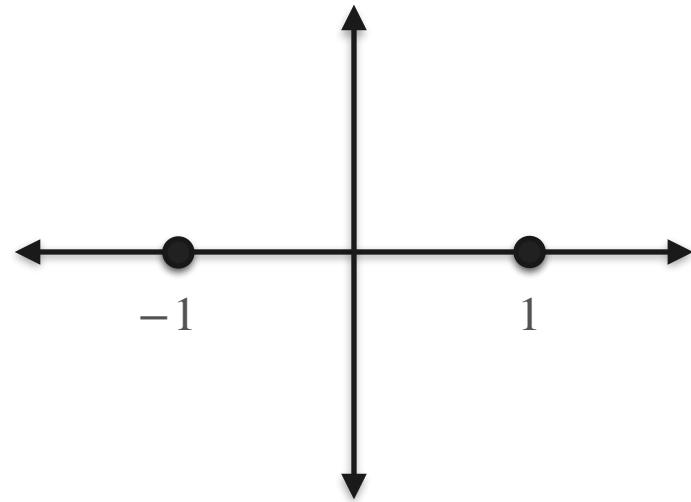
Candidates

$$\mathcal{N}(-1, 1^2)$$

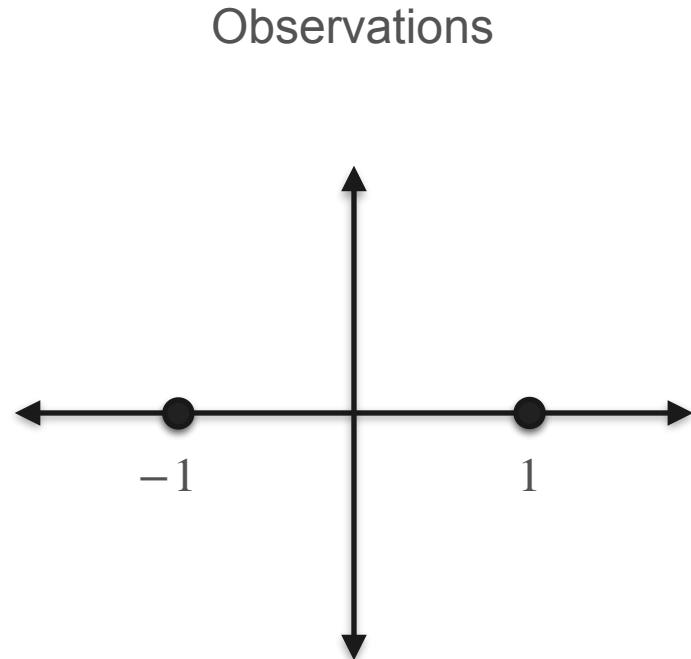
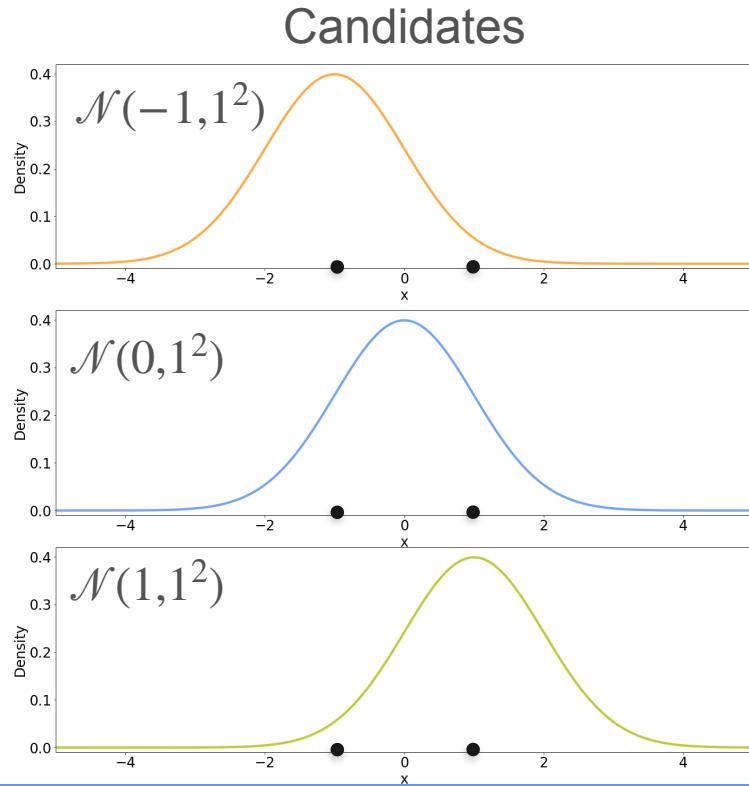
$$\mathcal{N}(0, 1^2)$$

$$\mathcal{N}(1, 1^2)$$

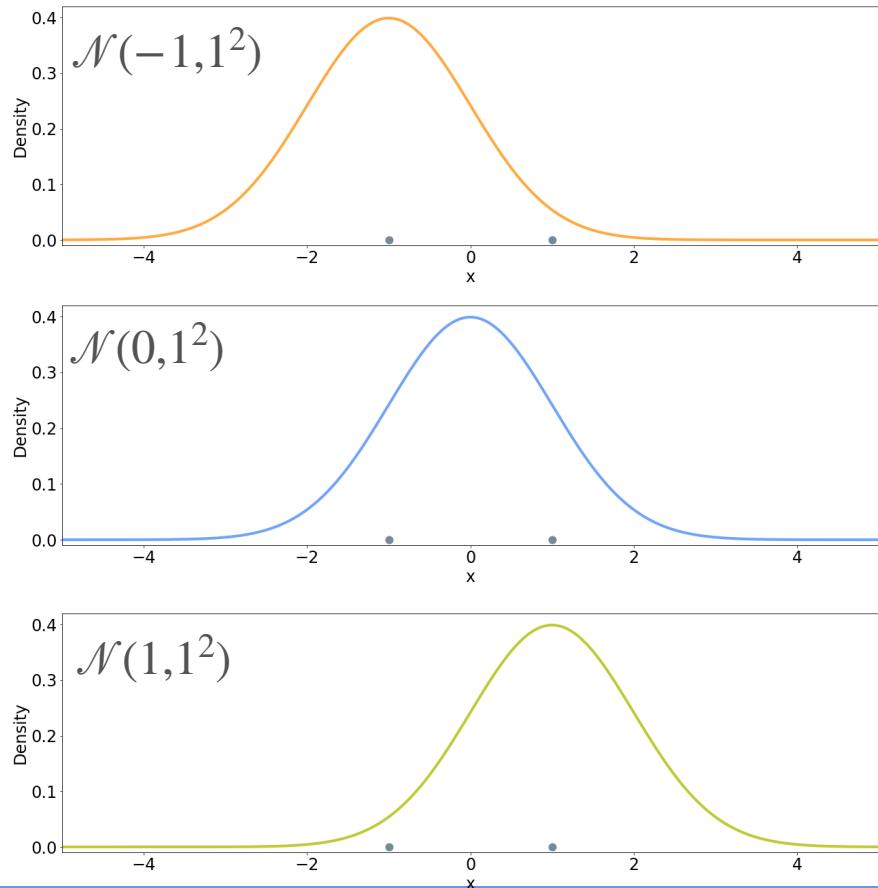
Observations



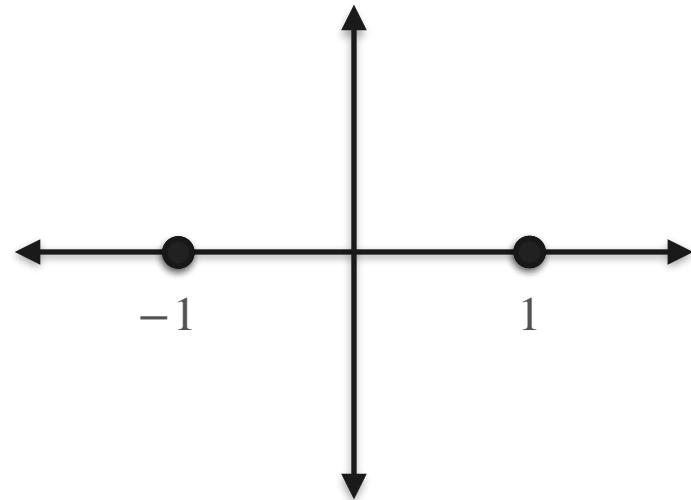
Gaussians With Three Different Means



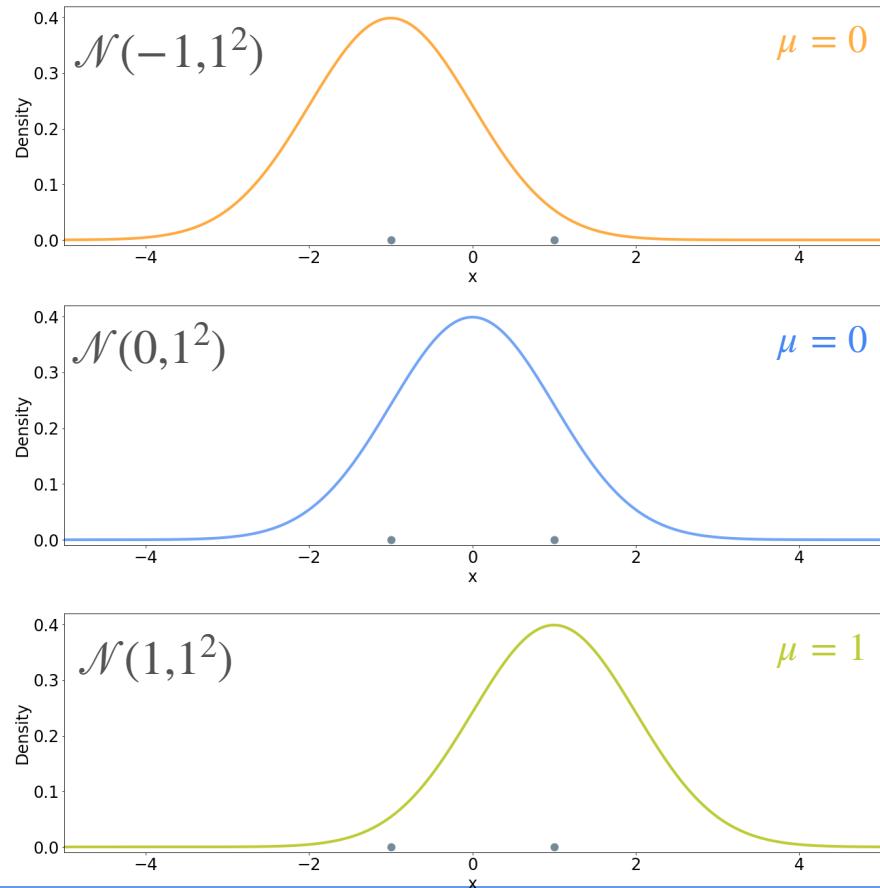
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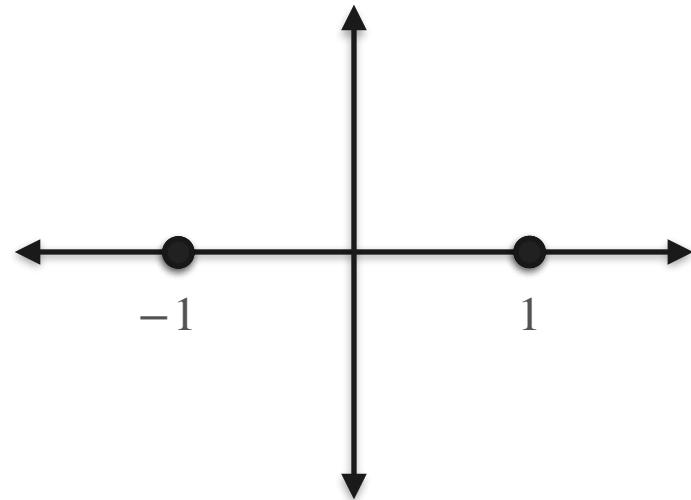
Observations



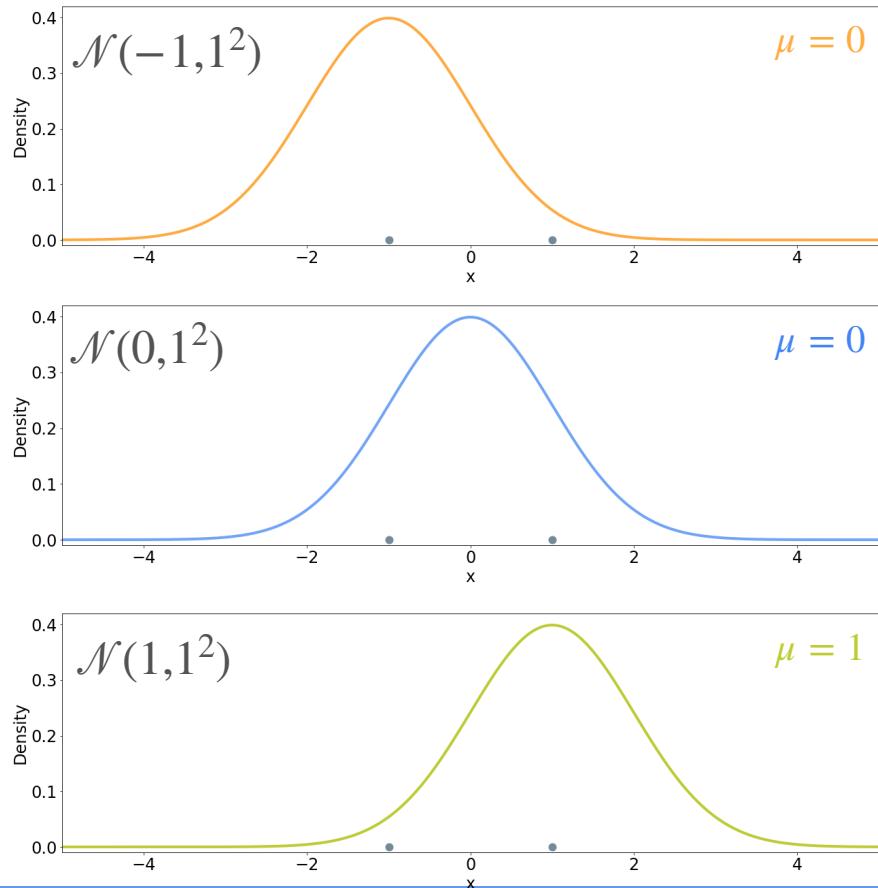
Candidates



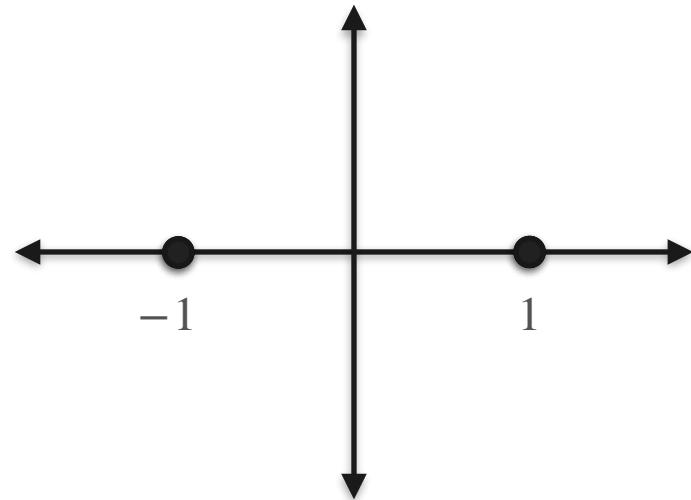
Observations



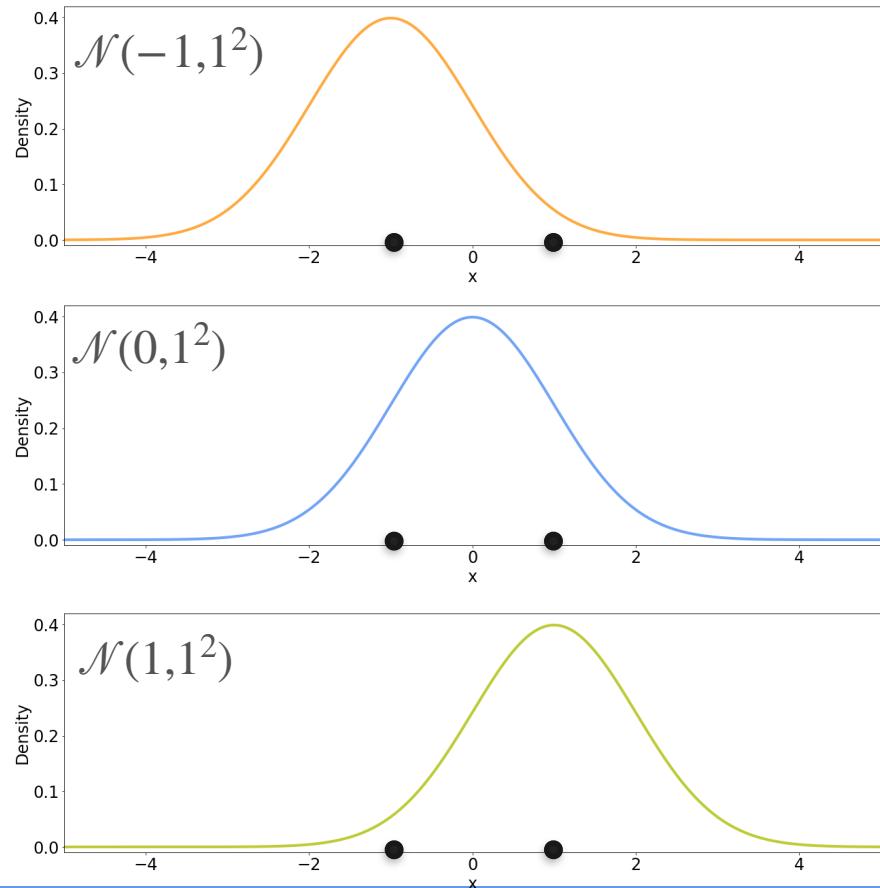
Candidates



Observations

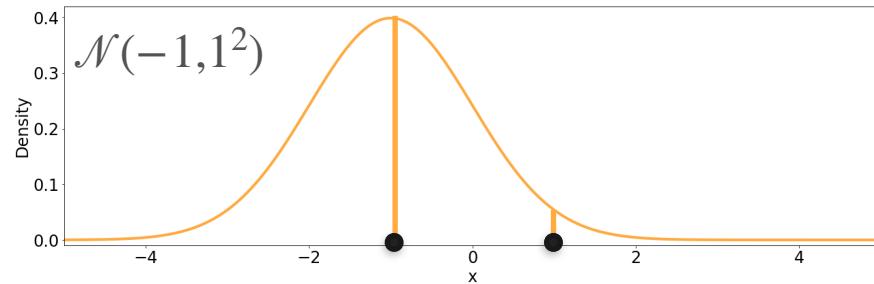


Candidates

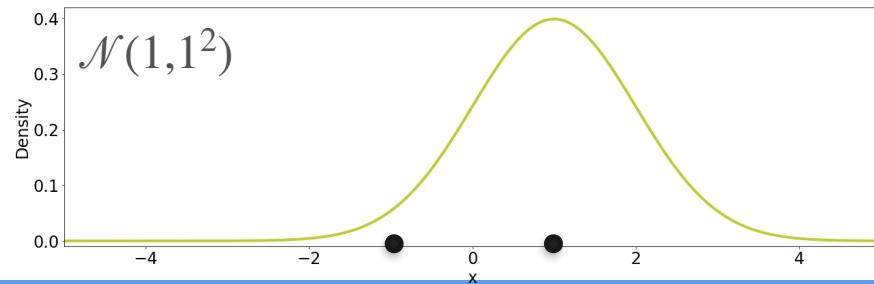
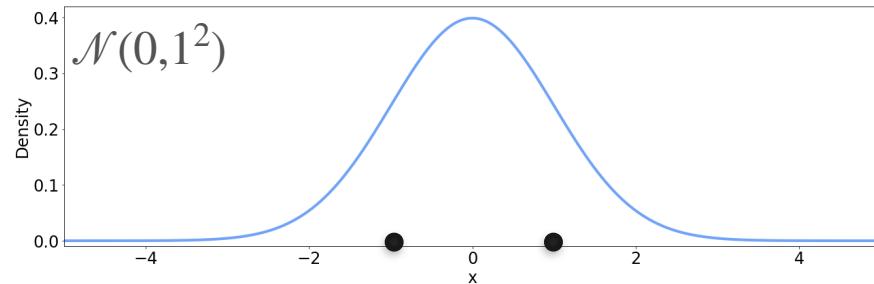


Observations

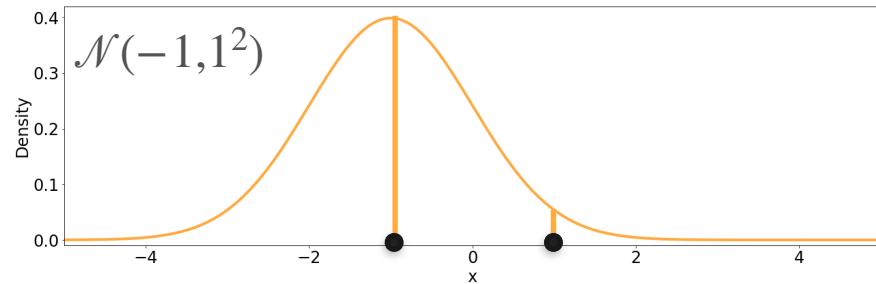
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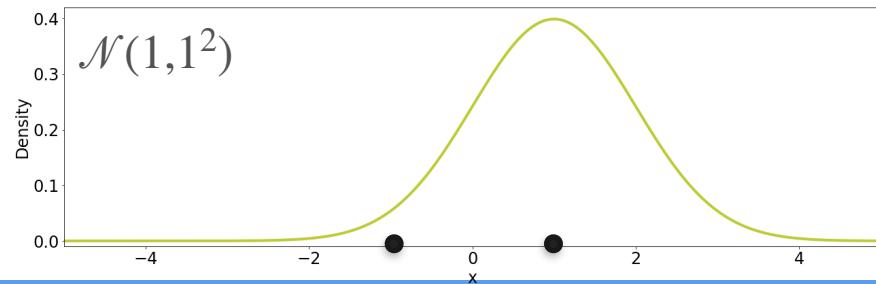
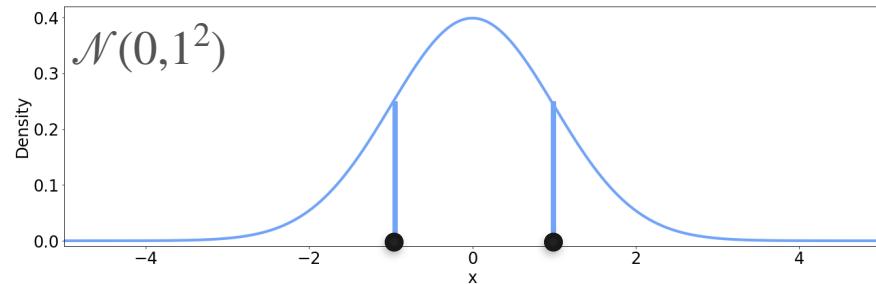
Observations



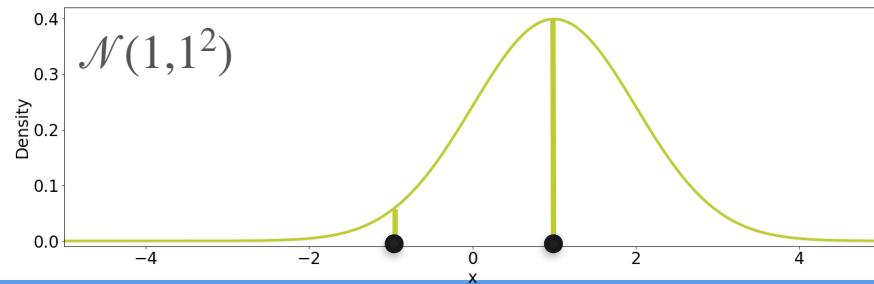
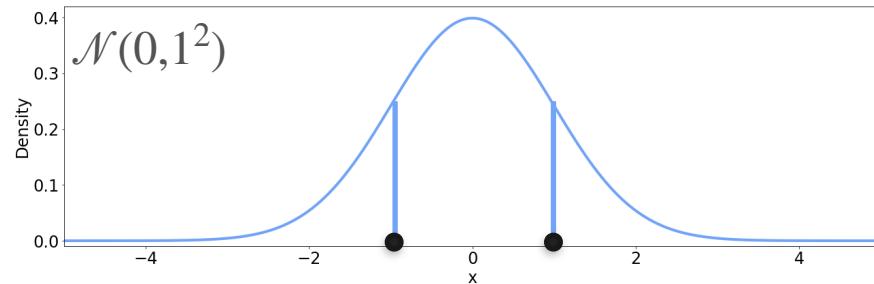
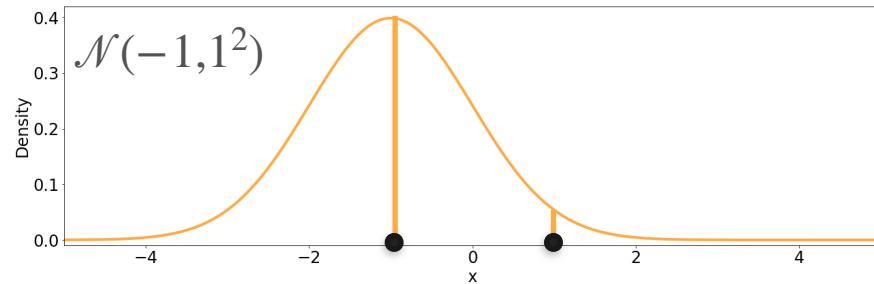
Candidates



Observations

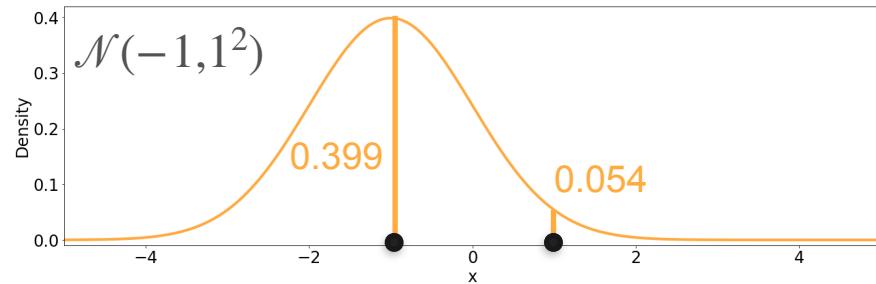


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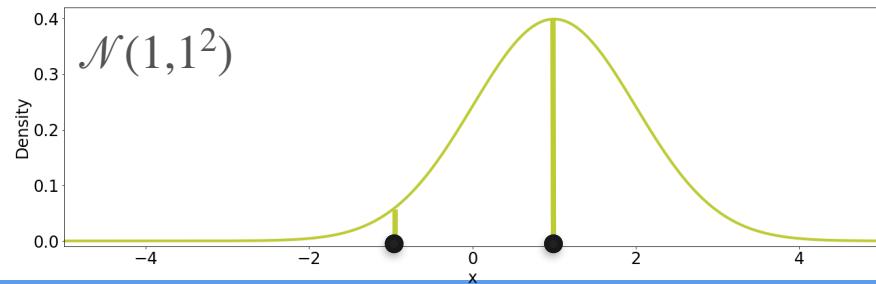
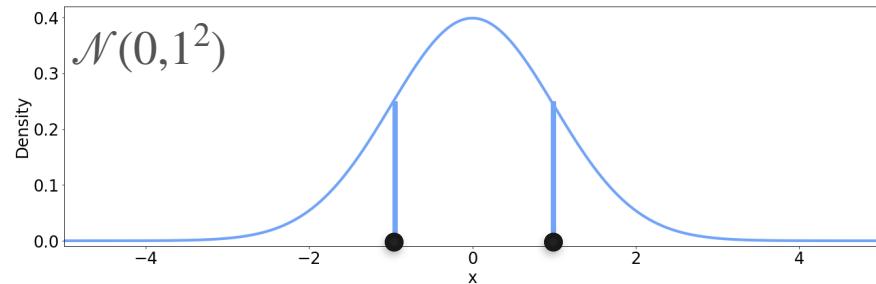


Observations

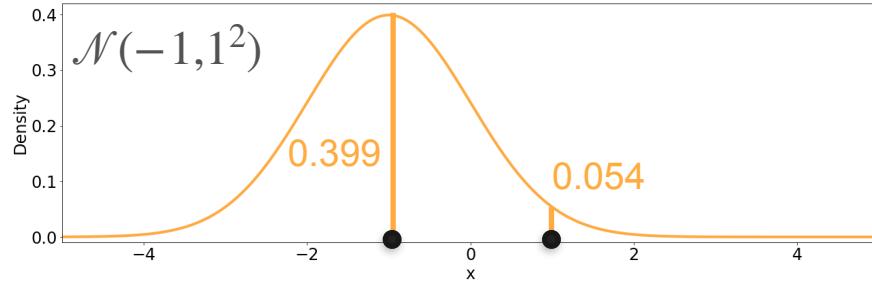
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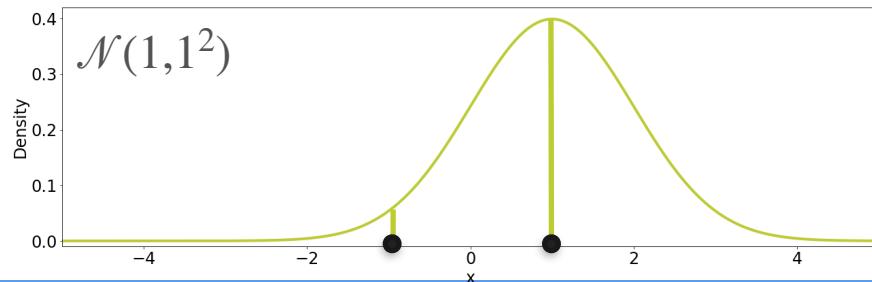
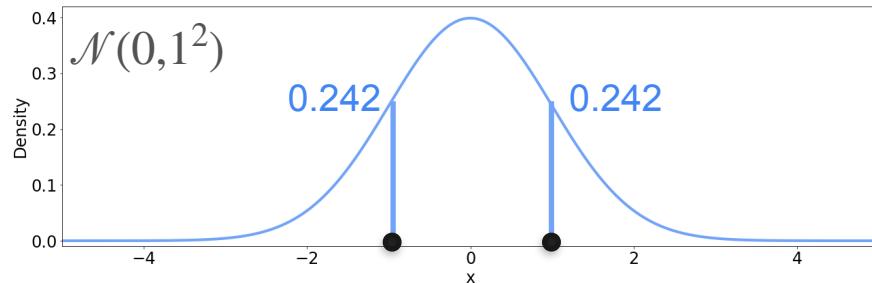
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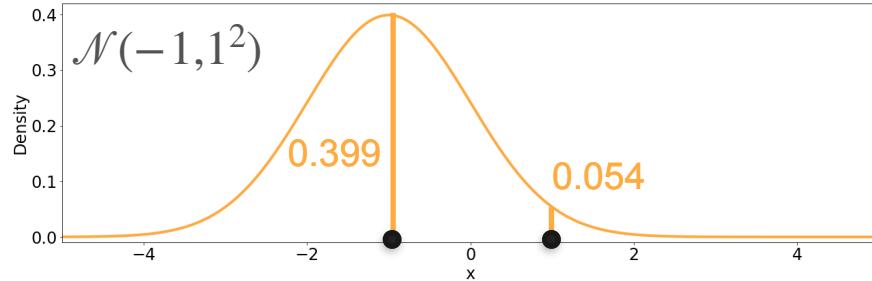
Candidates



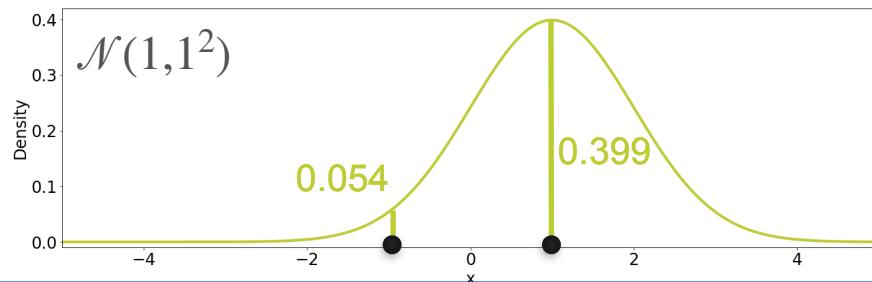
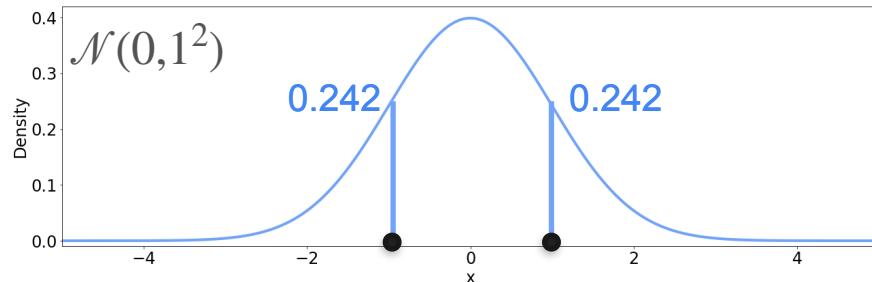
Observations



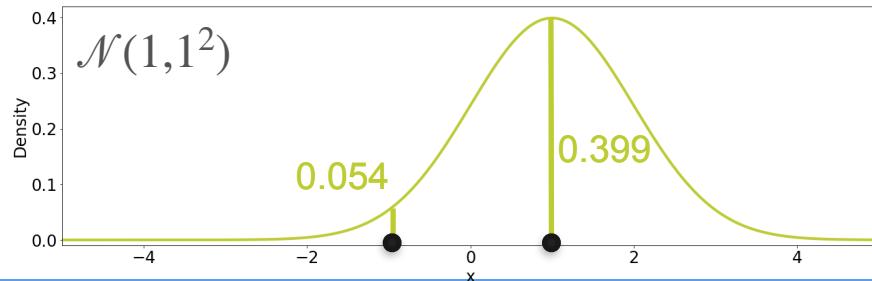
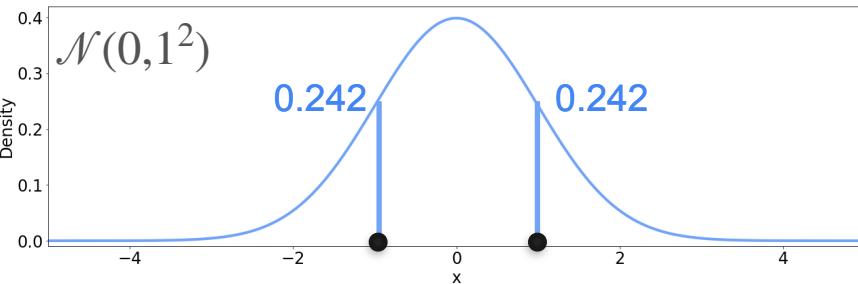
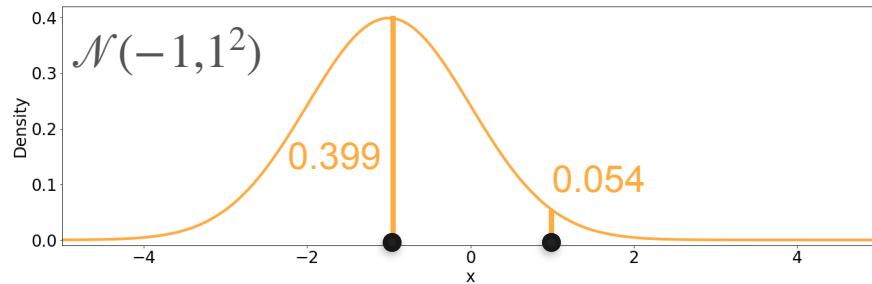
Candidates



Observations



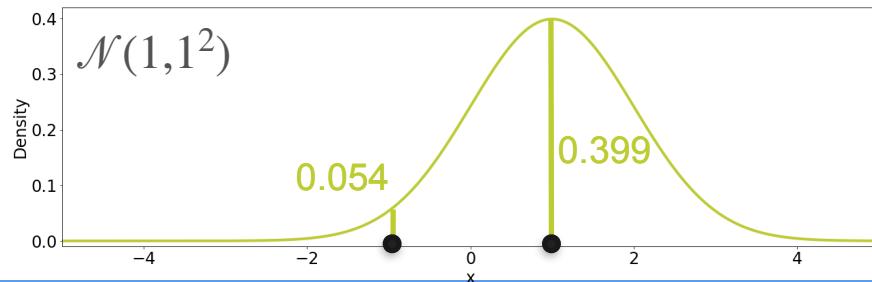
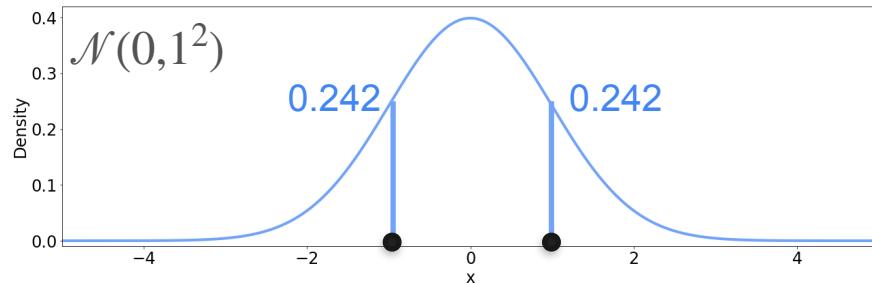
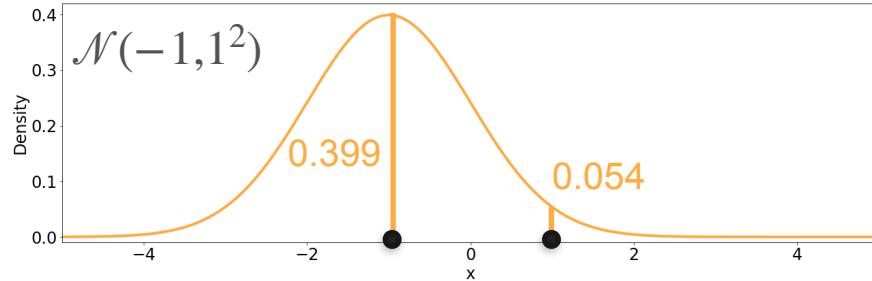
Candidates



Observations

$$0.399 \cdot 0.054 = 0.022$$

Candidates

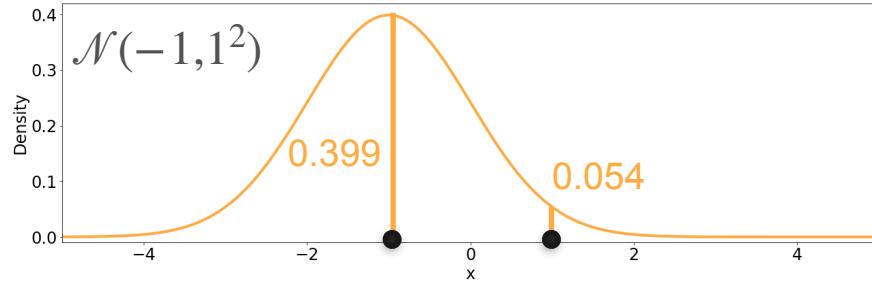


Observations

$$0.399 \cdot 0.054 = 0.022$$

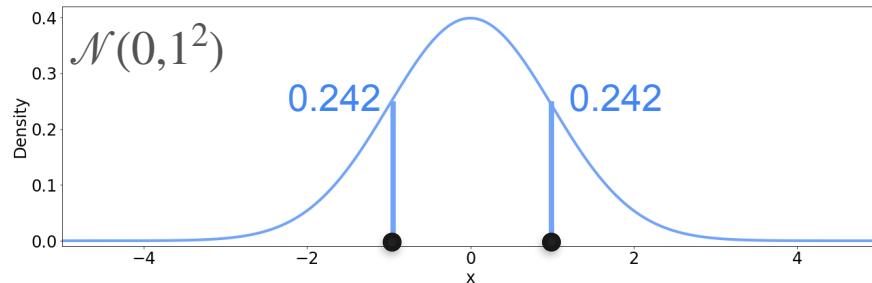
$$0.242 \cdot 0.242 = 0.059$$

Candidates

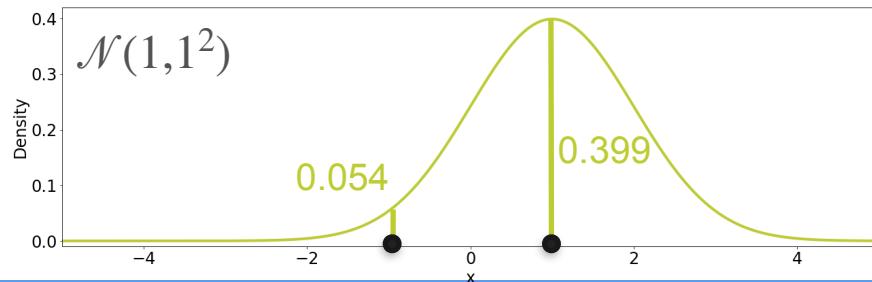


Observations

$$0.399 \cdot 0.054 = 0.022$$

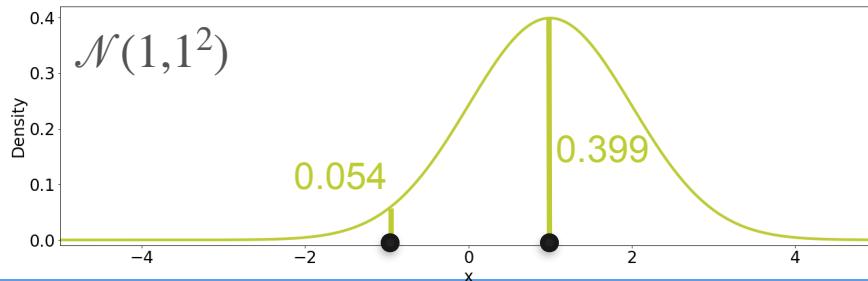
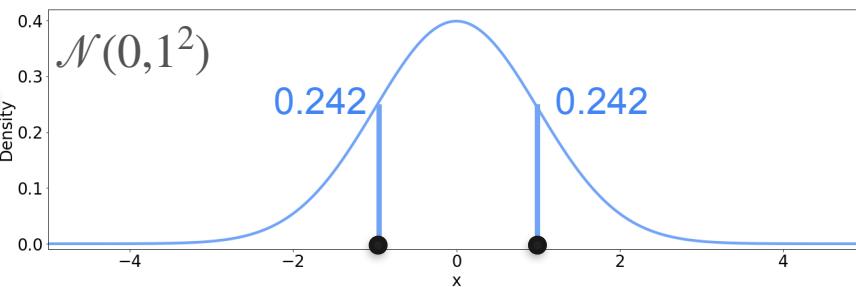
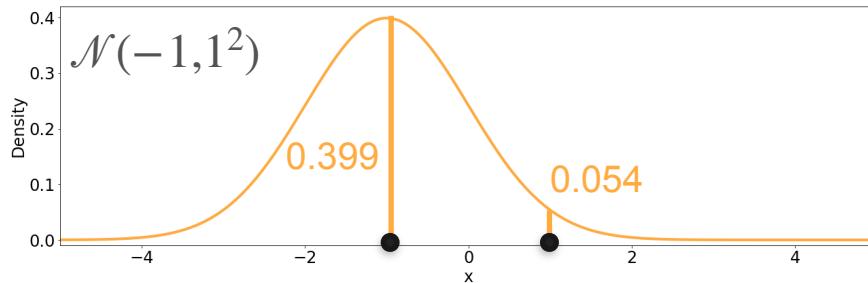


$$0.242 \cdot 0.242 = 0.059$$



$$0.054 \cdot 0.399 = 0.022$$

Candidates



Observations

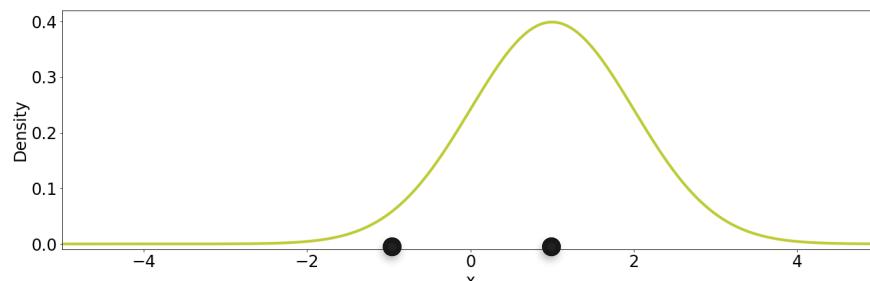
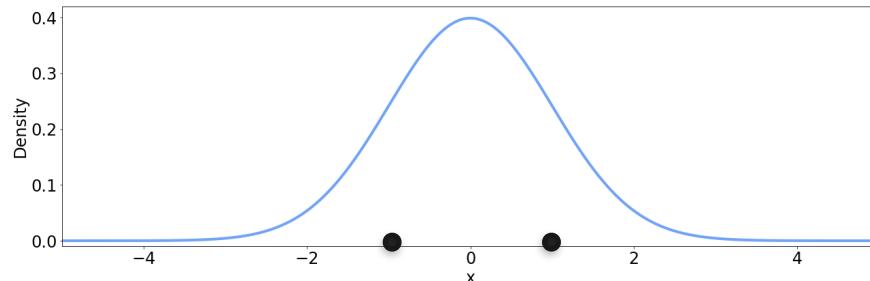
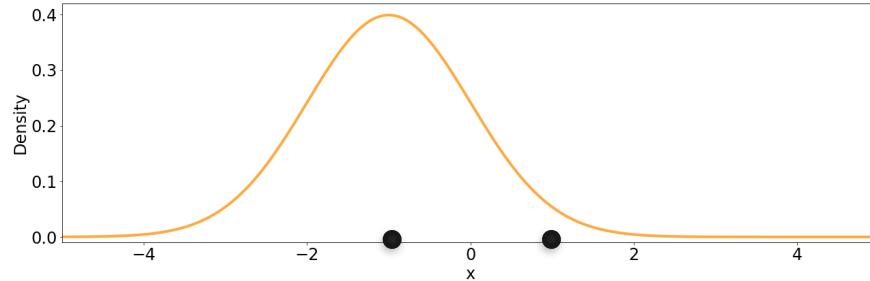
$$0.399 \cdot 0.054 = 0.022$$

The $\mathcal{N}(0, 1)$ is more likely!

$$0.242 \cdot 0.242 = 0.059$$

$$0.054 \cdot 0.399 = 0.022$$

Candidates

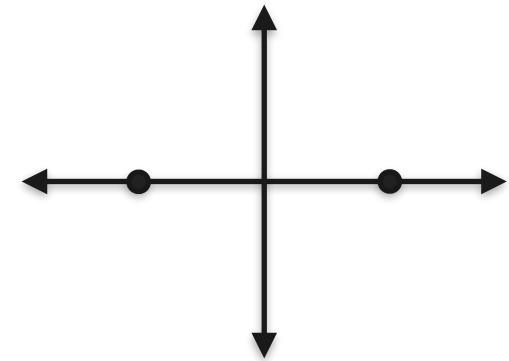


Likelihood = 0.022

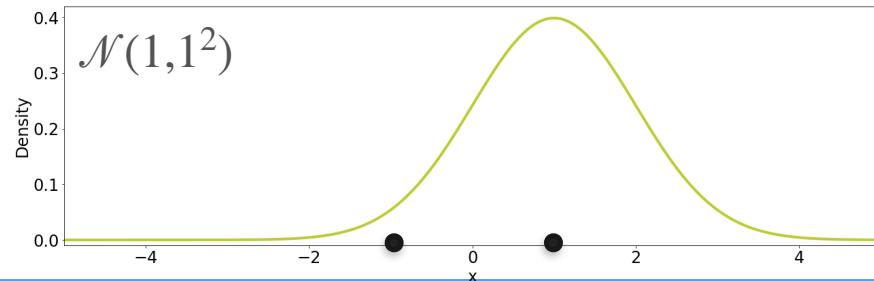
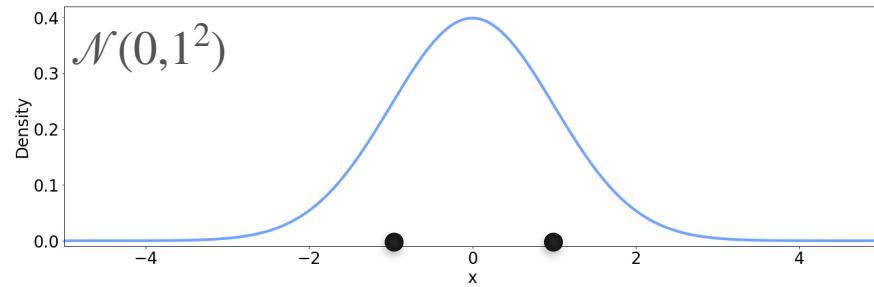
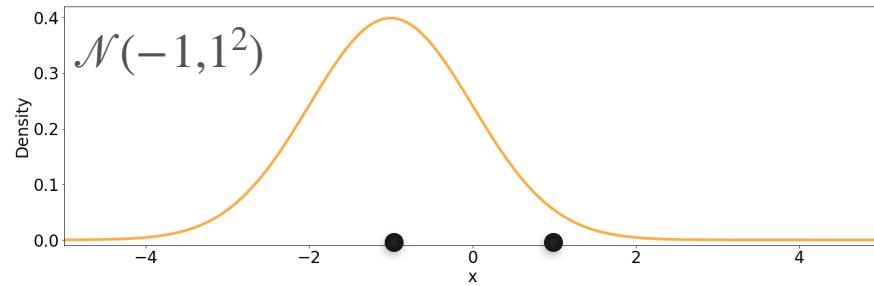
Likelihood = 0.059

Likelihood = 0.022

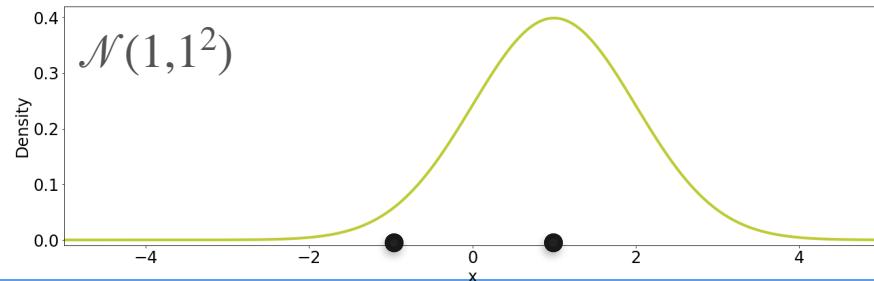
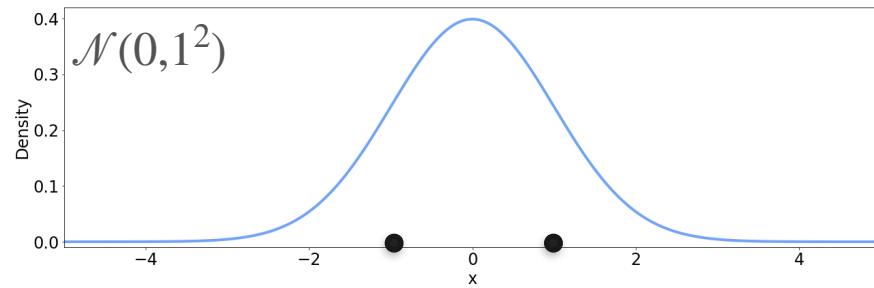
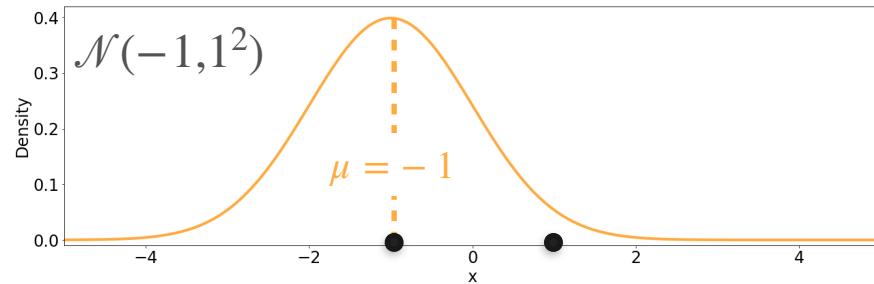
Observations



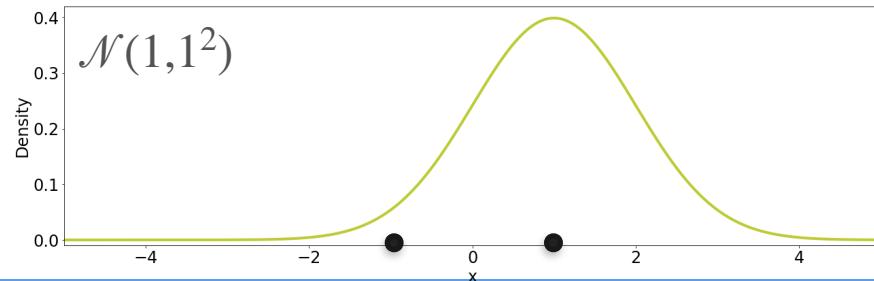
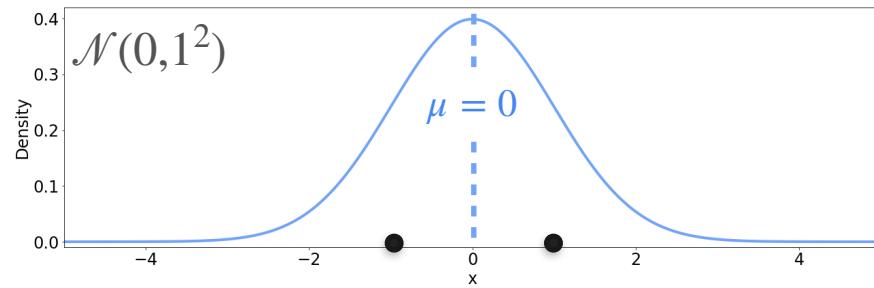
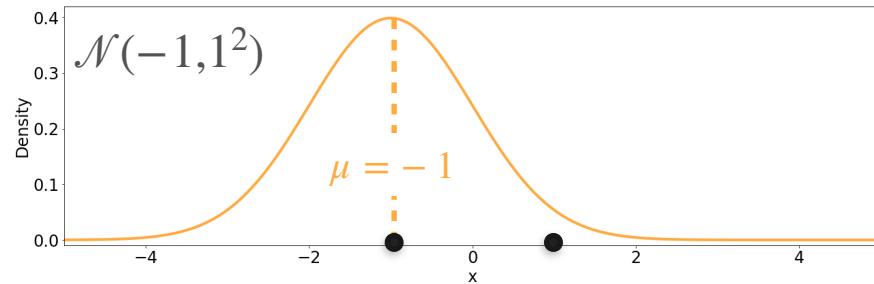
Candidates



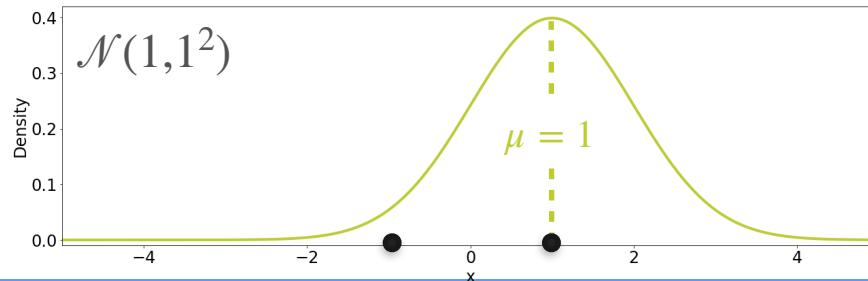
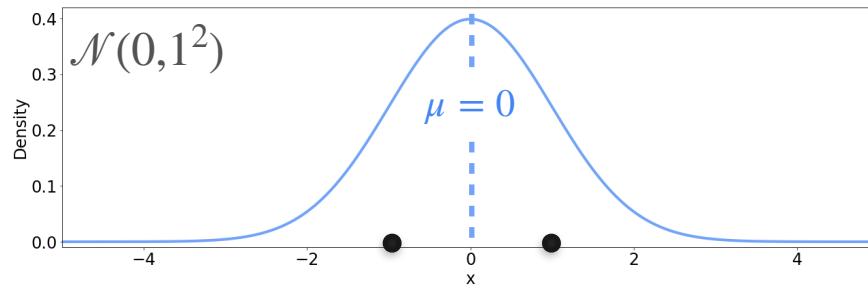
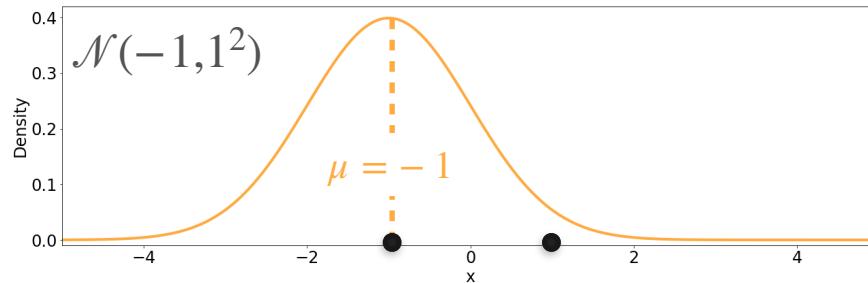
Candidates



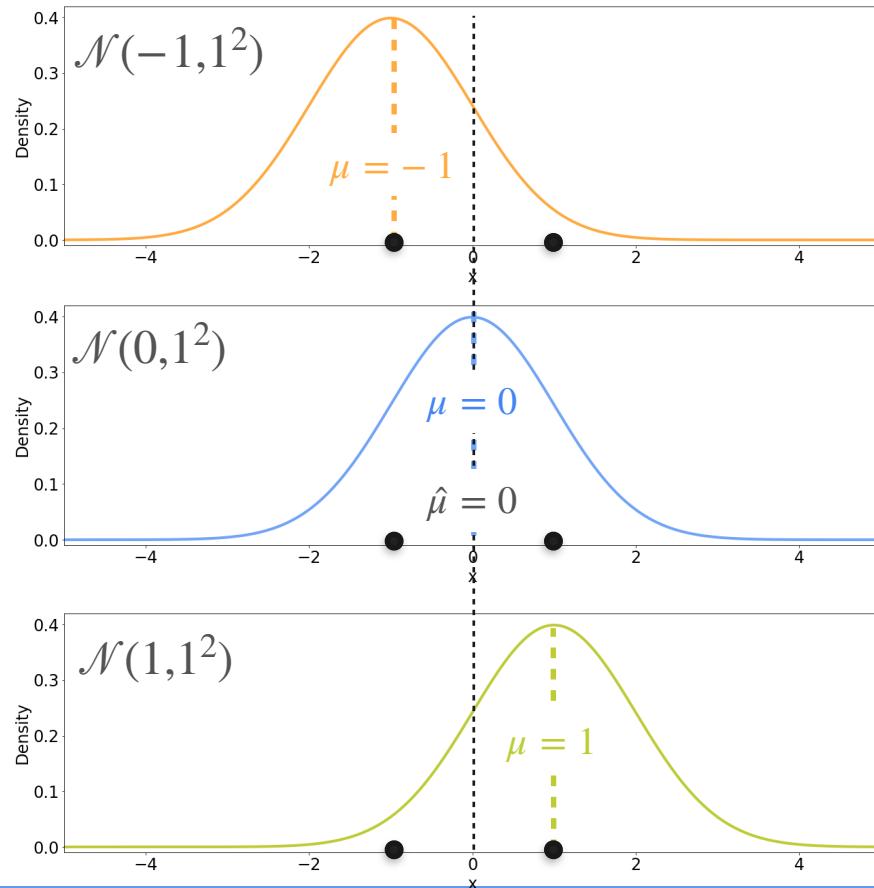
Candidates



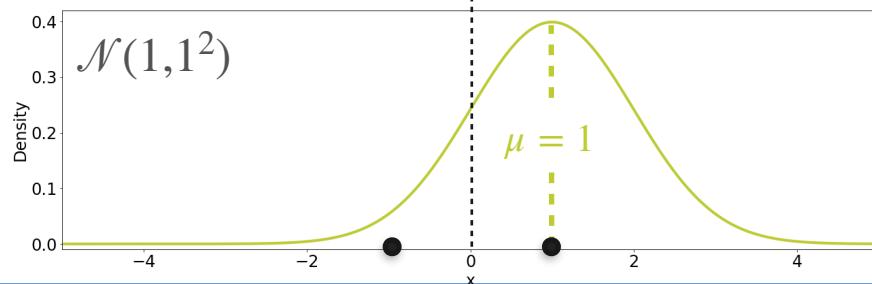
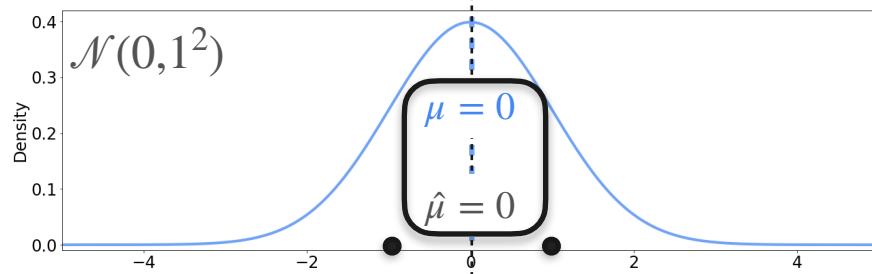
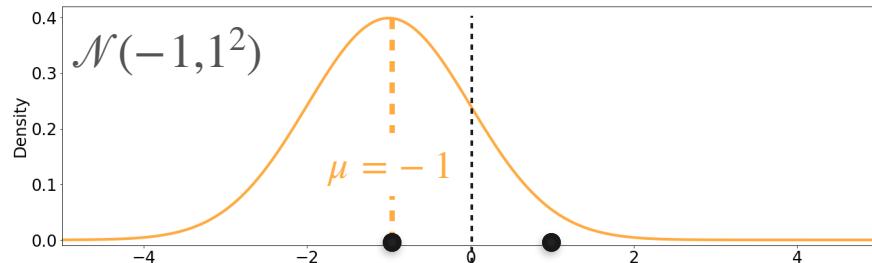
Candidates



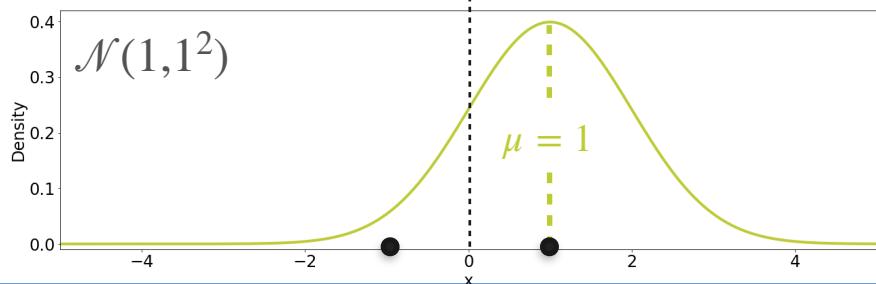
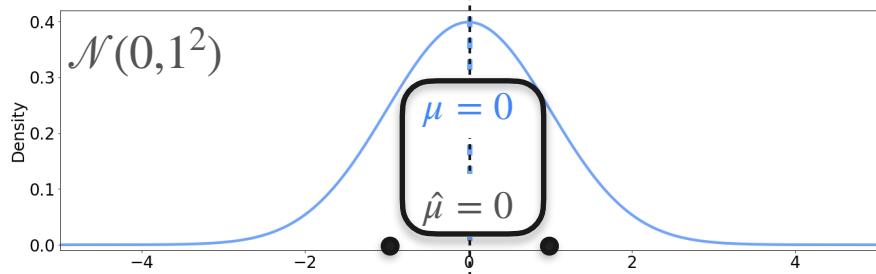
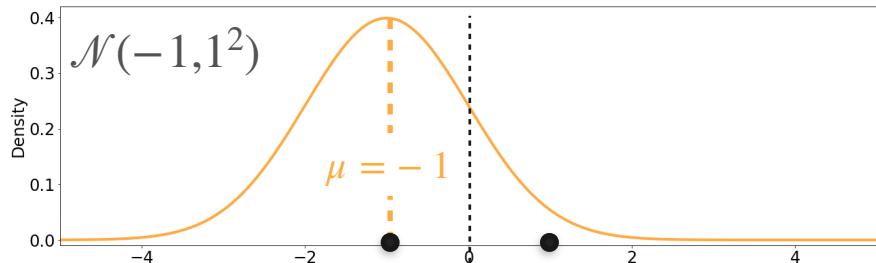
Candidates



Candidates



Candidates

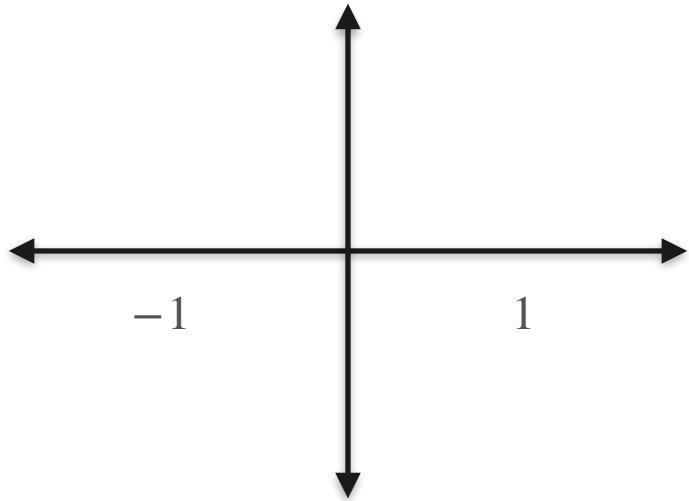


The best distribution is the one where
the **mean** of the distribution is the
mean of the sample

Gaussians With Three Different Variance

Candidates

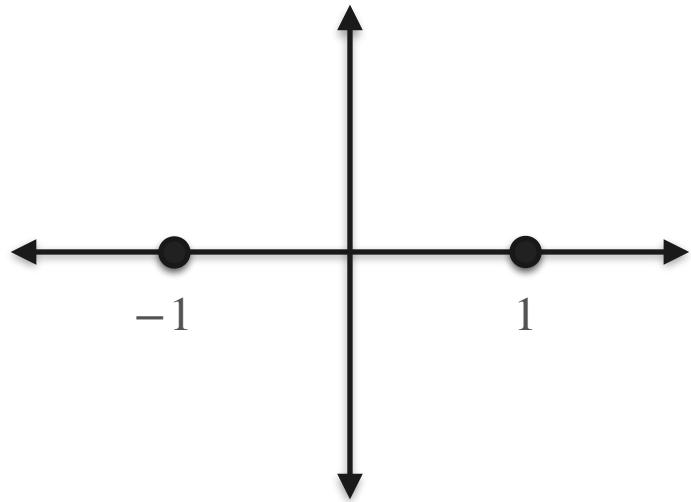
Observations



Gaussians With Three Different Variance

Candidates

Observations

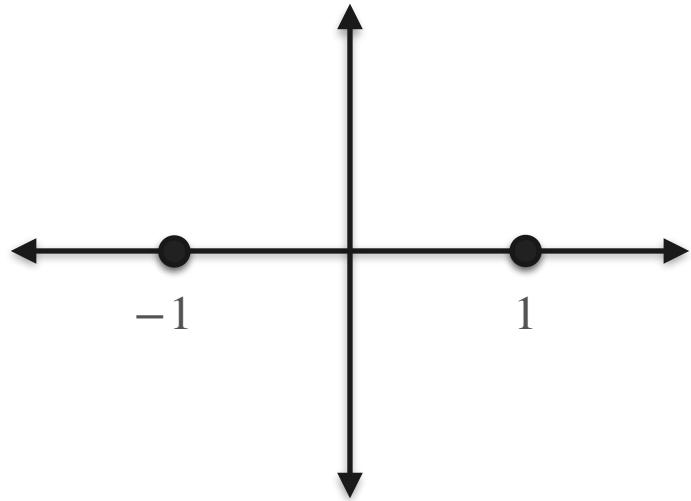


Gaussians With Three Different Variance

Candidates

$$\mathcal{N}(0, 0.5^2)$$

Observations



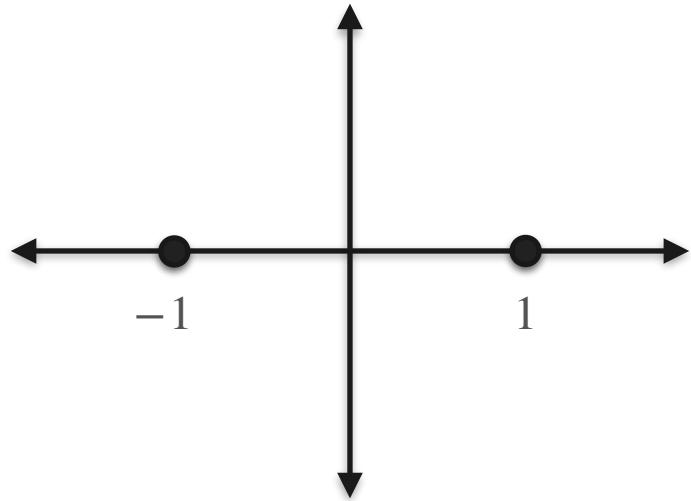
Gaussians With Three Different Variance

Candidates

$$\mathcal{N}(0, 0.5^2)$$

$$\mathcal{N}(0, 1^2)$$

Observations



Gaussians With Three Different Variance

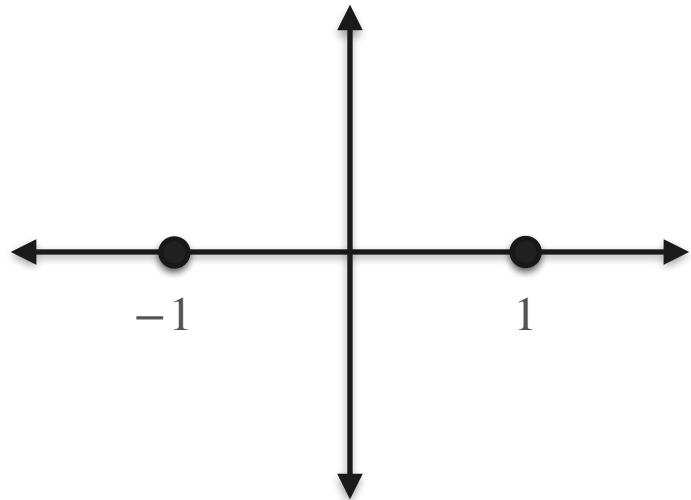
Candidates

$$\mathcal{N}(0,0.5^2)$$

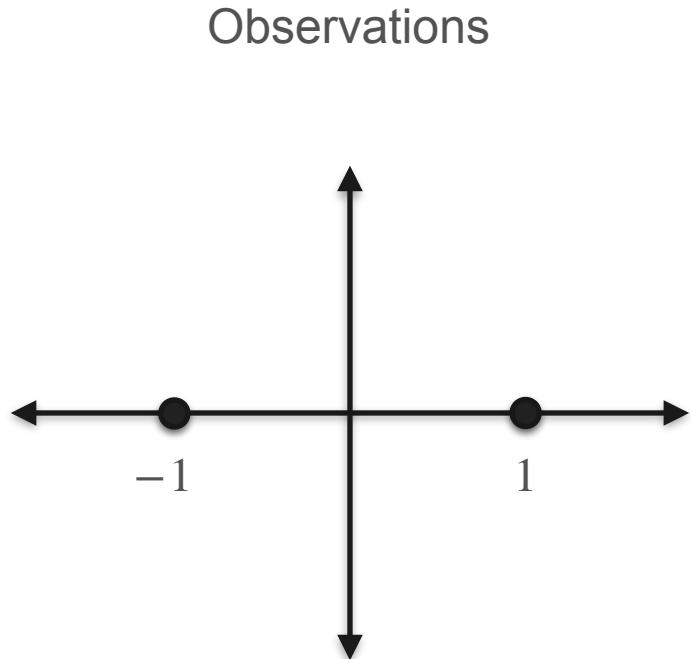
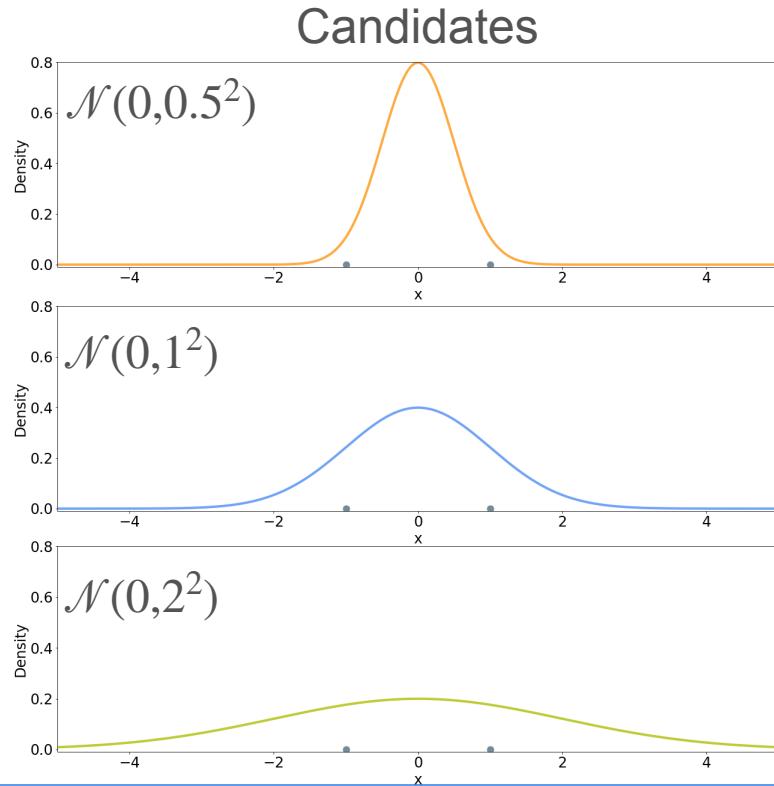
$$\mathcal{N}(0,1^2)$$

$$\mathcal{N}(0,2^2)$$

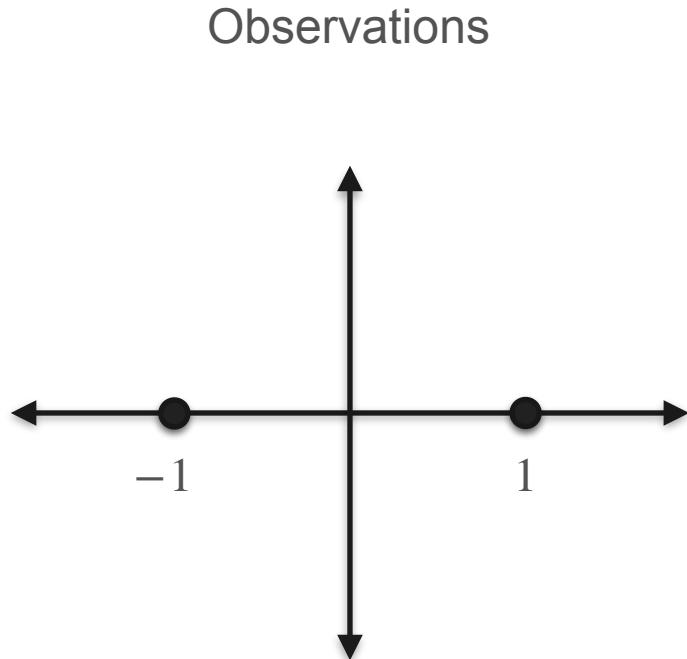
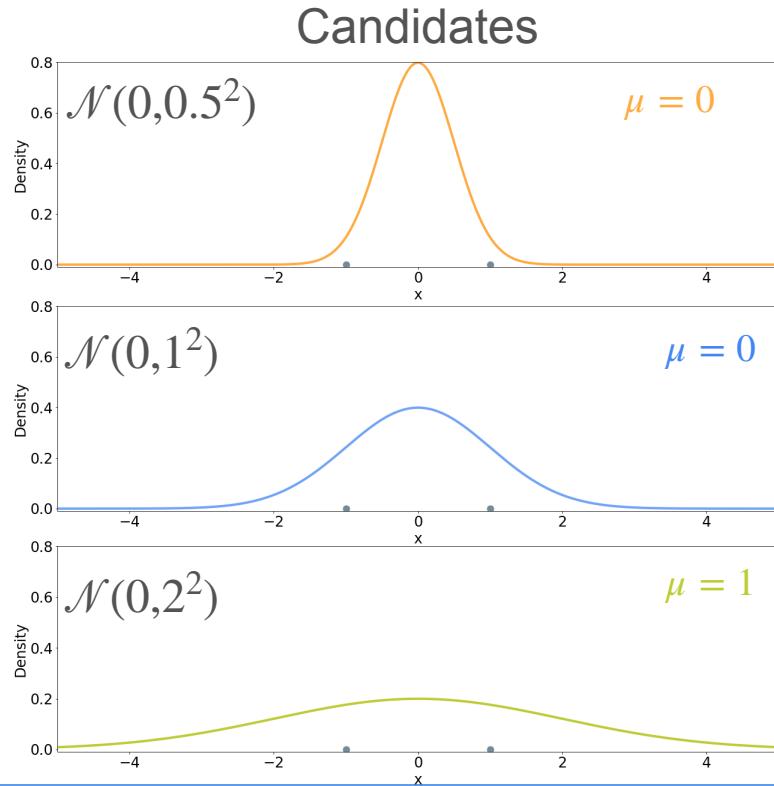
Observations



