Recursive Least Squares (RLS) of ARX (Transfer Function) Systems Based on Equation Error with Exponential Forgetting for Systems with Time Varying Parameters

Equations

Transfer function model

$$A(z)y(t) = z^{-d}B(z)u(t) + e(t)$$

Where

$$A(z) = z^{na} + a_1 z^{na-1} + \cdots + a_{na}\,, \quad B(z) = b_0 z^{nb} + b_1 z^{nb-1} + \cdots + b_{nb}$$
$$A(z^{-1}) = 1 + a_1 z^{-1} + \cdots + a_{na} z^{-na}\,, \quad B(z^{-1}) = b_0 + b_1 z^{-1} + \cdots + b_{nb} z^{-nb}$$

$e(t)$ is a white noise

Writing the difference equation, z is a shift operator $(zy(t) = y(t+1),\ z^{-1}y(t) = y(t-1))$

$$(1 + a_1 z^{-1} + \cdots + a_{na} z^{-na})y(t) = z^{-d}(b_0 + b_1 z^{-1} + \cdots + b_{nb} z^{-nb})u(t)$$
$$y(t) = -a_1 y(t-1) - a_2 y(t-2) - \cdots - a_{na} y(t-na) + b_1 u(t-d-1) + b_2 u(t-d-2) + \cdots + b_{nb} u(t-d-nb)$$
$$y(t) = \varphi^T(t-1)\theta^0$$

$$\varphi^T(t-1) = [-y(t-1) \ldots - y(t-na) \quad u(t-d-1) \ldots \quad u(t-d-nb)], \qquad \theta^0 = \begin{bmatrix} a_1 \\ a_2 \\ . \\ . \\ . \\ a_{na} \\ b_1 \\ b_2 \\ . \\ . \\ . \\ b_{nb} \end{bmatrix}$$

Algorithm (note replace each $\boldsymbol{\varphi(t)}$ in the standard RLS algorithm with $\boldsymbol{\varphi(t-1)}$ for the transfer function model)

At t = 0, P(0) = αI, where I is the identity matrix n × n and α is large number. θ(0) = 0. For t > 0 and 0 < λ < 1 ,do the following

1. $\varphi^T(t-1) = [-y(t-1) \ldots - y(t-na) \quad u(t-1) \ldots \quad u(t-nb)]$ Regressors
2. $P(t) = \frac{1}{\lambda}\left[P(t-1) - \frac{P(t-1)\varphi(t-1)}{\left(\lambda+\varphi^T(t-1)P(t-1)\varphi(t-1)\right)}\varphi^T(t-1)P(t-1))\right]$

   $= \frac{1}{\lambda}\left[\left(I - K(t)\varphi^T(t-1)\right)P(t-1)\right]$ Covariance matrix
3. $K(t) = P(t)\varphi(t) = \frac{P(t-1)\varphi(t-1)}{\left(\lambda+\varphi^T(t-1)P(t-1)\varphi(t-1)\right)}$ Weighting (scaling) factor
4. $\varepsilon(t) = y(t) - \varphi^T(t-1)\hat{\theta}(t-1)$ Predicted error
5. $\hat{\theta}(t) = \hat{\theta}(t-1) + K(t)\varepsilon(t)$ Estimation
6. $e(t) = y(t) - \varphi^T(t-1)\hat{\theta}(t)$ Residual error (not used here)

Where $\left(I + \varphi^T(t)P(t-1)\varphi(t)\right)$, is scalar for single output system.

Matlab file name = RLS_"na"_"nb"_"n"

na: order of the numerator.

nb: order of the denominator.

d: delay.

nu=na+nb+1: number of parameters to be estimated.