

Extended Recursive Least Squares (RELS) of ARMAX (Transfer Function) Systems Based on Equation Error

Equations

Transfer function model

$$A(z)y(t) = z^{-d}B(z)u(t) + C(z)e(t)$$

Where

$$\begin{aligned} A(z) &= z^{na} + a_1 z^{na-1} + \dots + a_{na}, & B(z) &= b_0 z^{nb} + b_1 z^{nb-1} + \dots + b_{nb} \\ A(z^{-1}) &= 1 + a_1 z^{-1} + \dots + a_{na} z^{-na}, & B(z^{-1}) &= b_0 + b_1 z^{-1} + \dots + b_{nb} z^{-nb} \\ C(z^{-1}) &= 1 + c_1 z^{-1} + \dots + c_{nc} z^{-nc}, & C(z) &= z^{nc} + c_1 z^{nc-1} + \dots + c_{nc} \end{aligned}$$

$e(t) = y(i) - \varphi(i)\theta(i-1)$ is a color noise

Writing the difference equation, z is a shift operator ($zy(t) = y(t+1)$, $z^{-1}y(t) = y(t-1)$)

$$\begin{aligned} (1 + a_1 z^{-1} + \dots + a_{na} z^{-na})y(t) &= z^{-d}(b_0 + b_1 z^{-1} + \dots + b_{nb} z^{-nb})u(t) + (1 + c_1 z^{-1} + \dots + c_{nc} z^{-nc})e(t) \\ y(t) &= -a_1 y(t-1) - a_2 y(t-2) - \dots - a_{na} y(t-na) + b_1 u(t-d-1) + b_2 u(t-d-2) + \dots \\ &\quad + b_{nb} u(t-d-nb) + e(t) + c_1 e(t-1) + \dots + c_{nc} e(t-nc) \\ y(t) &= \varphi^T(t-1)\theta^0 \end{aligned}$$

$$\varphi^T(t-1) = [-y(t-1) \dots -y(t-na) \quad u(t-d-1) \dots u(t-d-nb) \quad e(t-1) \dots e(t-nc)], \quad \theta^0 = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_{na} \\ b_1 \\ b_2 \\ \vdots \\ b_{nb} \\ c_1 \\ c_2 \\ \vdots \\ c_{nc} \end{bmatrix}$$

Algorithm (note replace each $\varphi(t)$ in the standard RLS algorithm with $\varphi(t-1)$ for the transfer function model)

At $t = 0$, $P(0) = \alpha I$, where I is the identity matrix $n \times n$ and α is large number. $\theta(0) = 0$. For $t > 0$ do the following

$$1. \varphi^T(t-1) = [-y(t-1) \dots -y(t-na) \quad u(t-d) \dots u(t-d-nb) \quad e(t-1) \dots e(t-nc)]$$

Regressors

$$2. P(t) = P(t-1) - \frac{P(t-1)\varphi(t-1)}{(I + \varphi^T(t-1)P(t-1)\varphi(t-1))} \varphi^T(t-1)P(t-1) = (I - K(t)\varphi^T(t-1))P(t-1)$$

Covariance matrix

$$3. K(t) = P(t)\varphi(t) = \frac{P(t-1)\varphi(t-1)}{(1 + \varphi^T(t-1)P(t-1)\varphi(t-1))}$$

Weighting (scaling) factor

$$4. \varepsilon(t) = y(t) - \varphi^T(t-1)\hat{\theta}(t-1)$$

$$5. \hat{\theta}(t) = \hat{\theta}(t-1) + K(t)\varepsilon(t)$$

$$6. e(t) = y(t) - \varphi^T(t-1)\hat{\theta}(t-1)$$

Predicted error

Estimation

Residual error

Where $(I + \varphi^T(t)P(t-1)\varphi(t))$, is scalar for single output system.

na: order of the numerator.

nb: order of the denominator.

nc: order of the noise characteristic polynomial.

d: delay.

nu=na+nb+nc+1: number of parameters to be estimated.