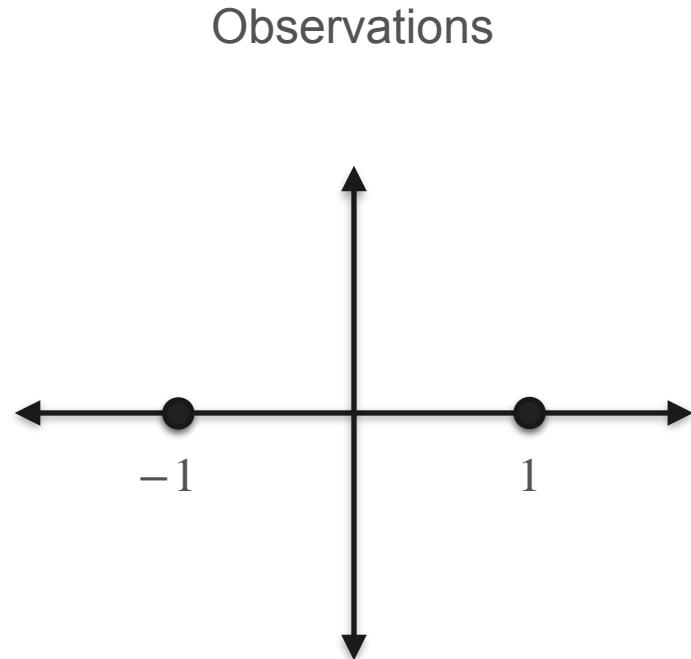
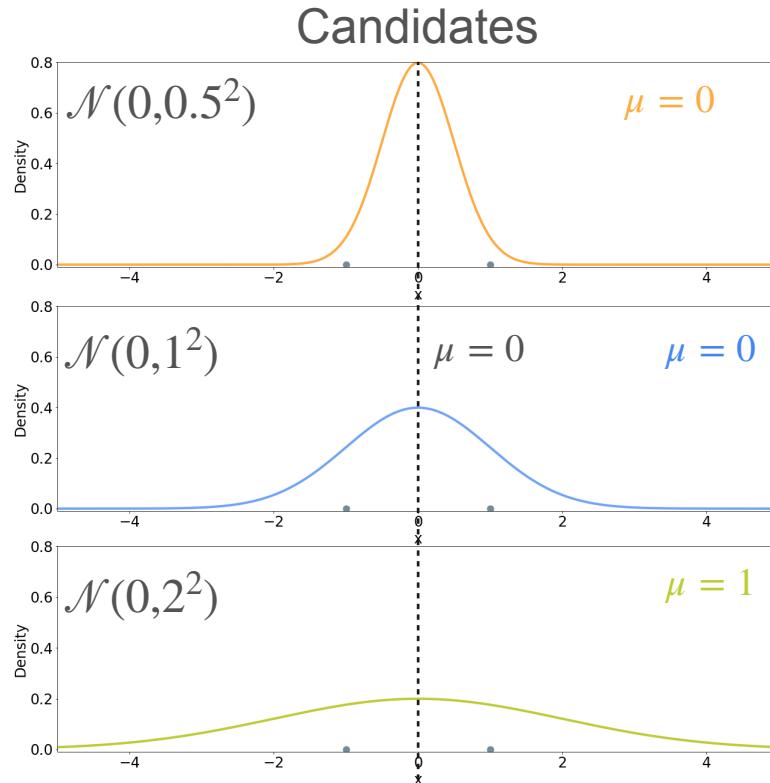
Gaussians With Three Different Variance



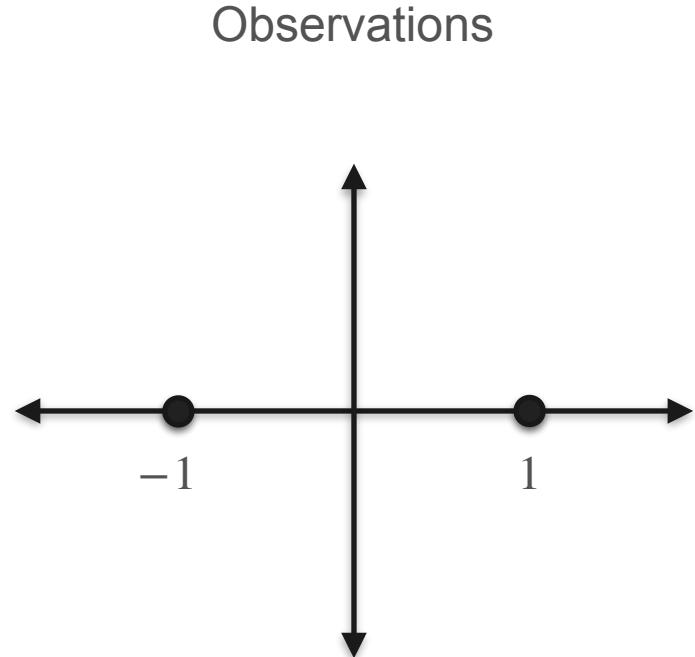
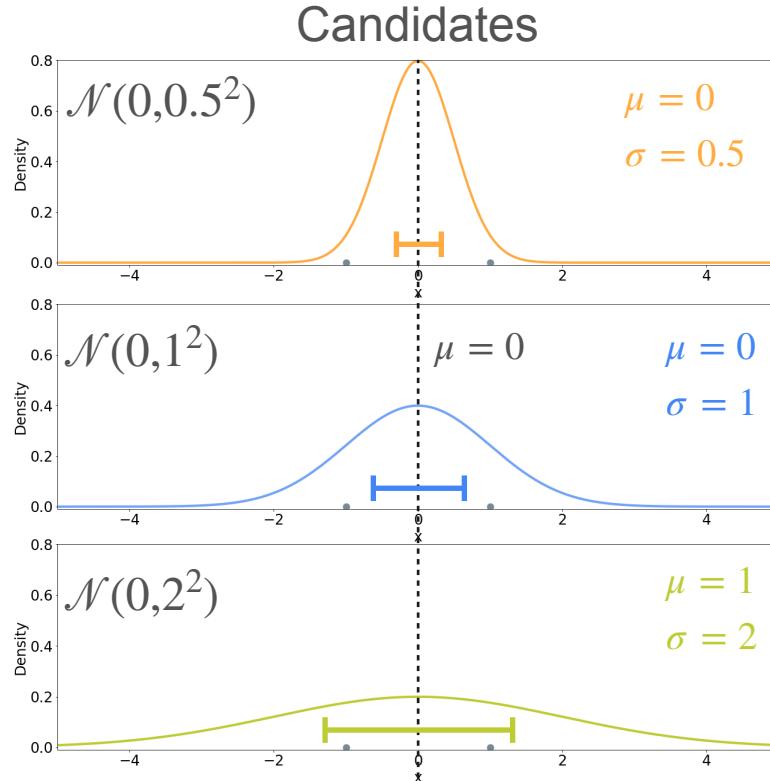
Gaussians With Three Different Variance



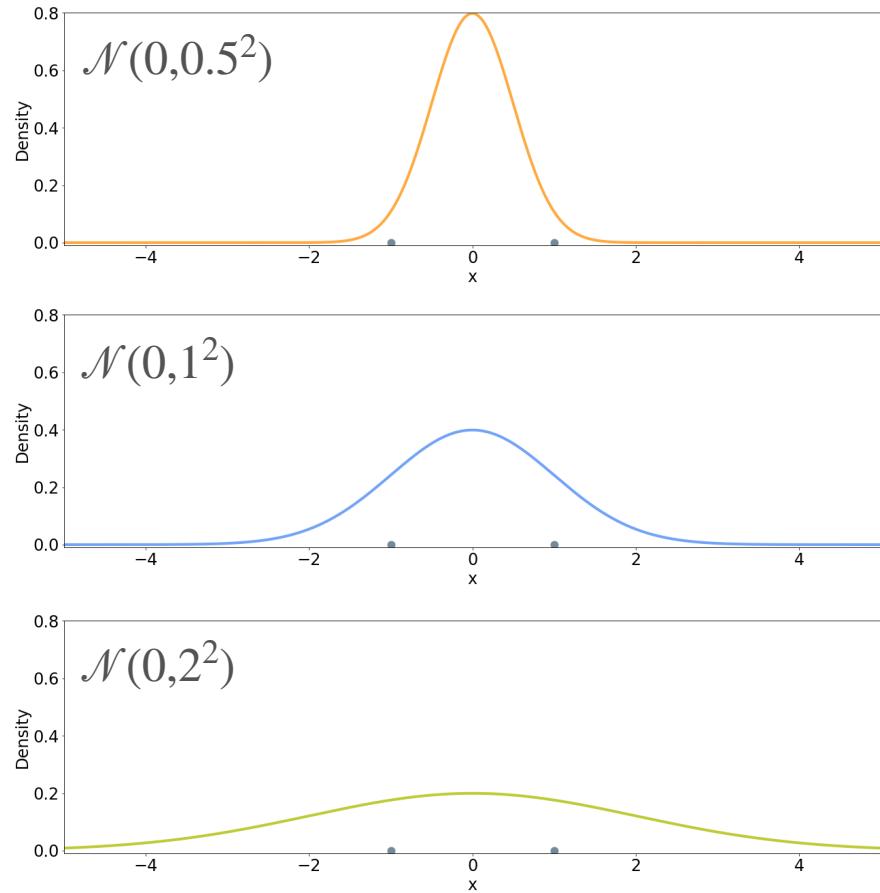
Gaussians With Three Different Variance



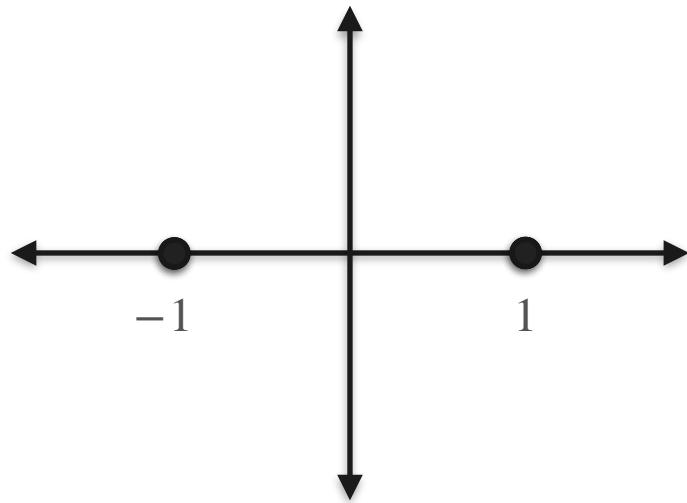
Gaussians With Three Different Variance



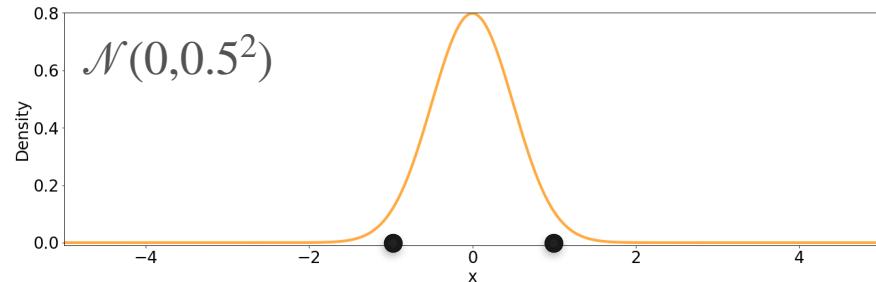
Candidates



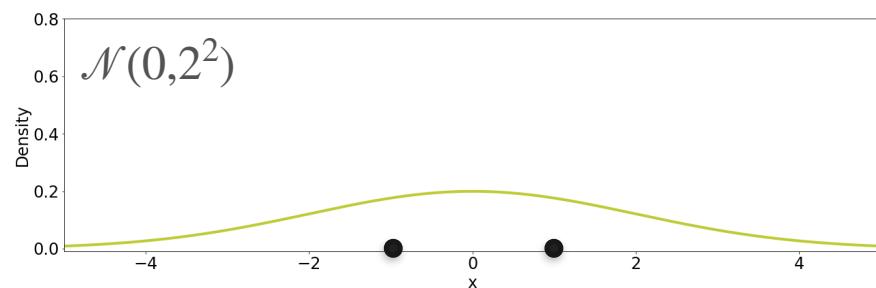
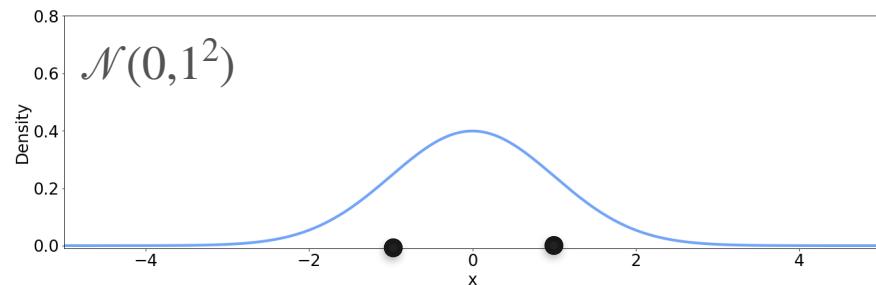
Observations



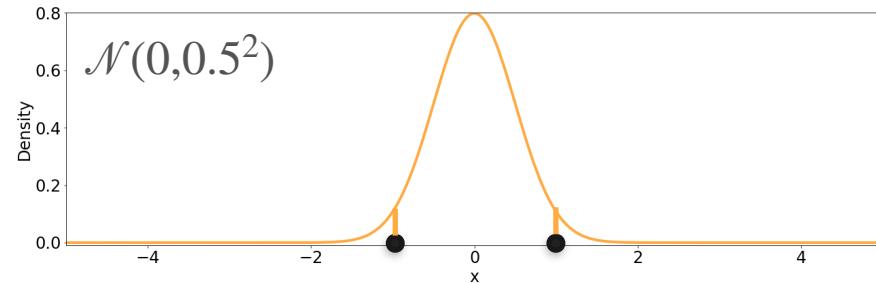
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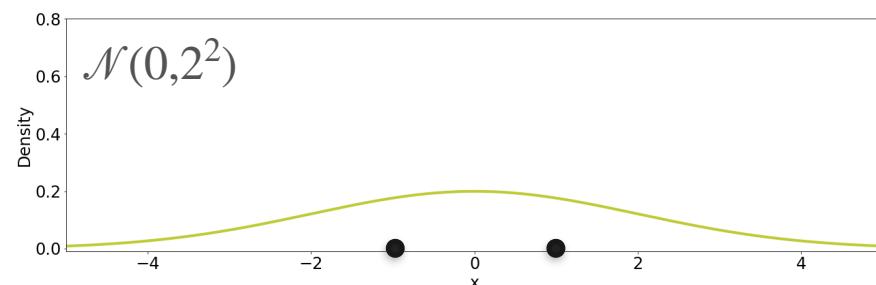
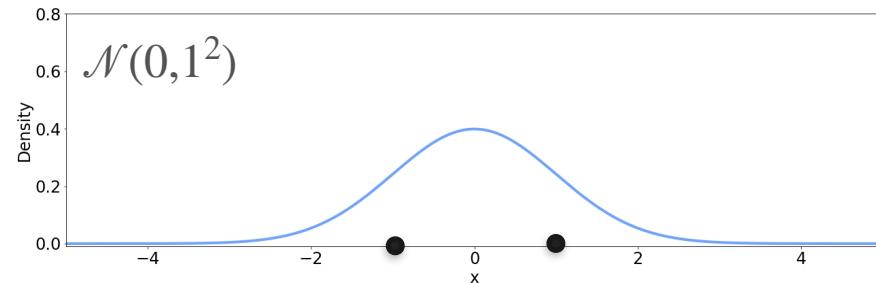
Observations



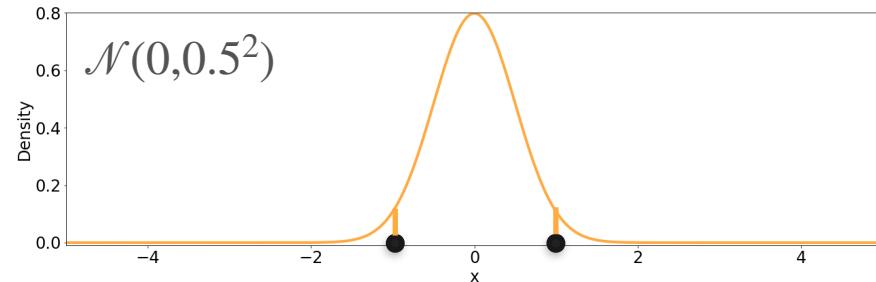
Candidates



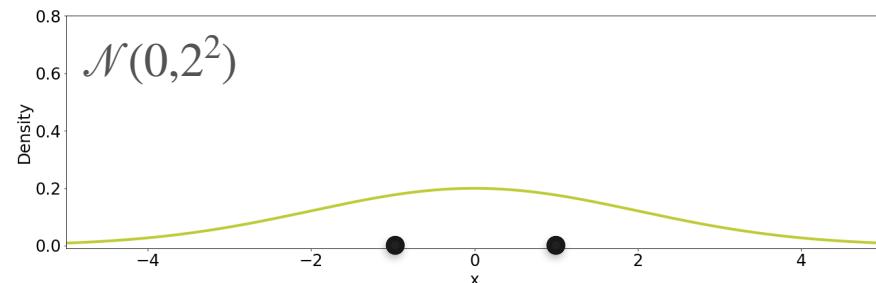
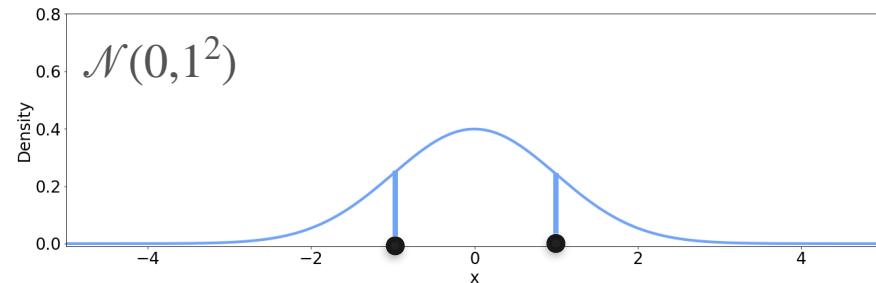
Observations



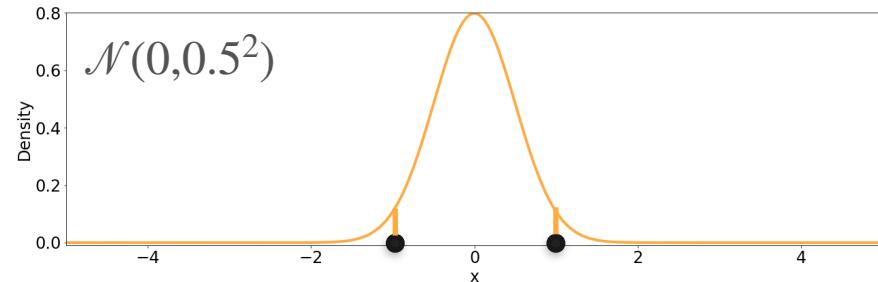
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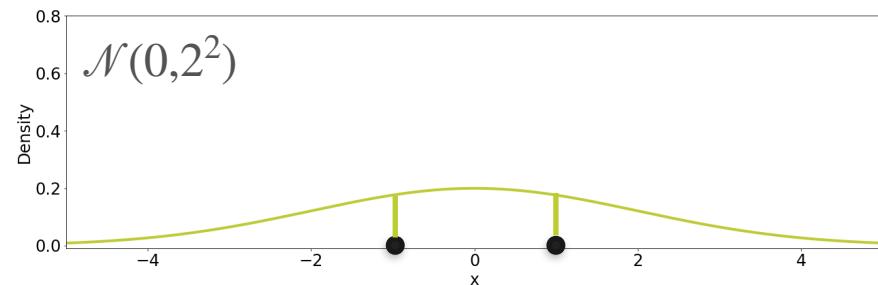
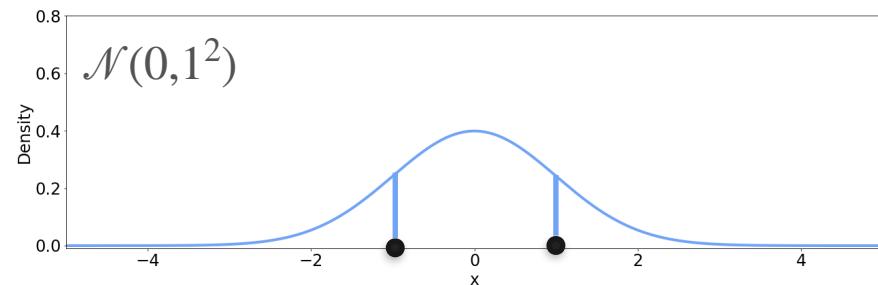
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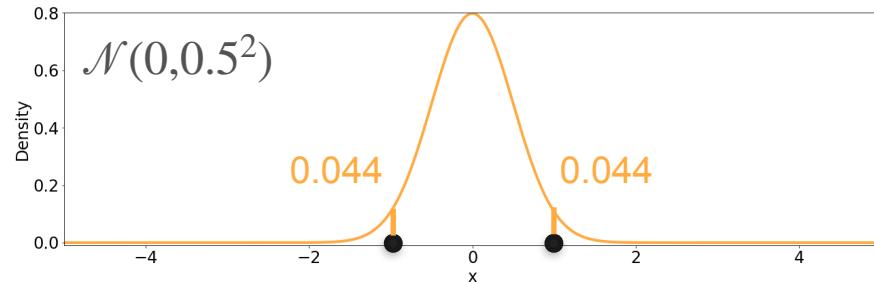
Candidates



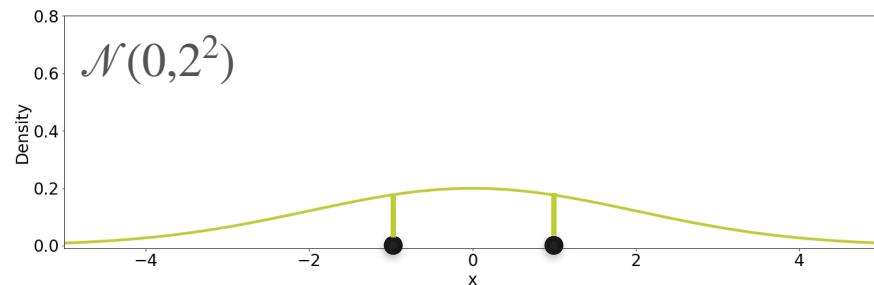
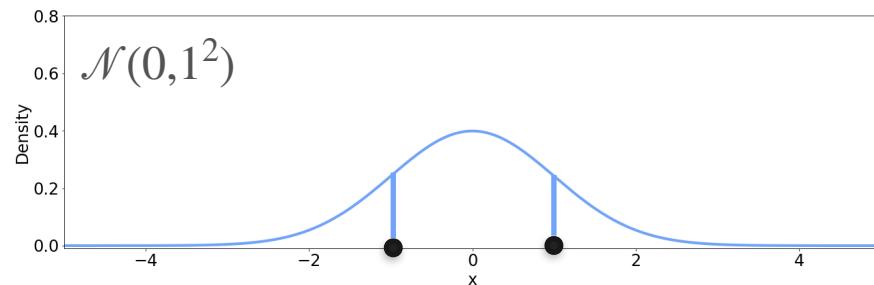
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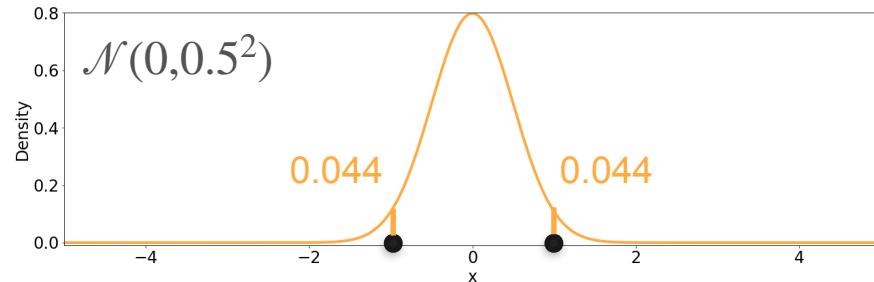
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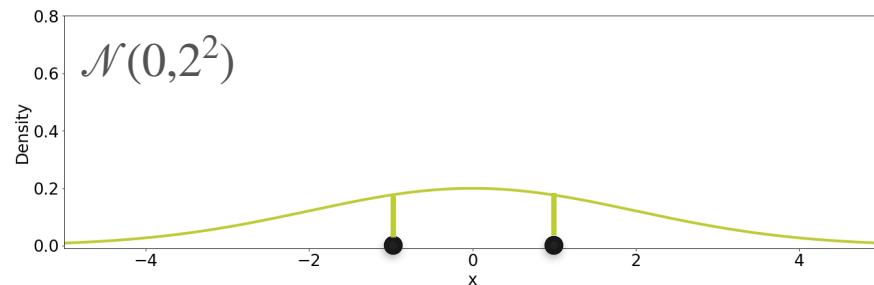
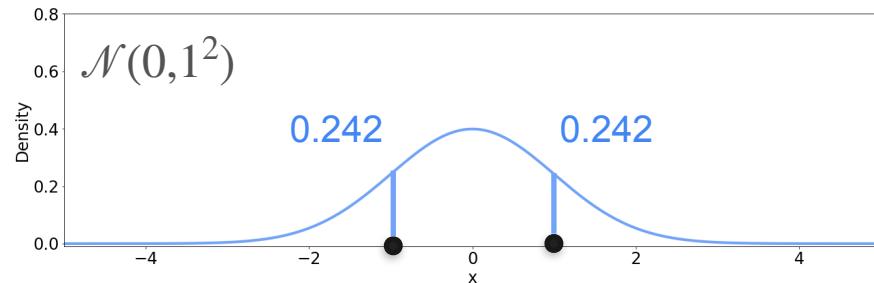
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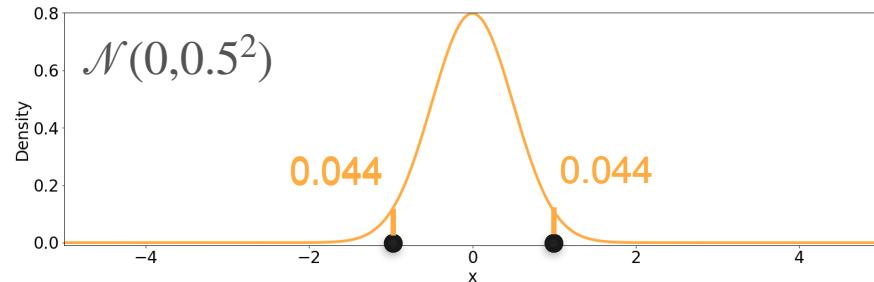
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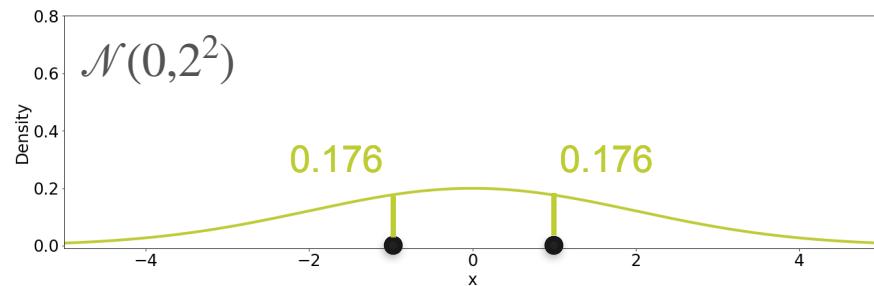
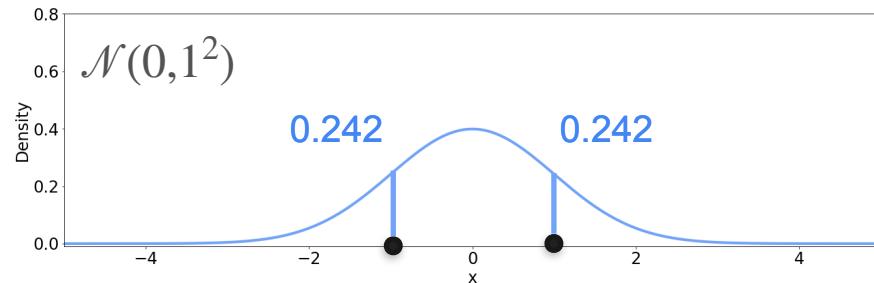
Observations



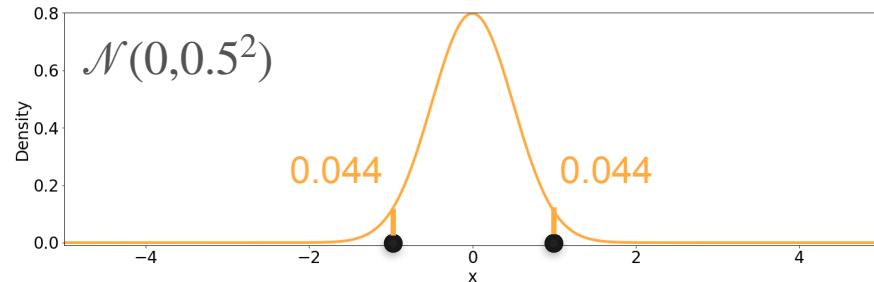
Candidates



Observations

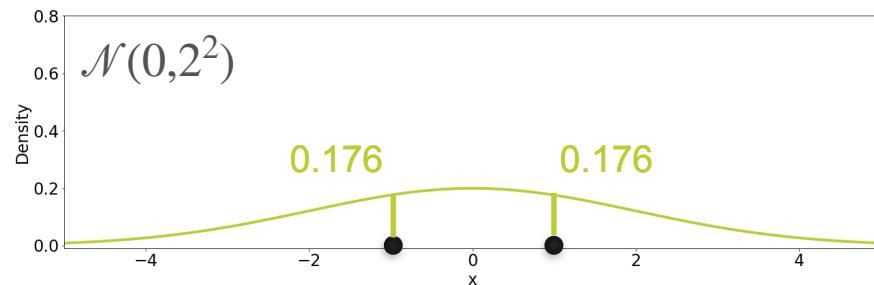
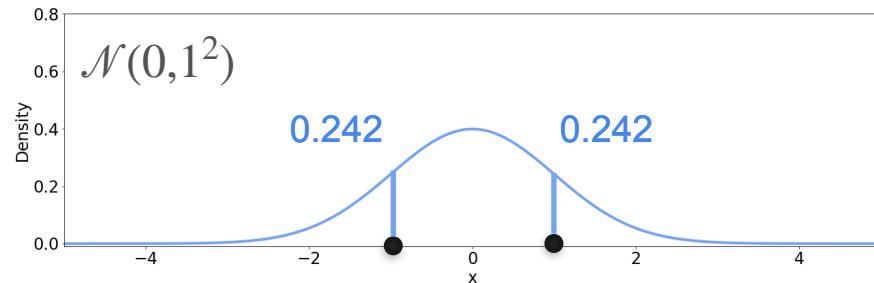


Candidates

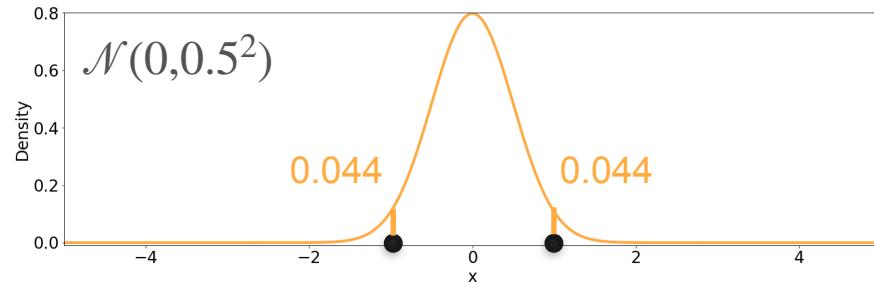


Observations

0.044 · 0.044 = 0.002

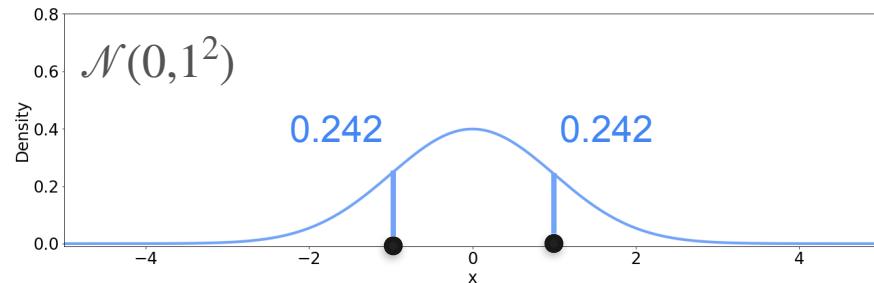


Candidates

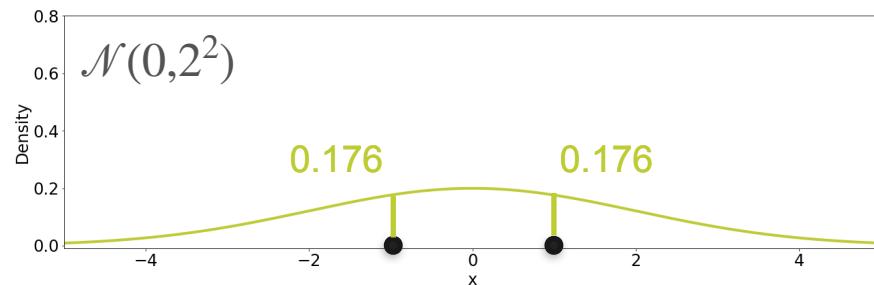


Observations

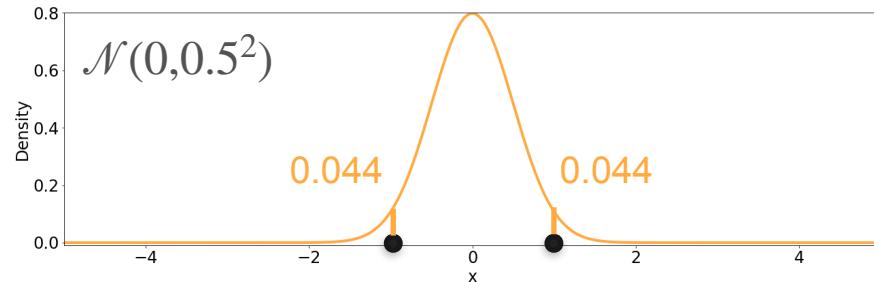
$$0.044 \cdot 0.044 = 0.002$$



$$0.242 \cdot 0.242 = 0.059$$

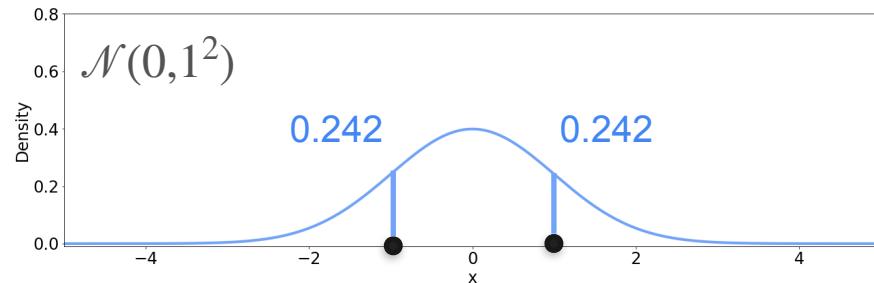


Candidates

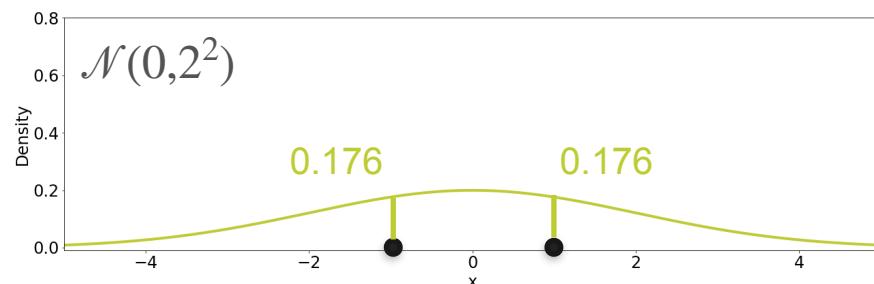


Observations

$$0.044 \cdot 0.044 = 0.002$$

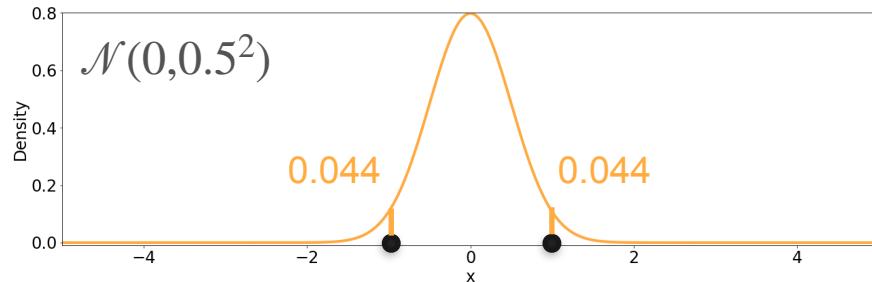


$$0.242 \cdot 0.242 = 0.059$$



$$0.176 \cdot 0.176 = 0.031$$

Candidates



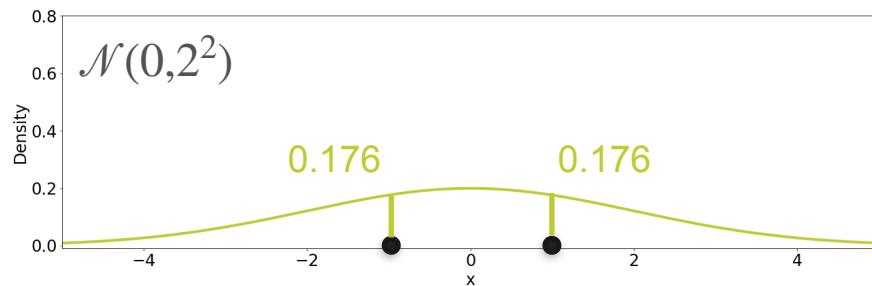
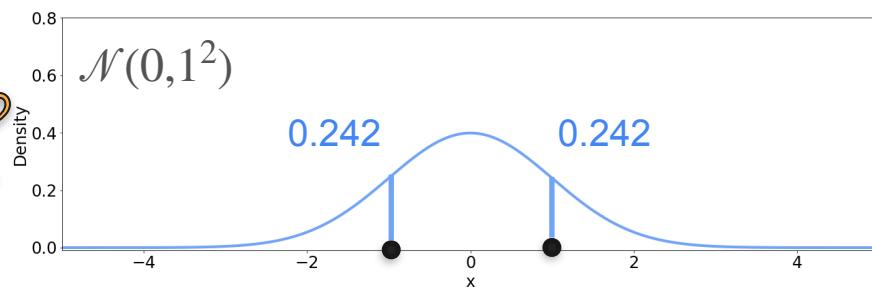
Observations

$$0.044 \cdot 0.044 = 0.002$$



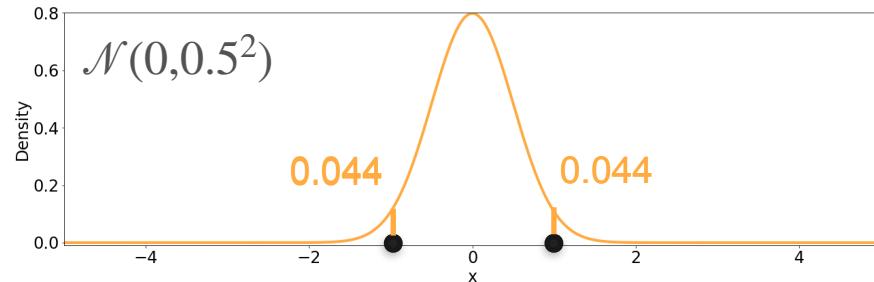
The $\mathcal{N}(0, 1^2)$ is more likely!

$$0.242 \cdot 0.242 = 0.059$$



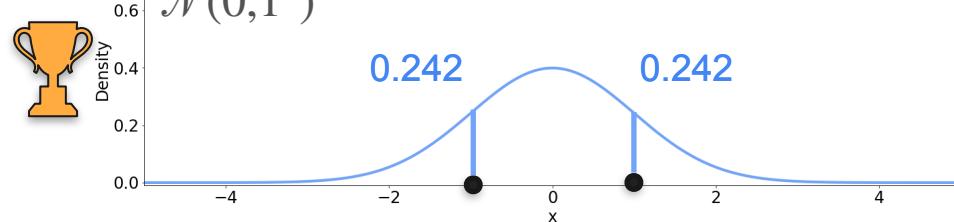
$$0.176 \cdot 0.176 = 0.031$$

Candidates



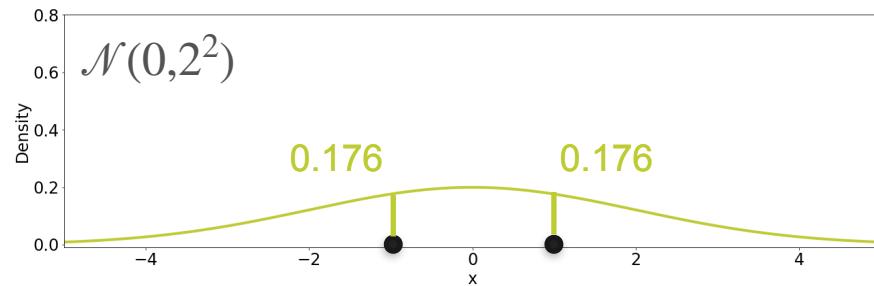
Observations

Likelihood = 0.002

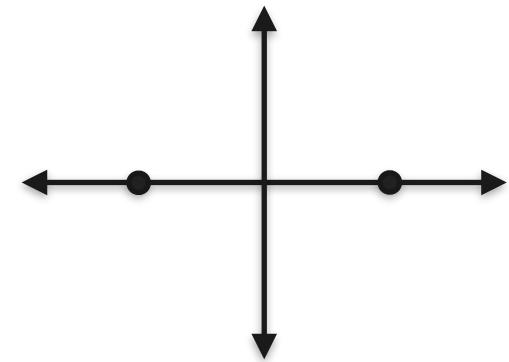


Observations

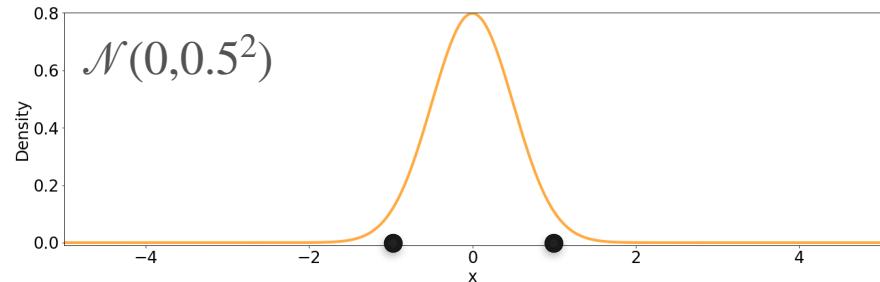
Likelihood = 0.059



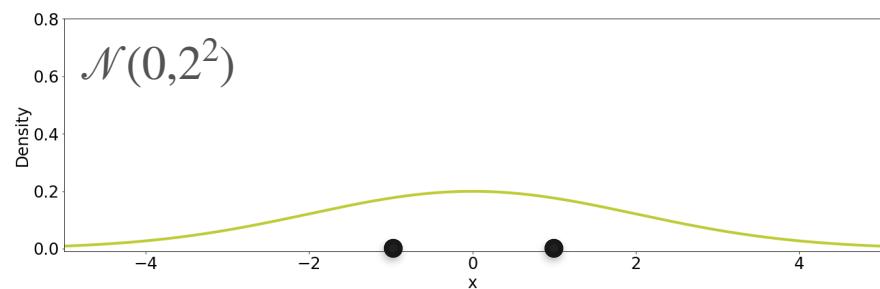
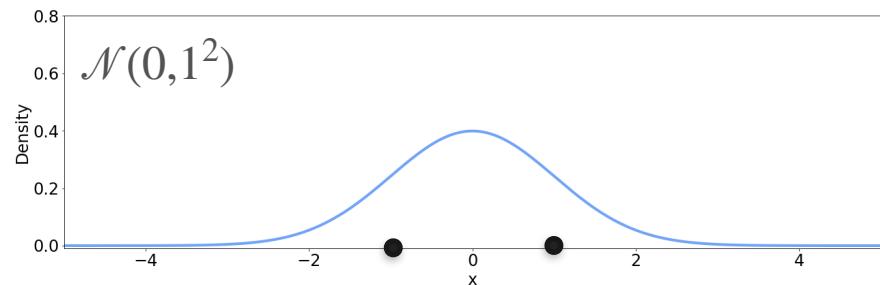
Likelihood = 0.031



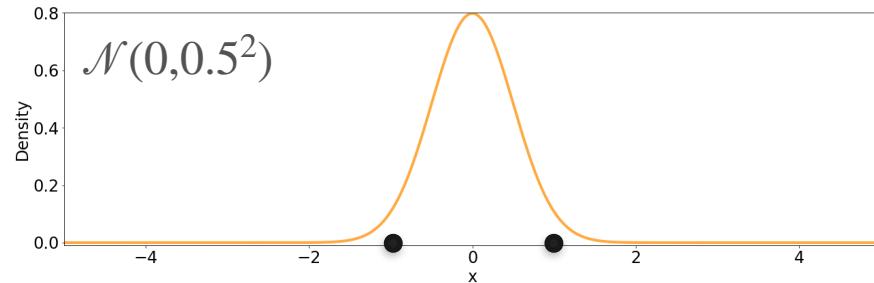
Candidates



Observations

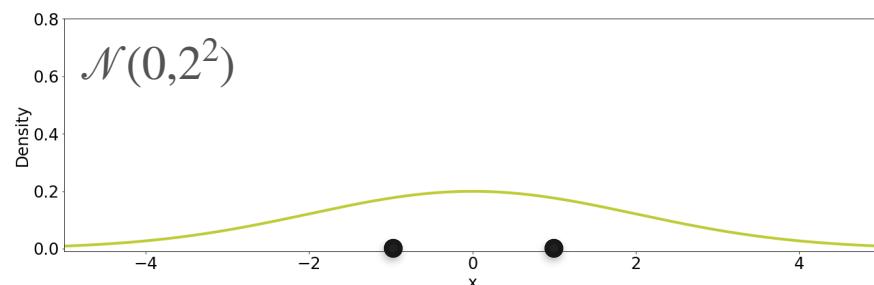
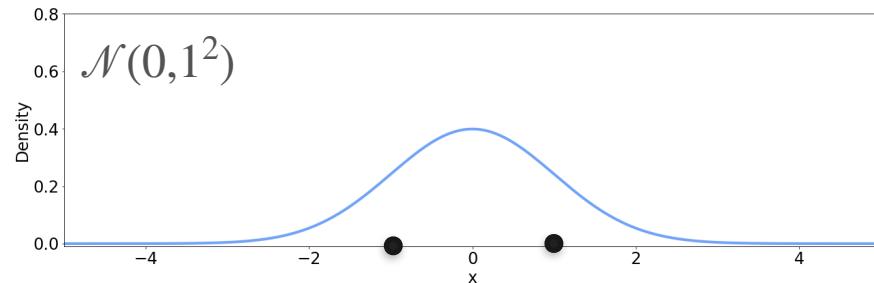


Candidates

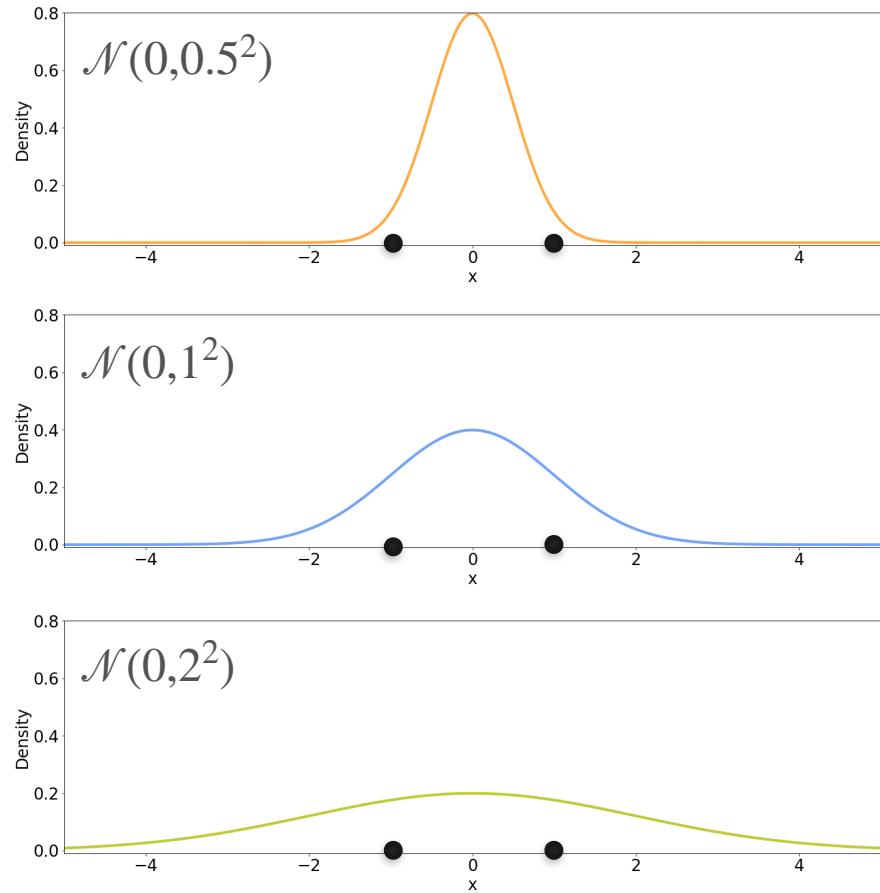


Observations

Variance of the observations



Candidates

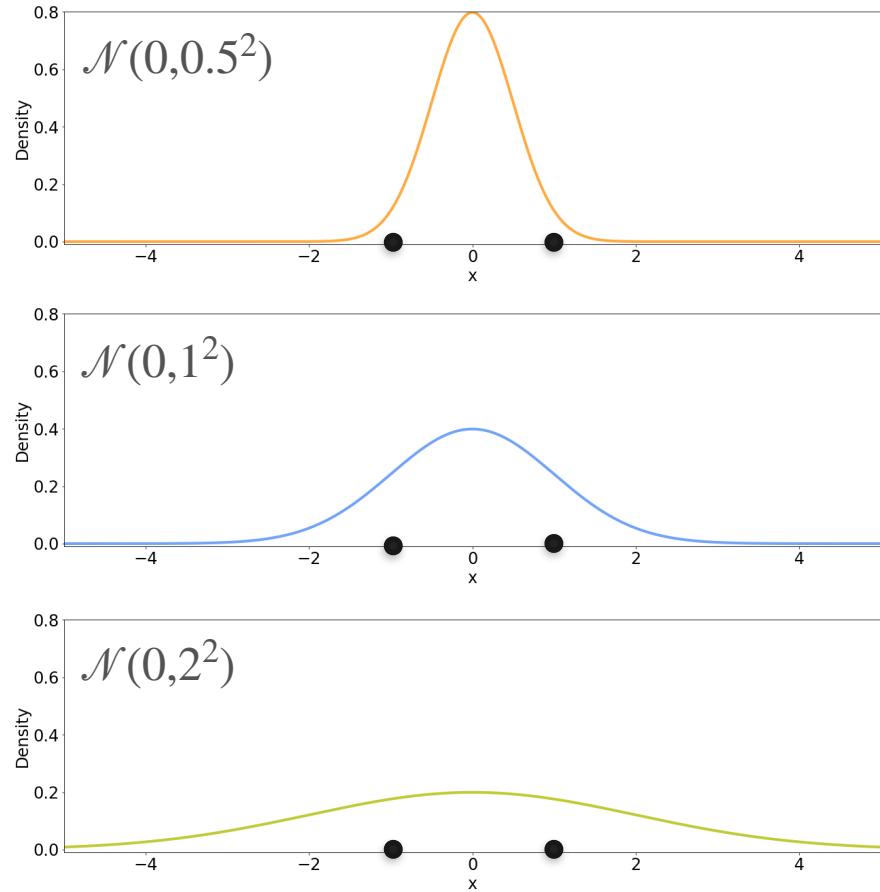


Observations

Variance of the observations

$$\widehat{\sigma}^2 = \frac{1}{2} ((0 - 1)^2 + (0 + 1)^2)$$

Candidates

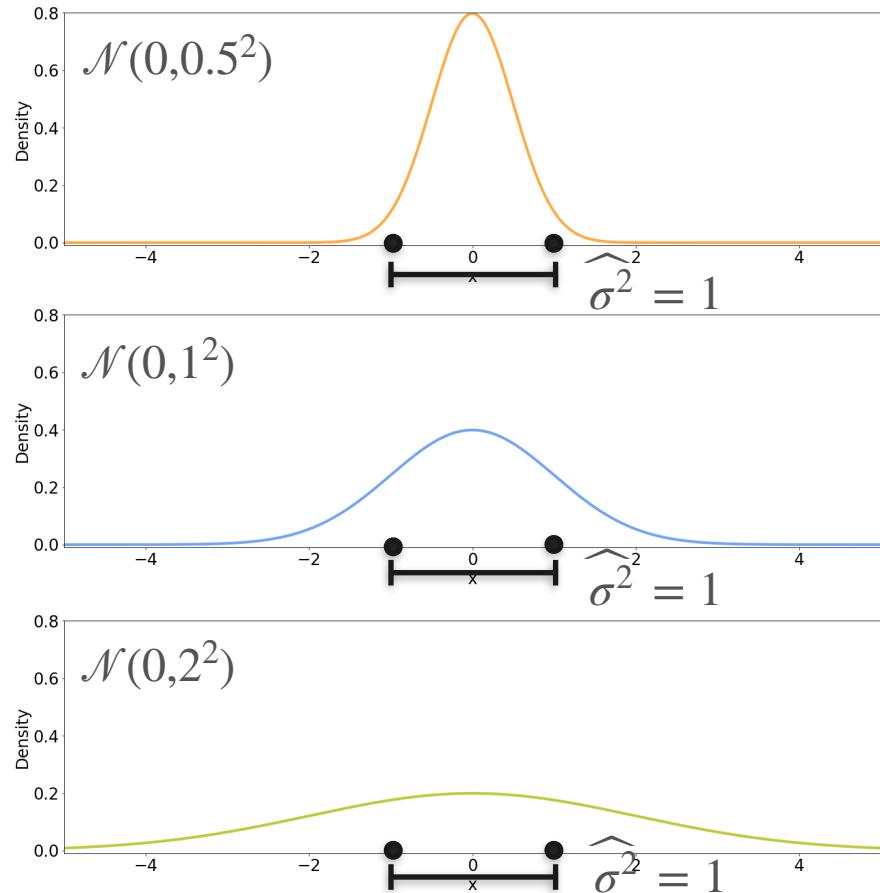


Observations

Variance of the observations

$$\widehat{\sigma}^2 = \frac{1}{2} ((0 - 1)^2 + (0 + 1)^2) = 1$$

Candidates

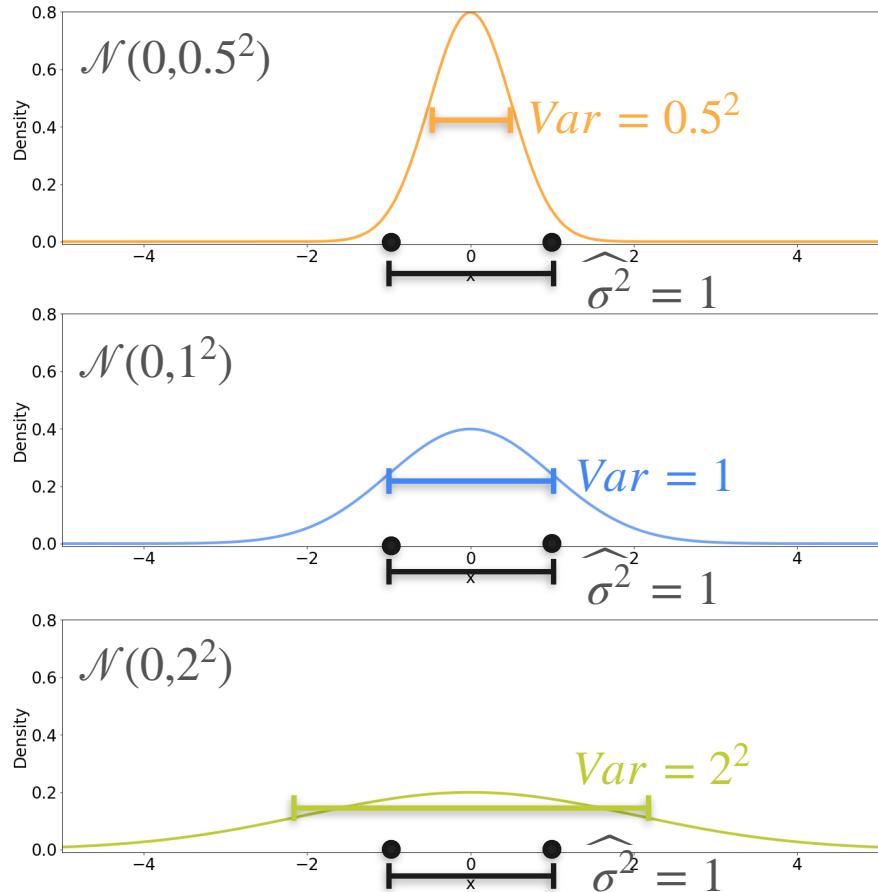


Observations

Variance of the observations

$$\hat{\sigma}^2 = \frac{1}{2} ((0 - 1)^2 + (0 + 1)^2) = 1$$

Candidates

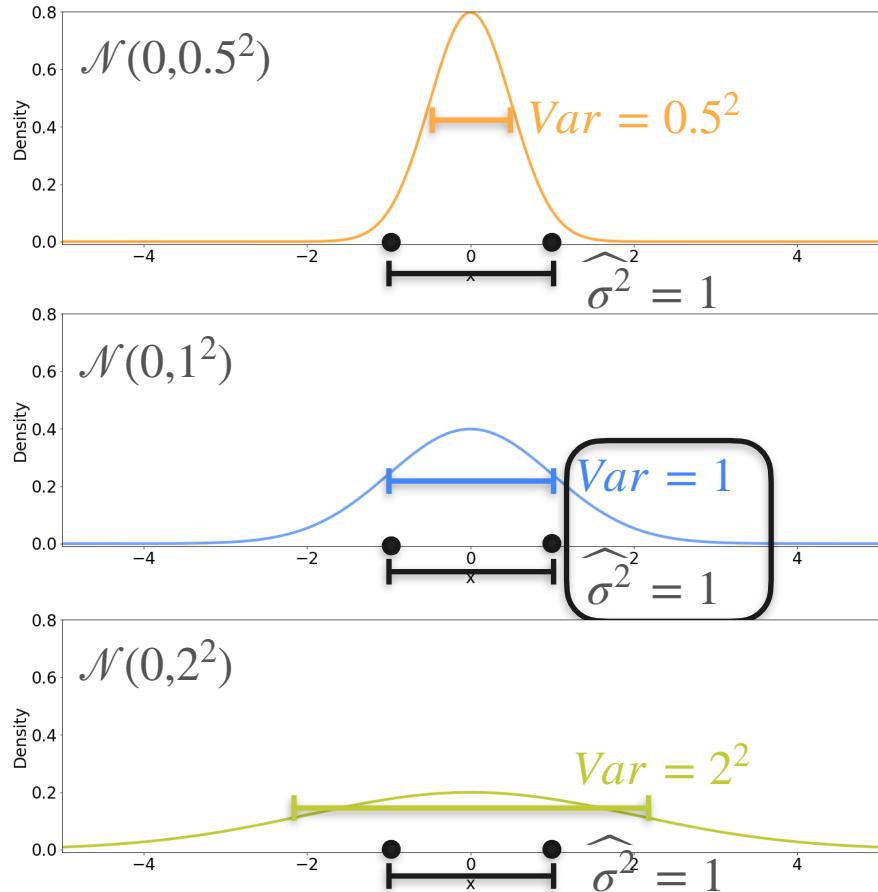


Observations

Variance of the observations

$$\hat{\sigma}^2 = \frac{1}{2} ((0 - 1)^2 + (0 + 1)^2) = 1$$

Candidates

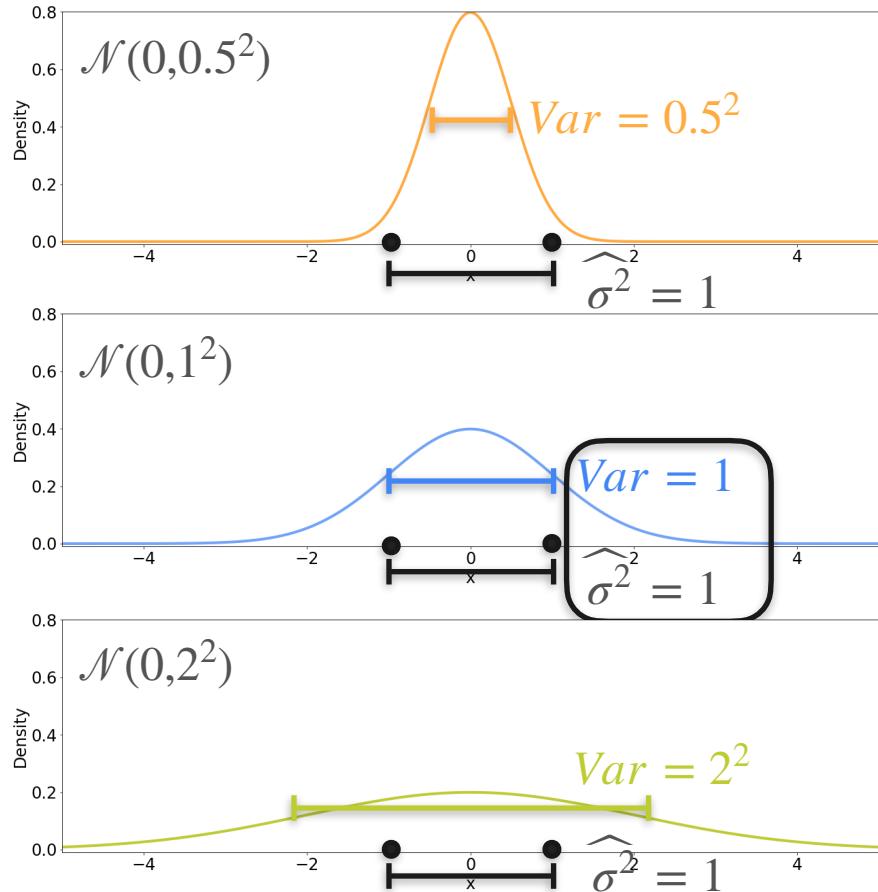


Observations

Variance of the observations

$$\hat{\sigma}^2 = \frac{1}{2} ((0 - 1)^2 + (0 + 1)^2) = 1$$

Candidates



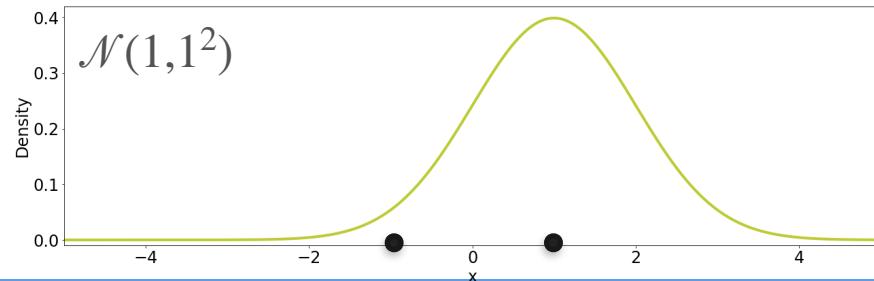
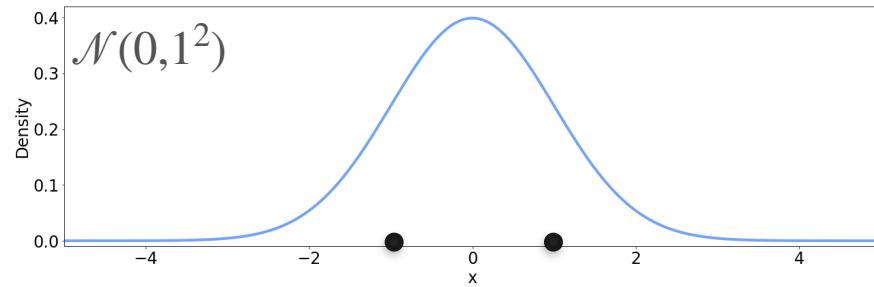
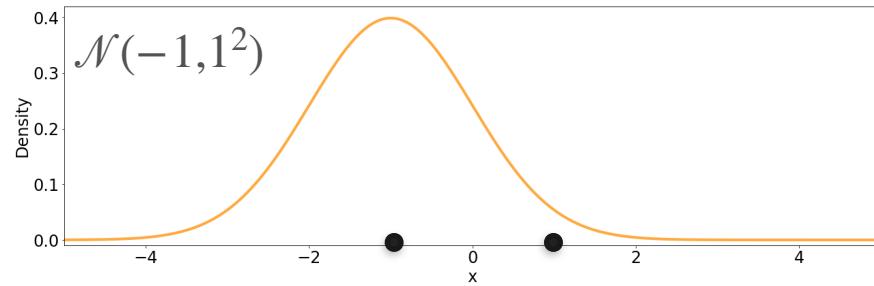
Observations

Variance of the observations

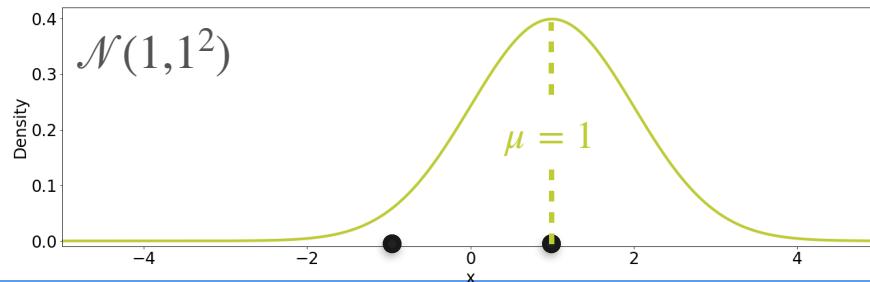
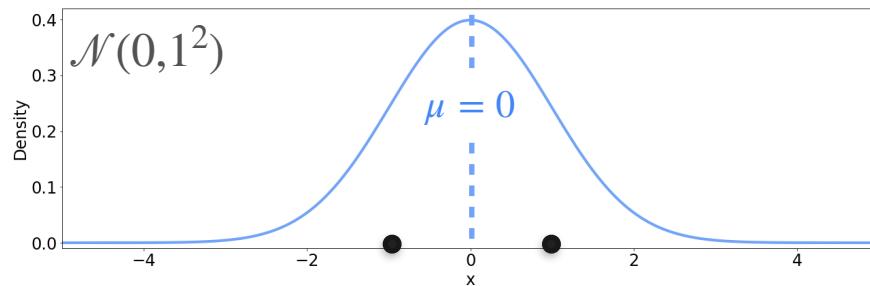
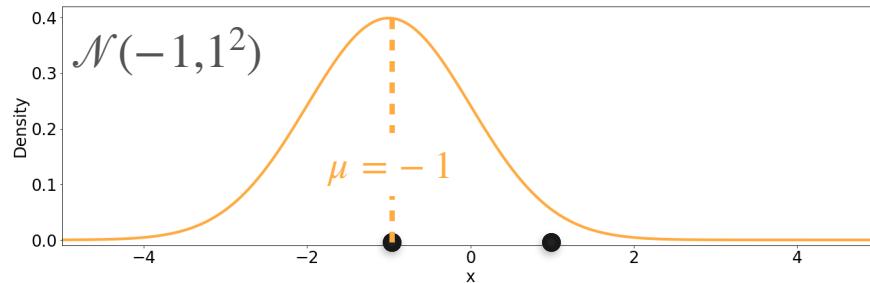
$$\widehat{\sigma}^2 = \frac{1}{2} ((0 - 1)^2 + (0 + 1)^2) = 1$$

The best distribution is the one where the **variance** of the distribution is the **variance** of the sample

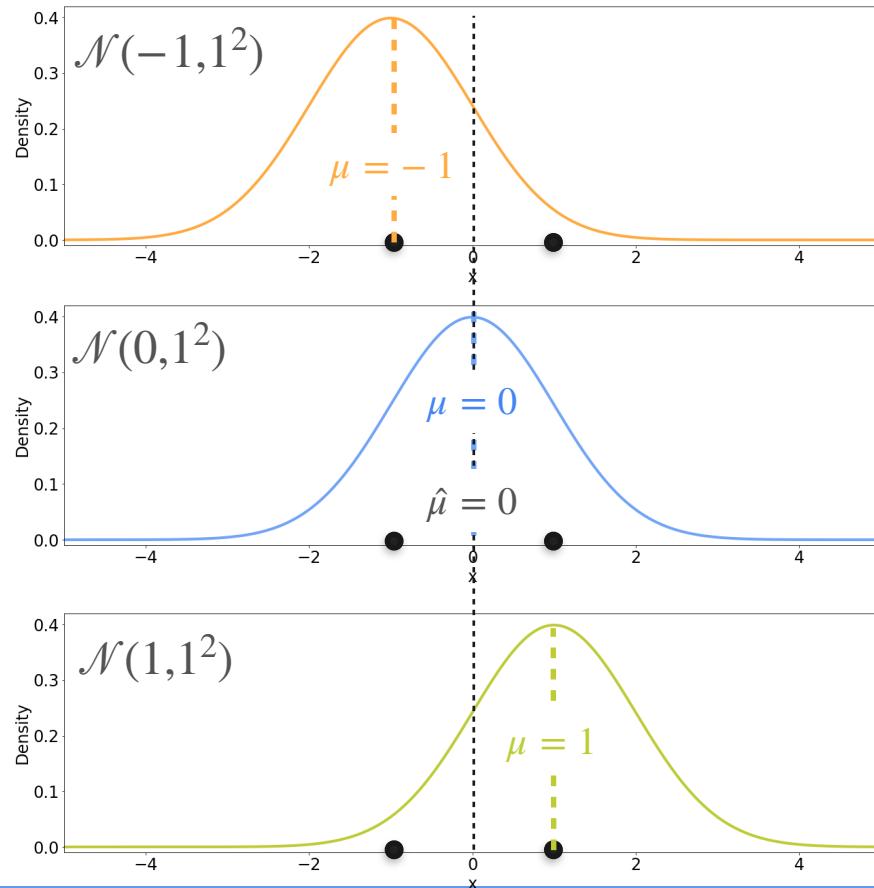
Candidates



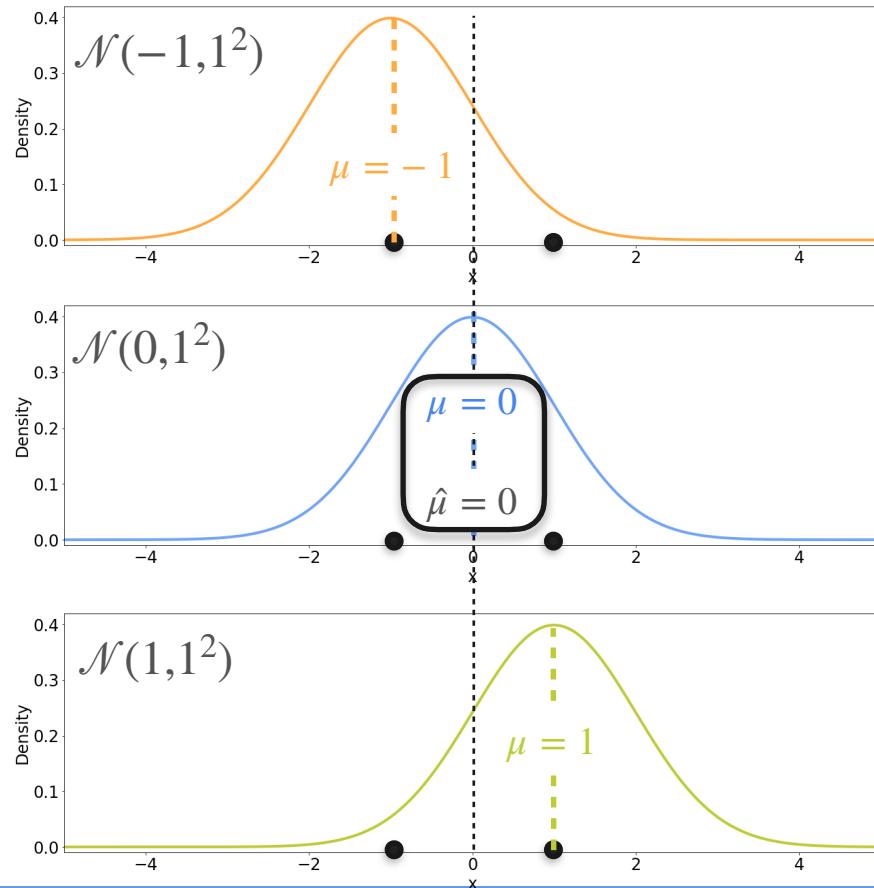
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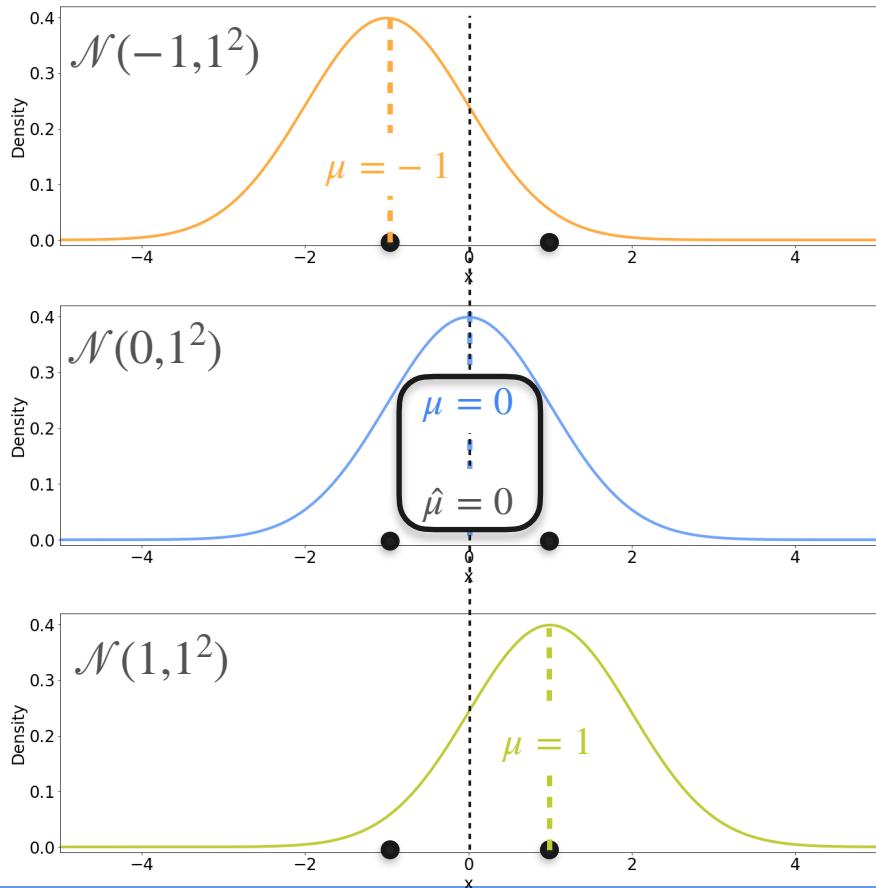
Candidates



Candidates

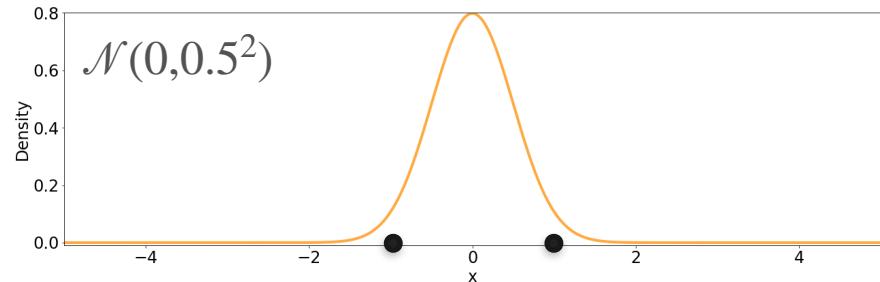


Candidates

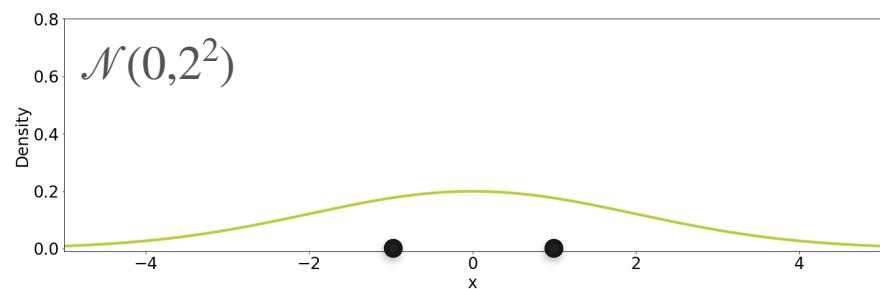
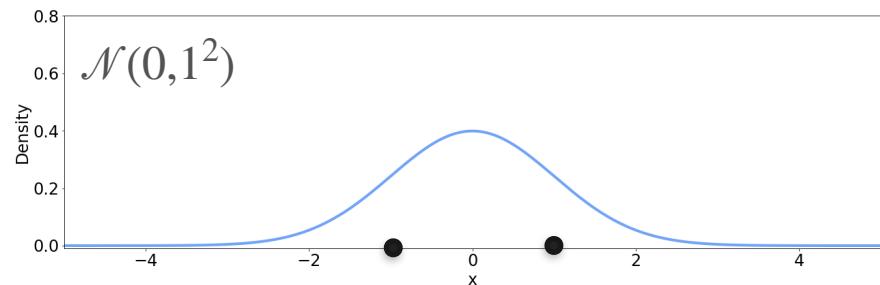


