Basic Concepts

CSE 425 Industrial Process Control

Lecture 2

Basic Concepts

- Mathematical Modeling
- Feedback loop
- Setpoint change vs. load upset
- Linearization

Need for Process Dynamic Models

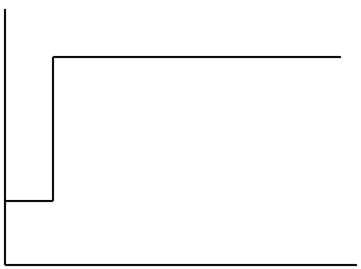
- Dynamic models give insight into the process to be controlled.
  - This enables us to determine what performance issues we can improve and what we can not.
- In addition, most controller design methods needs a model of the process.
- Question: if we possess a model for the process, do we still require feedback control?
  - Answer is YES!
  - Models are never perfect and there is always an "uncertainty" associated with any model
  - The key characteristic of **feedback control** is that control is likely to work in real situations where the true dynamics encountered differs within reasonable limits from the expected dynamic behaviour

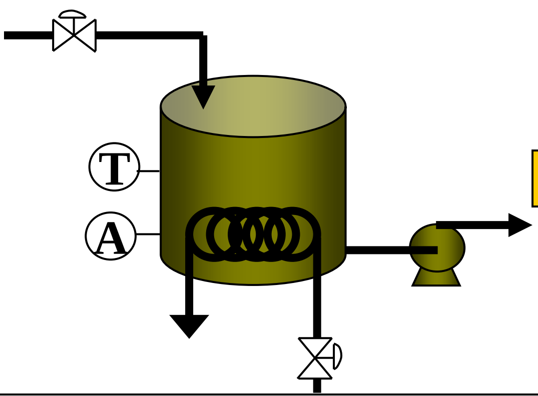
Mathematical Models

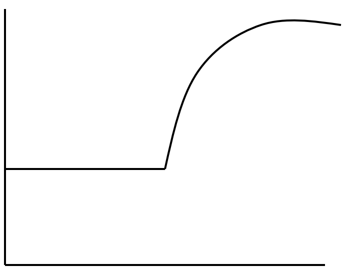
Input change, e.g., step in coolant flow rate

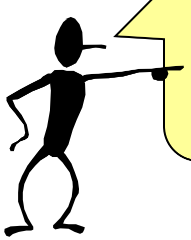
Process

Effect on output variable









Math models help us answer these questions!

How does the process influence the response?

- How far?
- How fast
- "Shape"

Basis for Modeling

- Physical laws such as Newton's method, Ohm's law, and material balances.

Overall Material

$$\{\text{Mass accumulation}\} = \{\text{mass}\}_{\text{in}} - \{\text{mass}\}_{\text{out}}$$

Component Material

$$\begin{Bmatrix}\text{Accumulation of} \\ \text{component mass}\end{Bmatrix} = \begin{Bmatrix}\text{component} \\ \text{mass in}\end{Bmatrix} - \begin{Bmatrix}\text{component} \\ \text{mass out}\end{Bmatrix} + \begin{Bmatrix}\text{generation of} \\ \text{component mass}\end{Bmatrix}$$

Energy

$$\{\text{Energy Accumulation}\} = \{\text{Energy}\}_{\text{in}} - \{\text{Energy}\}_{\text{out}}$$

