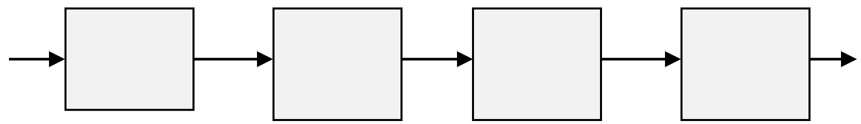
Dynamic System Structures

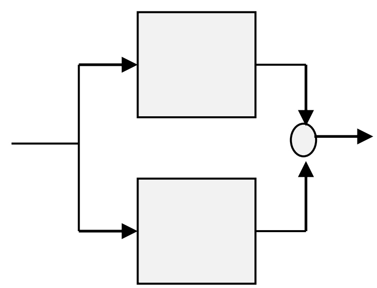
CSE 425 Industrial Process Control
Lecture 4

Process Structures

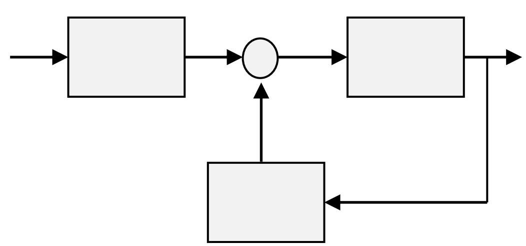
- Simple dynamic elements can yield complex dynamics when combined in typical process structures.



Series



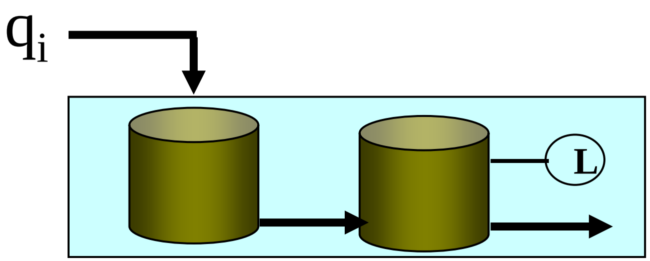
Parallel

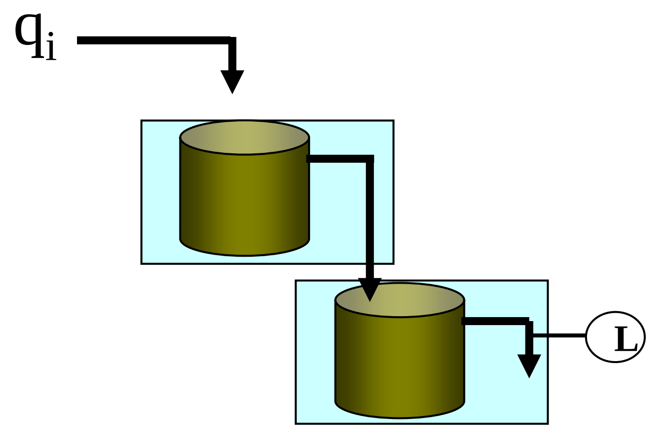


Recycle

Systems in Series

- **Examples:**





Interacting series:

Flow between tanks depends on the level in 2nd tank.

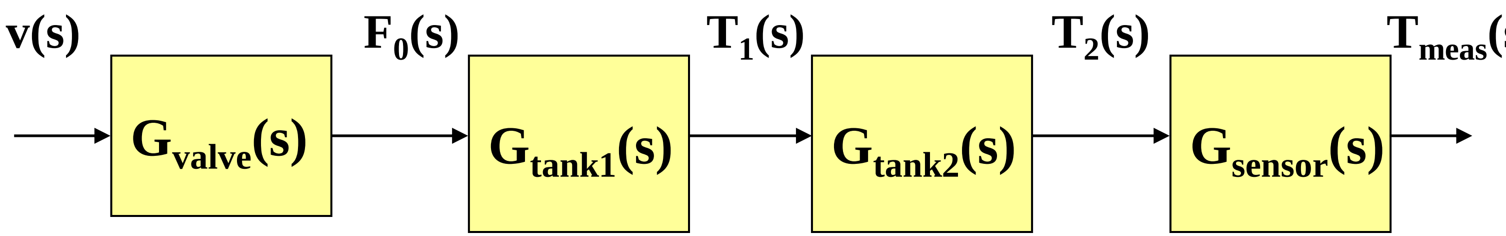
Non-interacting series:

Flow between tanks doesn't depend on the level in 2nd tank.