The best distribution is the one where
the **mean** of the distribution is the
mean of the sample

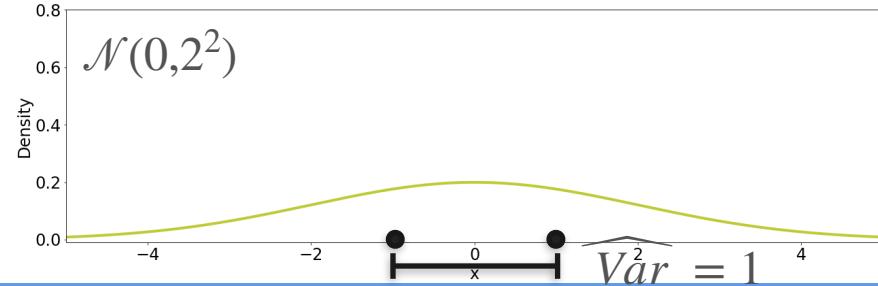
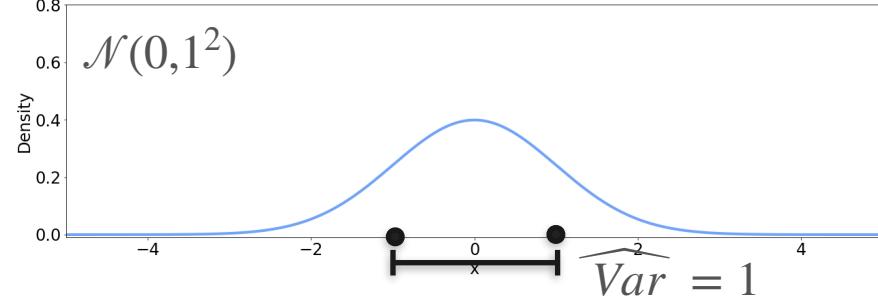
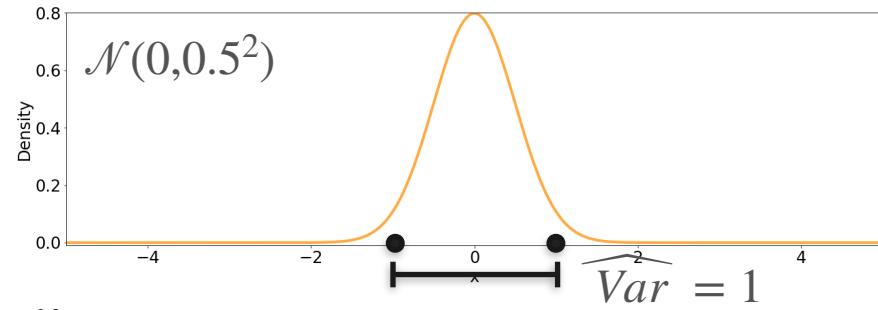
Candidates



Observations

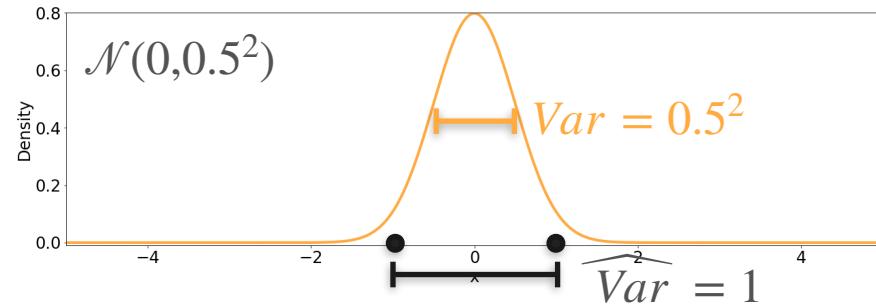


Candidates

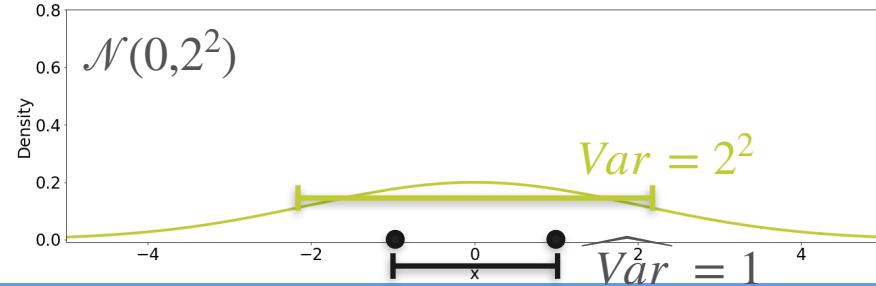
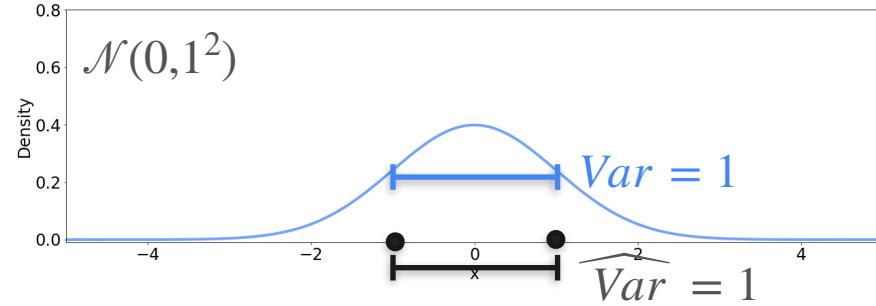


Observations

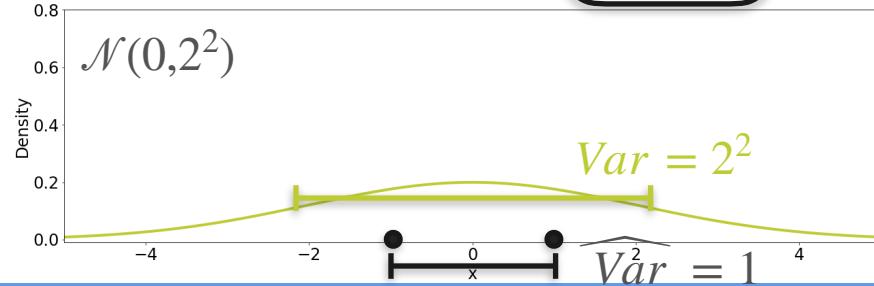
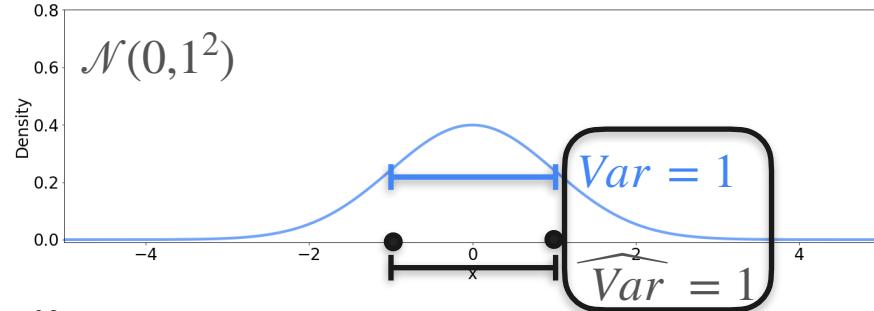
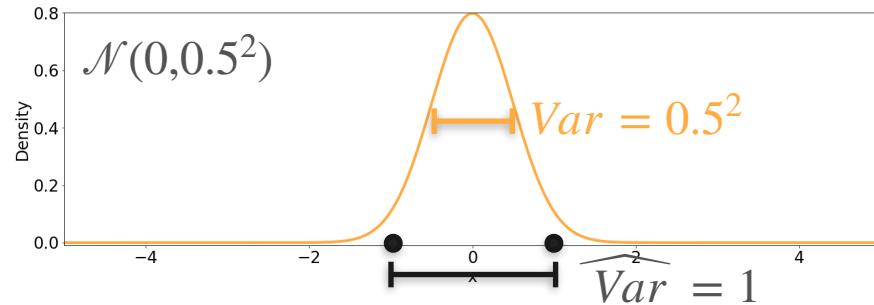
Candidates



Observations

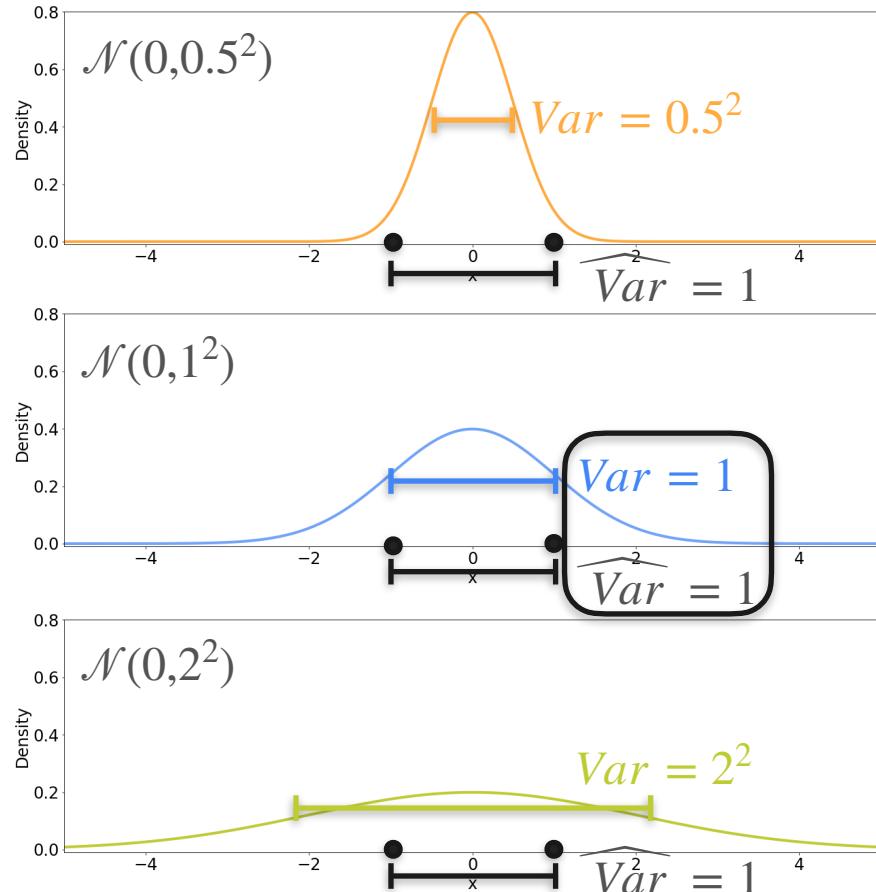


Candidates



Observations

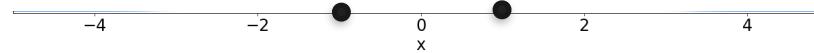
Candidates



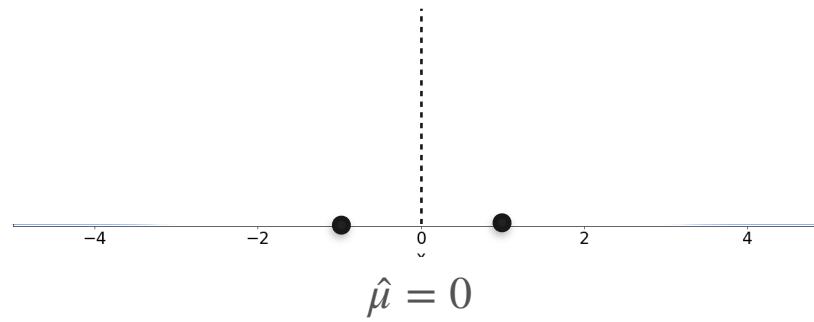
Observations

The best distribution is the one where the **variance** of the distribution is the **variance** of the sample

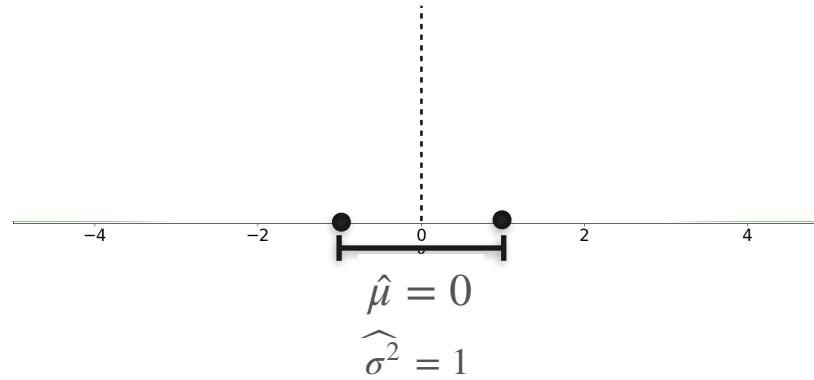
Best Gaussian Fit



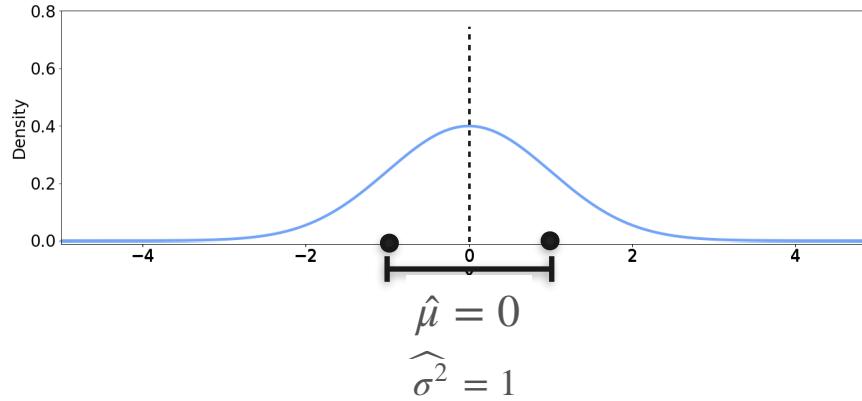
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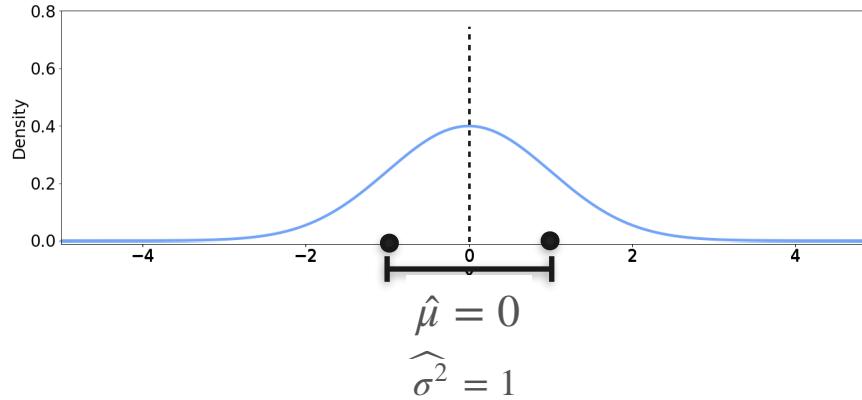
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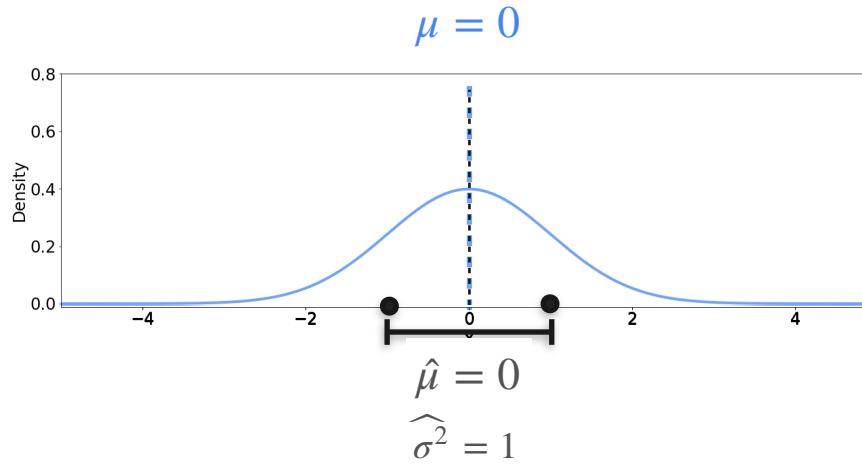


Best Gaussian Fit



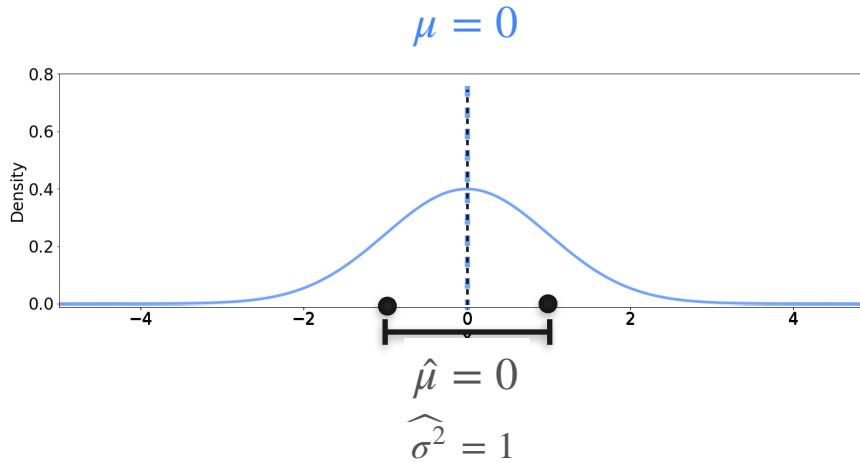
The best distribution is the one where
the **mean** of the distribution is the
mean of the sample

Best Gaussian Fit



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the **mean** of the distribution is the
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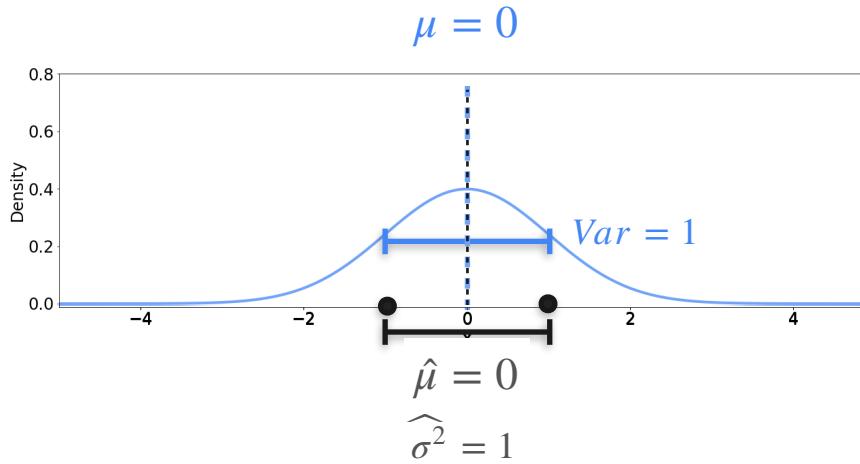
Best Gaussian Fit



The best distribution is the one where
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The best distribution is the one where
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variance of the sample

Best Gaussian Fit



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The best distribution is the one where
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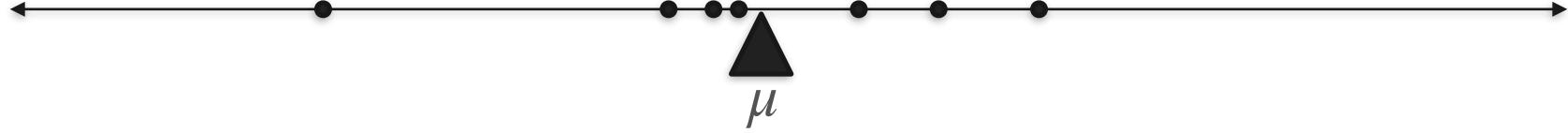
Best Gaussian Fit



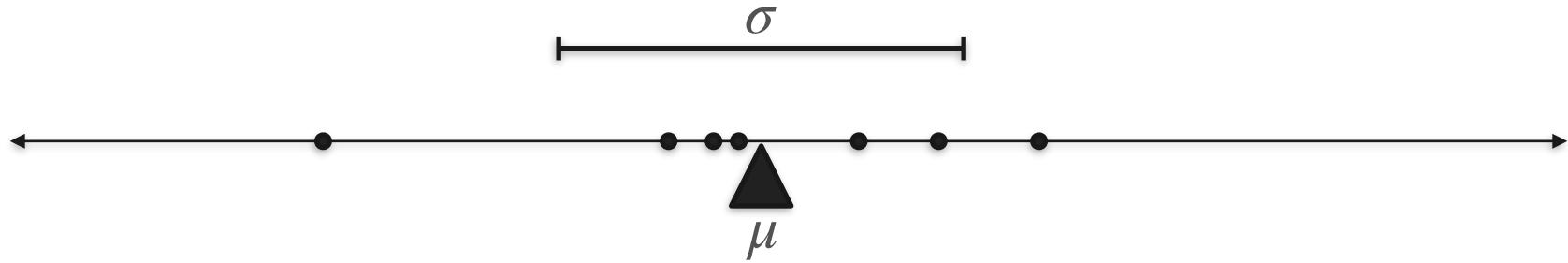
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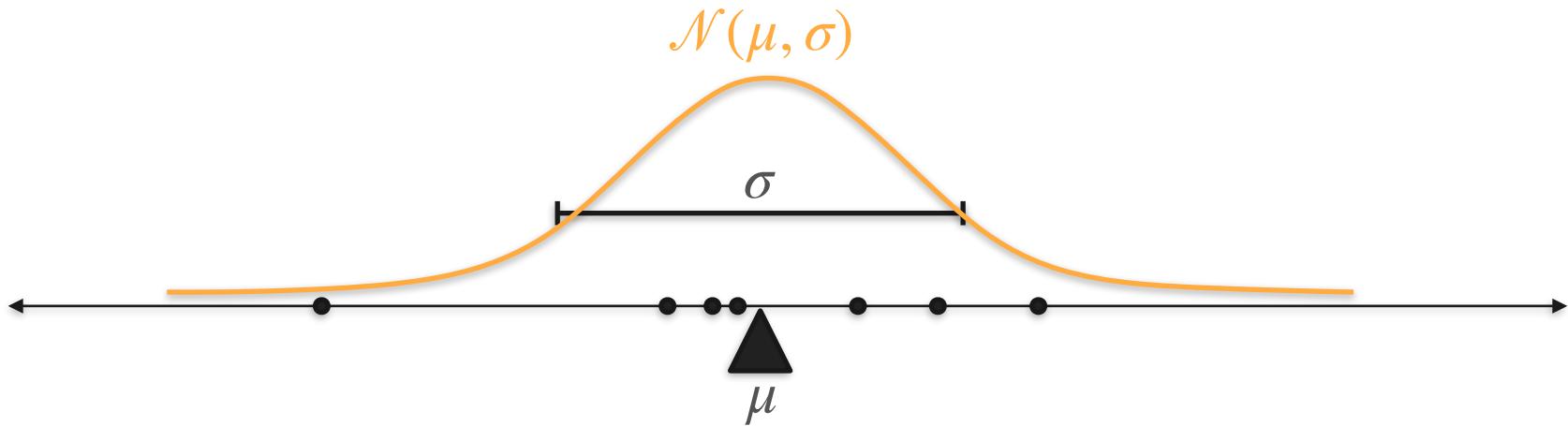
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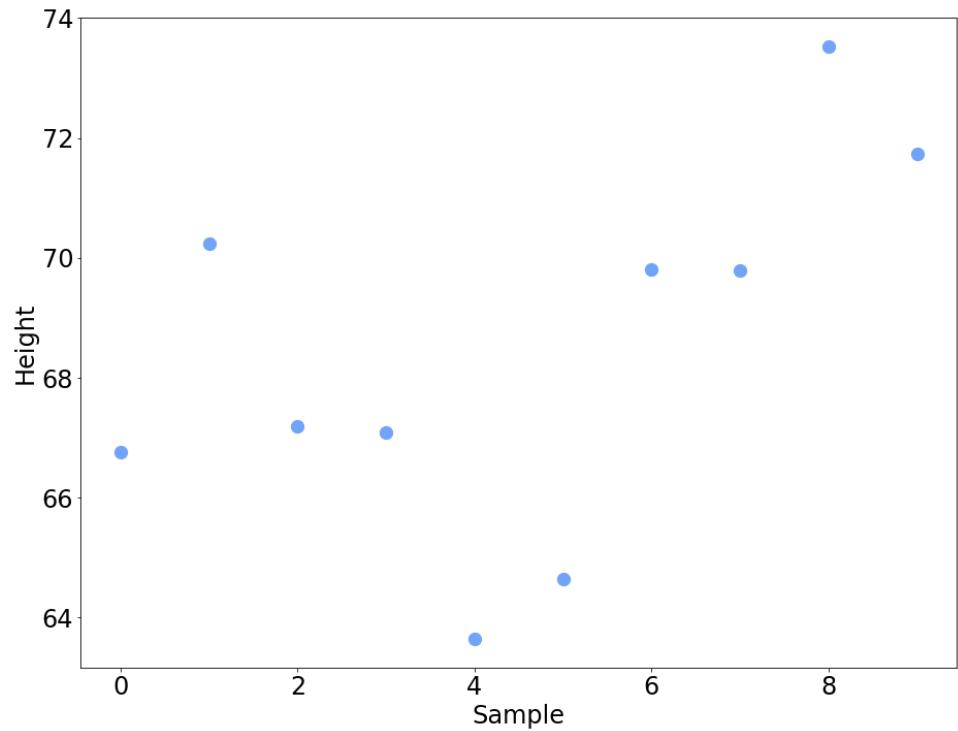
Maximum Likelihood: Gaussian Example

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X = "Height of an 18 year old"

You measure 10 people
(i.e. sample size =10)

$\mathbf{X} = (X_1, X_2, \dots, X_{10})$



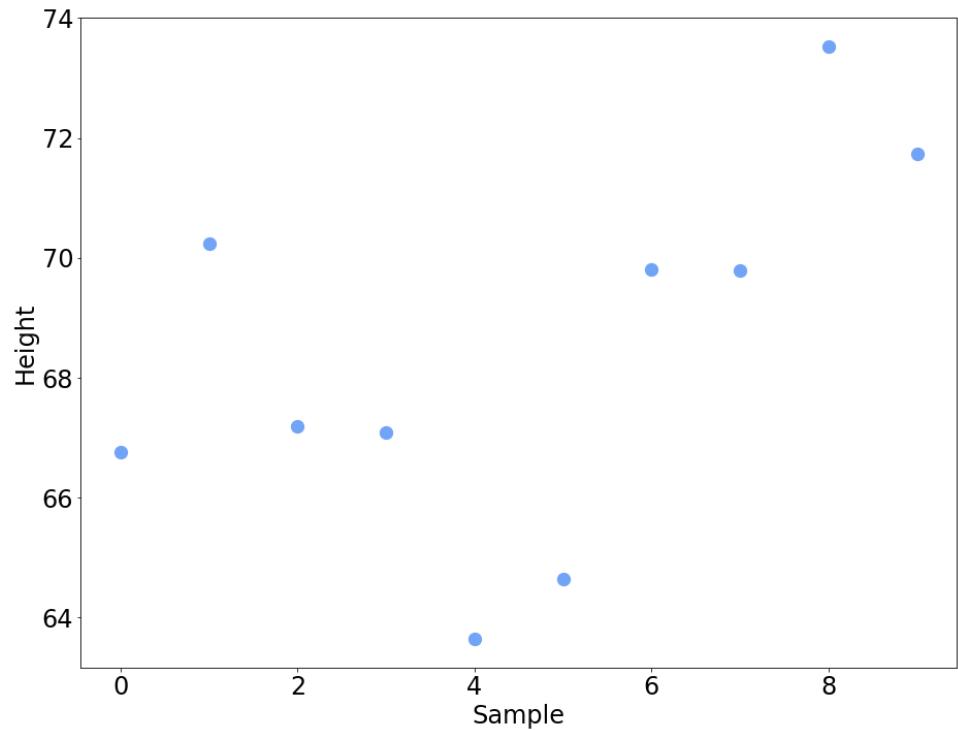
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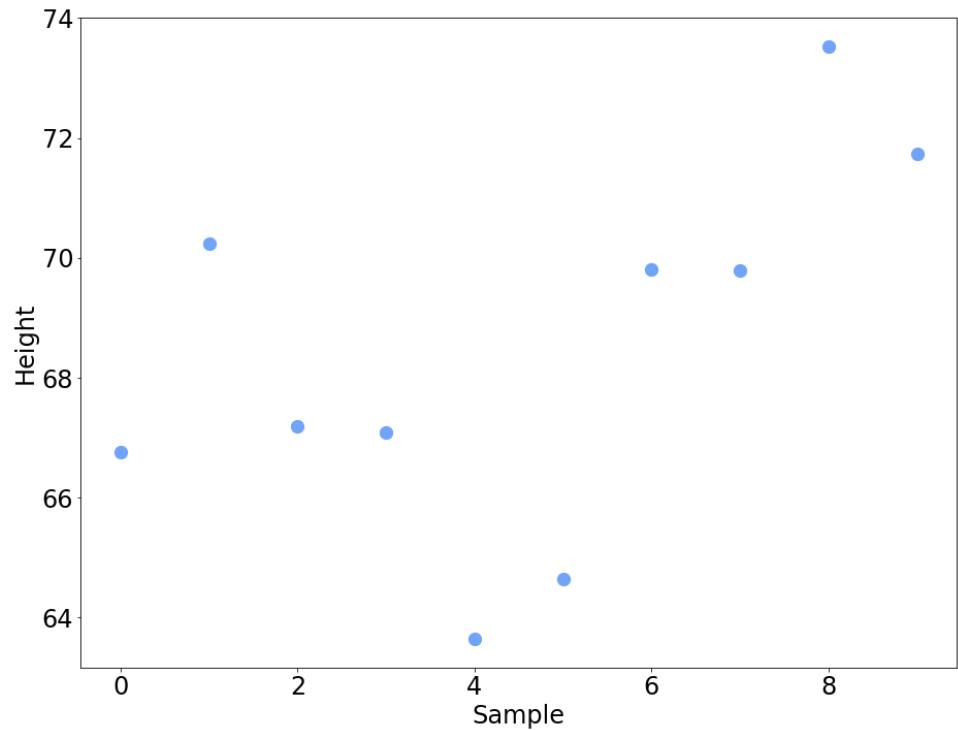
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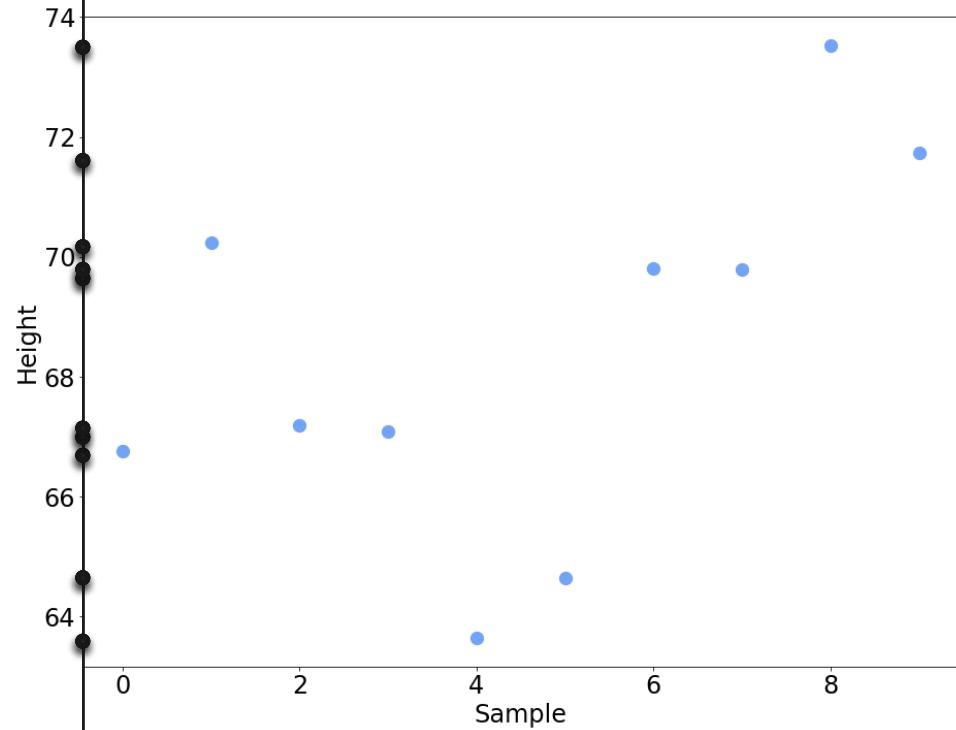
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$\mu?$





$\mu = 64$ $\mu = 68$  $\mu = 66$ $\mu = 70$ 

$$\begin{aligned}\mu &= 64 \\ \sigma &= 3.11\end{aligned}$$



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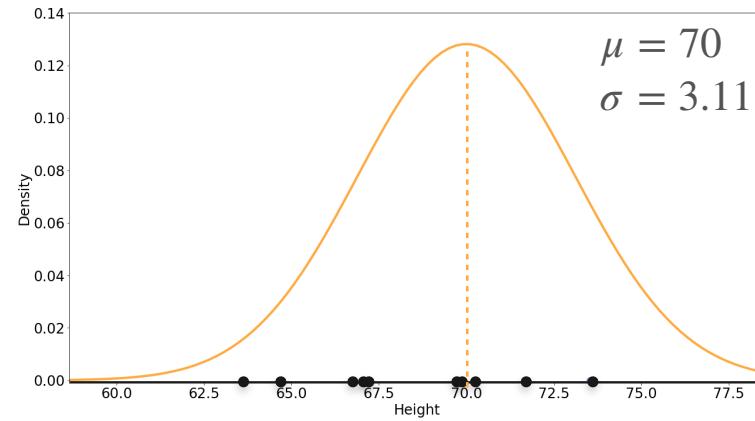
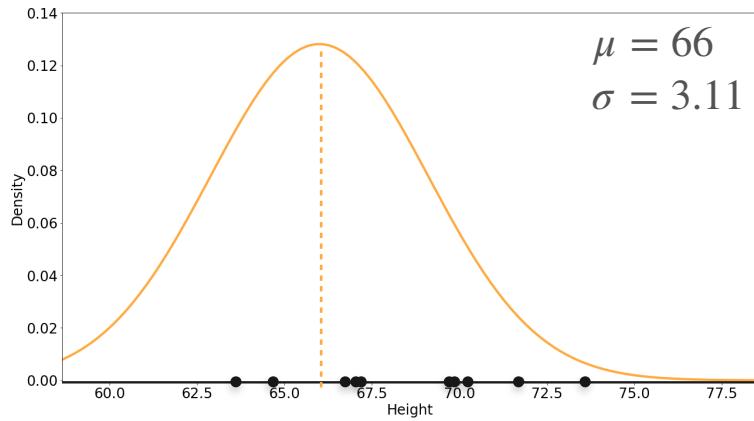
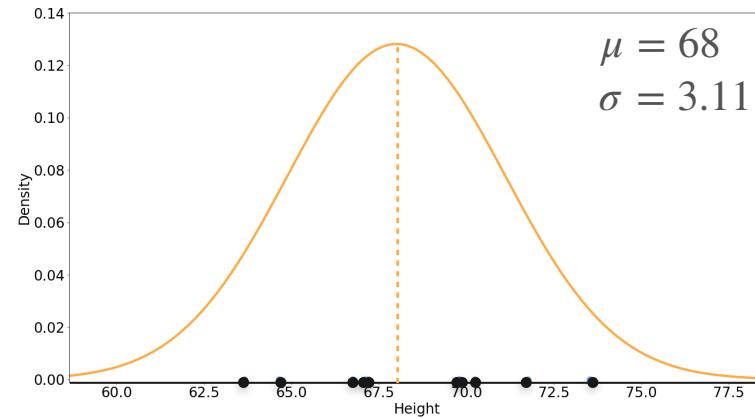
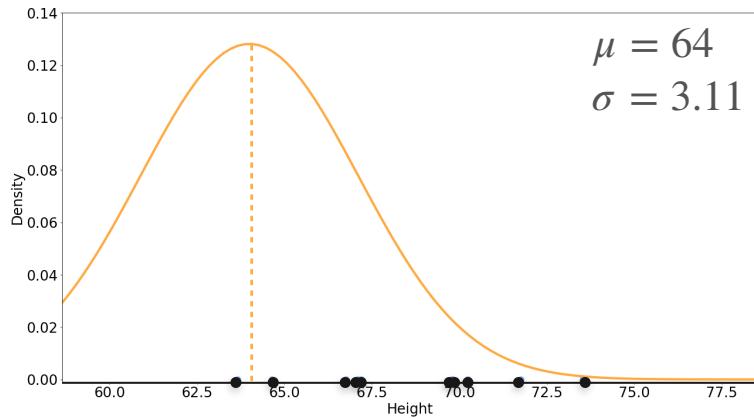


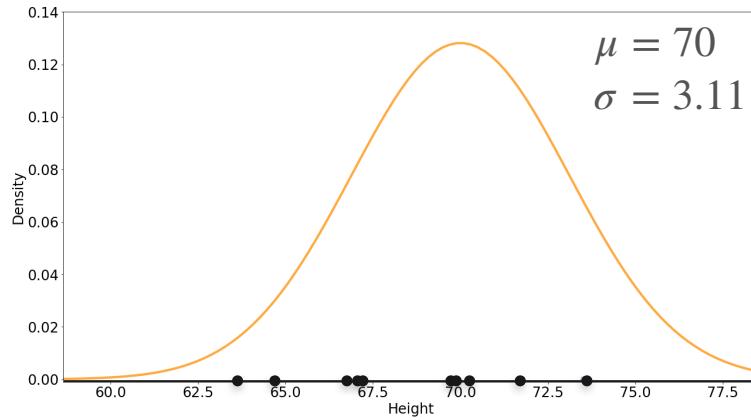
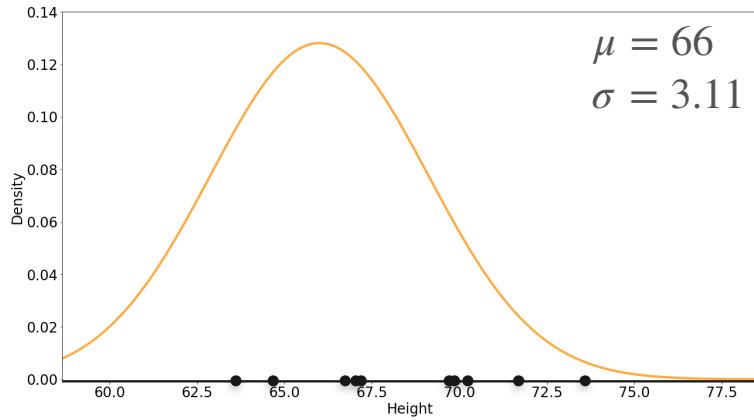
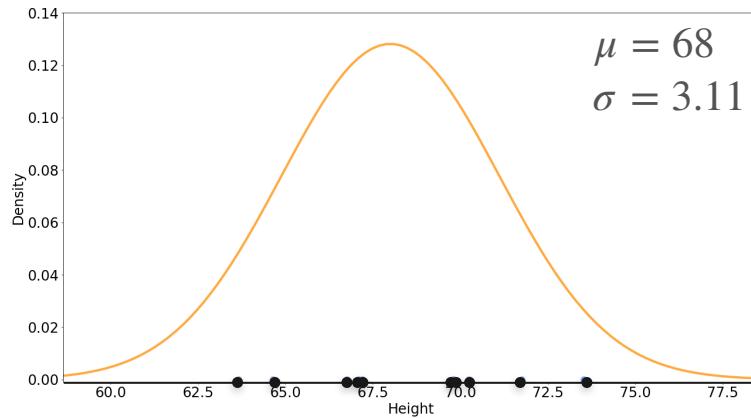
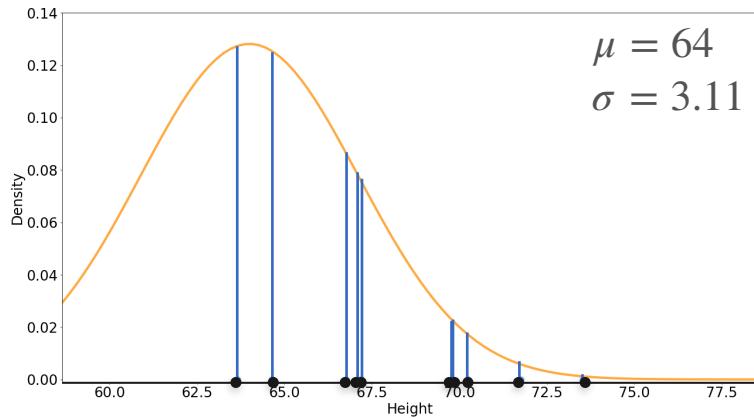
$$\begin{aligned}\mu &= 66 \\ \sigma &= 3.11\end{aligned}$$

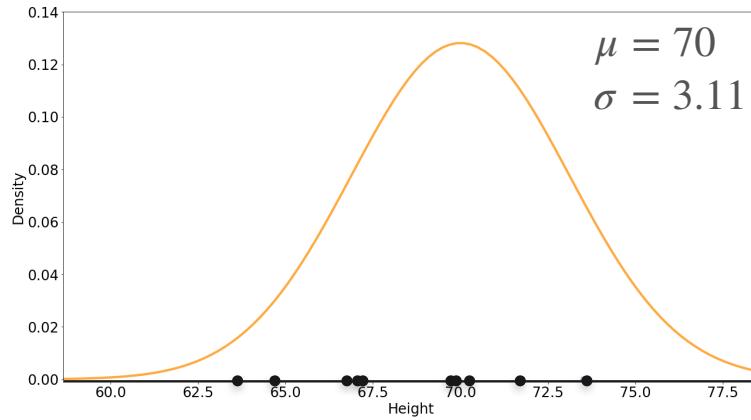
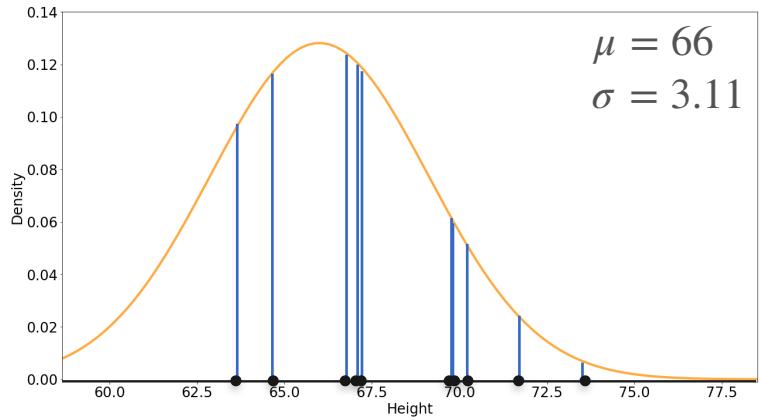
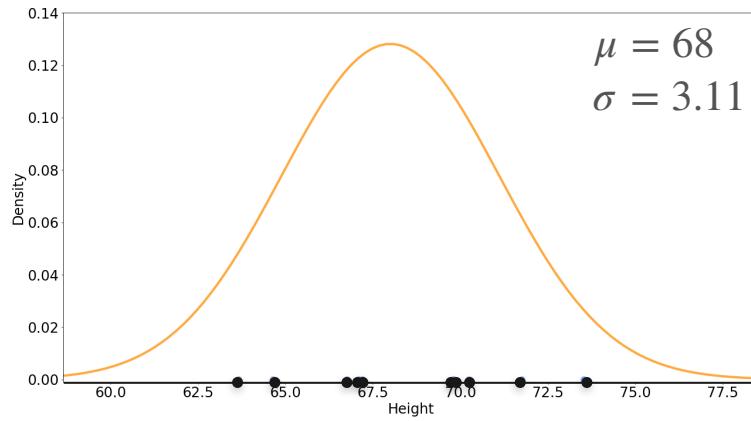
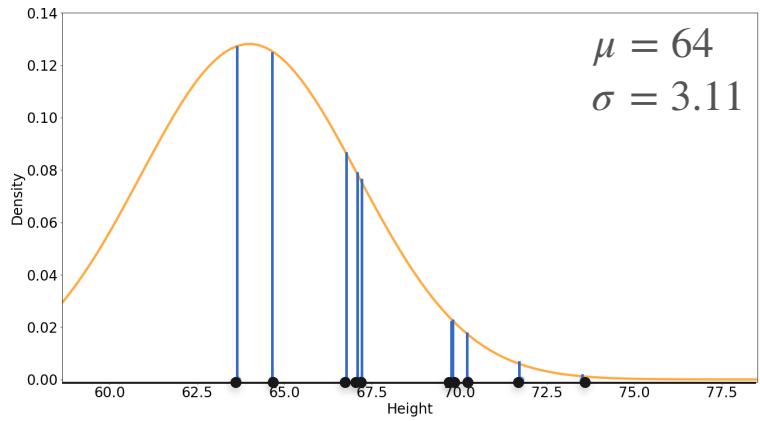


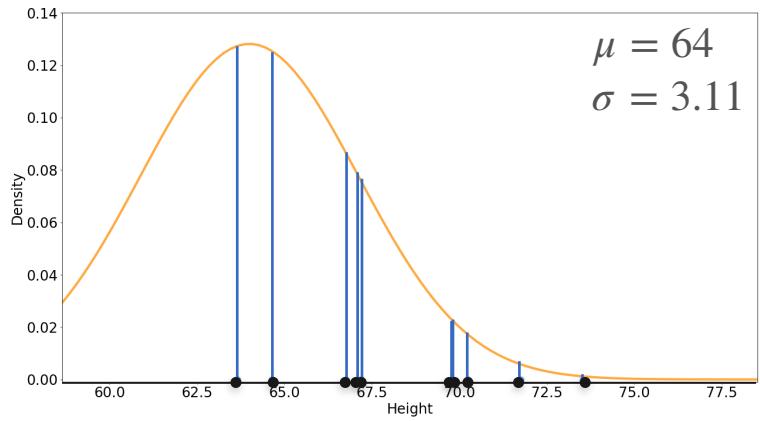
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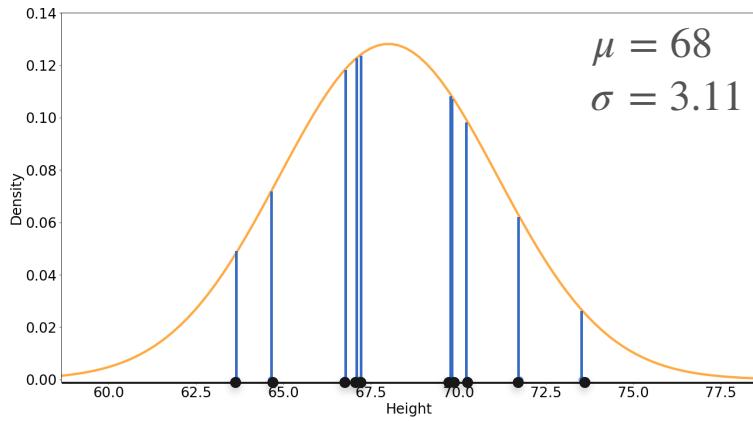




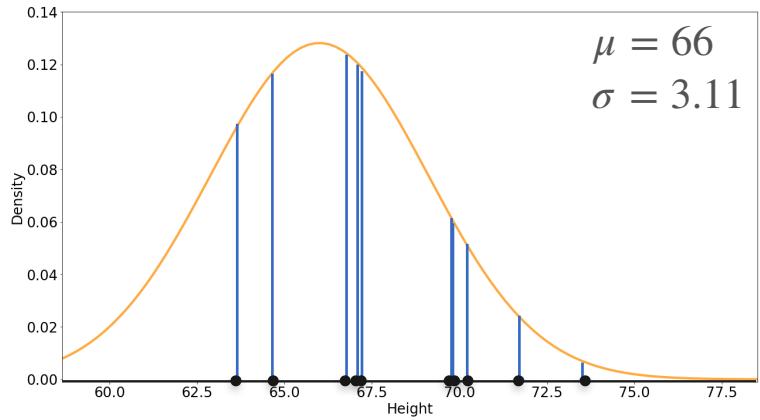




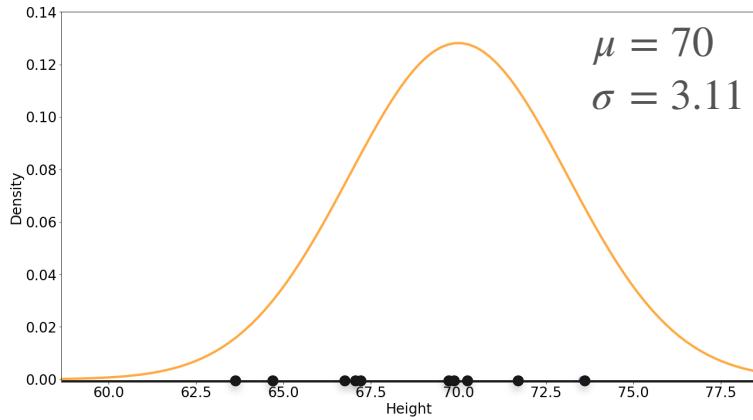
$$\mu = 64$$
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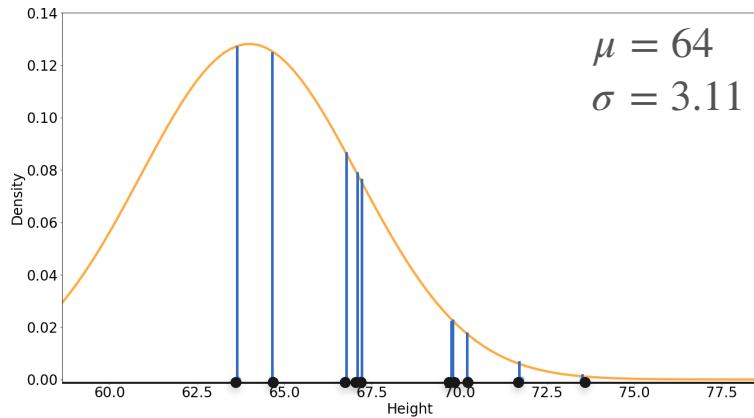
$$\mu = 68$$
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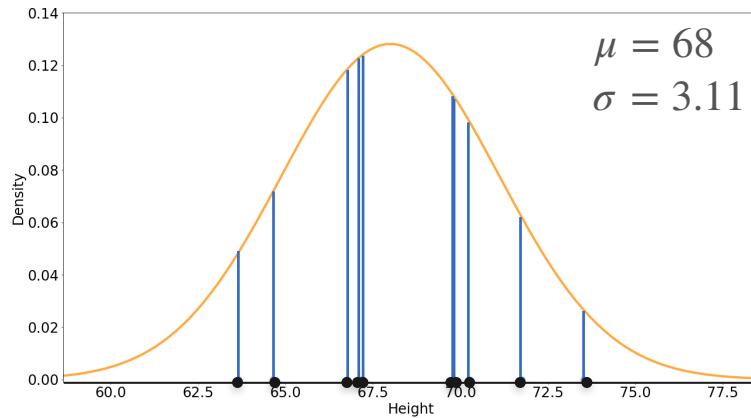
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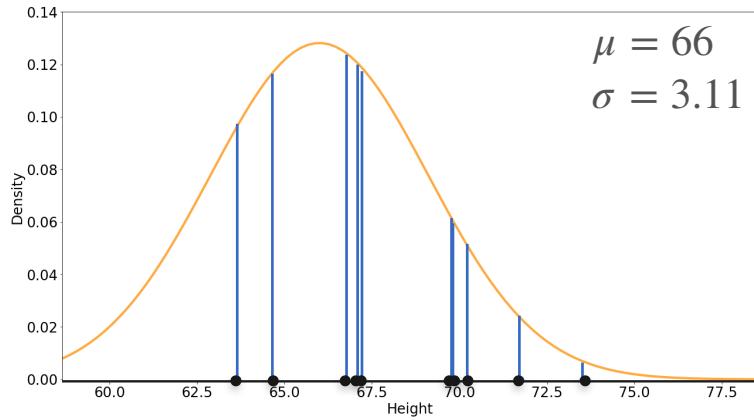
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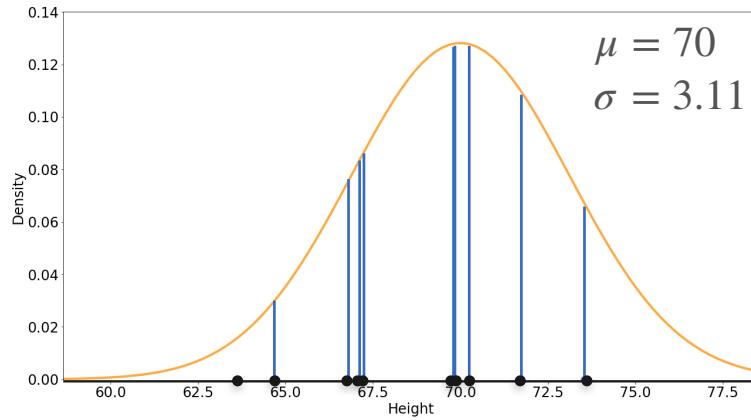
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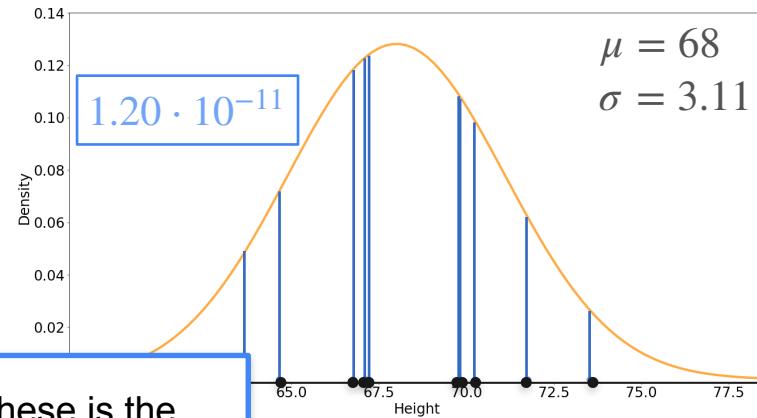
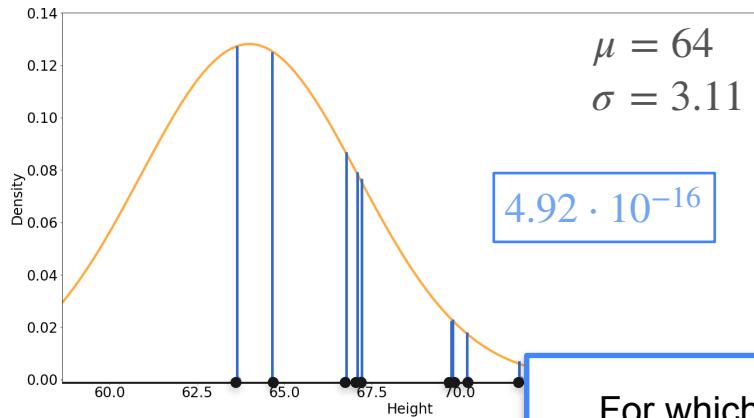
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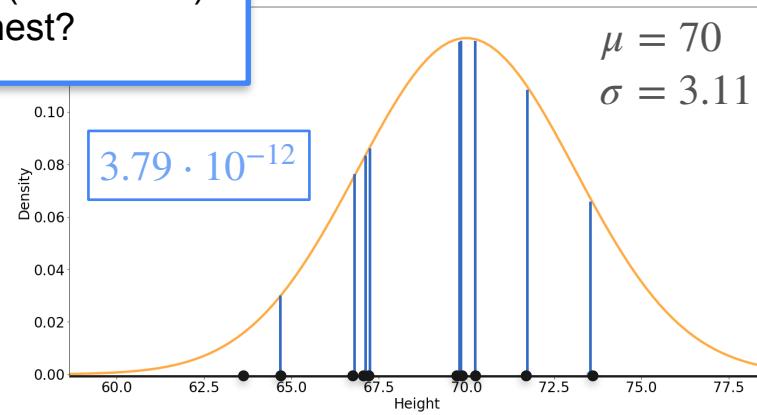
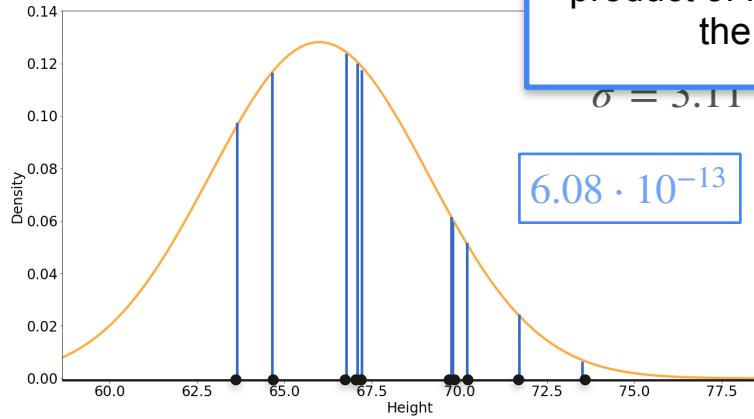
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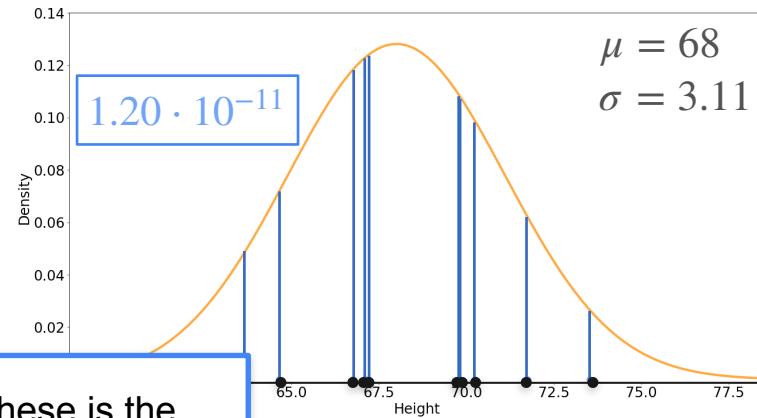
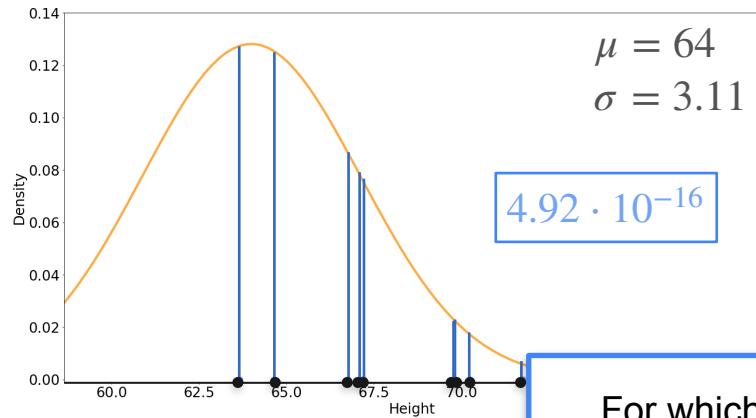


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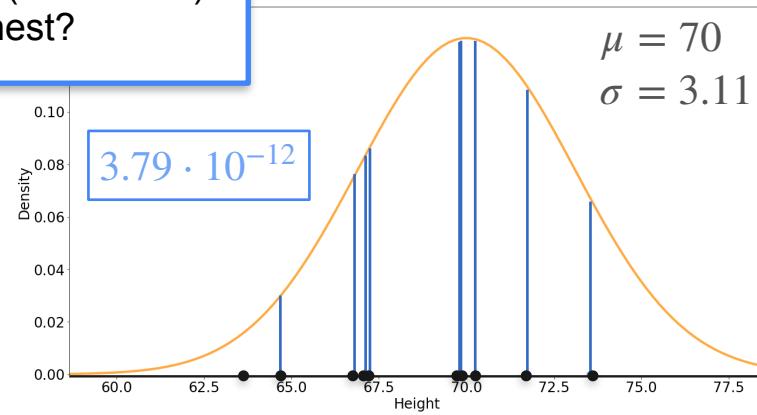
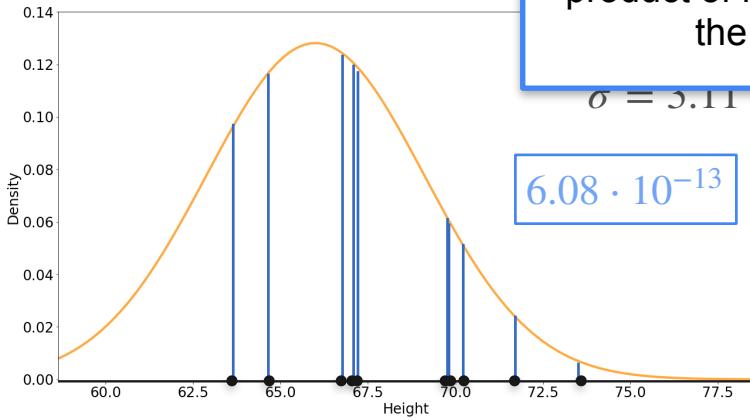


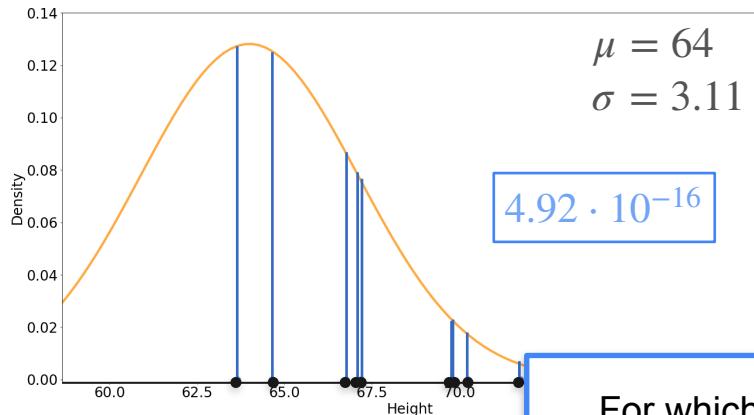
For which of these is the product of lines (likelihoods) the highest?





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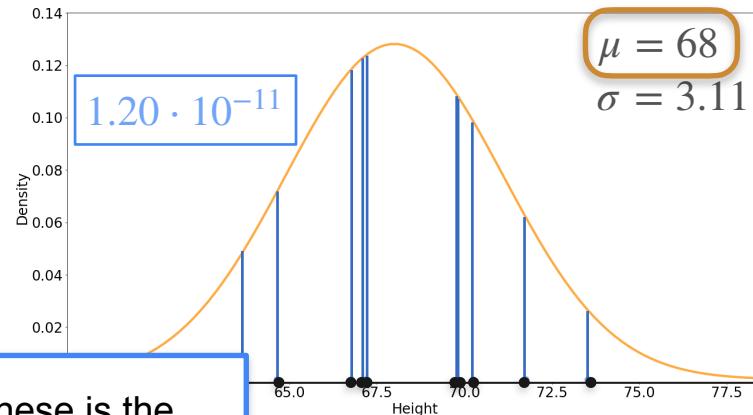




$$\mu = 64$$

$$\sigma = 3.11$$

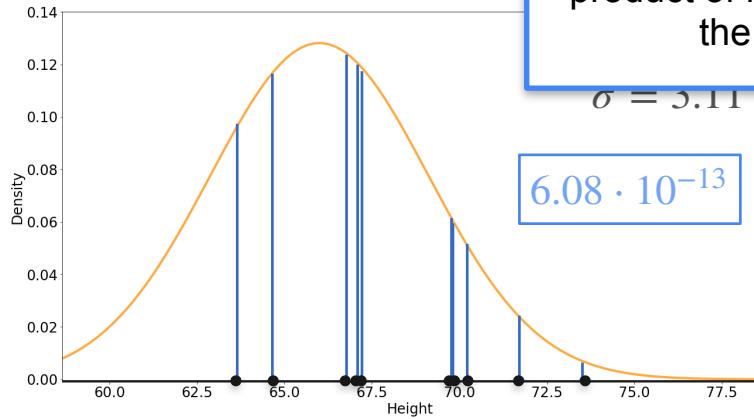
$$4.92 \cdot 10^{-16}$$



$$\mu = 68$$

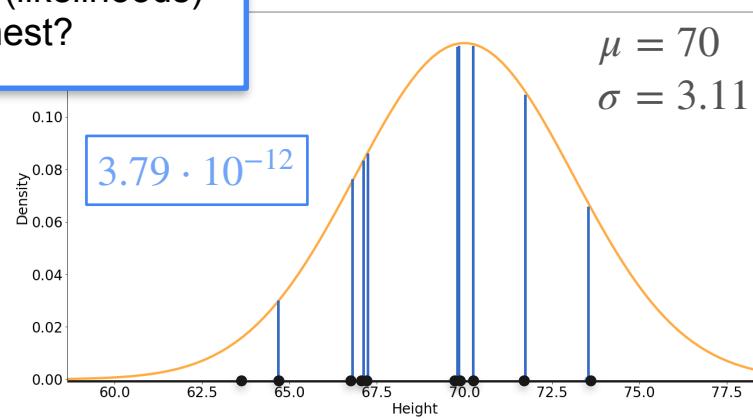
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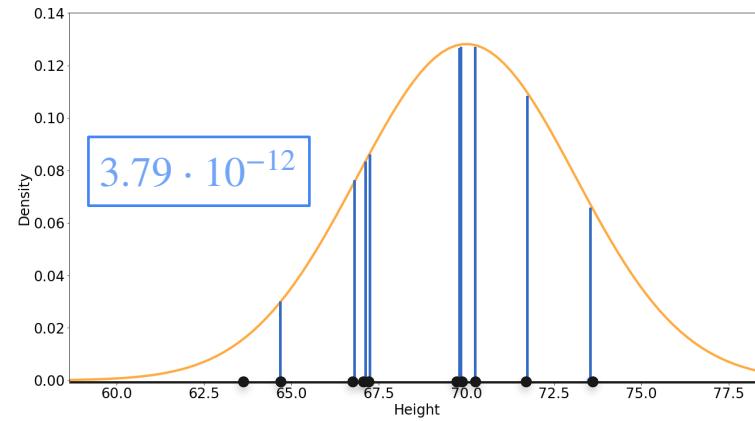
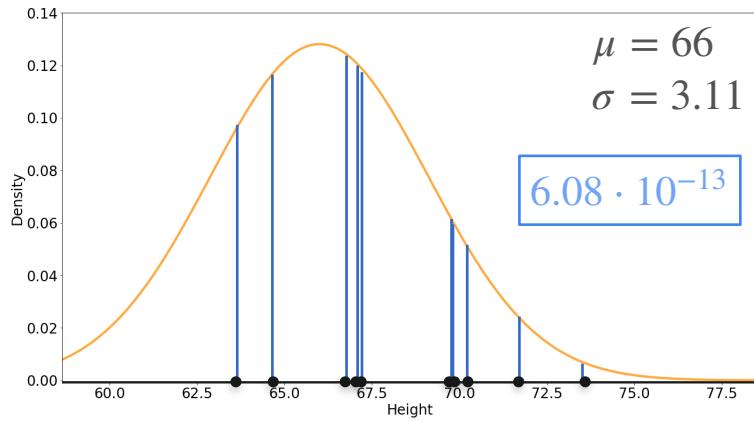
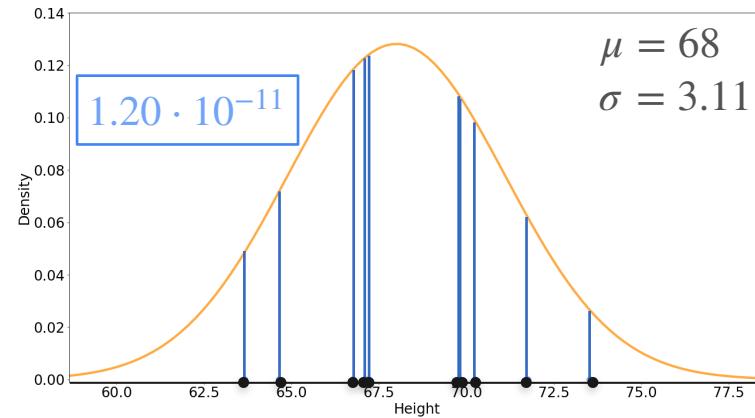
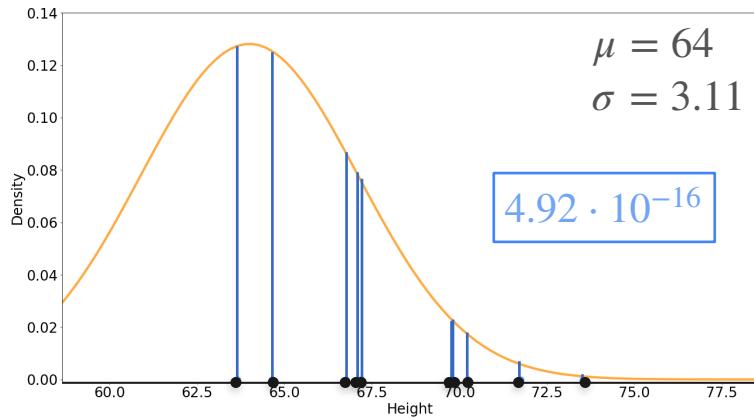
$$\sigma = 3.11$$

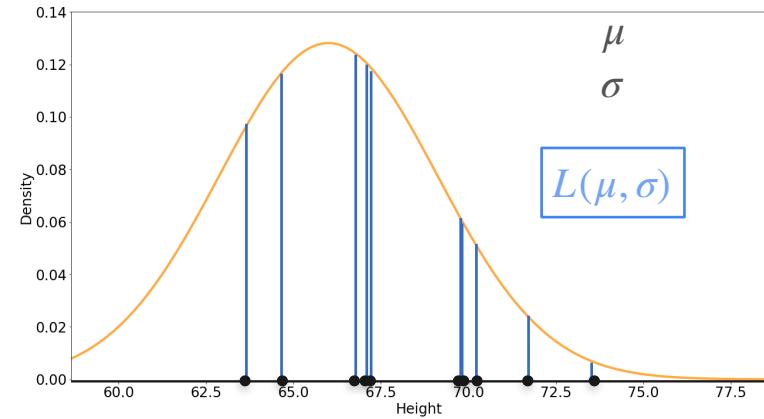
$$6.08 \cdot 10^{-13}$$



$$\mu = 70$$

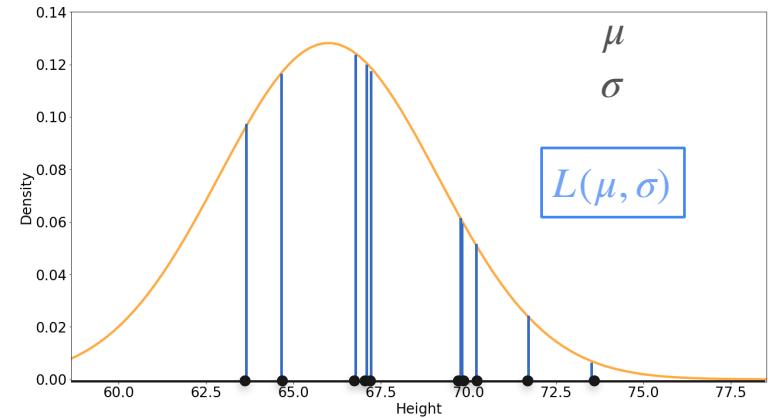
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$$\frac{d}{d\mu} \ell(\mu; \mathbf{x}) = \frac{d}{d\mu} \left[\log \left(\left(\frac{1}{\sqrt{2\pi}\sigma} \right)^{10} \right) - \frac{1}{2} \frac{\sum_{i=1}^{10} (x_i - \mu)^2}{\sigma^2} \right]$$

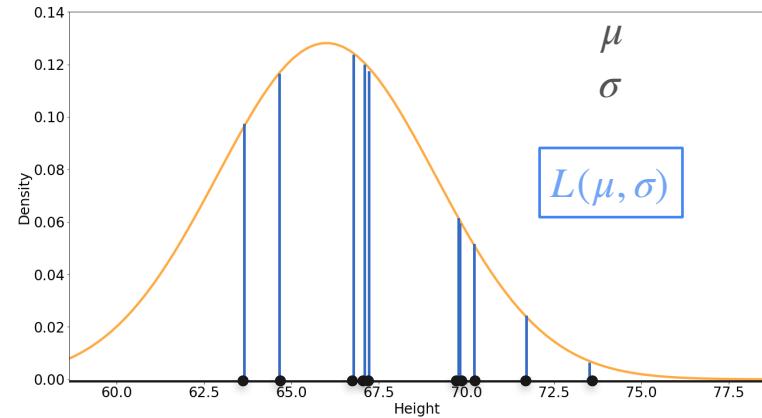
$$\text{Likelihood } L(\mu; \underline{x}) = \prod_{i=1}^{10} f_\mu(x_i)$$



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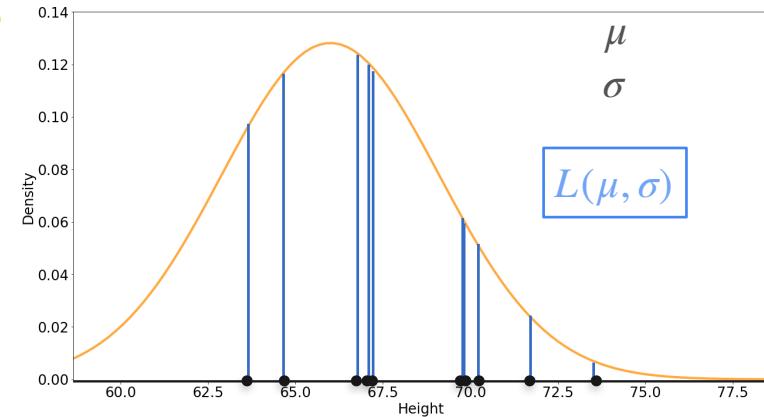


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Observations
 x_i are fixed!