Modeling procedure

1. Identify relevant variables.

2. Apply suitable conservation balances and formulate the model.

<u>Examples of variable selection</u>

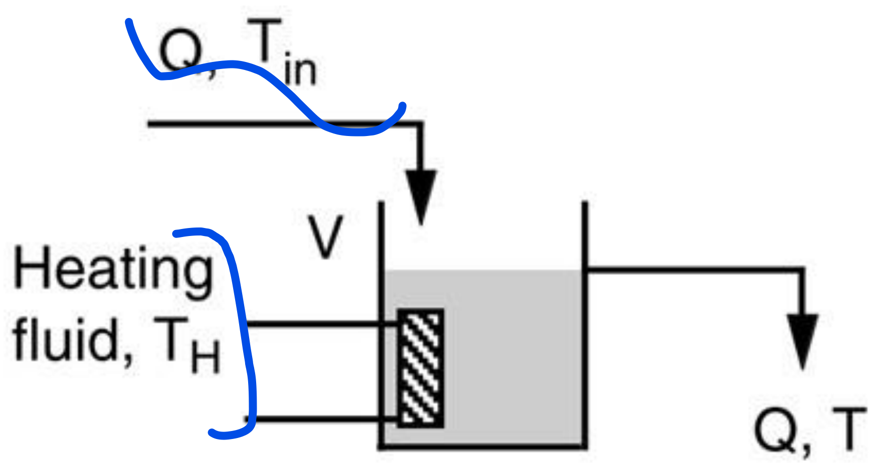
- **liquid level** $\rightarrow$ total liquid mass
- **temperature** $\rightarrow$ energy balance
- **concentration** $\rightarrow$ component mass balance

Example

- Stirred-tank heater is a common process example.
- We are interested in the control of temperature of the liquid inside the tank.
- By applying heat balance, we can arrive at a transfer function model:
- The process is described by two first-order transfer functions.

$$T(s) = \left(\frac{K_p}{\tau_p s + 1}\right) T_H(s) + \left(\frac{K_L}{\tau_p s + 1}\right) T_i(s)$$

$$Y(s) = G_P(s) U(s) + G_L(s) L(s)$$



The feedback loop

- The following is a possible feedback loop for the stirred tank heater.