- In both cases, the transfer function between inlet flow q_i and 2nd tank level is 2nd order.

Non-interacting Series

- The block diagram of a non-interacting series:



- The transfer function of the series is:

$$\frac{T(s)}{V(s)} = \prod_{i=1}^{n} G_i(s)$$

Multi-capacity processes

- If each element in the series is first order, the series is called *multicapacity* process:

$$\frac{Y(s)}{X(s)} = \prod_{i=1}^{n} \frac{K_i}{(\tau_i s + 1)}$$

- The overall gain is the product of gains of all elements.

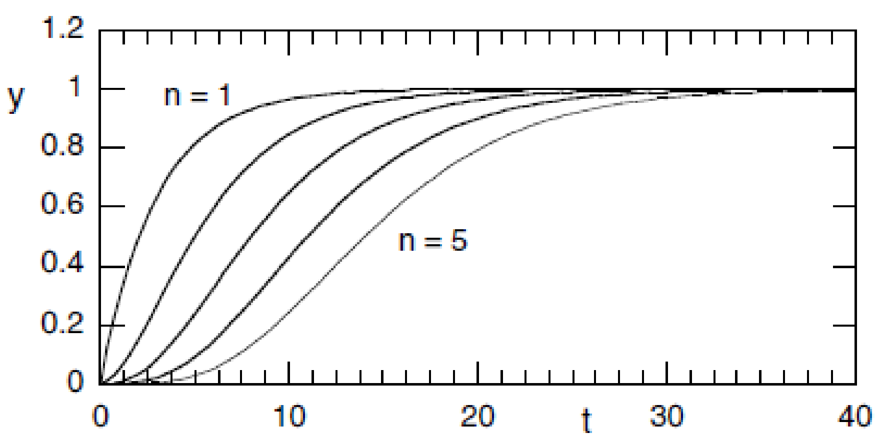
- The series is slower (more sluggish) than any single element. The more tanks we have in a series, the longer we have to wait until the last tank "sees" the changes that we have made in the first one.

Numerical Example

- Assume that all stages in a multi-capacity process have the same time constant `τ = 3`, then the whole system can be modeled as

$$\frac{Y_n(s)}{U(s)} = \frac{1}{(3s+1)^n}$$

- Let us simulate this system for $\boldsymbol{n}$ = 1, 2, 3, 4, 5.
- The response becomes more *sluggish* (slow) as the number of elements in the series increases.




```
G = tf(1,[3 1]);
step(G);
hold
step(G^2);
step(G^3);
step(G^4);
step(G^5);
```

The FOPDT Model

- In the previous figure, the initial response is small and can be ignored, specially for high order systems. Therefore, the initial part of the response can be approximated by pure **dead time.**

- In practice, high-order processes can be well approximated with the following first-order process plus dead-time (FOPDT) model:

$$\frac{Y(s)}{U(s)} = \frac{Ke^{-\theta s}}{\tau s + 1},$$

- FOPDT is the most common model used for approximating self-regulating processes.

Example

- Consider the following 4th order system:

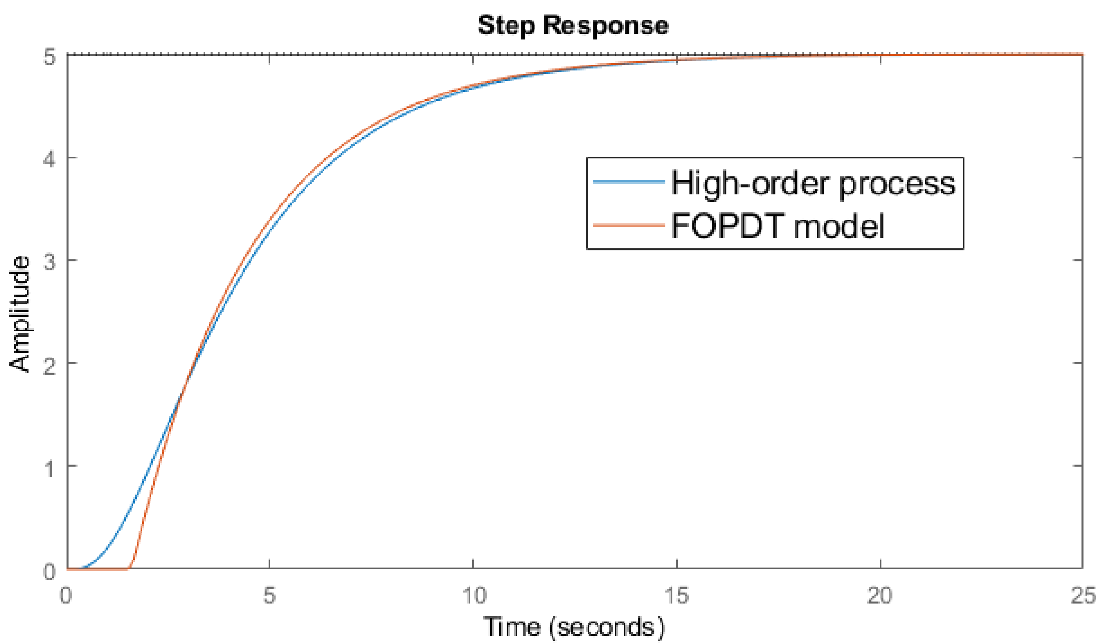
$$\frac{Y(s)}{U(s)} = \frac{5}{(0.1s+1)(0.5s+1)(s+1)(3s+1)}.$$

- The response of this system is dominated by the largest time constant 3 (dominant pole at -1/3).
- Accordingly, we may approximate the full-order function as

$$\frac{Y(s)}{U(s)} = \frac{5e^{-1.6s}}{3s+1},$$

- where the time delay 1.6 is the sum of smaller time constants 0.1, 0.5, and 1.
- Note also that the dc gain is 5.

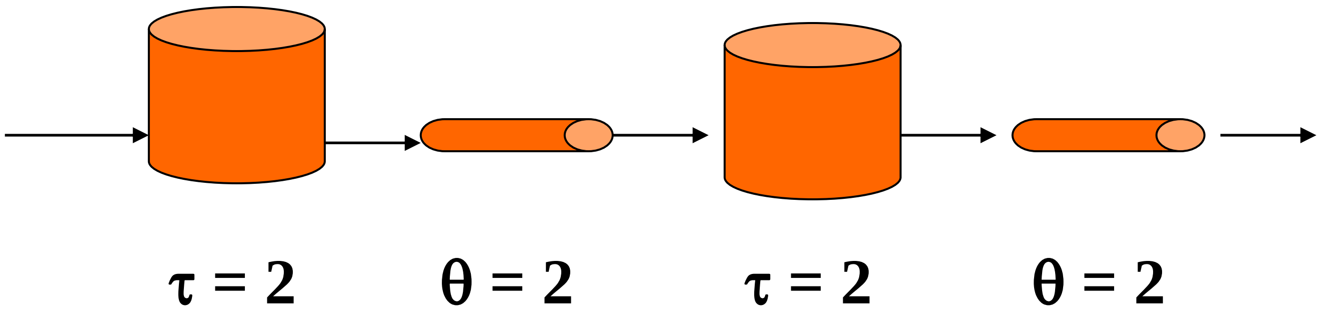
- Let us plot the step response of the 4th order process as well as its FOPDT approximation.
- As shown, the approximation is reasonable for large $\boldsymbol{t}$ where the pole at – 1/3 is dominant.