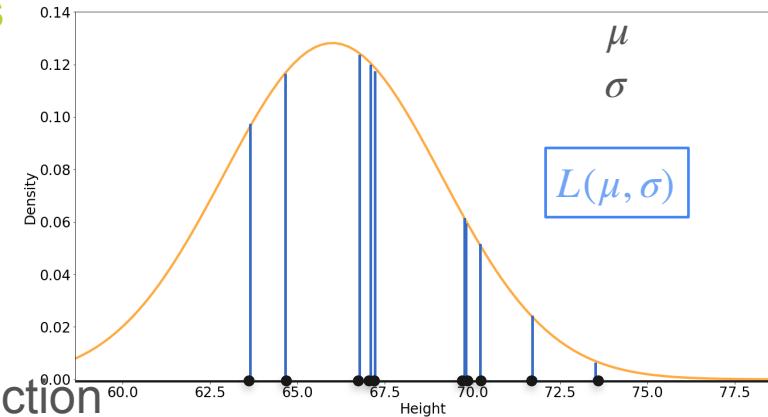
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It's a function
of μ

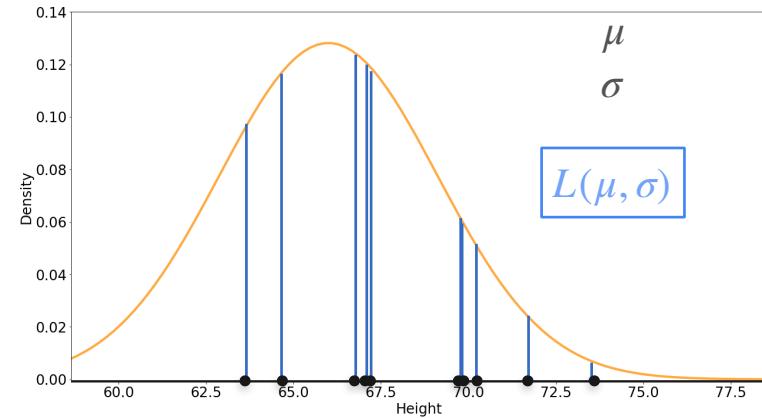


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$$\left(\frac{1}{\sqrt{2\pi} \sigma}\right)^{10}$$

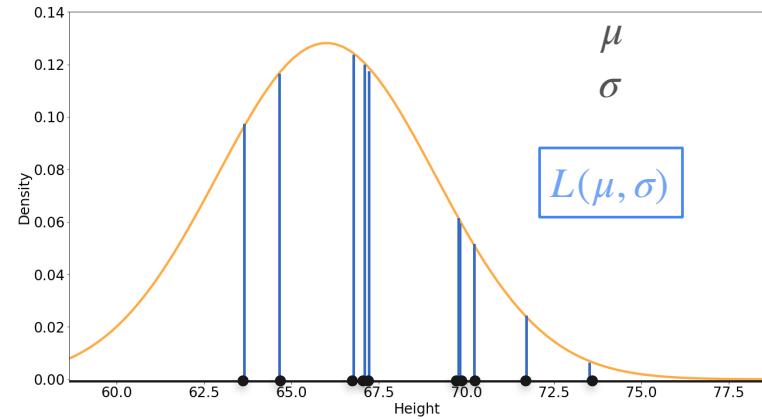


$$\frac{d}{d\mu} \ell(\mu; \mathbf{x}) = \frac{d}{d\mu} \left[\log \left(\left(\frac{1}{\sqrt{2\pi} \sigma} \right)^{10} \right) - \frac{1}{2} \frac{\sum_{i=1}^{10} (x_i - \mu)^2}{\sigma^2} \right]$$

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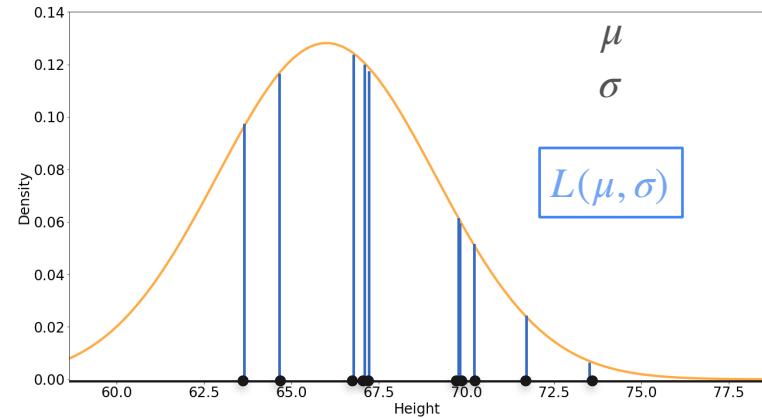


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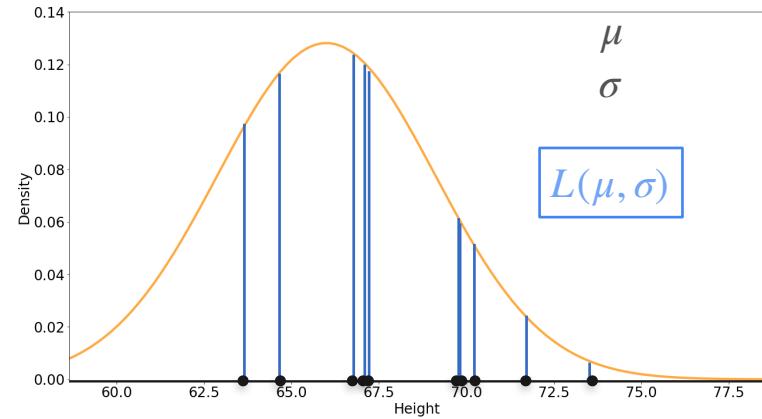


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$$\ell(\mu; \mathbf{x}) = \log(L(\mu; \mathbf{x}))$$

$$\frac{d}{d\mu} \ell(\mu; \mathbf{x}) = \frac{d}{d\mu} \left[\log\left(\left(\frac{1}{\sqrt{2\pi} \sigma}\right)^{10}\right) - \frac{1}{2} \frac{\sum_{i=1}^{10} (x_i - \mu)^2}{\sigma^2} \right]$$

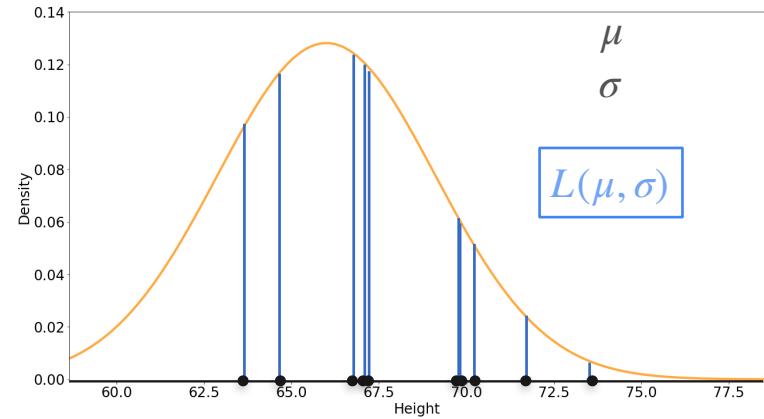
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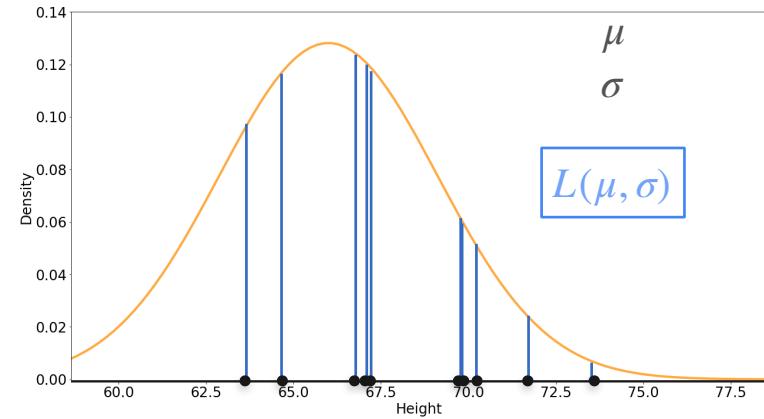
$$\ell(\mu; \mathbf{x}) = \log(L(\mu; \mathbf{x})) = \log\left(\left(\frac{1}{\sqrt{2\pi} \sigma}\right)^{10}\right) - \frac{1}{2} \frac{\sum_{i=1}^{10} (x_i - \mu)^2}{\sigma^2}$$

$$\frac{d}{d\mu} \ell(\mu; \mathbf{x}) = \frac{d}{d\mu} \left[\log\left(\left(\frac{1}{\sqrt{2\pi} \sigma}\right)^{10}\right) - \frac{1}{2} \frac{\sum_{i=1}^{10} (x_i - \mu)^2}{\sigma^2} \right]$$



$$\begin{aligned}
 \text{Likelihood } L(\mu; \underline{x}) &= \prod_{i=1}^{10} f_\mu(x_i) \\
 &= \prod_{i=1}^{10} \frac{1}{\sqrt{2\pi} \sigma} \exp\left(-\frac{1}{2} \frac{(x_i - \mu)^2}{\sigma^2}\right) \\
 &\quad \left(\frac{1}{\sqrt{2\pi} \sigma} \right)^{10} \exp\left(-\frac{1}{2} \frac{\sum_{i=1}^{10} (x_i - \mu)^2}{\sigma^2}\right)
 \end{aligned}$$

$$\begin{aligned}
 \text{Log-Likelihood } \ell(\mu; \mathbf{x}) &= \log(L(\mu; \mathbf{x})) = \log\left(\left(\frac{1}{\sqrt{2\pi} \sigma}\right)^{10}\right) - \frac{1}{2} \frac{\sum_{i=1}^{10} (x_i - \mu)^2}{\sigma^2} \\
 \frac{d}{d\mu} \ell(\mu; \mathbf{x}) &= \frac{d}{d\mu} \left[\log\left(\left(\frac{1}{\sqrt{2\pi} \sigma}\right)^{10}\right) - \frac{1}{2} \frac{\sum_{i=1}^{10} (x_i - \mu)^2}{\sigma^2} \right]
 \end{aligned}$$

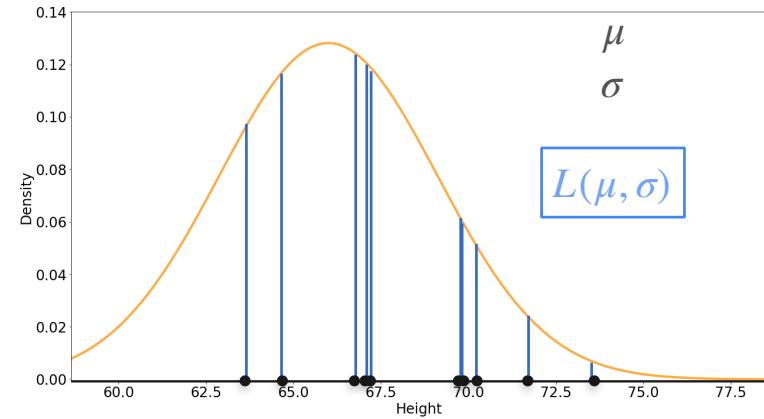


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Derivative

$$\frac{d}{d\mu} \ell(\mu; \mathbf{x}) = \frac{d}{d\mu} \left[\log\left(\left(\frac{1}{\sqrt{2\pi} \sigma}\right)^{10}\right) - \frac{1}{2} \frac{\sum_{i=1}^{10} (x_i - \mu)^2}{\sigma^2} \right]$$



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This needs to be zero

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$= \quad \quad \quad 0$

$$-\frac{1}{2} \frac{1}{\sigma^2} \sum_{i=1}^{10} 2(x_i - \mu)(-1)$$

$$\sum_{i=1}^{10} (x_i - \mu) = 0$$

$$\left(\sum_{i=1}^{10} x_i \right) - 10\mu = 0$$

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Log-Likelihood $\ell(\mu; \mathbf{x}) = \log(L(\mu; \mathbf{x})) = \log\left(\left(\frac{1}{\sqrt{2\pi}\sigma}\right)^{10}\right) - \frac{1}{2} \frac{\sum_{i=1}^{10} (x_i - \mu)^2}{\sigma^2}$

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Derivative

$$\frac{d}{d\mu} \ell(\mu; \mathbf{x}) = \frac{d}{d\mu} \left[\log\left(\left(\frac{1}{\sqrt{2\pi}\sigma}\right)^n\right) - \frac{1}{2} \frac{\sum_{i=1}^n (x_i - \mu)^2}{\sigma^2} \right] = 0$$

$$= 0$$

$$\boxed{-\frac{1}{2} \frac{1}{\sigma^2} \sum_{i=1}^n 2(x_i - \mu)(-\cancel{1})}$$

$$\sum_{i=1}^n (x_i - \mu) = 0$$

$$\left(\sum_{i=1}^n x_i \right) - n \mu = 0 \quad \rightarrow \quad \mu = \frac{1}{n} \sum_{i=1}^n x_i$$

This needs to be zero

The best distribution is the one where the **mean** of the distribution is the **mean** of the sample

Maximum Likelihood: Gaussian Example

Maximum Likelihood: Gaussian Example

Heights	66.75	70.24	67.19	67.09	63.65	64.64	69.81	69.79	73.52	71.74
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Maximum Likelihood: Gaussian Example

Heights	66.75	70.24	67.19	67.09	63.65	64.64	69.81	69.79	73.52	71.74
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$$\hat{\mu} = \frac{66.75 + 70.24 + 67.19 + 67.09 + 63.65 + 64.64 + 69.81 + 69.79 + 73.52 + 71.74}{10}$$

Maximum Likelihood: Gaussian Example

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---------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------

$$\hat{\mu} = \frac{66.75 + 70.24 + 67.19 + 67.09 + 63.65 + 64.64 + 69.81 + 69.79 + 73.52 + 71.74}{10}$$
$$= 68.442$$

Maximum Likelihood: Gaussian Example

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---------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------

Maximum Likelihood: Gaussian Example

What about the variance?

Heights	66.75	70.24	67.19	67.09	63.65	64.64	69.81	69.79	73.52	71.74
---------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------

Maximum Likelihood: Gaussian Example

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If you repeat the process, assuming unknown variance, you'll get that

Maximum Likelihood: Gaussian Example

What about the variance?

Heights	66.75	70.24	67.19	67.09	63.65	64.64	69.81	69.79	73.52	71.74
---------	-------	-------	-------	-------	-------	-------	-------	-------	-------	-------

If you repeat the process, assuming unknown variance, you'll get that

$$\hat{\sigma}^2 = \frac{\sum_{i=1}^{10} (x_i - \bar{x})^2}{10}$$

Maximum Likelihood: Gaussian Example

What about the variance?

Heights	66.75	70.24	67.19	67.09	63.65	64.64	69.81	69.79	73.52	71.74
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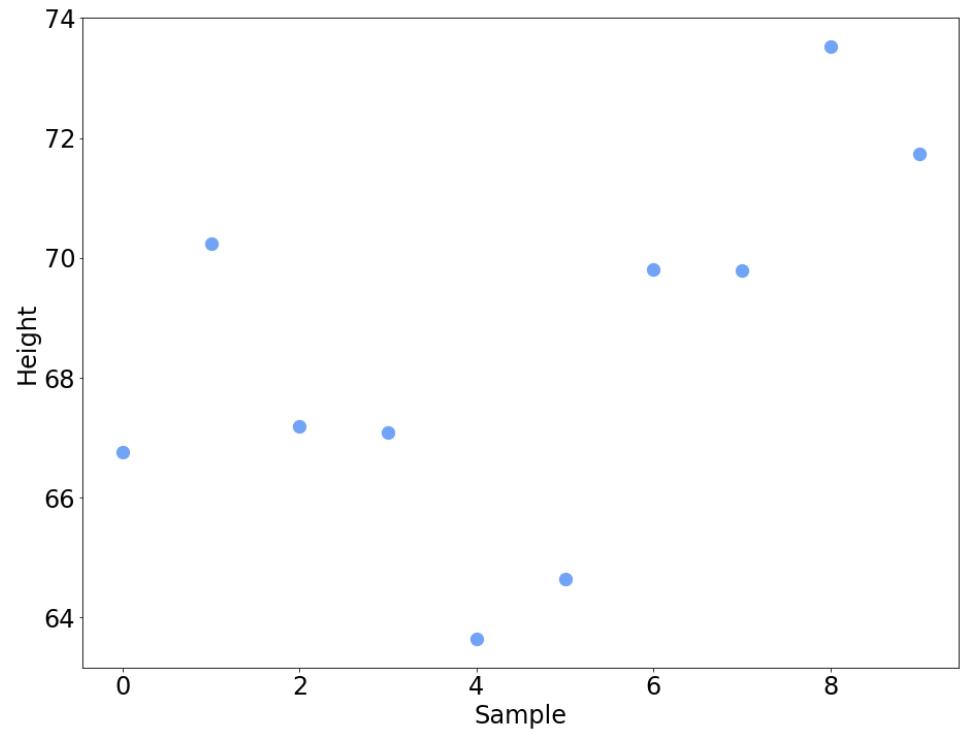
If you repeat the process, assuming unknown variance, you'll get that

$$\hat{\sigma}^2 = \frac{\sum_{i=1}^{10} (x_i - \bar{x})^2}{10} = \frac{1}{10} \left((66.75 - 68.442)^2 + (70.24 - 68.442)^2 + (67.19 - 68.442)^2 + (67.09 - 68.442)^2 + (63.65 - 68.442)^2 + (64.64 - 68.442)^2 + (69.81 - 68.442)^2 + (69.79 - 68.442)^2 + (73.52 - 68.442)^2 + (71.74 - 68.442)^2 \right) = 8.72$$

Maximum Likelihood: Gaussian Example

X = "Height of an 18 year old"

$\mathcal{N}(\mu, \sigma)$

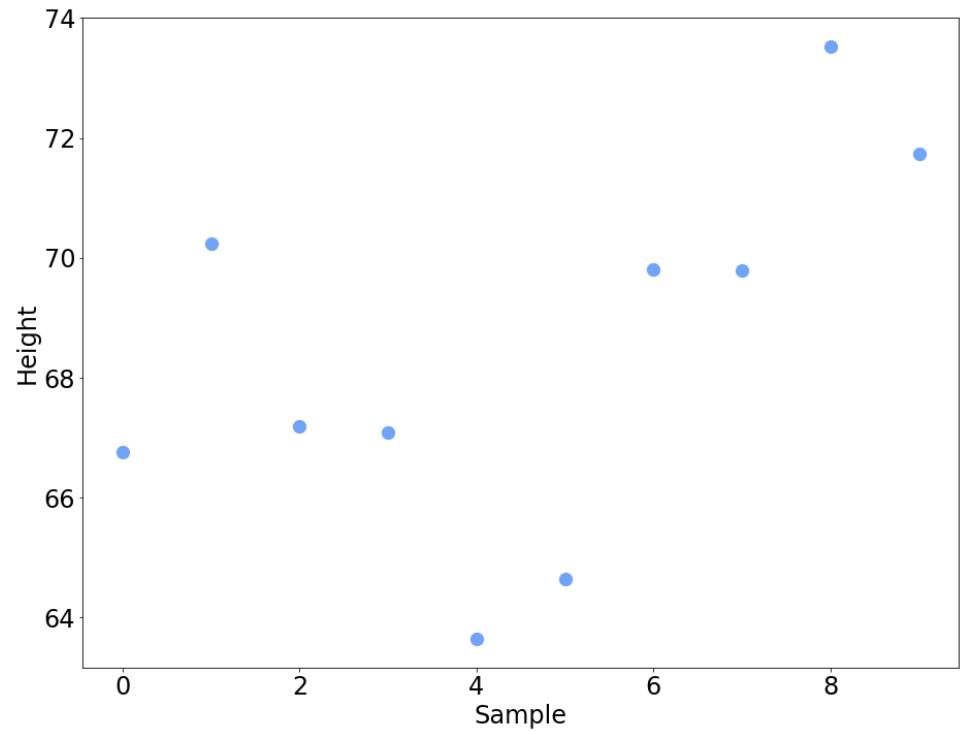


Maximum Likelihood: Gaussian Example

X = "Height of an 18 year old"

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$\hat{\mu} = 68.442$



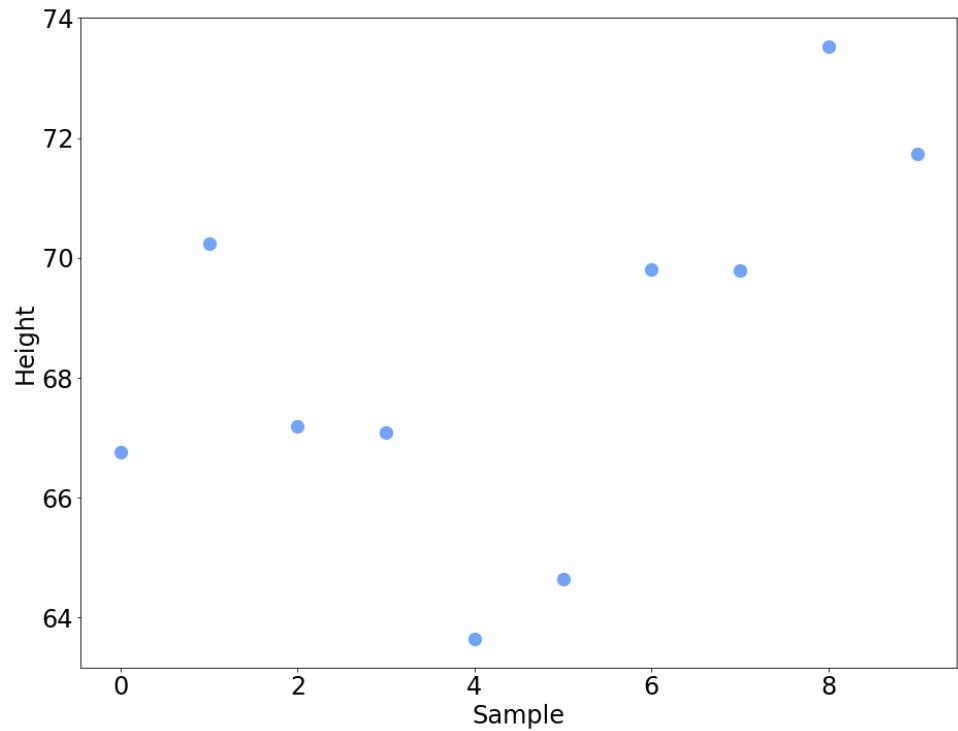
Maximum Likelihood: Gaussian Example

X = "Height of an 18 year old"

$$\mathcal{N}(\mu, \sigma)$$

$$\hat{\mu} = 68.442$$

$$\hat{\sigma} = 8.72$$





DeepLearning.AI

Point Estimation

MLE: Linear Regression

Maximum Likelihood

Maximum Likelihood



Data

Maximum Likelihood



Model 1



Data

Maximum Likelihood



Model 1



Model 2



Data

Maximum Likelihood



Model 1



Model 2

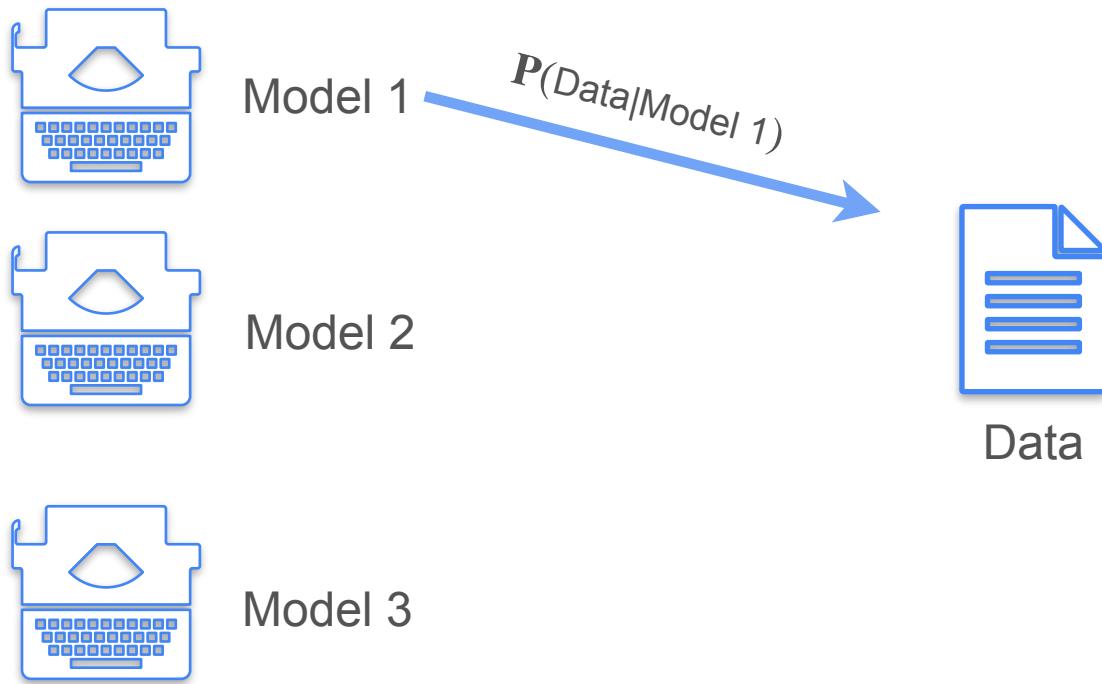


Model 3

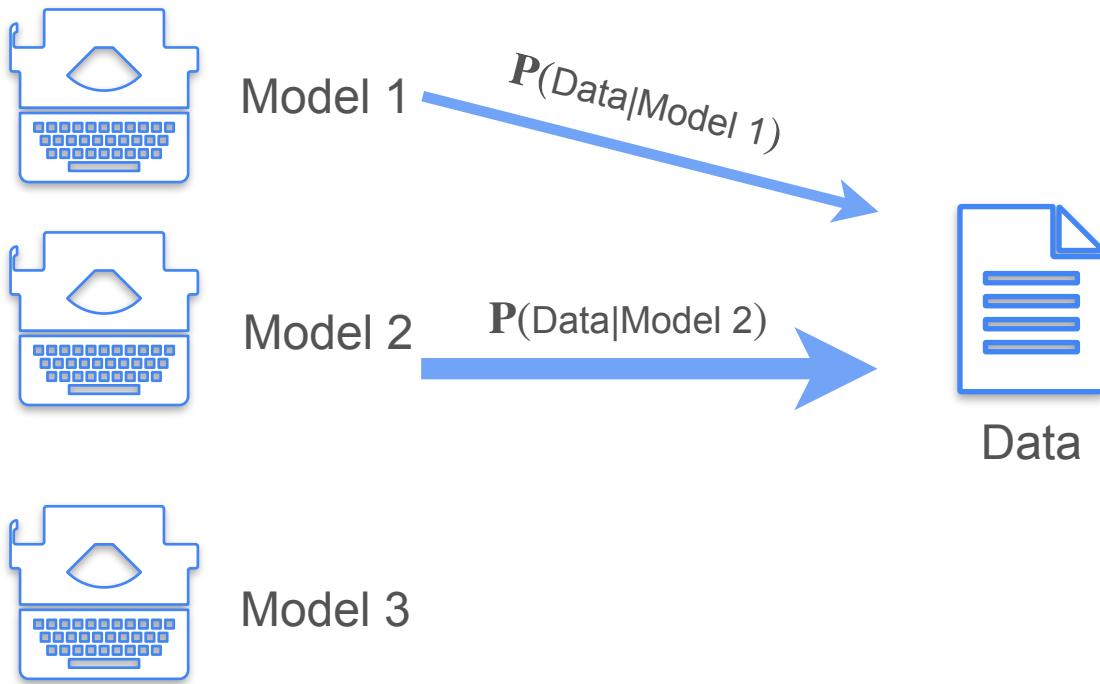


Data

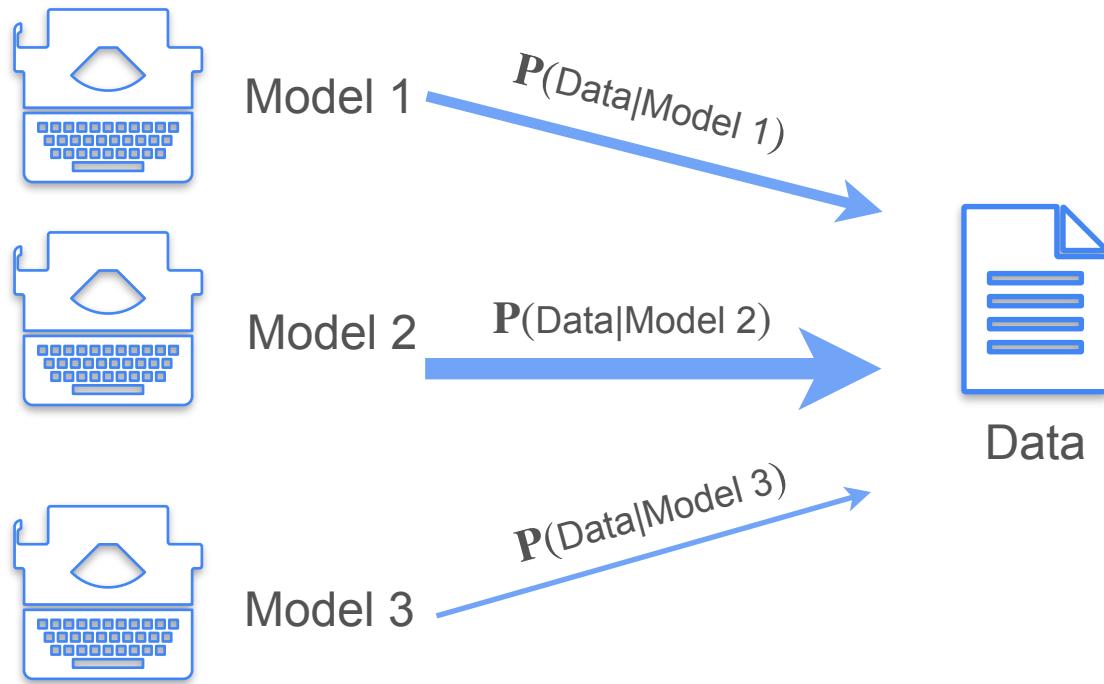
Maximum Likelihood



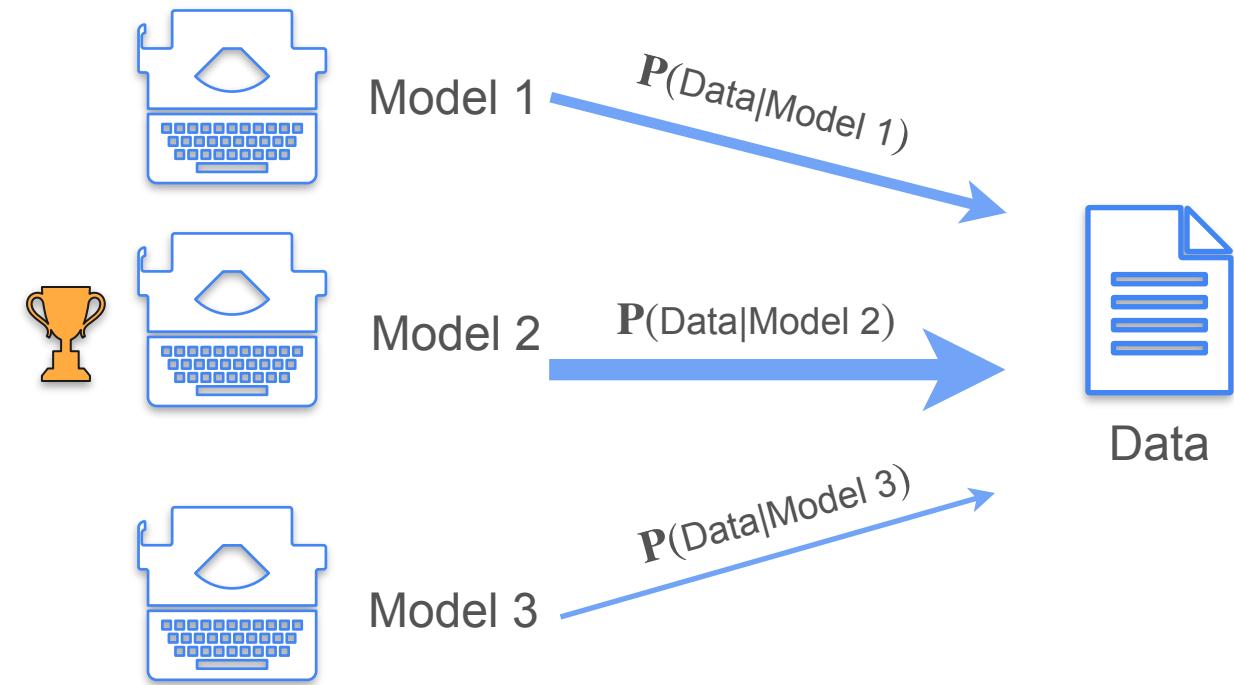
Maximum Likelihood



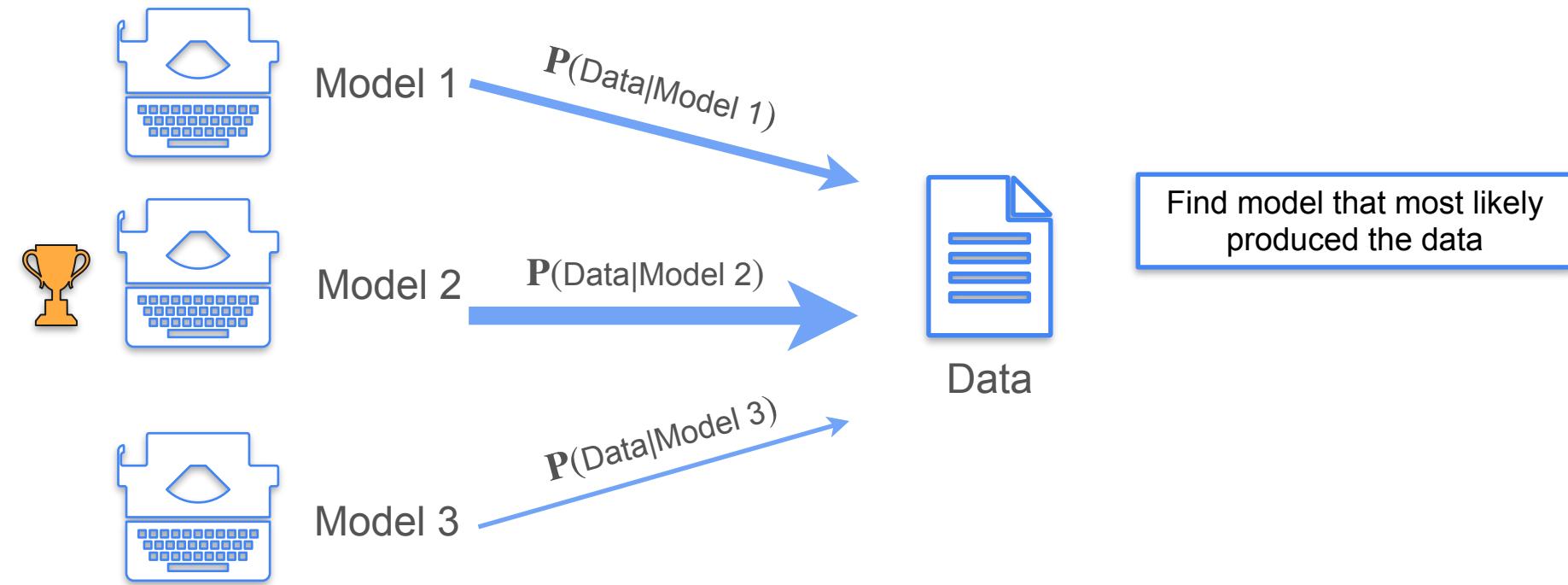
Maximum Likelihood



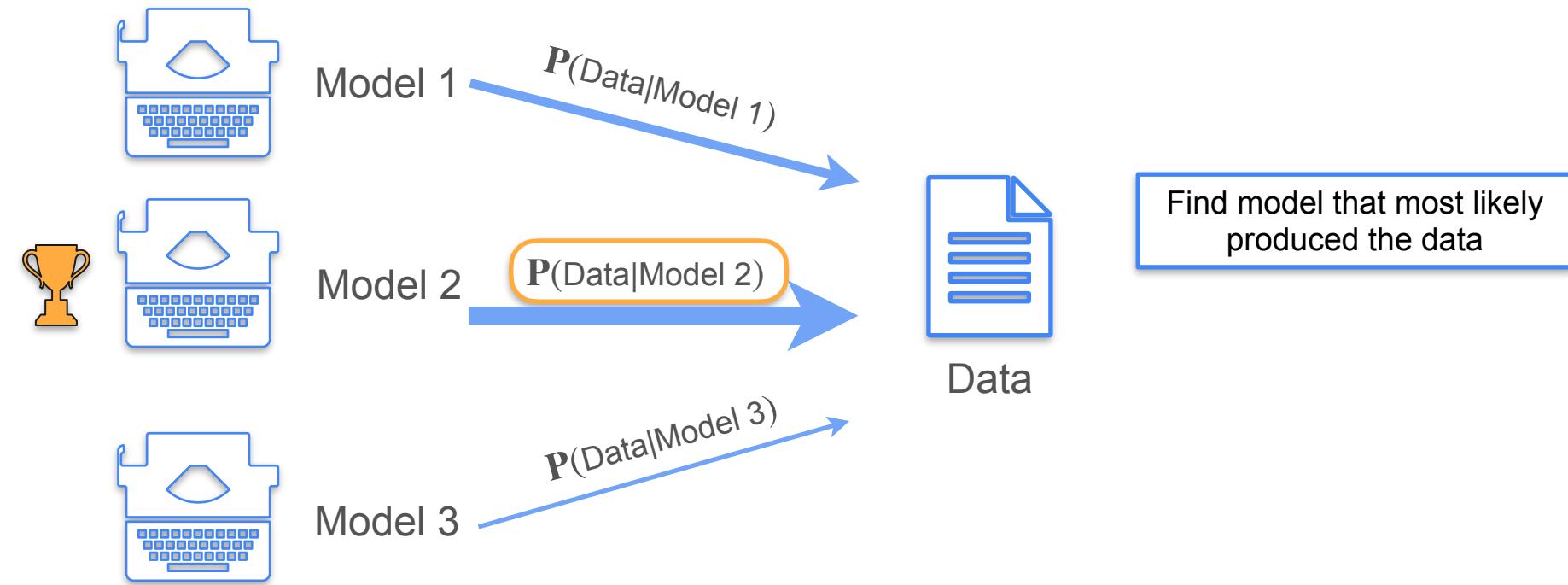
Maximum Likelihood



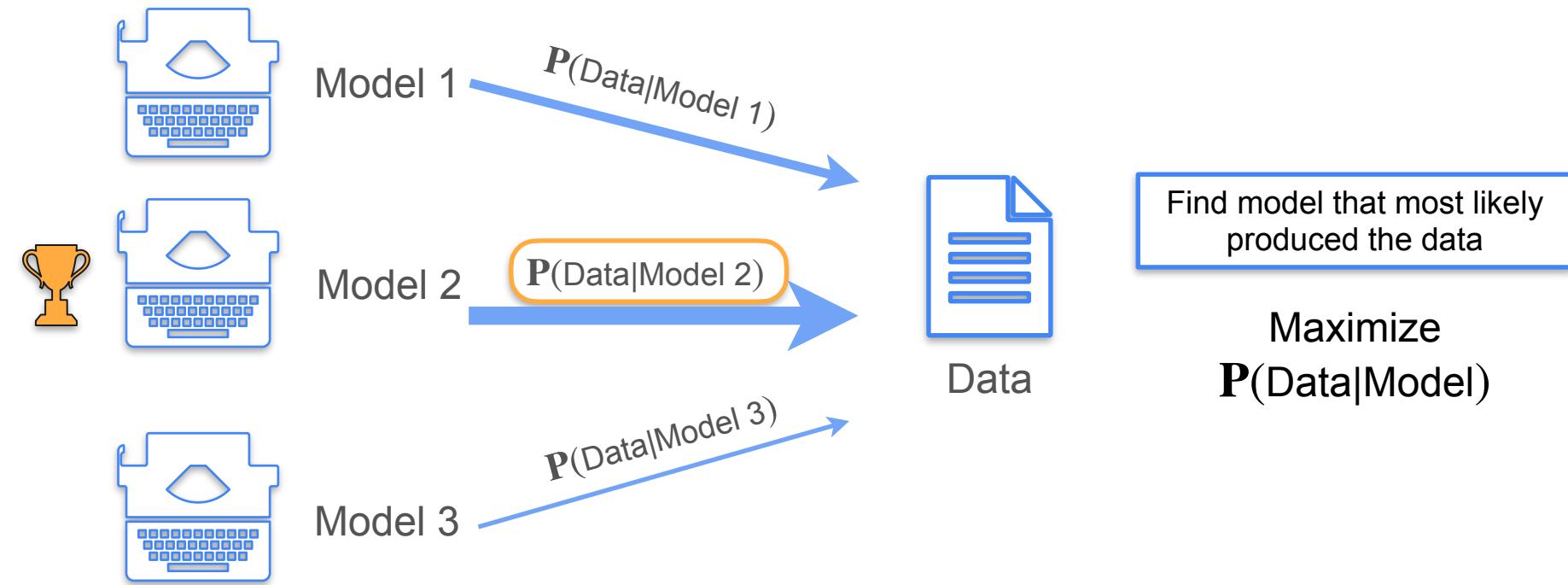
Maximum Likelihood



Maximum Likelihood

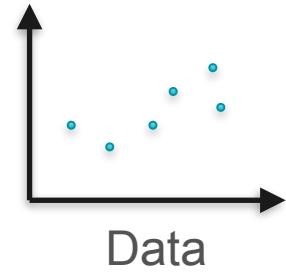


Maximum Likelihood

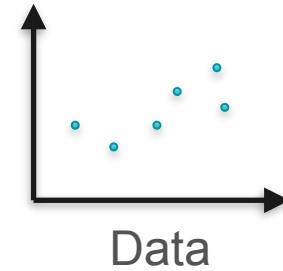
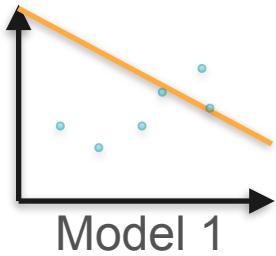


Example: Linear Regression

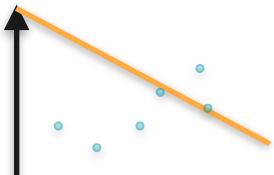
Example: Linear Regression



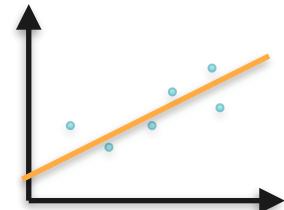
Example: Linear Regression



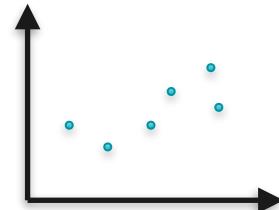
Example: Linear Regression



Model 1

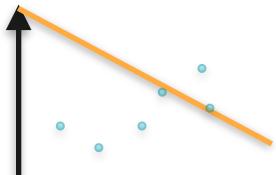


Model 2

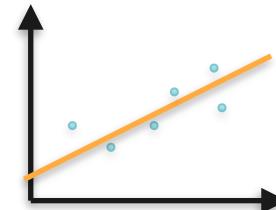


Data

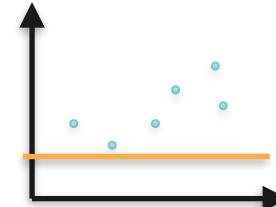
Example: Linear Regression



Model 1



Model 2

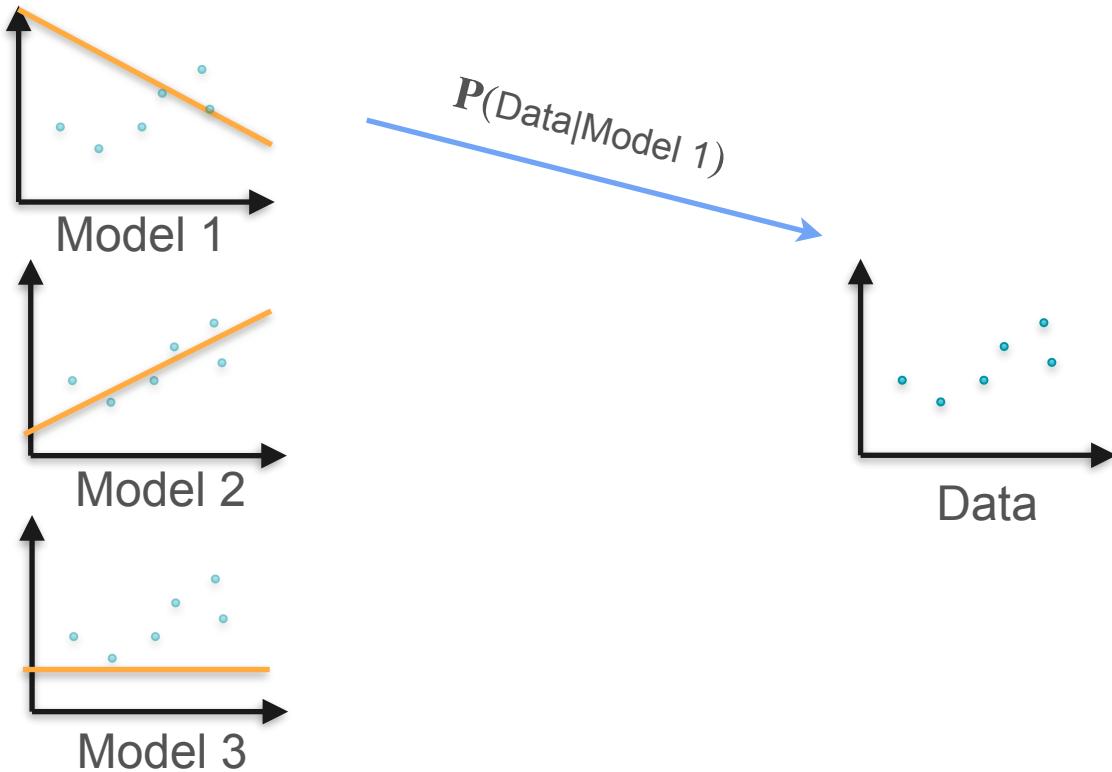


Model 3

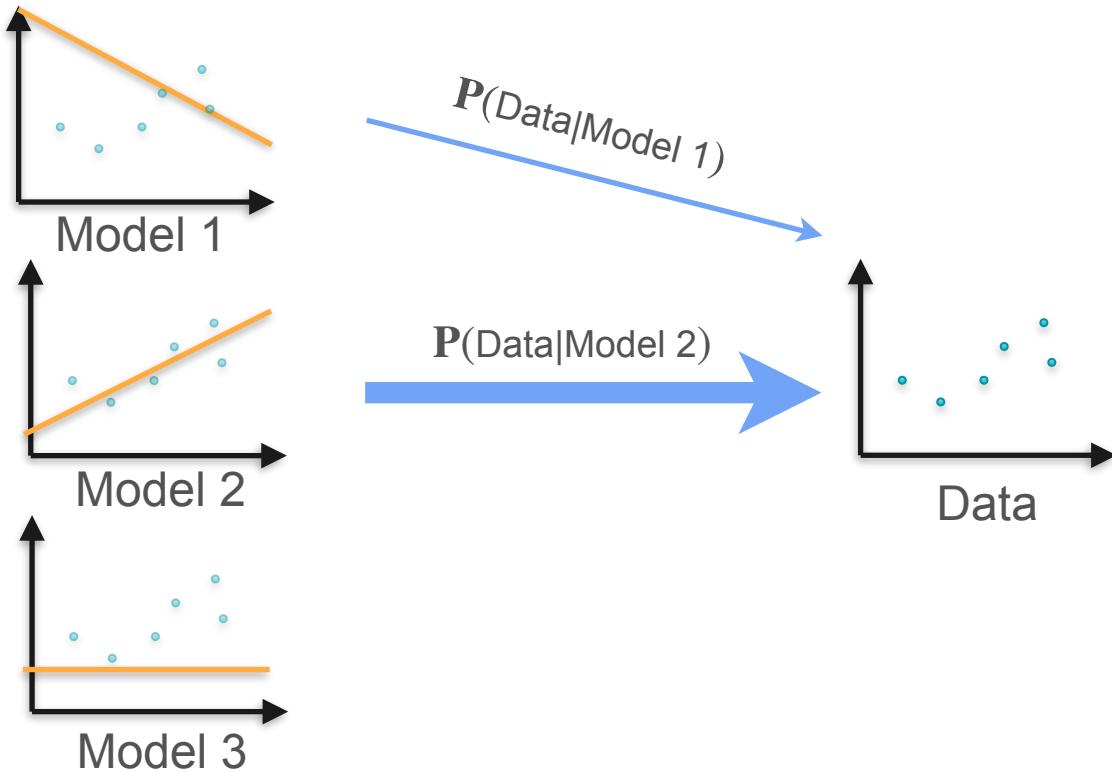


Data

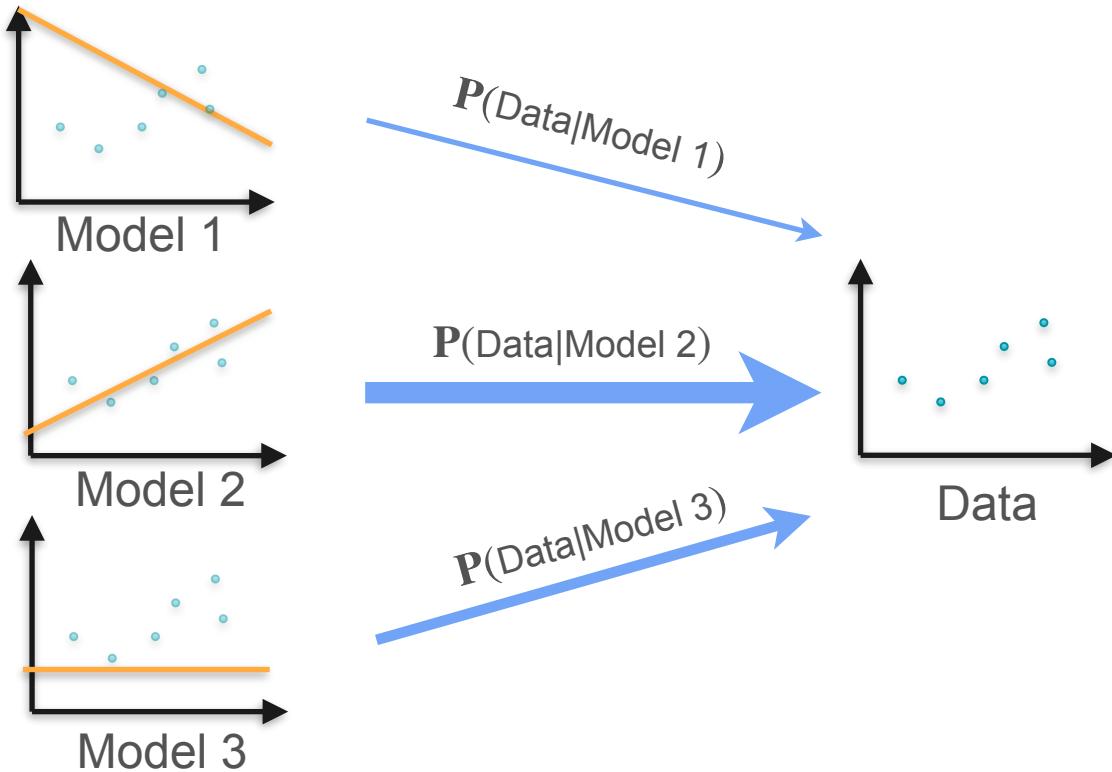
Example: Linear Regression



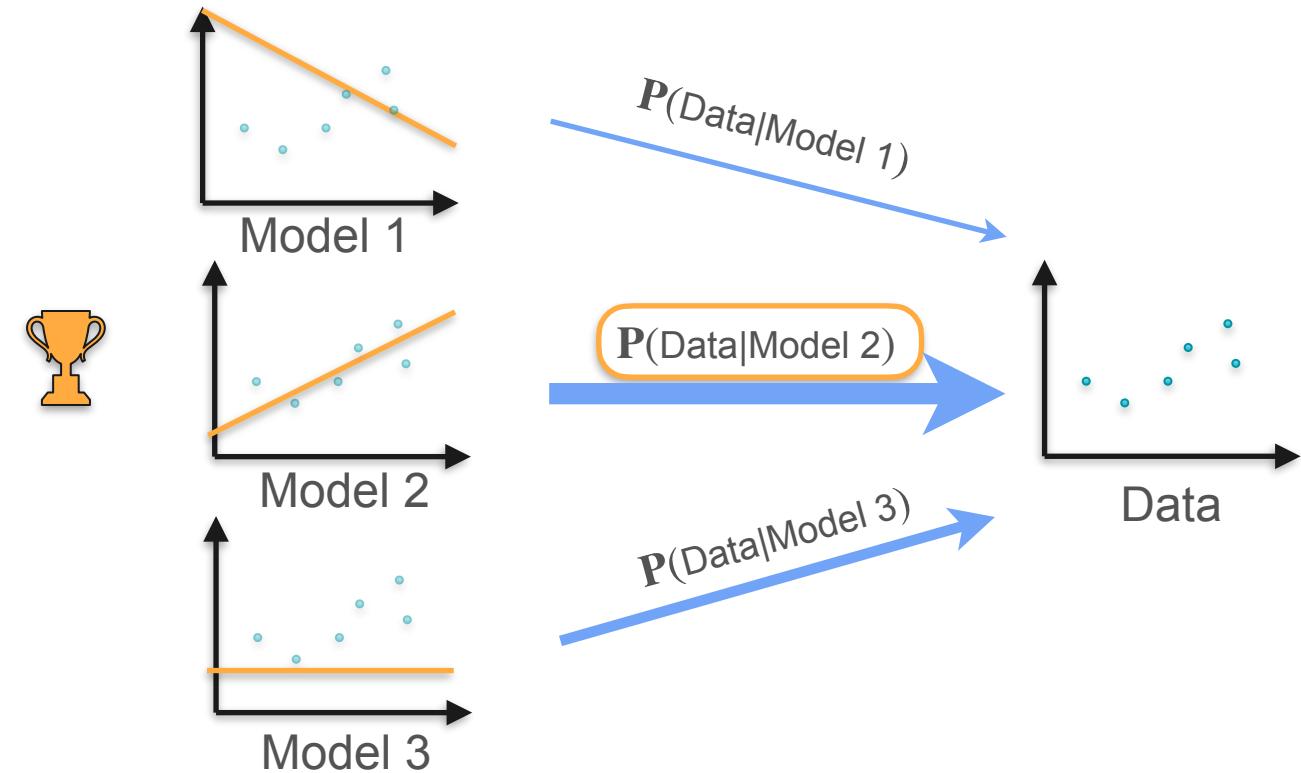
Example: Linear Regression



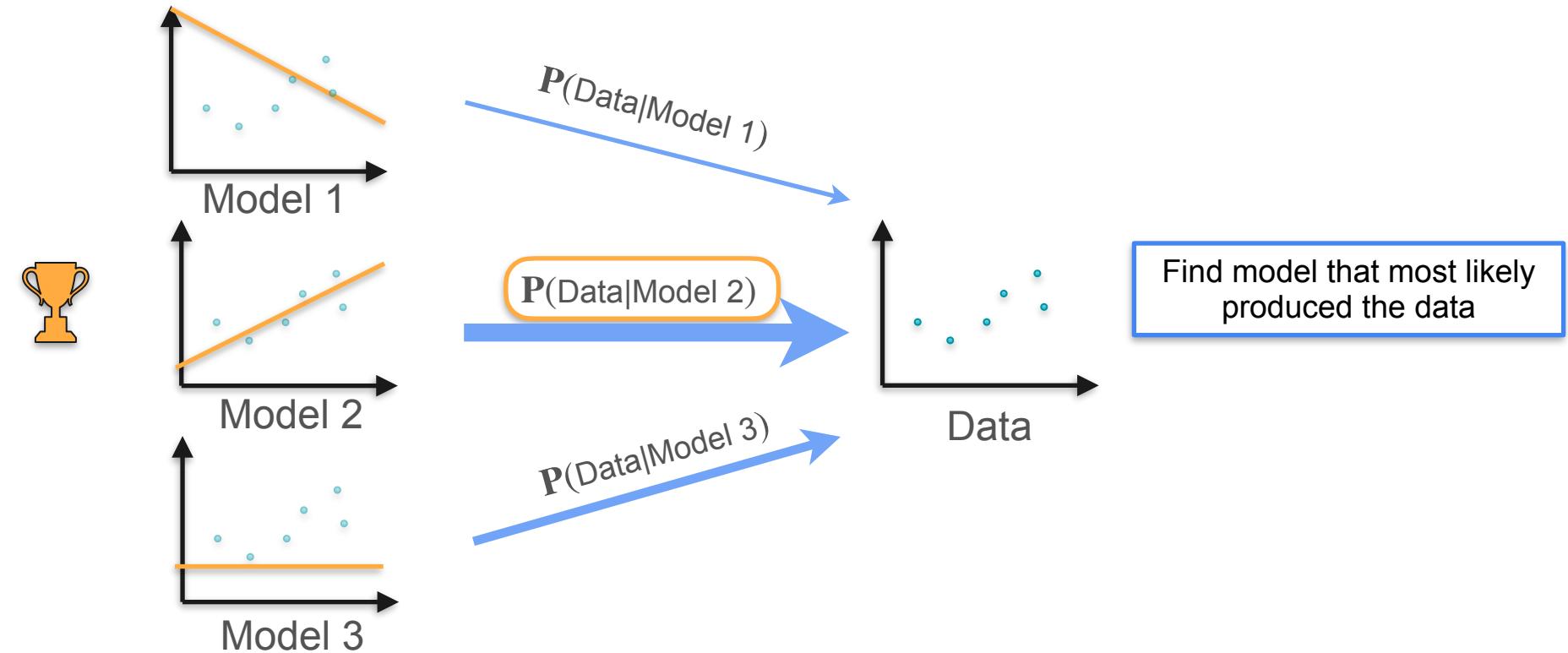
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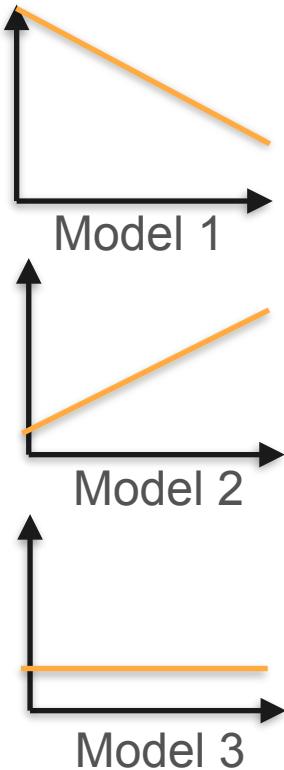
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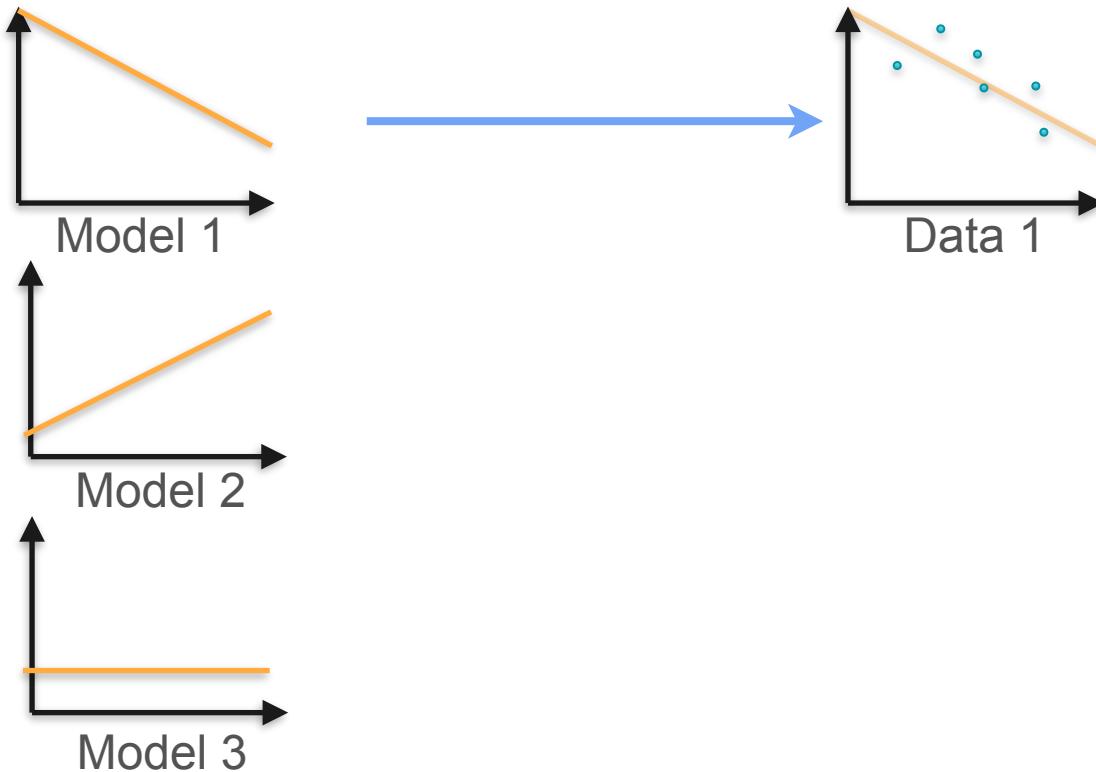
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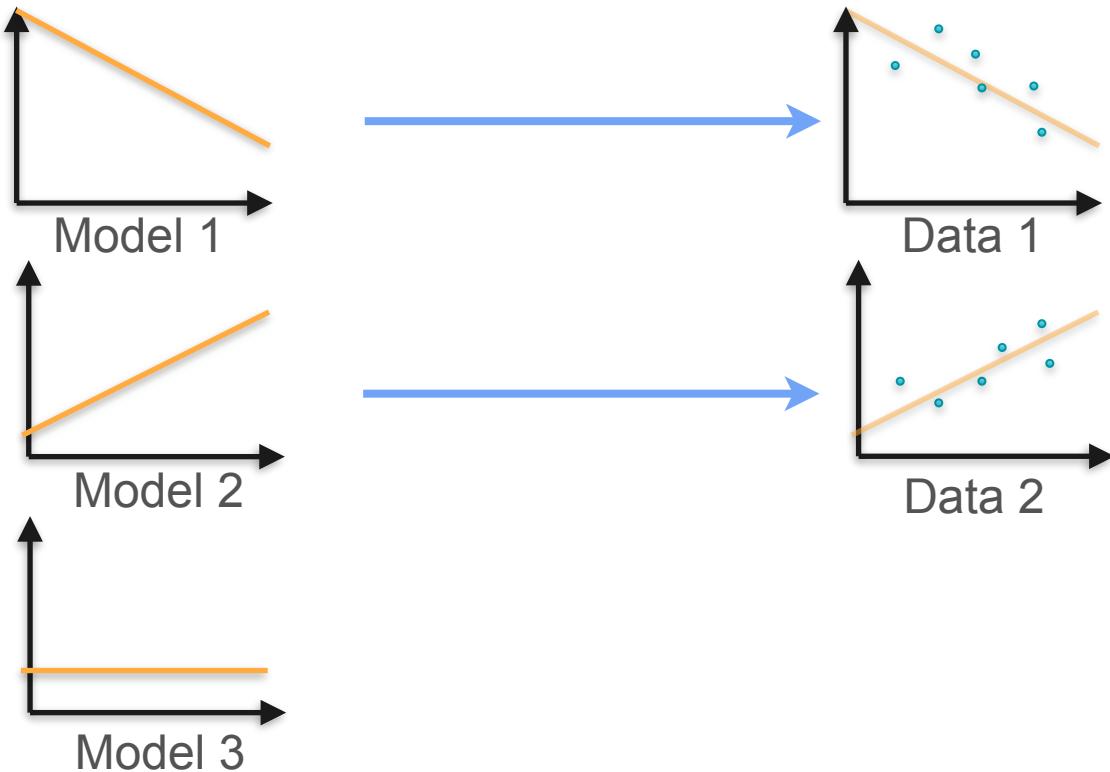
How Exactly Does a Line Produce Points?



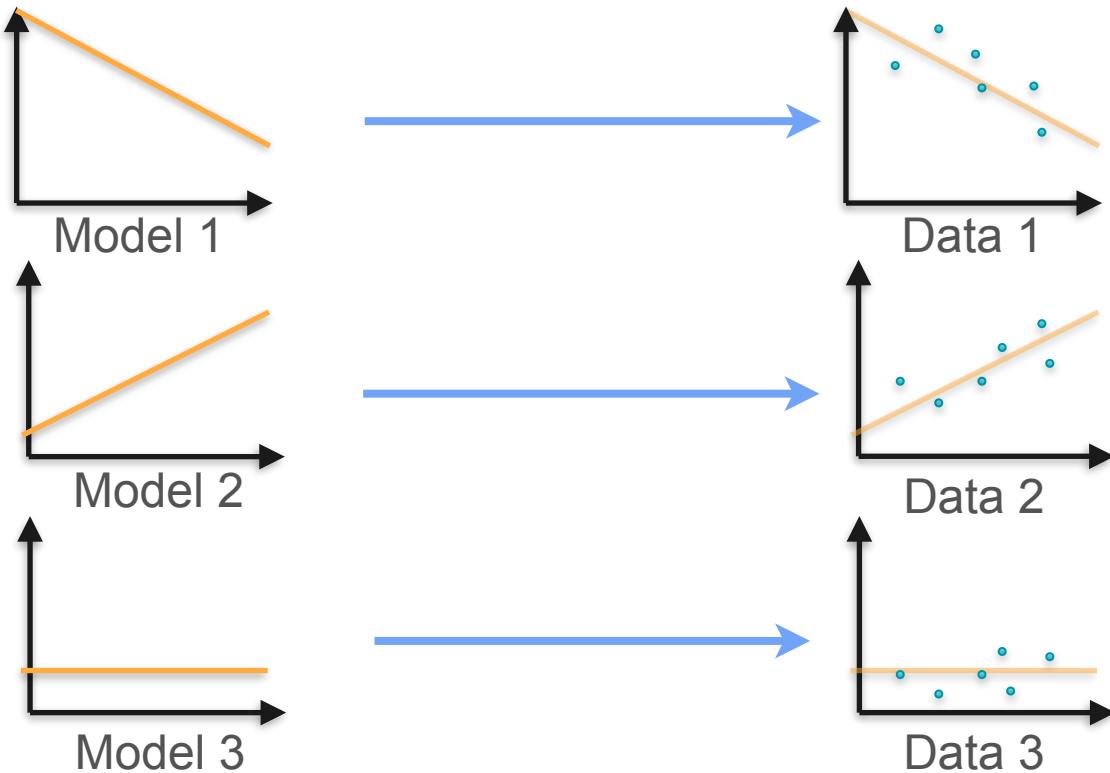
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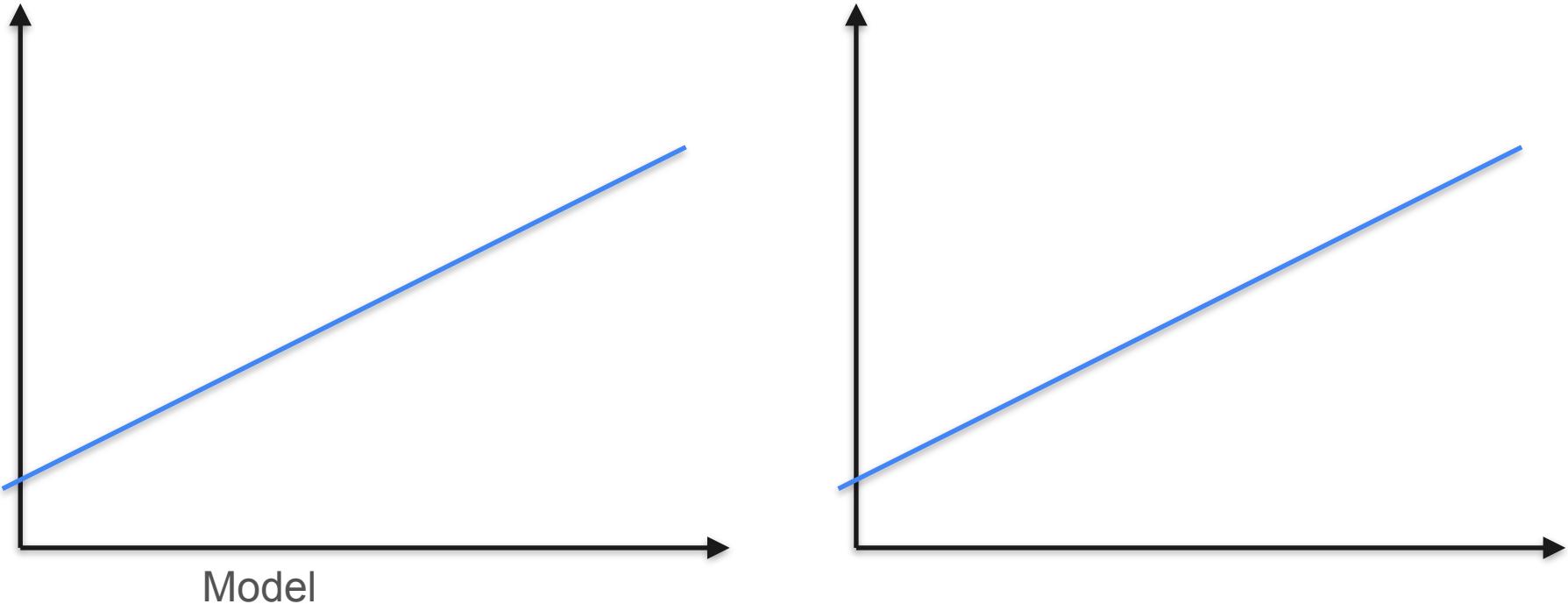
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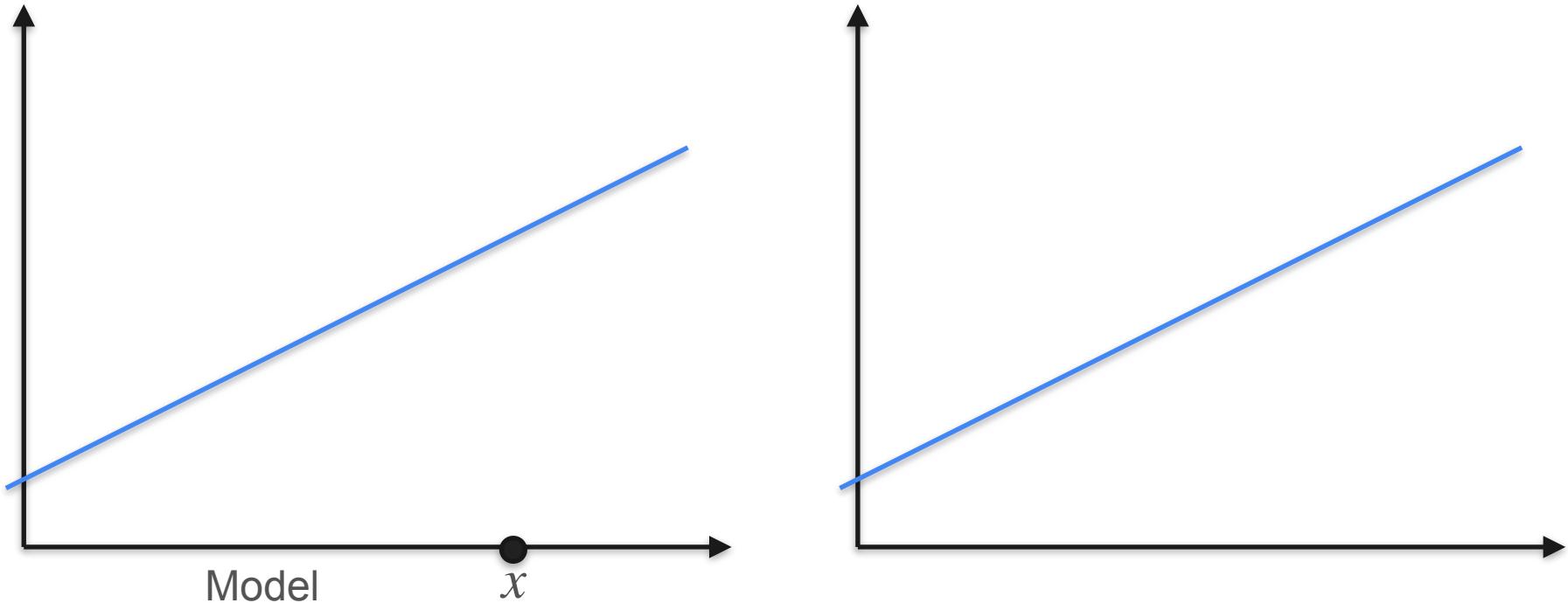
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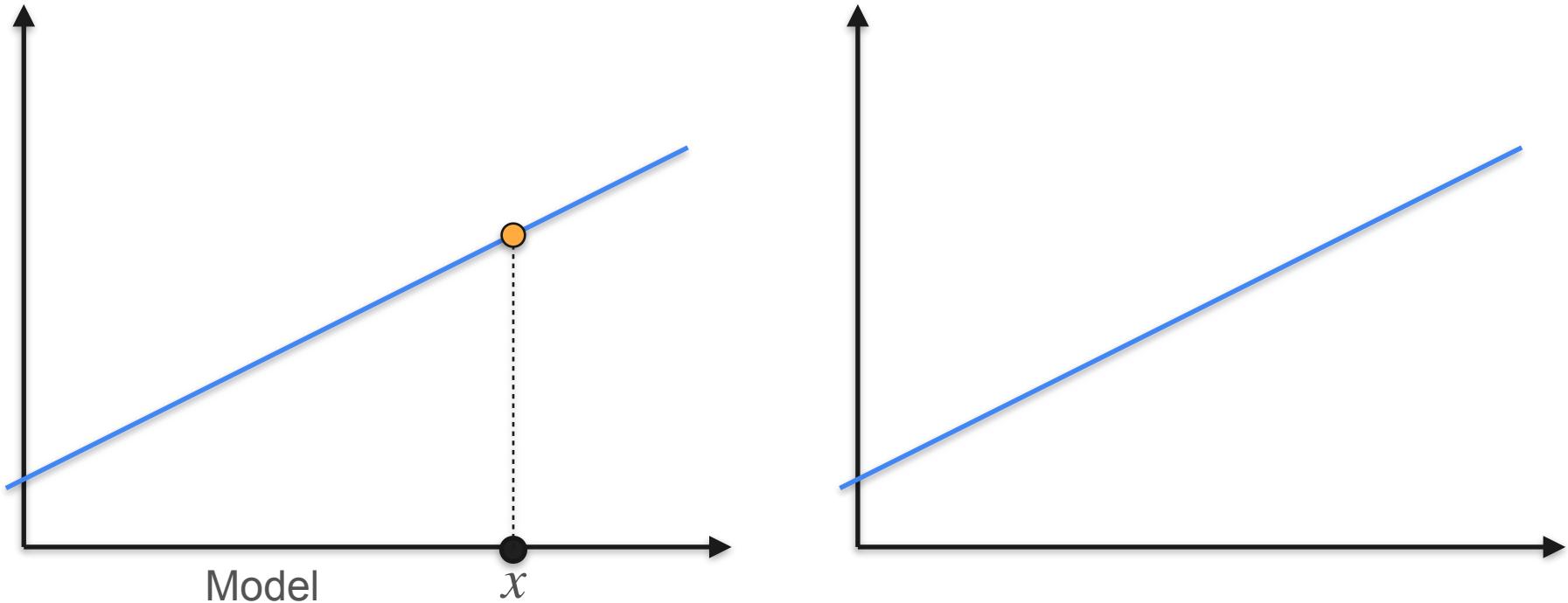
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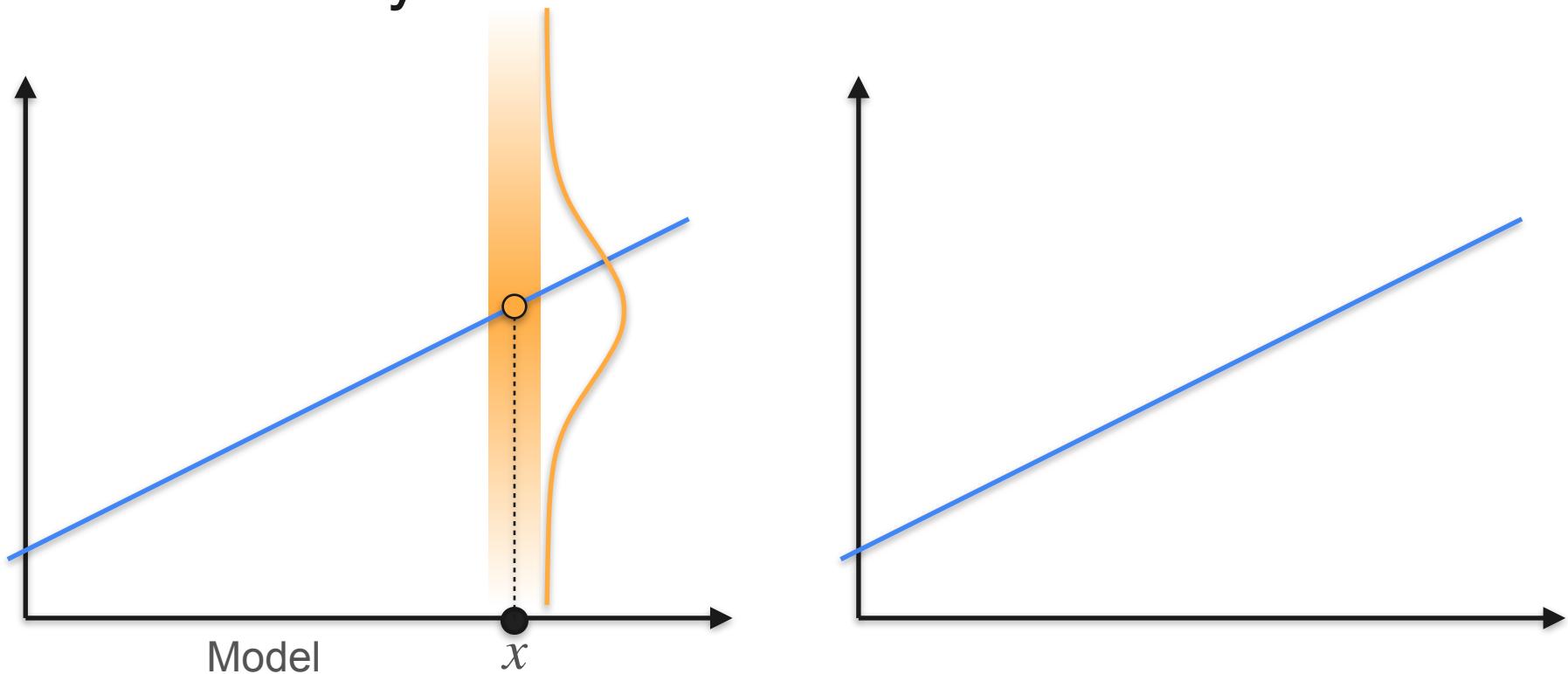
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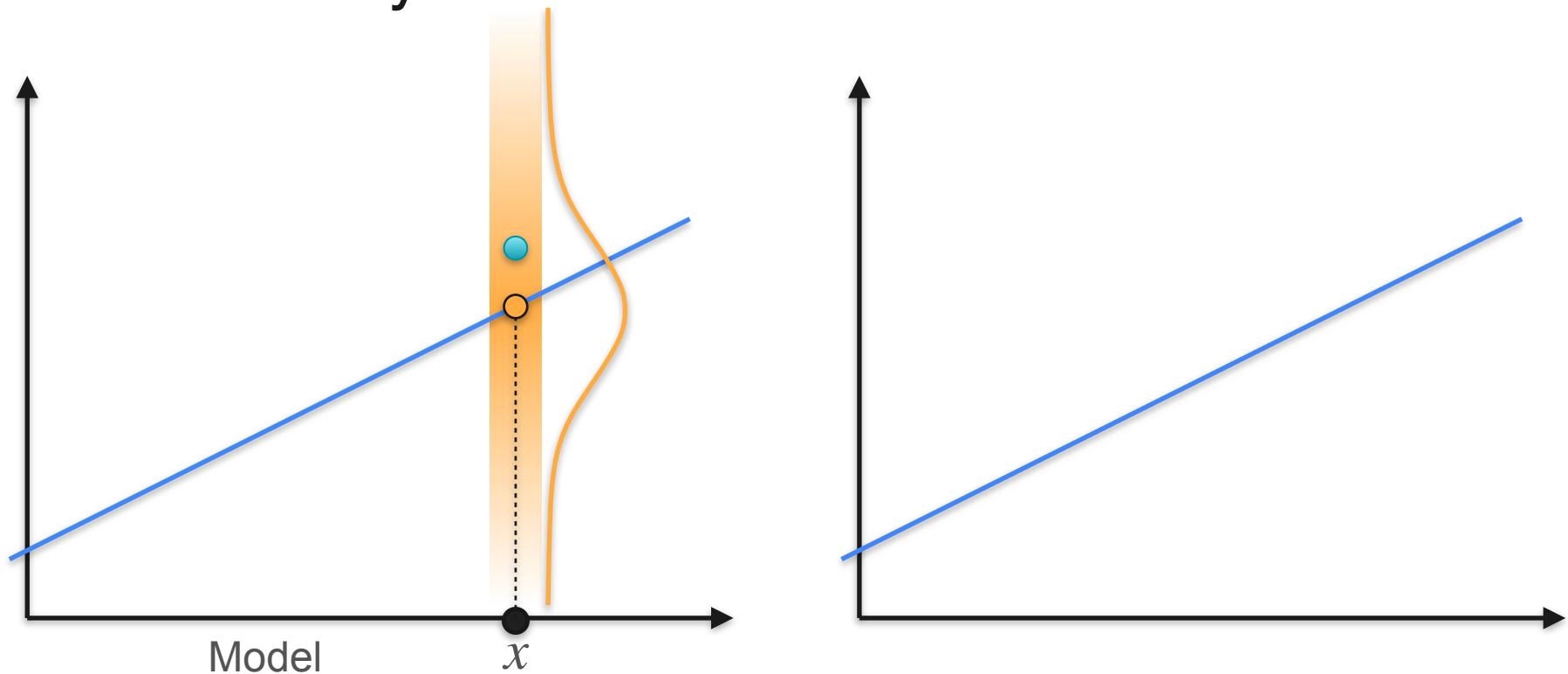
How Exactly Does a Line Produce Points?