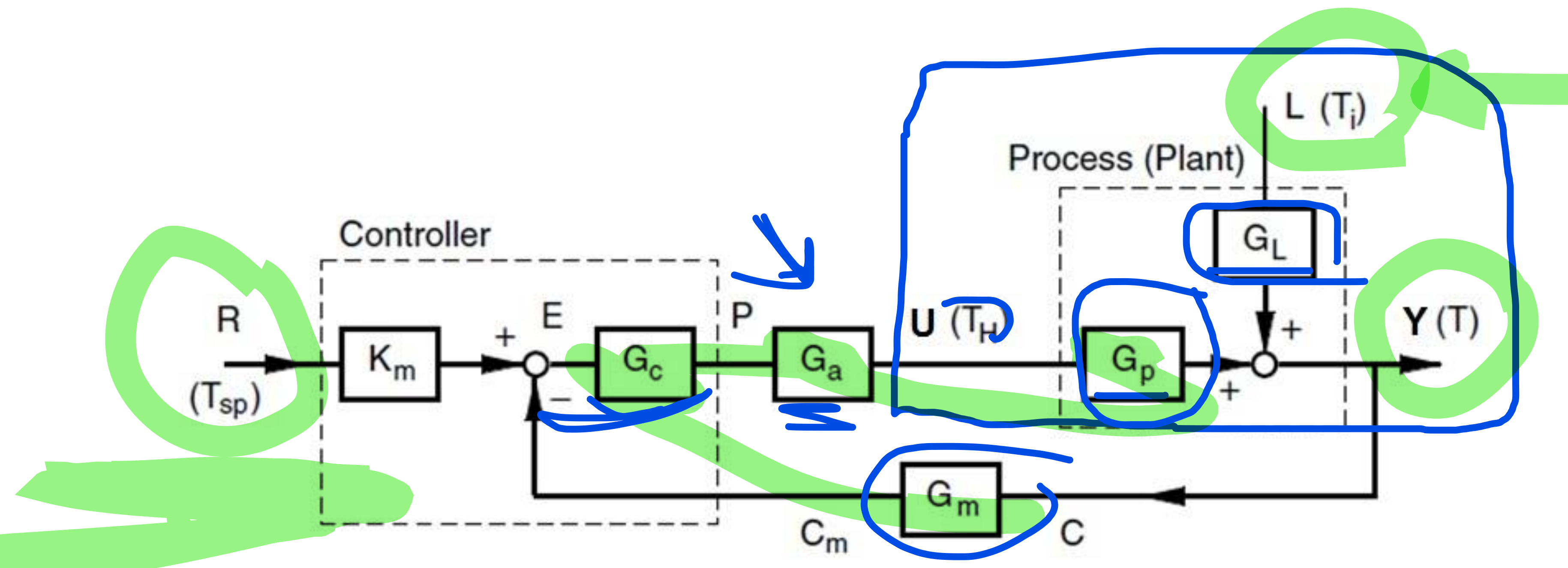


Figure 5.4. Block diagram of a simple SISO closed-loop system.

Components of a Control Loop

- **Four Main blocks:**
  - Process: the heart of the loop; we have to understand the dynamics of the process well without a controller.
  - Controller: this is the brain of the loop
  - Sensor or transmitter
  - Actuator or final control element
- **Four variables:**
  - Process Variable (PV): controlled variable
  - Desired or Reference value: Setpoint (SP)
  - Manipulated Variable
  - Disturbance or load

Transmitters & Final Control Elements

- **Transmitter**
  - Contains the sensor
  - Convert sensor output voltage into current
  - Called two-wire transmitter
  - Standard transmitter o/p range: 4-20 mA
  - Current is used (instead of voltage) to avoid voltage drop over the wire
  - There are binary sensors (output 0/1 or present/not present)
- **Final control element**
  - e.g. valve, heater, or a variable-speed Pump
  - Standard control signal for a pressure 3-15 psig
  - Occasionally, we need current -> psi (I/P converter)

Closed-Loop Transfer Functions

- The **closed loop** transfer functions relating the output **Y** to the set pint **R** and the disturbance **L** are given by:

$$Y = \left(\frac{K_m G_c G_a G_p}{1 + G_m G_c G_a G_p} \right) R + \left(\frac{G_L}{1 + G_m G_c G_a G_p} \right) L$$

Effect of setpoint changes (Servo problem)

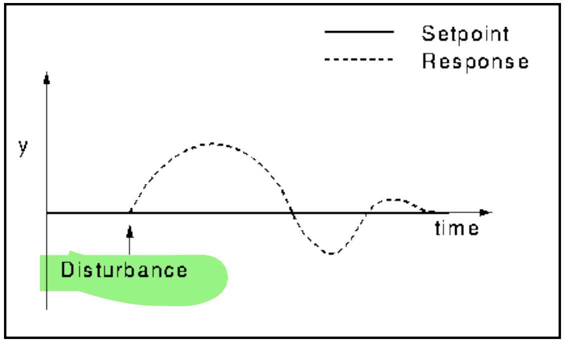
Effect of disturbance changes (Regulator problem)

$$\text{Transfer function between input and output} = \frac{\text{Product of all transfer functions between input and output}}{1+\ (\text{Product of all transfer function in the loop})}$$

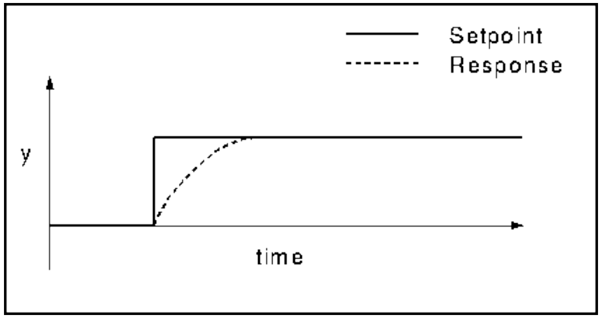
- The dynamics and stability of the closed-loop system are governed by the characteristic polynomial, $1+G_m G_c G_a G_p$, which is the same whether we are looking at set-point or disturbance changes.

