Solution

1. Prove: If x is an even integer then $3x + 5$ is an odd integer.

   Let $x = 2k$ by definition of an even number.

Proof:	$3(2k) + 5$	By substitution
	$6k + 5$	By algebra
	$6k + 4 + 1$	By algebra
	$=2(3k+2)+ 1$	By algebra

   Since $3k$ is an integer then $2(3k+2)+1$ is odd by the definition of odd

2. Prove: If the square of an integer x is even, then x is even.

   Assume x is odd; therefore, $x = 2k+1$, by the definition of odd, where k is a specific but arbitrarily chosen integer.

   Now, prove x^2 is odd.

Proof: $(2k+1)^2$	$= 4k^2+4k+1$	By substitution
	$= 2(2k^2+2k)+1$	By algebra

   Let $p = 2k^2+2k$, then $= 2p+1$, and that means it is odd. Therefore, if the contrapositive is true, the original proof is true (since the original theorem is logically equivalent to the contrapositive).

3. Prove: for all integers n,n^2+n+1 is *odd*

   (hint, use proof by cases, where one case starts with n is even, the second case starts with n is odd).

   Case #1: n is even, so $n = 2k$ By the definition of even

   Substitute n $=(2k)^2+2k+1$

   $=4k^2+2k+1$ By algebra

   $=2(2k^2+k)+1$ which is odd. $(2k^2+k)$ is an integer and $2(2k^2+k)+1$ is odd.

   Case #2: n is odd, so $n = 2k+1$ By the definition of odd

   Substitute n $=(2k+1)^2+(2k+1)+1$

   $=4k^2+4k+1+2k+1+1$ By algebra

   $=4k^2+6k+2+1$

   $=2(\underbrace{2k^2+3k+1}_{\text{integer}})+1$ which is odd.

   Since both cases are true, the original statement is proven to be true.