Solution

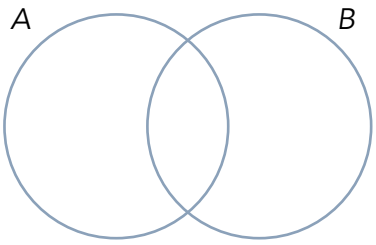
1. What are the possible values of n given this set notation: $\{n \in Z \mid n \text{ is a factor of } 8\}$?
   $\{2, 4\}$
2. Identify each of the following as true or false:
   a. $\{3\} \in \{1, 3, 5, 7\}$ False
   b. $\{3\} \subseteq \{1, 3, 5, 7\}$ True
   c. $\{3\} \in \{\{1\}, \{3\}, \{5\}, \{7\}\}$ True
   d. $\{3\} \subseteq \{\{1\}, \{3\}, \{5\}, \{7\}\}$ False

3. For all sets A, B, and C, prove: $A - (A \cap B) = A - B$ Identify the name of each law used.

$A - (A \cap B) = A \cap (\overline{A \cap B})$	by the Set Difference Law
$= A \cap (\overline{A} \cup \overline{B})$	by De Morgan's Law
$= (A \cap \overline{A}) \cup (A \cap \overline{B})$	by the Distributive Law
$= \varnothing \cup (A \cap \overline{B})$	by the Complement Law
$= (A \cap \overline{B}) \cup \varnothing$	by the Commutative Law
$= (A \cap \overline{B})$	by the Identity Law
$\therefore A - B$	by the Set Difference Law

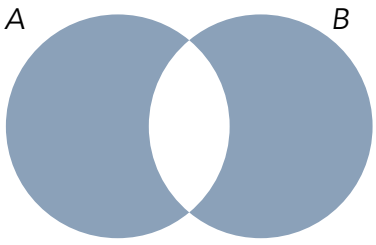
4. Let set A be a set of all the NASA employees and B is the set of all astronauts. Describe the following sets:
   a. $A \cap B$ = The set of NASA astronauts.
   b. $A \cup B$ = The set of NASA employees and all astronauts.
   c. $A - B$ = The set of NASA employees that are not astronauts.
   d. $B - A$ = The set of astronauts that are not NASA employees.

5. Illustrate the following using Venn Diagrams:
   a. $(A \cap B) - A$



$\varnothing$ empty set

b. $(A - AB) \cup (B - A)$



6. What is the power set of {a, b, c}?

   { { }, {a, b, c}, {a}, {b}, {c}, {a, b}, {a, c}, {b, c} }

   There should be 2^3=8 sets.

7. Interest rates are an example of a geometric sequence. In this example, you deposit $1,000 in a CD at your local bank, it earns 6% annual interest compounded monthly. What is the balance at the end of 12 months? (Hint: don't forget to find the monthly interest rate.)

   The initial value in the sequence a_0 = 1000, we want to find, a_{12}. The common ratio is the monthly interest rate: 6/12 = .005. Therefore, $a_{12} = 1000(1.005)^{12} \approx 1{,}061.68$

8. Find the value for the given summation:

$$\sum_{j=2}^{5}(2j-1)$$

$(2(2) – 1) + (2(3) – 1) + (2(4) – 1) + (2(5) – 1) = 3 + 5 + 7 + 9 = 24$

9. Rewrite the summation notation after separating out the last term:

$$\sum_{j=0}^{n} 2^j \qquad\qquad \sum_{j=0}^{n-1} 2^j + 2^n$$

10. Rewrite the summation by changing the summation index using $j = i+2$

$$\sum_{i=3}^{21}\frac{1}{i+3}=\sum_{j=5}^{23}\frac{1}{j+1}$$

11. Given the recursive definition for a function *g*:

    $g(0) = 0$

    $g(n) = g(n\text{-}1) + n^3$

    Find $g(3)=g(3\text{-}1)+ 3^3 = 27$

    $g(2\text{-}1)+ 2^3 = 8$

    $g(1\text{-}1)+ 1^3 = 1$

    $g(0) = 0$

    $=27+8+1= 36$