Servo vs. Regulatory Control

- **Regulatory Control**
  - Frequent disturbances to the processes prevents the process variables from staying at their optimal values
  - Regulatory control aims to compensate for effects of disturbances through changes in manipulated variables



- **Servo Control**
  - When we need to re-evaluate the setpoints for the controlled variable
  - Servo control loop responds to a change in setpoint and makes the controlled variable follow the setpoint



Control system: design issues

- **Determine the role of various variables:**
  - What we need to control,
  - What we need to manipulate,
  - What the sources of disturbances are.
- **State design objective and specifications:**
  - Servo or regulation,
  - Desired response
- **Design the control system:**
  - Select proper sensors, transmitters, and actuators.
  - Select proper controller or control strategy.
  - Tune the controller.

Process Examples

- Lime mud filter
- Paper machine basis weight
- Boiler combustion pressure
- Steam drum water level for a boiler
- Distillate composition in a distillation column
- Ammonia plant hydrogen-to-nitrogen

- What are the PV, MV, load disturbance for these processes?

Linearization

- The previous model of stirred-tank heater is linear. **This enables us to write a transfer function of the system.**

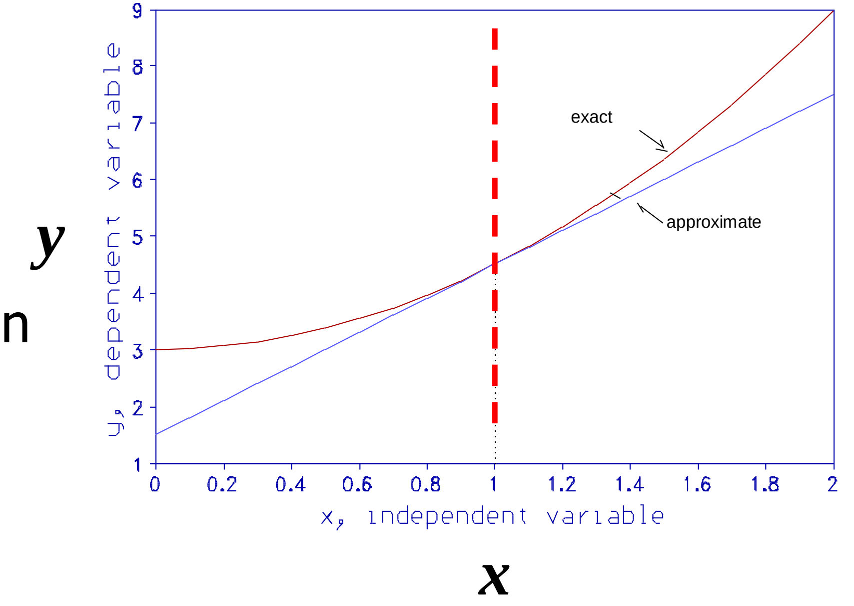
- However, for nonlinear systems, we can not write a transfer function.

- Fortunately, if we keep the changes around the operating point small, the nonlinear model can be approximated by a linear model. This is called **linearization**.

Linearization

- Consider a variable y which is related to another variable x through a nonlinear relationship.
- We are looking for a straight line approximation to the nonlinear function about some point x= x_s
- The accuracy of the approximation depends on
  - ❑ Shape of the non-linearity
  - ❑ Distance of x from x_s

$$y = 1.5\,x^2 + 3 \;\; \text{about } x_s = 1$$



Because *control system* maintains variables near desired values, *linearization* is often (but, not always) valid.

Linearization

To obtain the required approximation, expand the nonlinear relationship in Taylor Series and retain only constant and linear terms.