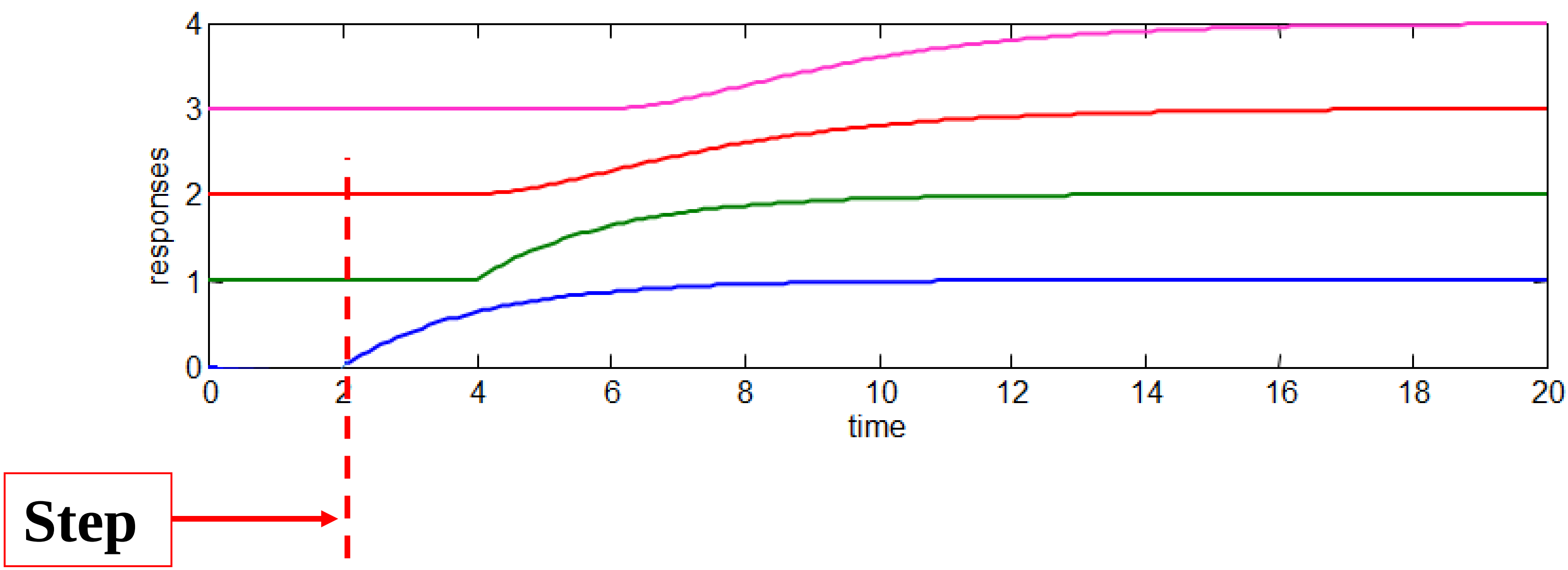


Class Exercise:

τ: time constant
θ: time delay

- Sketch the step response for the following system after each tank and pipe:

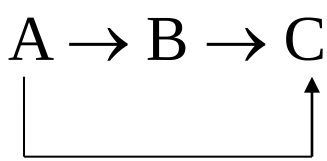


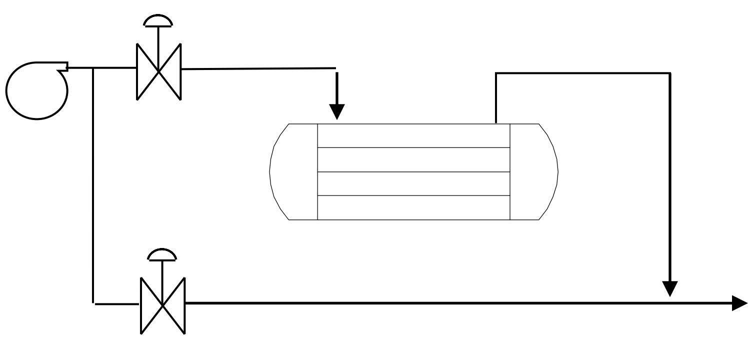


Parallel Structures

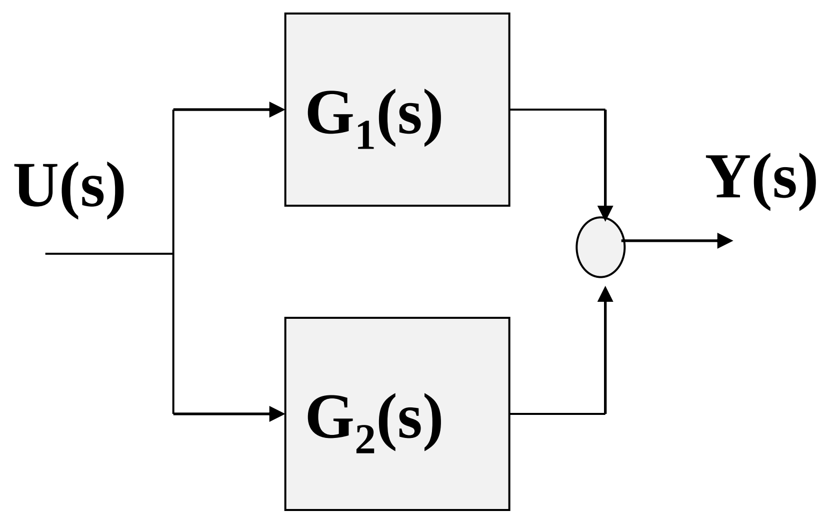
Parallel structures result when there are two paths between input and output, e.g. a flow split, where the paths have *different* time constants.

Example process systems





Block diagram



Transfer Function of a Parallel Structure

- Assume that both elements in parallel are first order, then the overall model is

$$\frac{Y}{U} = \frac{K_1}{\tau_1 s + 1} + \frac{K_2}{\tau_2 s + 1}$$

- Combining both terms gives a second-order function with a ***zero***

$$\boxed{\frac{Y}{U} = \frac{K(\tau s + 1)}{(\tau_1 s + 1)(\tau_2 s + 1)},}$$

- Where $$K = K_1 + K_2, \qquad \tau = \frac{K_1\tau_2 + K_2\tau_1}{K_1 + K_2}.$$

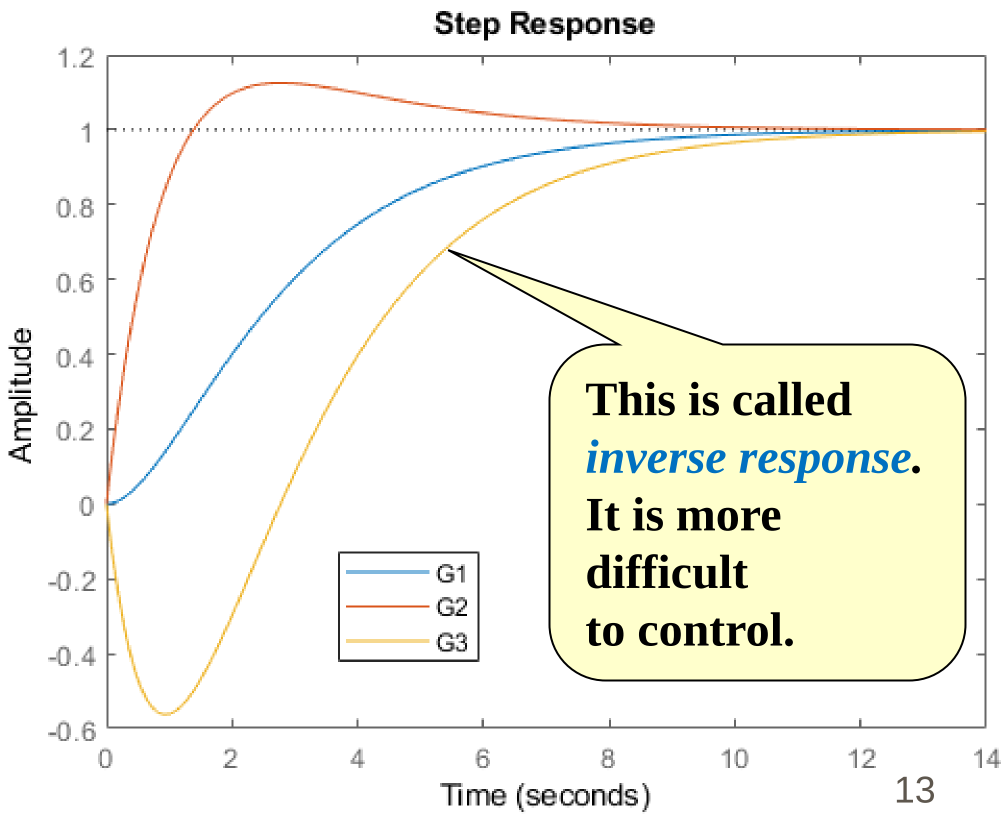
- As we know, the inherent dynamics is governed by the poles, however, the zeros have interesting effects on the response.

