Extended Recursive Least Squares (RELS) of ARMAX (Transfer Function) Systems Based on Equation Error

Equations

Transfer function model

$$A(z)y(t) = z^{-d}B(z)u(t) + C(z)e(t)$$

Where

$$A(z) = z^{na} + a_1 z^{na-1} + \cdots + a_{na}\,, \quad B(z) = b_0 z^{nb} + b_1 z^{nb-1} + \cdots + b_{nb}$$
$$A(z^{-1}) = 1 + a_1 z^{-1} + \cdots + a_{na} z^{-na}\,, \quad B(z^{-1}) = b_0 + b_1 z^{-1} + \cdots + b_{nb} z^{-nb}$$
$$C(z^{-1}) = 1 + c_1 z^{-1} + \cdots + c_{nc} z^{-nc}\,, \quad C(z) = z^{nc} + c_1 z^{nc-1} + \cdots + c_{nc}$$

$e(t) = y(i) - \varphi(i-1)\theta(i-1)$ is a color noise

Writing the difference equation, z is a shift operator ($zy(t) = y(t+1),\ \ z^{-1}y(t) = y(t-1)$)

$$(1 + a_1 z^{-1} + \cdots + a_{na} z^{-na})y(t) = z^{-d}(b_0 + b_1 z^{-1} + \cdots + b_{nb} z^{-nb})u(t) + (1 + c_1 z^{-1} + \cdots + c_{nc} z^{-nc})\mathrm{e(t)}$$
$$y(t) = -a_1 y(t-1) - a_2 y(t-2) - \cdots - a_{na} y(t-na) + b_1 u(t-d-1) + b_2 u(t-d-2) + \cdots$$
$$+ b_{nb} u(t-d-nb) + \mathrm{e(t)} + \mathrm{c_1 e(t-1)} + \cdots + \mathrm{c_{nc} e(t-nc)}$$
$$y(t) = \varphi^T(t-1)\theta^0$$

$$\varphi^T(t-1) = [-y(t-1) \ldots - y(t-na) \quad u(t-d-1) \ldots \quad u(t-d-nb) \quad e(t-1) \ldots e(t-nc)], \qquad \theta^0 = \begin{bmatrix} a_1 \\ a_2 \\ . \\ . \\ . \\ a_{na} \\ b_1 \\ b_2 \\ . \\ . \\ . \\ b_{nb} \\ c_1 \\ c_2 \\ . \\ . \\ . \\ c_{nc} \end{bmatrix}$$

Algorithm (note replace each $\boldsymbol{\varphi(t)}$ in the standard RLS algorithm with $\boldsymbol{\varphi(t-1)}$ for the transfer function model)

At t = 0, P(0) = αI, where I is the identity matrix n × n and α is large number. θ(0) = 0. For t > 0 do the following

1. $\varphi^T(t-1) = [-y(t-1) \ldots - y(t-na) \quad u(t-d) \ldots \quad u(t-d-nb) \quad e(t-1) \ldots e(t-nc)]$
   Regressors
2. $P(t) = P(t-1) - \frac{P(t-1)\varphi(t-1)}{\left(I+\varphi^T(t-1)P(t-1)\varphi(t-1)\right)}\varphi^T(t-1)P(t-1) = \left(I - K(t)\varphi^T(t-1)\right)P(t-1)$ Covariance matrix
3. $K(t) = P(t)\varphi(t) = \frac{P(t-1)\varphi(t-1)}{\left(1+\varphi^T(t-1)P(t-1)\varphi(t-1)\right)}$ Weighting (scaling) factor

4. $\varepsilon(t) = y(t) - \varphi^T(t-1)\hat{\theta}(t-1)$ Predicted error
5. $\hat{\theta}(t) = \hat{\theta}(t-1) + K(t)\varepsilon(t)$ Estimation
6. $e(t) = y(t) - \varphi^T(t-1)\hat{\theta}(t-1)$ Residual error

Where $\left(I + \varphi^T(t)P(t-1)\varphi(t)\right)$, is scalar for single output system.

na: order of the numerator.

nb: order of the denominator.

nc: order of the noise characteristic polynomial.

d: delay.

nu=na+nb+nc+1: number of parameters to be estimated.