This is the only variable

$$F(x) = \underbrace{F(x_s) + \left.\frac{dF}{dx}\right|_{x_s}}\;(x - x_s) + \cancel{\frac{1}{2!}\left.\frac{d^2F}{dx^2}\right|_{x_s}(x - x_s)^2} + R$$

Remember that these terms are constant because they are evaluated at x_s

We define the deviation variable: $x^* = (x - x_s)$

Example

Given the following nonlinear model

$$\frac{dy}{dt} + y = \sqrt{u}.$$

- Using linearization, draw the approximate **unit step response** of the system at u = 25.

- Show the initial and final values of the <u>exact</u> and <u>approximate</u> responses on the graph.

- Comment on the results.

Answer

- Let us define the operating point:

  $\mathbf{u_s}$: the steady state input ($\mathbf{u_s}$= 25)

  $\mathbf{y_s}$: the steady state output corresponding to $\mathbf{u_s}$

- Also define the deviation variables around the operating point.

$$\mathbf{y^* = y - y_s} \quad \text{and} \quad \mathbf{u^* = u - u_s}$$

- To find $\mathbf{y_s}$, substitute in the model by $\mathbf{u = u_s}$ = 25 and set the derivatives to zero

$$\frac{dy}{dt} + y = \sqrt{u} \qquad \Rightarrow \qquad y_s = \sqrt{u_s} = \sqrt{25} = 5$$

- Using Taylor expansion, the nonlinear term can be approximated around $\mathbf{u = u_s}$ as:

$$\sqrt{u} \approx \sqrt{u_s} + \frac{1}{2\sqrt{u_s}}(u - u_s) = \sqrt{u_s} + 0.1u^*$$

Answer

- Substituting in the model yields

because $y_s = \sqrt{u_s}$

$$\frac{dy}{dt} + y = \sqrt{u_s} + 0.1u^* \quad \Rightarrow \quad \frac{dy}{dt} + y^* = 0.1u^*$$

- Noting that:

$$\frac{dy^*}{dt} = \frac{d(y - y_s)}{dt} = \frac{dy}{dt} - \frac{dy_s}{dt} = \frac{dy}{dt}$$

because y_s *is constant*

- We can write:

$$\frac{dy^*}{dt} + y^* = 0.1u^*.$$

- This is a first order ordinary differential equation and hence, we can find its corresponding transfer function as