Example

- Let us compare the response of three systems:

$$G_1(s) = \frac{1}{(s+1)(2s+1)}, \qquad G_2(s) = \frac{3s+1}{(s+1)(2s+1)}, \qquad G_3(s) = \frac{-3s+1}{(s+1)(2s+1)}$$

- Inverse response process is caused by two competing processes – the faster of which takes the process first in a direction opposite to the steady state.

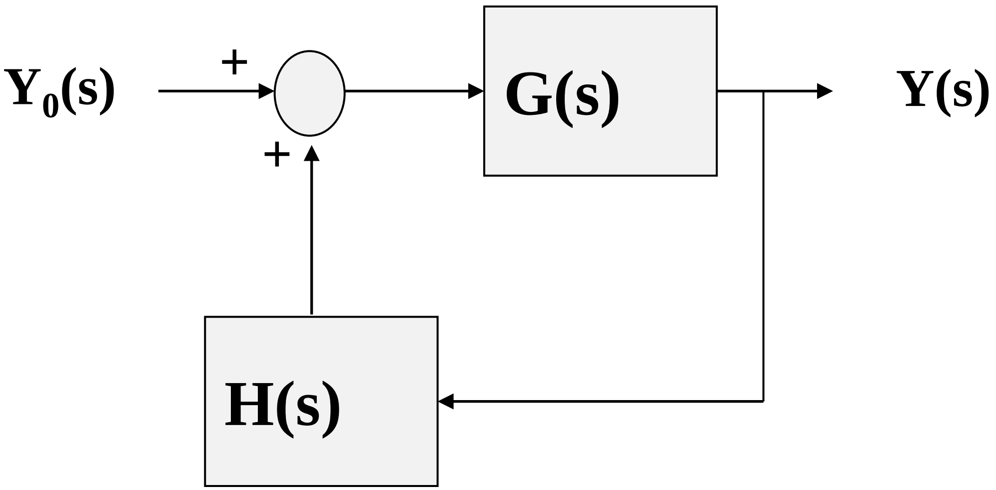


Recycle Structures

- In order to improve the quality of products (e.g. product concentration), products are occasionally fed back again to the process. This type of process is termed recycle structures.
- **Recycle** is a positive feedback mechanism which affects process dynamics.
- Comparing systems with recycle to original systems without recycle, the former has:
  - longer response time (larger time constants)
  - larger dc gain (to improve quality)

Example

Determine the effect of recycle on the dynamics of the given chemical reactor (faster or slower)? and the overall steady state gain.



