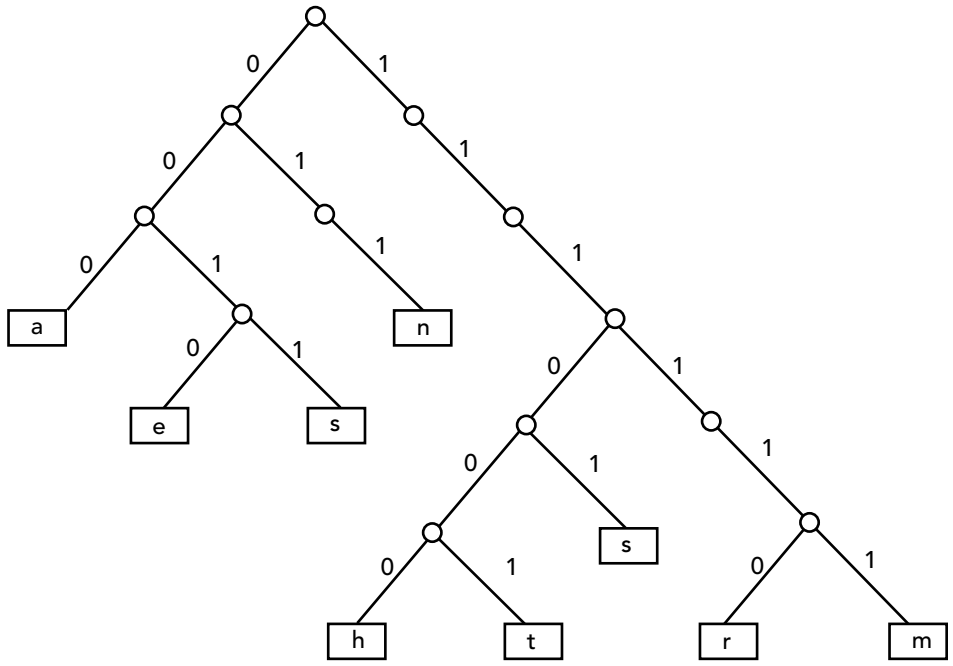
Solution

1. Given the following binary tree, encode and decode the prefix messages respectively:



0011 1111 1100 0111 1101 1100 1: smart
Math: 1111 1100 0111 0011 1100 0

2. Given this scenario:

   While watching the news, the meteorologist presents a three-day forecast in which there is a 50-50 chance of rain each day.

   Assuming that the meteorologist is correct, what is the probability that it rains on at most one of the three days?

   Answer: Because there is a 50-50 chance of rain on each of the three days, the sample space consists of eight equally likely outcomes:

   {RRR, RRN, RNR, RNN, NRR, NRN, NNR, NNN},

   where R indicates a rainy day and N is a day without rain.

   The event that it rains on at most one of the days is a subset containing these three possibilities:

   {RNN, NRN, NNR}.

   Hence, the probability that it rains on at most one of the three days is 3/8 or 37.5%.

3. Assume that an equally balanced coin is flipped three times.
   a. What is the probability that the first flip is heads?

      The subset where heads is the first flip includes {HHH, HTT, HHT, HTH}.

      Therefore, the probability is 4/8 or 50%.

   b. What is the probability that there are two consecutive heads?

      The subset where heads is the first flip is {HHH, HHT, THH}.

      Therefore, the probability is 3/8 or 37.5%.

   c. What is the probability that the first flip is heads and there are at least two consecutive heads? The subset where heads is the first flip is {HHH, HHT}.

      Therefore, the probability is 2/8 or 25%.

4. Use the Caesar cipher to encrypt the message (use an offset of 3).
   "Good morning"

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W

DLLAJLOKFKD

5. Use the Caesar cipher to decrypt this message (use an offset of 3).

AFPZOBQBJXQEFPCRK

DISCRETE MATH IS FUN