$$\frac{Y^*(s)}{U^*(s)} = \frac{0.1}{s+1}.$$

Answer, continued

- The nonlinear system can be approximated by the transfer function

$$\frac{Y^*(s)}{U^*(s)} = \frac{0.1}{s+1}.$$

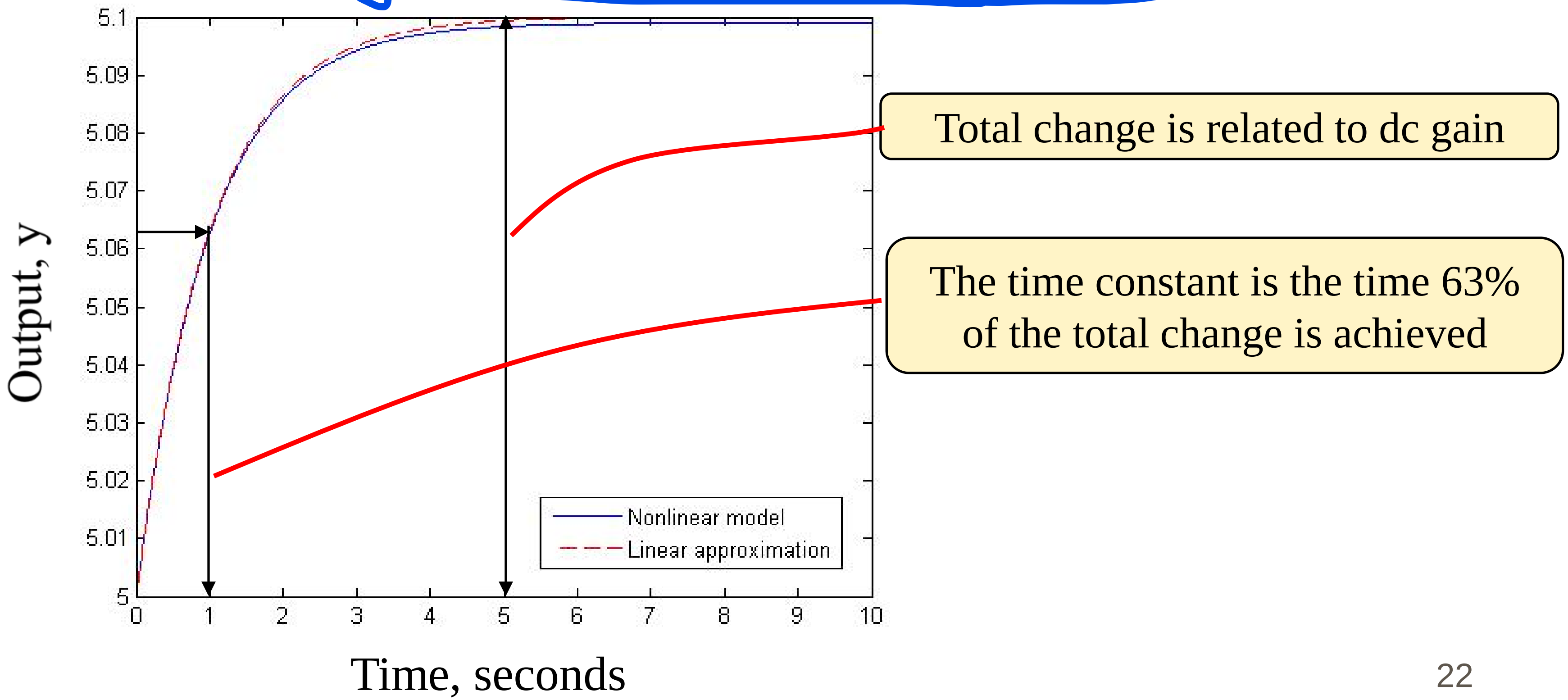
- Let us compare it to the standard first-order transfer function

$$\frac{Y^*(s)}{U^*(s)} = \frac{K}{\tau s + 1}.$$

- The dc gain of the process is approximately K = 0.1 and the time constant τ is 1 second.
- With these parameter values, we can draw the step response of the linearized model and compare it to the exact response of the original nonlinear system.

Comparison

- Let us compare the output of the linearized model to the original nonlinear model, assuming that the input u changes from 25 to 26 (unit step input).
- It is clear that the approximation is highly accurate.



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% This program shows how well the linear approximation compared to the nonlinear model
% ydot + y = sqrt(u)
% The linear model (around operating point us=25, ys=5) has the following  transfer function:


%                      0.1
%           Y*/U*= ----------
%                     s + 1
% where y* and u* are deviations around the operating point:
 us = 25;  ys = 5;

% Time range of simulation
dt=0.01;    t=0:dt:10;

% Simulating the nonlinear model for a step change du in the input around us = 25
du = 1;            % Try other values (e.g 2.0, 5.0, 10.0) to see how well the approximation is
y(1)=ys;
for i=1:length(t)-1
    ydot= -y(i)+sqrt(us+du);
    y(i+1) = y(i)+ydot*dt;
end
plot(t,y), hold on

% Simulating the linear approximation
y_sim=du*step(0.1,[1 1],t)
plot(t,y_sim+ys,'--r')
```

Linearization: general formula

- Consider a first-order ordinary differential equation:

$$\frac{dy}{dt} = f(u, y),$$

with $y(0) = y_s$.

- Then the corresponding linearized model can be written as:

$$\boxed{\frac{dy^*}{dt} = \left[\frac{\partial f}{\partial y}\right]_{u_s, y_s} y^* + \left[\frac{\partial f}{\partial u}\right]_{u_s, y_s} u^*.}$$