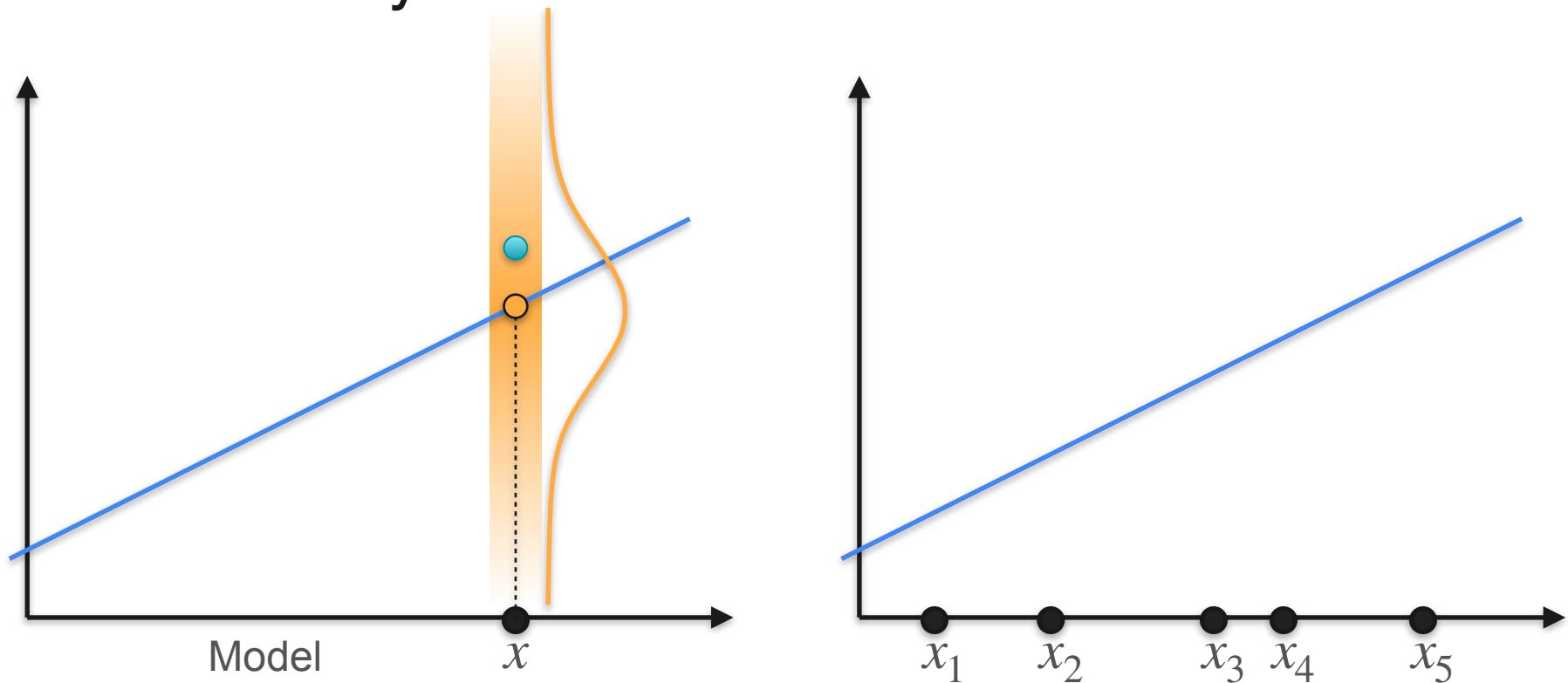
How Exactly Does a Line Produce Points?



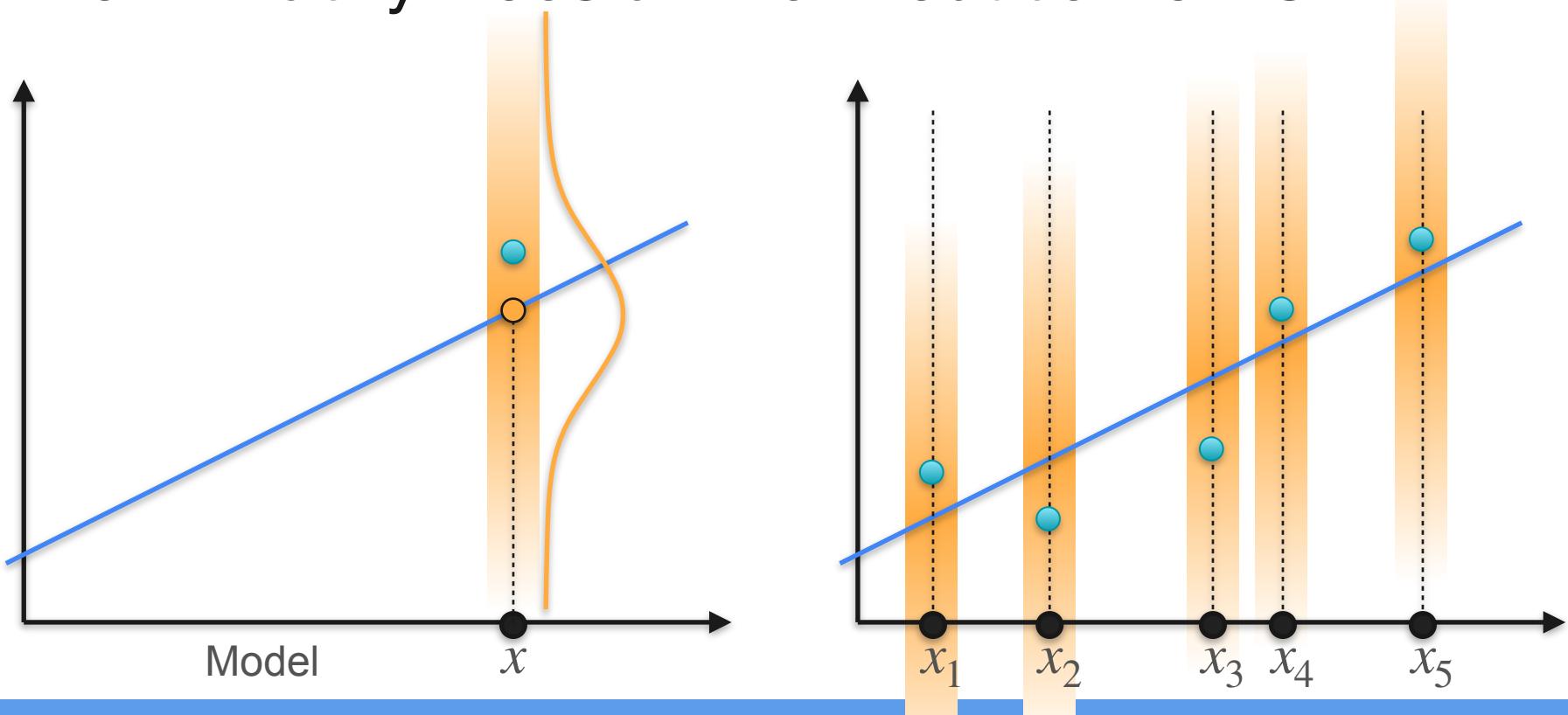
How Exactly Does a Line Produce Points?



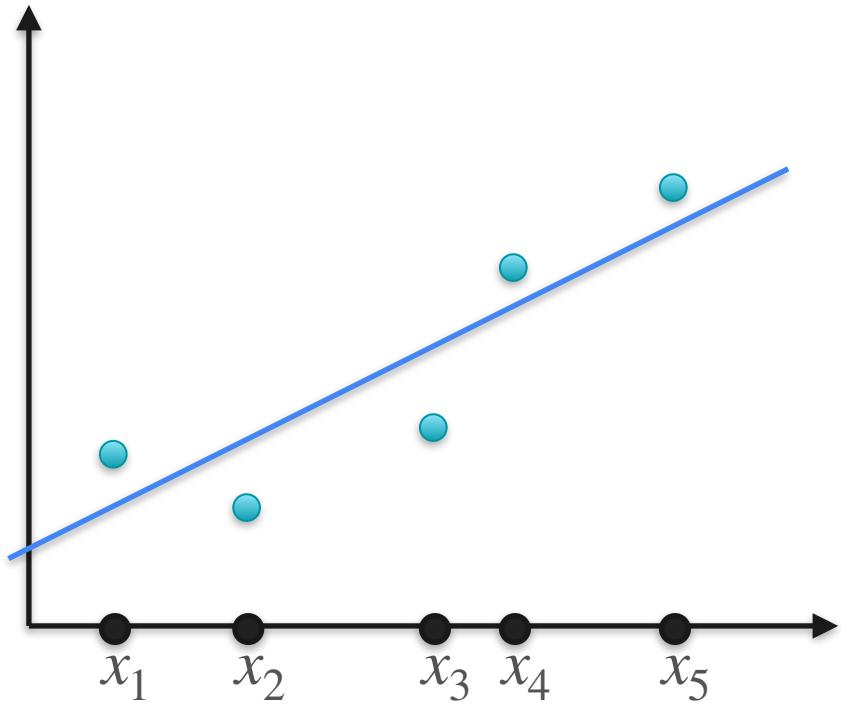
How Exactly Does a Line Produce Points?



How Exactly Does a Line Produce Points?

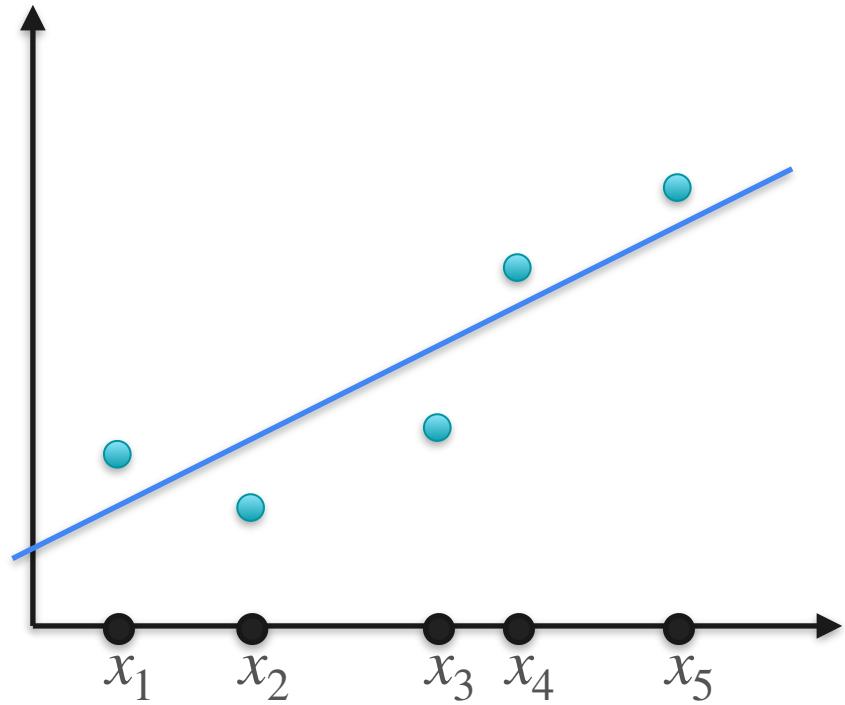


Linear Regression



Linear Regression

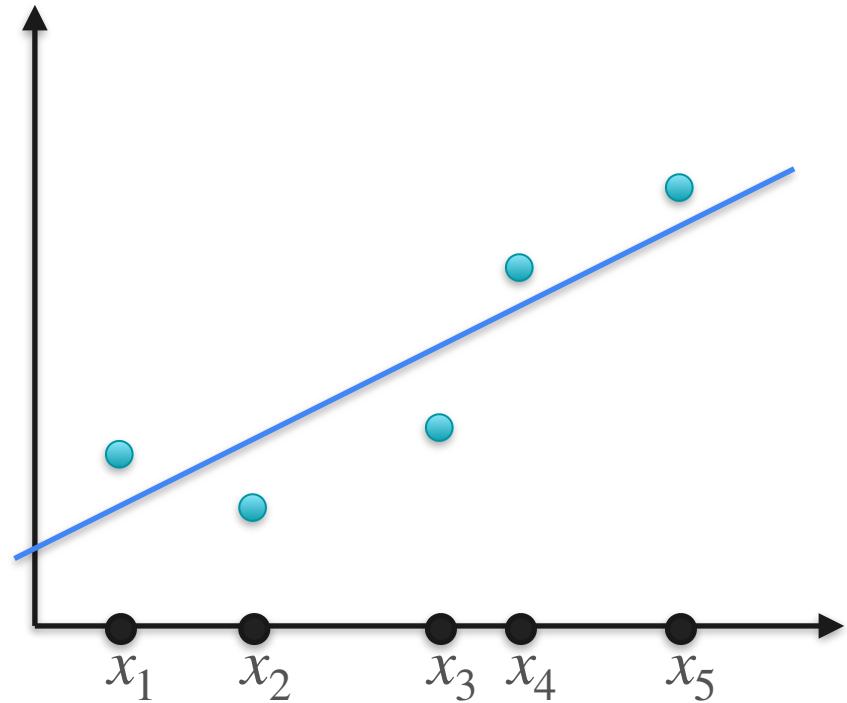
Line that best produced the points



Linear Regression

Line that best produced the points

Line that best fits the data
(linear regression)

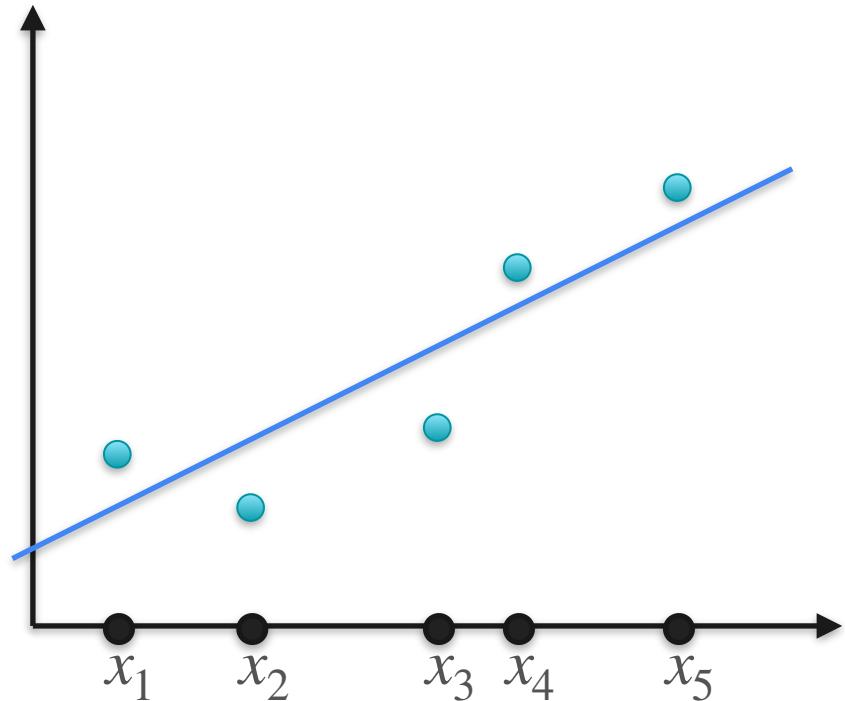


Linear Regression

Line that best produced the points



Line that best fits the data
(linear regression)

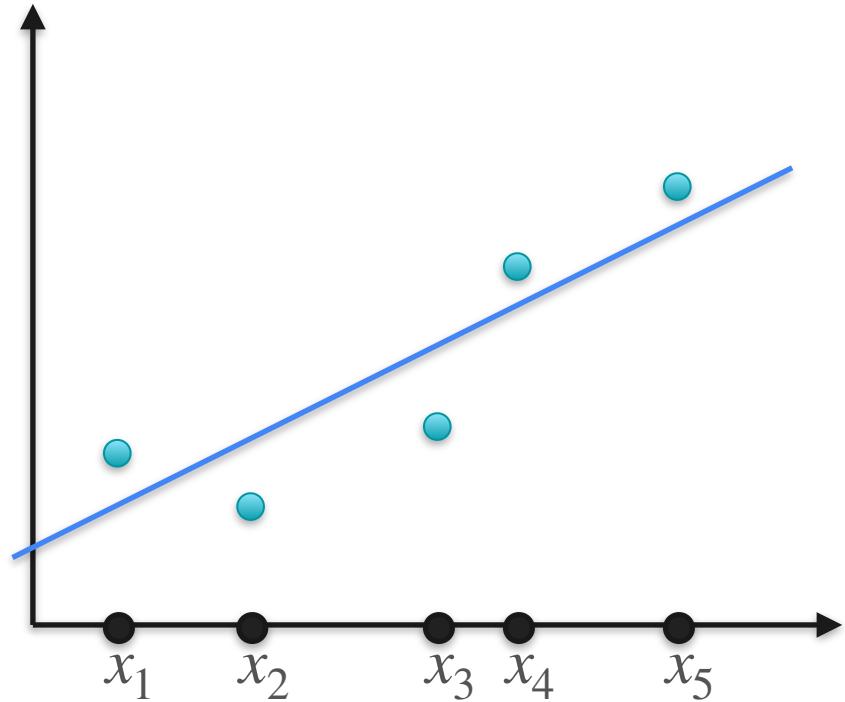


Linear Regression

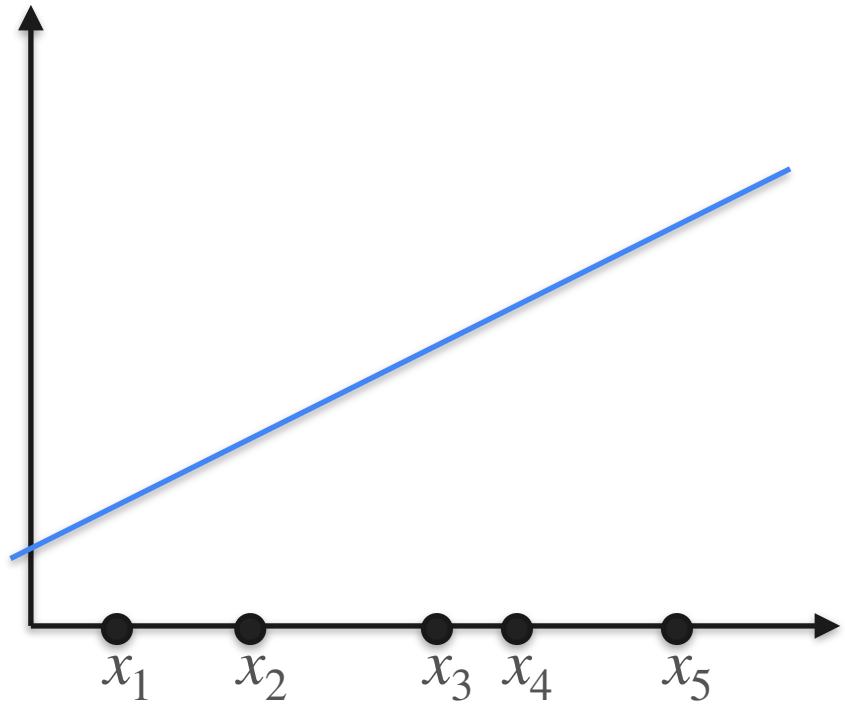
Line that best produced the points

How?

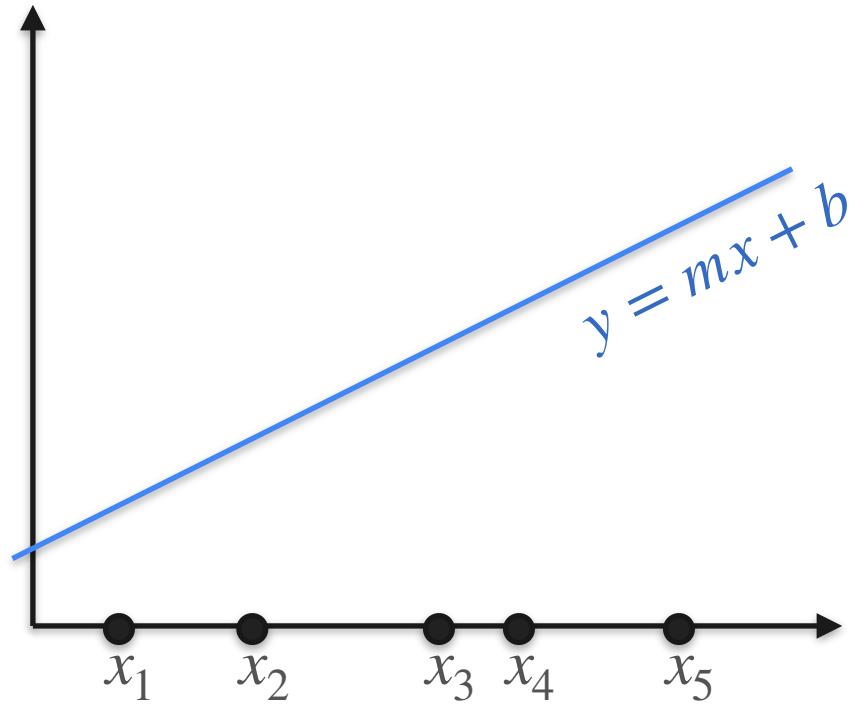
Line that best fits the data
(linear regression)



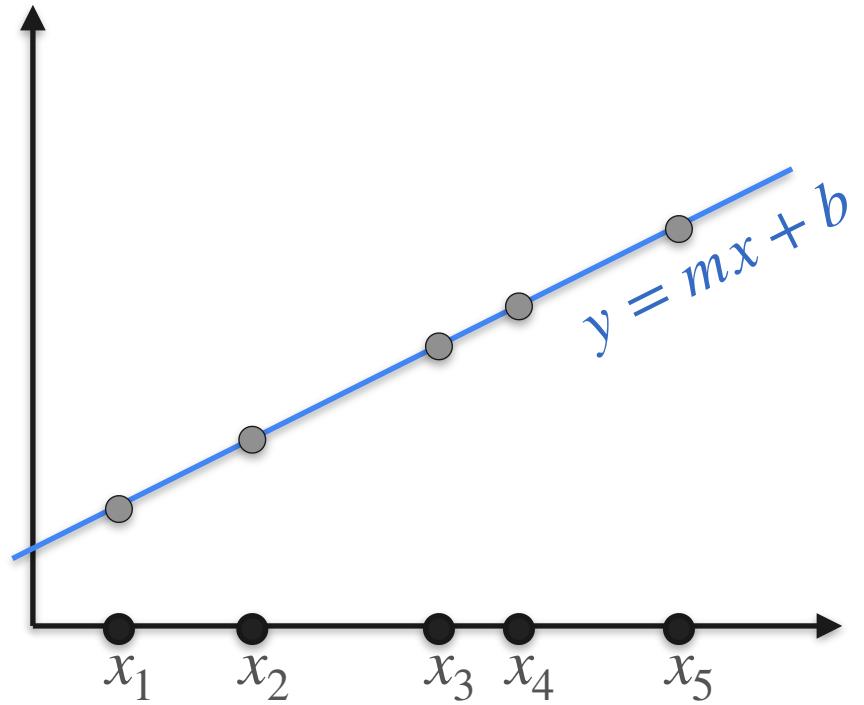
Linear Regression and Likelihood



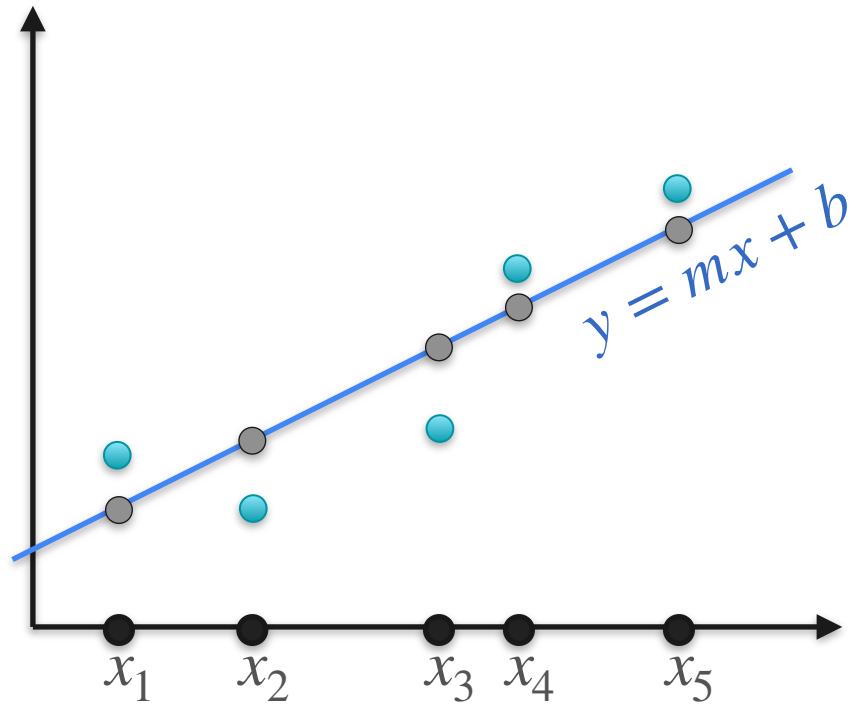
Linear Regression and Likelihood



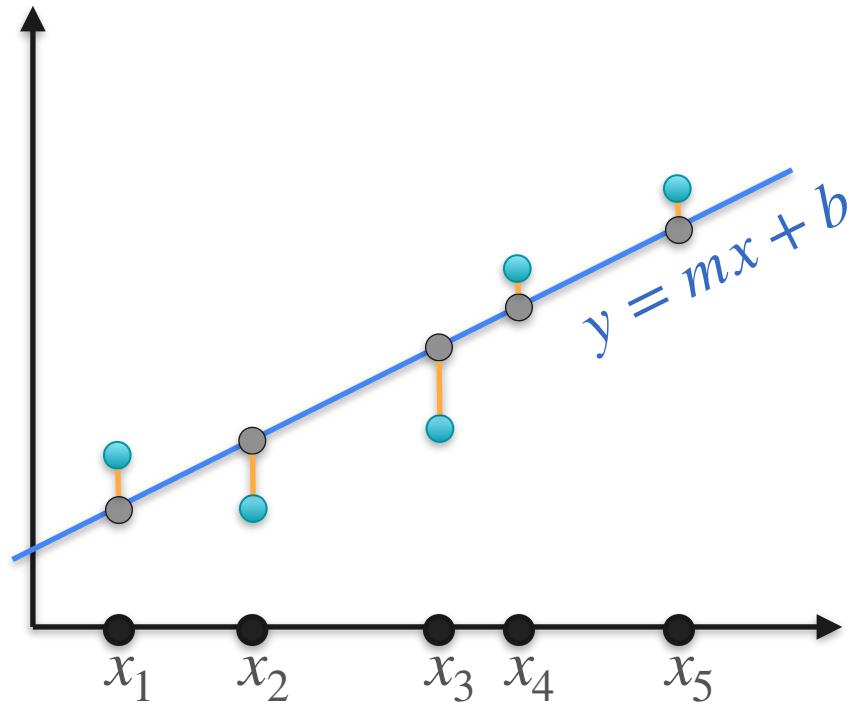
Linear Regression and Likelihood



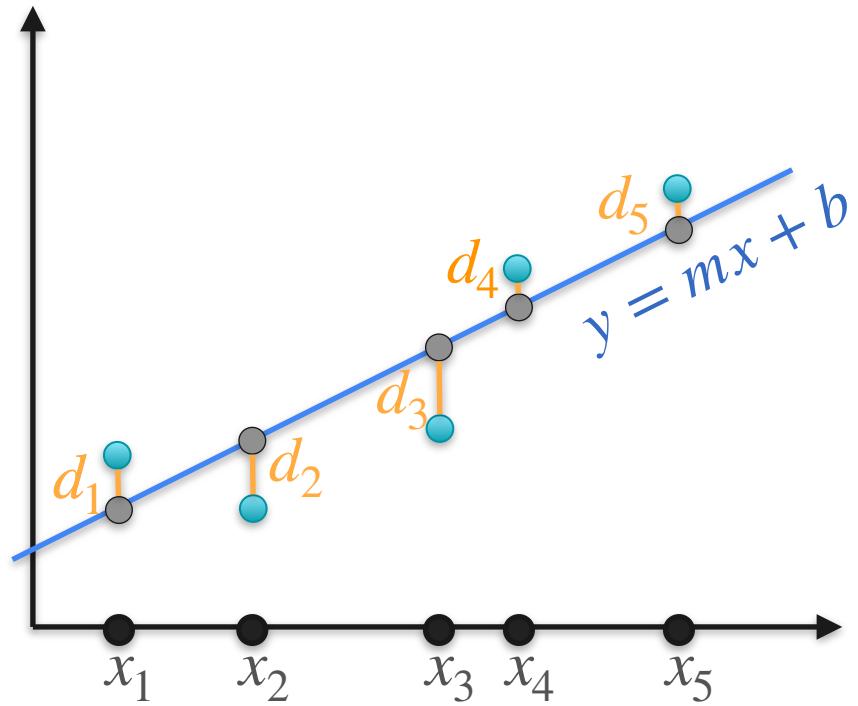
Linear Regression and Likelihood



Linear Regression and Likelihood

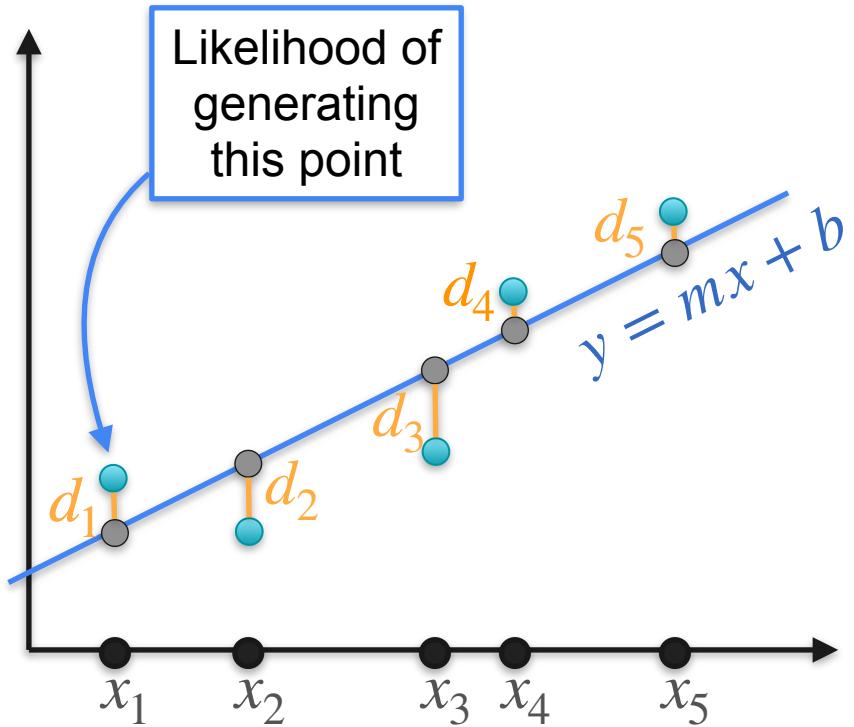


Linear Regression and Likelihood



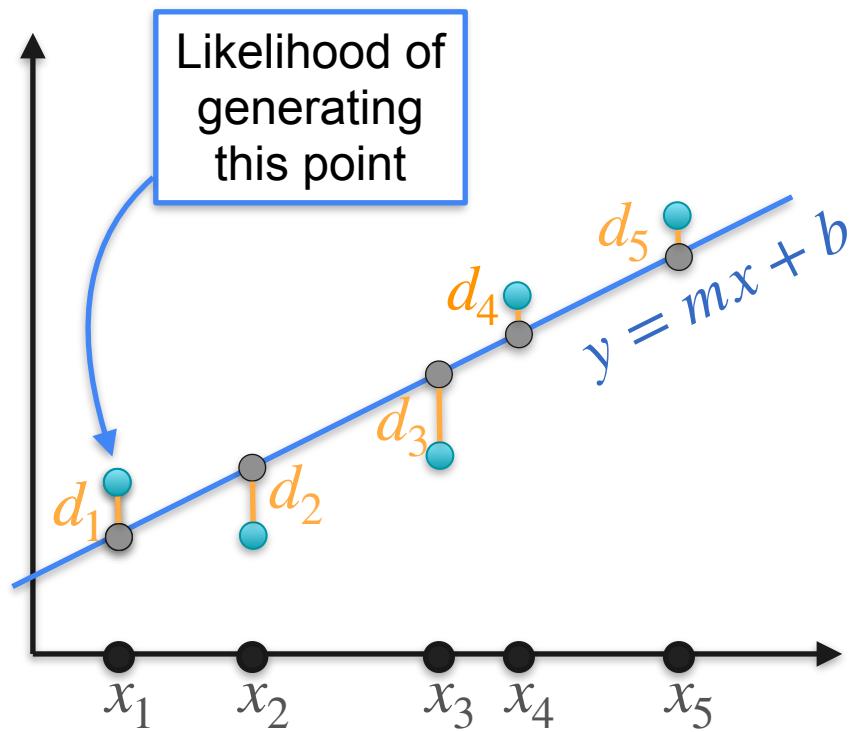
Linear Regression and Likelihood

Likelihood:



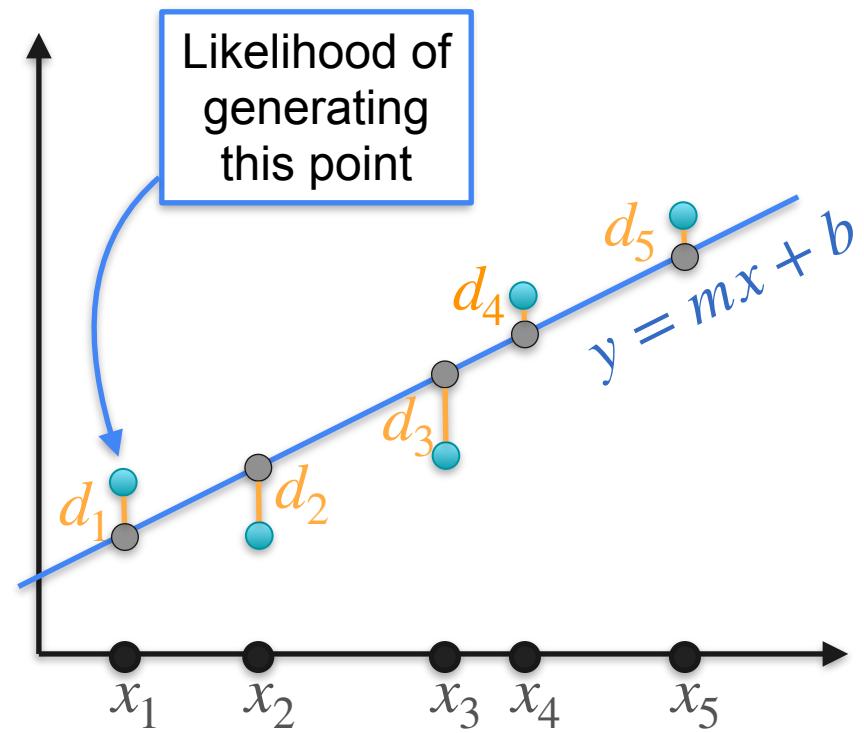
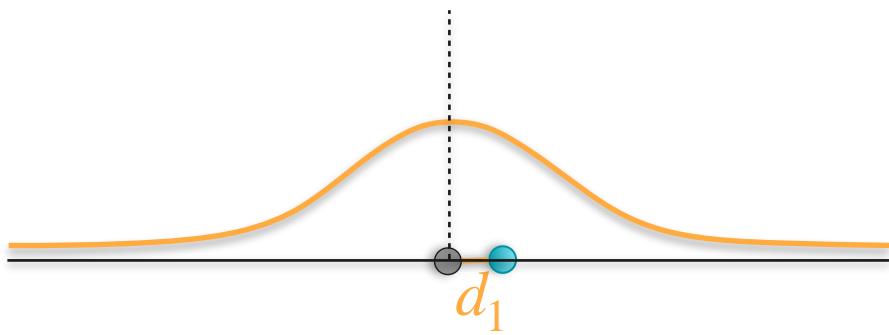
Linear Regression and Likelihood

Likelihood:



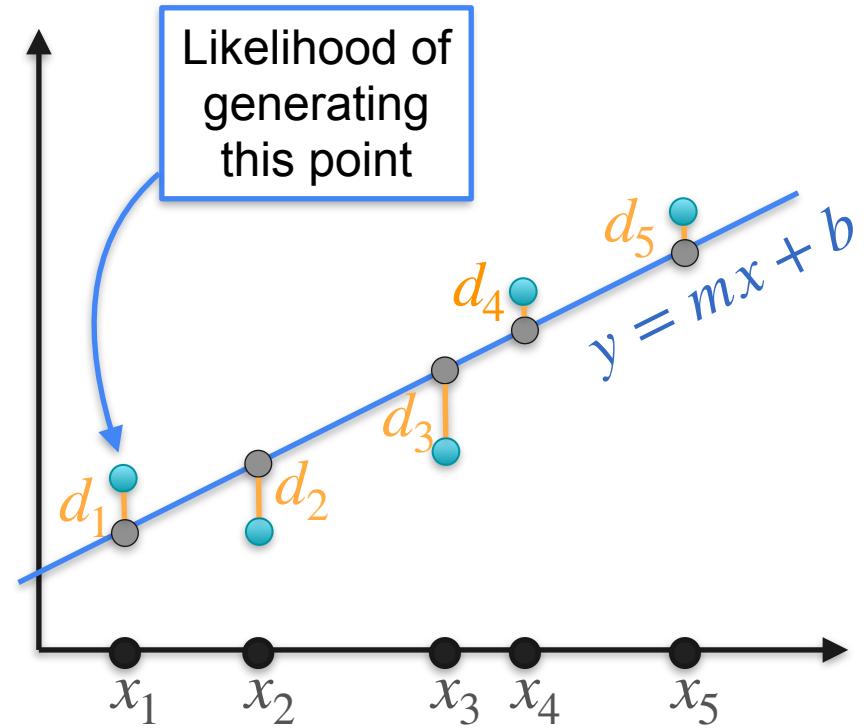
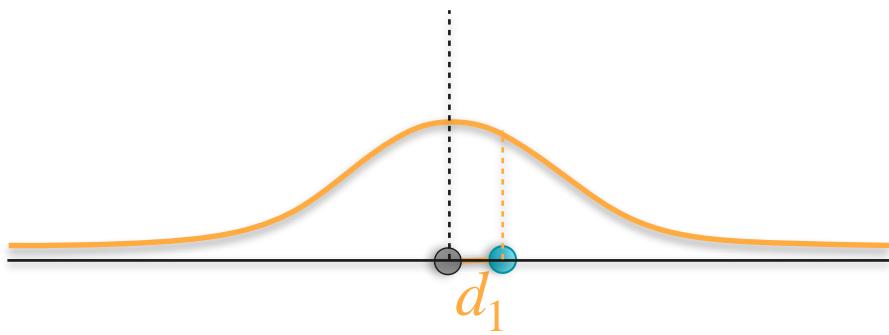
Linear Regression and Likelihood

Likelihood:



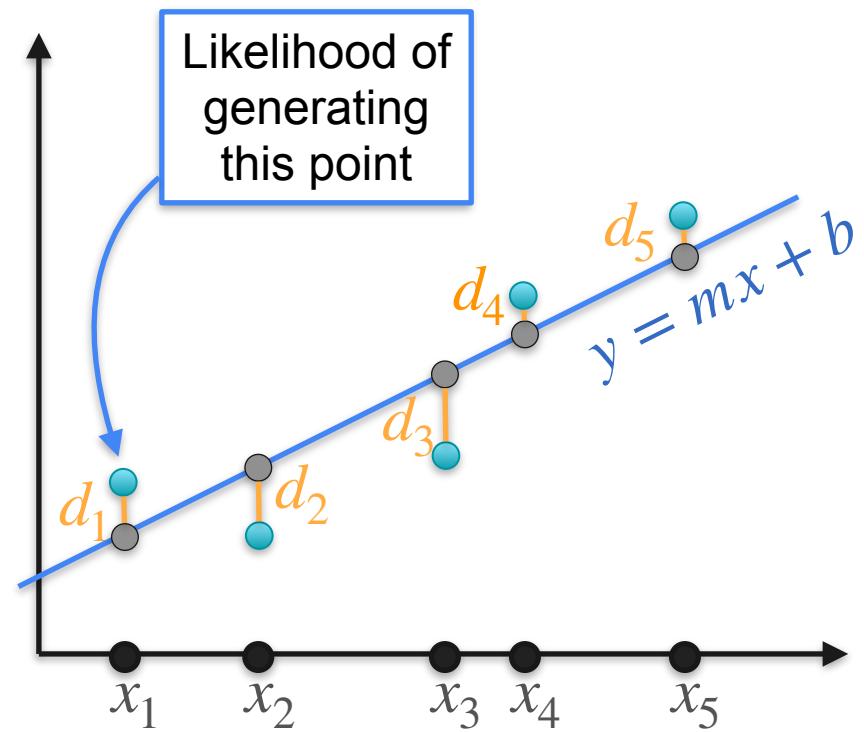
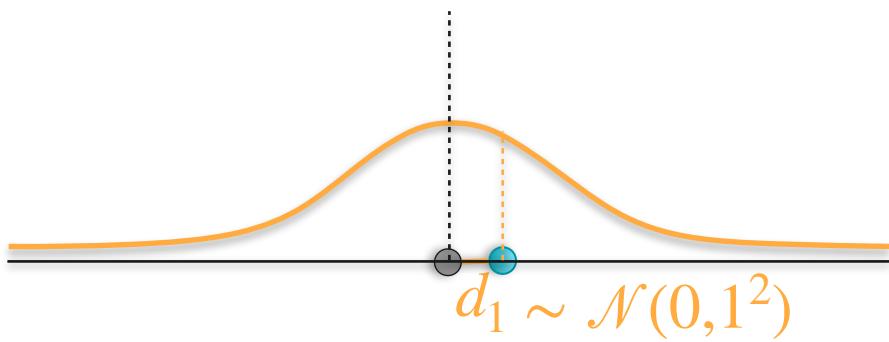
Linear Regression and Likelihood

Likelihood:



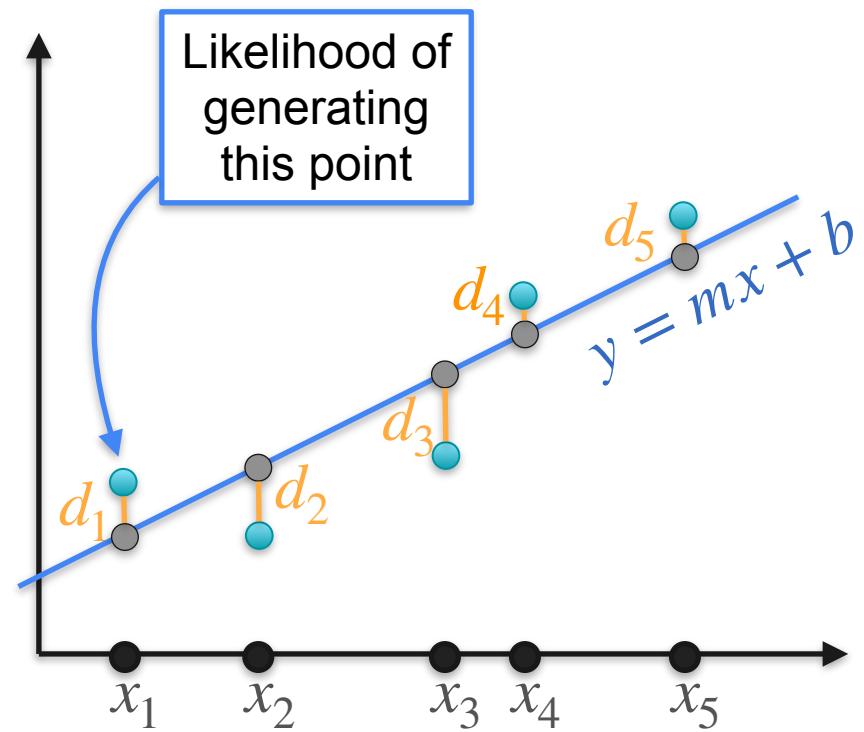
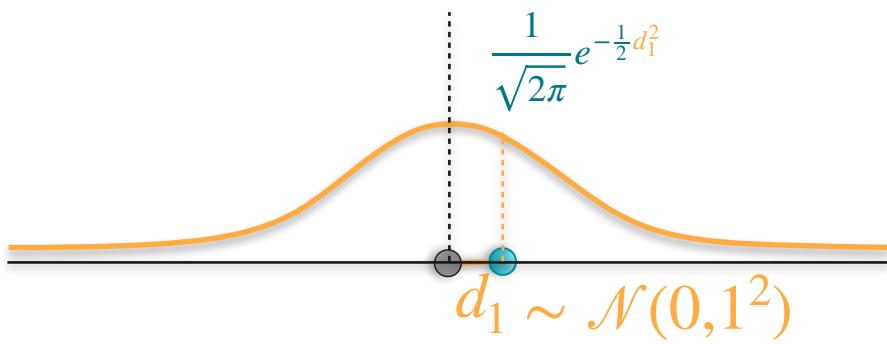
Linear Regression and Likelihood

Likelihood:



Linear Regression and Likelihood

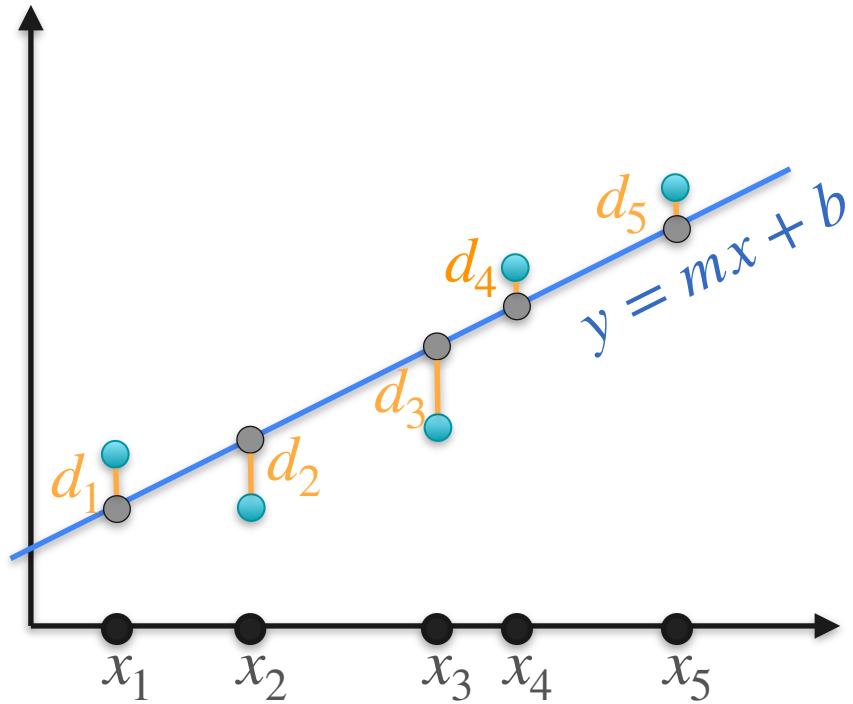
Likelihood:



Linear Regression and Likelihood

Likelihood:

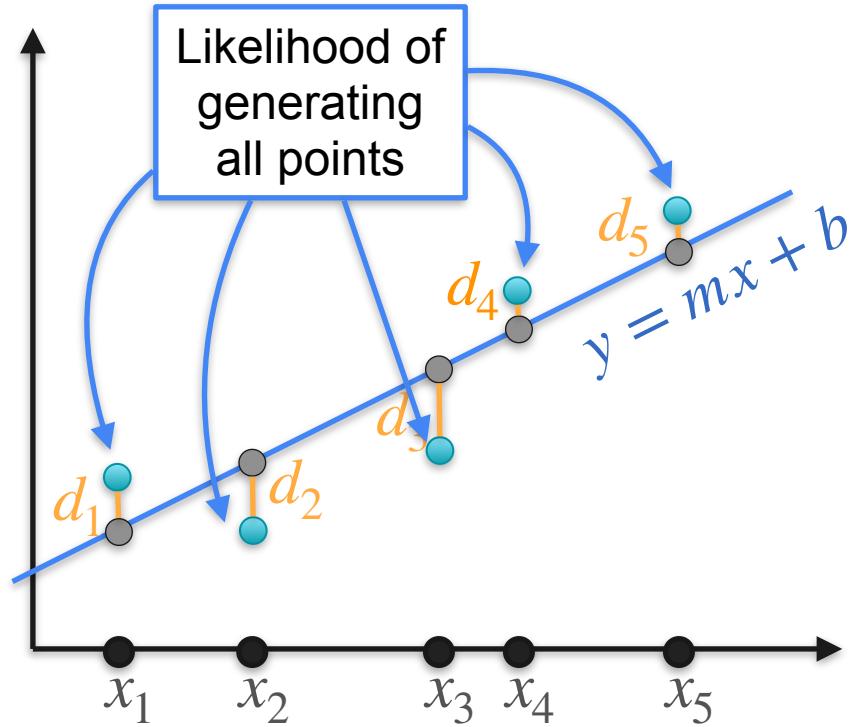
$$\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_i^2}$$



Linear Regression and Likelihood

Likelihood:

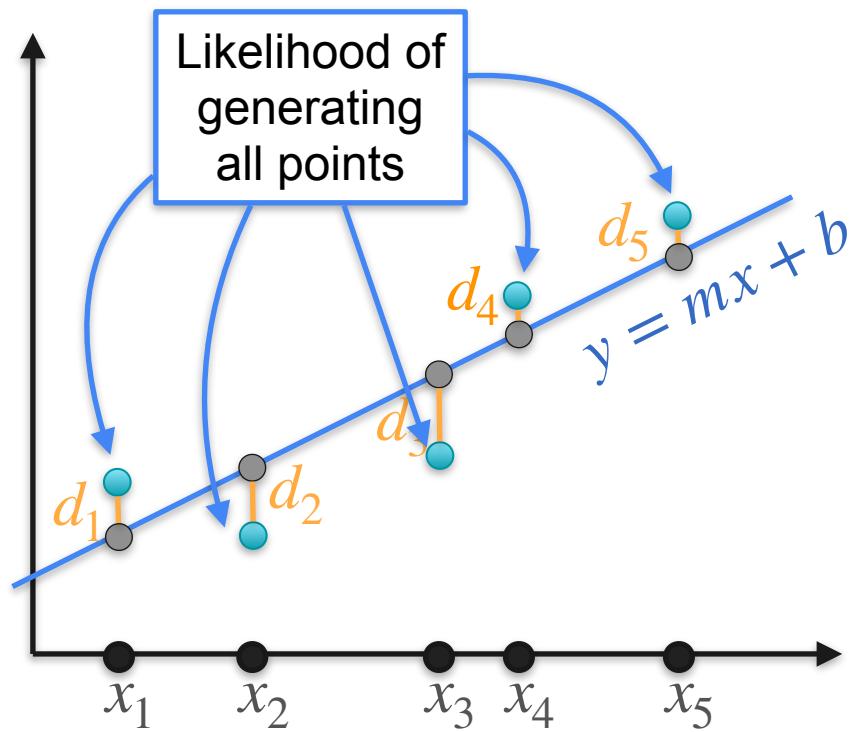
$$\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_i^2}$$



Linear Regression and Likelihood

Likelihood:

$$\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_1^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_2^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_3^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_4^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_5^2}$$

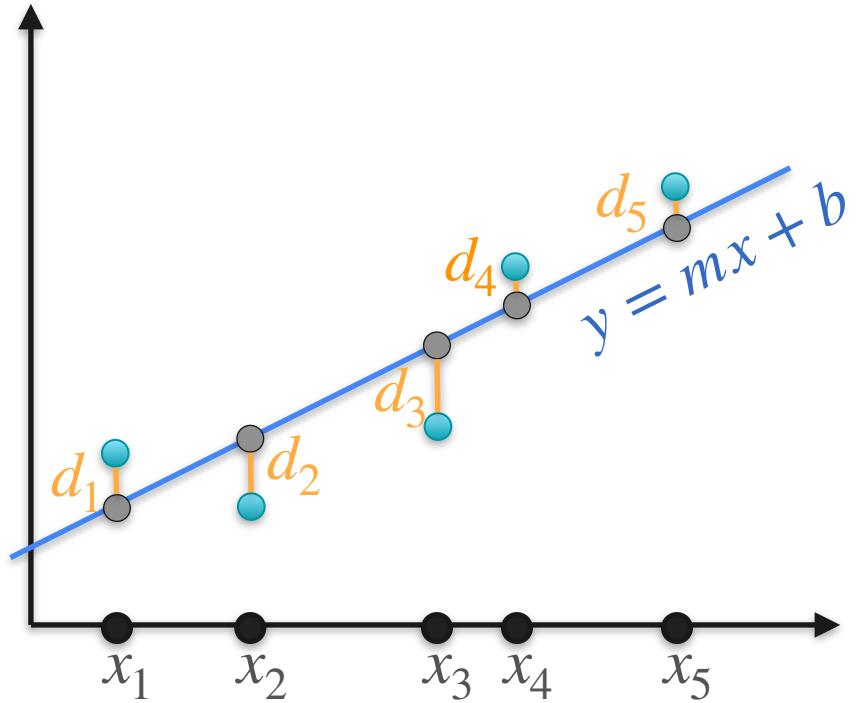


Linear Regression and Likelihood

Likelihood:

$$\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_1^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_2^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_3^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_4^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_5^2}$$

Maximize

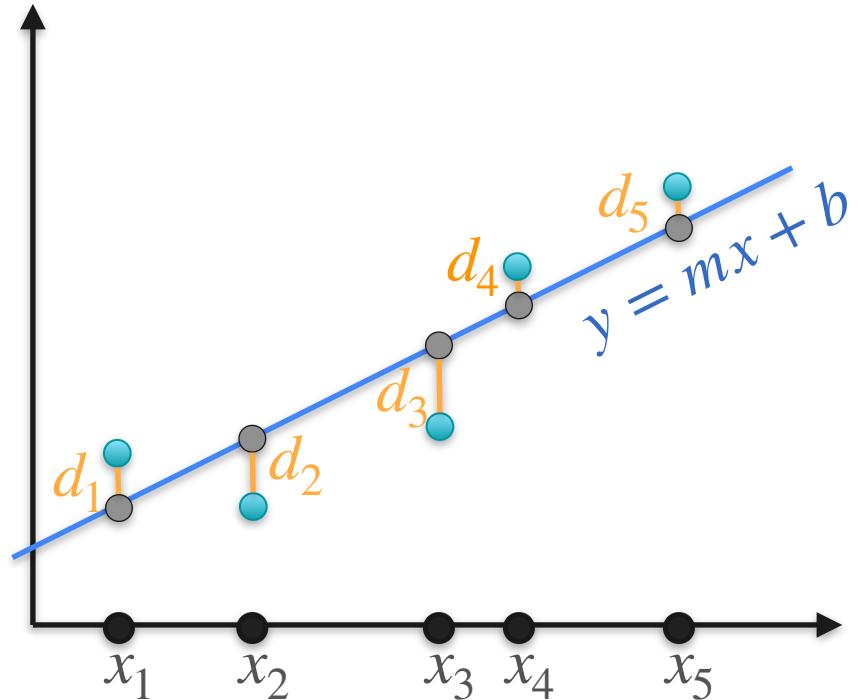


Linear Regression and Likelihood

Likelihood:

$$\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_1^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_2^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_3^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_4^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_5^2}$$

Maximize



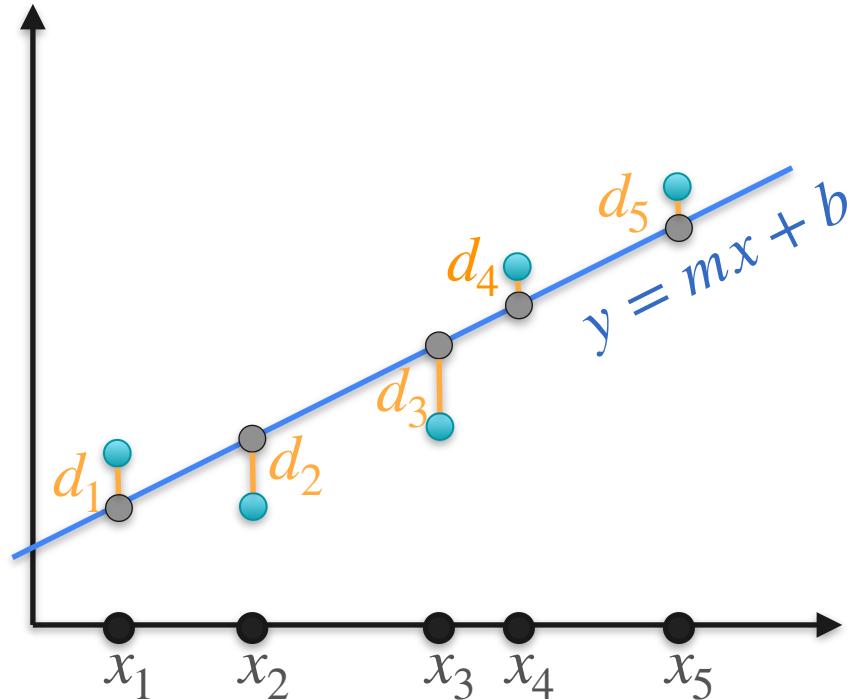
Linear Regression and Likelihood

Likelihood:

$$\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_1^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_2^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_3^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_4^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_5^2}$$

Maximize

$$e^{-\frac{1}{2}d_1^2} \cdot e^{-\frac{1}{2}d_2^2} \cdot e^{-\frac{1}{2}d_3^2} \cdot e^{-\frac{1}{2}d_4^2} \cdot e^{-\frac{1}{2}d_5^2}$$



Linear Regression and Likelihood

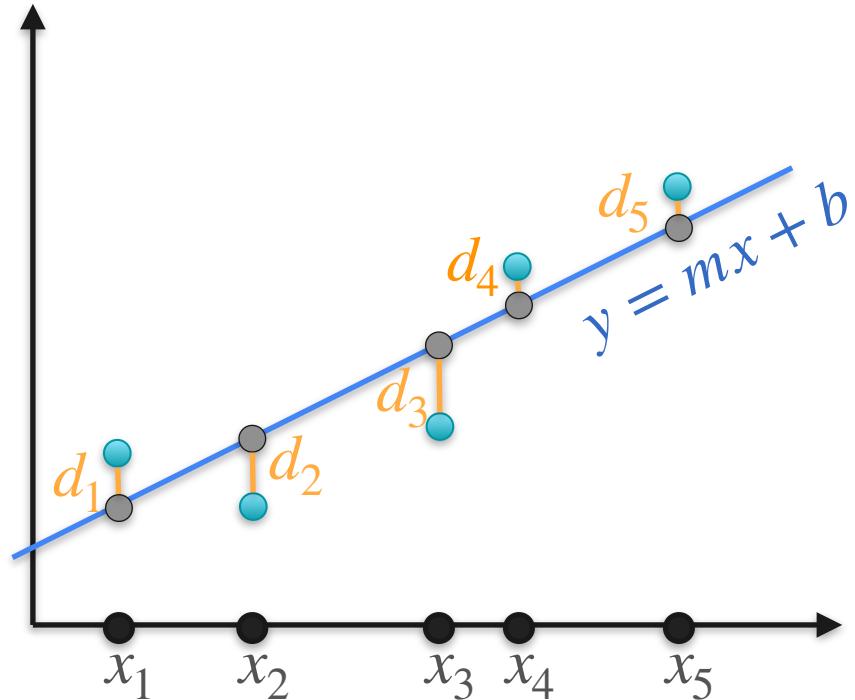
Likelihood:

$$\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_1^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_2^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_3^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_4^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_5^2}$$

Maximize

$$e^{-\frac{1}{2}d_1^2} \cdot e^{-\frac{1}{2}d_2^2} \cdot e^{-\frac{1}{2}d_3^2} \cdot e^{-\frac{1}{2}d_4^2} \cdot e^{-\frac{1}{2}d_5^2}$$

$$e^{-\frac{1}{2}(d_1^2 + d_2^2 + d_3^2 + d_4^2 + d_5^2)}$$



Linear Regression and Likelihood

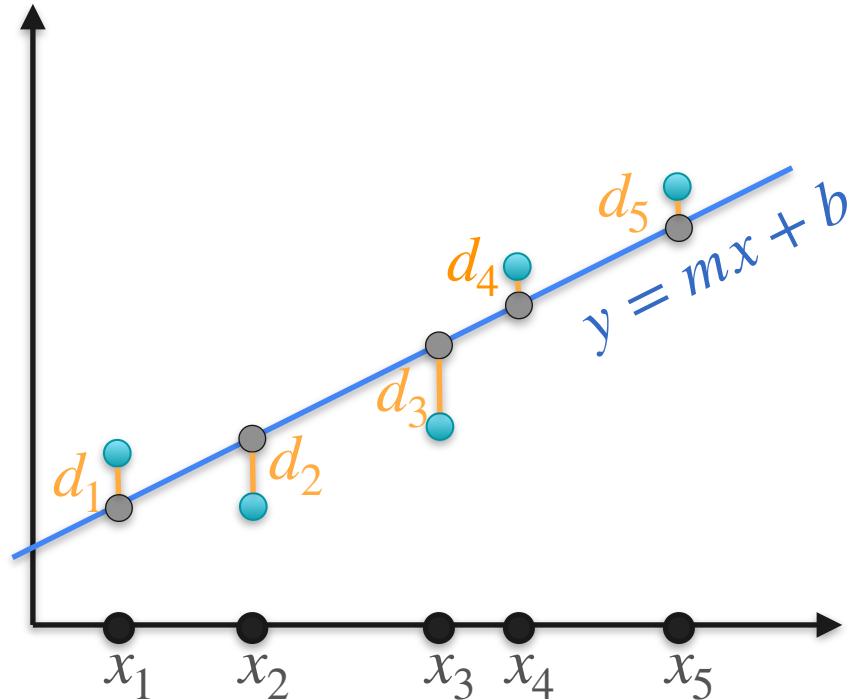
Likelihood:

$$\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_1^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_2^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_3^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_4^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_5^2}$$

Maximize

$$e^{-\frac{1}{2}d_1^2} \cdot e^{-\frac{1}{2}d_2^2} \cdot e^{-\frac{1}{2}d_3^2} \cdot e^{-\frac{1}{2}d_4^2} \cdot e^{-\frac{1}{2}d_5^2}$$

$$e^{-\frac{1}{2}(d_1^2 + d_2^2 + d_3^2 + d_4^2 + d_5^2)}$$



Linear Regression and Likelihood

Likelihood:

$$\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_1^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_2^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_3^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_4^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_5^2}$$

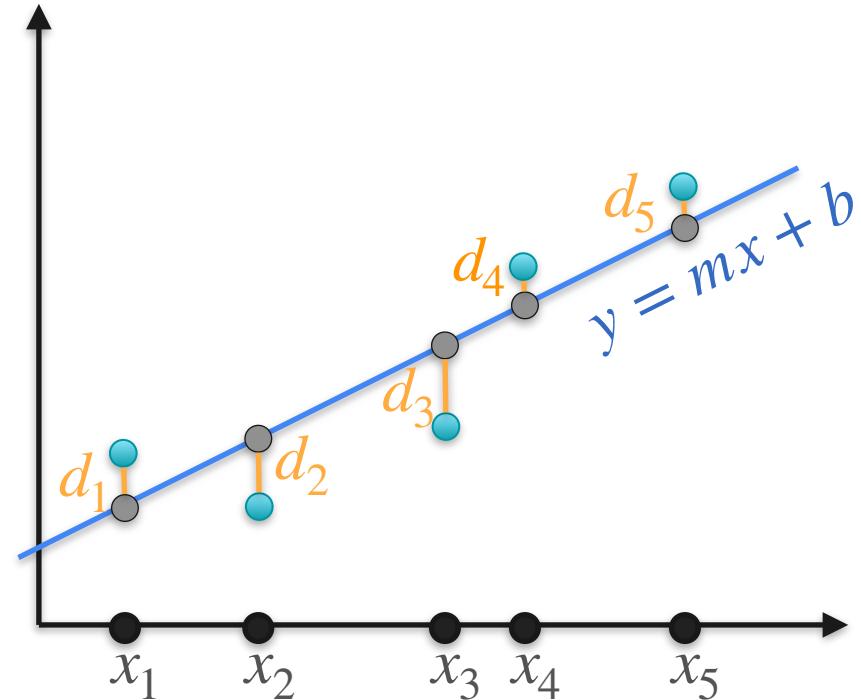
Maximize

$$e^{-\frac{1}{2}d_1^2} \cdot e^{-\frac{1}{2}d_2^2} \cdot e^{-\frac{1}{2}d_3^2} \cdot e^{-\frac{1}{2}d_4^2} \cdot e^{-\frac{1}{2}d_5^2}$$

~~$$e^{-\frac{1}{2}(d_1^2 + d_2^2 + d_3^2 + d_4^2 + d_5^2)}$$~~

Minimize

$$d_1^2 + d_2^2 + d_3^2 + d_4^2 + d_5^2$$



Linear Regression and Likelihood

Likelihood:

$$\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_1^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_2^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_3^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_4^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_5^2}$$

Maximize

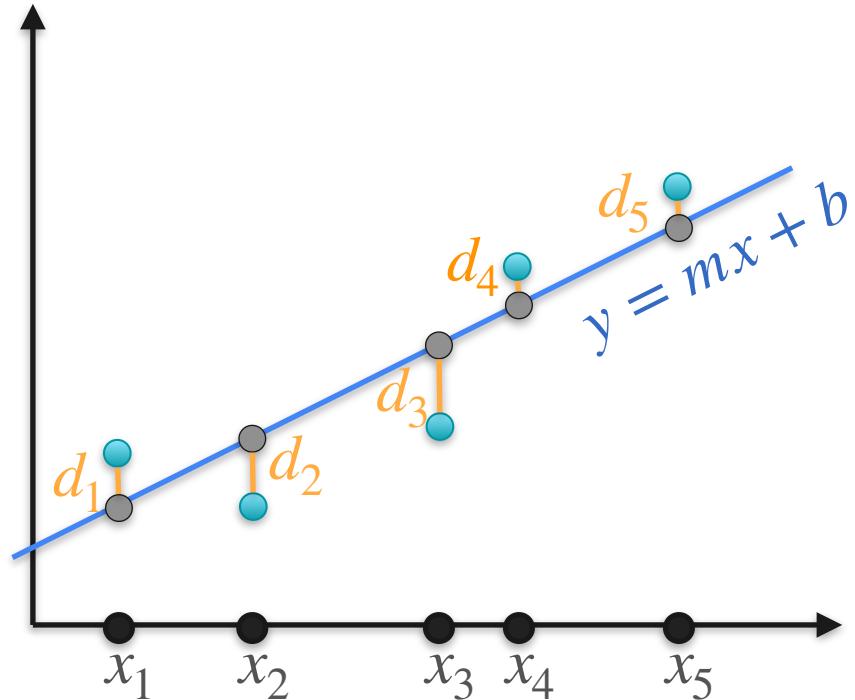
$$e^{-\frac{1}{2}d_1^2} \cdot e^{-\frac{1}{2}d_2^2} \cdot e^{-\frac{1}{2}d_3^2} \cdot e^{-\frac{1}{2}d_4^2} \cdot e^{-\frac{1}{2}d_5^2}$$

$$\cancel{e^{-\frac{1}{2}(d_1^2 + d_2^2 + d_3^2 + d_4^2 + d_5^2)}}$$

Minimize

$$d_1^2 + d_2^2 + d_3^2 + d_4^2 + d_5^2$$

Least squares error!



Linear Regression and Likelihood

Likelihood:

$$\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_1^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_2^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_3^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_4^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_5^2}$$

Maximize

$$e^{-\frac{1}{2}d_1^2} \cdot e^{-\frac{1}{2}d_2^2} \cdot e^{-\frac{1}{2}d_3^2} \cdot e^{-\frac{1}{2}d_4^2} \cdot e^{-\frac{1}{2}d_5^2}$$

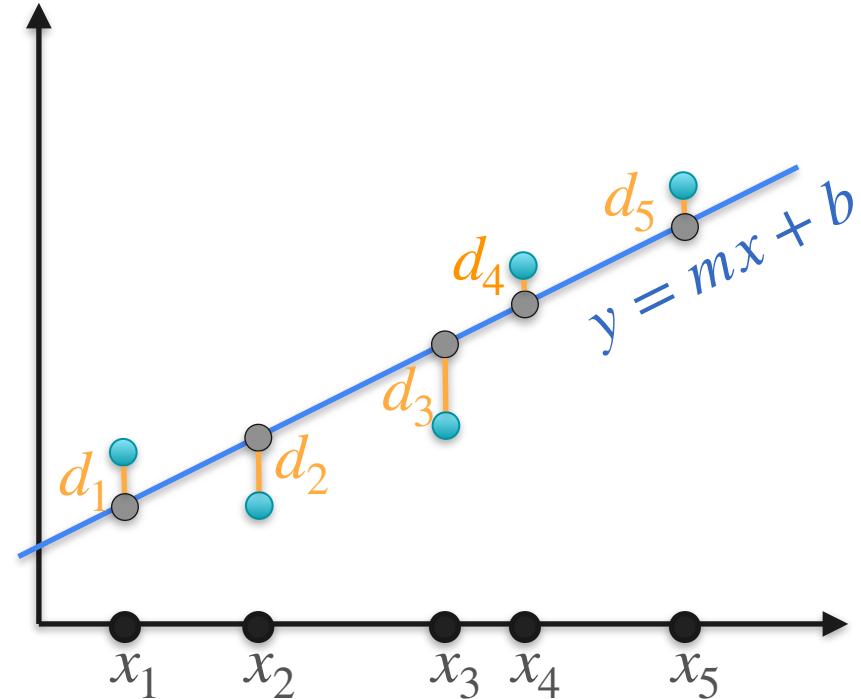
$$\cancel{e^{-\frac{1}{2}(d_1^2 + d_2^2 + d_3^2 + d_4^2 + d_5^2)}}$$

Minimize

$$d_1^2 + d_2^2 + d_3^2 + d_4^2 + d_5^2$$

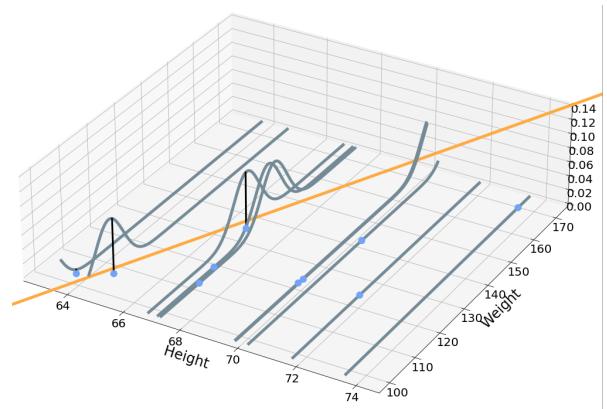
Linear regression!

Least squares error!

