

## Set Theory

$a \in A$	$a$ is an element of set $A$
$a \notin A$	$a$ is not an element of set $A$
$\{a_1, a_2, \dots, a_n\}$	Set with elements $a_1 \dots a_n$
$\{x \in D \mid P(x)\}$	The set of all $x$ in $D$ for which $P(x)$ is true
$R$	All real numbers
$Z$	All integer numbers
$Q$	All rational numbers
$W$	Whole numbers (counting numbers plus zero)
$N$	Natural numbers (counting numbers 1,2,3,4,5...)
$A \subseteq B$	$A$ is a subset of $B$
$A \not\subseteq B$	$A$ is NOT a subset of $B$
$A = B$	$A$ equals $B$
$A \cup B$	$A$ union $B$
$A \cap B$	$A$ intersects $B$
$B - A$	Elements in $B$ but not in $A$
$A^c$ or $\bar{A}$	Complement of $A$
$(x, y)$	Ordered pair
$A \times B$	Cartesian product
$\emptyset, \{ \}$	Empty set
$\wp(A)$	Power set of $A$

## Logical Equivalences

t = tautology

$\sim$  = NOT

$\wedge$  = AND

c = contradiction

$\vee$  = OR

$\equiv$  = logically equivalent

Name	Form 1	Form 2
Commutative Laws	$p \sim \equiv q \vee p$	$p \vee q \equiv q \vee p$
Associative Laws	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	$(p \vee q) \vee r \equiv p \vee (q \vee r)$
Distributive Laws	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
Identity Laws	$p \wedge t \equiv p$	$p \vee c \equiv p$
Negation Laws	$p \vee \sim p \equiv t$	$p \wedge \sim p \equiv c$
Double Negation Laws	$\sim \sim p \equiv p$	–
Idempotent	$p \wedge p \equiv p$	$p \vee p \equiv p$
Universal Bounds	$p \vee t \equiv t$	$p \wedge c \equiv c$
De Morgan's Laws	$\sim(p \wedge q) \equiv \sim p \vee \sim q$	$\sim(p \vee q) \equiv \sim p \wedge \sim q$
Absorption Laws	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$
Negations of t and c	$\sim t = c$	$\sim c = t$

## Rules of Inference with Propositions

<p>p</p> <p><math>p \rightarrow q</math>      <i>Modus ponens</i></p> <p><math>\therefore q</math></p>	<p>p</p> <p>q</p> <p><math>\therefore p \wedge q</math>      Conjunction</p>
<p><math>\neg q</math></p> <p><math>p \rightarrow q</math>      <i>Modus tollens</i></p> <p><math>\therefore \neg p</math></p>	<p><math>p \rightarrow q</math></p> <p><math>q \rightarrow r</math></p> <p><math>\therefore p \rightarrow r</math>      Hypothetical syllogism/transitivity</p>
<p>a) p      Addition/generalization</p> <p><math>p \vee q</math></p> <p>b) q</p> <p><math>\therefore p \vee q</math></p>	<p>a) <math>p \vee q</math></p> <p><math>\neg q</math></p> <p><math>\therefore p</math></p> <p>b) <math>p \vee q</math></p> <p><math>\neg p</math></p> <p><math>\therefore q</math>      Disjunctive syllogism/elimination</p>
<p>a) <math>p \vee q</math>      Simplification/specialization</p> <p><math>\therefore p</math></p> <p>b) <math>p \wedge q</math></p> <p><math>\therefore q</math></p>	<p><math>p \vee q</math></p> <p><math>\neg p \vee r</math></p> <p><math>\therefore q \vee r</math>      Resolution</p>
	<p><math>p \vee q</math></p> <p><math>p \rightarrow r</math></p> <p><math>q \rightarrow p</math></p> <p><math>\therefore r</math>      Proof by division into cases</p>