$$H(s) = 0.30$$

$$G(s) = \frac{3}{10s + 1}$$

$$\frac{Y(s)}{Y_0(s)} = \frac{G(s)}{1 - G(s)H(s)}$$

Mason's gain formula

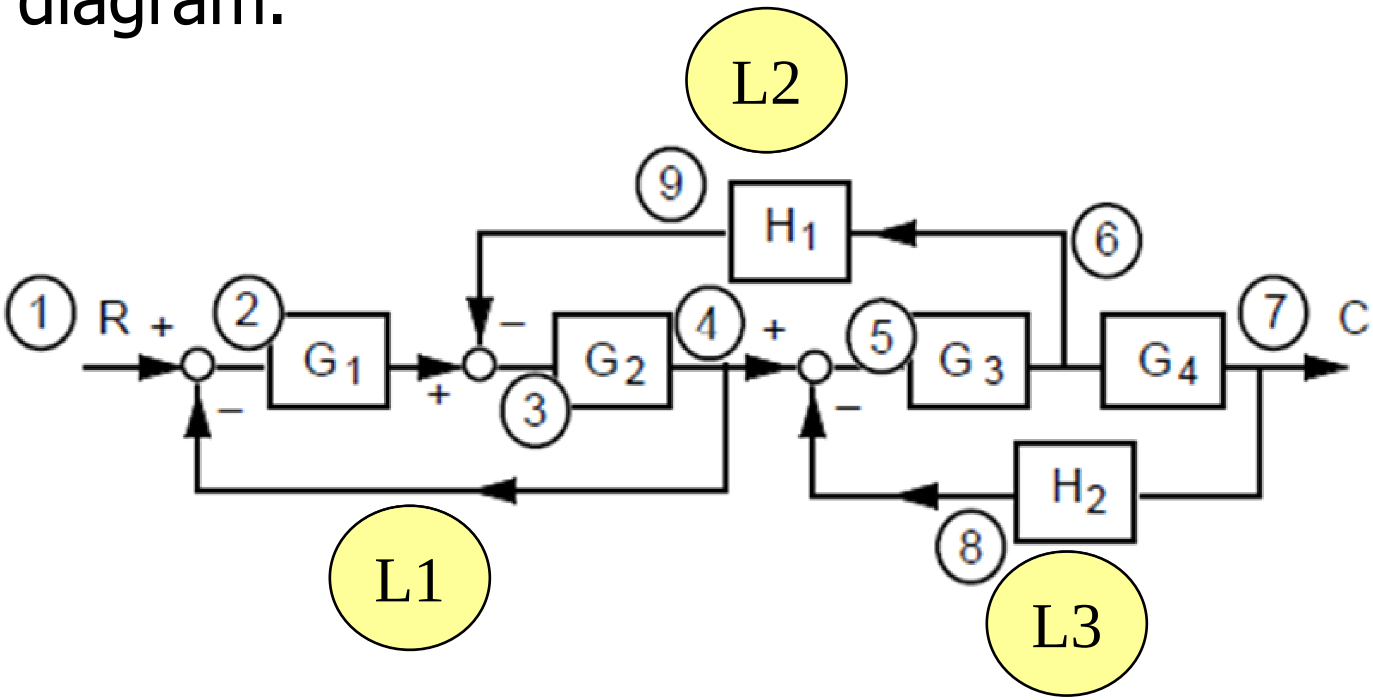
- Gives the transfer function between two variables in a much easier way than block diagram reduction.

$$\boxed{G(s) = \frac{\sum_i F_i \Delta_i}{\Delta}}$$

System determinant	$\Delta = 1 - \Sigma L_i + \Sigma L_i L_j - \Sigma L_i L_j L_k + \ldots$
Forward path gain	F_i = product of transfer functions along ith forward path from input to output
Forward path determinant	$\Delta_i = \Delta$ after deleting all loops touching F_i ($\Delta_i = 1$ if F_i touches all loops).
Loop path	A path starts from one variable and returns back to the same variable.
Non-touching loops	Two loops that don't share a common variable.

Example

- Find transfer function C/R in the following block diagram.



- We have one feedforward path and three loops.

Answer

- The feedforward path: $F_1 = G_1G_2G_3G_4$
- The loops

$$L_1 = -G_1G_2$$
$$L_2 = -H_1G_2G_3$$
$$L_3 = -H_2G_3G_4$$

- The determinant

$$\Delta = 1 - \sum L_i + \sum L_iL_j = 1 + G_1G_2 + H_1G_2G_3 + H_2G_3G_4 + G_1G_2H_2G_3G_4$$

L_1 and L_3 are not touching (they do not share a common variable).

- The determinant associated with F_1 is: $\Delta_1 = 1$

Because F_1 touches all loops.

- The transfer function

$$G(s) = \frac{F_1\Delta_1}{\Delta} = \frac{G_1G_2G_3G_4}{1 + G_1G_2 + H_1G_2G_3 + H_2G_3G_4 + G_1G_2H_2G_3G_4}$$