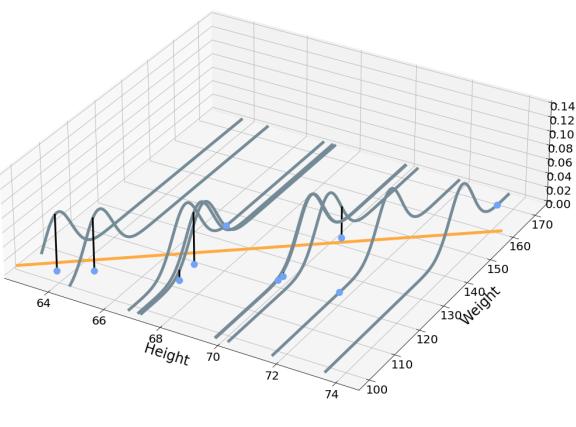
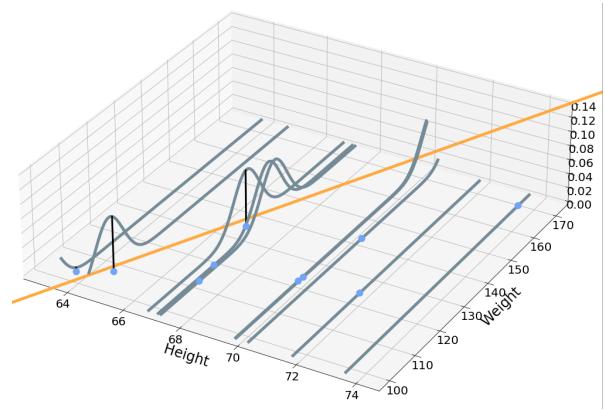


Picking the Right Model

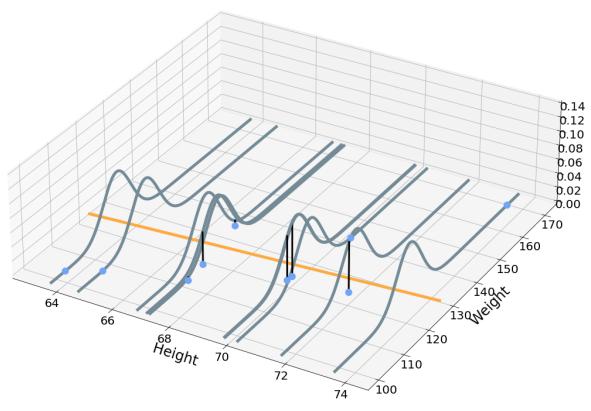
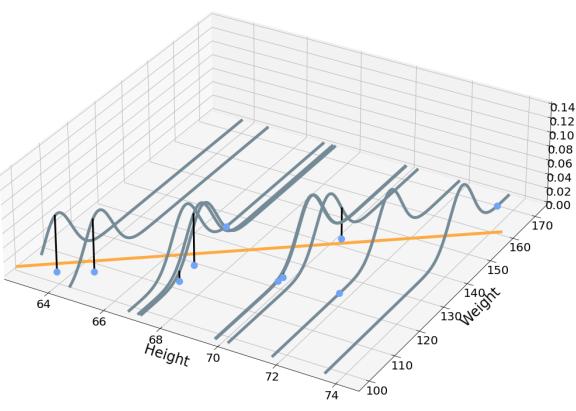
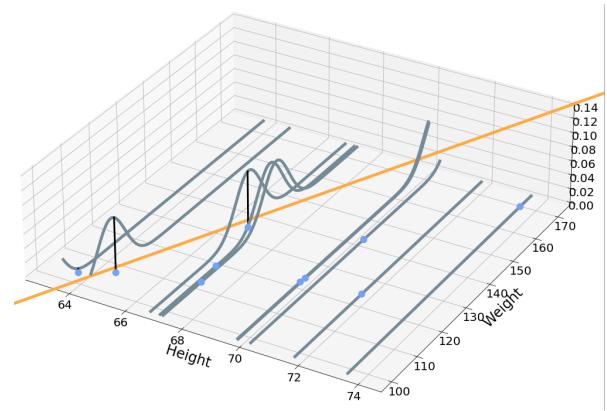
Picking the Right Model



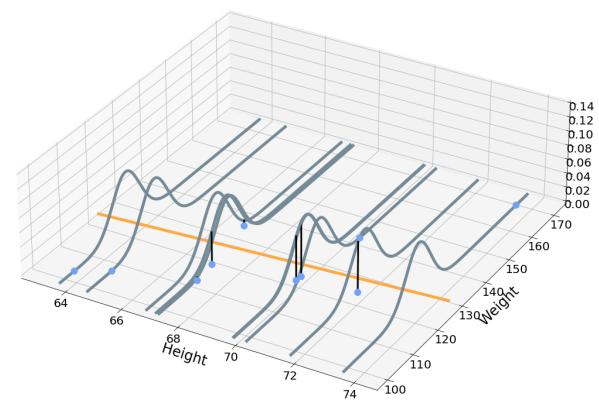
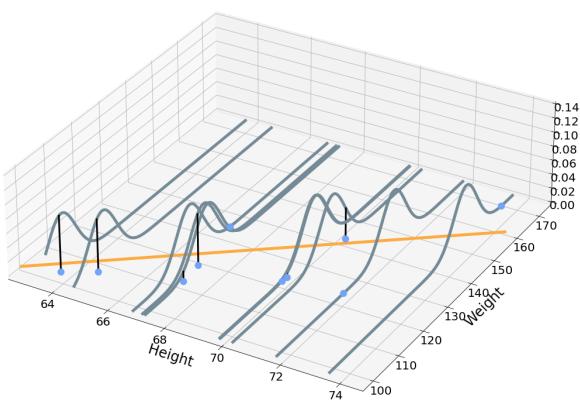
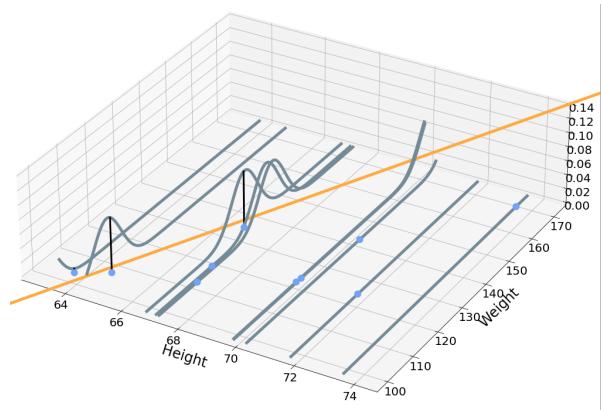
Picking the Right Model



Picking the Right Model



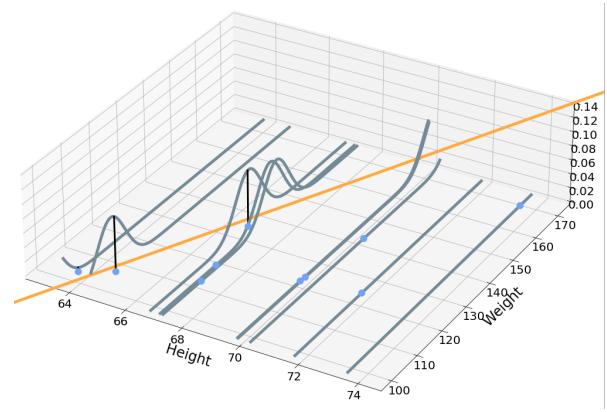
Picking the Right Model



Model 1:

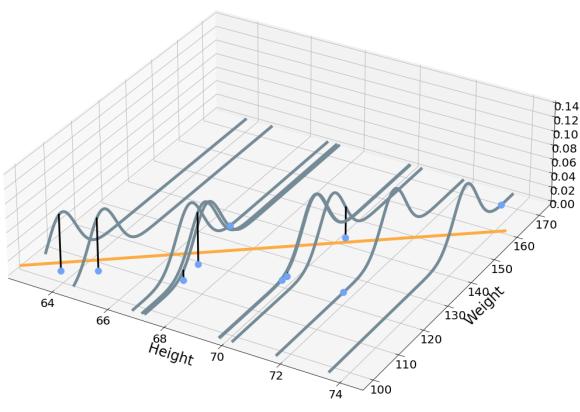
$$\text{Likelihood} = 4.91 \cdot 10^{-260}$$

Picking the Right Model



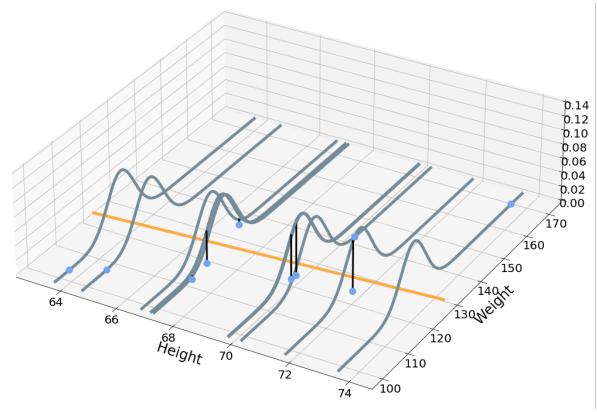
Model 1:

$$\text{Likelihood} = 4.91 \cdot 10^{-260}$$

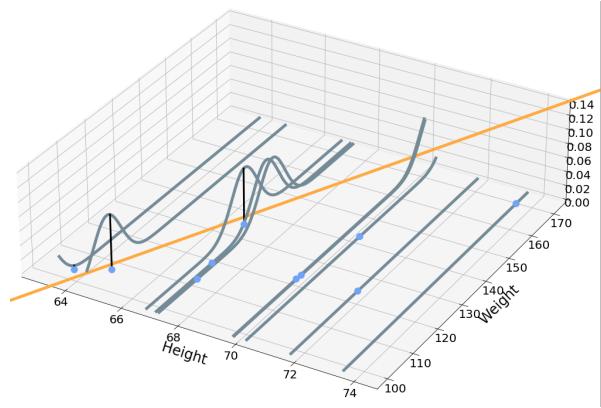


Model 2:

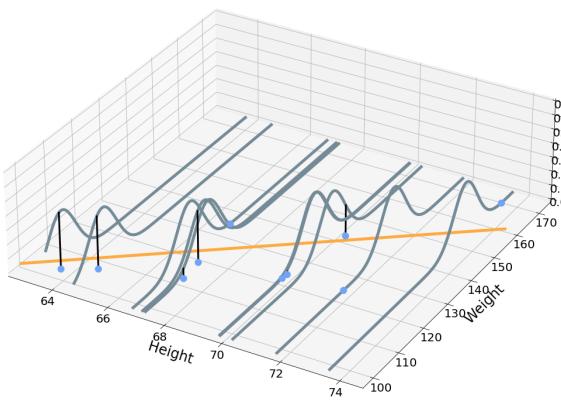
$$\text{Likelihood} = 8.16 \cdot 10^{-28}$$



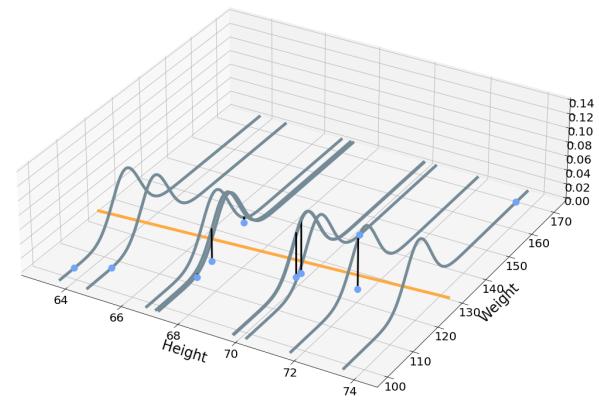
Picking the Right Model



Model 1:
Likelihood = $4.91 \cdot 10^{-260}$

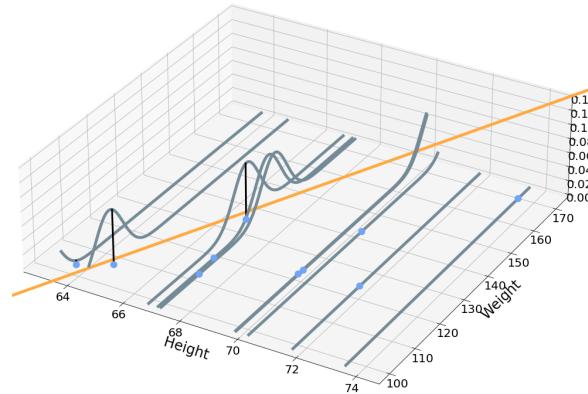


Model 2:
Likelihood = $8.16 \cdot 10^{-28}$



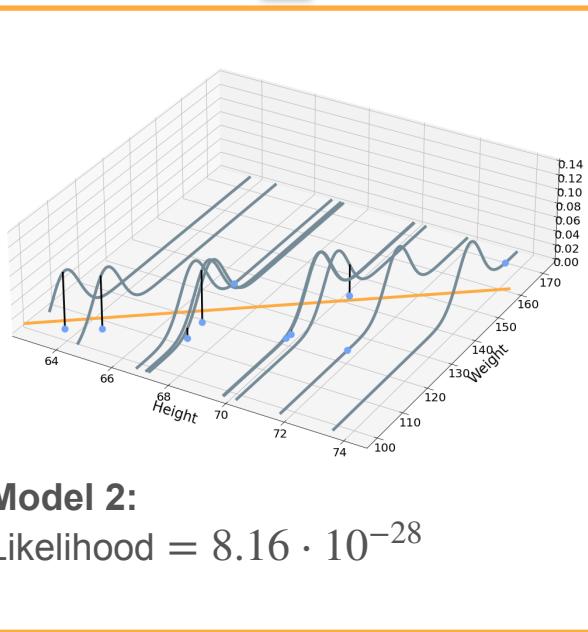
Model 3:
Likelihood = $3.48 \cdot 10^{-49}$

Picking the Right Model



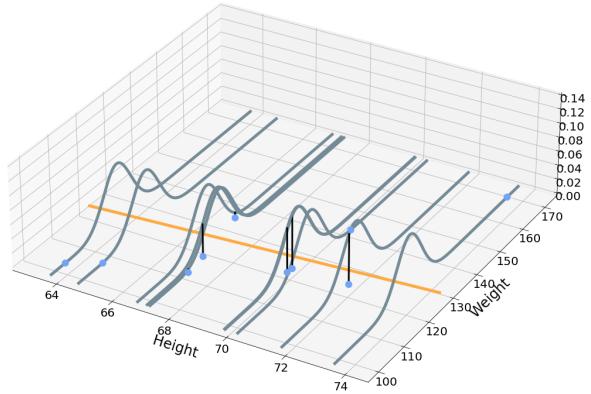
Model 1:

$$\text{Likelihood} = 4.91 \cdot 10^{-260}$$



Model 2:

$$\text{Likelihood} = 8.16 \cdot 10^{-28}$$



Model 3:

$$\text{Likelihood} = 3.48 \cdot 10^{-49}$$



DeepLearning.AI

Point Estimation

**Frequentist vs Bayesian
Statistics**

Frequentist Vs. Bayesian Statistics

Frequentist Vs. Bayesian Statistics

Frequentists

Bayesians

Frequentist Vs. Bayesian Statistics

Frequentists

- Probabilities represent long term frequency of events

Bayesians

Frequentist Vs. Bayesian Statistics

Frequentists

- Probabilities represent long term frequency of events

Bayesians

- Probabilities represent the degree of belief (or certainty)

Frequentist Vs. Bayesian Statistics

Frequentists

- Probabilities represent long term frequency of events
- Concept of Likelihood

Bayesians

- Probabilities represent the degree of belief (or certainty)

Frequentist Vs. Bayesian Statistics

Frequentists

- Probabilities represent long term frequency of events
- Concept of Likelihood

Bayesians

- Probabilities represent the degree of belief (or certainty)
- Concept of Prior

Frequentist Vs. Bayesian Statistics

Frequentists

- Probabilities represent long term frequency of events
- Concept of Likelihood

Bayesians

- Probabilities represent the degree of belief (or certainty)
- Concept of Prior
- Goal: update prior belief based on observations

Frequentist Vs. Bayesian Statistics

Frequentists

- Probabilities represent long term frequency of events
- Concept of Likelihood
- Goal: Find the model that most likely generated the observed data

Bayesians

- Probabilities represent the degree of belief (or certainty)
- Concept of Prior
- Goal: update prior belief based on observations

Frequentist Approach

Frequentist Approach



Frequentist Approach



p = probability of heads

Frequentist Approach



p = probability of heads

$$p = 0.8$$

Frequentist Approach



p = probability of heads

$$p = 0.8$$

Maximum likelihood

Bayesian Approach

p = probability of heads



Bayesian Approach

p = probability of heads



p could be anything between 0 and 1

Bayesian Approach

p = probability of heads



p could be anything between 0 and 1

p could still be anything between 0 and 1, but it may be closer to 1

Bayesian Approach

p = probability of heads



p could be anything between 0 and 1

p could still be anything between 0 and 1, but it may be closer to 1

p could still be anything between 0 and 1, but it's probably around 0.5

Bayesian Approach

p = probability of heads



p could be anything between 0 and 1

p could still be anything between 0 and 1, but it may be closer to 1

p could still be anything between 0 and 1, but it's probably around 0.5

p could still be anything between 0 and 1, but it's more likely closer to 0.66

Bayesian Approach

p = probability of heads



p could be anything between 0 and 1

p could still be anything between 0 and 1, but it may be closer to 1

p could still be anything between 0 and 1, but it's probably around 0.5

p could still be anything between 0 and 1, but it's more likely closer to 0.66

...

Bayesian Approach

p = probability of heads



p could be anything between 0 and 1

p could still be anything between 0 and 1, but it may be closer to 1

p could still be anything between 0 and 1, but it's probably around 0.5

p could still be anything between 0 and 1, but it's more likely closer to 0.66

...

p could still be anything between 0 and 1, but it's very probably close to 0.8

Bayesian Approach

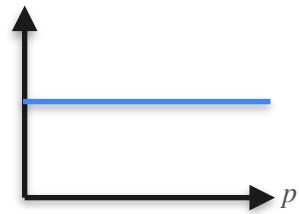


p = probability of heads

Bayesian Approach



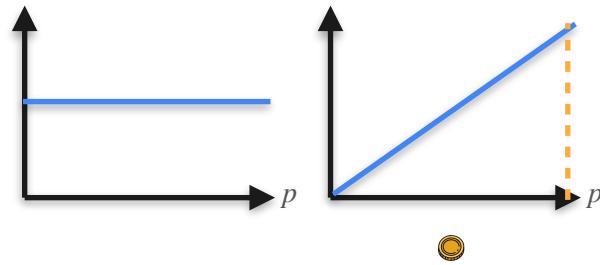
p = probability of heads



Bayesian Approach



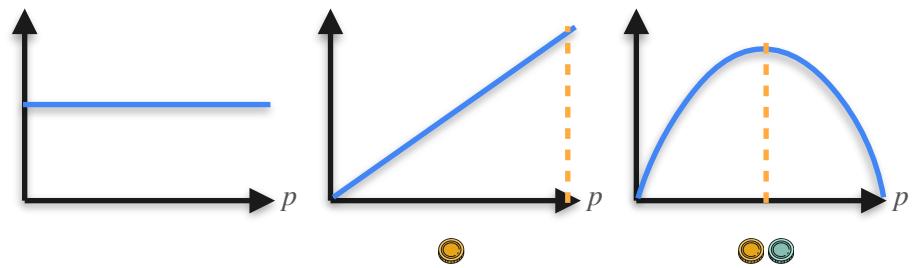
p = probability of heads



Bayesian Approach



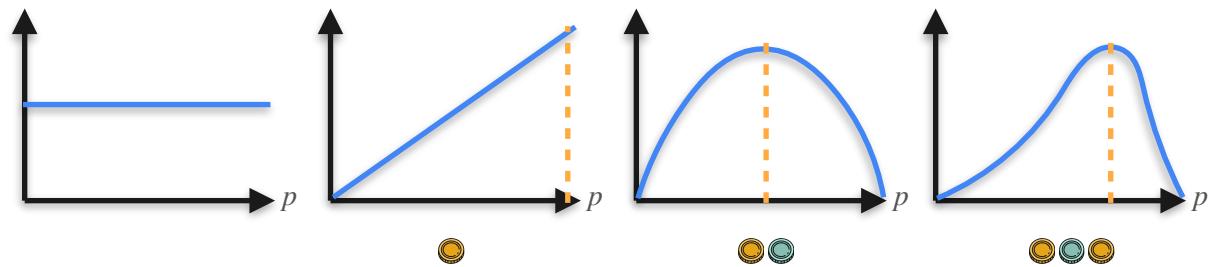
p = probability of heads



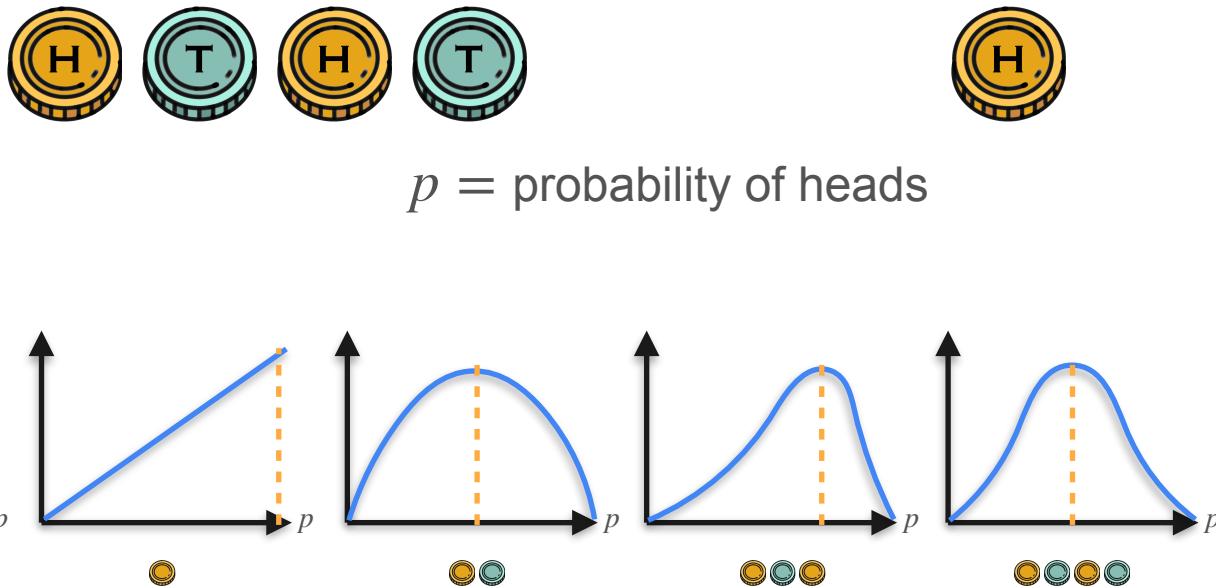
Bayesian Approach



p = probability of heads



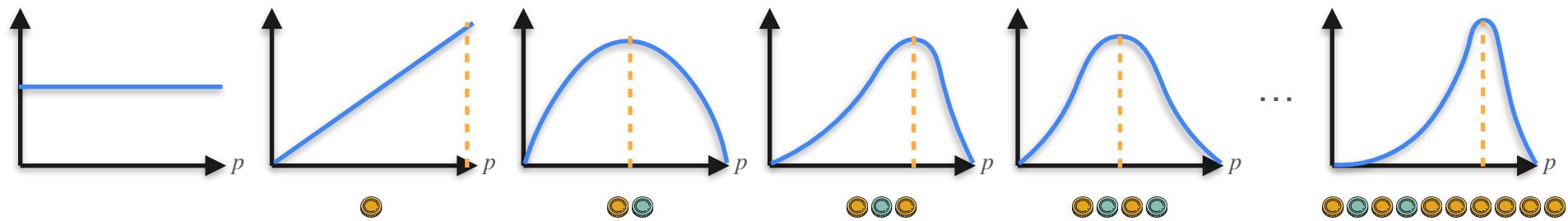
Bayesian Approach



Bayesian Approach



p = probability of heads



Bayesian Statistics

Bayesian Statistics

Remember Bayes' theorem? $\mathbf{P}(B|A) = \frac{\mathbf{P}(A|B)\mathbf{P}(B)}{\mathbf{P}(A)}$

Bayesian Statistics

Remember Bayes' theorem?

$$\boxed{P(B|A)} = \frac{P(A|B)P(B)}{P(A)}$$

↑
Posterior

Bayesian Statistics

Remember Bayes' theorem?

$$P(B|A) = \frac{P(A|B)P(B)}{P(A)}$$

↑
Posterior

← Prior

Bayesian Statistics

Remember Bayes' theorem?

$$P(B|A) = \frac{P(A|B)P(B)}{P(A)}$$

The diagram illustrates the components of Bayes' theorem:

- Posterior**: The term $P(B|A)$ is highlighted with an orange box and labeled "Posterior" with an orange arrow pointing to it.
- Prior**: The term $P(B)$ is highlighted with a blue box and labeled "Prior" with a blue arrow pointing to it.
- Normalizing constant**: The term $P(A)$ is highlighted with a green box and labeled "Normalizing constant" with a green arrow pointing to it.

Bayesian Statistics

The parameters
(or model) you
want to estimate

Remember Bayes' theorem?

$$P(B|A) = \frac{P(A|B)P(B)}{P(A)}$$

Prior

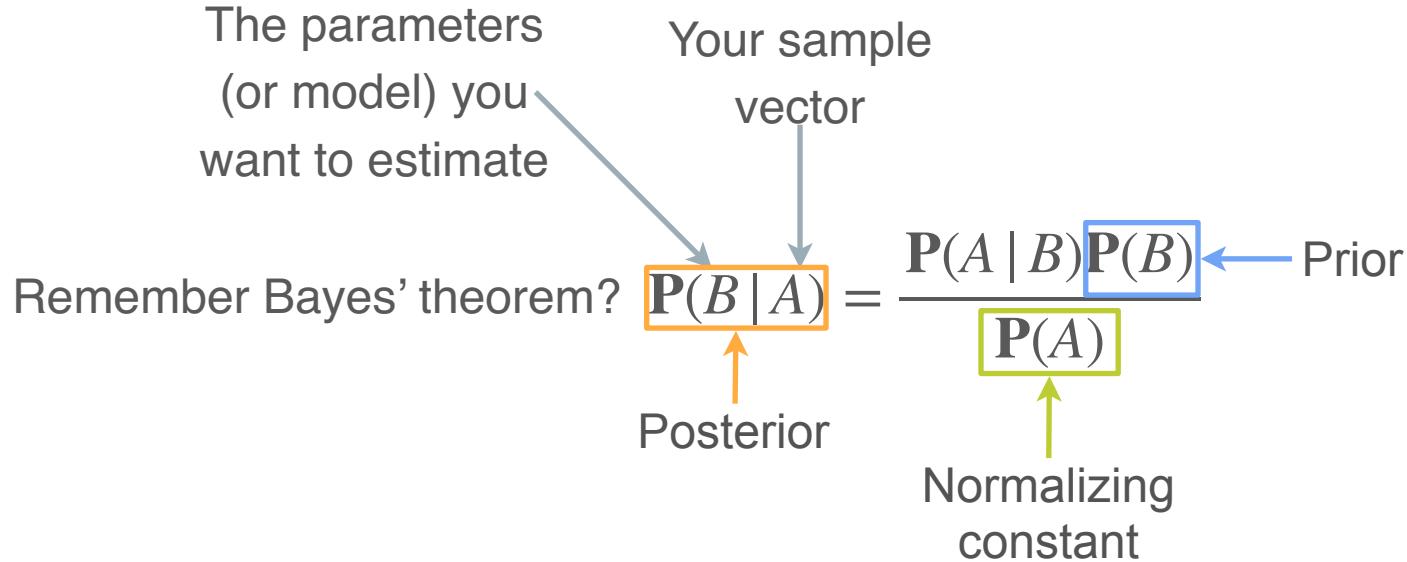
Posterior

Normalizing constant

The diagram illustrates Bayes' theorem with the following components and arrows:

- Posterior:** $P(B|A)$ is highlighted with an orange box and has an orange arrow pointing up to it from the text "The parameters (or model) you want to estimate".
- Prior:** $P(B)$ is highlighted with a blue box and has a blue arrow pointing left to it from the text "Remember Bayes' theorem?".
- Normalizing constant:** $P(A)$ is highlighted with a green box and has a green arrow pointing up to it from the text "Normalizing constant".

Bayesian Statistics



Bayesian Statistics

The parameters
(or model) you
want to estimate

Your sample
vector

Remember Bayes' theorem?

$$P(B|A) = \frac{P(A|B)P(B)}{P(A)}$$

Prior

Posterior

Normalizing
constant

The diagram illustrates the components of Bayes' theorem. At the top left, text asks 'Remember Bayes' theorem?'. To its right, an orange box contains the term $P(B|A)$. Above this box, a grey arrow points from the text 'The parameters (or model) you want to estimate'. Another grey arrow points from the text 'Your sample vector'. Below the orange box, a blue arrow points up to the fraction line, indicating the numerator $P(A|B)P(B)$. To the right of this arrow, the word 'Prior' is written. Below the fraction line, a green arrow points up to the denominator $P(A)$, labeled 'Normalizing constant' to its right.

Bayesian Statistics: Bernoulli Example

Remember Bayes' theorem? $P(B|A) = \frac{P(A|B)P(B)}{P(A)}$

Bayesian Statistics: Bernoulli Example

Remember Bayes' theorem? $P(B|A) = \frac{P(A|B)P(B)}{P(A)}$



Bayesian Statistics: Bernoulli Example

Remember Bayes' theorem? $P(B|A) = \frac{P(A|B)P(B)}{P(A)}$



$$\Theta = P(H)$$

Bayesian Statistics: Bernoulli Example

Remember Bayes' theorem? $P(B|A) = \frac{P(A|B)P(B)}{P(A)}$



Θ is a random variable $\rightarrow \Theta = P(H)$

Bayesian Statistics: Bernoulli Example

Remember Bayes' theorem? $P(B|A) = \frac{P(A|B)P(B)}{P(A)}$



Θ is a random variable $\rightarrow \Theta = P(H)$ $X_i = 1$ if $H, 0$ if T

Bayesian Statistics: Bernoulli Example

Remember Bayes' theorem? $P(B|A) = \frac{P(A|B)P(B)}{P(A)}$



Θ is a random variable $\rightarrow \Theta = P(H)$ $X_i = 1$ if $H, 0$ if T

Observations: $\sum_{i=1}^{10} X_i = 8$

Bayesian Statistics: Bernoulli Example

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Suppose that Θ takes on the particular value θ

Bayesian Statistics: Bernoulli Example

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$$P(A | B)$$

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Θ is a random variable $\rightarrow \Theta = P(H)$ $X_i = 1$ if $H, 0$ if T

Observations: $\sum_{i=1}^{10} X_i = 8 \leftarrow A$

Suppose that Θ takes on the particular value θ

$$P(A|B)$$

Bayesian Statistics: Bernoulli Example

Remember Bayes' theorem? $P(B|A) = \frac{P(A|B)P(B)}{P(A)}$



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Observations: $\sum_{i=1}^{10} X_i = 8 \leftarrow A$

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$$P(A|B)$$

Bayesian Statistics: Bernoulli Example

Remember Bayes' theorem? $P(B|A) = \frac{P(A|B)P(B)}{P(A)}$



Θ is a random variable $\rightarrow \Theta = P(H)$ $X_i = 1$ if $H, 0$ if T

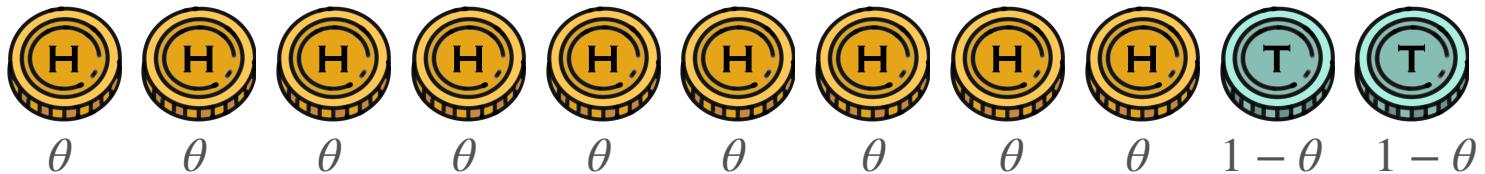
Observations: $\sum_{i=1}^{10} X_i = 8 \leftarrow A$

Suppose that Θ takes on the particular value $\theta \leftarrow B$

$$P(A|B) \rightarrow P\left(\sum_{i=1}^{10} X_i = 8 | \Theta = \theta\right) =$$

Bayesian Statistics: Bernoulli Example

Remember Bayes' theorem? $P(B|A) = \frac{P(A|B)P(B)}{P(A)}$



Θ is a random variable $\rightarrow \Theta = P(H)$ $X_i = 1$ if $H, 0$ if T

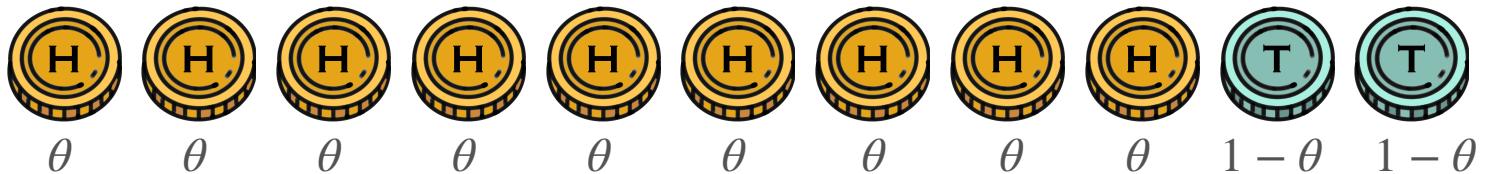
Observations: $\sum_{i=1}^{10} X_i = 8 \leftarrow A$

Suppose that Θ takes on the particular value $\theta \leftarrow B$

$$P(A|B) \rightarrow P\left(\sum_{i=1}^{10} X_i = 8 | \Theta = \theta\right) = \theta^8(1 - \theta)^2$$

Bayesian Statistics: Bernoulli Example

Remember Bayes' theorem? $P(B|A) = \frac{P(A|B)P(B)}{P(A)}$



Θ is a random variable $\rightarrow \Theta = P(H)$ $X_i = 1$ if $H, 0$ if T

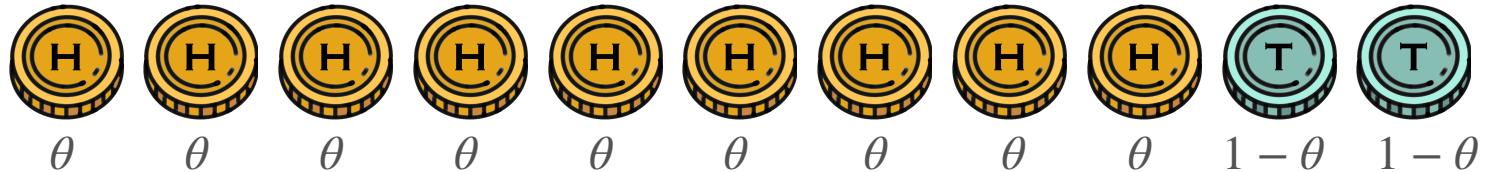
Observations: $\sum_{i=1}^{10} X_i = 8 \leftarrow A$

Suppose that Θ takes on the particular value $\theta \leftarrow B$

$$P(A|B) \rightarrow P\left(\sum_{i=1}^{10} X_i = 8 | \Theta = \theta\right) = \theta^8(1 - \theta)^2 \quad X_i | \Theta = \theta \sim Bernoulli(\theta)$$

Bayesian Statistics: Bernoulli Example

Remember Bayes' theorem? $P(B|A) = \frac{P(A|B)P(B)}{P(A)}$

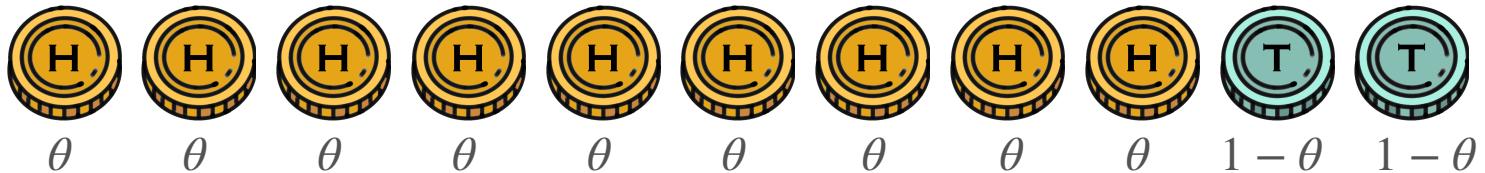


$$X_i | \Theta = \theta \sim \text{Bernoulli}(\theta)$$

How do you choose the prior?

Bayesian Statistics: Bernoulli Example

Remember Bayes' theorem? $P(B|A) = \frac{P(A|B)P(B)}{P(A)}$



$$X_i | \Theta = \theta \sim \text{Bernoulli}(\theta)$$

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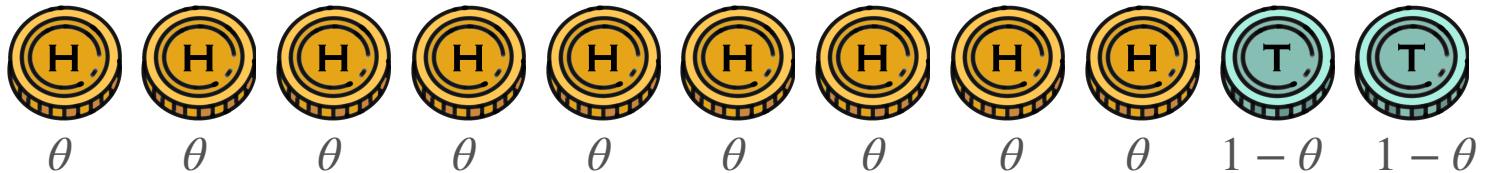
If before tossing the coins you have no idea whether the coin is fair or not, then your prior belief about Θ should not favor any value.

You can model this with a uniform distribution

$$\Theta \sim \text{Uniform}(0,1) \quad f_{\Theta}(\theta) = 1, \quad 0 \leq \theta \leq 1$$

Bayesian Statistics: Bernoulli Example

Remember Bayes' theorem? $P(B|A) = \frac{P(A|B)P(B)}{P(A)}$



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How do you choose the prior?

If before tossing the coins you have no idea whether the coin is fair or not, then your prior belief about Θ should not favor any value.

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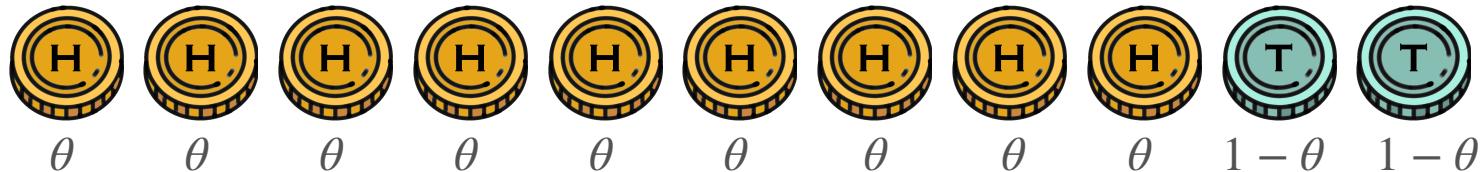
$$\Theta \sim \text{Uniform}(0,1)$$

$$f_{\Theta}(\theta) = 1, \quad 0 \leq \theta \leq 1$$

← Prior ($P(B)$)

Bayesian Statistics: Bernoulli Example

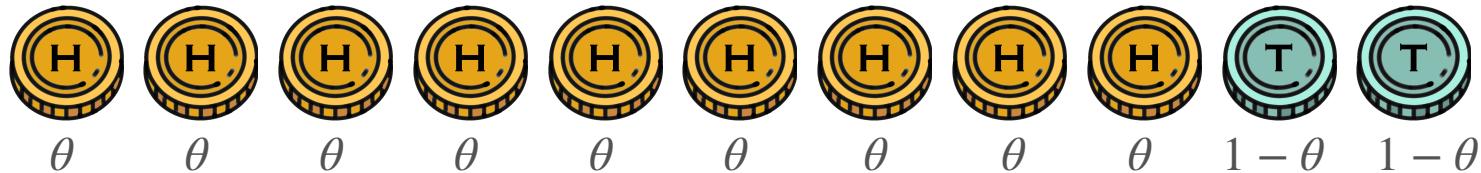
Remember Bayes' theorem? $P(B|A) = \frac{P(A|B)P(B)}{P(A)}$



$$X_i | \Theta = \theta \sim \text{Bernoulli}(\theta) \xleftarrow{\text{(P(A | B))}} \quad \Theta \sim \text{Uniform}(0,1) \xleftarrow{\text{Prior (P(B))}}$$

Bayesian Statistics: Bernoulli Example

Remember Bayes' theorem? $P(B|A) = \frac{P(A|B)P(B)}{P(A)}$

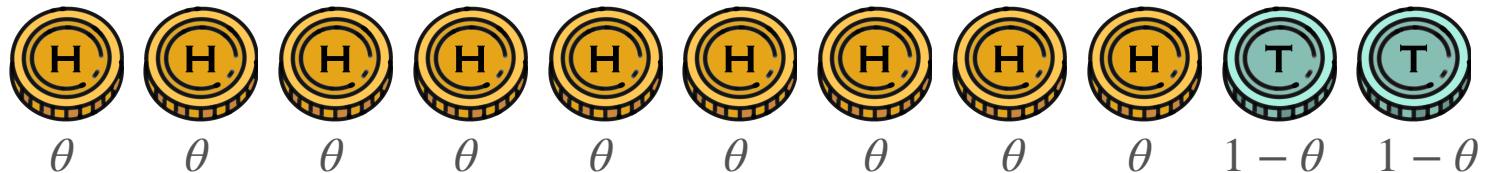


$$X_i | \Theta = \theta \sim \text{Bernoulli}(\theta) \xleftarrow{\text{(P(A | B))}} \quad \Theta \sim \text{Uniform}(0,1) \xleftarrow{\text{Prior (P(B))}}$$

How do you get the posterior? Bayes!

Bayesian Statistics: Bernoulli Example

Remember Bayes' theorem? $P(B|A) = \frac{P(A|B)P(B)}{P(A)}$



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How do you get the posterior? Bayes!

$$f_{\Theta|\mathbf{X}=\mathbf{x}}(\theta) = \frac{p_{\mathbf{X}|\Theta=\theta}(\mathbf{x})f_{\Theta}(\theta)}{p_{\mathbf{X}}(\mathbf{x})}$$

Bayesian Statistics: Bernoulli Example

Remember Bayes' theorem? $P(B|A) = \frac{P(A|B)P(B)}{P(A)}$



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How do you get the posterior? Bayes!

$$f_{\Theta|X=x}(\theta) = \frac{p_{X|\Theta=\theta}(x)f_{\Theta}(\theta)}{p_X(x)}$$

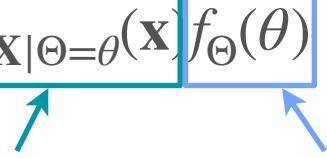
↑
Posterior ($P(B|A)$)

Normalizing constant

Bayesian Statistics: MAP Estimator

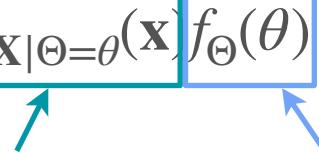
$$f_{\Theta|X=x}(\theta) \propto p_{X|\Theta=\theta}(x)f_{\Theta}(\theta)$$

Bayesian Statistics: MAP Estimator

$$f_{\Theta|X=x}(\theta) \propto p_{X|\Theta=\theta}(x) f_{\Theta}(\theta)$$


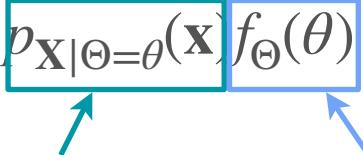
P(data | model) P(model)

Bayesian Statistics: MAP Estimator

$$f_{\Theta|X=x}(\theta) \propto p_{X|\Theta=\theta}(x) f_{\Theta}(\theta)$$


Does this look familiar? $\mathbf{P}(\text{data} \mid \text{model})$ $\mathbf{P}(\text{model})$

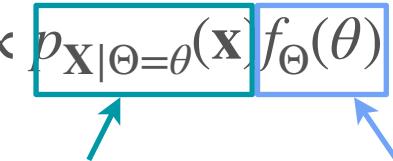
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Point estimators?

Bayesian Statistics: MAP Estimator

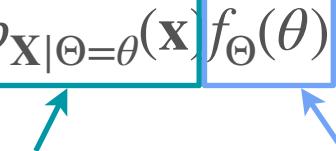
$$f_{\Theta|X=x}(\theta) \propto p_{X|\Theta=\theta}(x) f_{\Theta}(\theta)$$


Does this look familiar? $\text{P}(\text{data} | \text{model})$ $\text{P}(\text{model})$

Point estimators?

$$\hat{\theta} = \arg \max_{\theta} f_{\Theta|X=x}(\theta) = \arg \max_{\theta} p_{X|\Theta=\theta}(x) f_{\Theta}(\theta)$$

Bayesian Statistics: MAP Estimator

$$f_{\Theta|X=x}(\theta) \propto p_{X|\Theta=\theta}(x) f_{\Theta}(\theta)$$


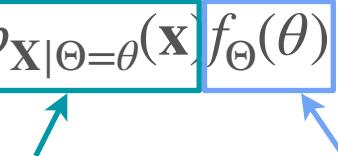
Does this look familiar? $\mathbf{P}(\text{data} \mid \text{model})$ $\mathbf{P}(\text{model})$

Point estimators?

$$\hat{\theta} = \arg \max_{\theta} f_{\Theta|X=x}(\theta) = \arg \max_{\theta} p_{X|\Theta=\theta}(x) f_{\Theta}(\theta)$$

This is called the Maximum a Posteriori (MAP) estimator

Bayesian Statistics: MAP Estimator

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Does this look familiar? $\text{P}(\text{data} | \text{model})$ $\text{P}(\text{model})$

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$$\hat{\theta} = \arg \max_{\theta} f_{\Theta|X=x}(\theta) = \arg \max_{\theta} p_{X|\Theta=\theta}(x) f_{\Theta}(\theta)$$

This is called the Maximum a Posteriori (MAP) estimator

Notice that if the prior is non informative, then MAP = MLE

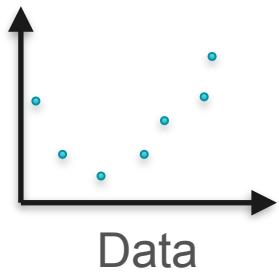


DeepLearning.AI

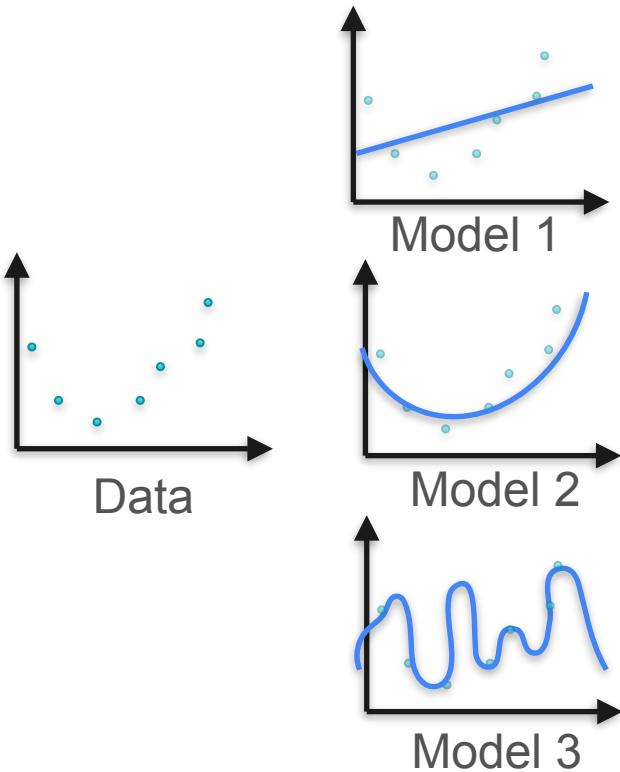
Point Estimation

Regularization

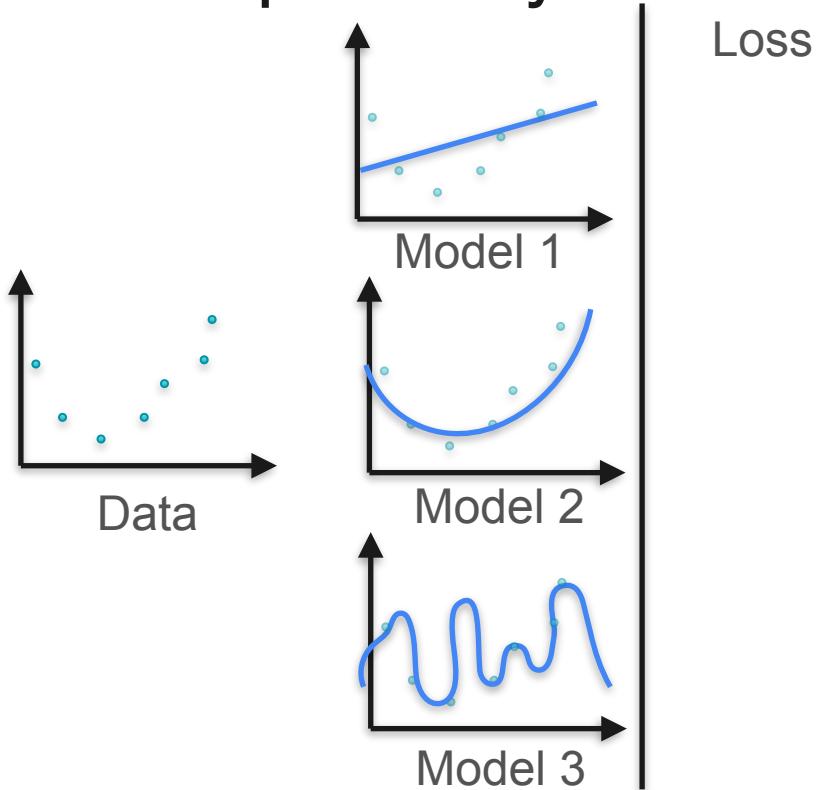
Example: Polynomial Regression



Example: Polynomial Regression

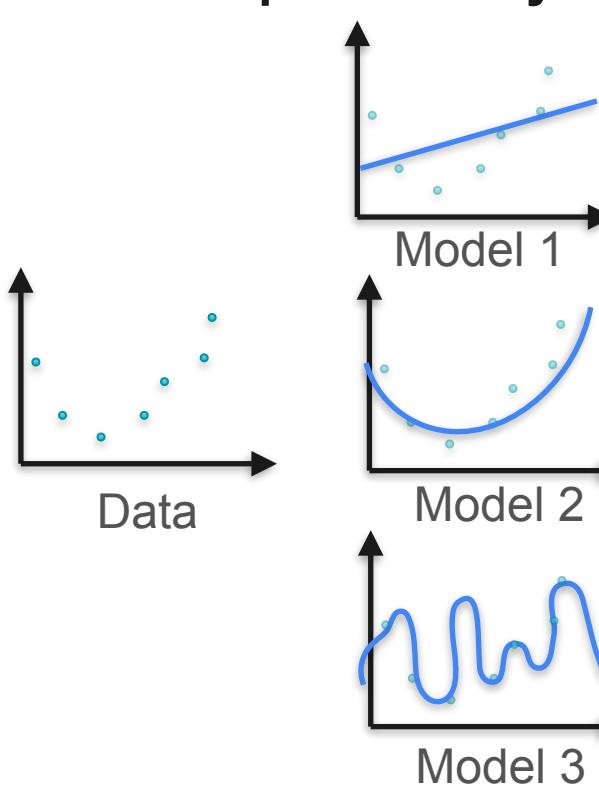


Example: Polynomial Regression



Loss

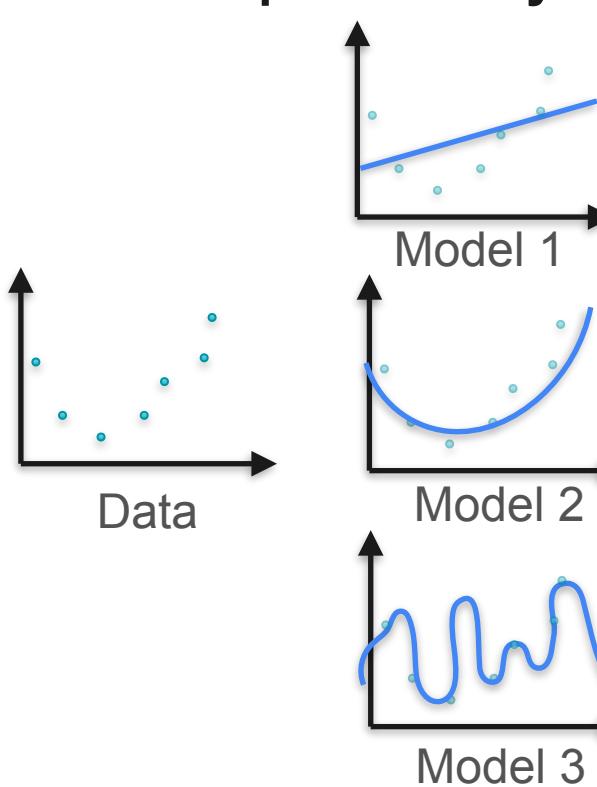
Example: Polynomial Regression



Loss

10

Example: Polynomial Regression

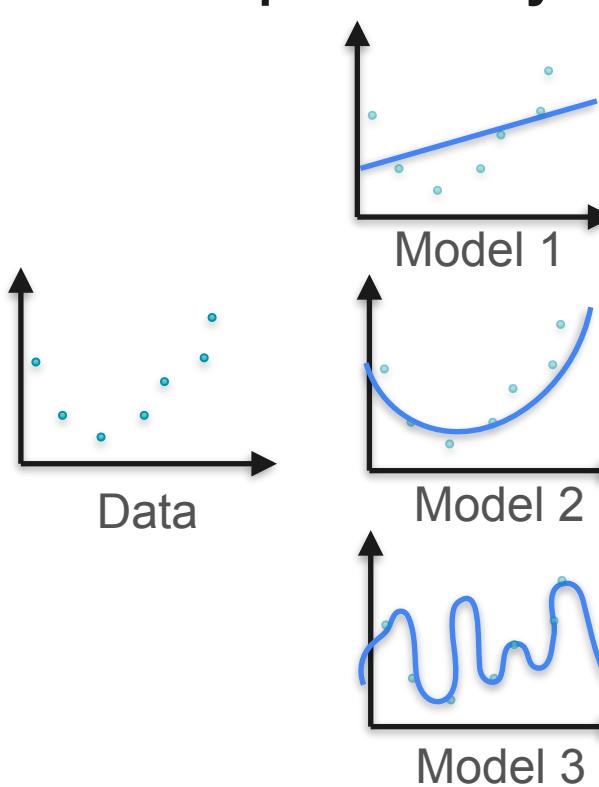


Loss

10

2

Example: Polynomial Regression



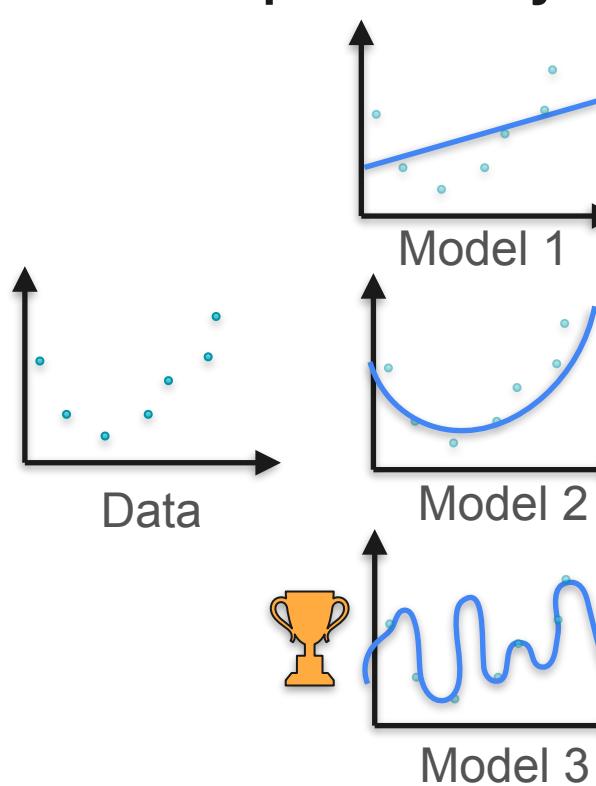
Loss

10

2

0.1

Example: Polynomial Regression



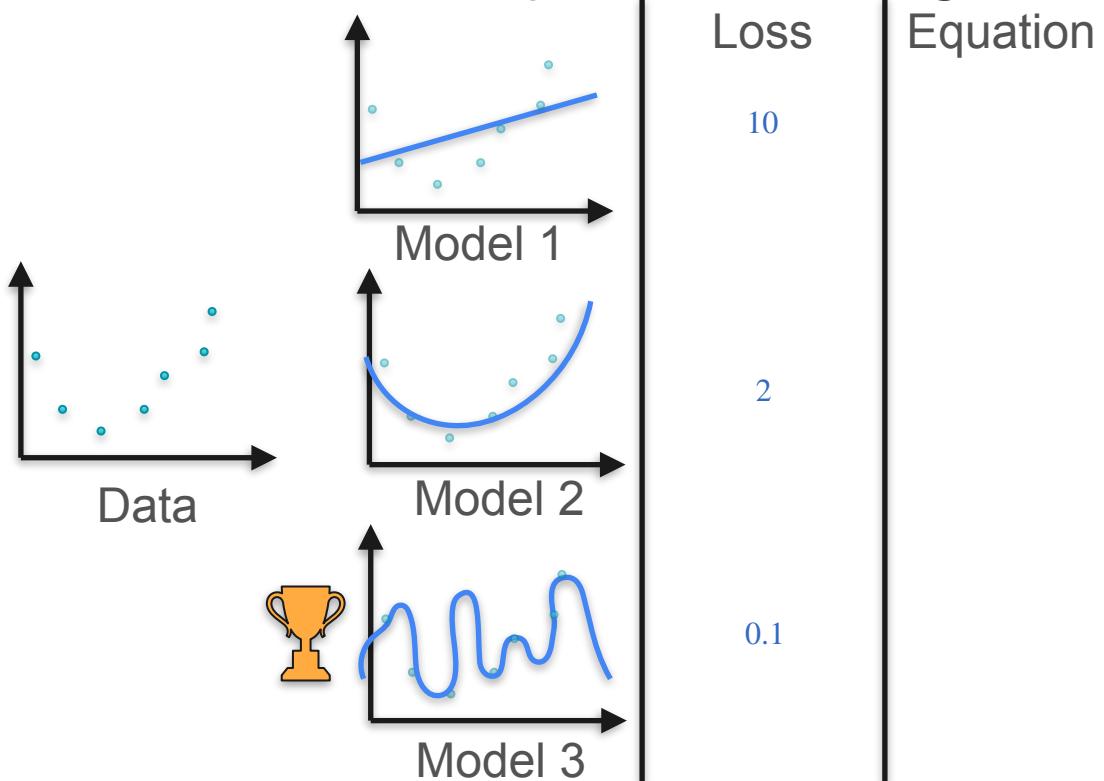
Loss

10

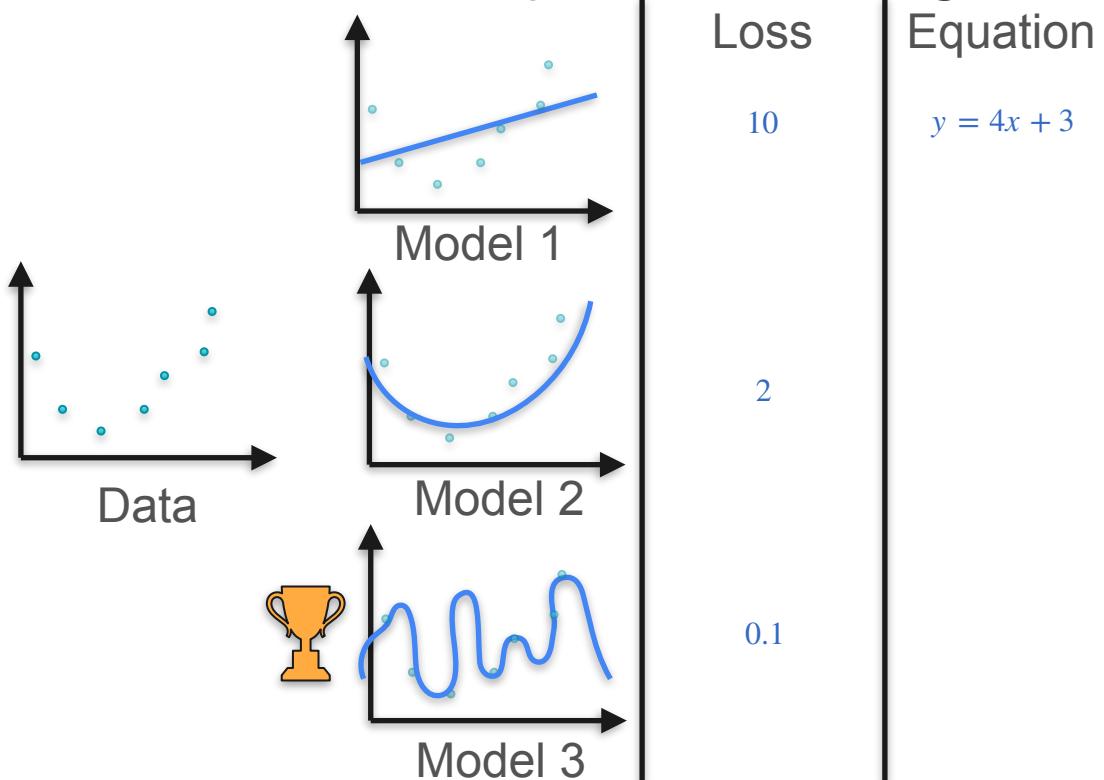
2

0.1

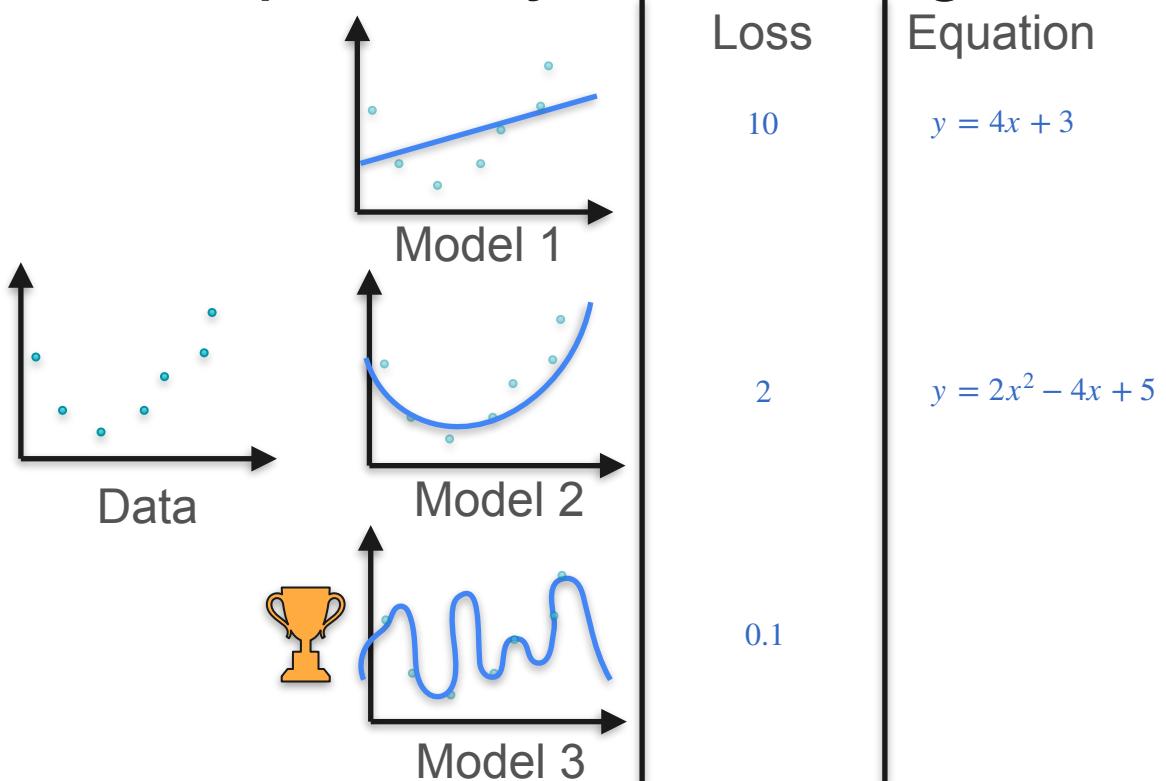
Example: Polynomial Regression



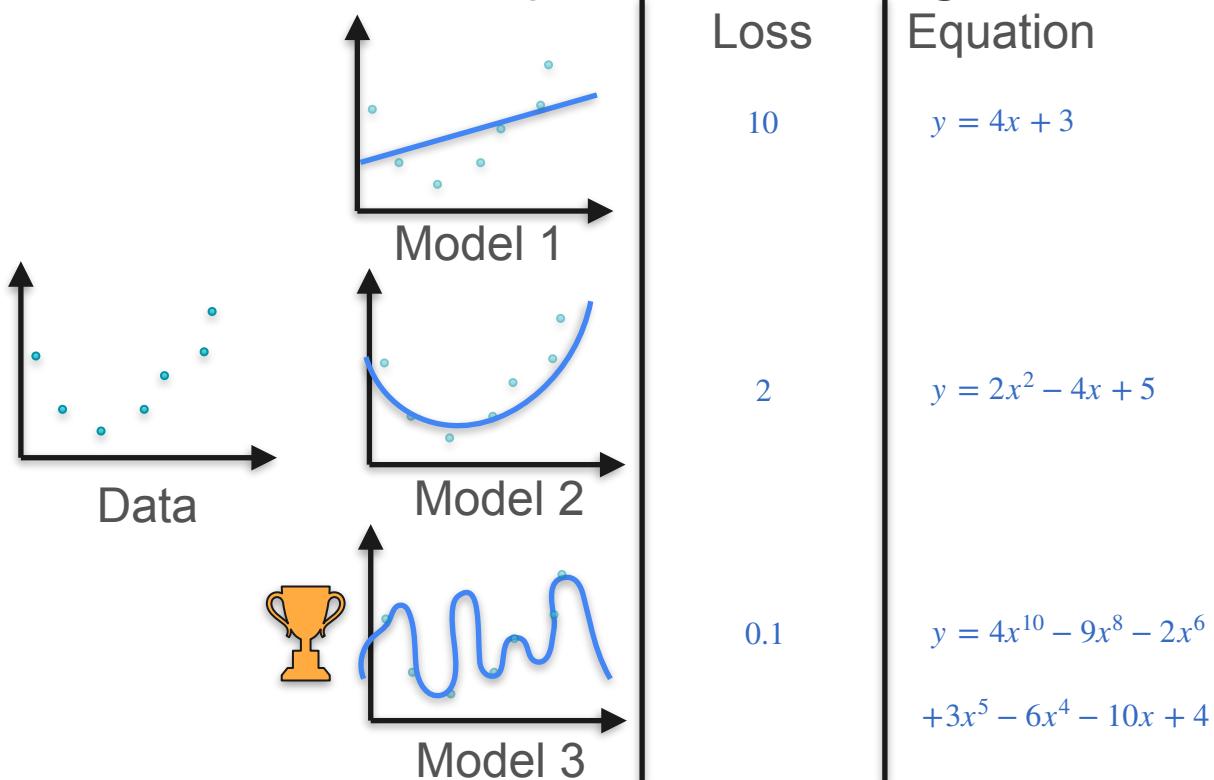
Example: Polynomial Regression



Example: Polynomial Regression



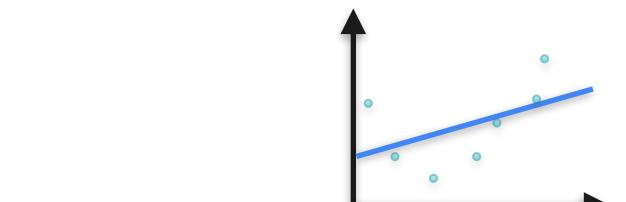
Example: Polynomial Regression



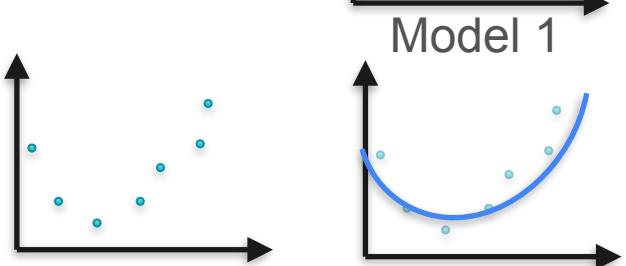
Example: Polynomial Regression



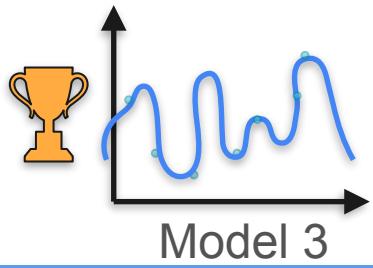
Data



Model 1



Model 2



Model 3

Loss

10

Equation

$$y = 4x + 3$$

2

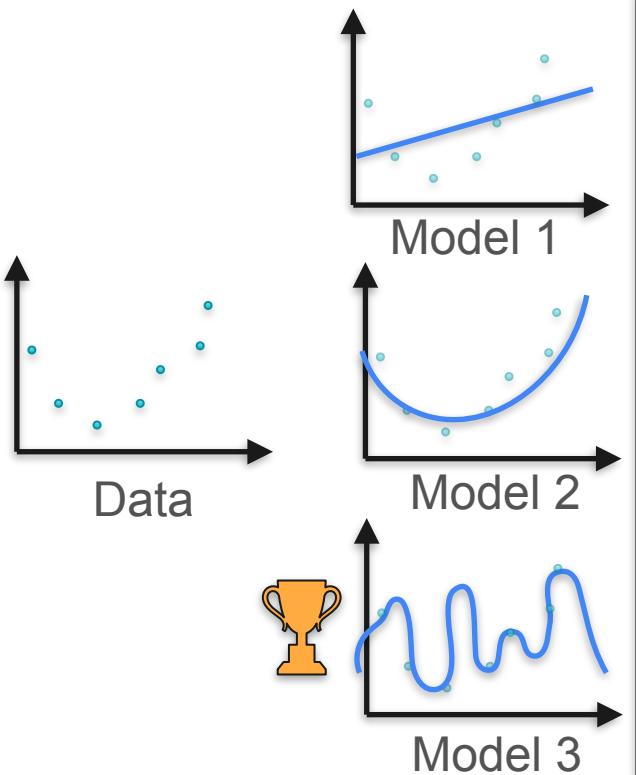
$$y = 2x^2 - 4x + 5$$

0.1

$$y = 4x^{10} - 9x^8 - 2x^6 \\ + 3x^5 - 6x^4 - 10x + 4$$

Penalty

Example: Polynomial Regression



Loss

10

Equation

$$y = 4x + 3$$

Penalty

$$L_2 = 4^2$$

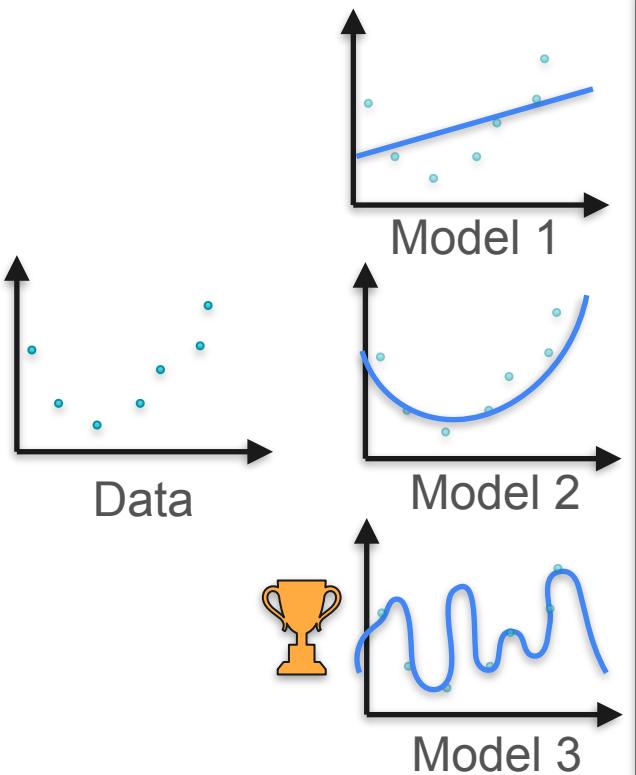
2

$$y = 2x^2 - 4x + 5$$

0.1

$$y = 4x^{10} - 9x^8 - 2x^6 \\ + 3x^5 - 6x^4 - 10x + 4$$

Example: Polynomial Regression



Loss

10

Equation

$$y = 4x + 3$$

Penalty

$$\begin{aligned}L_2 &= 4^2 \\&= 16\end{aligned}$$

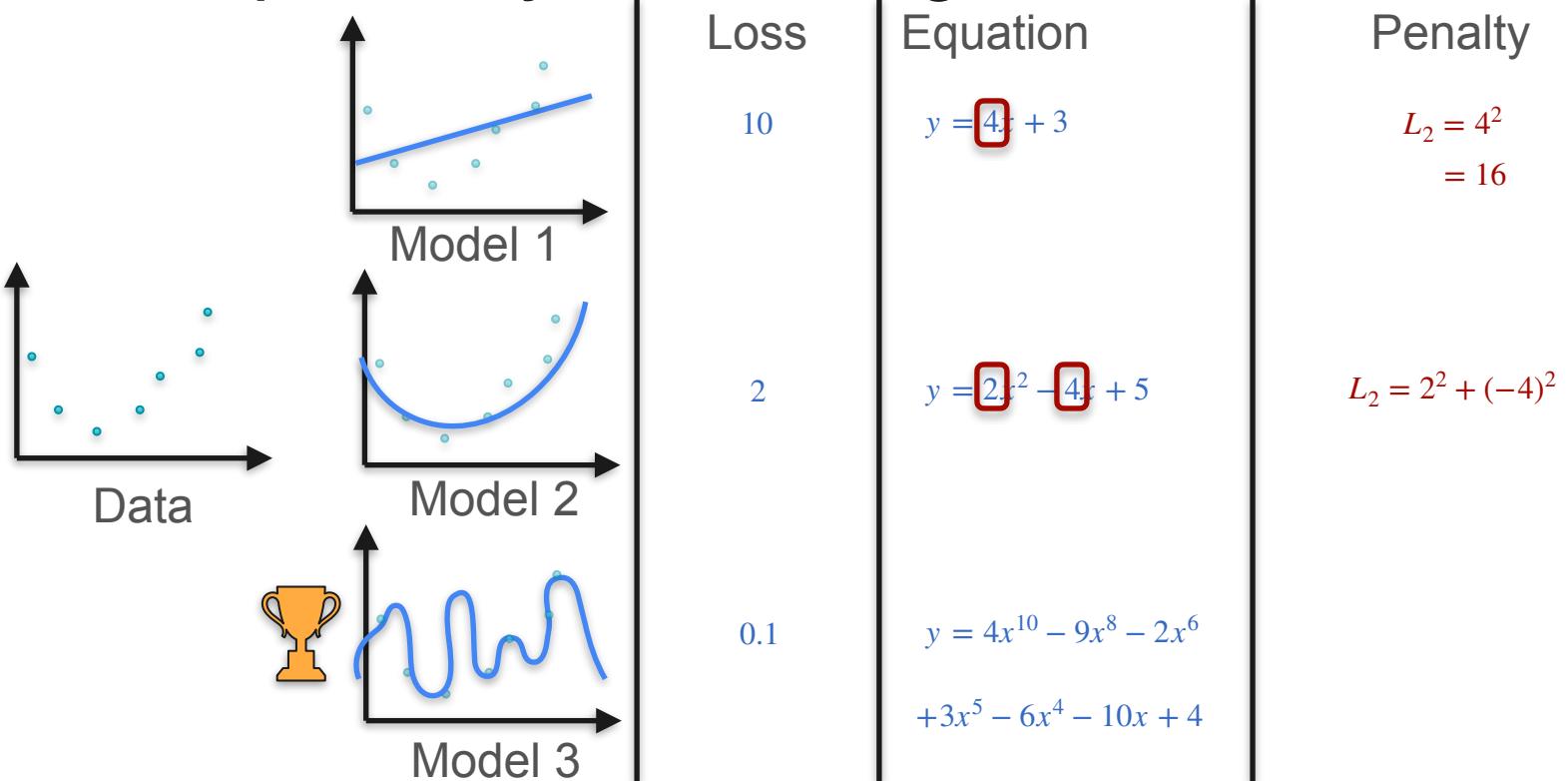
2

$$y = 2x^2 - 4x + 5$$

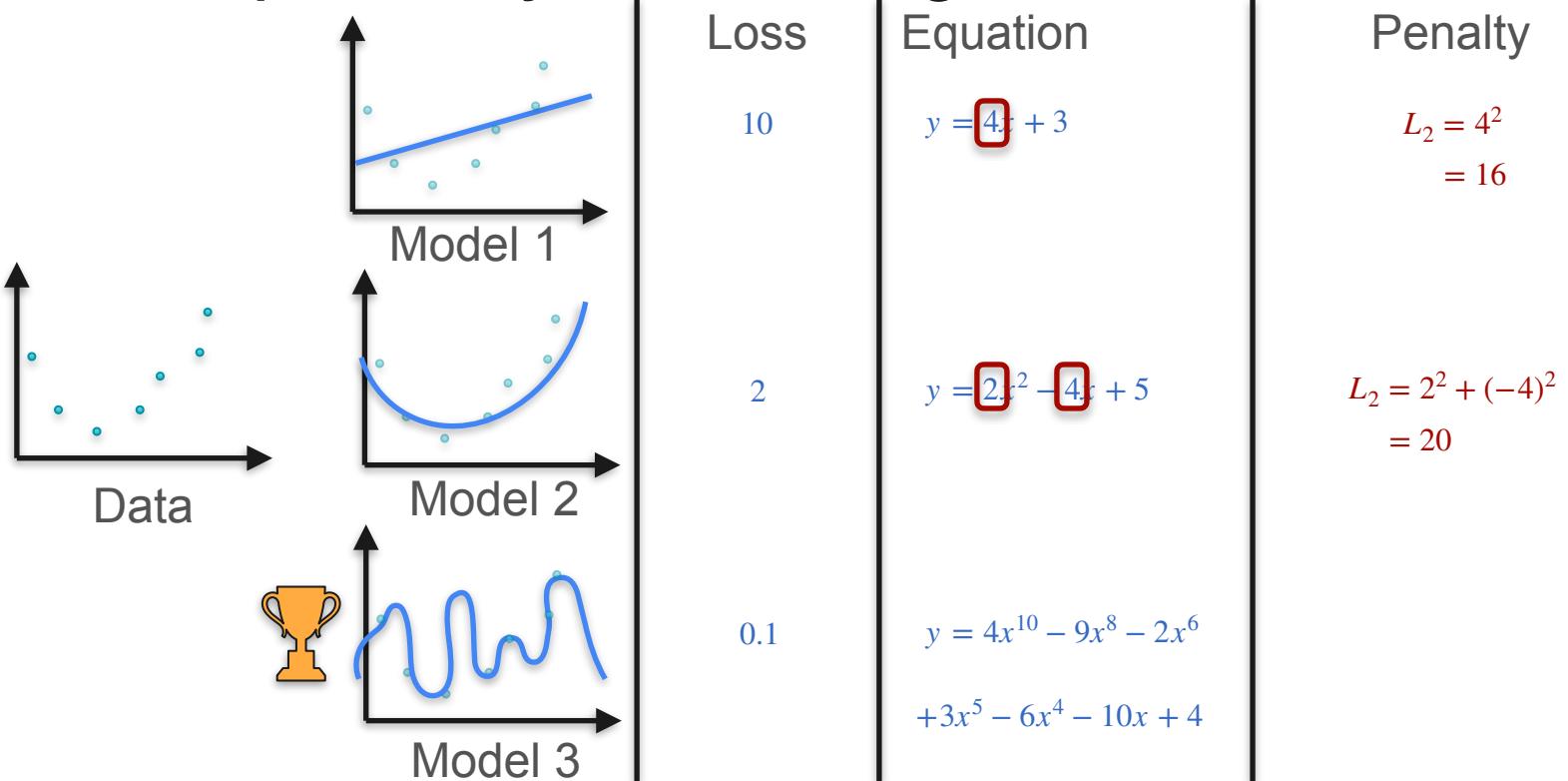
0.1

$$\begin{aligned}y &= 4x^{10} - 9x^8 - 2x^6 \\&\quad + 3x^5 - 6x^4 - 10x + 4\end{aligned}$$

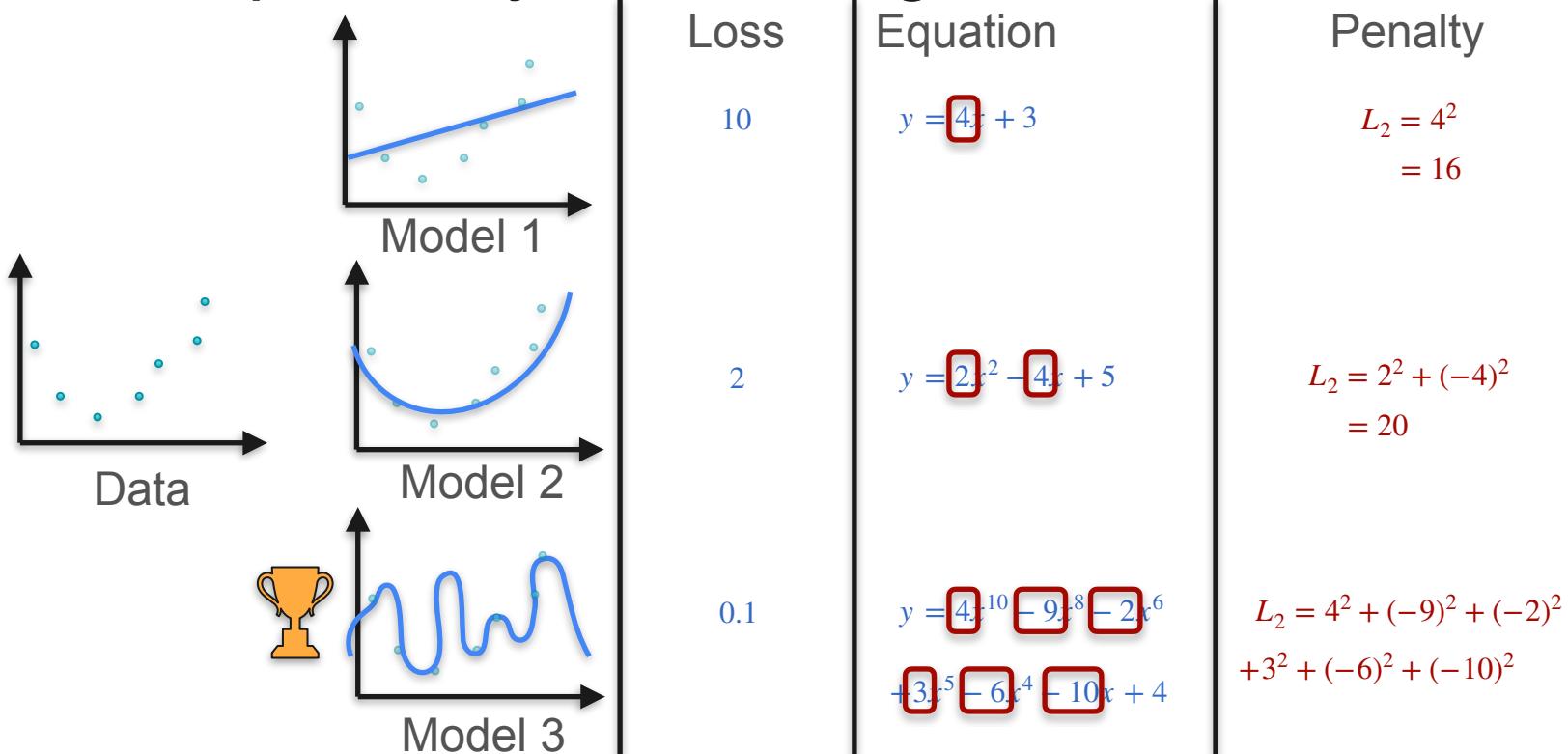
Example: Polynomial Regression



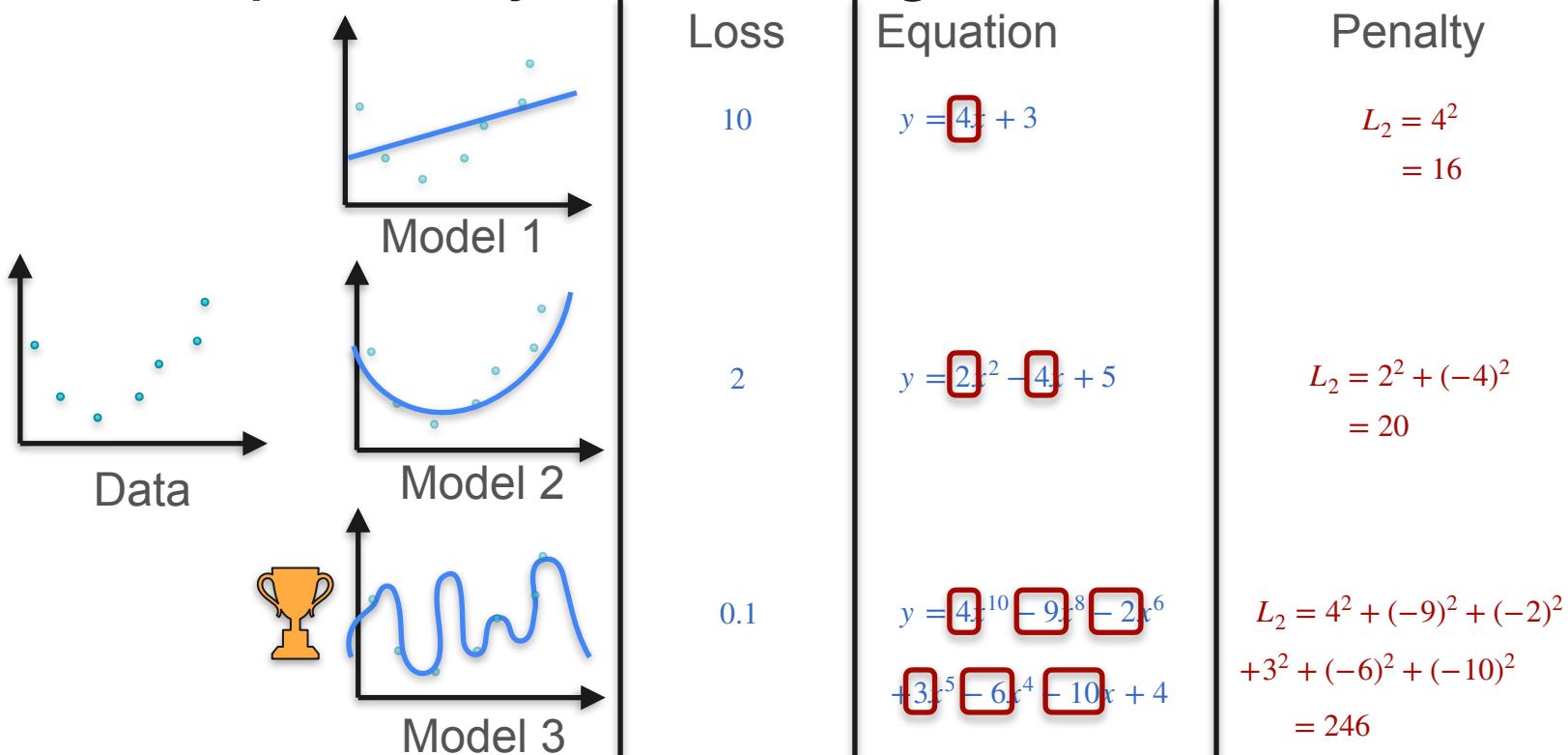
Example: Polynomial Regression



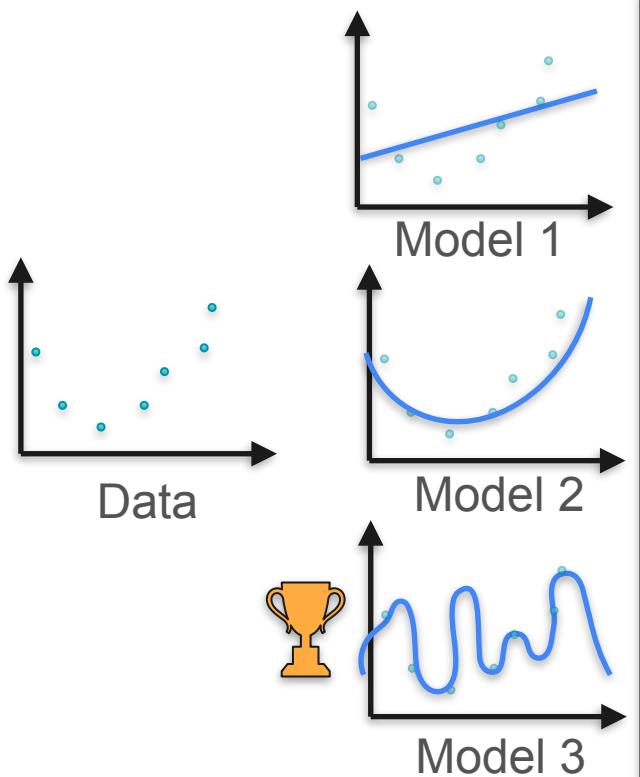
Example: Polynomial Regression



Example: Polynomial Regression

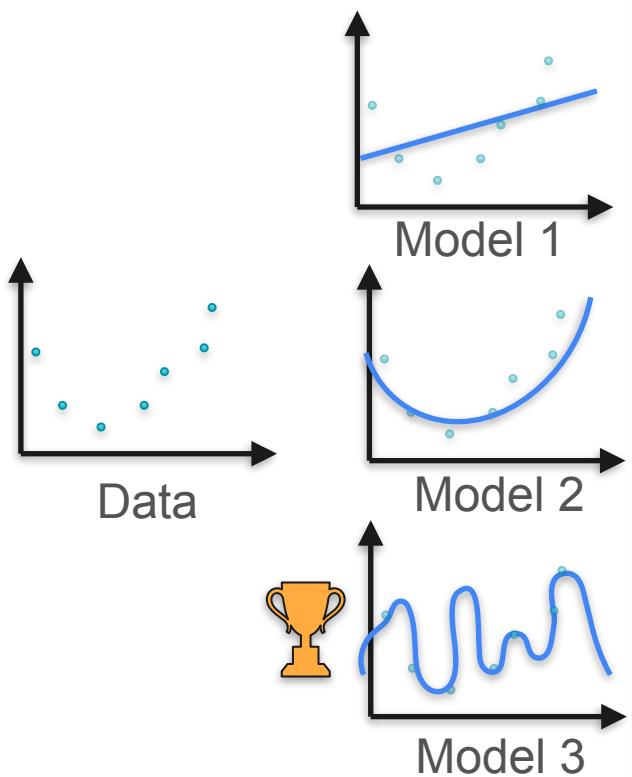


Example: Polynomial Regression



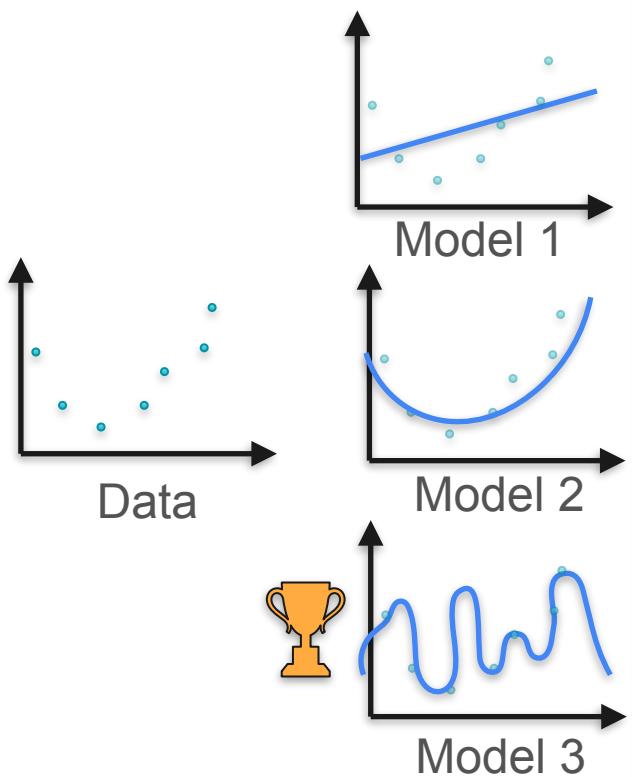
Loss	Equation	Penalty	New loss
10	$y = 4x + 3$	$L_2 = 4^2 = 16$	
2	$y = 2x^2 - 4x + 5$	$L_2 = 2^2 + (-4)^2 = 20$	
0.1	$y = 4x^{10} - 9x^8 - 2x^6 + 3x^5 - 6x^4 - 10x + 4$	$L_2 = 4^2 + (-9)^2 + (-2)^2 + 3^2 + (-6)^2 + (-10)^2 = 246$	

Example: Polynomial Regression



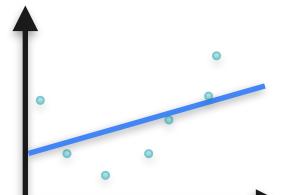
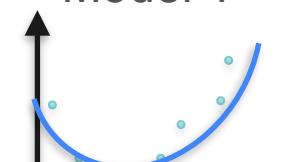
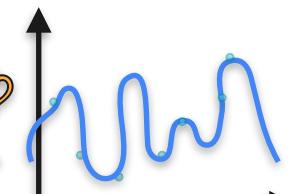
Loss	Equation	Penalty	New loss
10	$y = 4x + 3$	$L_2 = 4^2 = 16$	26
2	$y = 2x^2 - 4x + 5$	$L_2 = 2^2 + (-4)^2 = 20$	
0.1	$y = 4x^{10} - 9x^8 - 2x^6 + 3x^5 - 6x^4 - 10x + 4$	$L_2 = 4^2 + (-9)^2 + (-2)^2 + 3^2 + (-6)^2 + (-10)^2 = 246$	

Example: Polynomial Regression

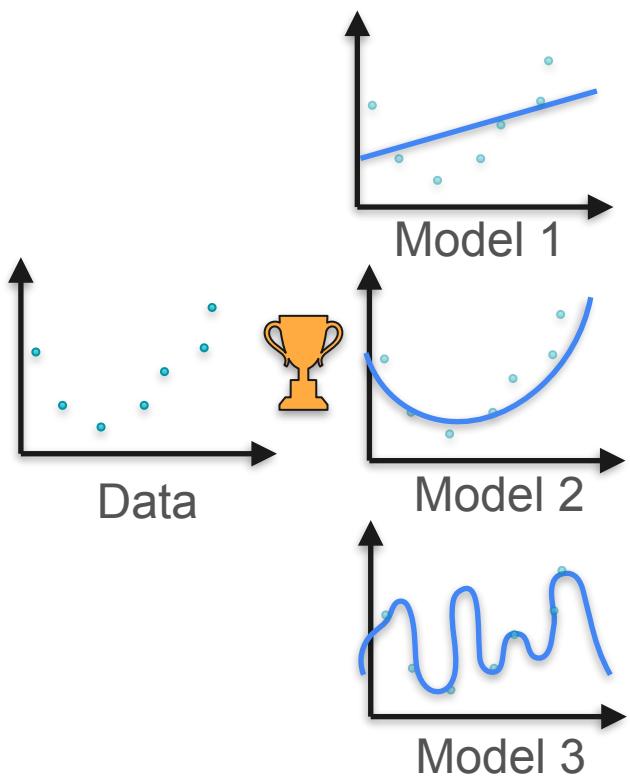


Loss	Equation	Penalty	New loss
10	$y = 4x + 3$	$L_2 = 4^2 = 16$	26
2	$y = 2x^2 - 4x + 5$	$L_2 = 2^2 + (-4)^2 = 20$	22
0.1	$y = 4x^{10} - 9x^8 - 2x^6 + 3x^5 - 6x^4 - 10x + 4$	$L_2 = 4^2 + (-9)^2 + (-2)^2 + 3^2 + (-6)^2 + (-10)^2 = 246$	

Example: Polynomial Regression

Data	Model 1	Loss	Equation	Penalty	New loss
		10	$y = 4x + 3$	$L_2 = 4^2 = 16$	26
		2	$y = 2x^2 - 4x + 5$	$L_2 = 2^2 + (-4)^2 = 20$	22
		0.1	$y = 4x^{10} - 9x^8 - 2x^6 + 3x^5 - 6x^4 - 10x + 4$	$L_2 = 4^2 + (-9)^2 + (-2)^2 + 3^2 + (-6)^2 + (-10)^2 = 246$	246.1

Example: Polynomial Regression



Loss	Equation	Penalty	New loss
10	$y = 4x + 3$	$L_2 = 4^2 = 16$	26
2	$y = 2x^2 - 4x + 5$	$L_2 = 2^2 + (-4)^2 = 20$	22
0.1	$y = 4x^{10} - 9x^8 - 2x^6 + 3x^5 - 6x^4 - 10x + 4$	$L_2 = 4^2 + (-9)^2 + (-2)^2 + 3^2 + (-6)^2 + (-10)^2 = 246$	246.1

Regularization Term

Regularization Term

Model: $y = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$

Regularization Term

Model: $y = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$

Log-loss: $\ell\ell$

Regularization Term

Model: $y = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$

Log-loss: $\ell\ell$

L2 Regularization Error: $a_n^2 + a_{n-1}^2 + \dots + a_1^2$

Regularization Term

Model: $y = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$

Log-loss: $\ell\ell$

L2 Regularization Error: $a_n^2 + a_{n-1}^2 + \dots + a_1^2$

Regularization parameter: λ

Regularization Term

Model: $y = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$

Log-loss: $\ell\ell$

L2 Regularization Error: $a_n^2 + a_{n-1}^2 + \dots + a_1^2$

Regularization parameter: λ

Regularization Term

Model: $y = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$

Log-loss: $\ell\ell$

L2 Regularization Error: $a_n^2 + a_{n-1}^2 + \dots + a_1^2$

Regularization parameter: λ

Regularized error:

Regularization Term

Model: $y = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$

Log-loss: $\ell\ell$

L2 Regularization Error: $a_n^2 + a_{n-1}^2 + \dots + a_1^2$

Regularization parameter: λ

Regularized error: $\ell\ell + \lambda (a_n^2 + a_{n-1}^2 + \dots + a_1^2)$



DeepLearning.AI

Point Estimation

Back to Bayesics

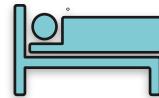
There's Popcorn on the Floor. What Happened?



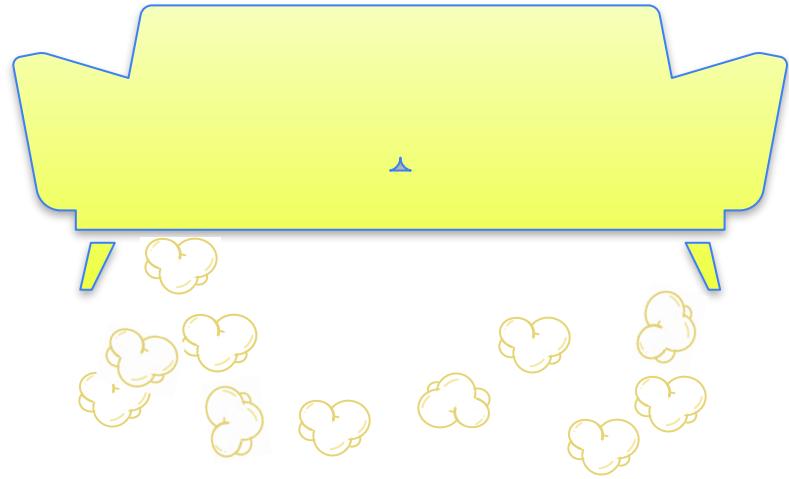
Movies



Board
Games



Nap



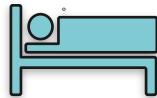
There's Popcorn on the Floor. What Happened?



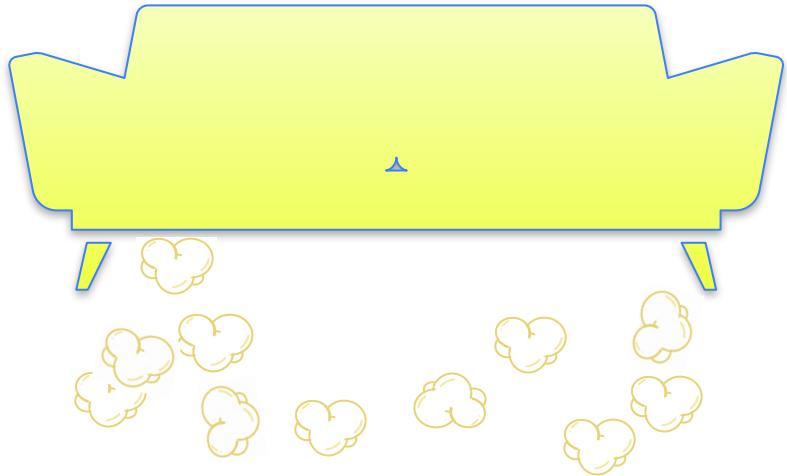
Movies *High*



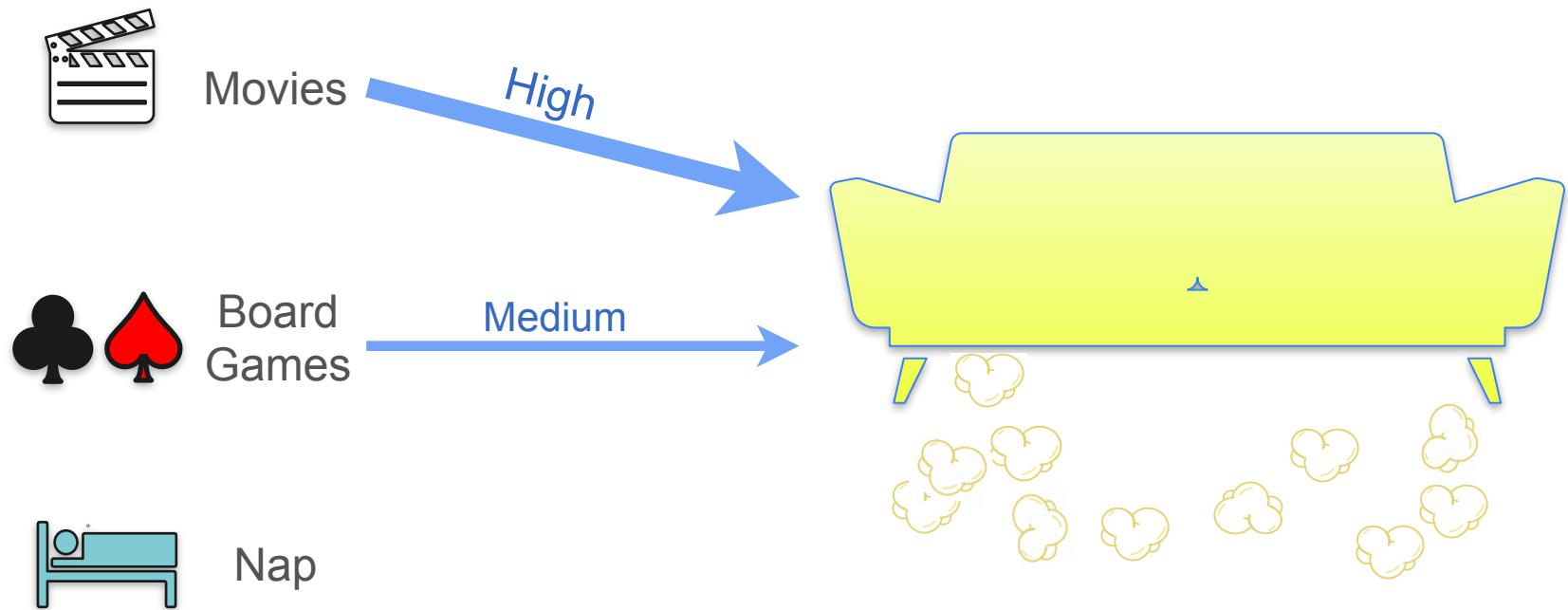
Board Games



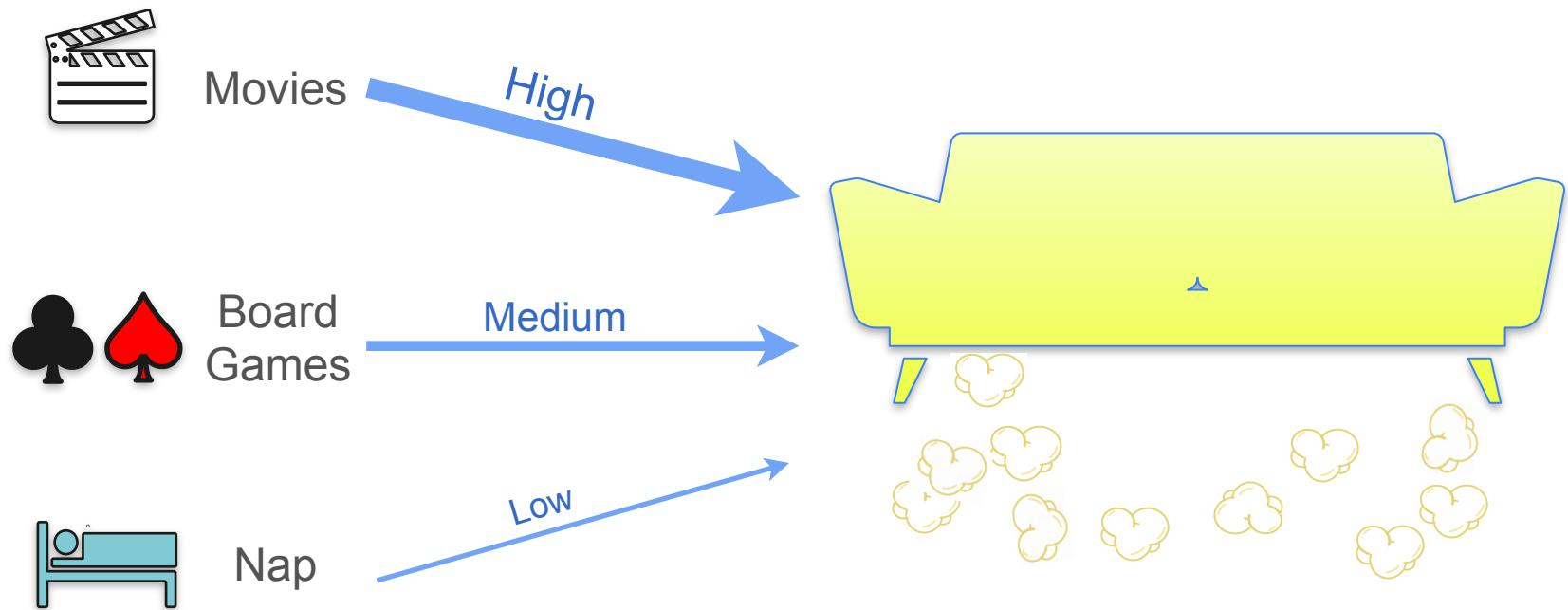
Nap



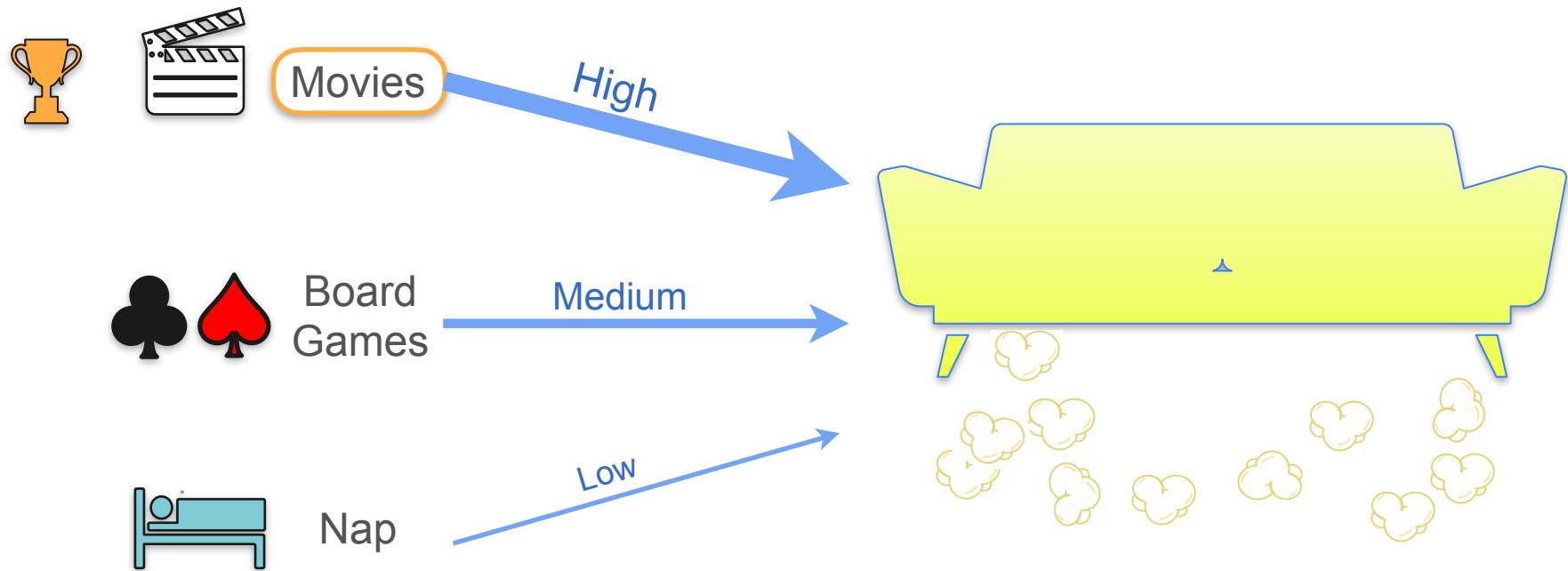
There's Popcorn on the Floor. What Happened?



There's Popcorn on the Floor. What Happened?



There's Popcorn on the Floor. What Happened?



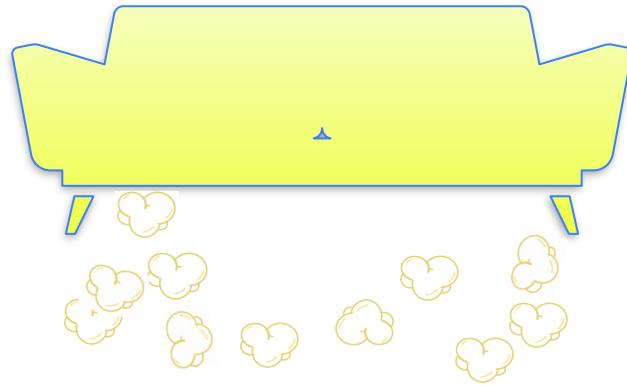
There's Popcorn on the Floor. What Happened?



Movies



Popcorn
throwing
contest



There's Popcorn on the Floor. What Happened?



Movies

High



Popcorn
throwing
contest



There's Popcorn on the Floor. What Happened?



Movies

High

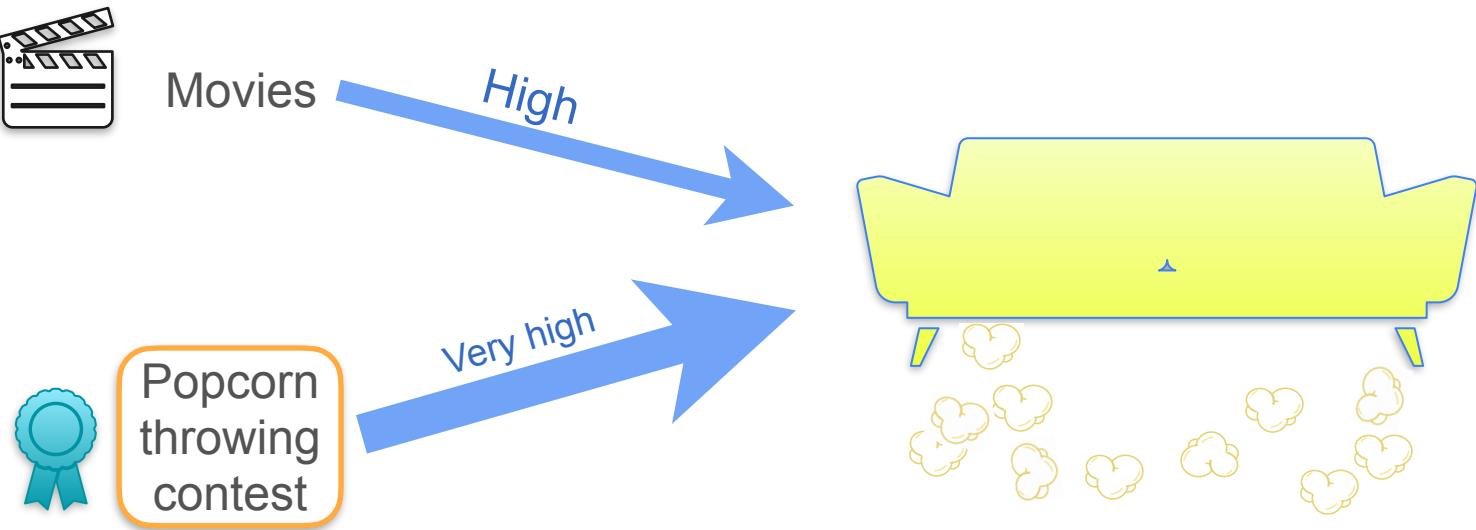


Popcorn
throwing
contest

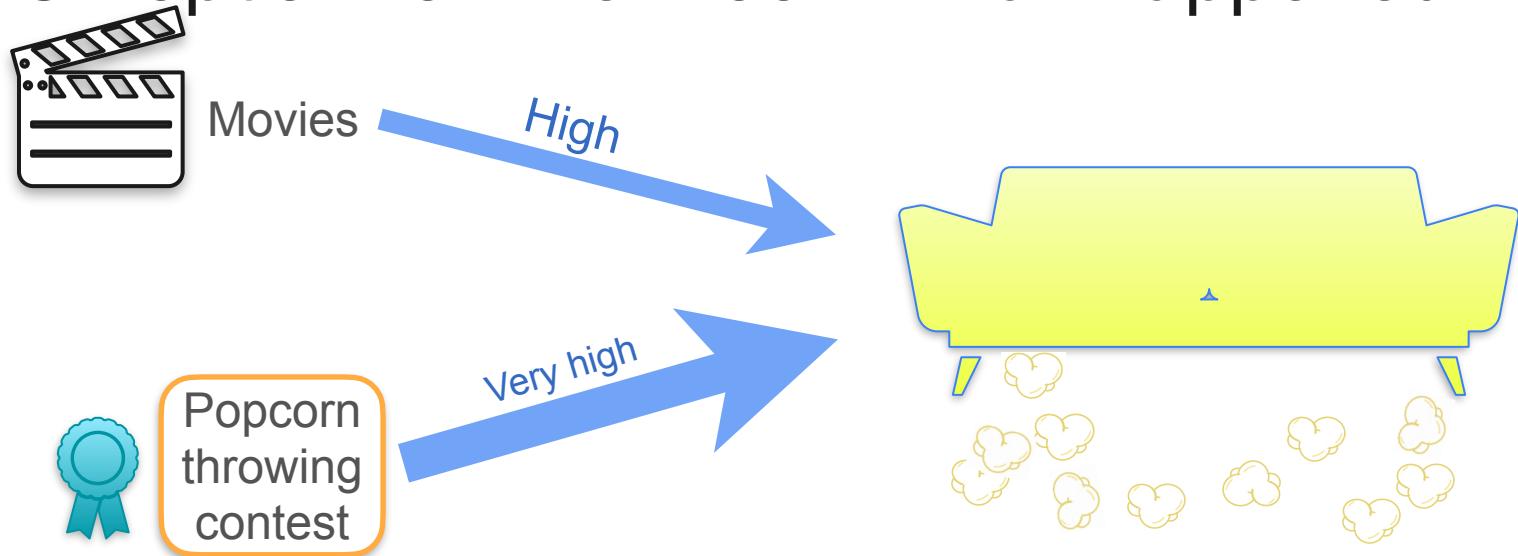
Very high



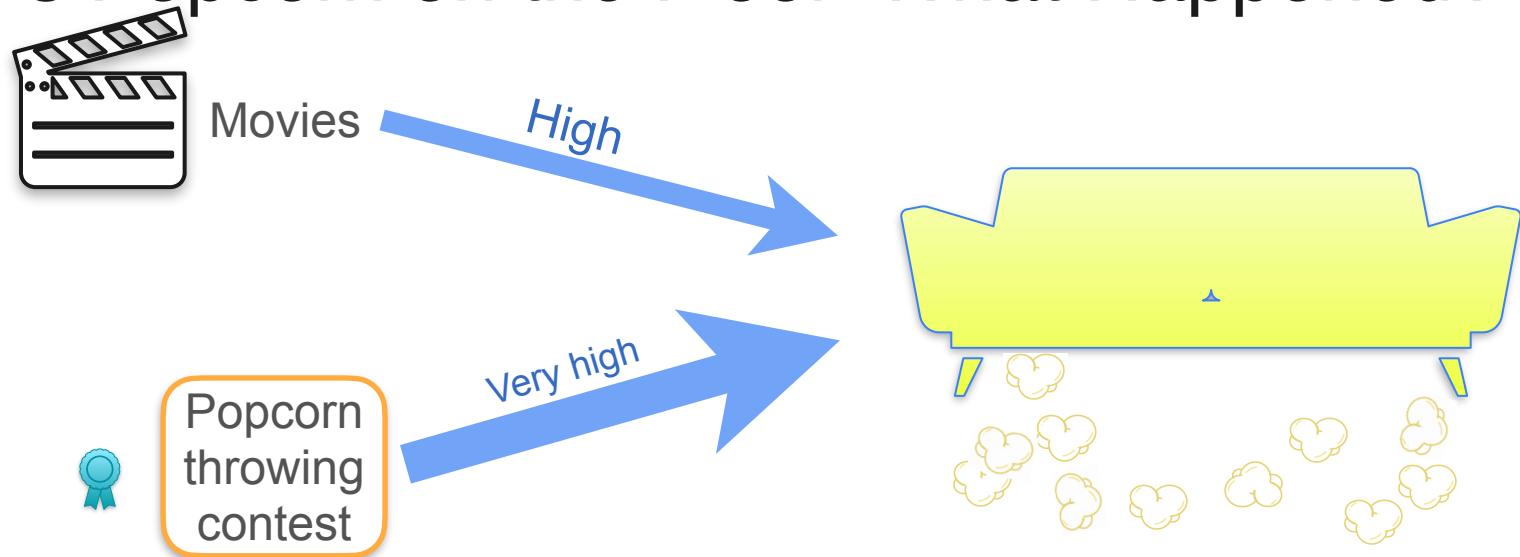
There's Popcorn on the Floor. What Happened?



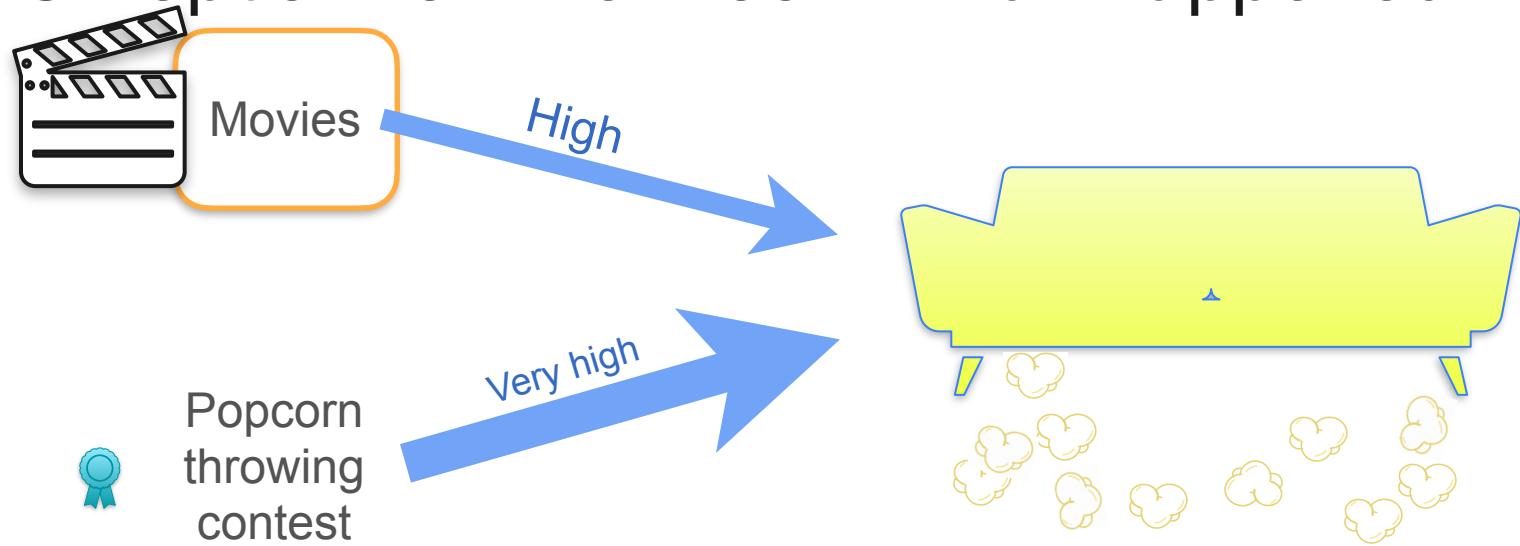
There's Popcorn on the Floor. What Happened?



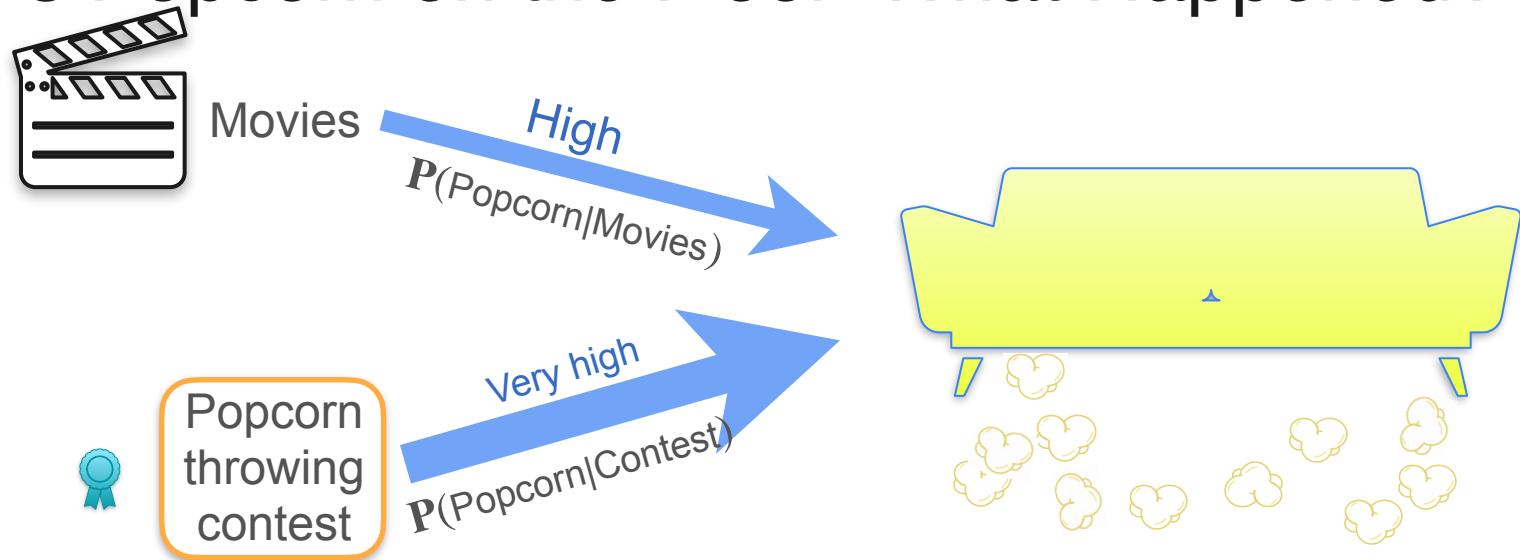
There's Popcorn on the Floor. What Happened?



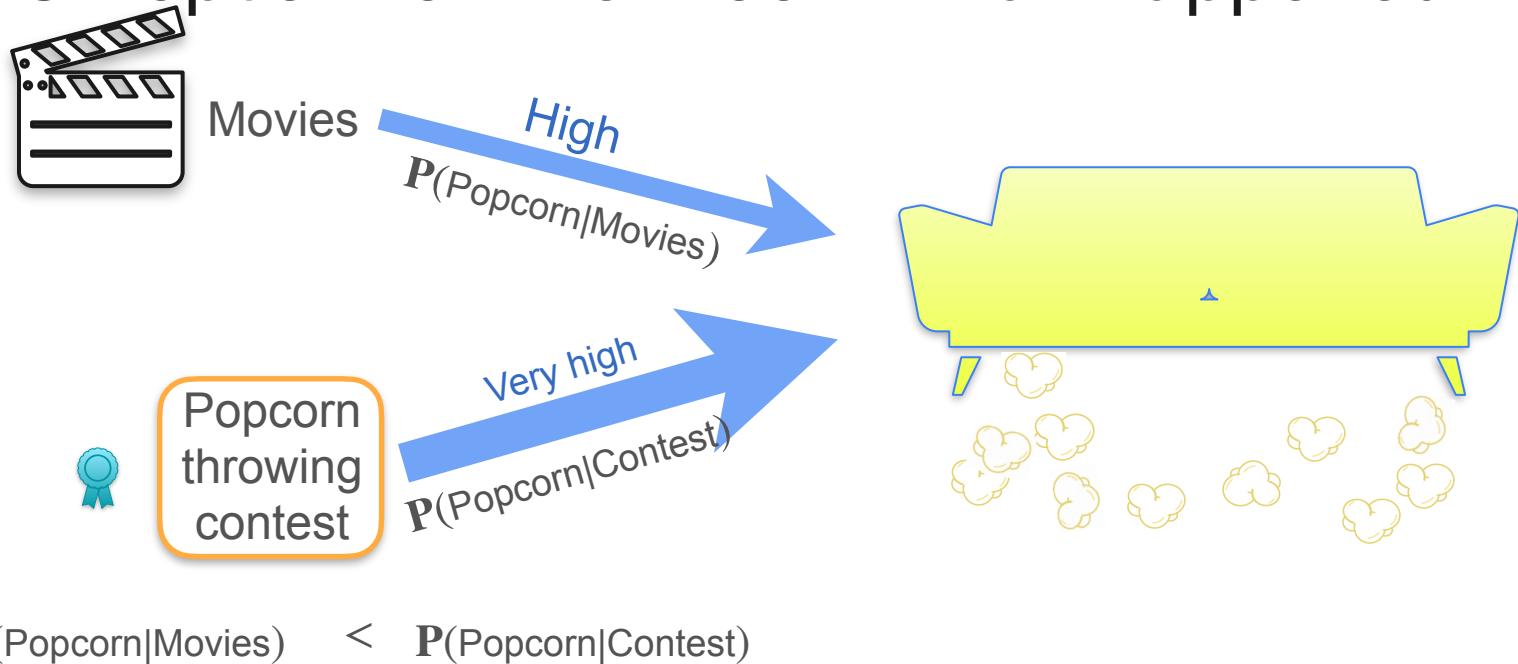
There's Popcorn on the Floor. What Happened?



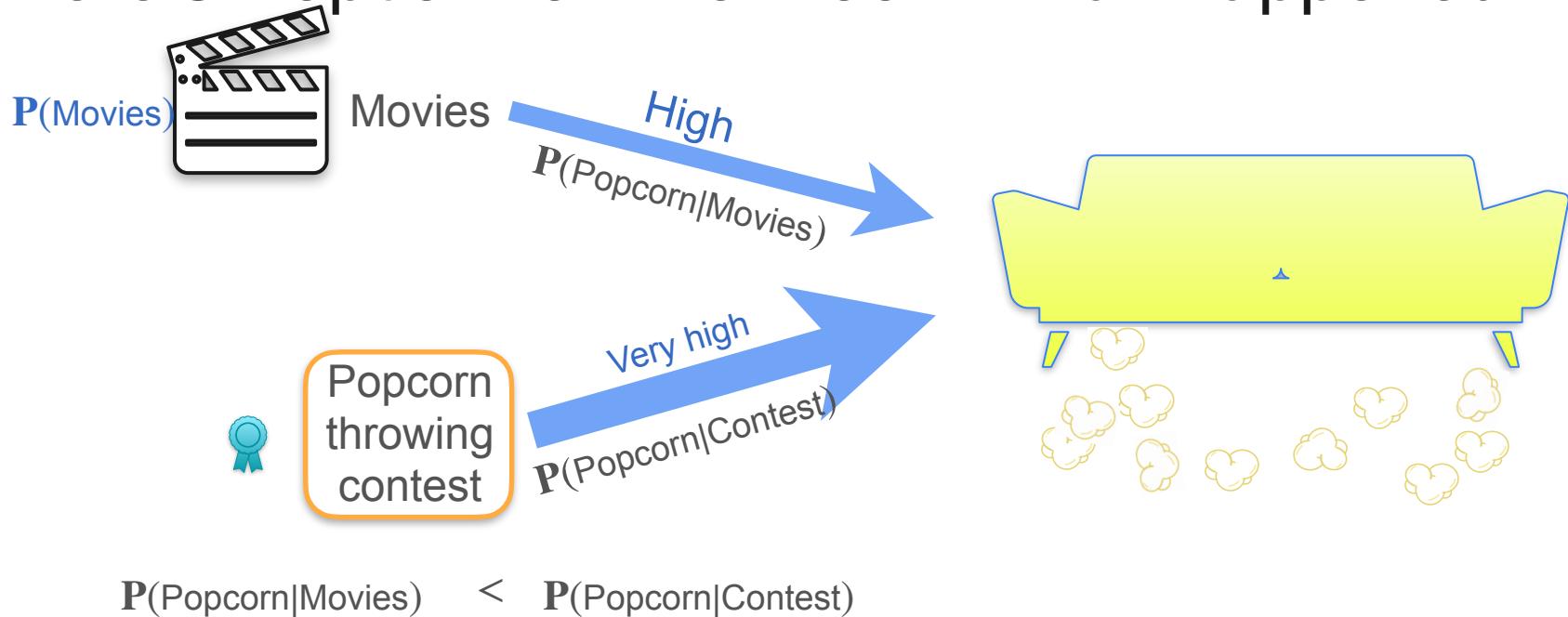
There's Popcorn on the Floor. What Happened?



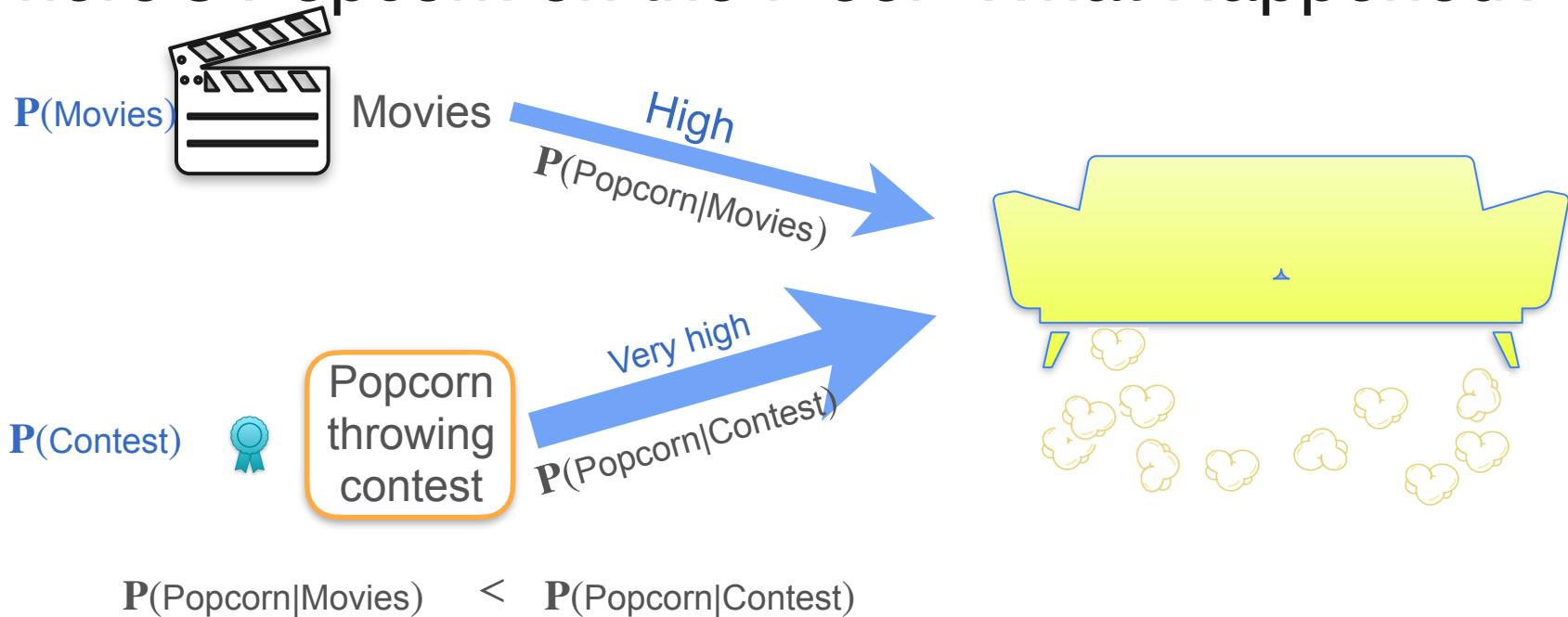
There's Popcorn on the Floor. What Happened?



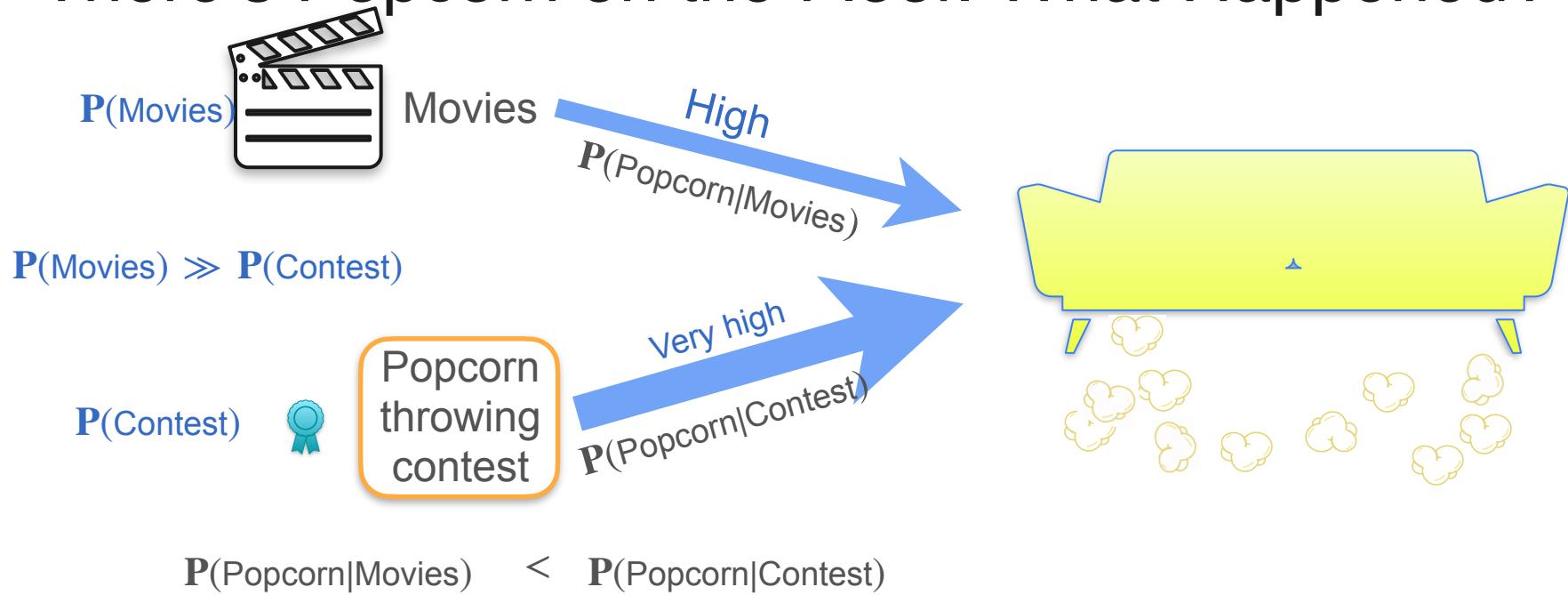
There's Popcorn on the Floor. What Happened?



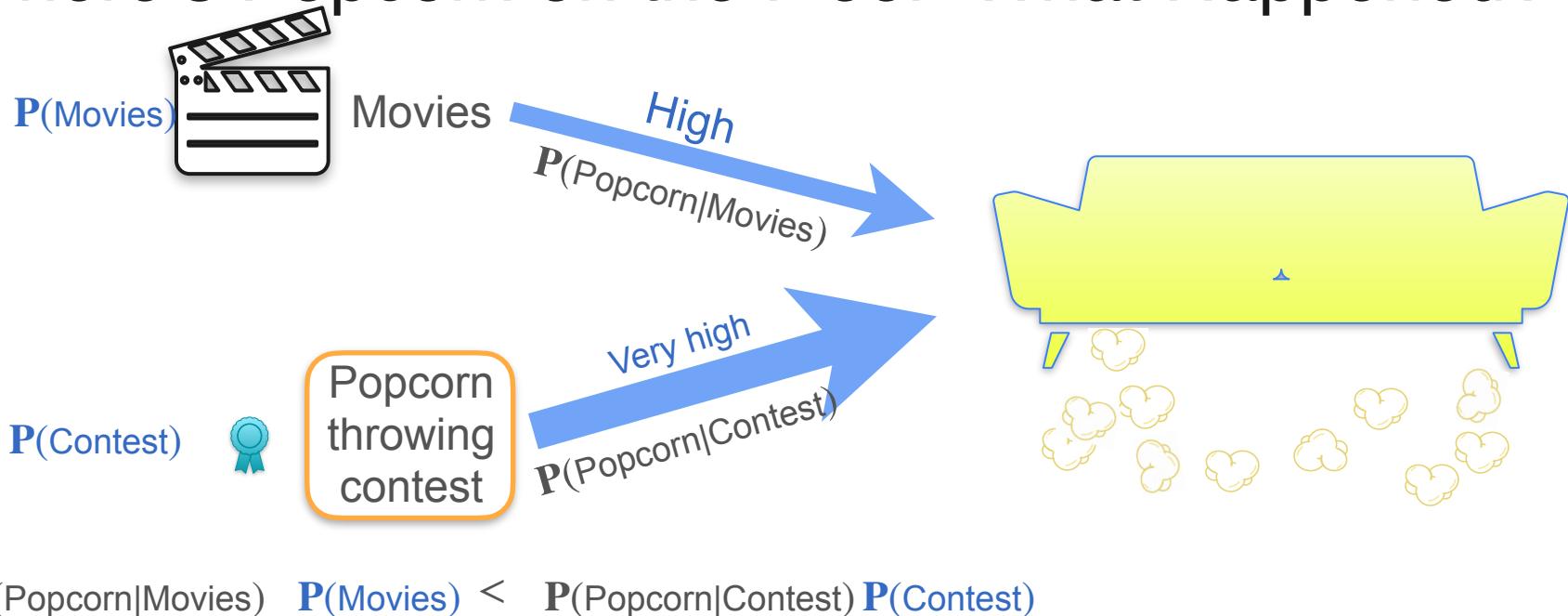
There's Popcorn on the Floor. What Happened?



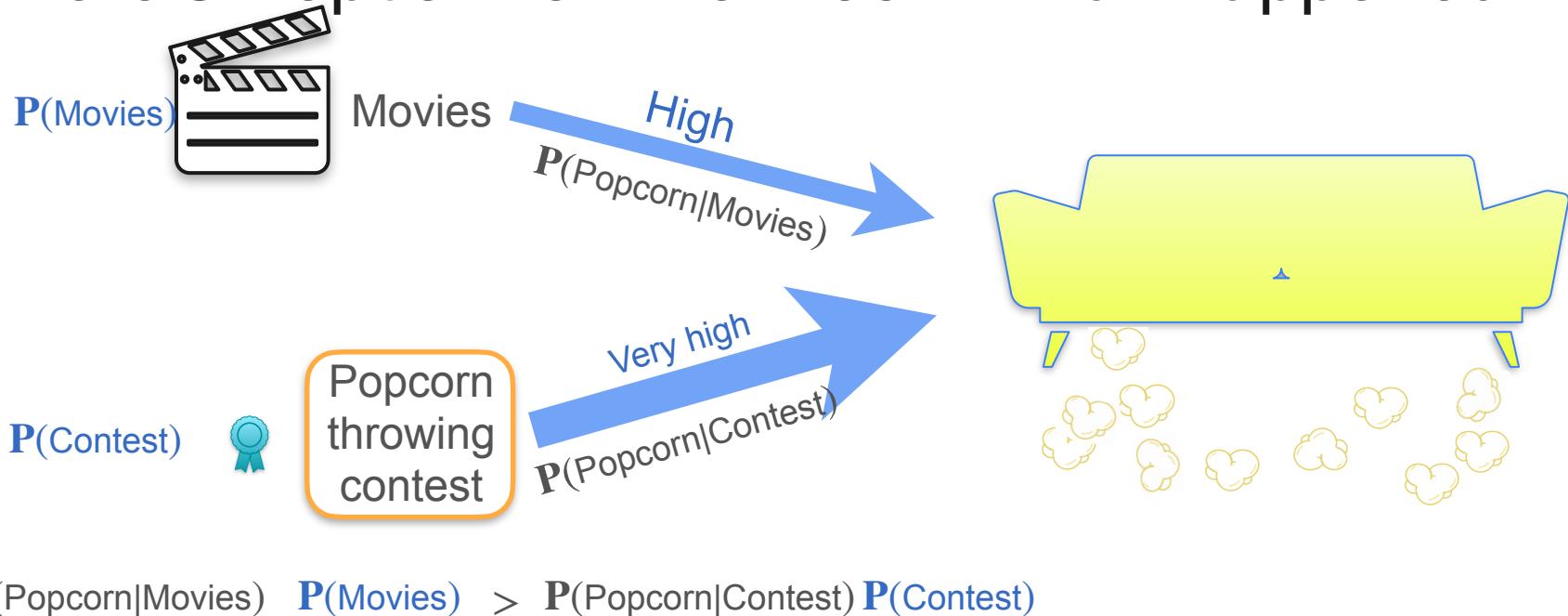
There's Popcorn on the Floor. What Happened?



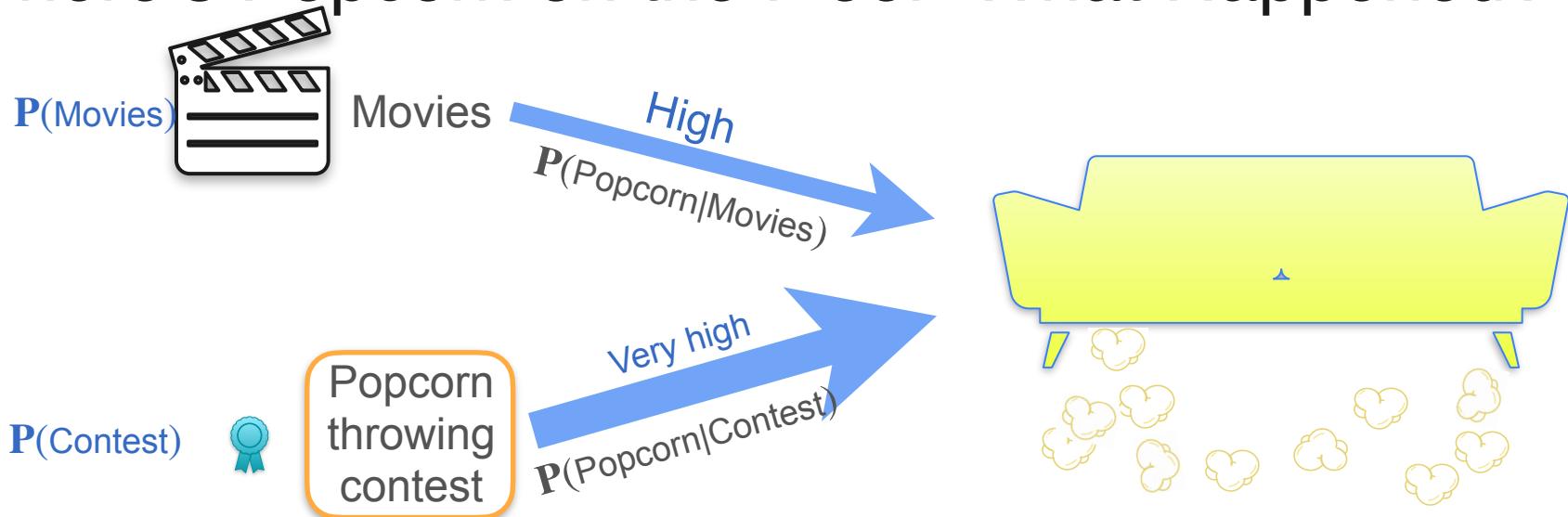
There's Popcorn on the Floor. What Happened?



There's Popcorn on the Floor. What Happened?



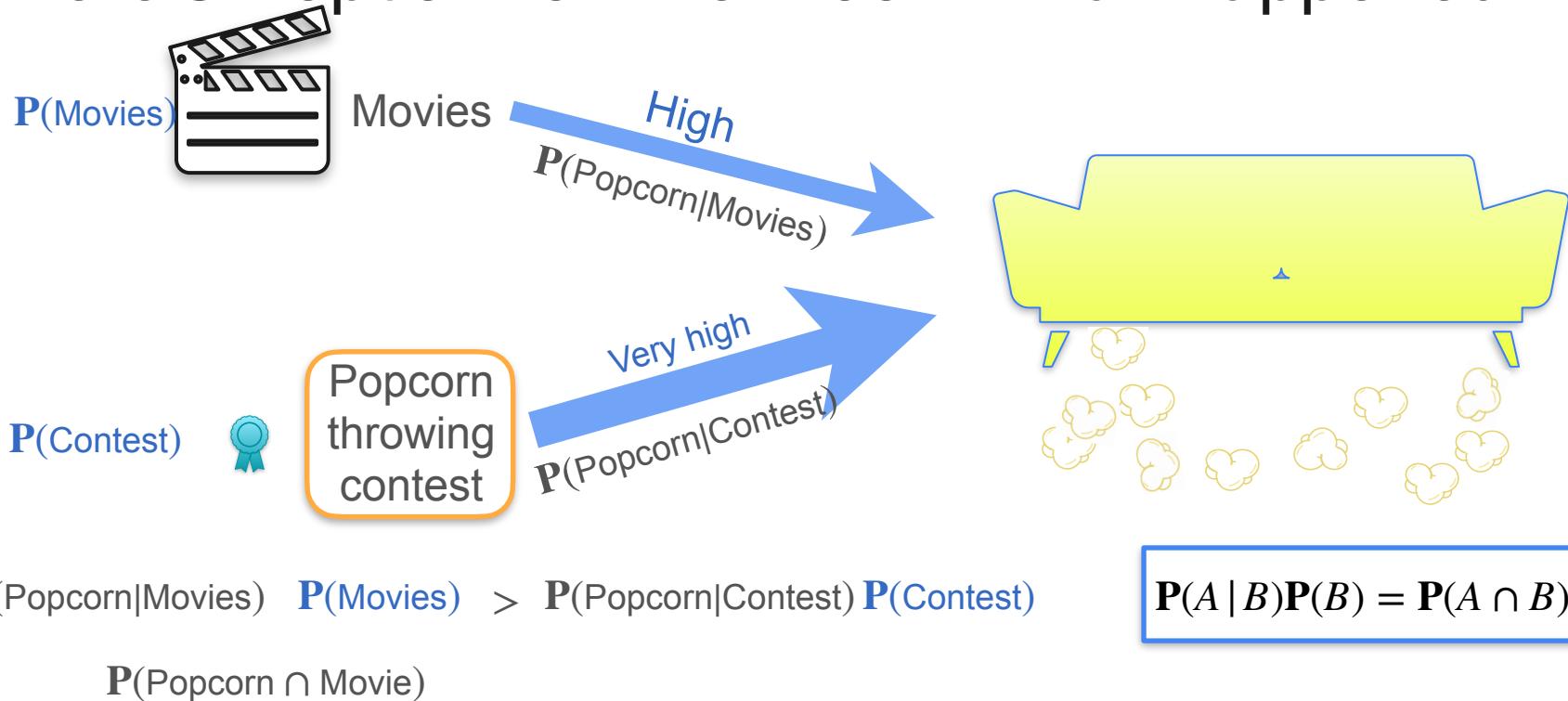
There's Popcorn on the Floor. What Happened?



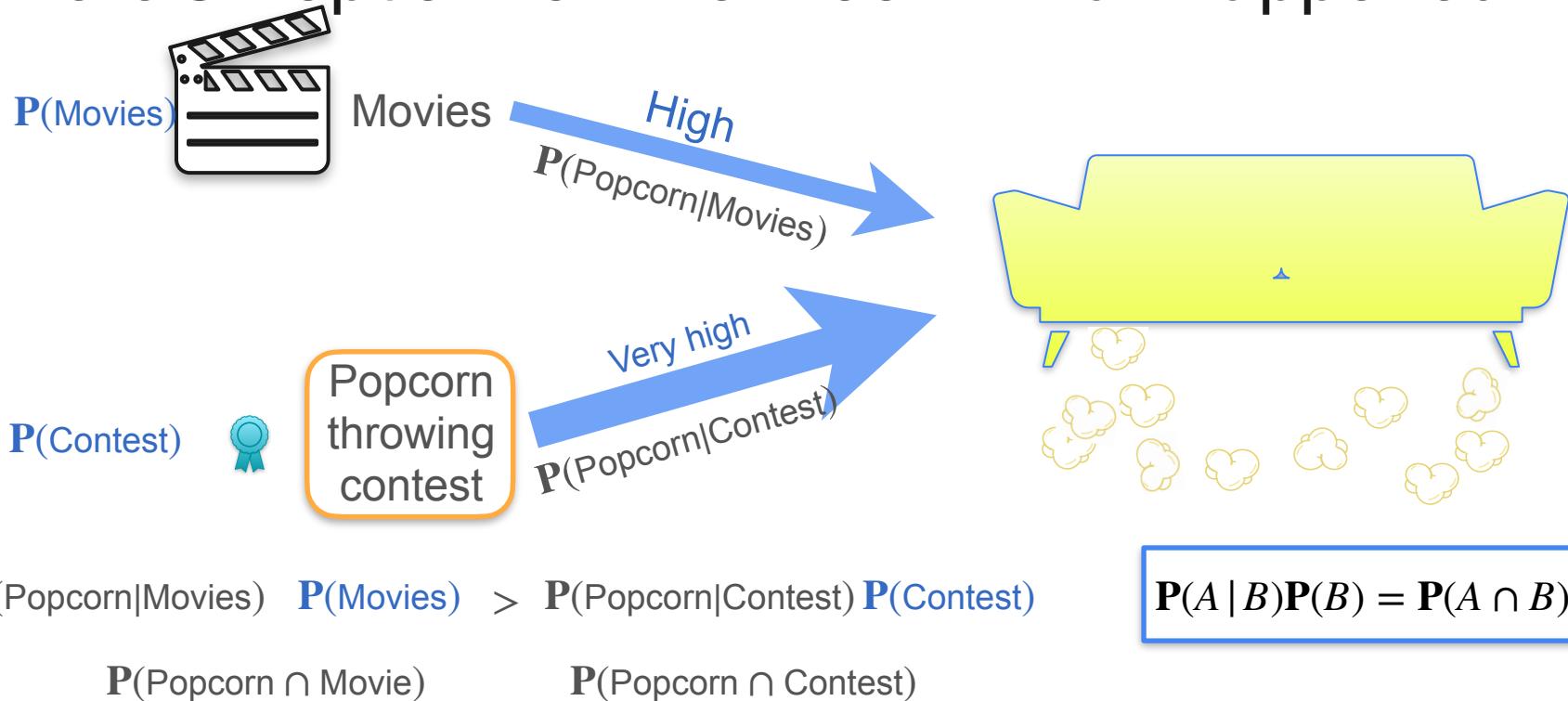
$$P(\text{Popcorn}|\text{Movies}) P(\text{Movies}) > P(\text{Popcorn}|\text{Contest}) P(\text{Contest})$$

$$P(A | B)P(B) = P(A \cap B)$$

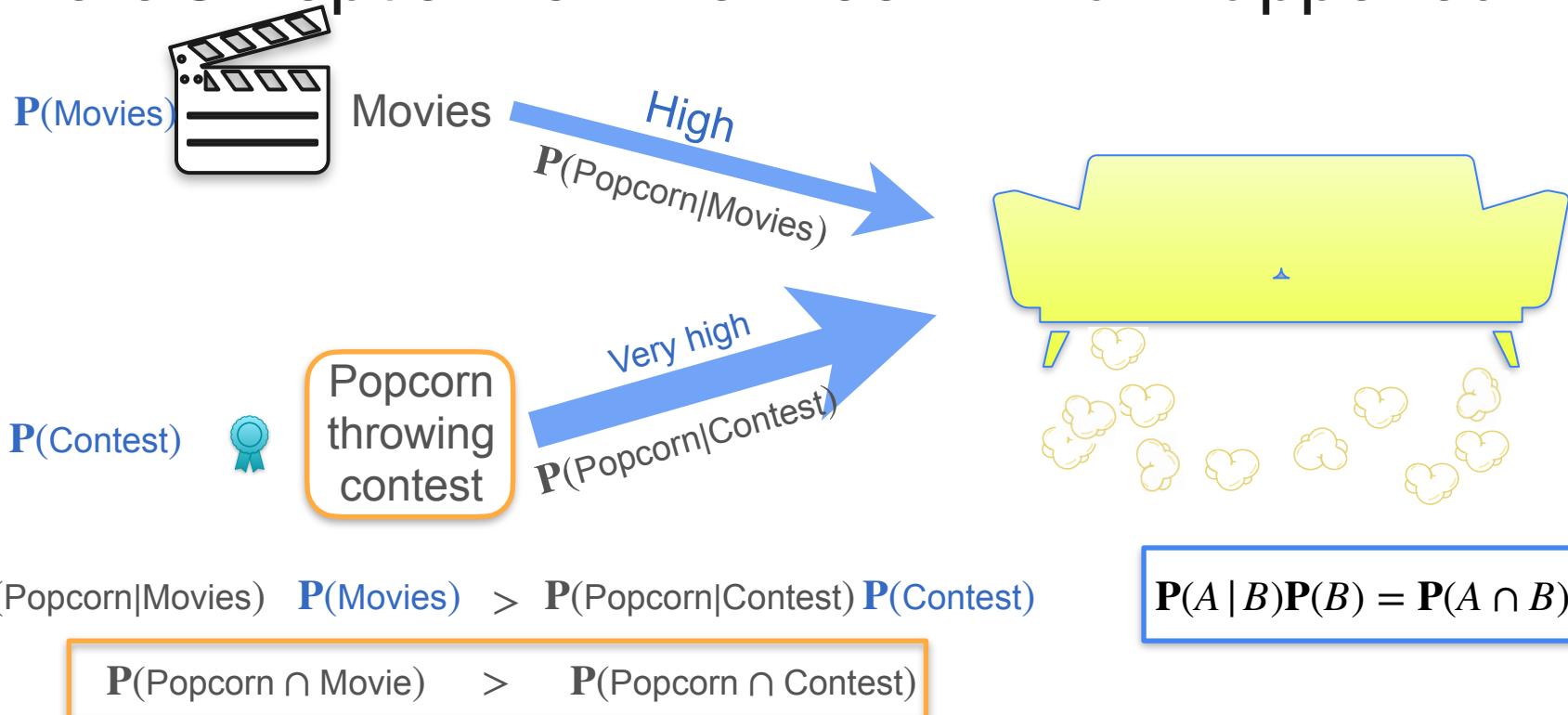
There's Popcorn on the Floor. What Happened?



There's Popcorn on the Floor. What Happened?



There's Popcorn on the Floor. What Happened?



What Does This Have To Do With Regularization?



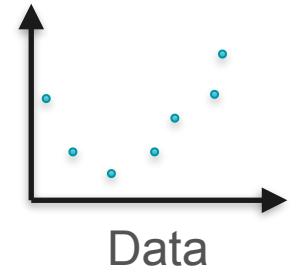
DeepLearning.AI

Point Estimation

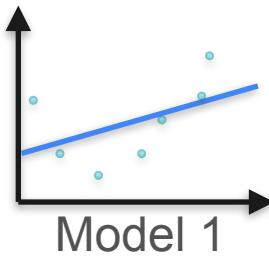
**Bayes Theorem and
Regularization**

Example: Polynomial Regression

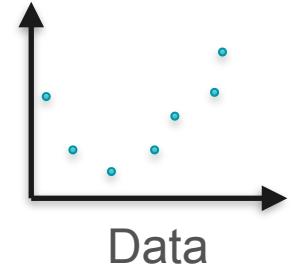
Example: Polynomial Regression



Example: Polynomial Regression

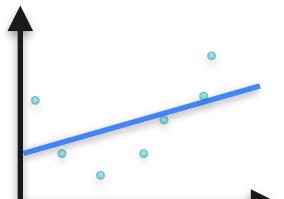


Model 1

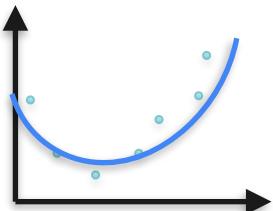


Data

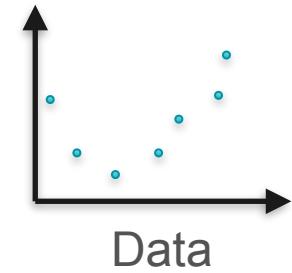
Example: Polynomial Regression



Model 1

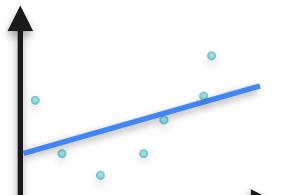


Model 2

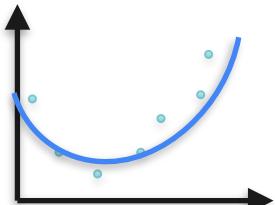


Data

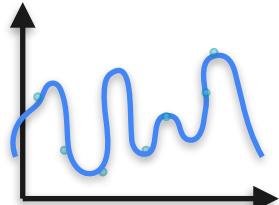
Example: Polynomial Regression



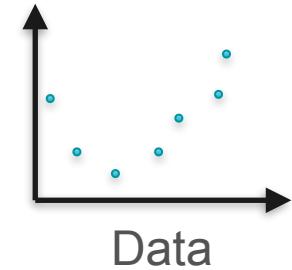
Model 1



Model 2

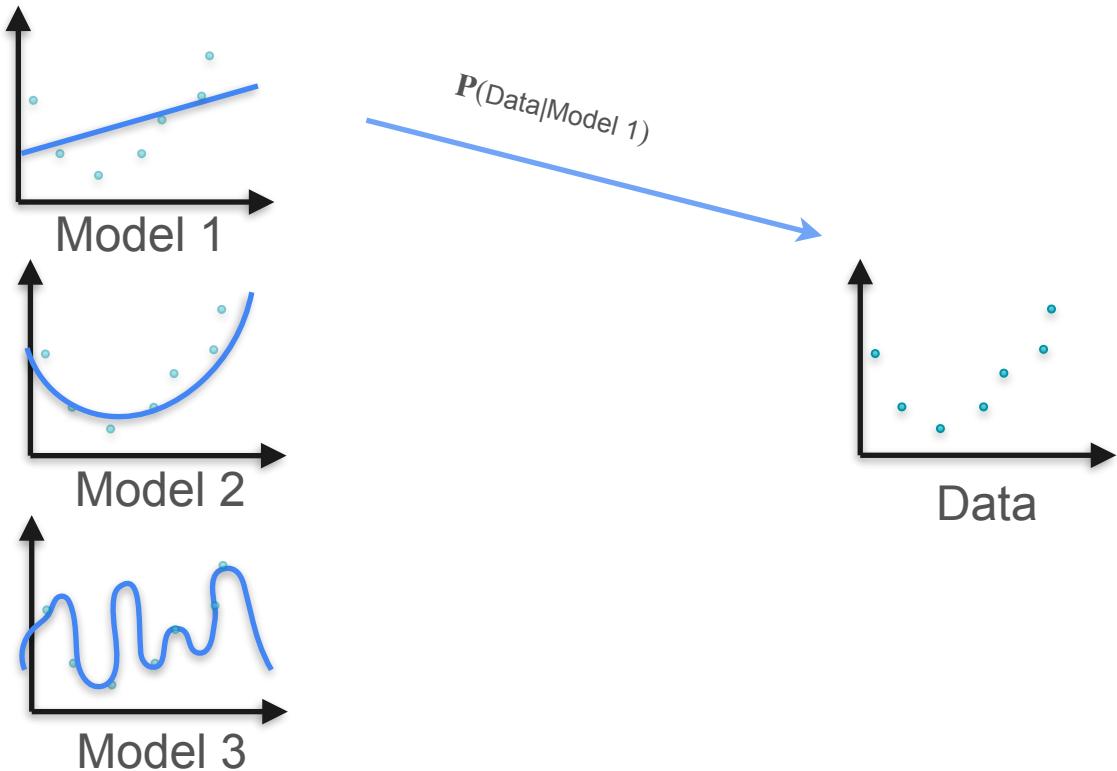


Model 3

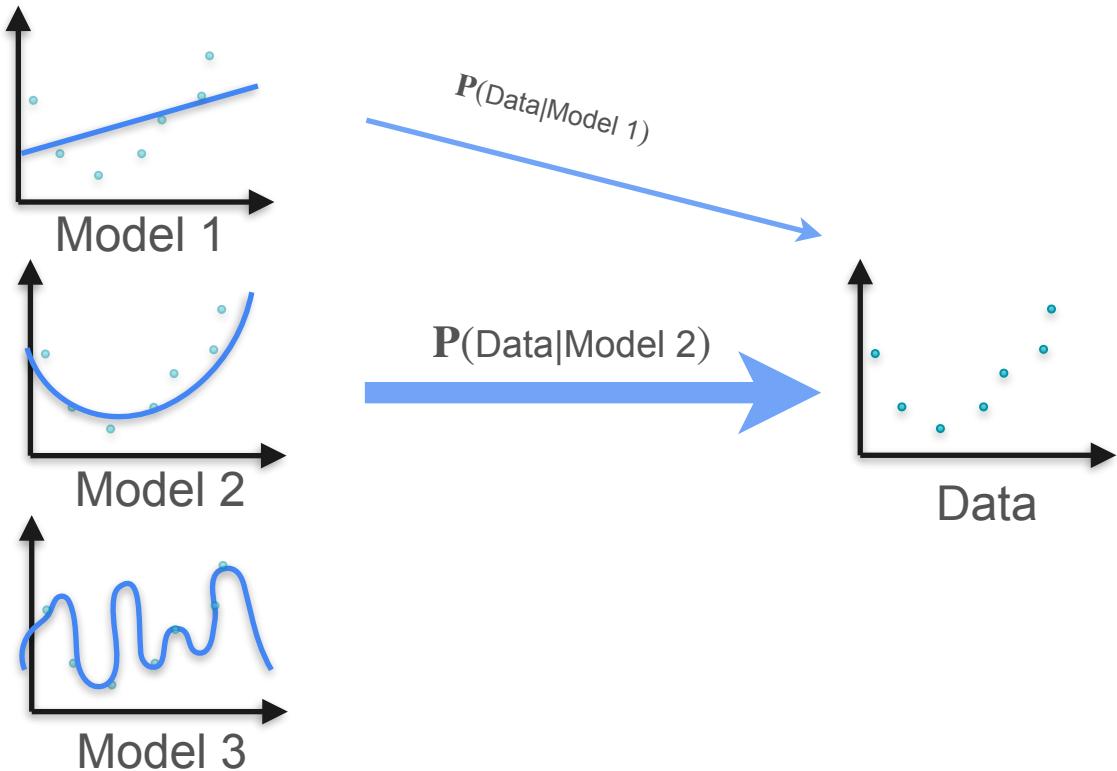


Data

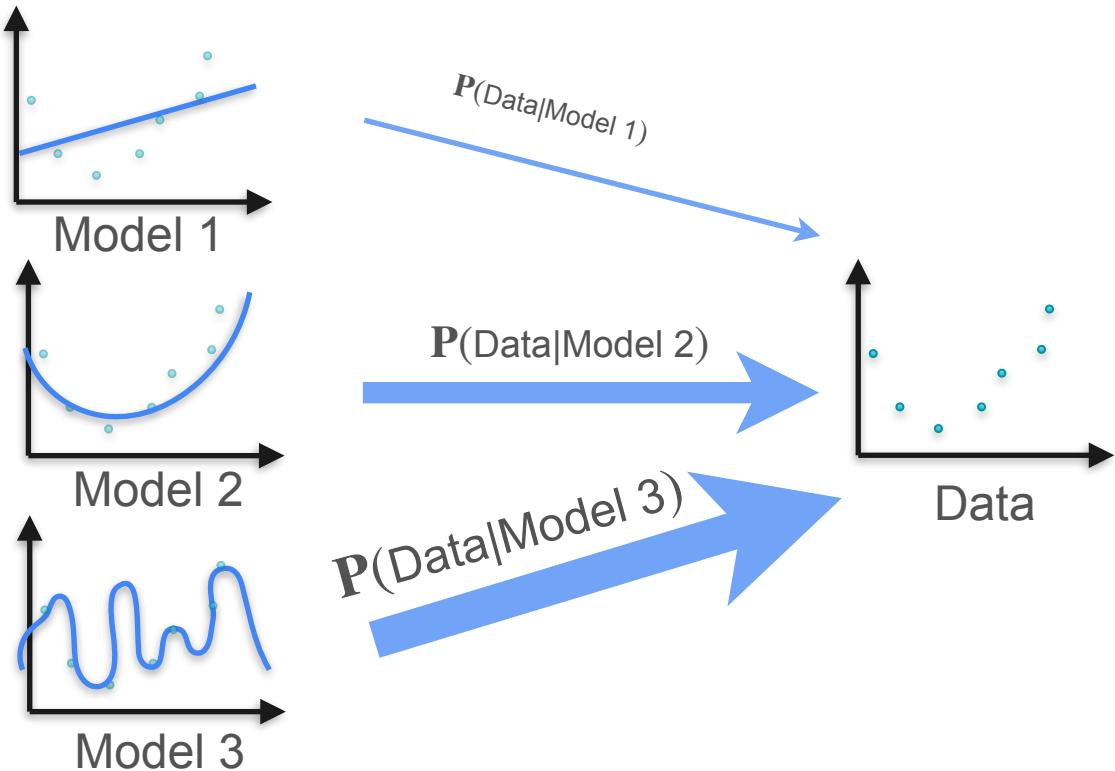
Example: Polynomial Regression



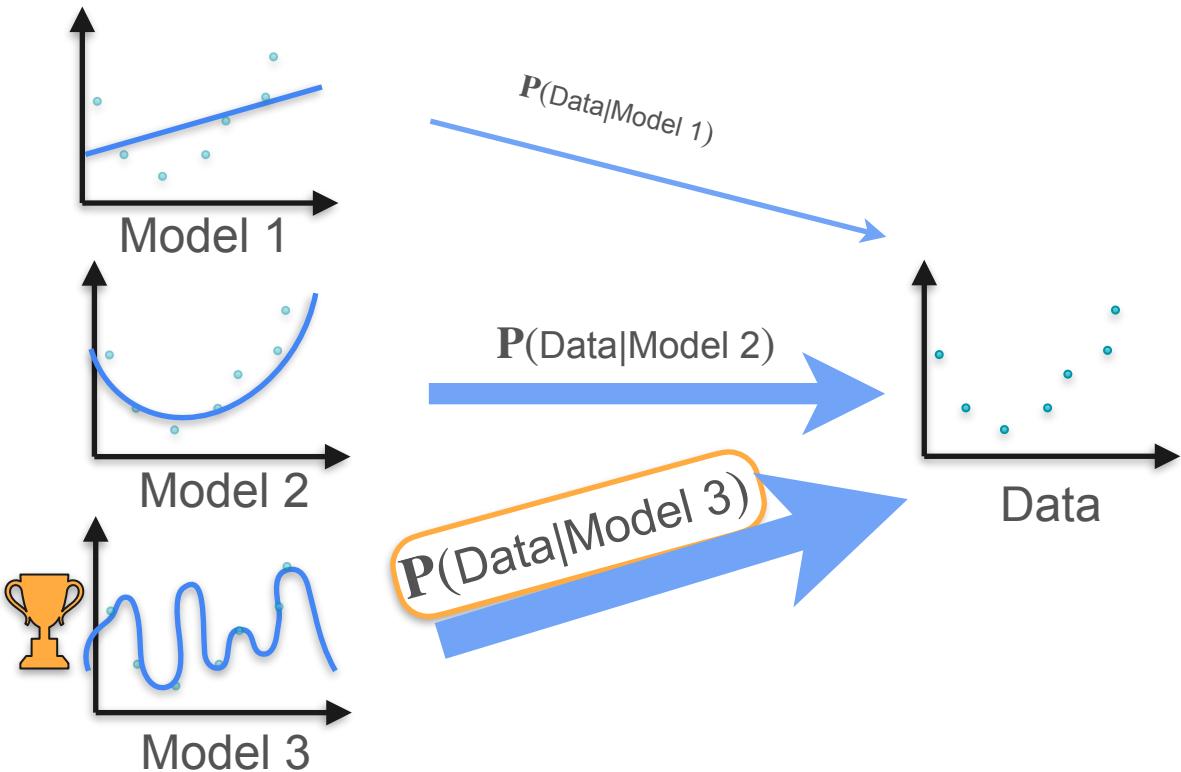
Example: Polynomial Regression



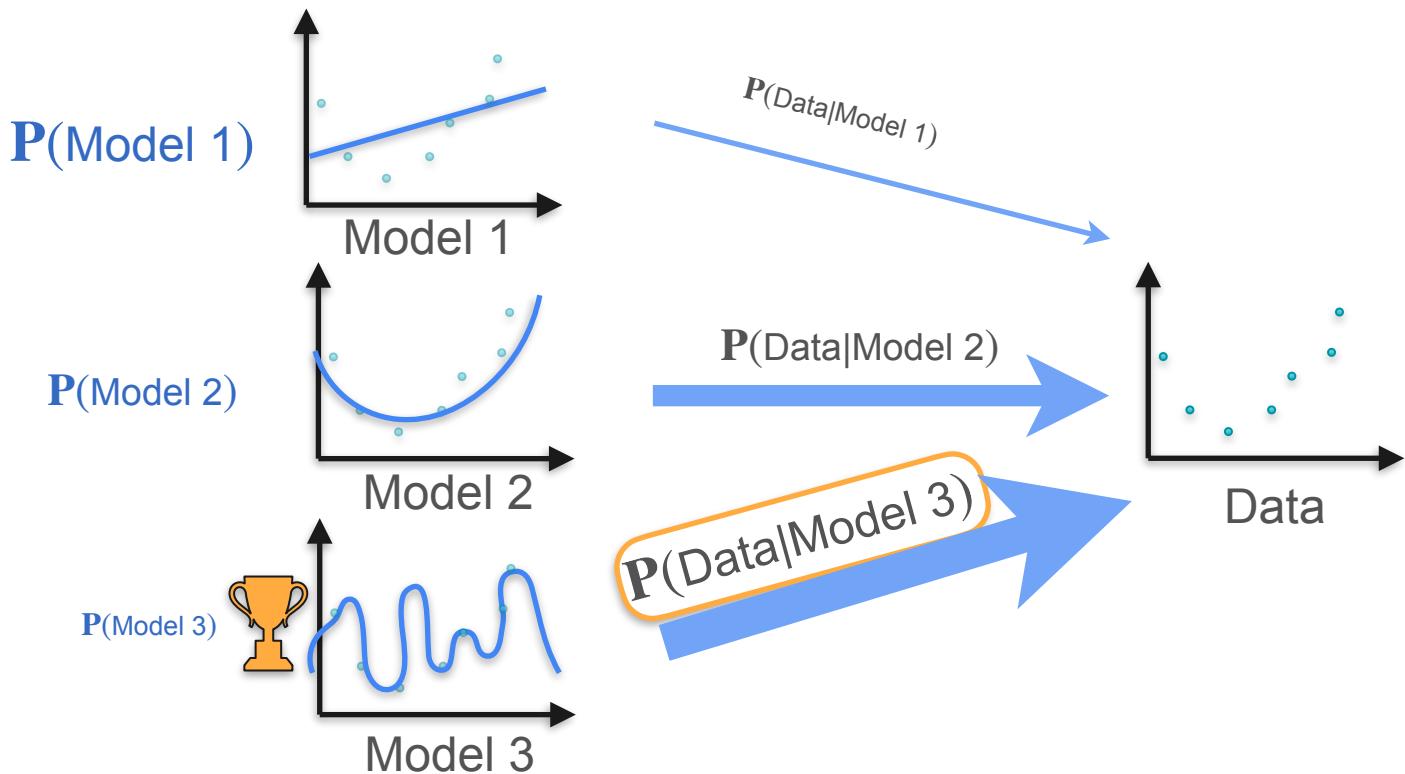
Example: Polynomial Regression



Example: Polynomial Regression

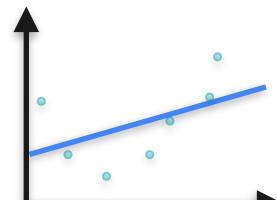


Example: Polynomial Regression

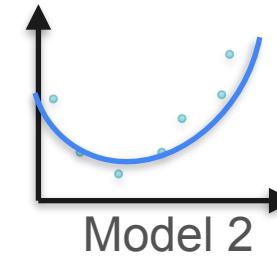


Example: Polynomial Regression

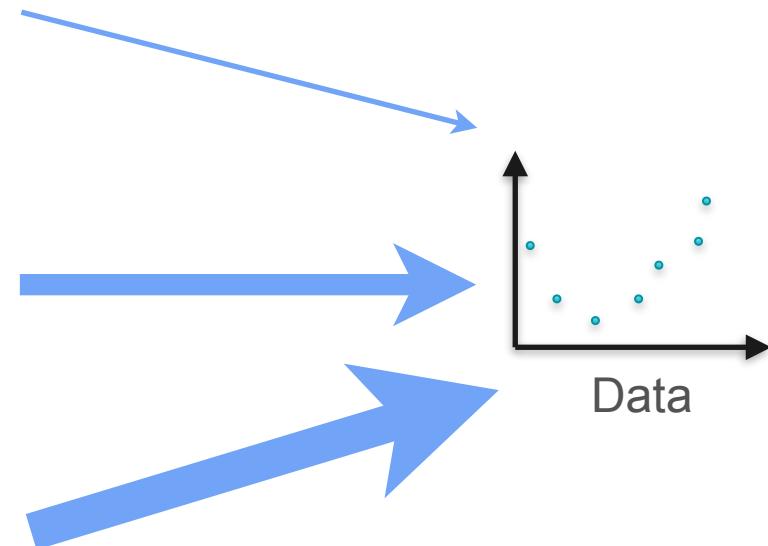
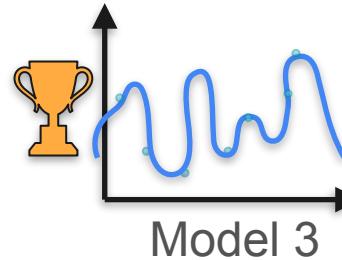
$P(\text{Model 1}) P(\text{Data}|\text{Model 1})$



$P(\text{Model 2}) P(\text{Data}|\text{Model 2})$

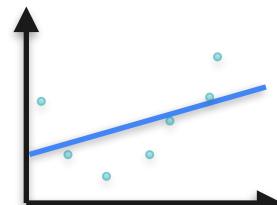


$P(\text{Model 3}) P(\text{Data}|\text{Model 3})$

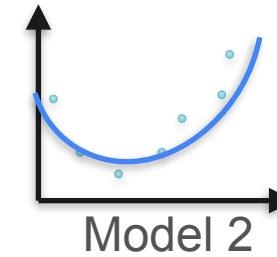


Example: Polynomial Regression

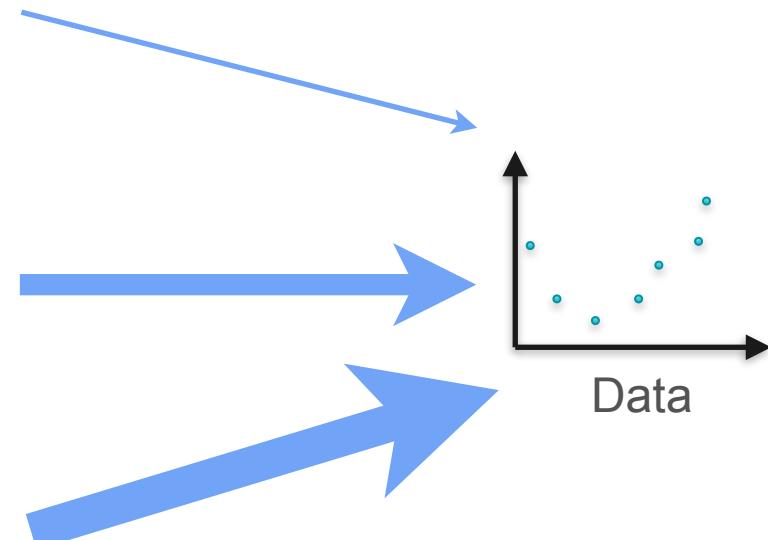
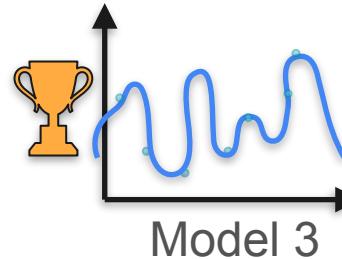
$P(\text{Model 1}) P(\text{Data}|\text{Model 1})$



$P(\text{Model 2}) P(\text{Data}|\text{Model 2})$

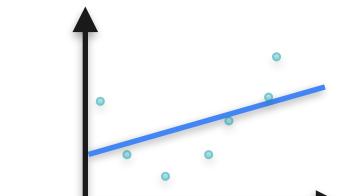


$P(\text{Model 3}) P(\text{Data}|\text{Model 3})$

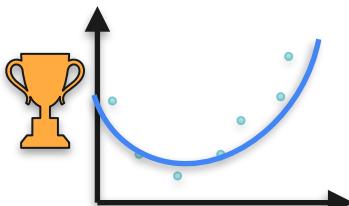


Example: Polynomial Regression

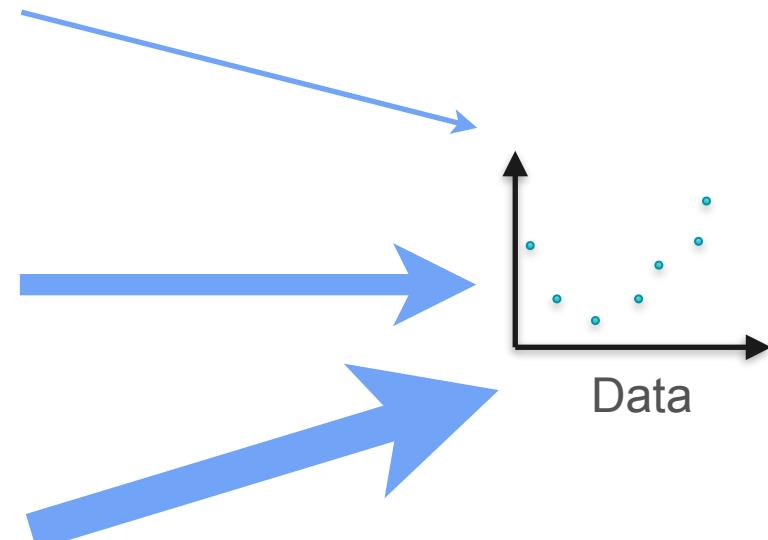
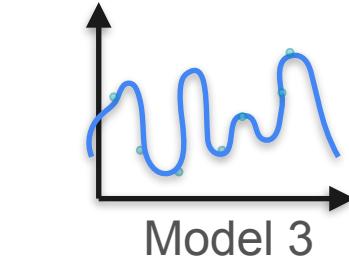
$P(\text{Model 1}) P(\text{Data}|\text{Model 1})$



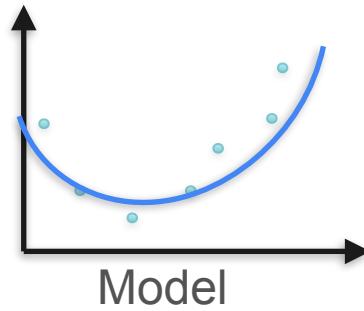
$P(\text{Model 2}) P(\text{Data}|\text{Model 2})$



$P(\text{Model 3}) P(\text{Data}|\text{Model 3})$

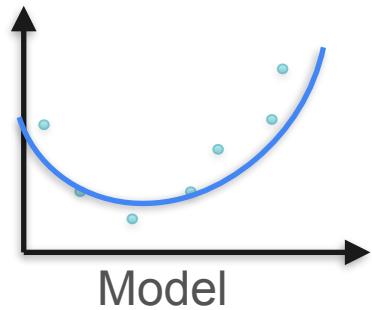


Maximum likelihood



Polynomial regression

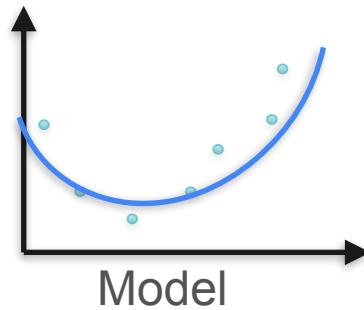
Maximum likelihood



Polynomial regression

Log-loss

Maximum likelihood

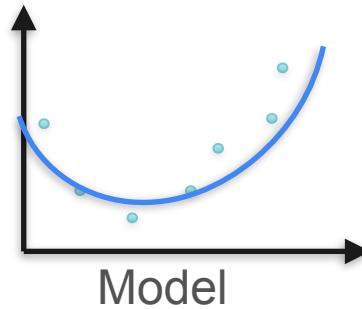


Polynomial regression

$P(\text{Data}|\text{Model})$

Log-loss

**Maximum likelihood
with Bayes**

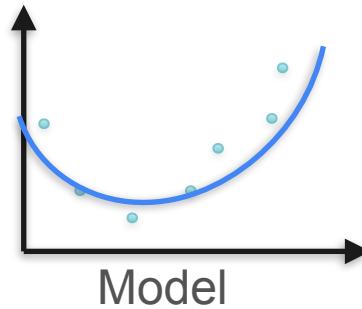


Polynomial regression

$P(\text{Data}|\text{Model})$

Log-loss

**Maximum likelihood
with Bayes**



Polynomial regression

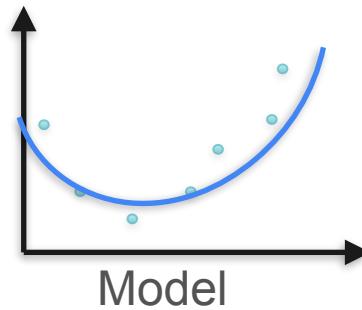
$P(\text{Data}|\text{Model})$

Log-loss

•

$P(\text{Model})$

**Maximum likelihood
with Bayes**



**Polynomial regression
with regularization**

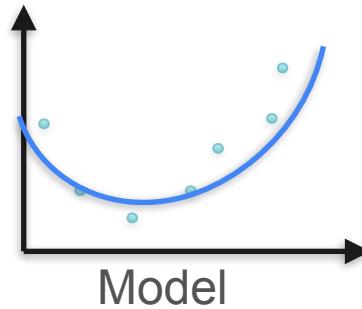
$P(\text{Data}|\text{Model})$

Log-loss

•

$P(\text{Model})$

**Maximum likelihood
with Bayes**



**Polynomial regression
with regularization**

$P(\text{Data}|\text{Model})$

•

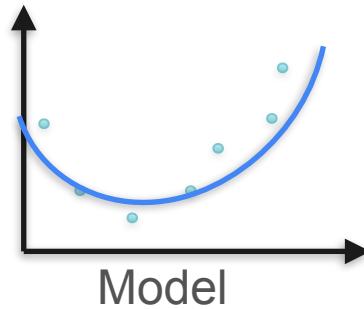
$P(\text{Model})$

Log-loss



Regularization term

Maximum likelihood
with Bayes



Polynomial regression
with regularization

$P(\text{Data}|\text{Model})$
•
 $P(\text{Model})$

Take logarithms!

Log-loss
+
Regularization term

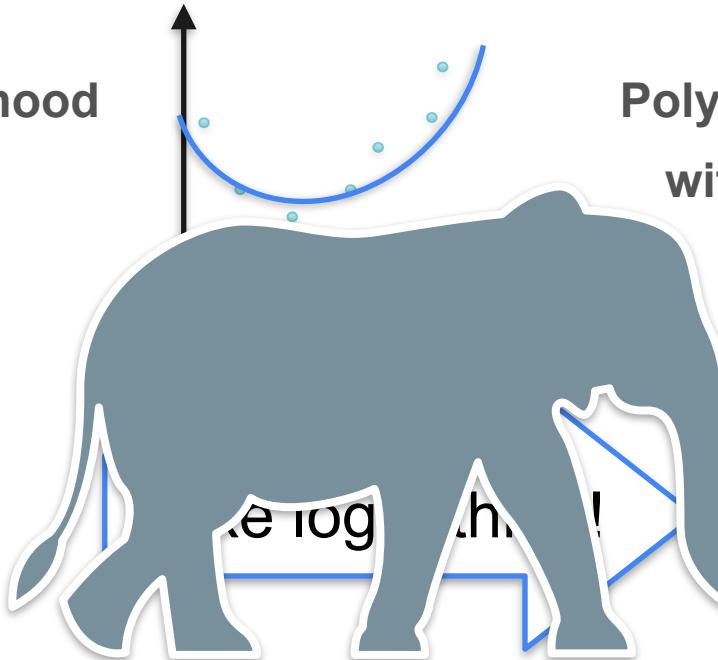
Maximum likelihood
with Bayes

Polynomial regression
with regularization

$P(\text{Data}|\text{Model})$

•

$P(\text{Model})$



Log-loss

+

Regularization term

Maximum likelihood
with Bayes

Polynomial regression
with regularization

$P(\text{Data}|\text{Model})$

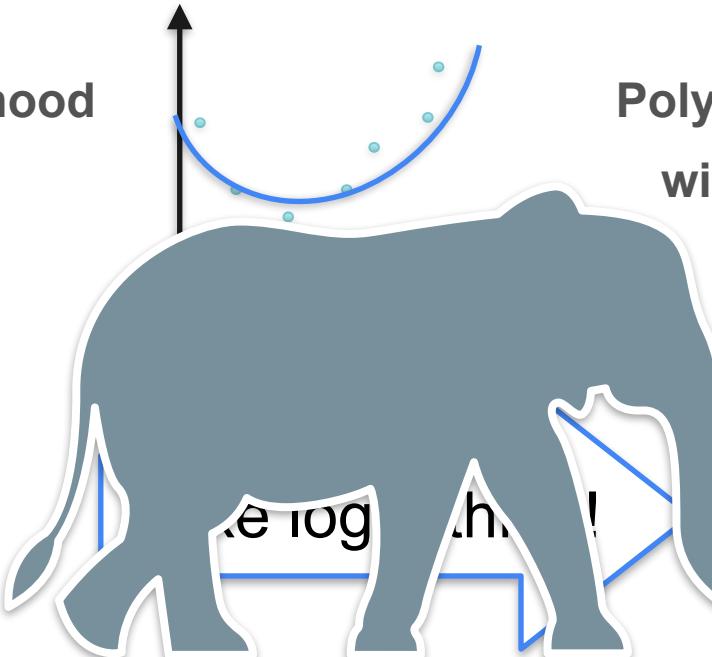
$P(\text{Model})$

?

Log-loss

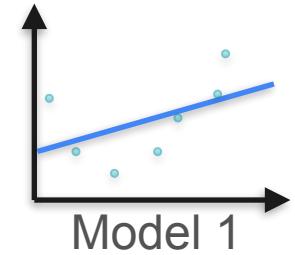
+

Regularization term

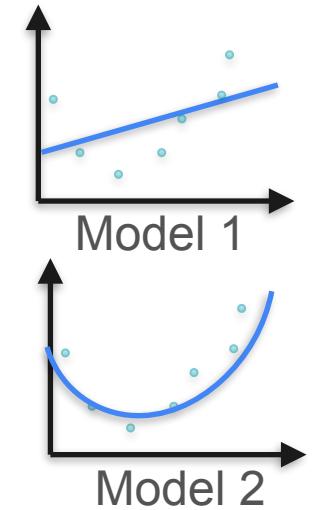


What Is the Probability of a Model?

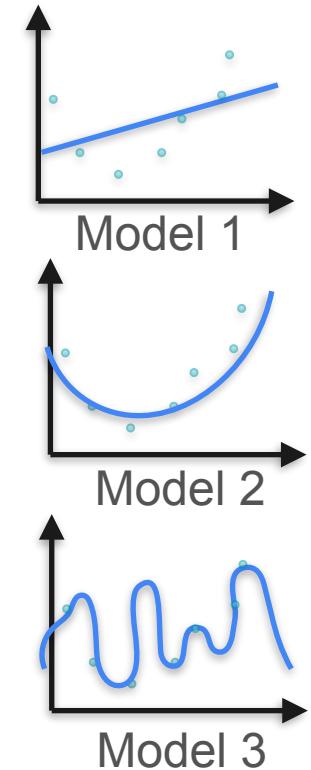
What Is the Probability of a Model?



What Is the Probability of a Model?



What Is the Probability of a Model?

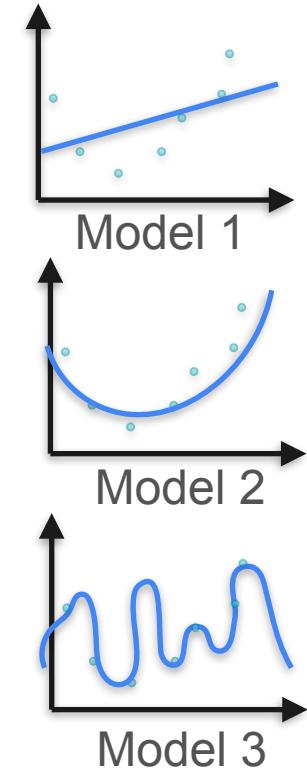


What Is the Probability of a Model?

$P(\text{Model 1})$

$P(\text{Model 2})$

$P(\text{Model 3})$



What Is the Probability of a Model?

$P(\text{Model 1})$

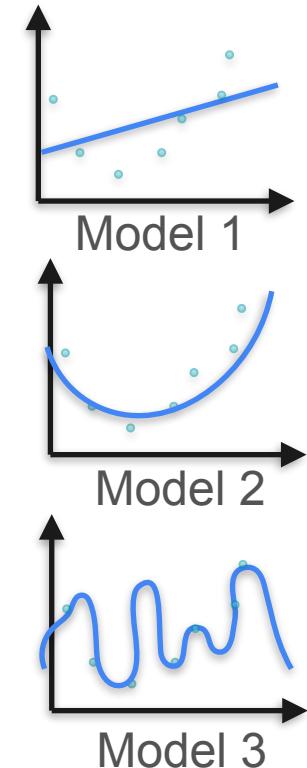
$$a_1x + b$$

$P(\text{Model 2})$

$$a_1x^2 + a_2x + b$$

$P(\text{Model 3})$

$$a_1x^{10} + a_2x^9 + \dots + a_{10}x + b$$



What Is the Probability of a Model?

$P(\text{Model 1})$

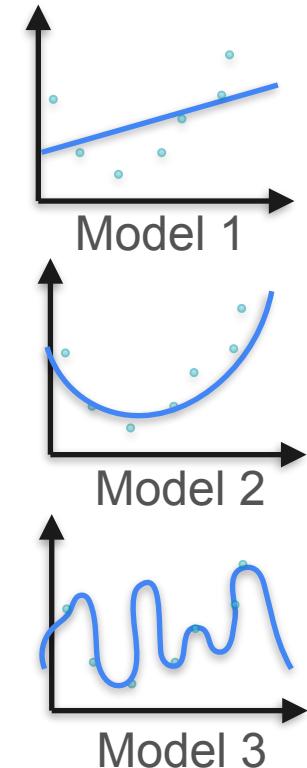
$$a_1x + b$$

$P(\text{Model 2})$

$$a_1x^2 + a_2x + b$$

$P(\text{Model 3})$

$$a_1x^{10} + a_2x^9 + \dots + a_{10}x + b$$



What Is the Probability of a Model?

P(Model 1)

$$a_1x + b$$

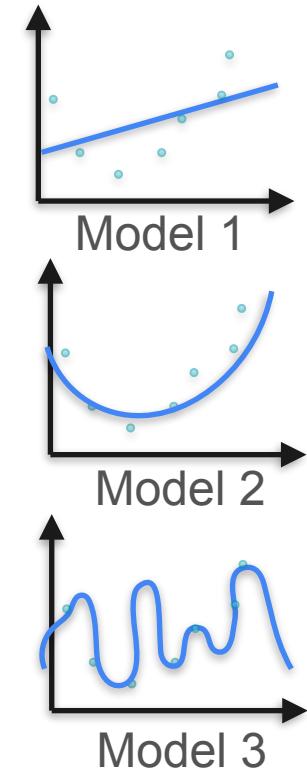
P(Model 2)

$$a_1x^2 + a_2x + b$$

P(Model 3)

$$a_1x^{10} + a_2x^9 + \dots + a_{10}x + b$$

$$a_i \sim \mathcal{N}(0,1)$$



What Is the Probability of a Model?

P(Model 1)

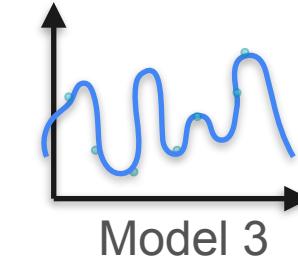
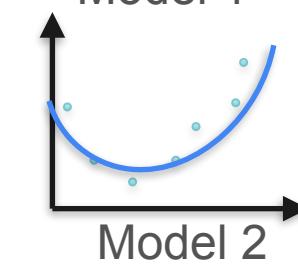
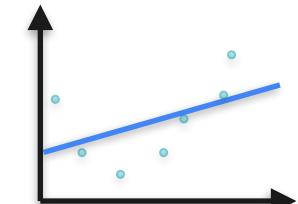
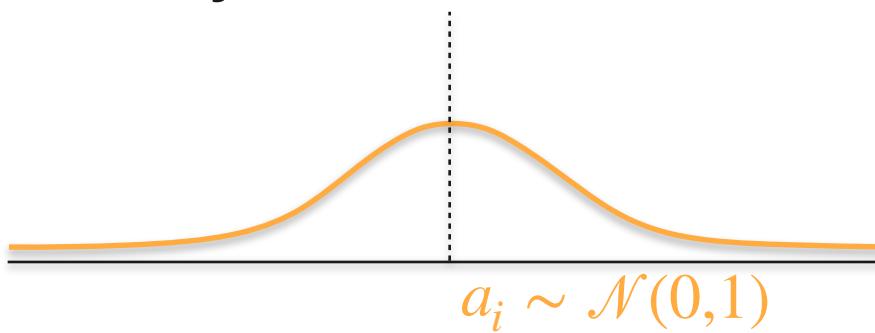
$$a_1x + b$$

P(Model 2)

$$a_1x^2 + a_2x + b$$

P(Model 3)

$$a_1x^{10} + a_2x^9 + \dots + a_{10}x + b$$



What Is the Probability of a Model?

P(Model 1)

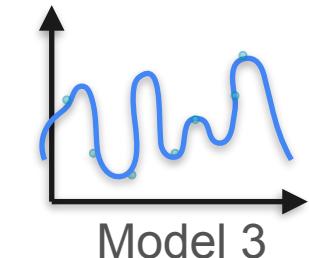
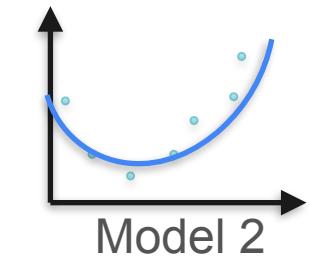
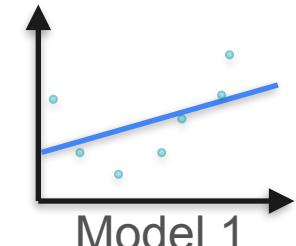
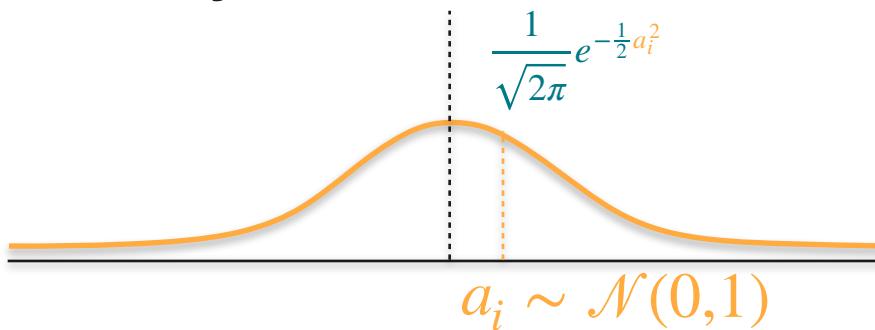
$$a_1x + b$$

P(Model 2)

$$a_1x^2 + a_2x + b$$

P(Model 3)

$$a_1x^{10} + a_2x^9 + \dots + a_{10}x + b$$



What Is the Probability of a Model?

$$P(\text{Model 1}) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}a_1^2}$$

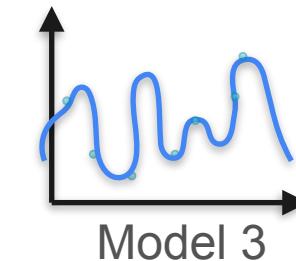
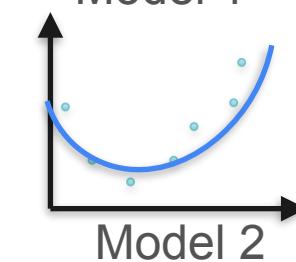
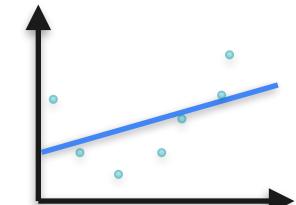
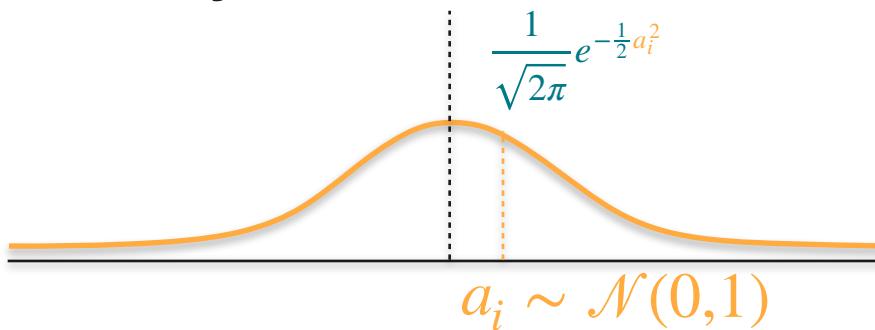
$a_1x + b$

$P(\text{Model 2})$

$$a_1x^2 + a_2x + b$$

$P(\text{Model 3})$

$$a_1x^{10} + a_2x^9 + \dots + a_{10}x + b$$



What Is the Probability of a Model?

$$P(\text{Model 1}) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}a_1^2}$$

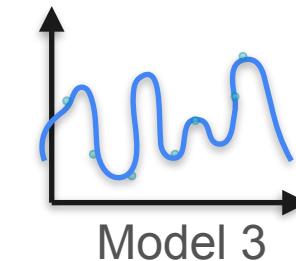
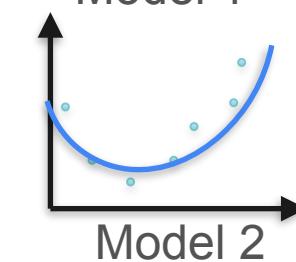
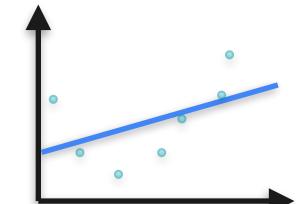
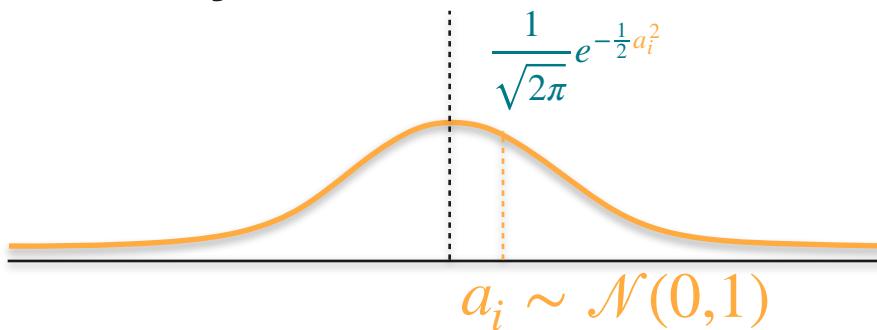
$a_1x + b$

P(Model 2)

$$a_1x^2 + a_2x + b$$

P(Model 3)

$$a_1x^{10} + a_2x^9 + \dots + a_{10}x + b$$



What Is the Probability of a Model?

$$P(\text{Model 1}) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}a_1^2}$$

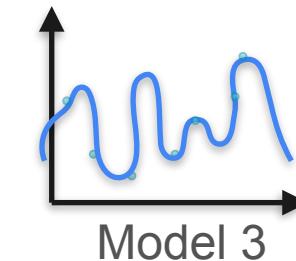
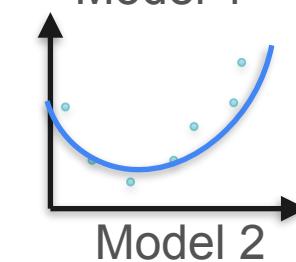
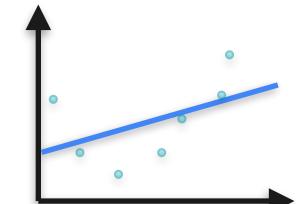
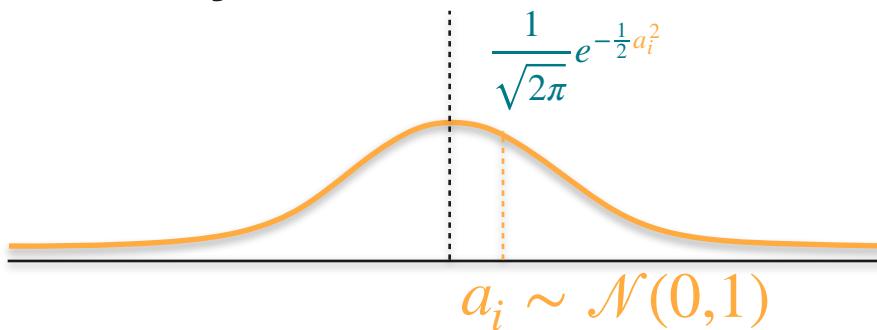
$a_1x + b$

P(Model 2)

$$a_1x^2 + a_2x + b$$

P(Model 3)

$$a_1x^{10} + a_2x^9 + \dots + a_{10}x + b$$



What Is the Probability of a Model?

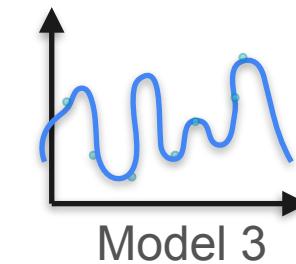
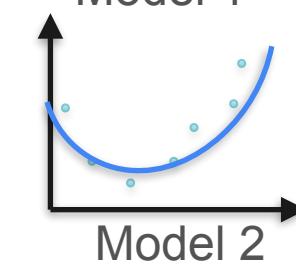
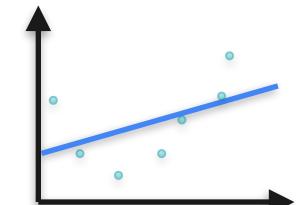
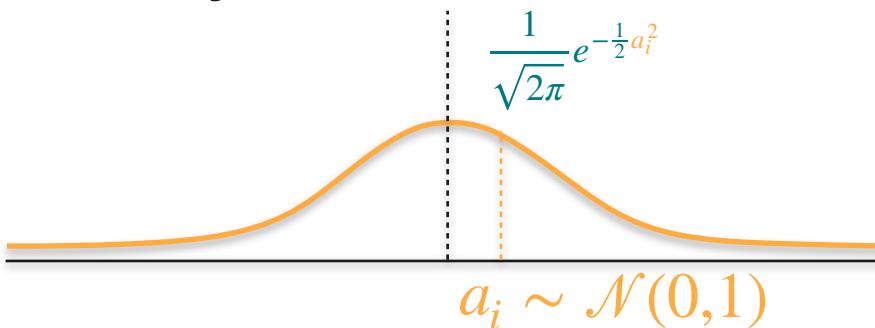
$$P(\text{Model 1}) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}a_1^2}$$

$a_1x + b$

$$P(\text{Model 2}) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}a_1^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}a_2^2}$$

$a_1x^2 + a_2x + b$

$$P(\text{Model 3}) = a_1x^{10} + a_2x^9 + \dots + a_{10}x + b$$



What Is the Probability of a Model?

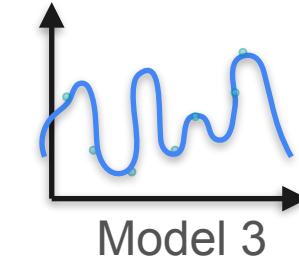
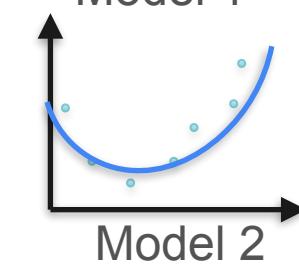
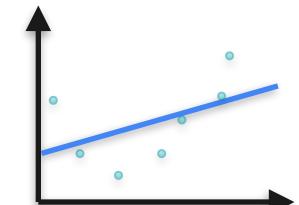
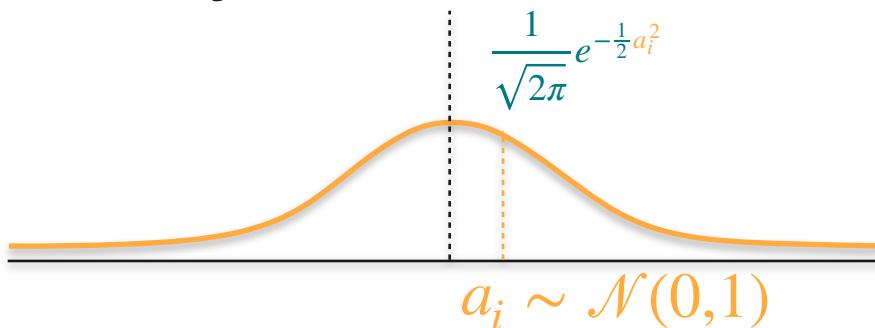
$$P(\text{Model 1}) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}a_1^2}$$

P(Model 2)

$$a_1x^2 + a_2x + b = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}a_1^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}a_2^2}$$

P(Model 3)

$$a_1x^{10} + a_2x^9 + \dots + a_{10}x + b$$



What Is the Probability of a Model?

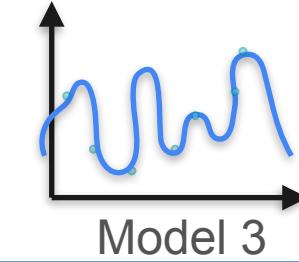
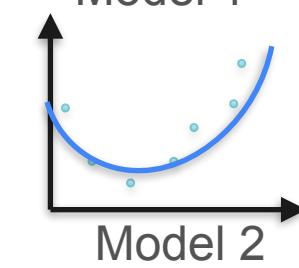
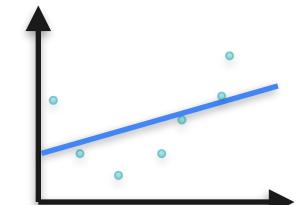
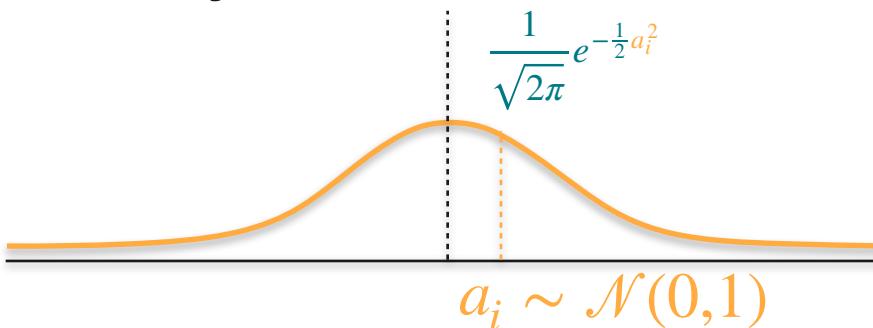
$$P(\text{Model 1}) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}a_1^2}$$

P(Model 2)

$$a_1x^2 + a_2x + b = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}a_1^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}a_2^2}$$

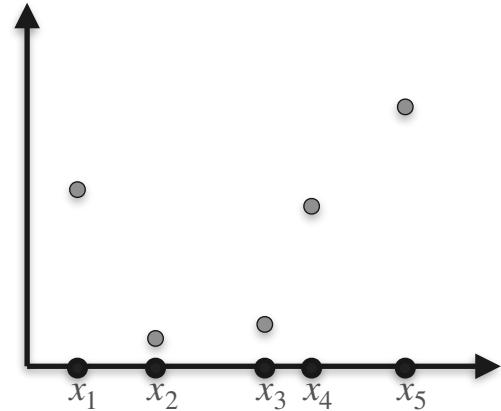
P(Model 3)

$$a_1x^{10} + a_2x^9 + \dots + a_{10}x + b = \prod_{i=1}^{10} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}a_i^2}$$

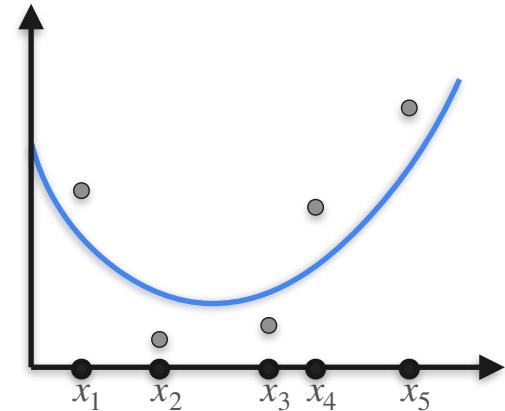


Bayes and Regularization

Bayes and Regularization

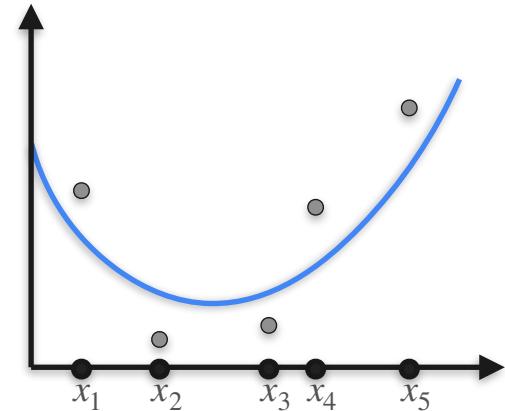


Bayes and Regularization



Bayes and Regularization

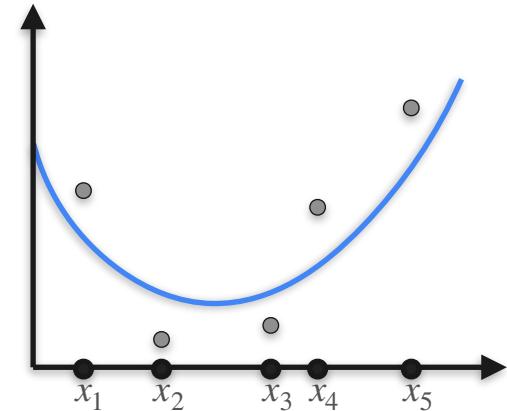
$P(\text{Data}|\text{Model})$



Bayes and Regularization

$P(\text{Data}|\text{Model})$

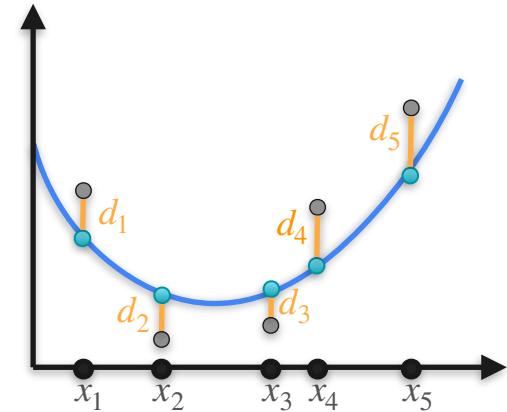
$P(\text{Model})$



Bayes and Regularization

$P(\text{Data}|\text{Model})$

$P(\text{Model})$

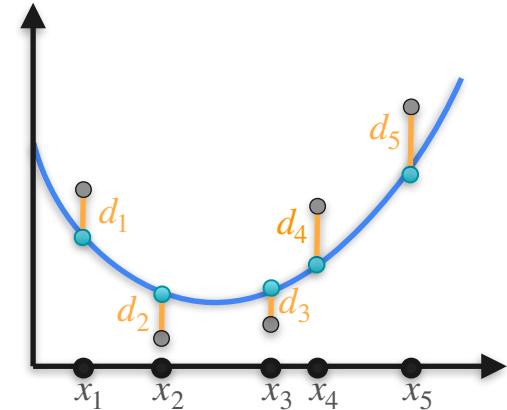


Bayes and Regularization

$\mathbf{P}(\text{Data}|\text{Model})$

$$\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_1^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_2^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_3^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_4^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_5^2}$$

$\mathbf{P}(\text{Model})$

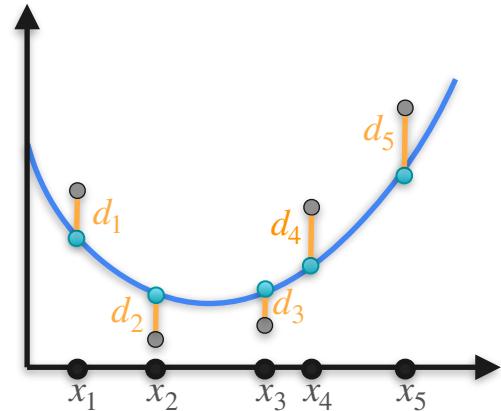


Bayes and Regularization

$\mathbf{P}(\text{Data}|\text{Model})$

$$\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_1^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_2^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_3^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_4^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_5^2}$$

$\mathbf{P}(\text{Model})$



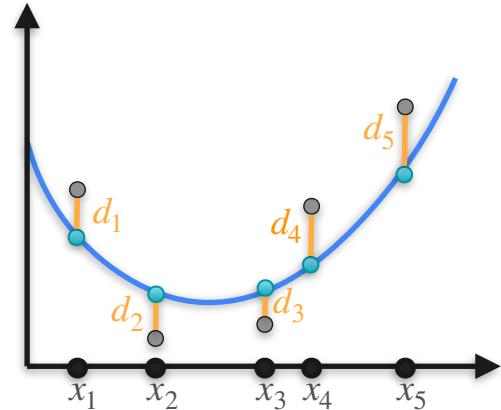
$$a_1x^2 + a_2x + b$$

Bayes and Regularization

$\mathbf{P}(\text{Data}|\text{Model})$

$$\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_1^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_2^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_3^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_4^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_5^2}$$

$\mathbf{P}(\text{Model})$



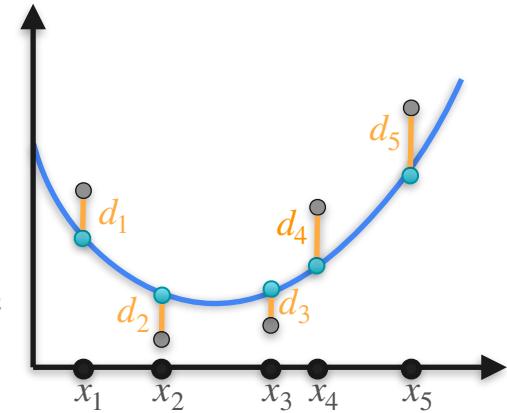
$$[a_1]x^2 + [a_2]x + b$$

Bayes and Regularization

$P(\text{Data}|\text{Model})$

$$\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_1^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_2^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_3^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_4^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_5^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}a_1^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}a_2^2}$$

$P(\text{Model})$



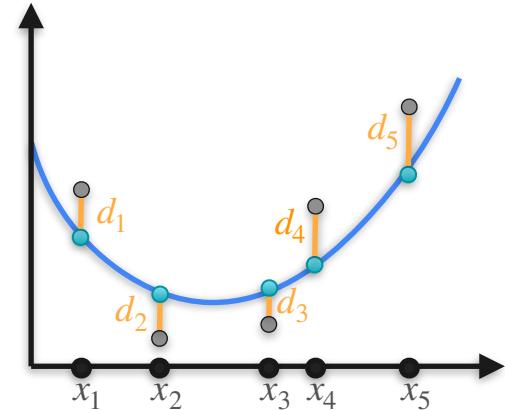
$$[a_1]x^2 + [a_2]x + b$$

Bayes and Regularization

$P(\text{Data}|\text{Model})$

$$\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_1^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_2^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_3^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_4^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_5^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}a_1^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}a_2^2}$$

$P(\text{Model})$



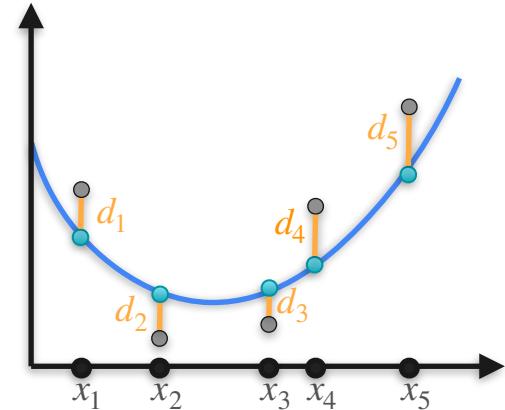
$$[a_1]x^2 + [a_2]x + b$$

Bayes and Regularization

$P(\text{Data}|\text{Model})$

$$\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_1^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_2^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_3^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_4^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_5^2}$$

$P(\text{Model})$



$$[a_1]x^2 + [a_2]x + b$$

Bayes and Regularization

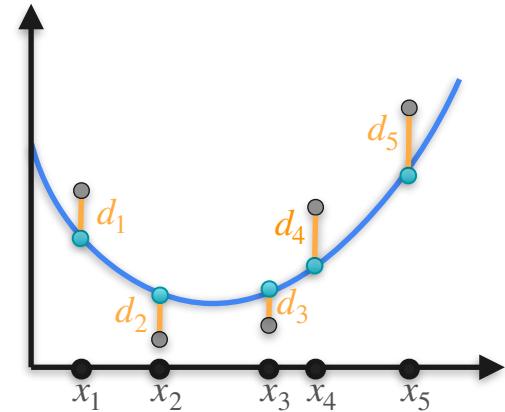
$P(\text{Data}|\text{Model})$

$$\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_1^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_2^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_3^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_4^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_5^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}a_1^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}a_2^2}$$

log

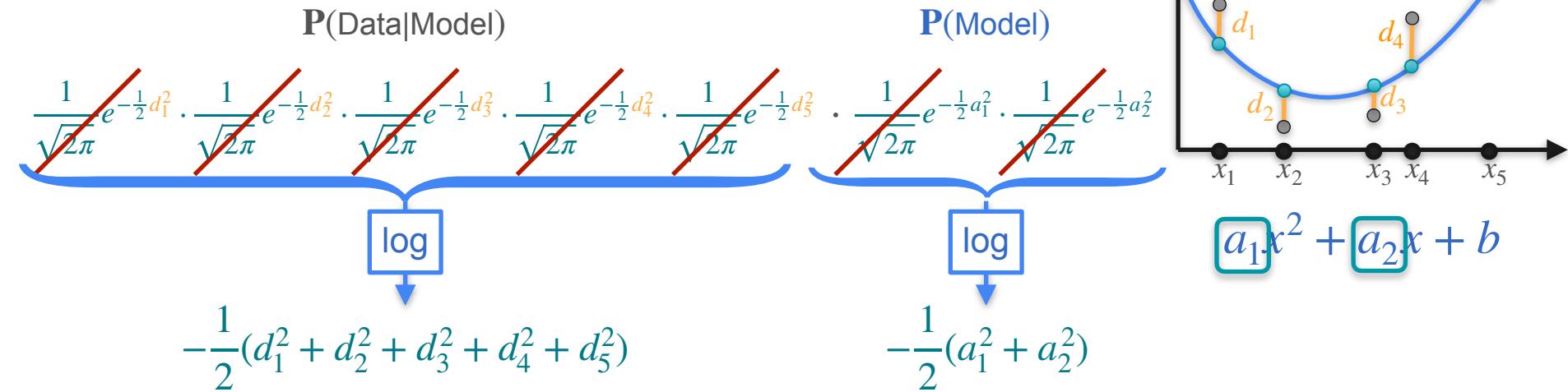
$$-\frac{1}{2}(d_1^2 + d_2^2 + d_3^2 + d_4^2 + d_5^2)$$

$P(\text{Model})$

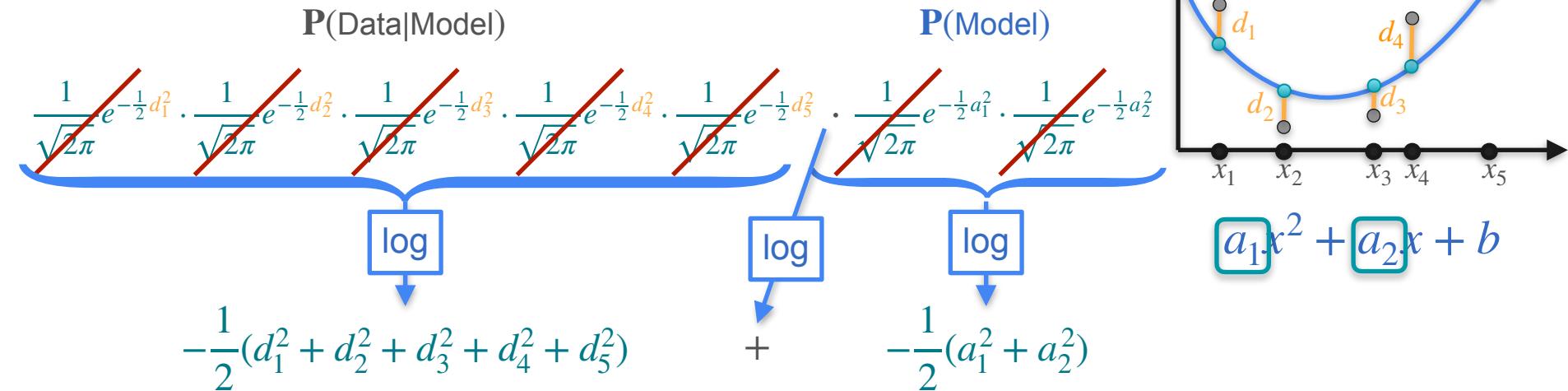


$$[a_1]x^2 + [a_2]x + b$$

Bayes and Regularization



Bayes and Regularization

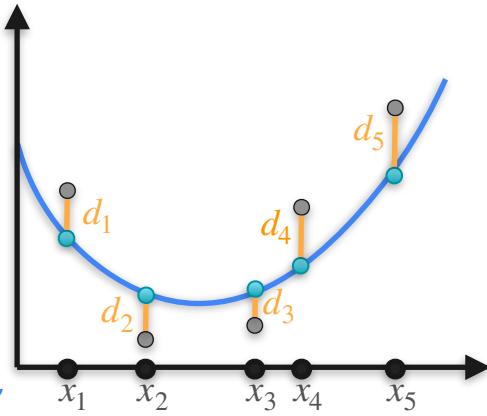


Bayes and Regularization

$P(\text{Data}|\text{Model})$

$$\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_1^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_2^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_3^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_4^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_5^2}$$

$P(\text{Model})$



Maximize

$$-\frac{1}{2}(d_1^2 + d_2^2 + d_3^2 + d_4^2 + d_5^2)$$

\log

\log

\log

+

$$-\frac{1}{2}(a_1^2 + a_2^2)$$

$$a_1x^2 + a_2x + b$$

Bayes and Regularization

$P(\text{Data}|\text{Model})$

$$\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_1^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_2^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_3^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_4^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_5^2}$$

$P(\text{Model})$

$$\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}a_1^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}a_2^2}$$

Maximize

$$-\frac{1}{2}(d_1^2 + d_2^2 + d_3^2 + d_4^2 + d_5^2)$$

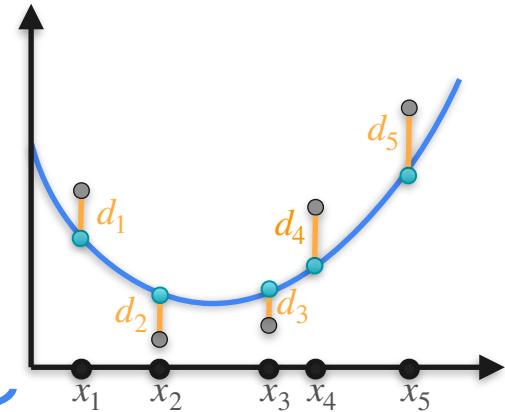
log

log

log

+

$$-\frac{1}{2}(a_1^2 + a_2^2)$$



$$a_1x^2 + a_2x + b$$

Minimize

Bayes and Regularization

$P(\text{Data}|\text{Model})$

$$\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_1^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_2^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_3^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_4^2} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_5^2}$$

$P(\text{Model})$

Maximize

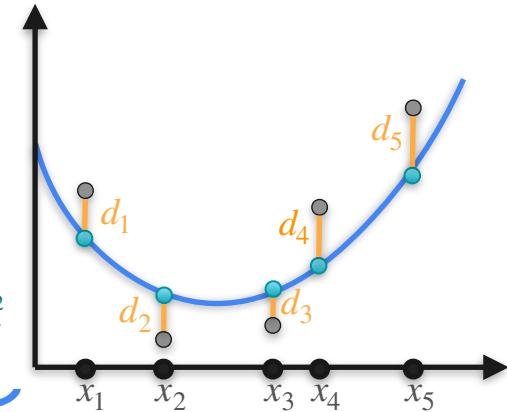
$$-\frac{1}{2}(d_1^2 + d_2^2 + d_3^2 + d_4^2 + d_5^2)$$

\log

\log

\log

$$+ \quad -\frac{1}{2}(a_1^2 + a_2^2)$$



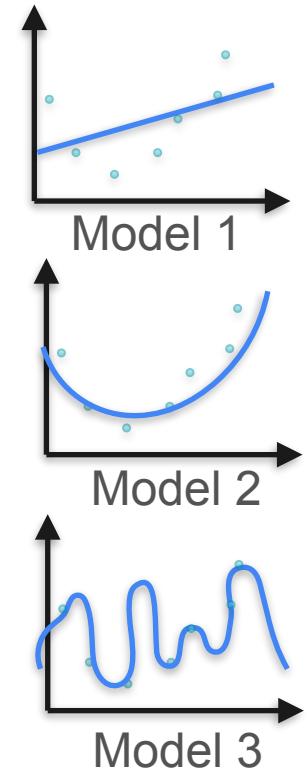
$$a_1 x^2 + a_2 x + b$$

Minimize

$$d_1^2 + d_2^2 + d_3^2 + d_4^2 + d_5^2 + a_1^2 + a_2^2$$

Regularization

Regularization

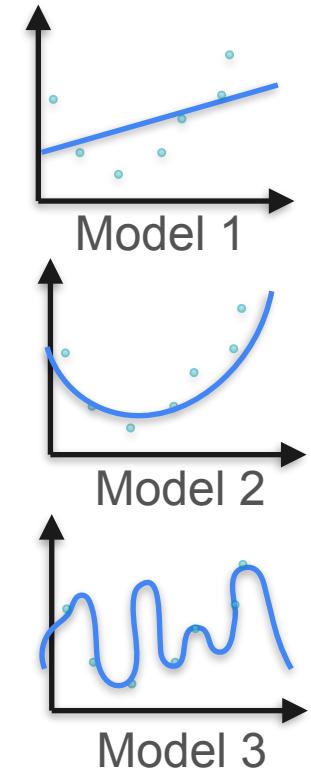


Regularization

$P(\text{Model 1})$

$P(\text{Model 2})$

$P(\text{Model 3})$



Regularization

P(Model 1)

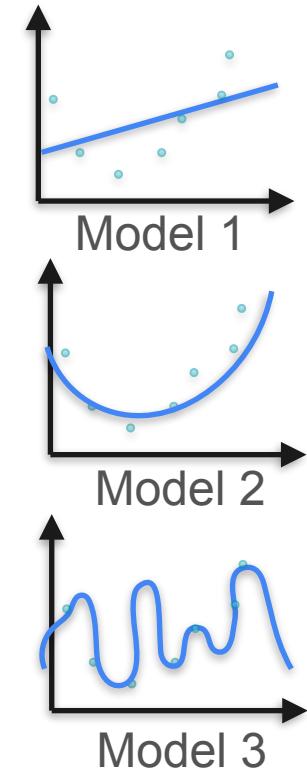
Minimize x_1^2

P(Model 2)

Minimize $x_1^2 + x_2^2$

P(Model 3)

Minimize $x_1^2 + \dots + x_{10}^2$



Regularization

$P(\text{Model 1})$

Minimize x_1^2

$P(\text{Model 2})$

Minimize $x_1^2 + x_2^2$

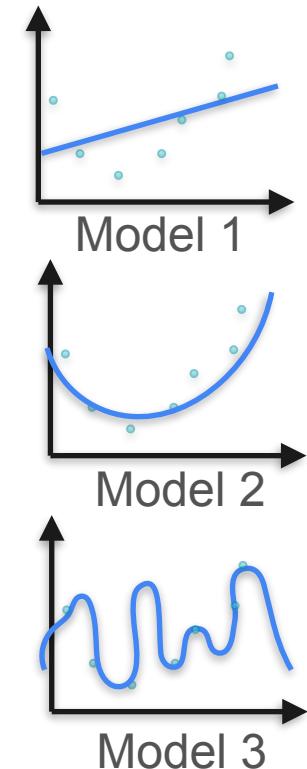
$P(\text{Model 3})$

Minimize $x_1^2 + \dots + x_{10}^2$

$P(\text{Data}|\text{Model 1})$

$P(\text{Data}|\text{Model 2})$

$P(\text{Data}|\text{Model 3})$



Regularization

P(Model 1)

Minimize x_1^2

P(Model 2)

Minimize $x_1^2 + x_2^2$

P(Model 3)

Minimize $x_1^2 + \dots + x_{10}^2$

P(Data|Model 1)

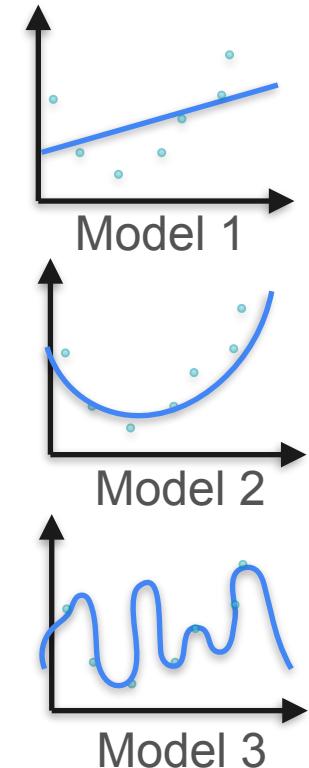
$$d_1^2 + d_2^2 + d_3^2 + d_4^2 + d_5^2$$

P(Data|Model 2)

$$d_1^2 + d_2^2 + d_3^2 + d_4^2 + d_5^2$$

P(Data|Model 3)

$$d_1^2 + d_2^2 + d_3^2 + d_4^2 + d_5^2$$



Regularization

P(Model 1)

Minimize x_1^2

P(Model 2)

Minimize $x_1^2 + x_2^2$

P(Model 3)

Minimize $x_1^2 + \dots + x_{10}^2$

New Loss

P(Data|Model 1)

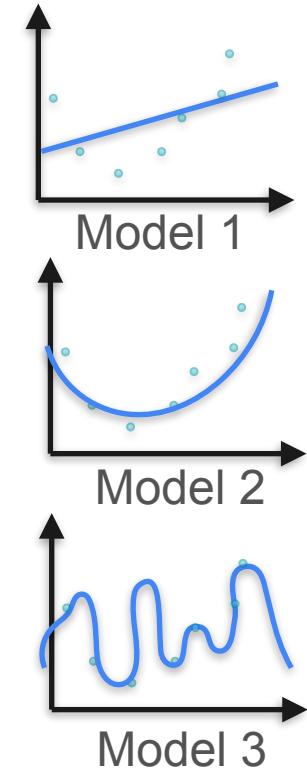
$$d_1^2 + d_2^2 + d_3^2 + d_4^2 + d_5^2$$

P(Data|Model 2)

$$d_1^2 + d_2^2 + d_3^2 + d_4^2 + d_5^2$$

P(Data|Model 3)

$$d_1^2 + d_2^2 + d_3^2 + d_4^2 + d_5^2$$



Regularization

P(Model 1)

$$\text{Minimize } x_1^2 + d_1^2 + d_2^2 + d_3^2 + d_4^2 + d_5^2$$

P(Model 2)

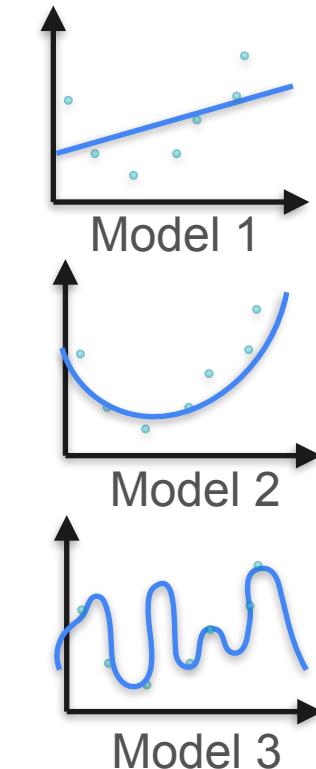
$$\text{Minimize } x_1^2 + x_2^2 + d_1^2 + d_2^2 + d_3^2 + d_4^2 + d_5^2$$

P(Model 3)

$$\text{Minimize } x_1^2 + \dots + x_{10}^2 + d_1^2 + d_2^2 + d_3^2 + d_4^2 + d_5^2$$

New Loss

P(Data|Model 1)





DeepLearning.AI

Point Estimation

Conclusion